GAM

Introduction to Trigonometry

Day 7

Review quiz: Analytic Geometry

- Given two points (2, -1) and (1, 2)
  1. What is the distance between the two points
  2. What is the midpoint between the two points
  3. What is the slope between the two points
  4. Find the equation, in point slope form, for a line passing through the two points
  5. Find the equation in General form
  6. Find the equation in slope-intercept form
  7. Find the y-intercept of the line
  8. Find the line equation which is parallel to the given line and with y intercept (0, -35)
  9. Find the line equation which is perpendicular to the given line and with y intercept (0, -35)

Chapter 10

Trigonometry

Objectives

- To define the sine and cosine of an acute angle
- To use sine and cosine in special right triangles
- To find measures of missing sides of triangles
- To graph trig functions

Sine and Cosine Ratios

- The rationale behind definitions of sine and cosine rely on similar triangle theory.

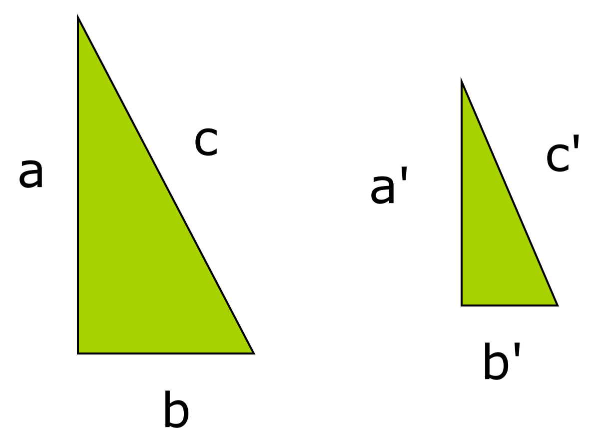
- Since corresponding parts of similar triangles are in the same ratio the following holds true regardless of the size of the triangles

Given two similar right triangles…

- When two (right) triangles are similar, the corresponding sides are proportional.

$$a/a' = c/c' \text{ and } b/b' = c/c'$$

$$\Rightarrow a/c = a'/c' \text{ and } b/c = b'/c'.$$

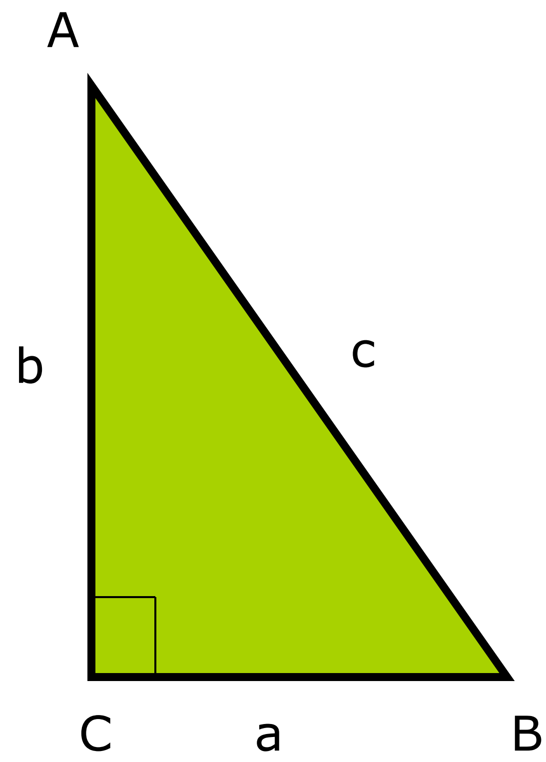


Definition of sine an cosine

- Let <A be an acute angle of the right triangle.
- sin A = $\frac{\text{opposite leg}}{\text{hypotenuse}} = \frac{a}{c}$
- cos A= $\frac{\text{adjacent leg}}{\text{hypotenuse}} = \frac{b}{c}$

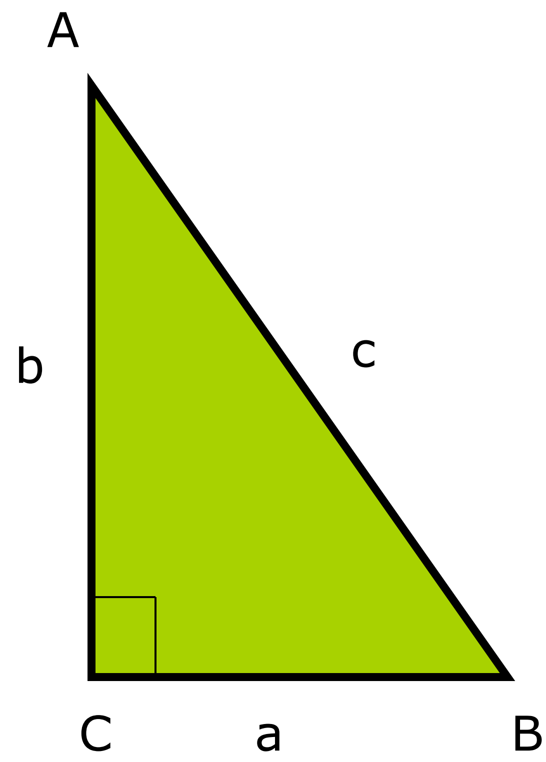
(side a is opposite <A)

(side b is adjacent to <A)



Definition of sine an cosine

- Let <A be an acute angle of the right triangle.
- $\sin B = \dfrac{\text{opposite leg}}{\text{hypotenuse}} = \dfrac{b}{c}$
- $\cos B = \dfrac{\text{adjacent leg}}{\text{hypotenuse}} = \dfrac{a}{c}$



(side b is opposite <B)

(side A is adjacent to <B)

Abbreviation of Sine and Cosine

- “sin” is usually used as an abbreviation for sine. It is still pronounced the same.

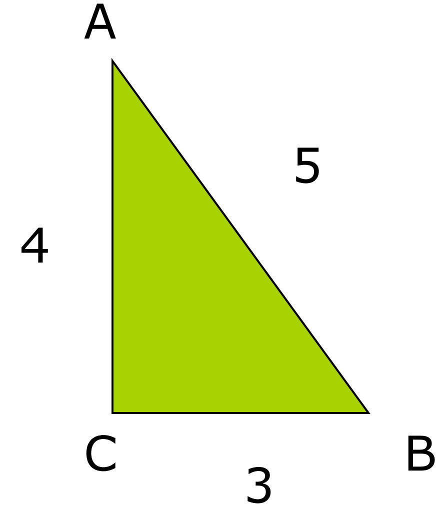
- “cos” is the abbreviation for cosine.

Note: <C in all future slides will be 90 degrees.

Application of the sin and cos definitions

- Determine sin A, cos A, sin B, cos B if the legs of the right triangle are 3 and 4.

1. Find the length of the hypotenuse using the Pythagorean Theorem.
2. Substitute the lengths of the legs and hypotenuse into the definitions



Answers:

sin A=   cos A =

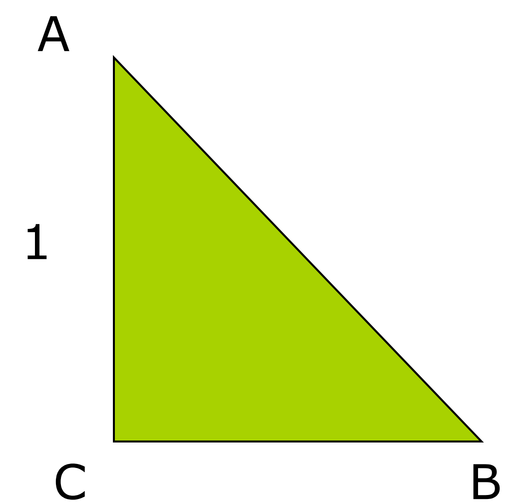
sin B=   cos B =

Application of the sin and cos

- Use your calculator to get
  - Sin 0^0
  - Cos 0^0
  - Sin 26.33^0
  - Cos 43.51^0
  - Sin 67^0
  - Cos 23^0
  - Sin 90^0
  - Cos 90^0

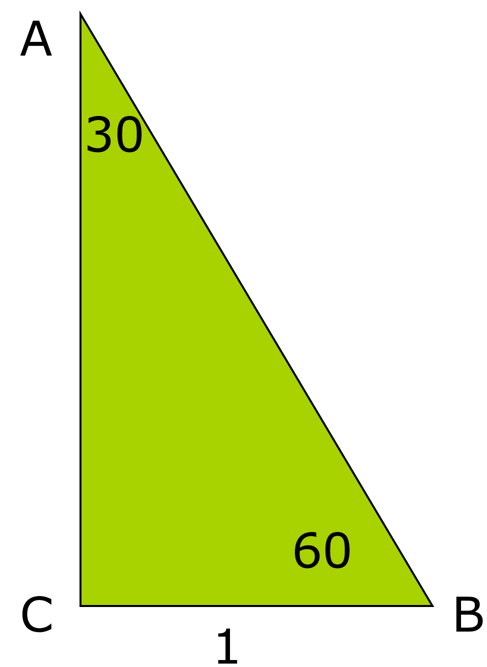
Sin and cos in 45-45-90 triangle

- If one leg of a 45-45-90 triangle measures 1″ find the lengths of the two remaining sides. Find sin 45 and cos 45.



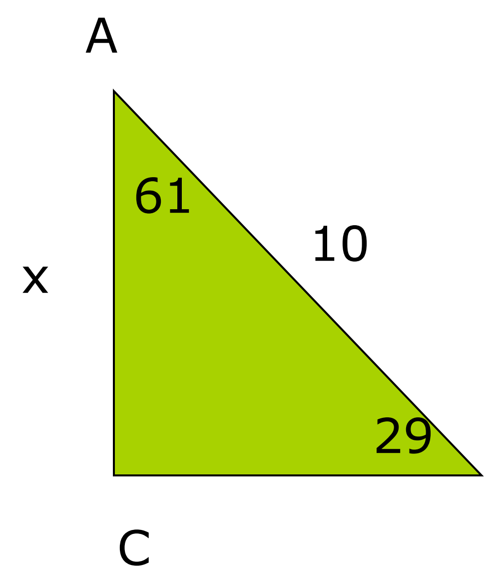
Sin and cos in 30-60-90 triangle

- If the short leg of a 30-60-90 triangle measures 1 inch find the lengths of the two remaining sides. Use these measurements to find sin 30, cos 30, sin 60, and cos 60.



Finding the measures of missing sides in a right triangle

- State two different equations that could be used to find x, correct to one decimal place.



Answer

1. sin 29 = x/10

   10sin 29=x

   10(.4848)=x

   4.8=x

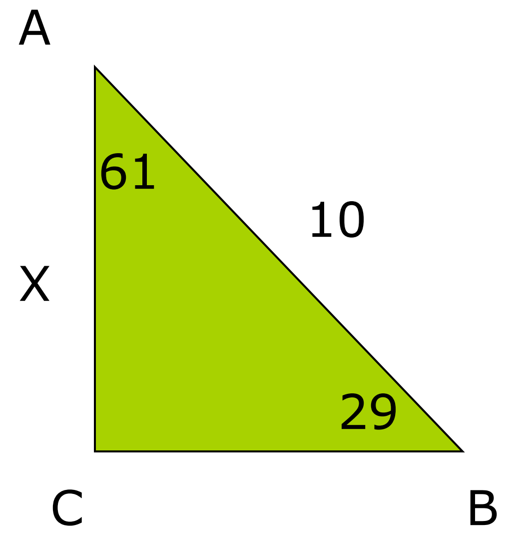
   or

2. cos 61 = x/10

   10 cos 61=x

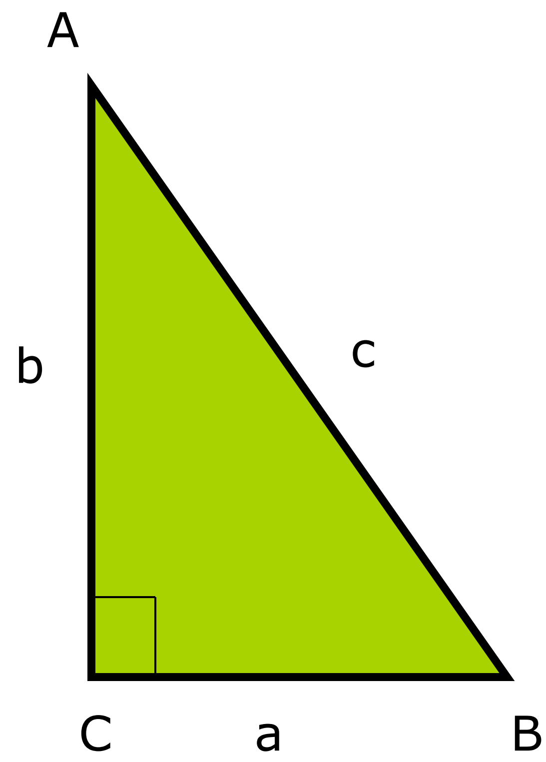
   10(.4848)=x

   4.8 = x



Definition of tangent ratio

- Let <A be and acute angle of the right triangle.
- $\tan A = \dfrac{\text{opposite leg}}{\text{adjacent leg}} = \dfrac{a}{b}$



(side a is opposite <A)

(side b is adjacent to <A)

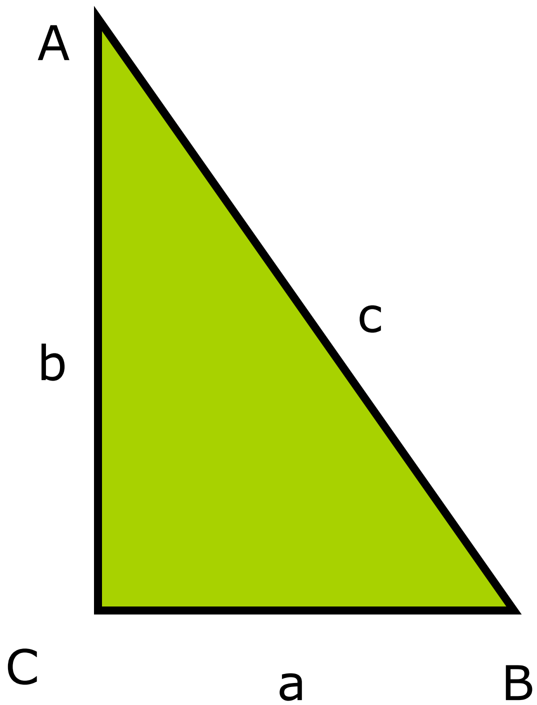
Tan is the abbreviation for Tangent.

Definition of remaining trig functions

$$\csc A = \frac{\text{hypotenuse}}{\text{opposite leg}} = \frac{c}{a}$$

$$\sec A = \frac{\text{hypotenuse}}{\text{adjacent leg}} = \frac{c}{b}$$

$$\cot A = \frac{\text{adjacent leg}}{\text{opposite leg}} = \frac{b}{a}$$



Csc is the abbreviation for cosecant. Sec is Secant and cotangent is cot.

Complementary Relationships

- csc A = 1/sin A
- sec A = 1/cos A
- cot A = 1/tan A

Complementary relationships

- $\sin A = \cos(90 - A)$
- $\tan A = \cot(90 - A)$
- $\sec A = \csc(90 - A)$

Simple chart of trig functions

angle	sin	cos
0 °	$\frac{1}{2}\sqrt{0}$	$\frac{1}{2}\sqrt{4}$
30 °	$\frac{1}{2}\sqrt{1}$	$\frac{1}{2}\sqrt{3}$
45 °	$\frac{1}{2}\sqrt{2}$	$\frac{1}{2}\sqrt{2}$
60 °	$\frac{1}{2}\sqrt{3}$	$\frac{1}{2}\sqrt{1}$
90 °	$\frac{1}{2}\sqrt{4}$	$\frac{1}{2}\sqrt{0}$

Inverse Trig functions

- The inverse sin function is the inverse of the restricted sin function y=sin x
- You must restrict the sin function so the inverse is a function.
- The symbolism used is:

$$x = \arcsin y \text{ or } x = \sin^{-1} y$$

It can be interpreted as

"x is the angle whose sin value is y

Important

- Note: $\sin^{-1}(y) \neq 1 / \sin(y)$

Inverse Trig functions

If $\sin(x) = y$

Then $x = \sin^{-1}(y)$

Where $-90^0 \le x \le 90^0$ and $-1 \le y \le +1$

If $\cos(x) = y$

Then $x = \cos^{-1}(y)$

Where $0 \le x \le 180^0$ and $-1 \le y \le +1$

Summary of Inverse Trig functions

Function	Inverse	*x*	y
$y = \sin(x)$	$x = \sin^{-1}(y)$	$-90^0 \le x \le 90^0$	$-1 \le y \le +1$
$y = \cos(x)$	$x = \cos^{-1}(y)$	$0 \le x \le 180^0$	$-1 \le y \le +1$
$y = \tan(x)$	$x = \tan^{-1}(y)$	$-90^0 \le x \le 90^0$	$-\infty < y < +\infty$
$y = \cot(x)$	$x = \cot^{-1}(y)$	$0 < x < 180^0$	$-\infty < y < +\infty$
$y = \sec(x)$	$x = \sec^{-1}(y)$	$0 \le x \le 180^0$ $x \ne 90^0$	$y \le -1$ $y \ge +1$
$y = \csc(x)$	$x = \csc^{-1}(y)$	$-90^0 \le x \le 90^0$ $x \ne 0^0$	$y \le -1$ $y \ge +1$

Inverse Trig functions

If $\sin(25.63°) = .4327$

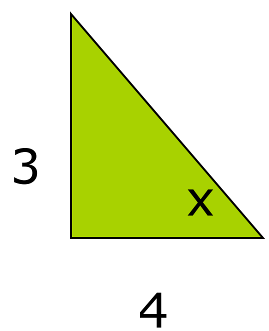
Then $\sin^{-1}(.4327) = 25.63°$

Solve the following using calculator

- Sin^{-1} .23
- Sin^{-1} .77
- Cos^{-1} .51
- Cos^{-1} .29
- Tan^{-1} 12.58
- Tan^{-1} 1.92

Exercises

- Use $\sin^{-1}$, $\cos^{-1}$, or $\tan^{-1}$ to find x.

1.


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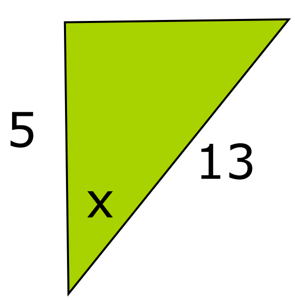

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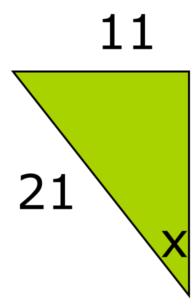

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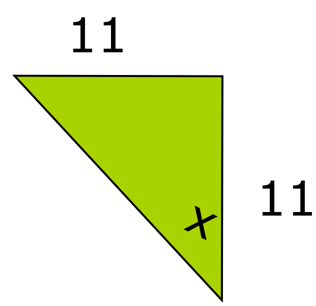


Solve the following problem

If $\sin A = 5/8$

What is $\cos A$ and $\tan A$?

Solve the following problem

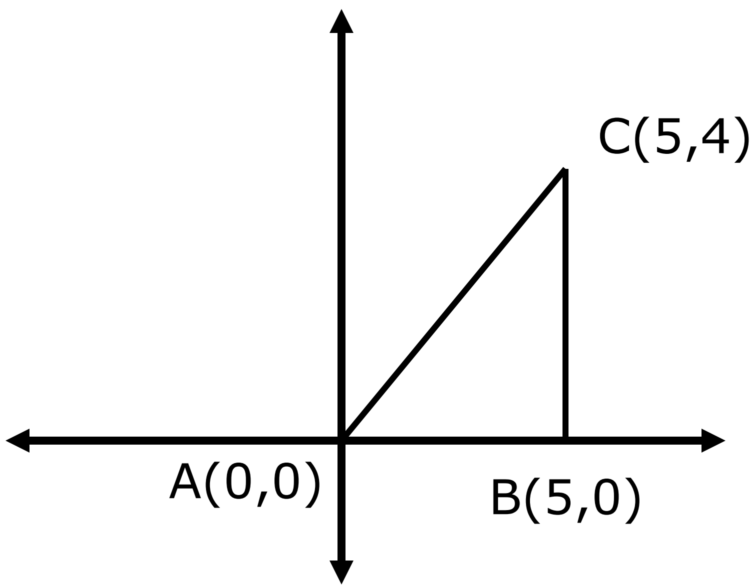
- If tan A = 9/5
- What is sin A and cos A ?

Exercises (slope and tan x)

A. Find the slope of line segment AC

B. Find tan A

C. Make a conjecture about slope of $\overline{AC}$.



Solving right triangles

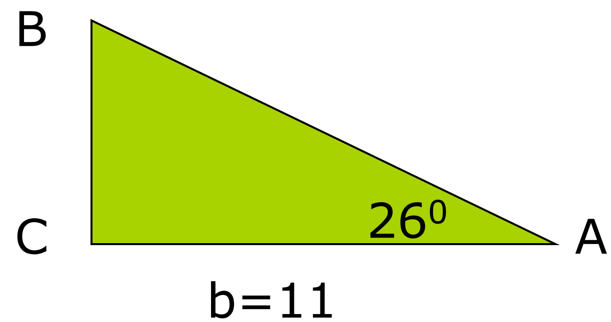
- The six parts of any triangle can be found when the measures of individual parts are given. This process is called solving the triangle.

- A right triangle can be solved when two of its sides are known or if an acute angle and a side are known.

Exercise

- Solve the right triangle (find angles and sides) with $m\angle A=26^0$ and $b=11$. Round the answers to the nearest whole number.

1. What is m<B ?
2. What is BC ?
3. What is AB ?



Solution

m<B = 90 – 26 = 64°

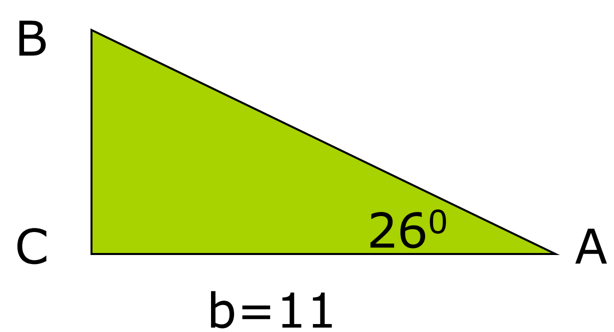
tan A = a/b

.4873 = a/11

11 (.4873) = a

5.36 = a

BC = 5.36



Solution

To find c you could use the Pythagorean Theorem but it is easier to use cos A.

$\cos 26 = 11/c$

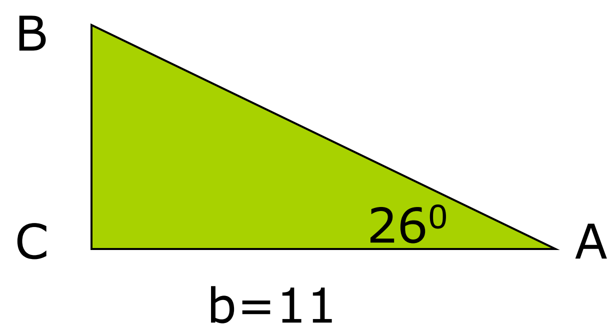
$.8987 = 11/c$

$c\,(.8987) = 11$

$c = 11/(.8987)$

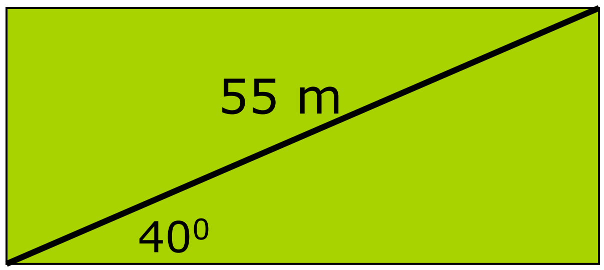
$c = 12.23$

$AB = 12.23$



Applied problem # 1

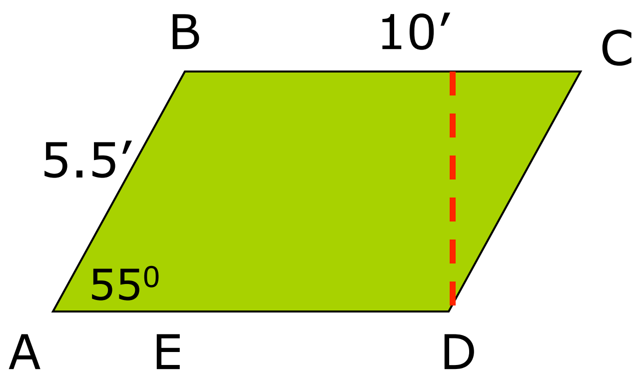
1. Find the perimeter of the rectangle.



Applied problem #2

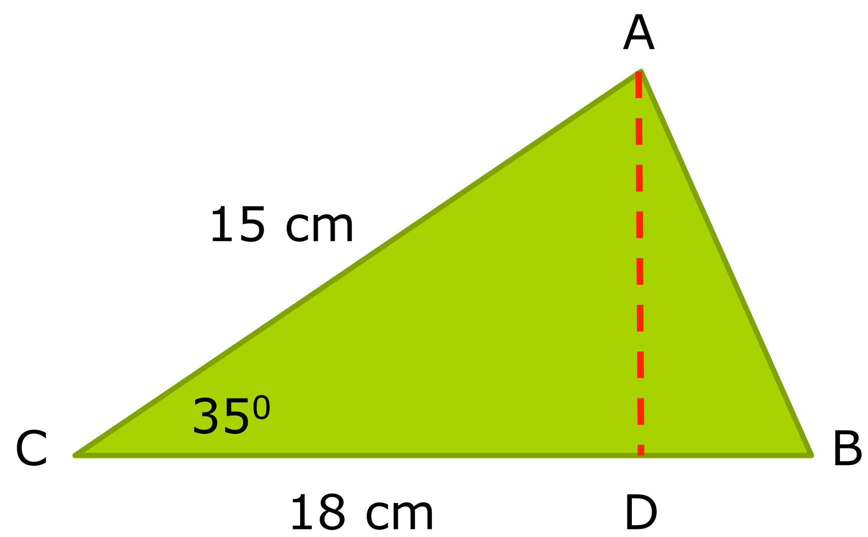
2. BA = 5.5′, BC = 10′

   Find the area of the parallelogram.



Applied problem #3

- AC = 15cm, BC =18 cm. Find the area of the triangle.



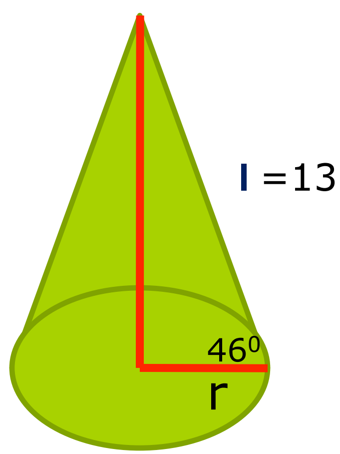
Applied problem #4

- Given a right circular cone with slant height **l** =13 inches. The angle formed by the slant height and the radius is 46°. Find the lateral area of the cone.

cos 46^0= r/ **l**, 0.695 = r/13, r = 9.035

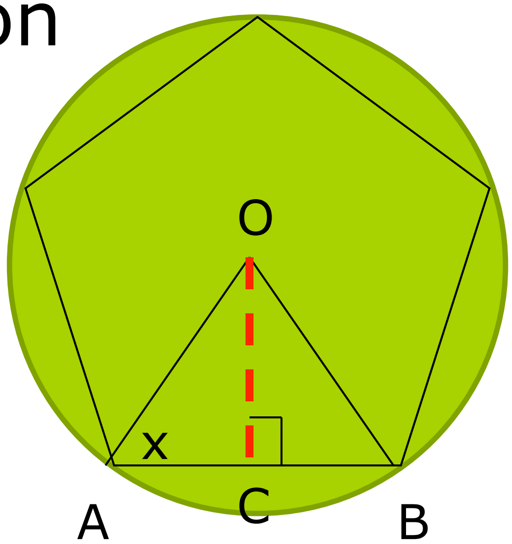
LA = ½ PL = ½ (2 π r) ***l*** = π r **l**

= (3.14) (9.035) (13) = 368.81



Applied problem # 5

- Find the length of an apothem $\overline{OC}$ in a regular pentagon whose radius is 12 inches.
- (Apothem is the line segment from the center of the regular polygon perpendicular to the side.)
- Find the area of the regular pentagon.



Solution

Given OA = OB = 12 inch

m<AOB = 360/5 = 72^0

m<OAB = m<OBA = (180 – 72)/2 = 54^0

sin x = OC/OA, cos x = AC/OA

$\sin 54^0$ = OC/12, 0.81 = OC/12, OC = 9.72

$\cos 54^0$ = AC/OA, 0.59 = AC/12, AC = 7.08

AB = AC + BC = 7.08 + 7.08 = 14.16

Area of ΔOAB = ½ (14.16 * 9.72) = 68.8

The area of the pentagon = 5 * 68.8 = 344

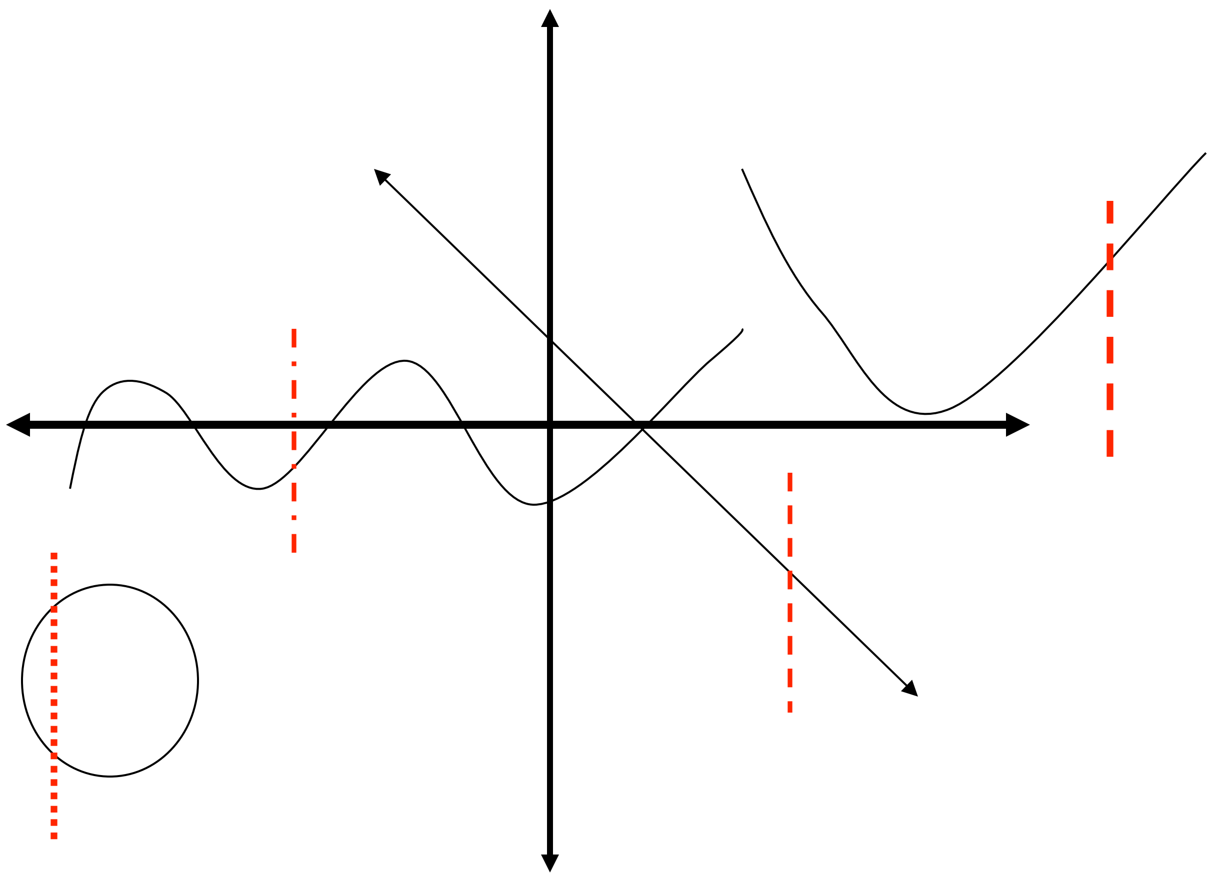
Functions

- A function is a partnership of two sets of values where every value of the first set has one and only partner value in the second set.

- A graph of a function is easy to identify by the <u>vertical line</u> test. If a vertical line can be dropped anywhere on the graph and only hit the graph in one point then the graph is the graph of a function.

Function or not?

How many times does the red line hit the graph?

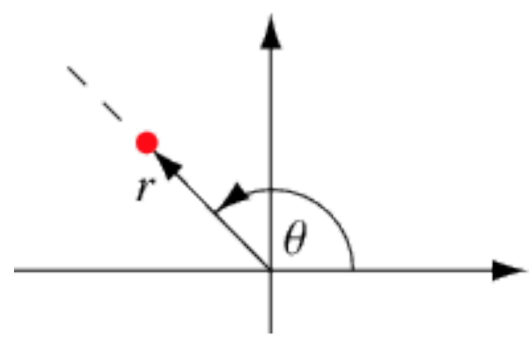


Graphing

- To graph trig functions use the table method where you chose values for x and solve for y.

Polar Coordinates

- Angle (θ) starts at positive x-axis
- Distance (r) is relative to the origin
- Polar coordinates are written (r, θ)



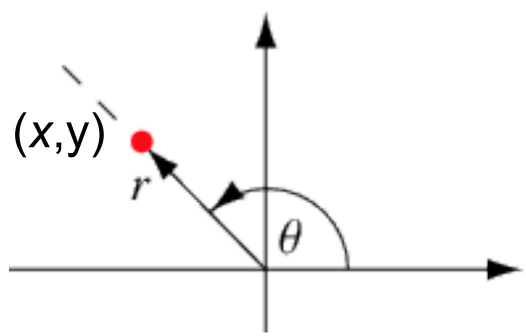
To Polar Coordinates

- Given Cartesian coordinates (x,y)

  $r = \sqrt{(x)^2+(y)^2}$

  $\theta = \tan^{-1}(y/x)$

  - θ will be given as angle from x-axis
  - you need to adjust the value for the correct Quadrant



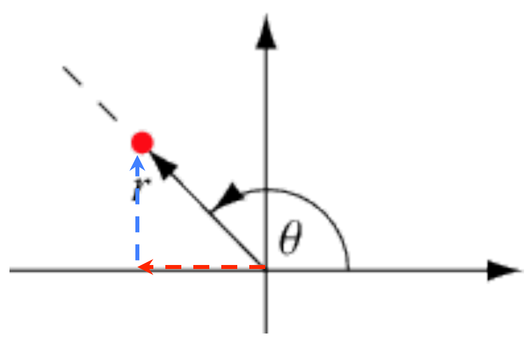
From Polar Coordinates

- Given Polar coordinates (r, θ)

  $x = r\cos(\theta)$

  $y = r\sin(\theta)$

- Cartesian results will be written as (x,y)



Today you learned

- Definition of the trig functions
- Solving right triangles
- Inverse trig functions
- Graphing simple trig functions
- Polar coordinates

Review for Final

- Use your calculator to get
  - sin 32.70^0, cos 55.42^0, tan 38.59^0
  - Sin^{-1} 0.26, $\cos^{-1}$ 0.50, $\tan^{-1}$ 23.19
- If sin A = 5/13, find cos A and tan A
- How to find the surface area of a cylinder?
- How to find the area of a triangle?
- How to find distance, midpoint, slope, equation for a line
- What are the slopes for parallel lines, perpendicular lines?

Final (next class)

- Final covers entire book