GAM Day 3

Quadrilaterals and Similar Polygons

Quiz: From the last lesson

1. What are the 3 ways of proving triangles congruent?
2. What is a median, an altitude, and an angle bisector?
3. What is CPCTC? When is CPCTC used?
4. What is concurrent?
5. What is a transversal?
6. How can you prove that two lines are parallel?
7. What is the sum of interior angles of a 10-side polygon

Answer

1. SAS, ASA, and SSS
2. CPCTC: Corresponding Parts of Congruent Triangles are Congruent
3. CPCTC is used after you have proved two triangles are congruent
4. Two or more lines are concurrent if they intersect in one and only one point.

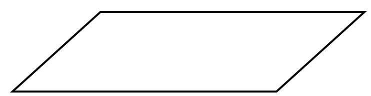
Answer

5. A transversal is a line that intersects two or more distinct lines in different points.
6. Alternate interior/exterior angles are congruent
7. S = (10-2) 180 = 1440

Chapter 4

Quadrilaterals

4.1 Parallelograms

- A quadrilateral is a four sided polygon.
- A parallelogram is a quadrilateral whose opposite sides are parallel.
- The symbol  represents the word parallelogram.

4.1 Parallelograms

- Th. 4.1 Each diagonal divides a parallelogram into two congruent triangles.
- Cor. 4.2 The opposite sides and opposite angles of a parallelogram are congruent.
- Th. 4.3 Consecutive angles of a parallelogram are supplementary.

Theorems on Parallelograms

- Th. 4.4 The diagonals of a parallelogram bisect each other.

- Th. 4.5 If both pairs of opposite sides of a quadrilateral are congruent, then the quadrilateral is a parallelogram. (Use this to prove a quad. is a parallelogram.)

Theorems to use to prove a quad. is a parallelogram.

- Th. 4.6 If both pairs of opposite angles of a quadrilateral are congruent, then the quadrilateral is a parallelogram.
- Th. 4.7 If two opposite sides of a quadrilateral are congruent and parallel, then the quadrilateral is a parallelogram.
- Th. 4.8 If the diagonals of a quadrilateral bisect each other, then the quadrilateral is a parallelogram.

To prove a quadrilateral is a parallelogram:

1. Show both pair of opposite sides are parallel.
2. Show both pairs of opposite sides congruent.
3. Show both pairs of opposite angles congruent.
4. Show one pair of opposite sides congruent and parallel
5. Show diagonals bisect each other.

Summary-Properties of a Parallelogram

Parallelogram

- Opposite sides are congruent
- Opposite angles are congruent
- Diagonals bisect each other
- Consecutive angles between parallel sides are supplementary

4.2 Rhombus

- A rhombus is a parallelogram that has two equal adjacent congruent sides.
- Th. 4.9 All four sides of a rhombus are congruent.
- Th. 4.10 The diagonals of a rhombus are perpendicular.

Rhombus Theorems

- Th. 4.11 If the diagonals of a parallelogram are perpendicular, then the parallelogram is a rhombus.

- Th. 4.12 The diagonals of a rhombus bisect the angles of the rhombus.

Summary-Properties of a Rhombus

1. Opposite sides are parallel.
2. Diagonals divide it into two congruent Δ’s.
3. Opposite sides are congruent.
4. Opposite <‘s are congruent.
5. Consecutive <‘s are supplementary
6. Diagonals bisect each other.
7. All sides are congruent.
8. Diagonals are perpendicular.
9. Diagonals bisect angles of the rhombus.

4.3 Rectangles

- Def- A rectangle is a parallelogram with one right angle.
- Th. 4.15 All angles of a rectangle are right angles.

Theorems on Rectangles

- Th. 4.16 The diagonals of a rectangle are congruent.

- Th. 4.17 If the diagonals of a parallelogram are congruent, then the parallelogram is a rectangle.

Summary-Properties of a Rectangle

Rectangle

- All angles are congruent
- All angles are right angles
- The diagonals of a rectangle are congruent
- Has all the properties of a parallelogram

Squares

- A square is a rhombus with one right angle.
- A square has all the properties of a rhombus and a rectangle.
- Th. 4.18 Two parallel lines are always the same distance apart.

Summary-Properties of a Square

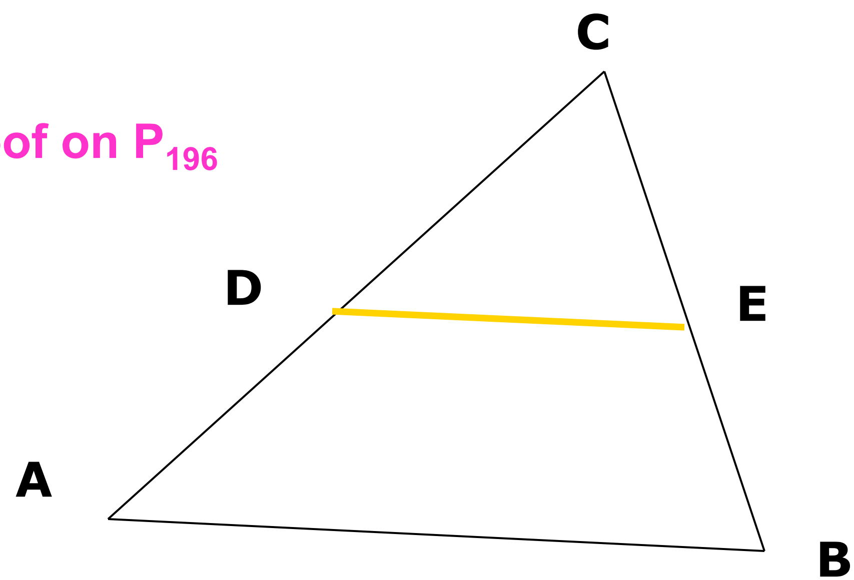
Square

- Has all the properties of the rhombus and rectangles

New Theorem on Triangles

- Th. 4.19 The segment joining the midpoints of two sides of a triangle is parallel to the third side and its length is one-half the length of the third side.

See Proof on P_{196}



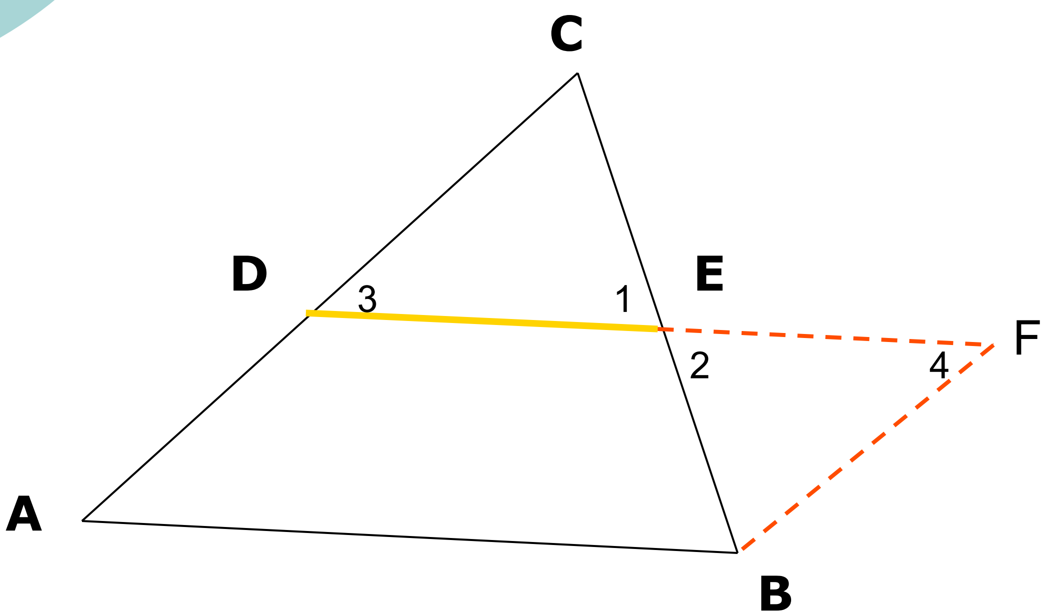
D is the midpoint of AC

E is the midpoint of CB

DE || AB

DE=½AB

Proof of Theorem 4.19



Extend $\overline{DE}$ to $\overline{DF}$ with DE = EF

Connect $\overline{BF}$

m <1 = m <2, DE = EF, CE = BE

$\Delta DCE \cong \Delta FBE$ (SAS)

m <3 = m <4 (CPCTC)
==> AD || BF
BF = DC = AD
one pair of opposite sides congruent and parallel ==> ADFB is a parallelogram

==> DF = AB

==> DE = ½ DF = ½ AB

4.4 Trapezoids

- A trapezoid is a quadrilateral with exactly one pair of parallel sides. The parallel sides are called bases and the nonparallel sides are called legs.

- A pair of angles of a trapezoid are called base angles if they include the same base.

Isosceles Trapezoid

- If the legs of a trapezoid are congruent, the trapezoid is an isosceles trapezoid.

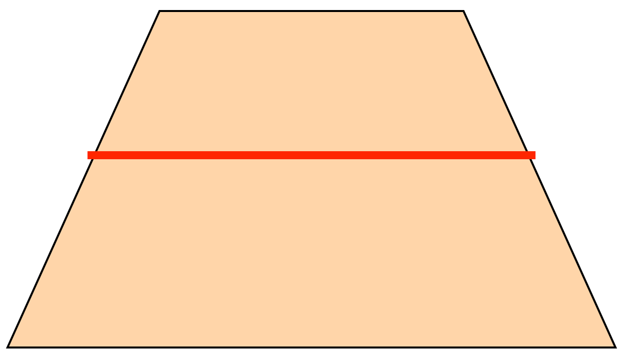
Theorems on Trapezoids

- Th. 4.20 The base angles of an isosceles trapezoid are congruent.

- Th. 4.21 The diagonals of an isosceles trapezoid are congruent.

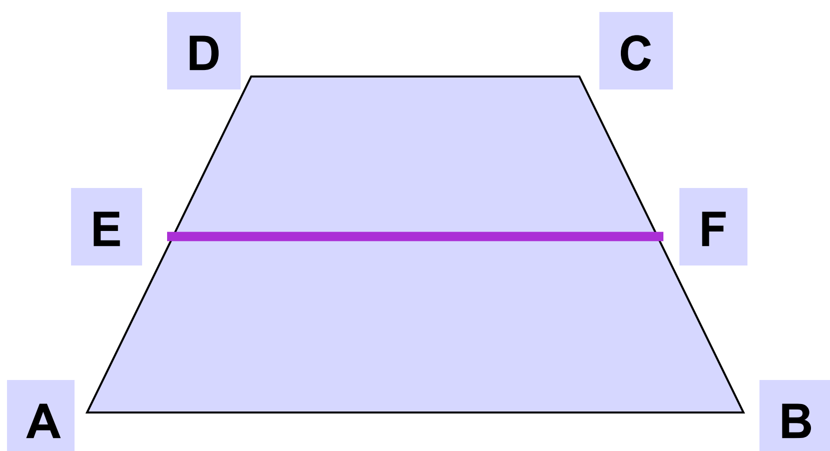
Median of a Trapezoid

- Def. The segment joining the midpoints of the legs of a trapezoid is the median of the trapezoid.



Theorems on Trapezoids

- Th.4.22 The median of a trapezoid is parallel to the bases and equal to one-half their sum.



E is the midpoint of AD
F is the midpoint of BC

Then

EF || DC || AB

EF = ½ (DC + AB)

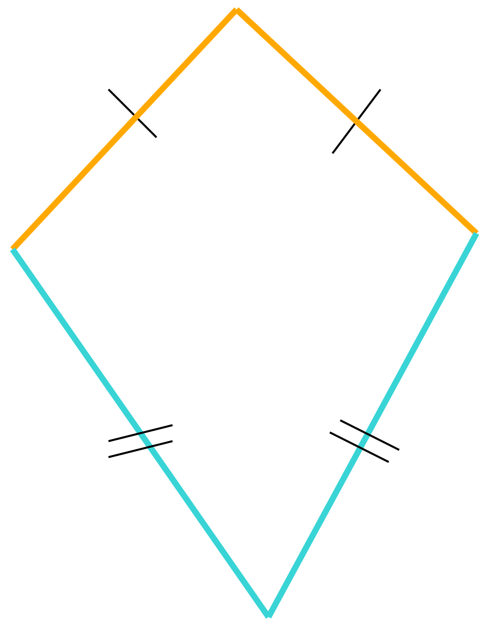
Summary-Properties of a Trapezoid

Trapezoid

- Consecutive angles between parallel sides are supplementary

Kite

- A kite is a quadrilateral with exactly two distinct pairs of congruent consecutive sides.
- A kite is not a parallelogram!



Theorems on Kites

- Th. 4.13 If a quadrilateral is a kite, one pair of opposite angles is congruent.

Theorems on Kites

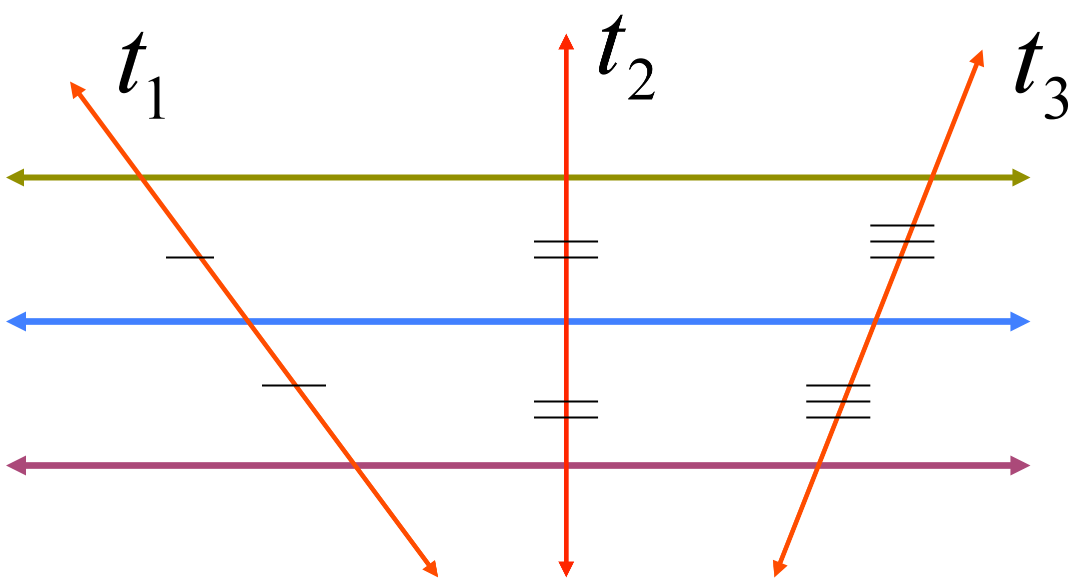
- Th. 4.14 If a quadrilateral is a kite, one diagonal is the perpendicular bisector of the other diagonal.

Summary-Properties of Kites

- 1. It is a quadrilateral but not a parallelogram with two pair of congruent, consecutive sides.
- 2. One pair of opposite angles are congruent.
- 3. One diagonal is the perpendicular bisector of the other diagonal.
- 4. One diagonal bisects two of the kite's angles.

Transversal

- Th. 4.23 If three or more parallel lines intercept congruent segments on one transversal, then they intercept congruent segments on all transversals.



Summary - Properties of Polygons

Trapezoid

- Consecutive angles between parallel sides are supplementary

Parallelogram

- Opposite sides are congruent
- Opposite angles are congruent
- Diagonals bisect each other
- Consecutive angles between parallel sides are supplementary

Properties of Polygons (Cont.)

Rhombus

- Diagonals are perpendicular to each other
- Diagonals bisect each other
- Diagonals bisect opposite angles
- Consecutive angles between parallel sides are supplementary
- Diagonals are perpendicular
- Diagonals bisects opposite angles

Properties of Polygons (cont.)

Rectangle

- All angles are congruent
- All angles are right angles
- Has all the properties of a parallelogram

Square

- Has all the properties of the rhombus and rectangles

Chapter 5

Similar Polygons and the Pythagorean Theorem

5.1 Ratio and Proportion

The ratio of one number *a* to another number *b*, $b \neq 0$, is the fraction *a*/*b*.

- The ratio of *a* to *b* is sometimes written as *a:b* and read "*a is to b*."
- $a : b = a/b$ (*where* $b \neq 0$)

Proportions

- An equation showing that two ratios are equal is called a proportion.

$$\frac{a}{b} = \frac{c}{d}$$ is a proportion (where $b \neq 0$, $d \neq 0$)

The number *a* and *d* are called the extremes and *b* and *c* are called the means.

- $a : b = c : d \iff \frac{a}{b} = \frac{c}{d}$

Solve the following problem

If $(x - 1) : 2 = 3x : 7$

what is x?

Means-Extremes Property

- Th. 5.1: In a proportion, the product of the means is equal to the product of the extremes.

$$\text{if} \quad \frac{a}{b} = \frac{c}{d}$$

Then ad = bc

====================

If a : b = c : d

Then ad = bc

Solve the following problem

If $(2x - 1) : (x + 2) = 5 : 3$

What is x?

Geometric Mean or Mean Proportional

- Definition: In the proportion a : b = b : c, b is called the geometric mean or mean proportional between a and c.

- If b is the geometric mean between a and c, then $b^2 = ac$

- Since 4:10 = 10:25, 10 is the geometric mean between 4 and 25

Solve the following problem

1. What is the geometric mean between 25 and 49?

2. What is the arithmetic mean between 25 and 49

Reciprocal Property of Proportions

Th 5.2:

If $$\frac{a}{b} = \frac{c}{d}$$

Then $$\frac{b}{a} = \frac{d}{c}$$

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If a : b = c : d, then b : a = d : c

Means Property of Proportions

Th: 5.3: If

$$\frac{a}{b} = \frac{c}{d}$$

Then

$$\frac{a}{c} = \frac{b}{d}$$

====================

If $a : b = c : d$

Then $a : c = b : d$

Extremes Property of Proportions

Th 5.4: If

$$\frac{a}{b} = \frac{c}{d}$$

Then

$$\frac{d}{b} = \frac{c}{a}$$

===================

If $a : b = c : d$

Then $d : b = c : a$

Addition Property of Proportions

Th 5.5: If

$$\frac{a}{b} = \frac{c}{d}$$

Then

$$\frac{a + b}{b} = \frac{c + d}{d}$$

Subtraction Property of Proportions

Th 5.6: If

$$\frac{a}{b} = \frac{c}{d}$$

Then

$$\frac{a - b}{b} = \frac{c - d}{d}$$

Extended ratios ★

- An extended ratio compares more than two quantities and is written as $a:b:c$.
- If a:b:c = p:q:r

  then a:p = b:q = c:r
- *Example: If the sides of a triangle are 8. 10 and 12 inches long then the sides are in the ratio of 4 to 5 to 6.*
  - *8:10:12 = 4:5:6 or*
  - *8:4 = 10:5 = 12:6*

Proportional segments

- If lengths of segments are proportional, the segments are called proportional segments.
- Segments $\overline{AB}$ and $\overline{CD}$ are proportional to segments $\overline{EF}$ and $\overline{GH}$ we have $\frac{AB}{CD} = \frac{EF}{GH}$

Complete 12-26 odd on p. 223.

Review

- Today you learned
  - Properties of Quadrilaterals
  - Ratio and proportions

Assignments

- Read Chapter 4 (omit kite)
- Read chapter 5.1

Assignments

- Read the following chapters for next class
  - Chapter 5.2 – 5.4
  - Chapter 6

END