Circuit Theory

Problem Set 4

Operational Amplifier Circuits

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23

(a) For an ideal op-amp, the input current is zero so that:

$$\frac{V_{\mathrm{IN}} - 0}{R_1} = \frac{0 - V_{\mathrm{OUT}}}{R_2} \tag{1}$$

so that:

$$V_{\mathrm{OUT}} = -\frac{R_2}{R_1} V_{\mathrm{IN}} \tag{2}$$

When $R_2 = 4R_1$ and $V_{\mathrm{OUT}} = 2\,\mathrm{V}$, we have $V_{\mathrm{IN}} = -0.5\,\mathrm{V}$.

(b) We have:

$$\frac{V_{\mathrm{OUT}} - V_{\mathrm{IN}}}{R_2} = \frac{V_{\mathrm{IN}} - 0}{R_1} \tag{3}$$

so that:

$$V_{\mathrm{OUT}} = \frac{R_1 + R_2}{R_1} V_{\mathrm{IN}} \tag{4}$$

When $R_2 = 4R_1$ and $V_{\mathrm{OUT}} = 2\,\mathrm{V}$, we have $V_{\mathrm{IN}} = 0.4\,\mathrm{V}$.

(c) We have:

$$\begin{aligned} \frac{V_{\mathrm{ref}} - V}{R_2} &= \frac{V - V_{\mathrm{OUT}}}{R_1} \\ \frac{V_{\mathrm{IN}} - V}{R_1} &= \frac{V}{R_2} \end{aligned} \tag{5}$$

where V is the input voltage of the op-amp.

Solving for V_{OUT} yields:

$$V_{\mathrm{OUT}} = \frac{R_2}{R_1}(V_{\mathrm{IN}} - V_{\mathrm{ref}}) \tag{6}$$

When $R_2 = 4R_1$ and $V_{\mathrm{OUT}} = 2\,\mathrm{V}$, we have $V_{\mathrm{IN}} = 1.5\,\mathrm{V}$.

•

24

For there to be no current through the op-amp, we have:

$$\frac{V_{\mathrm{IN}}}{R} = \frac{\mathrm{d}q}{\mathrm{d}t} \tag{7}$$

where $q(t)$ is the charge on left-plate of the capacitor.

Hence the output of the first op-amp satisfies $V'_{\mathrm{OUT}} = -q/C$. Further imposing the no-current condition on the second op-amp yields:

$$\frac{V'_{\mathrm{OUT}}}{R} + \frac{V_1}{R} = \frac{0 - V_{\mathrm{OUT}}}{R} \tag{8}$$

This leads to:

$$V_{\mathrm{OUT}} = -V'_{\mathrm{OUT}} - V_1 = \frac{1}{RC}\int_0^t V_{\mathrm{IN}}\,\mathrm{d}t - V_1 \tag{9}$$

Given that $V_{\mathrm{IN}} = 1\,\mathrm{V}$, we have the numerical value of V_{OUT} to be $(t-10)$. This grows arbitrarily large as t tends to infinity, which is not physically possible.

•

25

Imposing the no-current condition on the op-amp yields:

$$\frac{V_{\mathrm{IN}}}{R} = \frac{\mathrm{d}q_C}{\mathrm{d}t} + i_R \tag{10}$$

where q_c is the charge on the capacitor satisfying $V_{\mathrm{OUT}} = -q_C/C$ and i_R is the current through the resistor satisfying $V_{\mathrm{OUT}} = -i_R R$.

This yields a differential equation for $V_{\mathrm{OUT}}(t)$:

$$\frac{\mathrm{d}V_{\mathrm{OUT}}}{\mathrm{d}t} + \frac{V_{\mathrm{OUT}}}{RC} + \frac{V_{\mathrm{IN}}}{RC} = 0 \tag{11}$$

Now let the input voltage take the complex form $V_{\mathrm{IN}} = \tilde{V}e^{\mathrm{i}\omega t}$, where ω is the angular frequency. Applying an integrating factor of $e^{t/CR}$ yields:

$$\begin{aligned}\frac{\mathrm{d}}{\mathrm{d}t}\left(e^{t/CR}V_{\mathrm{OUT}}\right) &= -\frac{\tilde{V}}{CR}e^{(1/CR+i\omega)t}\\ V_{\mathrm{OUT}}(t) &= -\frac{\tilde{V}}{1+i\omega CR}e^{i\omega t} = -\frac{1}{1+i\omega CR}V_{\mathrm{IN}}(t)\end{aligned} \tag{12}$$

Taking the ratio leads to $|V_{\mathrm{OUT}}/V_{\mathrm{IN}}| = 1/\sqrt{1+(\omega CR)^2}$. This equals $1/\sqrt{2}$ when $\omega = 1/(CR)$, which is the usual definition of the resonance frequency.

•

26

First note that the current output by V_{IN} is constant and does not alter when any of the switches changes. Hence we can calculate the voltages V_0, V_1, V_2, V_3 above the switches without ambiguity.

First focus on the node above the last switch S_3. The two $2R$ resistors combine to give a R resistor, which means $V_3 = V_2/2$. Note that this argument can be made for every node above a switch, which means $V_i = V_{i-1}/2$ for $i = 1, 2, 3$. But V_0 is just the input voltage, and we thus know all the node voltages.

Applying the no-current condition on the op-amp yields:

$$\frac{0 - V_{\mathrm{OUT}}}{R} = I = \sum_{i=0}^{3} \frac{S_i}{2R} V_i \tag{13}$$

Therefore:

$$V_{\mathrm{OUT}} = -V_{\mathrm{IN}} \left(\frac{S_0}{2} + \frac{S_1}{4} + \frac{S_2}{8} + \frac{S_3}{16} \right) \tag{14}$$

The range of the output is from $0\,\mathrm{V}$ to $-9.375\,\mathrm{V}$. To make the output non-inverting, we can use the circuit in Question 23 Part (b), replacing R_1 with the present network and R_2 with R. This will lead to the output expression:

$$V_{\mathrm{OUT}} = V_{\mathrm{IN}} \left(1 + \frac{S_0}{2} + \frac{S_1}{4} + \frac{S_2}{8} + \frac{S_3}{16} \right) \tag{15}$$

(*this seems weird...*)