GAM

Geometry and Measurement

Day 1

Welcome

Course Description:

Geometry and Measurement emphasizes geometric concepts that build upon algebra skills developed in College Mathematics. The curriculum focuses on deductive thinking, the axiomatic system, and properties of two and three dimensional figures. Trigonometry is introduced as a content link to Fundamentals of Physics.

Course/Attendance Policies

- ***Attendance***- More then **6 hours** absent will result in failure.
- ***Excused absences*** - require documentation from a doctor or administration.
- (**Absent + Excused**) **≥ 12** hours will result in failure.
- ***Sickness*** -Please call instructor (407-679-0100 extension 8937) before class and leave answering machine message.
- ***Tardiness***- Attendance will be taken at the beginning of class and directly after break. Missing more than fifteen minutes of any period will be counted as a 2-hour absence.
- ***GPS***-Most absences and tardy count against your GPS score.

Course Policies (cont.)

Classroom Management- Students that misbehave will be dismissed from class. Prior to returning to class a meeting must be scheduled with the Course Director.

- ***No food or drink*** is allowed in class except bottled water.
- ***No cell phone usage*** is permitted during class. Please place all cell phones in vibrate mode or turn off
- ***Problem Solving***- Any problems or issues with scheduling, course requirements, or related issues should be addressed to the Course Director during office hours.

Labs

- ***Labs are generally held the same day as lecture*** in the same room and last 4 hours.
- ***All Labs must be attended***.

- ***Uncompleted work*** will be taken home and completed. It must be handed in at the beginning of next lab or students will receive a zero for that lab.

- ***Book, paper, pencil and calculator*** must be brought to lab and class every day. Being unprepared will count against your GPS score.

- ***No Laptops are allowed*** during lectures or labs.

- ***Homework*** is to read the chapters in the book pertaining to the next lecture.

Professionalism

Students should conduct themselves professionally at all times. Lateness or absence, inappropriate language, sleeping in class, disrespectful attitude, distracting others, and using laptops will not be tolerated.

Excessive talking will result in loss of GPS points.

***Students are responsible for the information in the syllabus and are expected to handle absences and assignments in the manner described.**

Instructor Information

- Jenn-Leun Chu
- Extension 8937
- Office Hours (will be updated every month)
  - Monday:
  - Tuesday:
  - Wednesday:
  - Thursday:
  - Friday:
- Extra help
  - Schedule with teacher during break
  - Schedule with lab specialists

Class Tests

- Test 1 is on the 3rd class
  - Covers chapters 1, 2 and 3
- Test 2 is on the 5th class
  - Covers chapters 4, 5, and 6
- Test 3 = Final is on the 8th class
  - Covers all the materials (chapters 1 to 10)
- Everyone has to take tests, No excuses.

Class 8
FINAL EXAM

- Comprehensive test on all course content.
- Test is on the 8th class (2nd class day of the week 4).
- Everyone has to take final examination, No excuses.

Quizzes and Construction Activities in Labs

- Lab 1: Construction Activity 1
- Lab 2: Construction Activity 2
  Quiz 1
- Lab 3: Construction Activity 3
- Lab 4: Quiz 2
- Lab 7: Quiz 3

Grading Policy

- **Structure:**

  15% - Test 1

  15% - Test 2

  20% - Test 3 (Cumulative Final)

  10% - GPS

  40% - Lab work and Quizzes

  - 12% - Quizzes (4% each)
  - 12% - Construction activities (4% each)
  - 16% - Home work

- **Scale:**

A+	95 – 100	4.0
A	90 – 94	3.5
B+	85 – 89	3.0
B	80 – 84	2.5
C+	76 – 79	2.0
C	73 – 75	1.5
D	70 – 72	1.0
Fail	0 – 69.99999999999999999999999%	0.0

Course Outline

- History of Geometry
- Foundations of Geometry
- Geometric Proofs
- Constructions (discussed in Labs)
- Triangles
- Parallel Lines and Polygons
- Quadrilaterals

Course Outline (cont.)

- Similar Polygons and Pythagorean Theorem
- Circles
- Areas of Polygons and Circles
- Solid Geometry
- Analytic Geometry
- Introduction to Trigonometry

The following GAM course materials are on propeller

- Syllabus
- PPT Slides for each class
- Formula Sheets

Lab (40%)

- Labs are extension of class lectures
- Some of the book materials discussed in labs are not necessarily covered in the lectures (e.g. the constructions)

Brief History Of Geometry

Chapter 0

Brief History Of Geometry

- As early as 25,000 B.C. evidence was found that basic geometric designs were drawn
- The earliest recorded explorations of geometry come from the Egyptians and Babylonians around 3000 B.C.

Egyptians and Geometry

- The Egyptians used geometry for
  - dividing land (due to Nile River flooding)
  - architecture
  - Astronomy/constellations

Ancient Achievements

- Use empirical trial-and-error approach
  - Egyptians had developed formula for calculating volumes of pyramids
  - Babylonians had a trigonometry table
  - Egyptians and Babylonians had intuitively discovered the Pythagorean theorem

Geometry from Greek

- The geometry we will study was Greeks' work between about 500 B.C. and 600 A.D.
- The Greeks viewed geometry
  - As the ultimate in perfect reasoning
  - Having a strong connection with philosophical truth
- Between 600 A.D. and 1600 A.D. Geometry was largely ignored (in favor of algebra)

Pythagoras 580/572 BC – 500/490 BC

- A Greek mathematician, scientist and mystic
- Pythagoras, in about 525 B.C., proved deductively
  - $a^2 + b^2 = c^2$
  - Called Pythagorean Theorem

Plato 428/427 BC – 348/347 BC

- Plato studied with students of Pythagoras
- Plato believed that students in geometry should use nothing but a compass and straightedge (no marked rulers)

Euclid about 325 BC - about 265 BC

- Euclid wrote one of the first geometry books "*The Elements*"
- This book represented the Greek's ideas on geometry
- This book includes definitions and five axioms
- Greeks believed that the five axioms were self-evident and needed no proof
- The geometry we will study is based on Euclid's work

Rene Descartes

March 31, 1596 – February 11, 1650

- Rene Descartes, a mathematician and philosopher in the 17th century, developed Analytic Geometry
- Analytic Geometry is the study of geometry using the principles of algebra

The Foundations of Geometry

Chapter 1

1.1 Inductive and Deductive Reasoning

- We use inductive reasoning when we reach a general conclusion (called generalization) based on a limited collection of specific observations.

- The limitation of inductive reasoning is that there are no guarantees that the conclusion drawn is always correct or that it is the only possible conclusion.

Inductive Reasoning

- Social scientists frequently use inductive reasoning, a general conclusion (called generalization) is drawn from a collection of observations

Axiomatic system

- An axiomatic system consists of four parts:
  1. Undefined terms
  2. Definitions
  3. Axioms or postulates
  4. Theorems

- Geometry is an axiomatic system

Undefined Terms

- Undefined terms are the starting points in a system.
- It is impossible to define every term because definitions are also formed with words that have meaning.
- Some terms must be assumed to go forward.

Definitions

- Definitions are statements that give meaning to new terms that will be used in a system.
- The words used to form a definition are either undefined terms or previously defined terms.

Postulates

- Postulates or axioms are statements about undefined terms and definitions that are *accepted as true without verification* or proof.
- They also serve as a starting point in a system.

Theorems

- A theorem is a statement that we can prove by using definitions, postulates, and the rules of deduction and logic.
- Many theorems are expressed as ***if…then …*** statements
- The phrase following ***if*** is the hypothesis and includes given information.
- The phrase following ***then*** includes the statement to be proved and is called the conclusion of the theorem.

Examples

- Undefined Terms: happy, pleasant, person
- Definition: “Teri” is a happy person.
- Postulate: Every happy person is pleasant.
- Theorem: “Teri” is pleasant.

Deductive Reasoning

- Deductive reasoning is the process of reaching a specific conclusion based on a collection of accepted true statements/assumptions (such as: undefined terms, definitions, postulates/axioms, and/or previously proved theorems).
- Deductive reasoning is the basis of an axiomatic system
- Mathematicians usually use deductive reasoning to prove a theorem

Deductive Reasoning

- When we reason deductively, we start with one or more premises (undefined terms, definitions, axioms or postulates, or previously proved theorems) and attempt to arrive at a conclusion that logically follows if the premises are accepted.

Fallacy

- A fallacy is a conclusion that does not necessarily follow from the premises.

Undefined Terms

- In geometry, we begin with 4 undefined terms
  - Set
  - Point
  - Line
  - Plane

1.2 Set

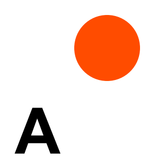
- A set is a collection of objects.

- Example:
  - S = {…, -3, -2, -1, 0, 1, 2, 3, …}
  - T = { Tom, table_A, a_cow, FullSail_University, 52 }
  - U = { }

Points

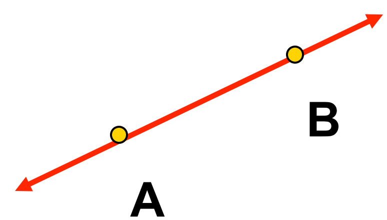
- A point is an object that determines a position but that has no dimension.
- A point is named with a capital letter.

Point A



Lines

- A line is a set of points in a one-dimensional straight figure that extends in opposite directions without ending.
- To name a line, we use any two points on the line.
- Notation: Line AB or $\overleftrightarrow{AB}$



Planes

- A plane is a set of points on a flat surface having two dimensions and extending without boundary.

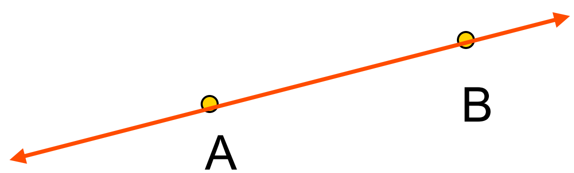
Plane ***P***

Space – a defined term

- Definition: The set of all points is called space.
- Any set of points, lines, or planes in space is called a geometric figure.

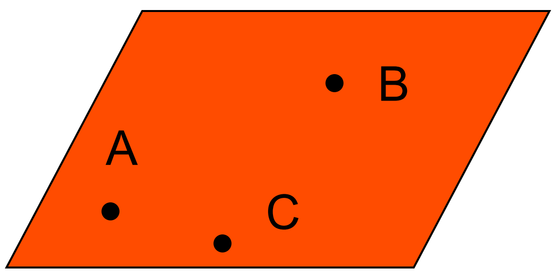
The Postulates
(Two types-Geometric and Algebraic)

- Post. 1.1 Given any two distinct points in space, there is exactly one line that passes through them.



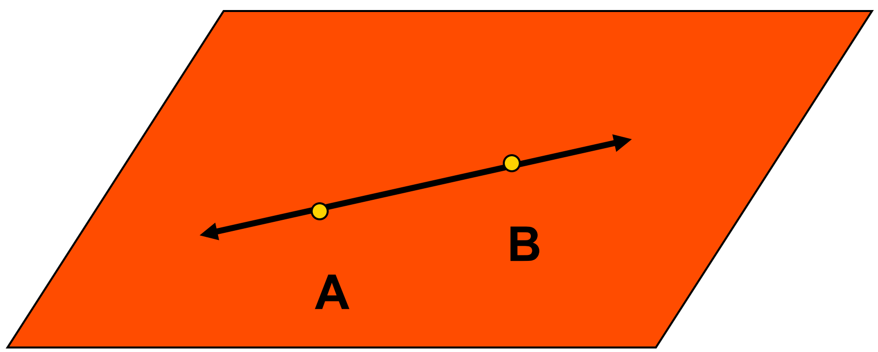
Plane

- Post. 1.2 Given any three distinct points in space not on the same line, there is exactly one plane that passes through them.



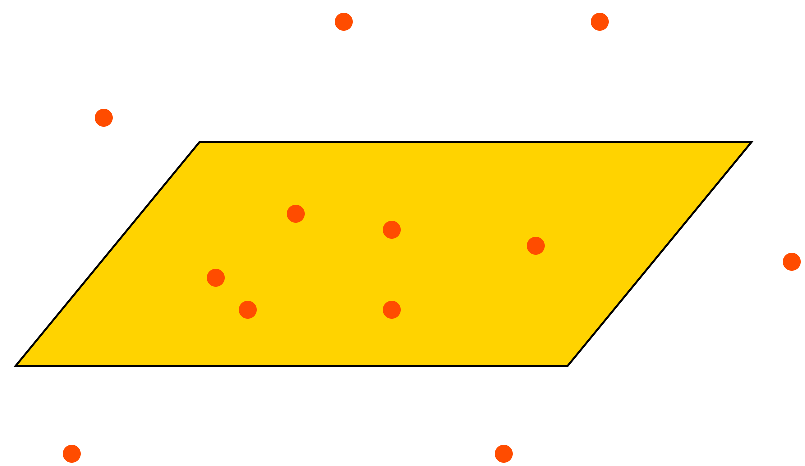
(Geometric) Postulates

- Post. 1.3 The line determined by any two distinct points in a plane is also contained in the plane.



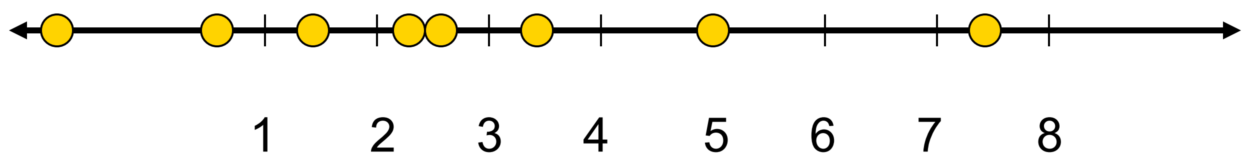
(Geometric) Postulates

- Post. 1.4 No plane contains all points of space.



Ruler Postulate

- Post 1.5 Ruler Postulate -There is a one-to-one correspondence between the set of all points on a line and the set of real numbers.



(Algebraic) Postulates

- Post. 1.6 <u>The Reflexive Law</u> : Any quantity is equal to itself. $x = x$.

- Post. 1.7 <u>The Symmetric Law</u> : If $x = y$, then $y = x$.

- Post 1.8 <u>The Transitive Law</u> : If $x = y$, and $y = z$, then $x = z$.

(Algebraic) Postulates

- Post 1.9 The Addition-Subtraction Law

  If w, x, y and z are any four quantities with $w = x$ and $y = z$,

  then $w + y = x + z$ and $w - y = x - z$.

- Post. 1.10 The Multiplication-Division Law

  If w, x, y and z are any four quantities with

  $w = x$ and $y = z$,

  then $wy = xz$ and $w/y = x/z$ (provided $y \neq 0$ and $z \neq 0$).

(Algebraic) Postulates

- Post 1.11 <u>The Substitution Law</u>

  If x and y are any two quantities with $x = y$, then x can be substituted for y in any expression containing y.

- Post 1.12 <u>The Distributive Law</u>

  If x, y and z are any three quantities, then $x (y + z)= xy + xz$.

(Algebra) Arithmetic Operations

- Multiplication notations

  $$a \bullet b = c$$

  $$(a)(b) = c$$

  $$a \times b = c$$

- a and b are called factors
- c is called the product
- We will avoid using the sign “x” for multiplication, since it may be confused with other algebraic symbols

(Algebra) Arithmetic Operations

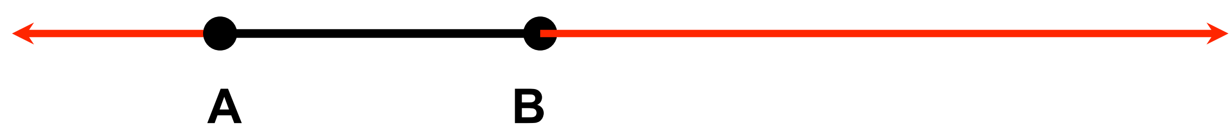
- Division

  $a \div b = c$ (with remainder d)

  $a / b = c$ (with remainder d)
- a is the dividend
- b is the divisor
- c is the quotient
- d is the Remainder

1.3 Segments

- Let A and B be two distinct points on a line. The geometric figure consisting of all points between A and B, including A and B, is called a line segment or segment (denoted as $\overline{AB}$)
- The points A and B are called the endpoints of $\overline{AB}$.

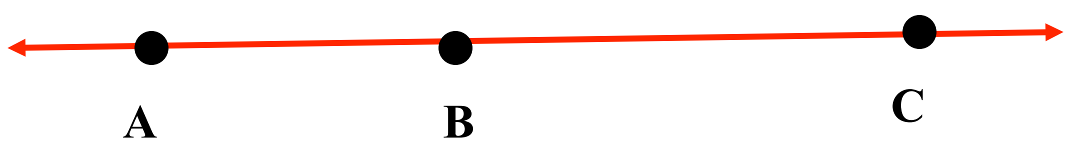


- The length of segment $\overline{AB}$ is the distance between the endpoints A and B and is denoted as AB.

Segment Addition

- Post. 1.13 <u>Segment Addition Postulate</u> - Let A, B, and C be three points on the same line with B between A and C.

  Then AC = AB + BC, BC = AC - AB, and AB = AC - BC.

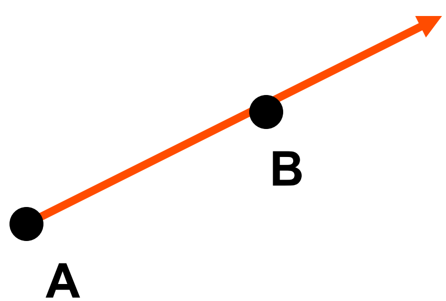


Collinear

- Points A, B, and C are said to be collinear which means they are on the same line.

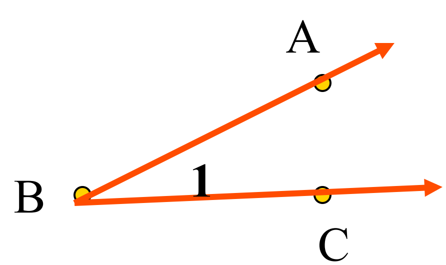
Rays

- Let A and B be two distinct points on a line. The geometric figure consisting of the point A together with all points on $\overleftrightarrow{AB}$ on the same side of A as B is called a ray, denoted by $\overrightarrow{AB}$.
- The point A is called the endpoint of $\overrightarrow{AB}$.



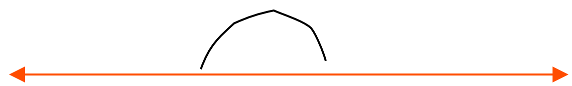
Angles

- An angle is a geometric figure consisting of two rays that share a common endpoint, called the vertex of the angle. The rays are called the sides of the angle.
- Angles are named in the following ways: <B, <ABC, <CBA, and <1.

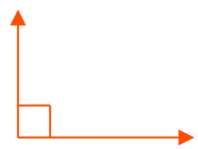


Facts about angles

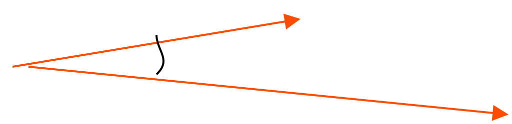
- Angles are measured in degrees. 1 degree is 1/360 of a complete rotation.
- Straight angle - an angle with a measure of 180º.



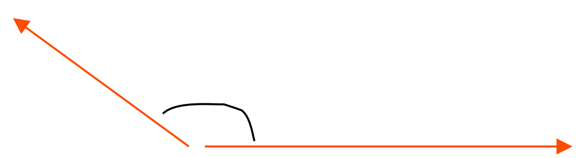
- Right angle - an angle with a measure of 90º.



- Acute angles have measures between 0º and 90º.



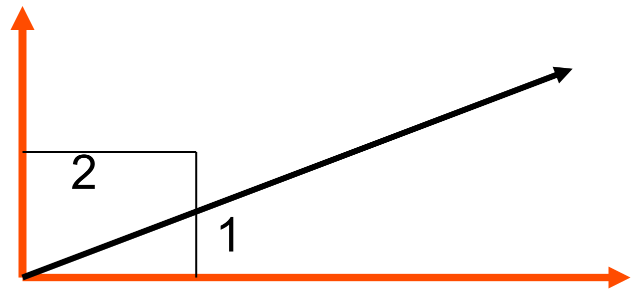
- Obtuse angles have measures between 90º and 180º.



Complementary Angles

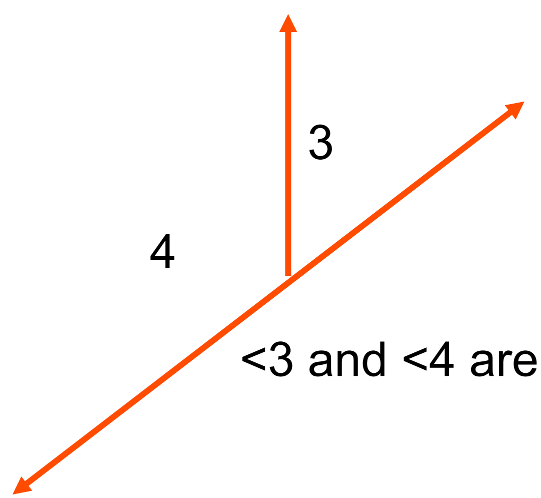
- Two angles whose measures total 90º are complementary angles and each is called the complement of the other.

<1 and <2 are complementary.



Supplementary Angles

- Two angles whose measures total 180º are supplementary angles, and each angle is called the supplement of the other.



<3 and <4 are supplementary.

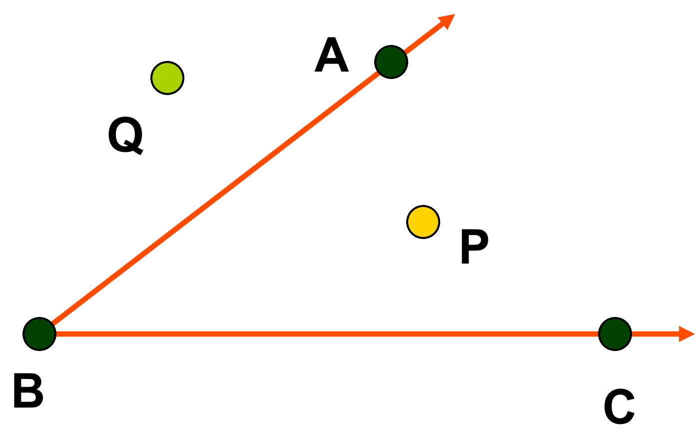
Example

- If m<P=(2y-9) and m<Q= (7y) and <P and <Q are supplementary, find y.

- Complete 13-26 on Page 43-44

More Undefined Terms

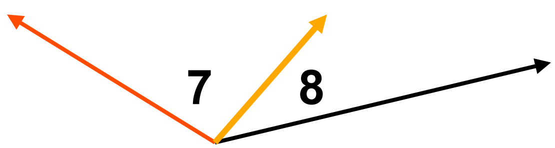
- Point P is in the interior of < ABC.



- Point Q is exterior to <ABC.
- B is the vertex of <ABC

Adjacent Angles

- Two angles are <u>adjacent angles</u> if they have a common vertex, share a common side, and have no interior points in common. (The angles don't overlap.)



<7 and <8 are adjacent.

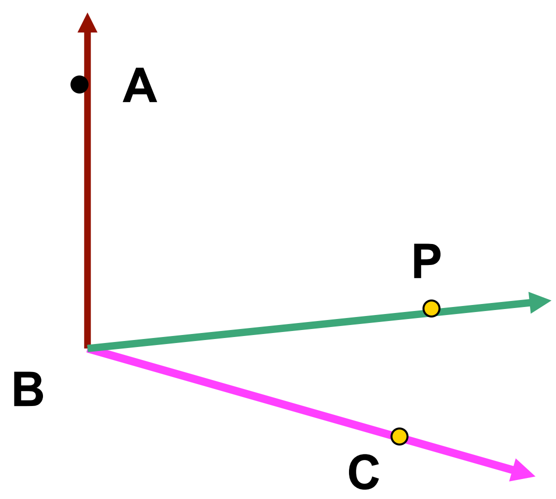
Angle Addition

- Post. 1.14 Angle addition postulate

  m<ABC= m<ABP + m<PBC,

  m<PBC=m<ABC - m<ABP and

  m<ABP=m<ABC-m<PBC.



Deductive Proofs

- A theorem is usually stated in the form of "If P then Q", called conditional statement
- A proof of a theorem involves deductive reasoning rather than inductive reasoning
- Mathematicians use deductive reasoning to prove that a theorem is true based on accepted assumptions and previously proved theorems

1.4 Conditional Statements

- Statements that can be written in "*if P then Q*" form are called Conditional Statements.

Example of a Conditional Statement:

If the sun is shining, then I can see my shadow.

- Where P = *the sun is shining*
- *Where Q = I can see my shadow*
- "*the sun is shining*" is the hypothesis.
- "*I can see my shadow*" is the conclusion.
- "*if P then Q*" is symbolically represented as "P→Q", which is read "P implies Q"

Converse Statement

- By exchanging the hypothesis with the conclusion a converse is formed represented $Q \rightarrow P$ symbolically.
- Even if the conditional statement is true the converse may not be true so the two statements are not equivalent.
- Using the example on the last slide the converse would be:

If I can see my shadow, then the sun is shining.

Negation

- The negation of a statement P is represented as ~P
- If P is true then “not P” is false.

Inverse Statement

- The <u>inverse</u> statement is formed by negating the hypothesis and the conclusion. $\sim P \rightarrow \sim Q$
- Using the same example the inverse is:

  If the sun is not shining then I can not see my shadow.

Contrapositive Statement

- The contrapositive statement exchanges the negation of the hypothesis and the negation of the conclusion or $\sim Q \rightarrow \sim P$.

If I don't see my shadow then the sun is not shining.

Conditional Statements

- $P \rightarrow Q$ Conditional Statement given
- $Q \rightarrow P$ Converse
- $\sim P \rightarrow \sim Q$ Inverse
- $\sim Q \rightarrow \sim P$ Contrapositive
- $\sim P$ Negation of statement P

1.5 Formalizing Geometric Proof

- The classic format of a direct proof of a theorem $P \rightarrow Q$ shows a series of statements, starting with the hypothesis P.
- Each statement follows from the preceding one using the reasoning of the preceding statement, and the final statement is the conclusion.

Formalizing Geometric Proof for $P \rightarrow Q$,

where Q_1, Q_2, …, Q_{n-1} are intermediate steps

Statements	Reasons
1. P	1. Given
2. Q_1	2. Reason for Q_1
3. Q_2	3. Reason for Q_2
…	…
n. Q_{n-1}	n. Reason for Q_{n-1}
n+1. Q Therefore $P \rightarrow Q$	n+1. Reason for Q

The Reasons

- The reasons given for the truth of each statement, written to the right of the statement, must be **accepted** or **previously proved statements.**

What you can use as reasons

- The following information can be used as reasons for each statement.
  - Given
  - Postulates
  - Definitions
  - Theorems (previously proved)

Format of Direct Proof of $P \rightarrow Q$

- Given: P (Hypothesis)
- Prove: Q (Conclusion)
  - Suppose we have $P \rightarrow Q_1$, $Q_1 \rightarrow Q_2$, $Q_2 \rightarrow Q_3$, and $Q_3 \rightarrow Q$ as accepted or previously proved statements.

Statements	Reasons
1. P	1. Given
2. Q_1	2. Reason for $P \rightarrow Q_1$
3. Q_2	3. Reason for $Q_1 \rightarrow Q_2$
4. Q_3	4. Reason for $Q_2 \rightarrow Q_3$
5. Q Therefore $P \rightarrow Q$	5. Reason for $Q_3 \rightarrow Q$

Format of Direct Proof of P→Q

Statements	Reasons
1. P	1. Given
2. Q1	2. Reason for Q1
3. Q2	3. Reason for Q2
4. Q3	4. Reason for Q3
5. Q	5. Reason for Q

Steps in completing a proof

1. Draw and "mark" the figure with given information.
2. Write the information stated in the hypothesis as the first statement and for the reason write "*Given*", using all the symbols from your diagram.
3. Then write Prove and state what it is to be proved.

Proof (cont.)

4. Write Proof and head two columns with the words “Statement” and “Reasons”

Note: The first statements are usually taken from the ***given*** statements and the final statement will always be the ***prove*** statement.

Completing a proof

5. Work down the proof from statement to statement where every step follows from the one before it.
6. The last statement is the conclusion.

- **Given information, undefined terms, postulates, definitions, and previously proved theorems are used as reasons in a proof.**

The First Theorem

- **Theorem 1.1** <u>Addition Theorem for Segments</u>

  If B is a point between A and C on segment $\overline{AC}$, Q is a point between P and R on segment $\overline{PR}$, if AB = PQ, and BC = QR, then AC=PR. (Also works for subtraction Th. 1.2)

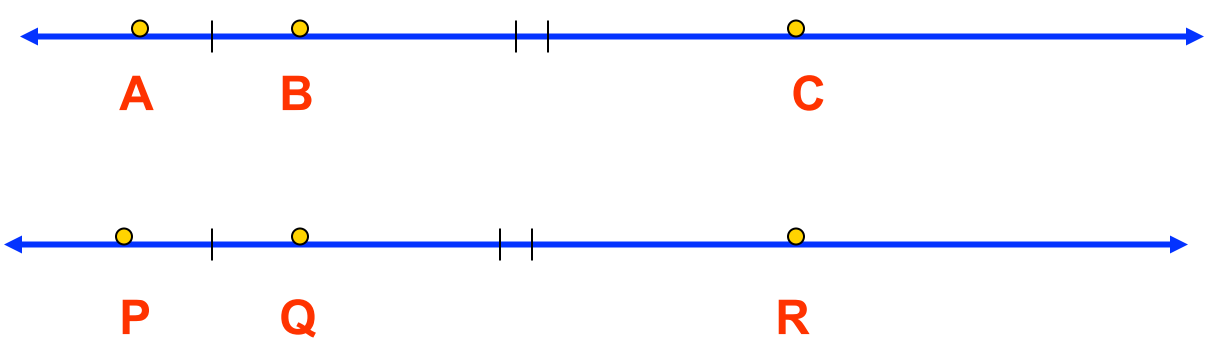
Note: **Hypothesis** **Conclusion**

- If a statement for a proof is not in If ...Then form then reword it.

1. Draw and mark the figure from the given information.

- From the description in the hypothesis you can draw the following figure.

The slash marks indicate the line segments are equal lengths.

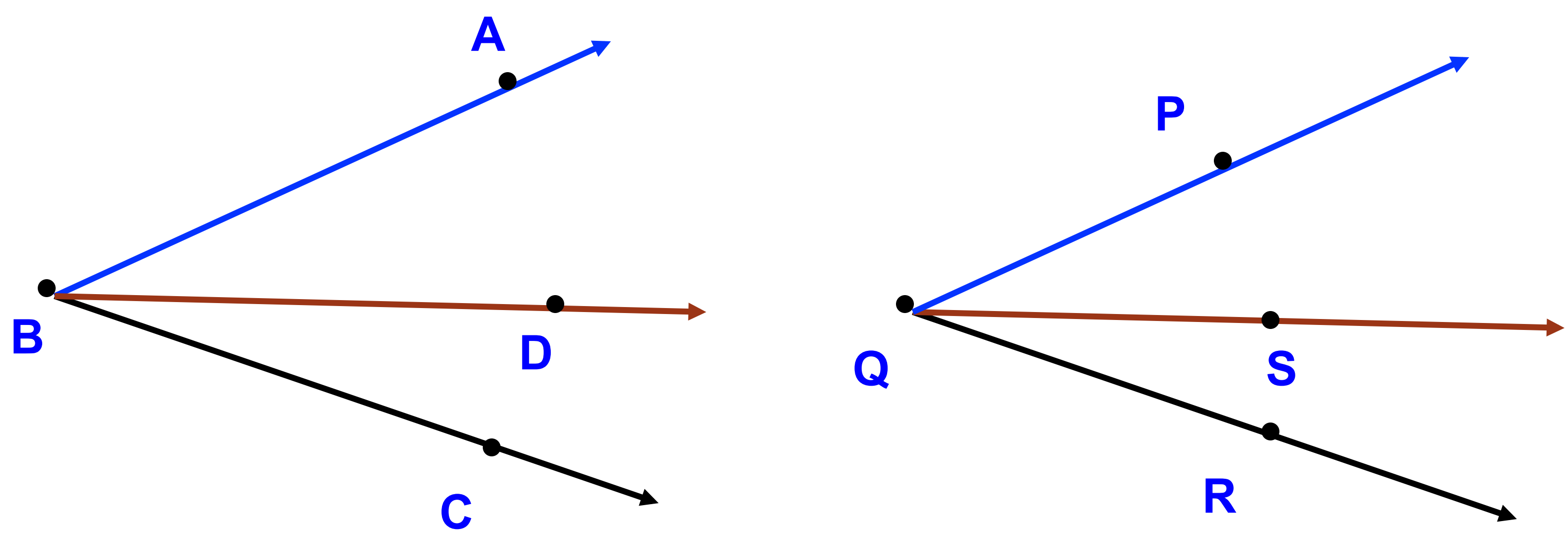


2. Write the information stated in the hypothesis as the first statement and for the reason write *given.*

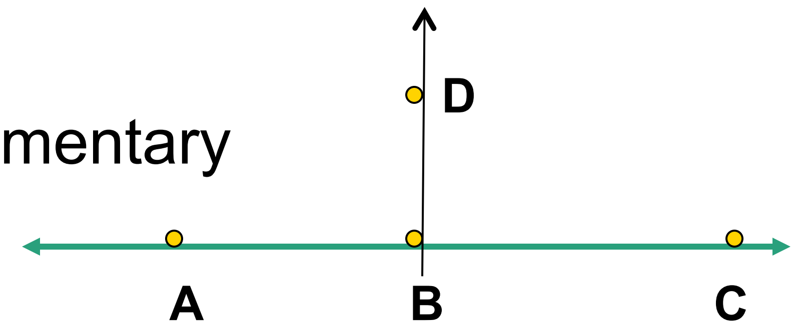
Statements	Reasons
1. B is between A & C on AC	1. Given
2. Q is between P & R on PR	2. Given
3. AB =PQ	3. Given
4. BC=QR	4. Given
5. AB + BC = PQ + QR	5. Addition-Subtraction Law
6. AC = AB + BC and PR = PQ + QR	6. Segment Addition Postulate
7. AC = PR	7. Substitution Law

Theorem 1.3 Addition Theorem for Angles

If D is a point in the interior of <ABC, S is a point in the interior of <PQR, if m<ABD = m<PQS, and m<DBC=m<SQR, then m<ABC = m<PQR. (Also works for subtraction of angles, Th. 1.4)



- Th. 1.5 Two equal supplementary angles are right angles.



Statements	Reasons
1. m<ABD=m<DBC	
2. <ABD supp to <DBC	
3. m<ABD+m<DBC=180	
4. m<ABD+m<ABD=180	
5. 2m<ABD=180	
6. m<ABD=90	
7. m<DBC=90	
8. <ABD and <DBC are rt. <‘s	

Theorems & corollaries on complementary angles

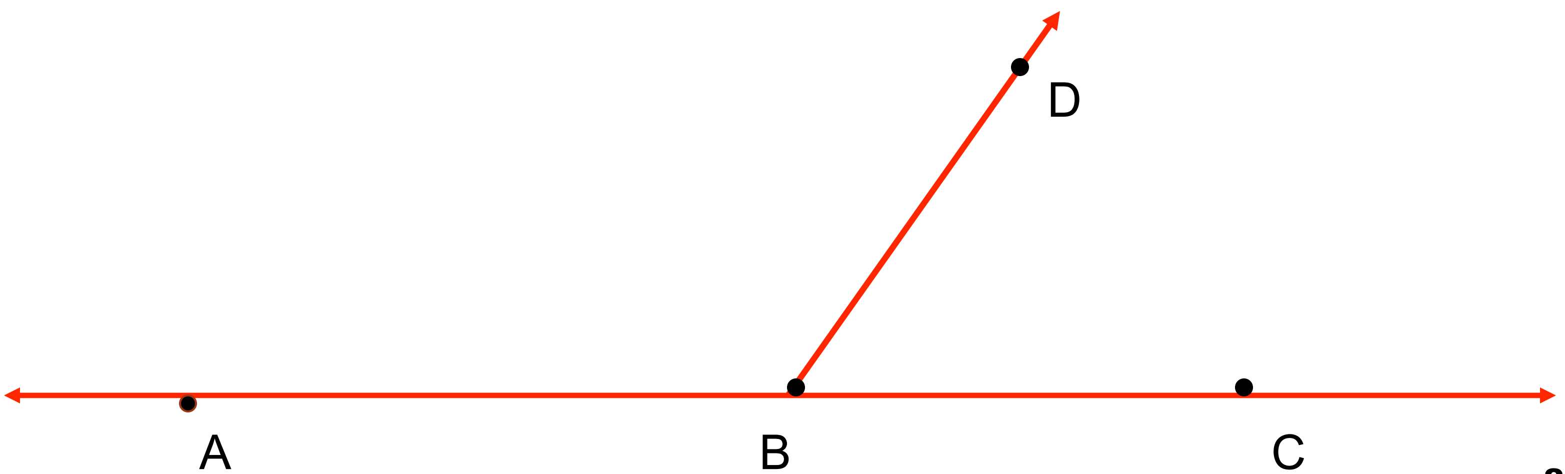
- Th. 1.6 Complements of equal angles are equal in measure.
- A corollary is a theorem that is easy to prove as a direct result of a previously proved theorem.
- Cor. 1.7 Complements of the same angle are equal in measure.

Theorems on supplementary angles

- Th. 1.8 Supplements of equal angles are equal in measure.
- Cor. 1.9 Supplements of the same angle are equal in measure.

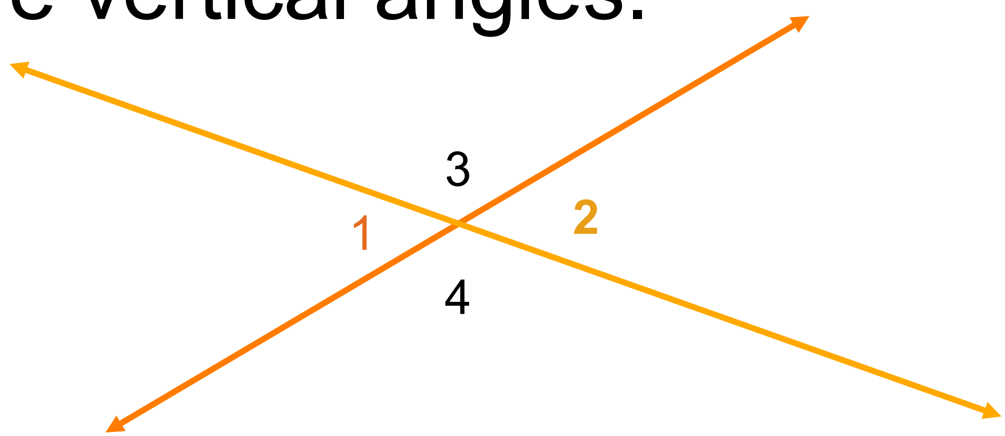
Theorems and corollaries on supplementary angles

Th. 1.10 If A, B, and C are three points on a line, with B between A and C, and <ABD and <DBC are adjacent angles, then <ABD and <DBC are supplementary.



Vertical angles

- Vertical angles are two non adjacent angles formed by two intersecting lines.
- <1 and <2 are vertical angles.



- Th. 1.11 Vertical angles are equal in measure.

1.6 Line Segment Bisection

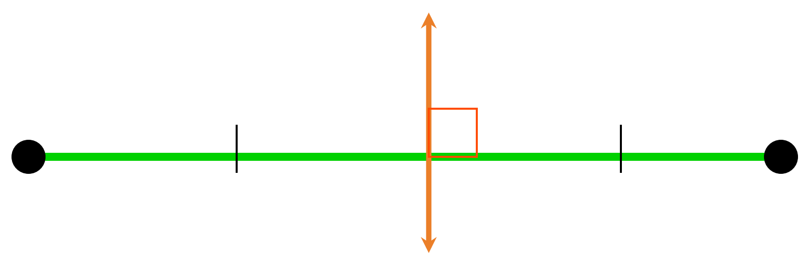
- Let $\overline{AB}$ be a line segment. To bisect $\overline{AB}$ is to identify a point C between A and B such that AC=CB.
- Point C is called the midpoint of $\overline{AB}$. The midpoint is a point on the line segment that separates the segment into two equal parts.
- A line segment that contains the midpoint C but no other point of $\overline{AB}$ is call a bisector of $\overline{AB}$.

Midpoints and Perpendiculars

- **Post. 1.15** Midpoint Postulate – Each line segment has exactly one midpoint.
- Two lines are perpendicular if they intersect and form equal adjacent angles. The angles formed are right angles.
- Th. 1.12 All right angles are equal in measure.

Bisectors and Perpendiculars

- A line that both bisects and is perpendicular to a given line segment is called a perpendicular bisector of the segment.



- Post. 1.16 Perpendicular Bisector Postulate - Each given line segment has exactly one perpendicular bisector.

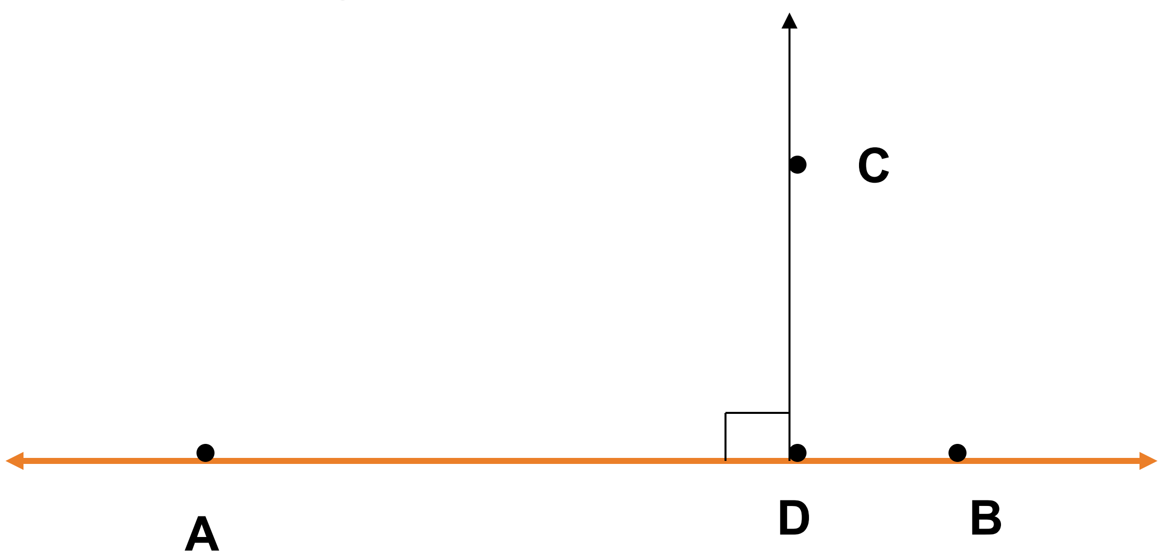
Postulates on perpendiculars

- Post. 1.17 There is exactly one line perpendicular to a given line passing through a given point on the line.

- Post. 1.18 There is exactly one line perpendicular to a given line passing through a point not on the line.

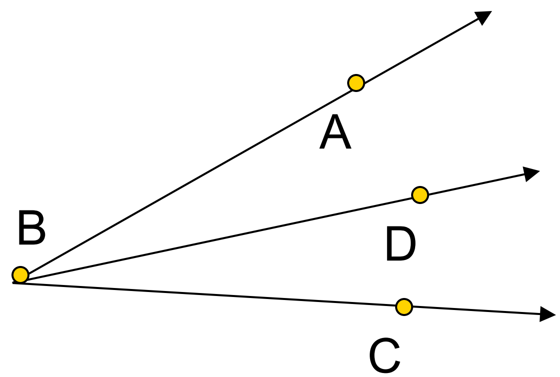
Distance from a point to a line

- Definition: Let $\overleftrightarrow{AB}$ be a line with C a point not on $\overleftrightarrow{AB}$. If D is the point on $\overleftrightarrow{AB}$ such that $\overleftrightarrow{CD} \perp \overleftrightarrow{AB}$, the <u>distance from C to $\overleftrightarrow{AB}$</u> is CD, the length of $\overline{CD}$.



Angle Bisector

- Let <ABC be an angle. To bisect <ABC is to identify $\overrightarrow{BD}$ where D is in the interior of <ABC and m<ABD = m<DBC, $\overrightarrow{BD}$ is called the angle bisector of <ABC.



- Post. 1.19 Each angle has exactly one bisector.

Homework

- Study all definitions, theorems, postulates and corollaries.
- Read Chapter 1
- Read Chapters 2 (2.1 to 2.5) for next class
- Read Chapters 3.1 for next class