GAM 0702

Day 5

Quiz: From last lesson…

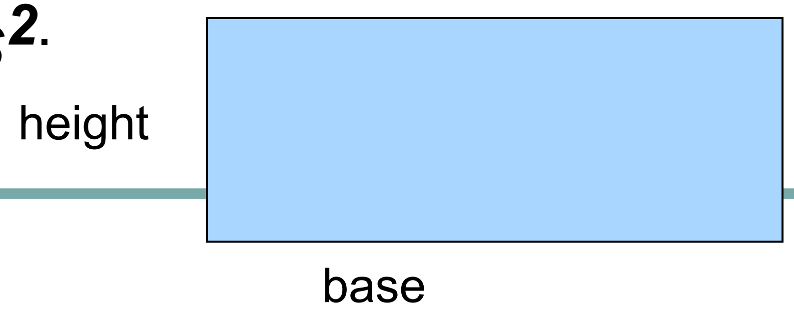
- What is the measure of a central angle with respect to its intercepted arc?
- What is the measure of an inscribed angle with respect to its intercepted arc?
- What is the geometric mean between 4 and 9?
- What is true about the corresponding sides of similar polygons?
- How to prove that two triangles similar?
- Are congruent triangles similar?

Chapter 7

Areas of Polygons and Circles

7.1 Areas of Quadrilaterals

- Obj.- To apply area formulas of rectangles and triangles.
- Post 7.1- The area of a rectangle with base *b* and height *h* is determined with the formula $\boldsymbol{A=bh}$.
- Corollary 7.1 The area of a square with sides of length *s* is determined by the formula $\boldsymbol{A= s^2}$.



Postulates relating to area

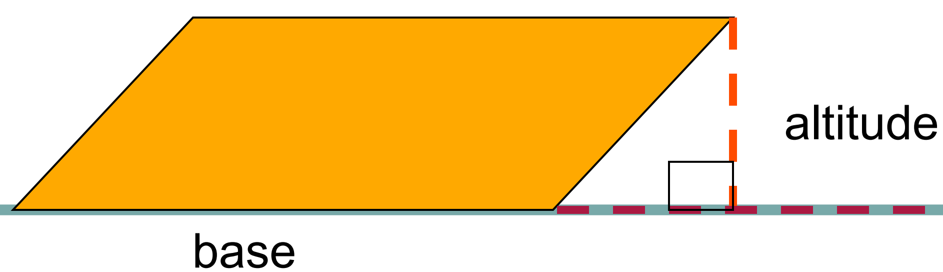
- Post. 7.2 Additive Property of Areas

  If lines divide a given area into several smaller non-overlapping areas, the given area is the sum of the smaller areas.

- Post 7.3 Two congruent polygons have the same area.

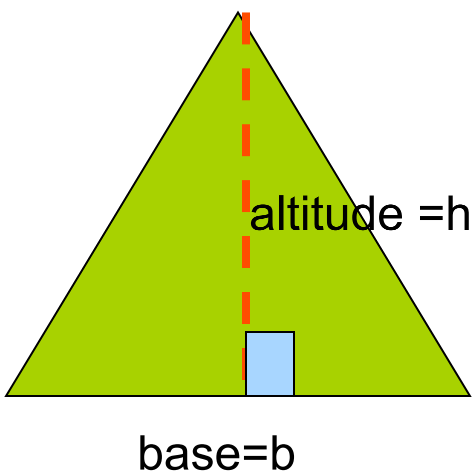
Parallelograms

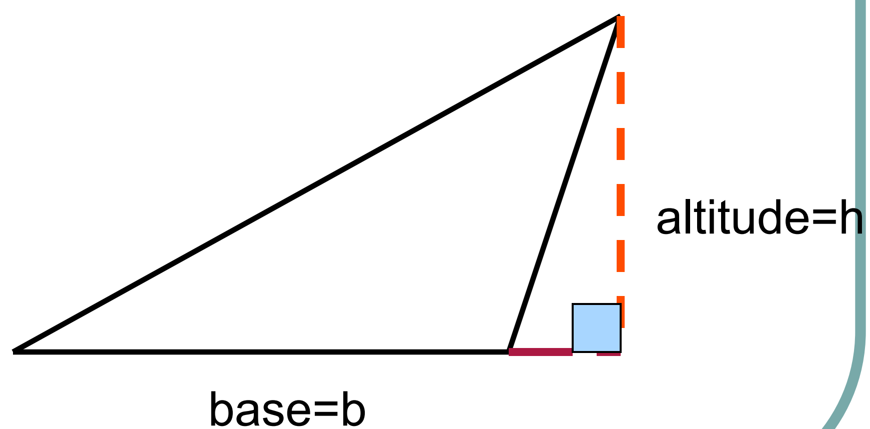
- An <u>altitude</u> of a parallelogram is a segment from a vertex perpendicular to a non-adjacent side. The length of an altitude is called the <u>height</u> of the parallelogram and the side to which it is drawn is called the <u>base</u>.
- Th. 7.2 The area of a parallelogram with base length *b* and height *h* is ***A=bh.***



Triangles

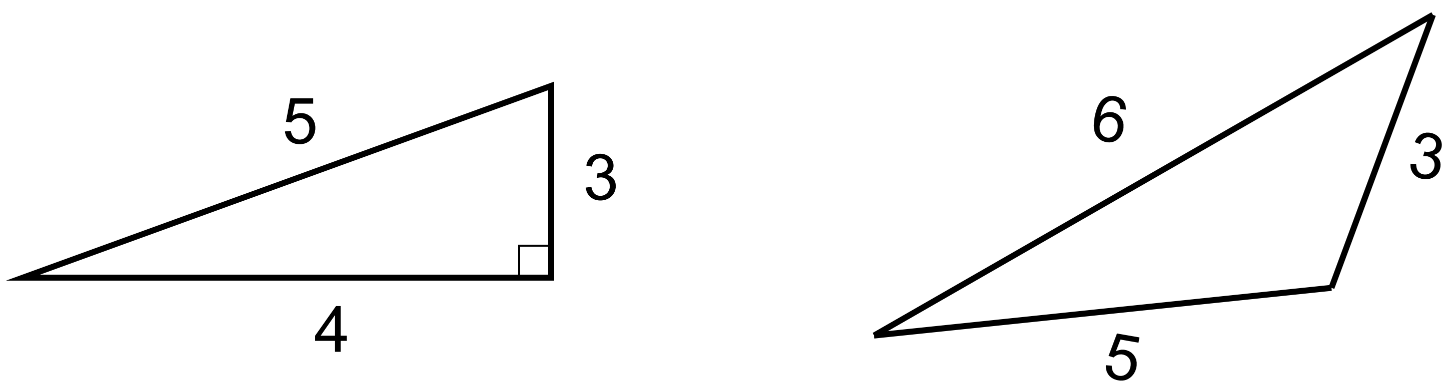
- The area of a triangle with length of base *b* and height *h* is determined with the formula ***A=½bh***.





Heron’s formula

- Find areas of the following two triangles



Heron's formula

- Used for triangles where three side lengths are known but the altitude is not known.
- The <u>semiperimeter (s)</u> is half the perimeter.
- Th. 7.4 If the three sides of a triangle have lengths a, b, and c, the area is

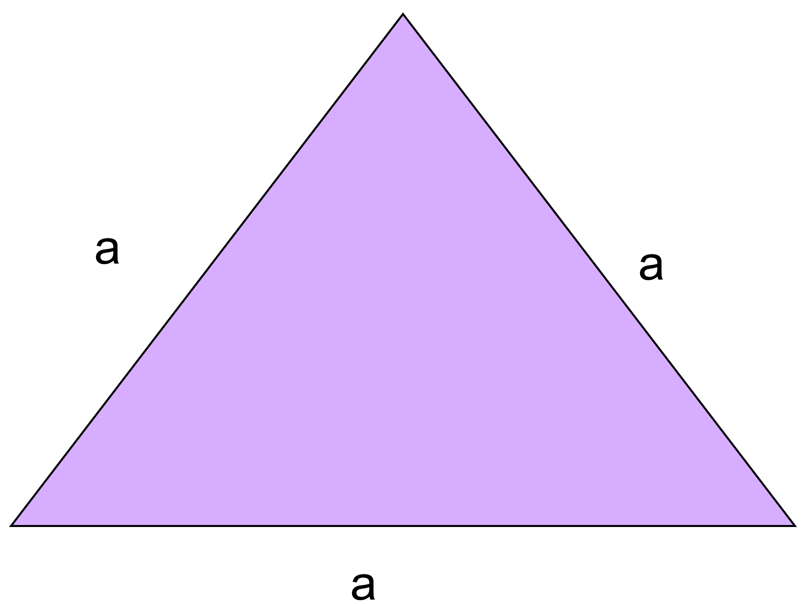
$$A=\sqrt{s(s-a)(s-b)(s-c)}$$

where $s=\dfrac{a+b+c}{2}$

Area of an Equilateral Triangle

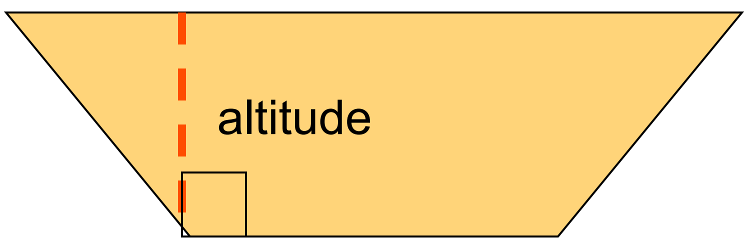
- Cor. 7.5 The area of an equilateral triangle with sides length a is

$$A = \frac{a^2\sqrt{3}}{4}$$



Trapezoid

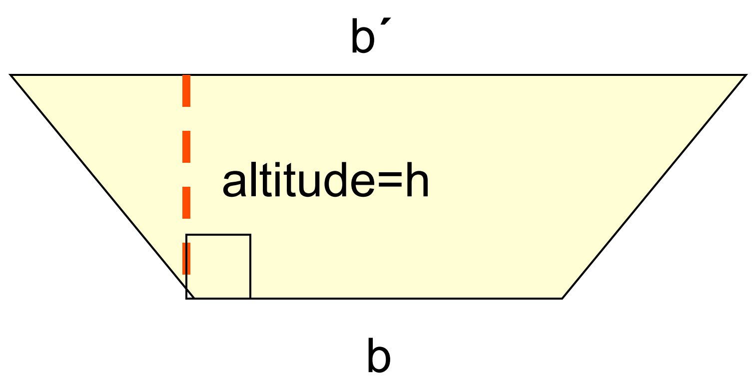
- Obj. To find areas of trapezoids
- The altitude of a trapezoid is the segment from a vertex of the trapezoid perpendicular to the nonadjacent base. The length of the altitude is the height.



Trapezoid Area Formula

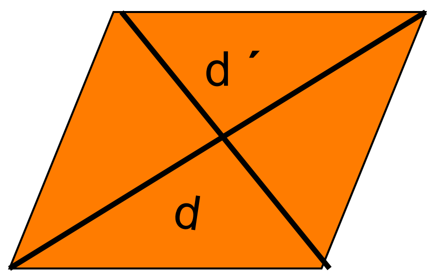
- Th. 7.6 The area of a trapezoid with length of bases b and b´ and height h is determined with the formula

$$\mathbf{A = \tfrac{1}{2}\mathit{h}(\mathit{b} + b')}$$



Rhombus Area Formula

- Th. 7.7 The area of a rhombus with diagonals of length d and d´ is determined by the formula **A= *½dd ´***



The proof is on 354.

- **You may create a reference card to use on the test of the formula summary on page 351 and 420.**

Circles

- Obj.-To find circumference and areas of circles.

- The circumference of a circle is the distance around the circle.
- The ratio of the circumference of a circle to the diameter is π.

$$\frac{C}{d} = \pi$$

- A French software engineer Fabrice Bellard took 131 days on 01/08/2010 calculated π to 2,699,999,990,000-digit
- Nearly 2.7 trillion decimal places
- The 131 days comprise
  - 103 days for the computation in binary digits,
  - 13 days for verification,
  - 12 days to convert the binary digits to a base of 10
  - 3 final days to check the conversion.

π with 835 digits

- π ≈ 3.
1415926535897932384626433832795028841971693993751058
2097494459230781640628620899862803482534211706798214
8086513282306647093844609550582231725359408128481117
4502841027019385211055596446229489549303819644288109
7566593344612847564823378678316527120190914564856692
3460348610454326648213393607260249141273724587006606
3155881748815209209628292540917153643678925903600113
3053054882046652138414695194151160943305727036575959
1953092186117381932611793105118548074462379962749567
3518857527248912279381830119491298336733624406566430
8602139494639522473719070217986094370277053921717629
3176752384674818467669405132000568127145263560827785
7713427577896091736371787214684409012249534301465495
8537105079227968925892354201995611212902196086403441
8159813629774771309960518707211349999998372978049951
0597317328160963185950244594553469083026425223082533
44…..

Circle Formulas

- Post. 7.4 The circumference of a circle with radius r and diameter d is determined with the formula

$$\mathbf{C=2\pi r \text{ or } C= \pi d}$$

- Post 7.5 The area of a circle with radius r is determined with the formula

$$\mathbf{A= \pi r^2}$$

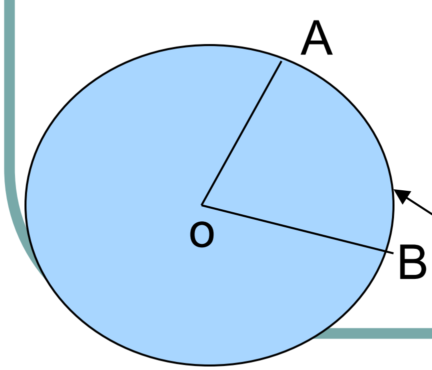
Area of a Sector

- Obj.-To find area and arc length of sectors.
- A sector of a circle is a region bounded by two radii of the circle and the arc of the circle determined by the radii.
  - Post.7.6 The area of a sector of a circle with radius r whose arc has a measure of m˚ is determined with the formula

$$A = \frac{m \pi r^2}{360}$$

Arc Length of a Sector

- A <u>sector</u> is a piece of a circle. The arc length is the length of the boundary of the sector.
- Post7.7 The length of an arc measuring m˚ in a circle with radius r is determined with the formula.

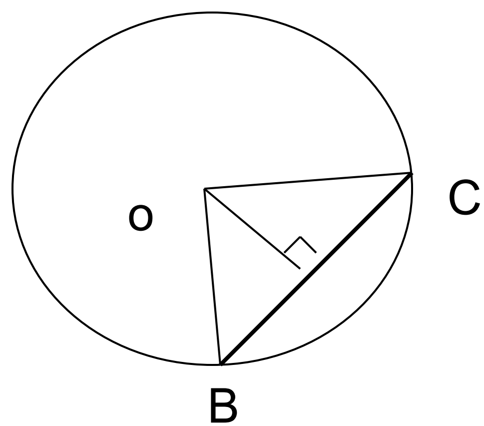


$$L = \frac{m(2\pi r)}{360} \text{ or } \frac{m\pi r}{180}$$

Measurement of arc AB = m°

Area of a Segment of a Circle

- A <u>segment</u> of a circle is a region bounded by a chord of the circle and the arc formed by the chord.
- How do you find the area?



Chapter 8

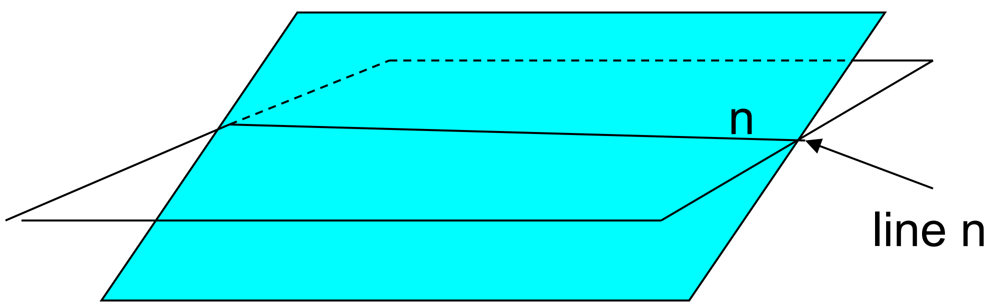
Solid Geometry

Solid Geometry

- Obj.-Determine the behavior of lines and planes in space.
- A line is <u>parallel to a plane</u> if it does not intersect the plane.
- A line is <u>perpendicular to a plane</u> if each line in the plane that passes through the point of intersection is perpendicular to the line.

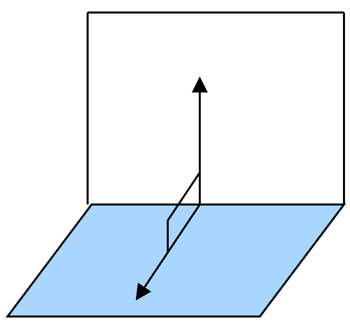
Planes in Space

- Two <u>planes are parallel</u> if they do not intersect.
- Post. 8.1 The intersection of two planes is a line.



Planes

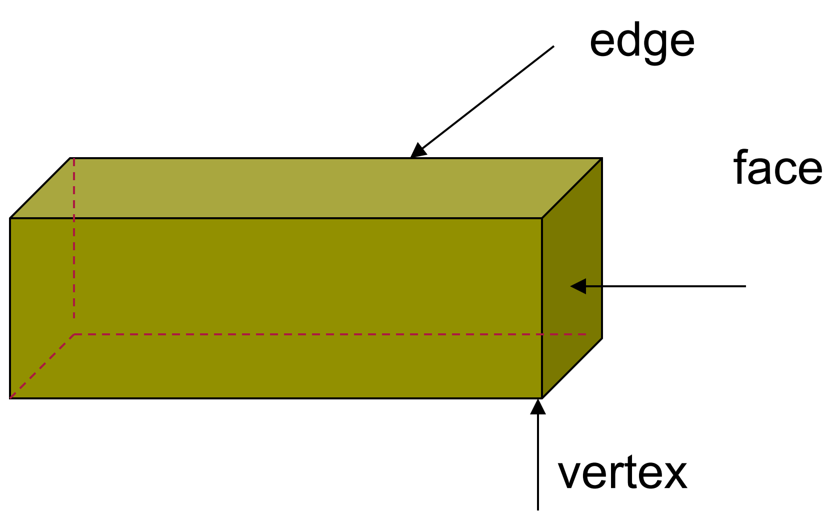
- Two planes are perpendicular if either plane contains a line that is perpendicular to the other plane.



- If two planes or a line and a plane intersect but are not perpendicular, they are called <u>oblique.</u>

Polyhedron

- A solid figure formed by the intersection of planes is called a polyhedron.



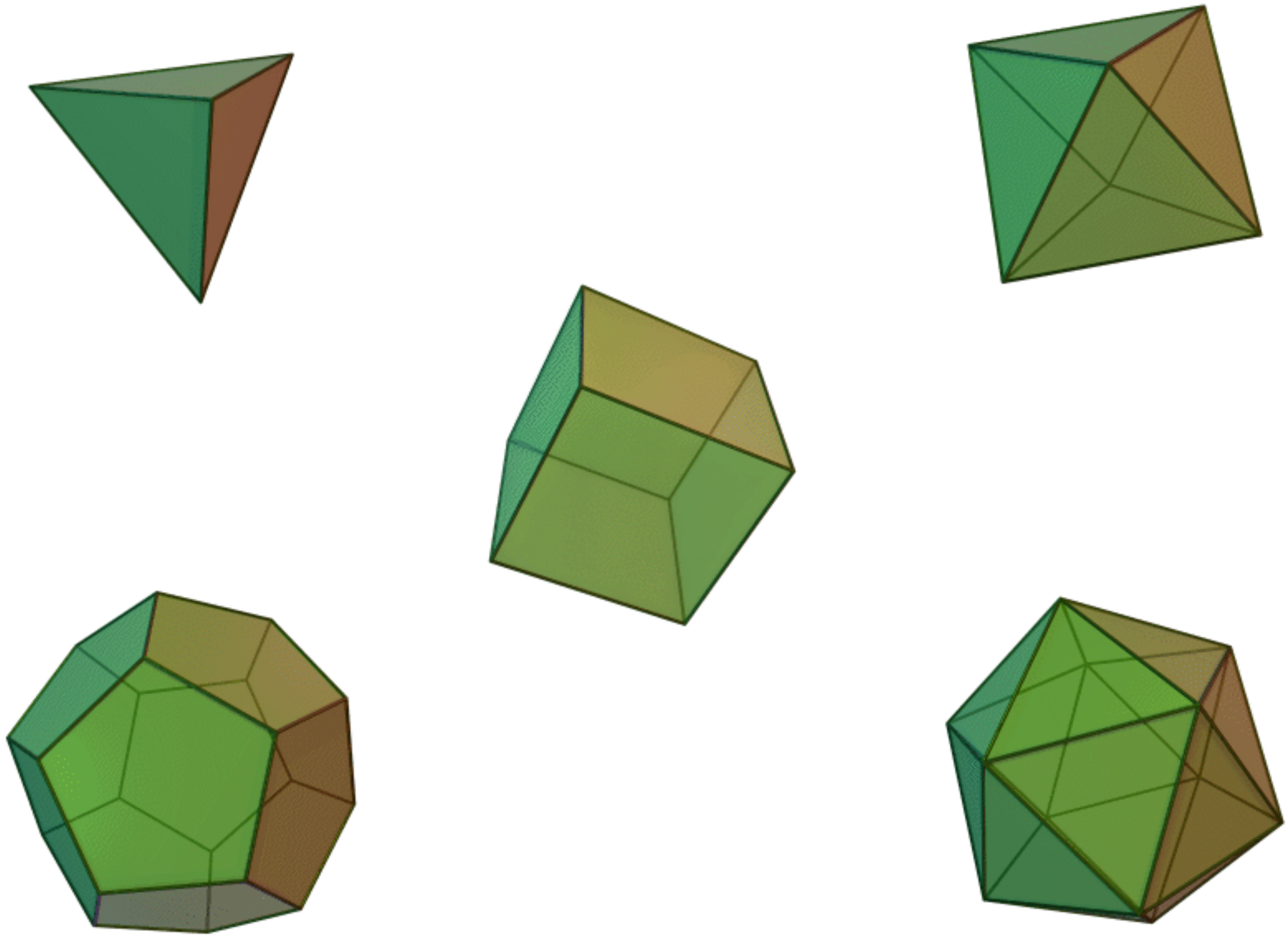
- A regular polyhedron is a solid figure in which all faces are congruent regular polygons.

Regular Polygon

- A polygon is a regular polygon if all its sides are congruent and all its angles are congruent.

Platonic Solids

- There are only five (P. 384) possible regular polyhedrons which are also called platonic solids.
- Tetrahedron (faces are equilateral triangles)
- Hexahedron (another name for a cube)
- Octahedron (faces are equilateral triangles)
- Dodecahedron (faces are regular pentagons)
- Icosahedrons (faces are equilateral triangles)



Euler's Equation ★

- In around 1750, Euler derived the well known formula, called **Euler's Formula**, to describe the relationships of vertices, faces, and edges of any polyhedrons:

$$V + F - E = 2$$

Where V is the number of vertices, F is the number of faces, and E is the number of edges.

Example

- Example 1. A cube is a polyhedron with $V = 8$, $F = 6$, and $E = 12$, and

$$V + F - E = 8 + 6 - 12 = 2$$

- The Euler formula is satisfied by the cube.

Example

- Example 2. Find the number of vertices of a polyhedron that has 12 faces and 30 edges.

$$V + F - E = 2$$

$$V + 12 - 30 = 2$$

$$V - 18 = 2$$

$$V = 2 + 18$$

$$V = 20$$

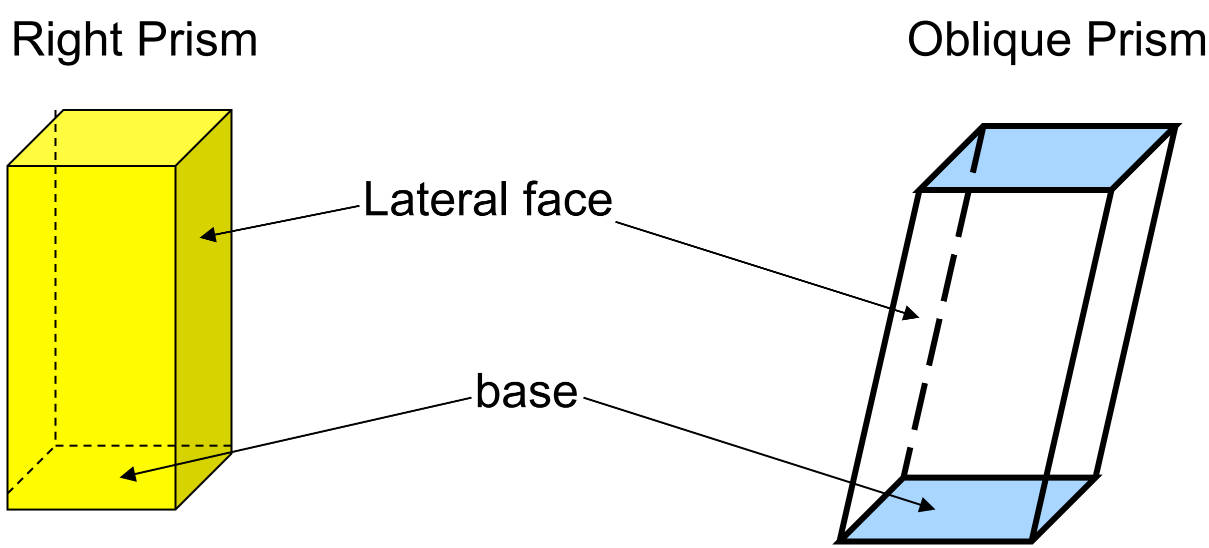
Thus, the polyhedron has 20 vertices.

Prisms

- Obj.- To define prism, lateral surface area, surface area, and volume
- A solid figure formed by joining two parallel congruent polygonal regions with parallelograms (as sides) is called a prism.
- The polygonal regions are called bases and the other surfaces are lateral faces.

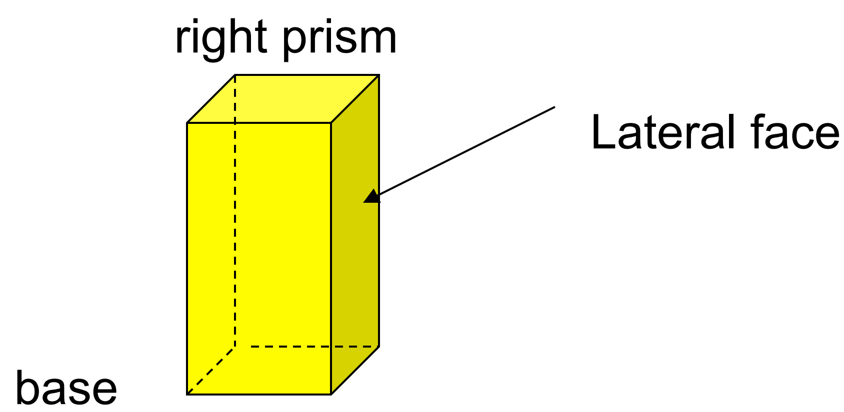
Right/Oblique Prism

- If the lateral faces of a prism are rectangles, then the prism is a right prism;
- otherwise it is an oblique prism.



Lateral Surface Area

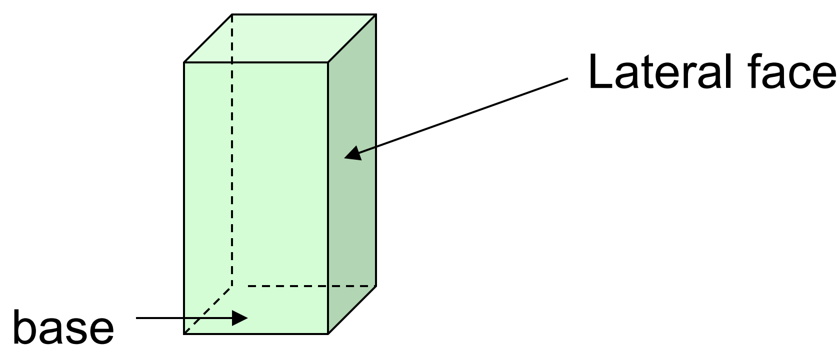
- The <u>lateral area</u> (lateral surface area) LA of a right prism is determined with the formula **LA = ph** where p is the perimeter of the base and h is the height of the prism.



(Total) Surface Area

- Surface area is the sum of the areas of the surfaces of a solid.
- The <u>surface area</u> (SA) of the prism is the lateral surface area plus the area of the two bases.

SA = LA + 2B



Regular Prism

- A regular prism is a prism whose bases are regular polygons.

- A regular right prism is a prism whose bases are congruent regular polygons and the lateral faces are rectangles

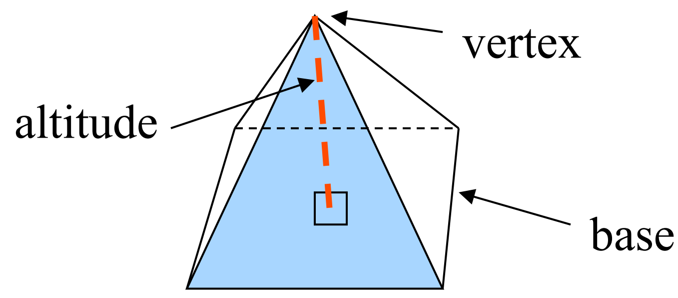
Volume of a Prism

- Volume is measured in cubic units. ex. cm^3

- Th. 8.2 The volume of a right prism is determined with the formula **V=Bh,** where B is the area of a base, and h is the height.

Pyramids

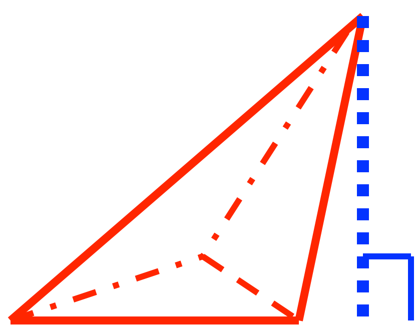
- The solid figure formed by connecting a polygon with a point not in the plane of the polygon is called a pyramid. The polygonal region is called the base and the point the vertex. The line segment from the vertex perpendicular to the plane of the base is the altitude.



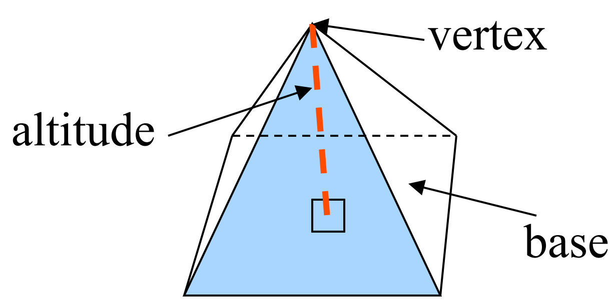
Pyramid

Types of Pyramids ★

- A <u>regular pyramid</u> has a regular polygon as a base, congruent isosceles triangles for the lateral surfaces. It's altitude passes through the center of the base.

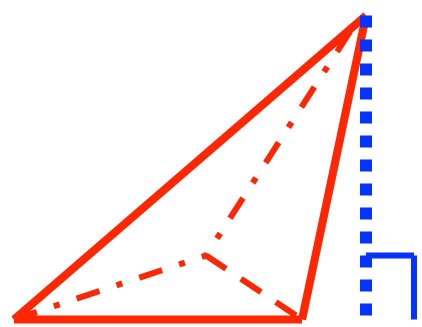


oblique pyramid



regular pyramid

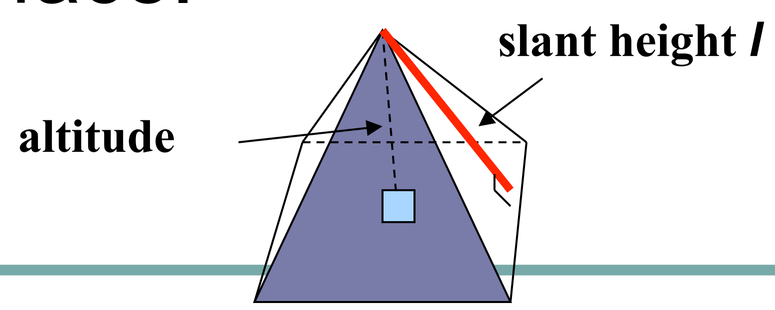
- An oblique pyramid's altitude does not pass through the center of the base.



oblique pyramid

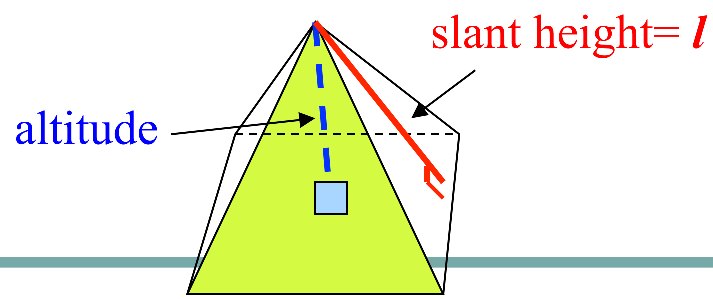
Parts of a Pyramid ★

- The distance ***l*** is called the slant height of the lateral surfaces of a regular pyramid. The distance ***l*** is also the height of the triangular face of the pyramid. Use the formula for area of a triangle (A=½bh) to find the area of each lateral face.



Lateral Surface Area of Pyramids

- Th. 8.3The lateral area of a regular pyramid is determined with the formula $LA=\frac{1}{2}p\boldsymbol{l}$ where p is the perimeter of the base and ***l*** is the slant height.
- The <u>surface area</u> of a regular pyramid is determined by the formula **SA= LA + B**

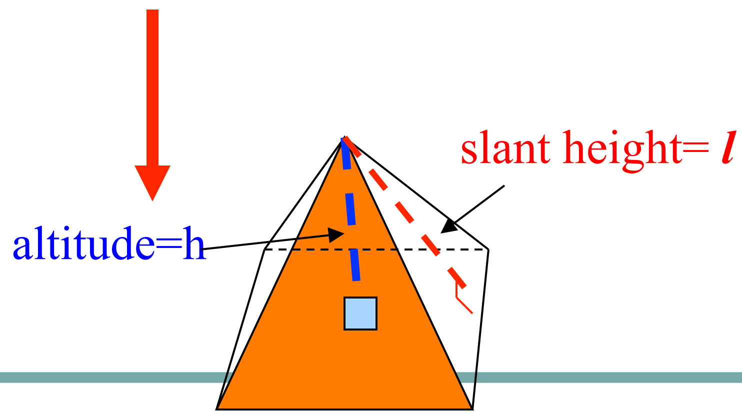


B is the area of the base.
p is the perimeter of the base.

Volume of Pyramids ★

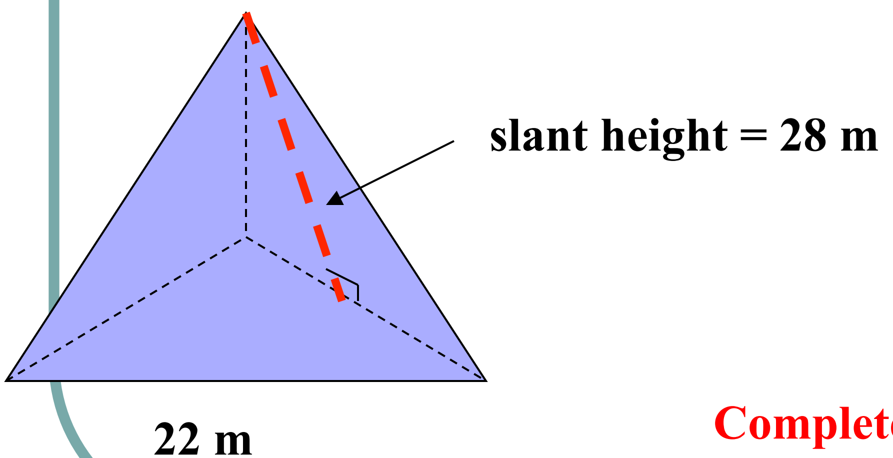
- Th. 8.4 The volume of a regular pyramid is determined with the formula $V=\frac{1}{3}Bh$ where B is the area of the base.

Note: To find the volume of a pyramid you **do not** use the slant height. You use the altitude of the solid.



Examples and problems ★

- Find the lateral surface area and surface area of a regular pyramid of an equilateral triangle base with side 22m; slant height 28 m.



Complete problem 5 on p. 401.

Solution ★

- SA = area of the base equilateral triangle
  + area of lateral surfaces of 3 triangles

Use Heron's formula for base area

$B = ((33)\ (11)\ \ (11)\ \ (11))^{1/2} = 11^2\ (3)^{½}$

$= (121)\ \ (1.732) = 209.57\ m^2$

Lateral surface area of 3 triangles

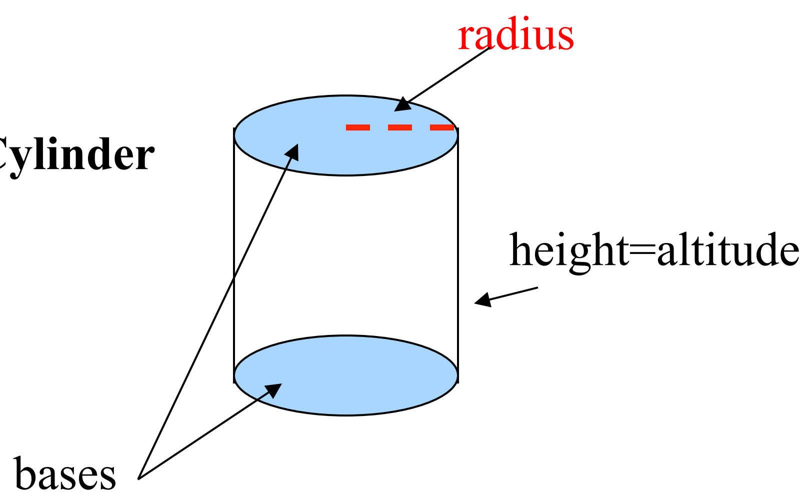
$LA = 3\ (½\ (22)\ (28)) = 3\ (308) = 924\ m^2$

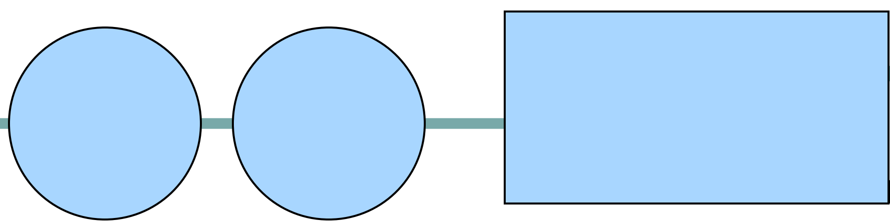
$SA = LA + B = 924 + 209.57 = 1133.57\ m^2$

Cylinders

- The solid figure formed by joining two congruent circles in parallel planes is called a <u>cylinder</u>.

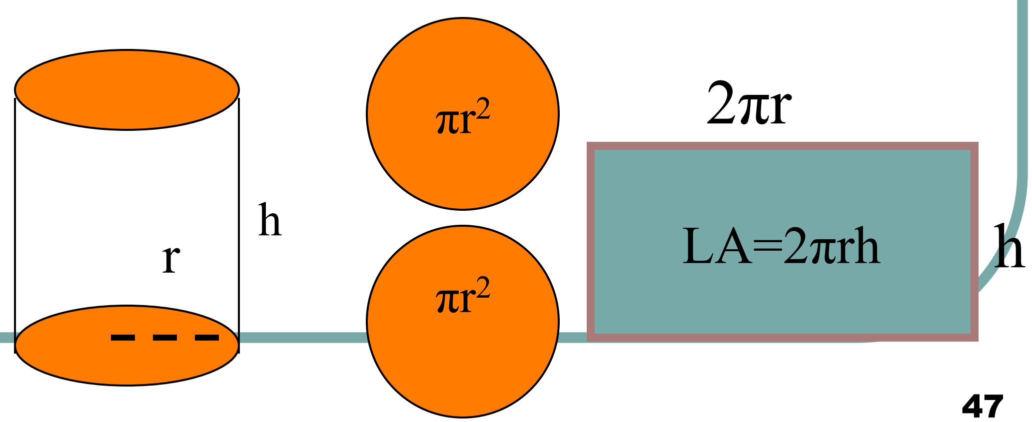
Right Circular Cylinder





Lateral Area of Cylinders

- Th. 8.5 The <u>lateral area</u> of a right circular cylinder is determined with the formula LA= $2\pi rh$ where r is the radius of a base and h is the height, the length of the altitude. Add the area of the bases to find the total surface area.
- SA= $2\pi rh + 2\pi r^2$

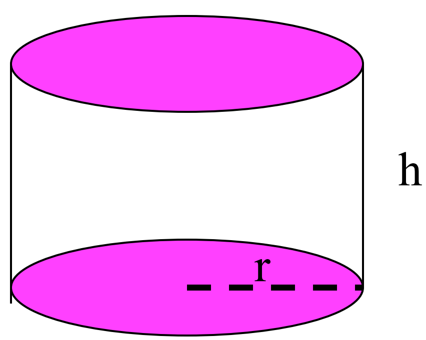


Volume of a Cylinder

- Th. 8.6 The volume of a right circular cylinder is determined with the formula

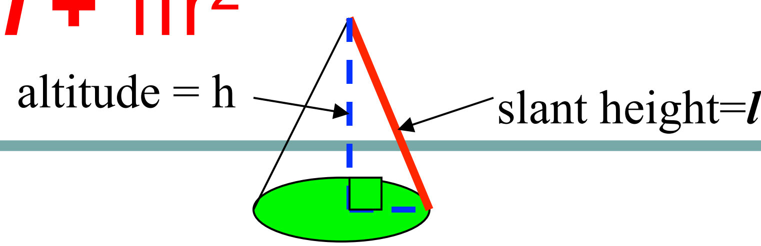
$$V = \pi r^2 h = Bh$$

where r is the radius and h is the height.



Cones

- The solid figure formed by connecting a circle with a point (vertex) not in the plane of the circle is called a <u>cone</u>.
- Th. 8.7 The <u>lateral area</u> of a right cone is determined with the formula $LA=\pi r\boldsymbol{l}$ where r is the radius of the base and ***l*** is the slant height.
- The formula for the surface area of a cone is $SA= \pi r\boldsymbol{l} + \pi r^2$

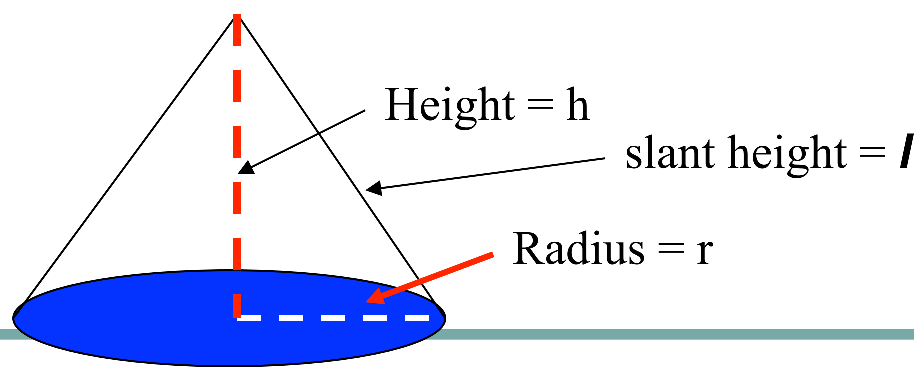


Volume of a Cone

- Th. 8.8 The volume of a right circular cone is determined with the formula

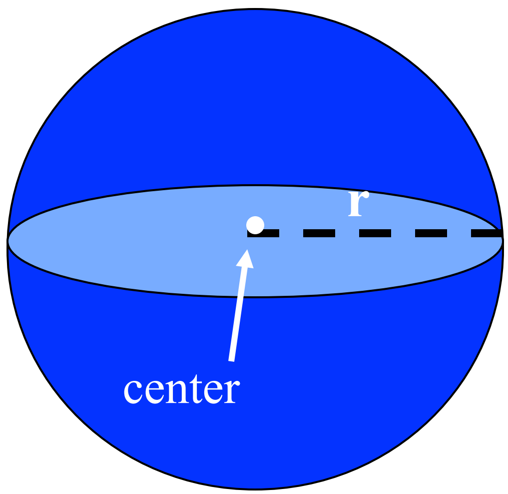
$$V = \tfrac{1}{3} Bh \text{ or } \tfrac{1}{3} \pi r^2 h$$

where r is the radius of the base, B is the area of the base, and h is the height of the cone.



Spheres

- A sphere is the set of all points in space a given distance called the radius, from a given point, called the center.



$SA=4\pi r^2$, is the surface area

And

$V= (4/3)\ \pi r^3$, is the volume formula

- A hemisphere is half a sphere.

Examples

- 1. A cube and a sphere have a surface area of 150 sq. in. Find the volume of each and determine which has the larger volume.

Solution

A cube has 6 congruent faces, each face is a square

let the area of each face be s^2

Then $6\ s^2 = 150 \Rightarrow s^2 = 25$, $s = 5$

The volume of a cube is $s^3 = 5^3 = 125\ in^3$

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A sphere has an area of $4\pi r^2 = 150 \Rightarrow r^2 = 150 / 4\pi = 11.942$

$r = 3.46$

A sphere has a volume $V = (4/3)\ \pi r^3 = 4\ \pi r^3/3$

$= 4\ \pi r^2\ r/3 = 150\ (3.46/3) = 150\ (1.15) = 172.5\ in^3$

++++++++++++++++++++++++++++++++++++++

=>A sphere has more volume then a cube if constructed with the same amount of material

Examples

2. A chemical-storage tank is a cylinder with a hemisphere cap on each end. If the height of the cylindrical portion is 16 ft and the radius of the cylinder and hemispheres is 3 ft, how many cubic feet of a chemical will the tank hold?

Assignments

- Read the following chapters
  - Chapter 7.1 to 7.3
  - Chapter 8.1 to 8.2, 8.4 to 8.5

For next lesson

- Read the following chapters for next class
  - Read chapter 9
  - Study for final which will be held the last day of class.

- End of Lesson 5