GAM

Day 4

Pythagorean Theorem and Circles

Chapter 5

Similar Polygons and the Pythagorean Theorem

Review: From the last Lesson

1. Find the geometric mean between 25 and 144.
2. What is true about the median of a trapezoid?
3. Which quadrilateral has all the properties of a parallelogram, rhombus, and rectangle?
4. Is kite a parallelogram?
5. What is a ratio?
6. What is a proportion?

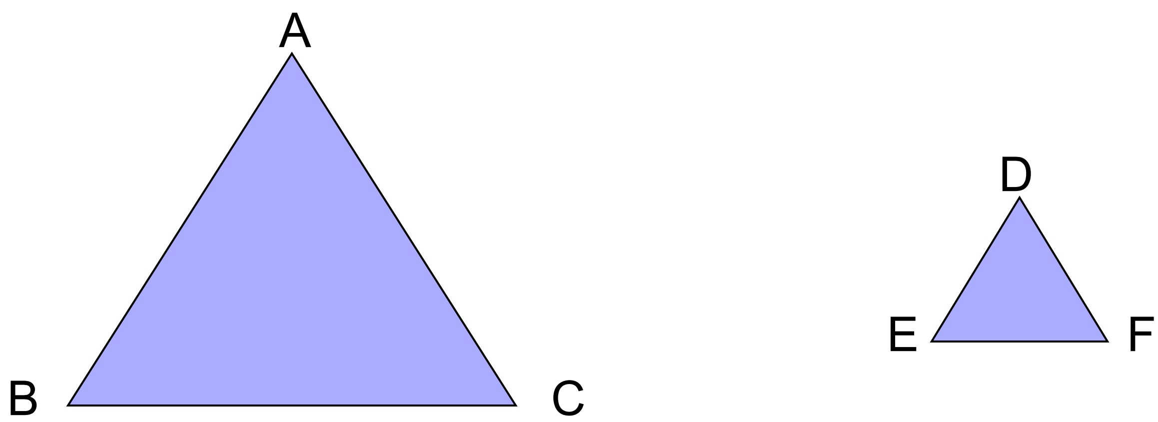
Answer

- 60
- =(base1 + base2) / 2
- Square
- No
- A ratio is a fraction
- An equation showing that two ratios are equal is called a proportion.

5.2 Similar Polygons

- Two polygons are similar if their vertices can be paired in such a way that
  - the corresponding angles are congruent and
  - the corresponding sides are proportional.

$\Delta ABC \sim \Delta DEF$



$\Delta ABC \sim \Delta DEF$ if and only if (<==>)

1. $\angle A \cong \angle D$, $\angle B \cong \angle E$, $\angle C \cong \angle F$

2. $\dfrac{AB}{DE} = \dfrac{AC}{DF} = \dfrac{BC}{EF}$

Similar Triangles

- Post. 5.1 AAA

  Two triangles are similar if three angles of one triangle are congruent to the corresponding three angles of the other triangle.

AA

- Th. 5.8 AA

Two triangles are similar if two angles of one triangle are congruent to the corresponding two angles of the other triangle.

- AA → AAA

Theorems involving similarity

- Th. 5.9: If $\Delta ABC \cong \Delta DEF$,

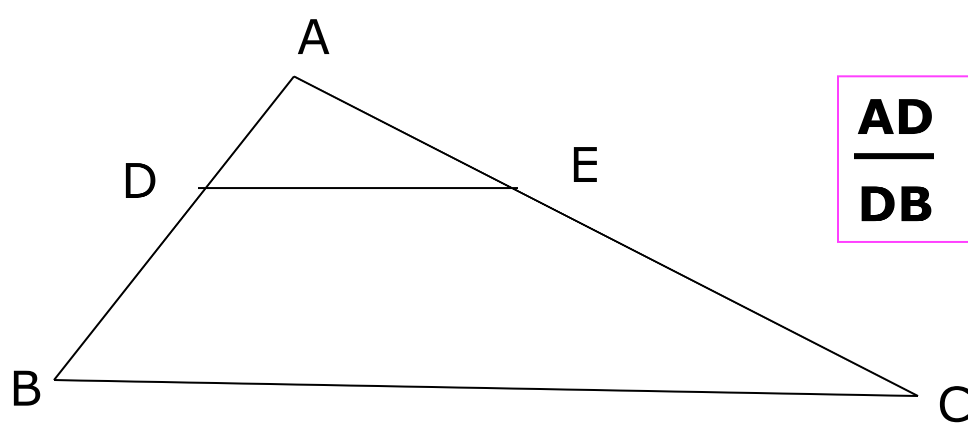
  then $\Delta ABC \sim \Delta DEF$

- Th. 5.10 Transitive Law for similar triangles:

  If $\Delta ABC \sim \Delta DEF$, and $\Delta DEF \sim \Delta GHI$, then $\Delta ABC \sim \Delta GHI$.

More theorems using similarity

- Th. 5.11 Triangle Proportionality Theorem A line parallel to one side of a triangle that intersects the other two sides divides the two sides into proportional segments.



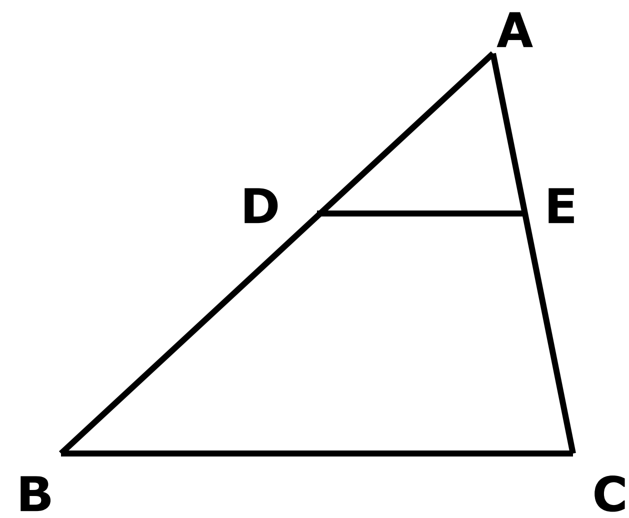
$$\frac{AD}{DB} = \frac{AE}{EC}$$

Example

$\overline{DE} \parallel \overline{BC}$.

AC= 9 yd, AB = 10 yd. AD=4 yd. Find AE and EC.

Hint: let AE=x



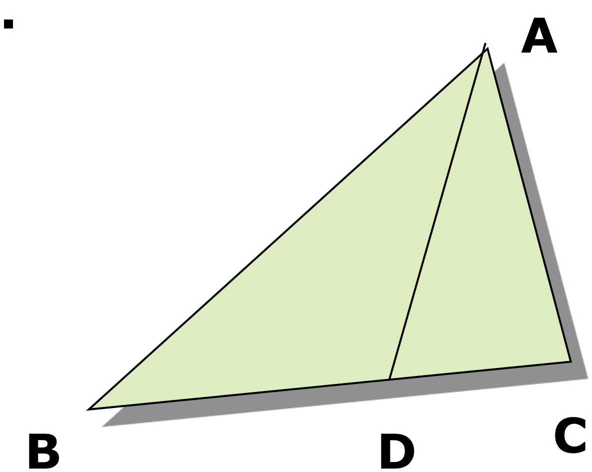
Triangle angle-Bisector Theorem ★

- Th. 5.12 The bisector of one angle of a triangle divides the opposite side into segment that are proportional to the other two sides.

$$\boxed{\frac{BD}{CD} = \frac{AB}{AC}}$$

Or

$$\boxed{\frac{BA}{BD} = \frac{CA}{CD}}$$



Proof of theorem 5.12 ★

$\overline{AD}$ is a bisector of < BAC

make $\overline{BE}$ || $\overline{AD}$, and extend $\overline{CA}$ to $\overline{CE}$

m<1 = m<3, m<2 = m<4

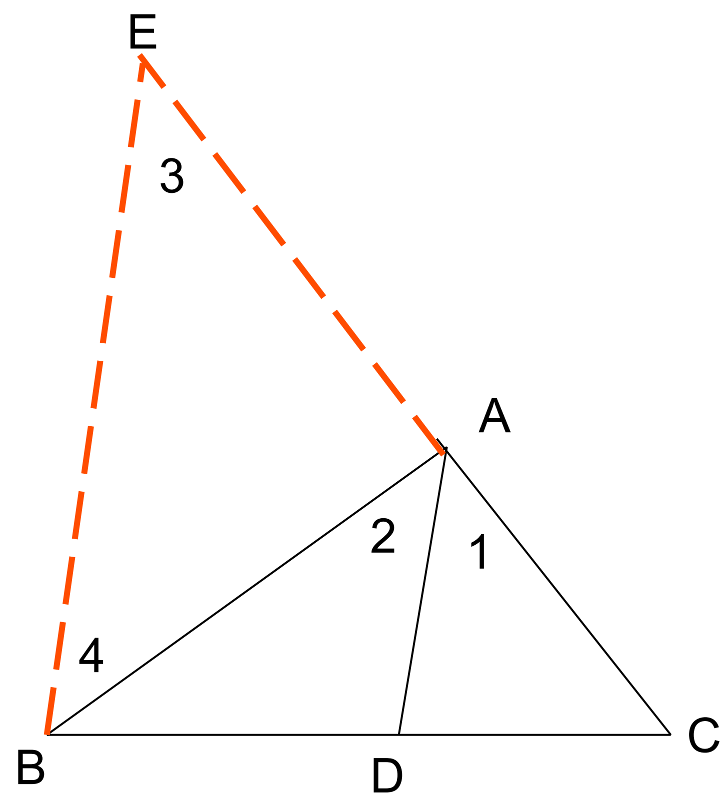
Since m<1 = m<2 ==> m<3 = m<4,

==> AE = AB

since AD || BE, in triangle CBE,

$$\frac{CA}{AE} = \frac{CD}{DB} \Longrightarrow \frac{CA}{CD} = \frac{AE}{DB}$$

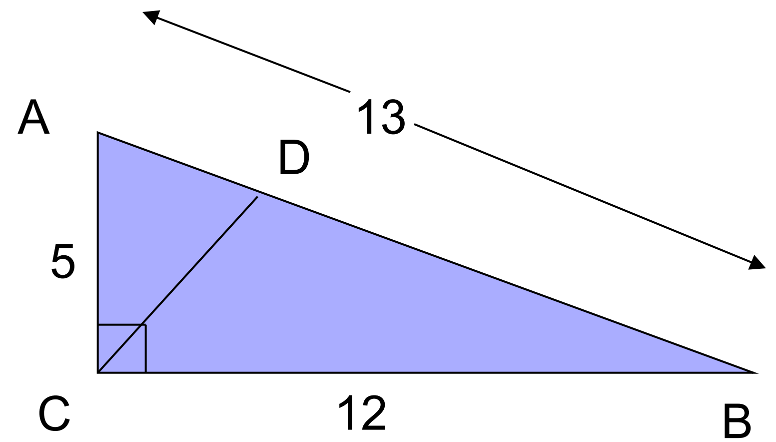
$$\Longrightarrow \frac{CA}{CD} = \frac{AB}{DB} \qquad \Longrightarrow \frac{CA}{CD} = \frac{BA}{BD}$$



Example

The right ΔABC has legs 5 ft and 12 ft and hypotenuse 13 ft. $\overline{CD}$ bisects right <C, Find AD.

Hint: let AD=x



Answer ★

AC : AD = BC : BD

Let AD = x, then BD = 13 – x

Given that AC = 5, BC = 12,

We have

$$5 : x = 12 : (13 - x)$$

$$12x = 5 * (13 - x)$$

$$12x = 65 - 5x$$

$$12x + 5x = 65$$

$$17x = 65$$

$$x = 3.8$$

$$BD = 9.2, AD = 3.8$$

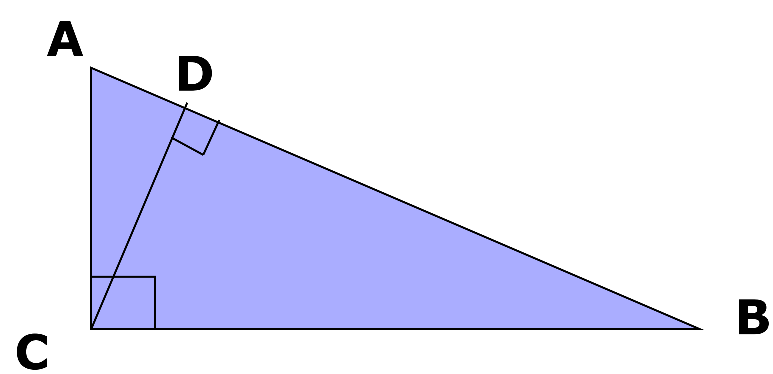
5.3 Properties of Right Triangles

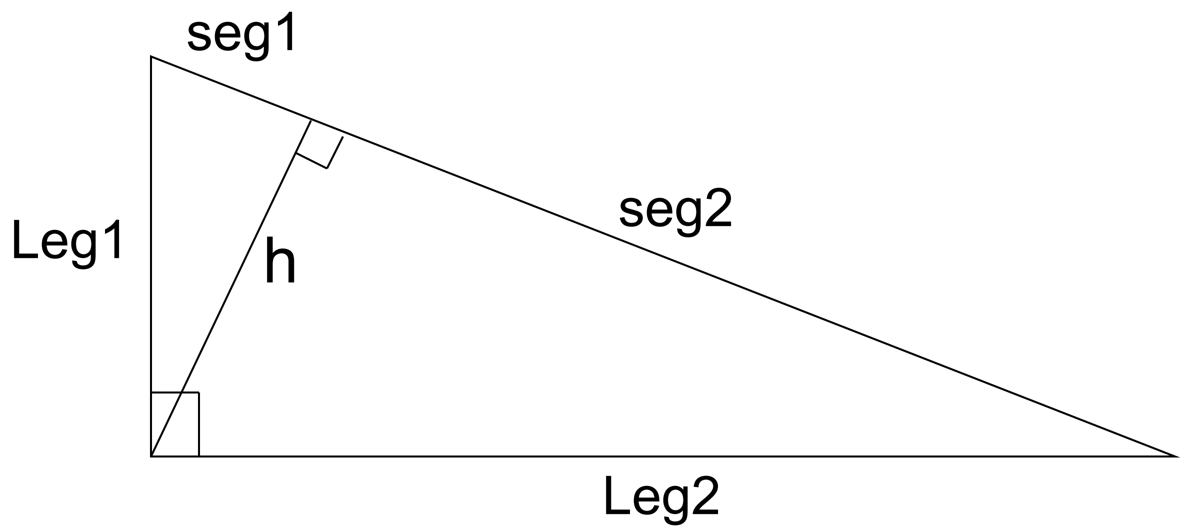
- Th. 5.13 The altitude from the right angle to the hypotenuse in a right triangle forms two right triangles that are similar to each other and to the original triangle.

ΔABC ~ ΔACD

ΔABC ~ ΔCBD

ΔACD ~ ΔCBD

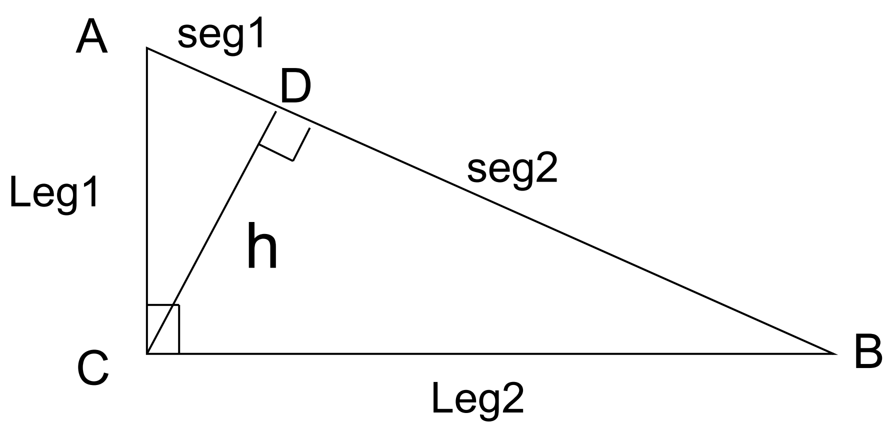




- $h^2 = (seg1)\ (seg2)$
- $Leg1^2 = seg1\ (seg1 + seg2)$
- $Leg2^2 = seg2\ (seg1 + seg2)$
- seg1 + seg2 = hypotenuse

Corollaries

- Cor. 5.14 The altitude h from the right angle to the hypotenuse in a right triangle is the geometric mean or mean proportional between the segments of the hypotenuse.



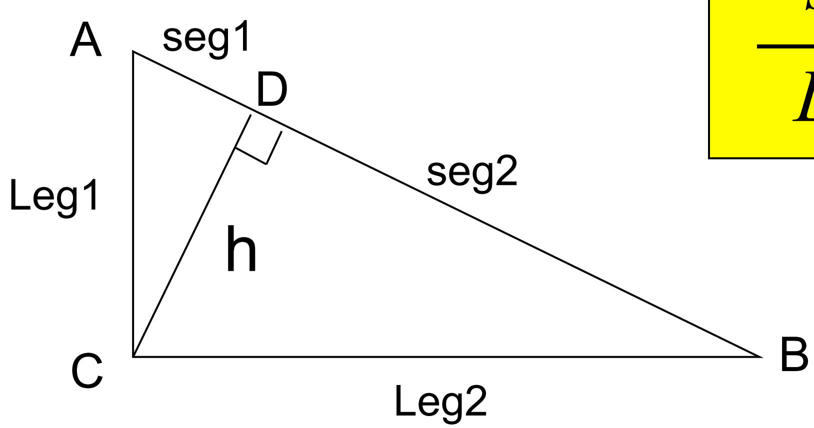
$$\Delta ACD \sim \Delta CBD$$

$$\frac{seg1}{h} = \frac{h}{seg2}$$

$$h^2 = (seg1)\,(seg2)$$

Corollaries

- Cor. 5.15 If the altitude is drawn from the right angle to the hypotenuse in a right triangle, then each leg is the geometric mean proportional between the hypotenuse and the segment of the hypotenuse adjacent to the leg. (p. 240)



ΔABC ~ ΔACD

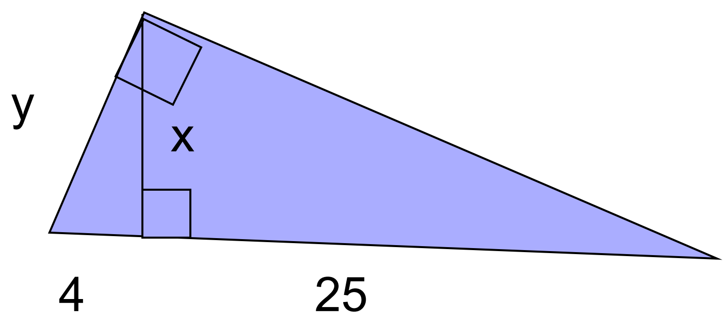
$$\frac{seg1}{Leg1} = \frac{Leg1}{seg1 + seg2}$$

ΔABC ~ ΔCBD

$$\frac{seg2}{Leg2} = \frac{Leg2}{seg1 + seg2}$$

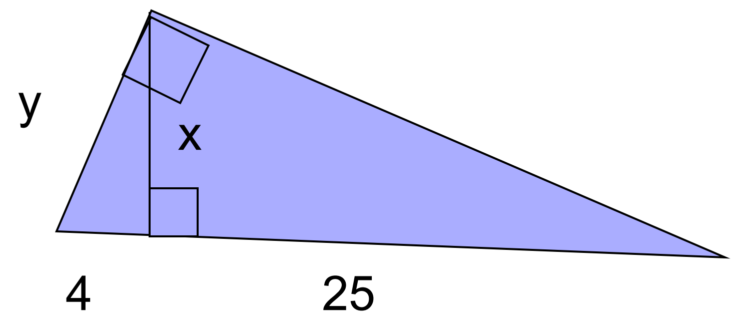
Example

- Find x and y.



Example

- Find x and y.



$x^2 = 4 * 25$

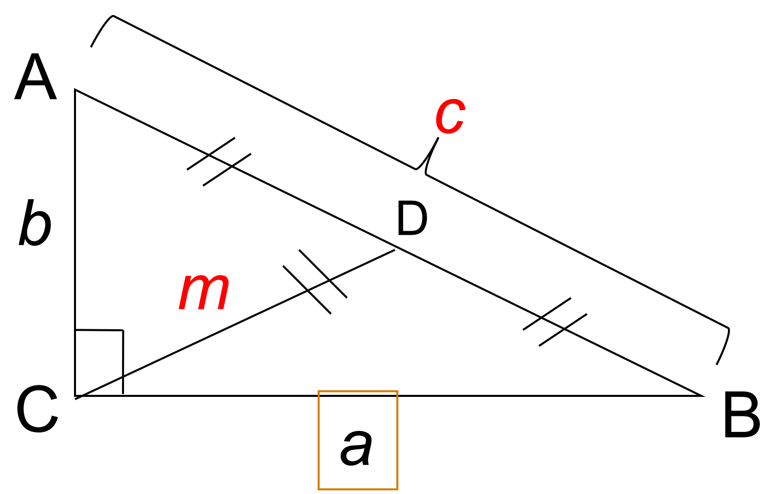
$y^2 = 4 * (4 + 25)$

Theorem

- Th. 5.16 The median from the right angle in a right triangle is one-half the length of the hypotenuse.

$$m = \frac{1}{2}c$$

$\boxed{CD = AD = BD}$



- Do problems on p. 243

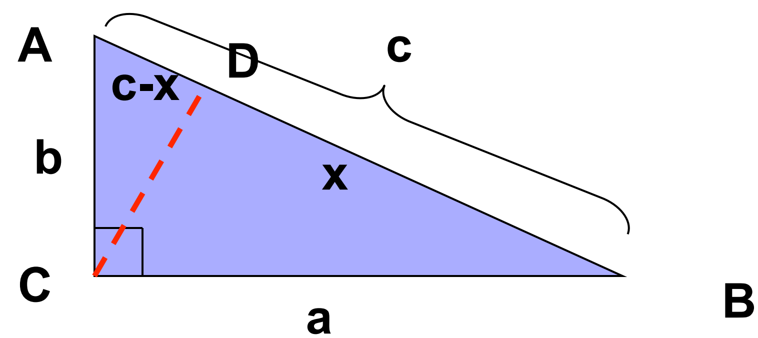
5.4 The Pythagorean Theorem

- Th. 5.17 The Pythagorean Theorem
  In a right triangle, the square of the length of the hypotenuse is equal to the sum of the squares of the lengths of the legs. (p. 244)

Proof of Pythagorean Theorem

Given: Rt. ΔABC

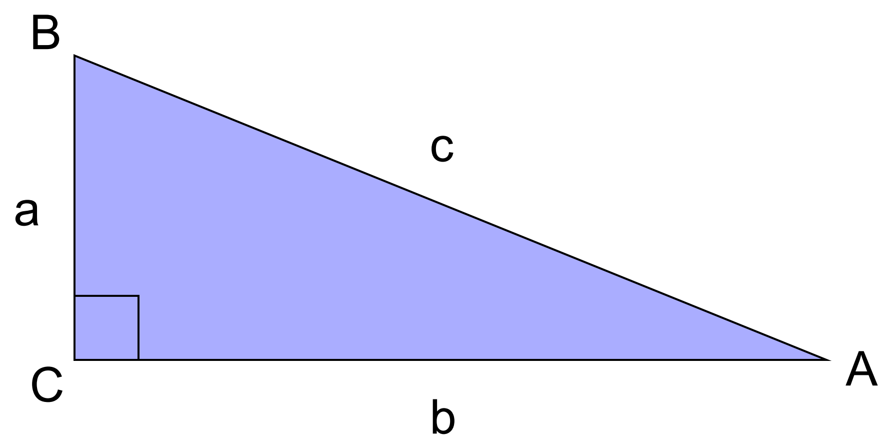
Prove: $a^2 + b^2 = c^2$



1. ΔABC is a right triangle	1.
2. $\overline{CD}$ is the altitude from $\angle C$ to AB	2. Auxiliary line-From a pt. not on a line one perpendicular line can be drawn to the line through the point.
3. $\frac{c}{a} = \frac{a}{x}$ and $\frac{c}{b} = \frac{b}{c-x}$	3.
4. $a^2 = cx$ and $b^2 = c(c-x)$	4.
5. $a^2 + b^2 = cx + c(c-x)$	5.
6. $a^2 + b^2 = c^2$	6.

Examples and problems

- In right triangle $\triangle ABC$, c=32 ft and a=18 ft. find b.



- Do problems 2-8 even on page 251.

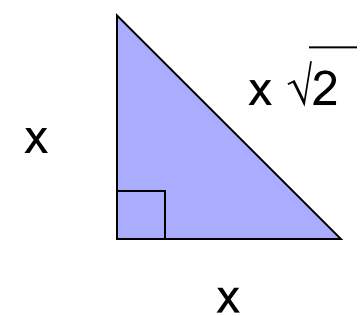
Converse of the Pythagorean Theorem

- Th. 5.18 If the sides of a triangle have lengths a, b and c and $a^2 + b^2 = c^2$, then the triangle is a right triangle.
- Do problems 10-14 even on p. 251.

Special Right Triangles

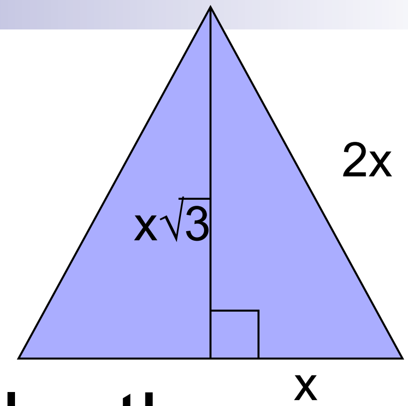
45°– 45°– 90° Theorem

- Th. 5.19 In a 45°– 45°– 90° triangle, the hypotenuse is $\sqrt{2}$ times as long as each (congruent) leg.
- Proof:



- Do problem 15 and 17 on p. 252.

30°– 60°– 90° Theorem



- Th. 5.20 In a 30°– 60°– 90° triangle, the length of the hypotenuse is twice the length of the short leg, and the length of the long leg is $\sqrt{3}$ times as long as the length of the short leg.
- Proof:

Chapter 6

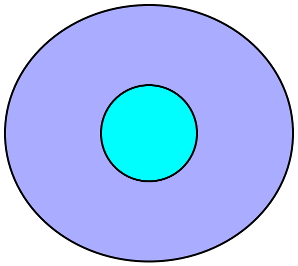
Circles

6.1 Circles and Radii

- A circle is the set of all points in a plane that are located a fixed distance from a point called its center. A line segment joining the center of a circle to one of its points is called the radius of the circle.

More about Circles

- Th. 6.1 The diameter *d* of a circle is twice the radius *r* of the circle. $d=2r$
- Post. 6.1 If two circles are congruent, then their radii and diameters are congruent. Conversely, if the radii or diameters are congruent, then two circles are congruent.
- Circles that lie in the same plane and have a common center are concentric circles.



Arcs and Semicircles

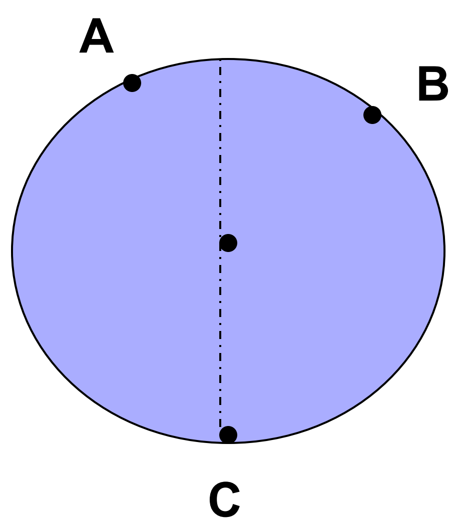
- An arc of a circle forms a continuous part of the circle.
- An arc of a circle whose endpoints are the endpoints of a diameter of the circle is called a semicircle.

Major Arc and Minor Arc

- An arc that is longer then a semicircle is called a major arc of the circle,
- an arc that is shorter than a semicircle is called a minor arc of the circle.

Naming Arcs

- Arcs are named using three points on the arc (usually if it is a major arc) or two points (if it is a minor arc.)



$\overset{\frown}{AB}$ = minor arc

$\overset{\frown}{ABC}$= major arc

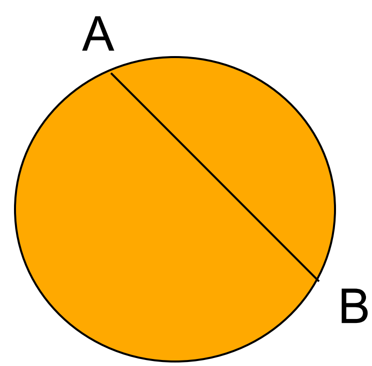
$\overset{\frown}{AC}$ = minor arc

Chord, Arc, and Diameter

- A line segment joining two distinct points on a circle is called a chord of the circle.

  $\overline{AB}$ is a chord

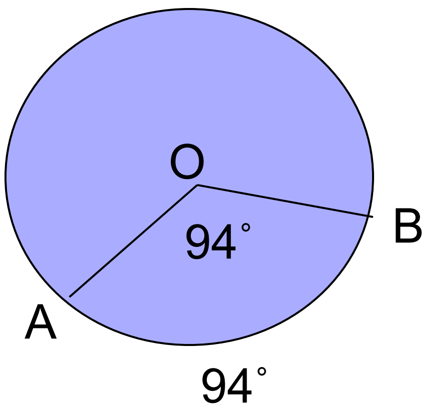
  $\overset{\frown}{AB}$ is an arc



- A diameter is a chord that contains the center of the circle

Central Angles and Arcs

- An angle with sides that are radii of a circle and vertex the center of the circle is called a <u>central angle</u>.



if O is the center of the circle

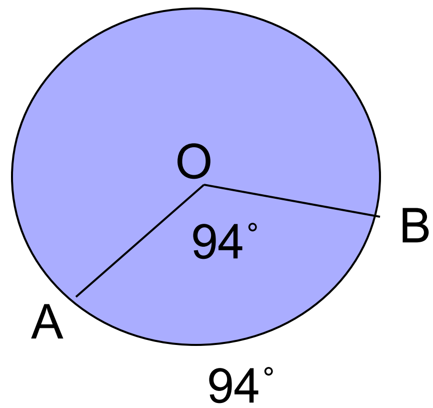
then <AOB is a central angle

m<AOB = 94°

m$\widehat{AB}$ = 94°

Central Angles and Arcs

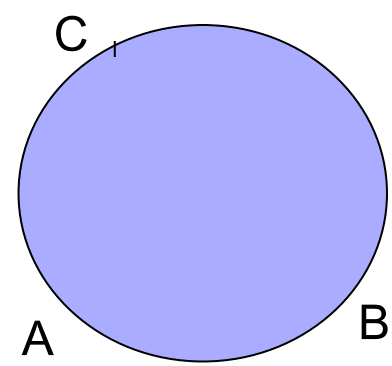
- The measure of an arc is the number of degrees in the central angle that intercepts the arc.



$m\overset{\frown}{AB} = 94^\circ = m\angle AOB$

Arcs

- Congruent arcs are arcs with the same measure.



- Post. 6.2 $m\overset{\frown}{ACB} = m\overset{\frown}{AC} + m\overset{\frown}{CB}$

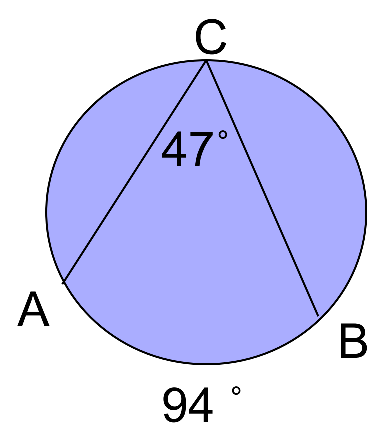
$$m\overset{\frown}{BC} = m\overset{\frown}{ACB} - m\overset{\frown}{AC}$$

and $m\overset{\frown}{AC} = m\overset{\frown}{ACB} - m\overset{\frown}{BC}$

Inscribed Angles

- An angle whose vertex is on a circle and whose sides intersect the circle in two other points is called an inscribed angle.

<ACB is an inscribed angle



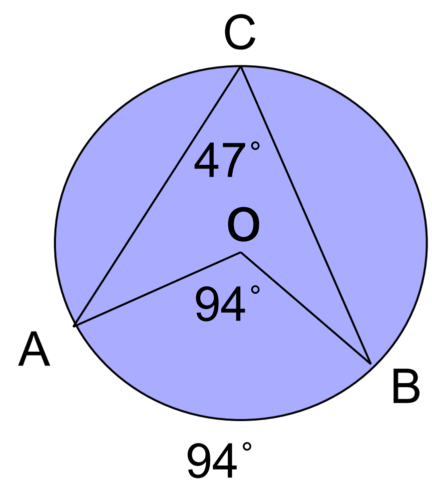
Inscribed Angles

- Th. 6.2 The measure of an inscribed angle is one-half the measure of its intercepted arc.

  $m\angle ACB = 47^0$

  $m\angle AOB = 94^0$ $m\,\overset{\frown}{AB} = 94^0$

  $m\angle ACB = \frac{1}{2}\, m\angle AOB$

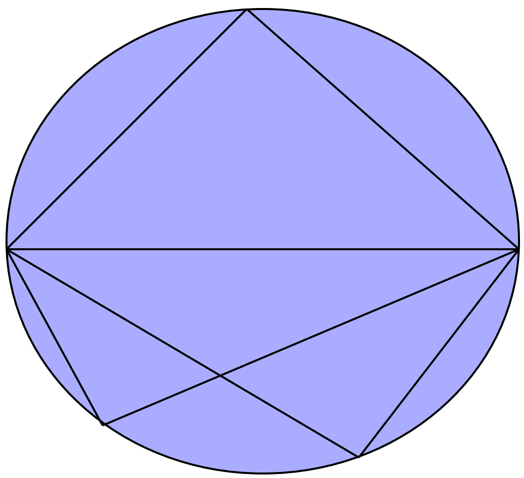


Inscribed Angles

- Cor. 6.3 Inscribed angles that intercept the same or congruent arcs are congruent.

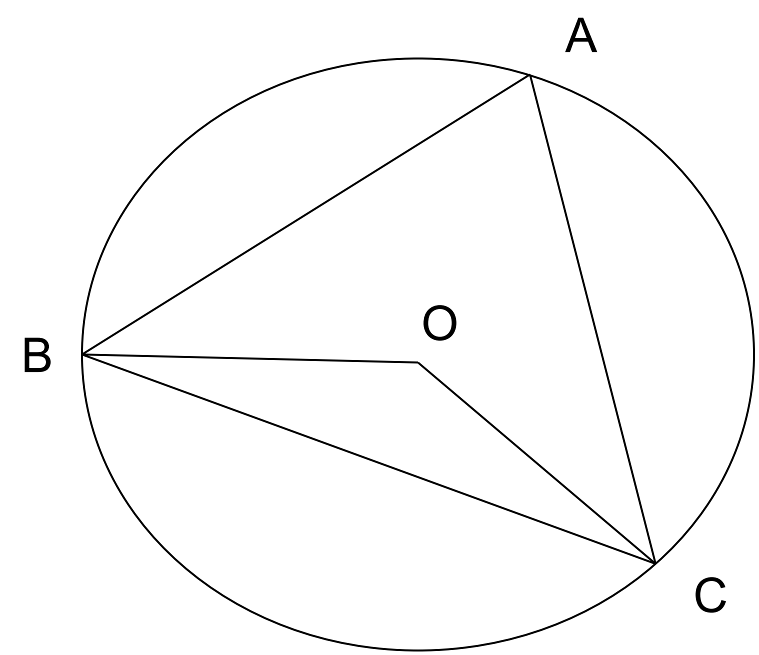
Inscribed Angles

- Cor. 6.4 Every angle inscribed in a semicircle is a right angle.



Solve the following

If m $\overset{\frown}{BC} = 130^0$, and
If AB = AC. Then what
are the measurements of $\overset{\frown}{BAC}$,
<BAC, <BOC, <ABC, <ACB,
<OBC, and <OCB.



Solution

If m $\overset{\frown}{BC} = 130^0$ (given), then

m $\overset{\frown}{BAC} = 360^0 - 130^0 = 230^0$

m $\angle BAC = 130^0 / 2 = 65^0$ (inscribed angle)

m $\angle BOC = 130^0$ (central angle)

Since AB = AC (given),

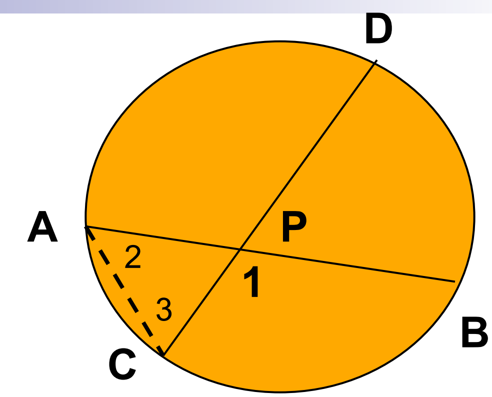
ΔABC is an isosceles triangle,

Then, m $\angle ABC$ = m $\angle ACB = (180^0 - 65^0)/2 = 57.5^0$

Since OB = OC, ΔOBC is an isosceles triangle,

m $\angle OBC$ = m $\angle OCB = (180^0 - 130^0)/2 = 25^0$

6.2 Chords and Angles



- Th. 6.5 When two chords of a circle intersect, the measure of each angle formed is one-half the sum of the measures of its intercepted arc and the arc intercepted by its vertical angle. (p.285)

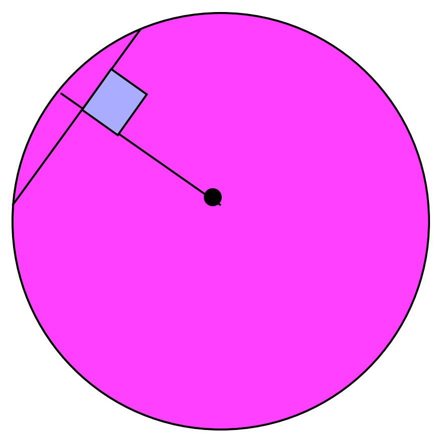
$$\boxed{m\angle 1 = \tfrac{1}{2}(\, m\overset{\frown}{CB} + m\overset{\frown}{AD})}$$

Theorems

- Th. 6.6 In the same circle, the arcs formed by congruent chords are congruent.
- Th. 6.7 In the same circle, the chords formed by congruent arcs are congruent.
- Def.-A line that divides an arc into two arcs with the same measure is called a bisector of the arc.

Theorems

- Th. 6.9 A line drawn from the center of a circle to the midpoint of a chord (not the diameter) or to the midpoint of the arc formed by the chord is perpendicular to the chord.



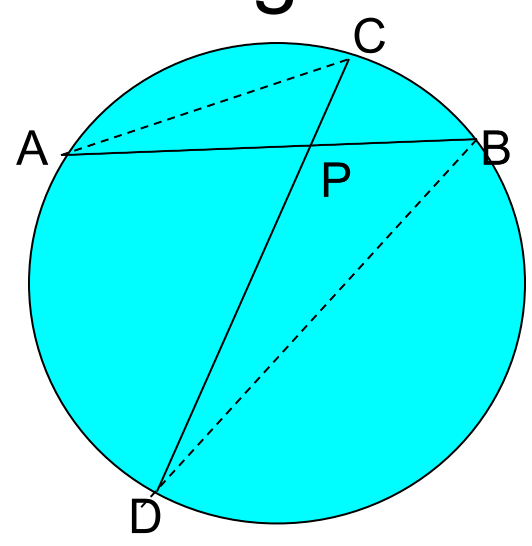
Theorems

- Th. 6.10 In the same circle, congruent chords are equidistant from the center of the circle.
- Th. 6.11 In the same circle, chords equidistant from the center of the circle are congruent.
- Th. 6.12 The perpendicular bisector of a chord passes through the center of the circle.

Chord Length Theorems

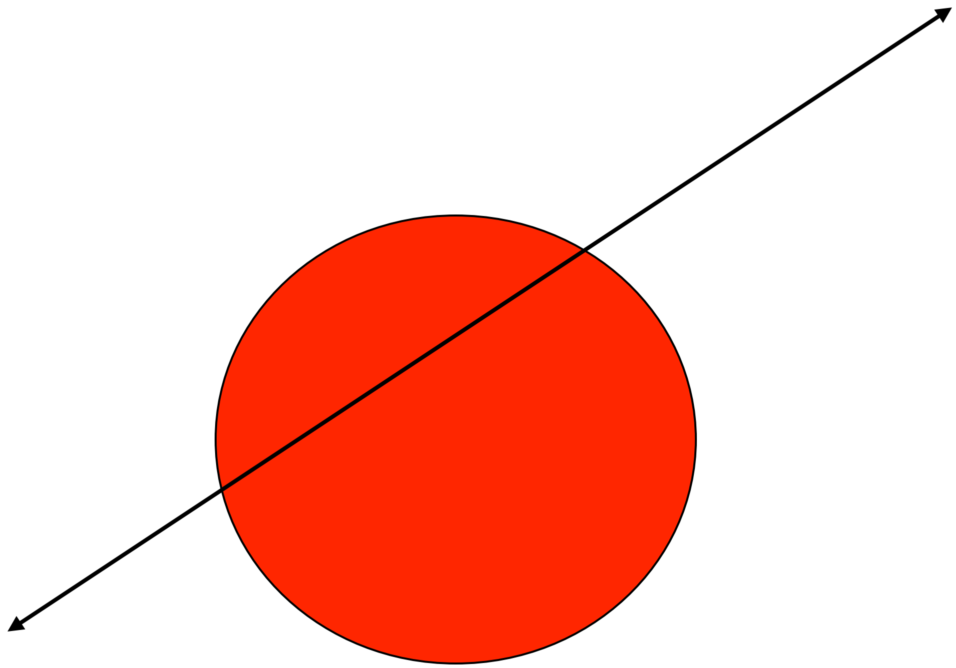
- Th. 6.13 If two chords intersect inside a circle, the product of the lengths of the segments of one chord is equal to the product of the lengths of the segments of the other. (p. 291)

(PA)(PB)=(PC)(PD)



Secants

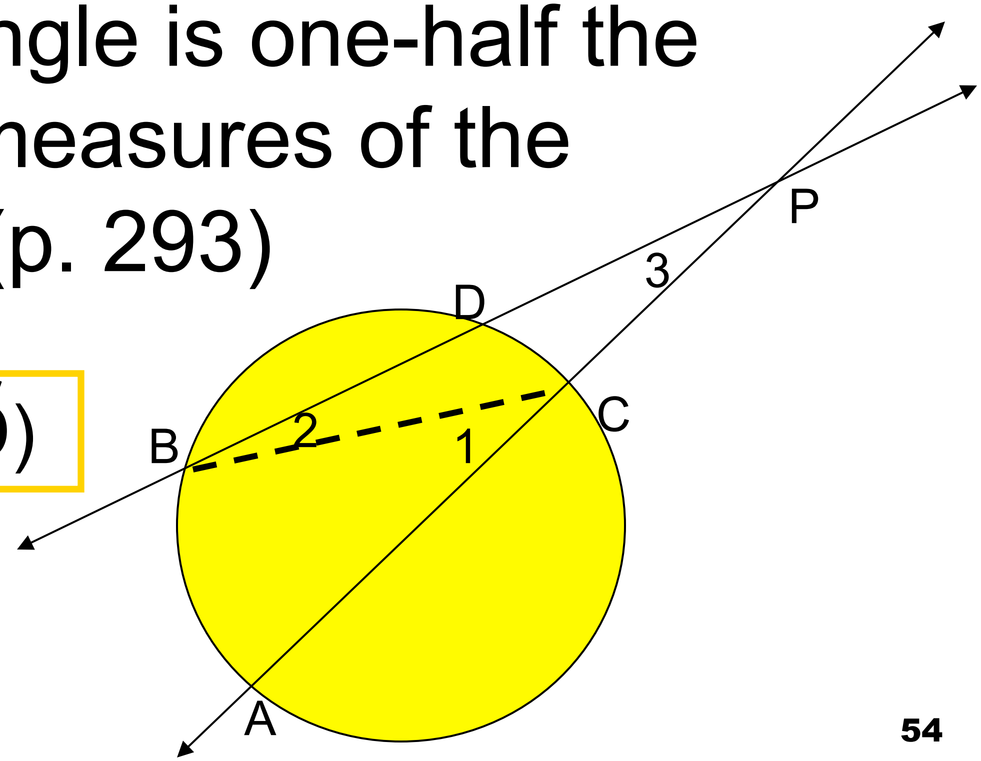
- If a line intersects a circle in two points the line is called a secant.



Secant Theorems

- Th. 6.14 If two secants intersect forming an angle outside the circle, then the measure of this angle is one-half the difference of the measures of the intercepted arcs. (p. 293)

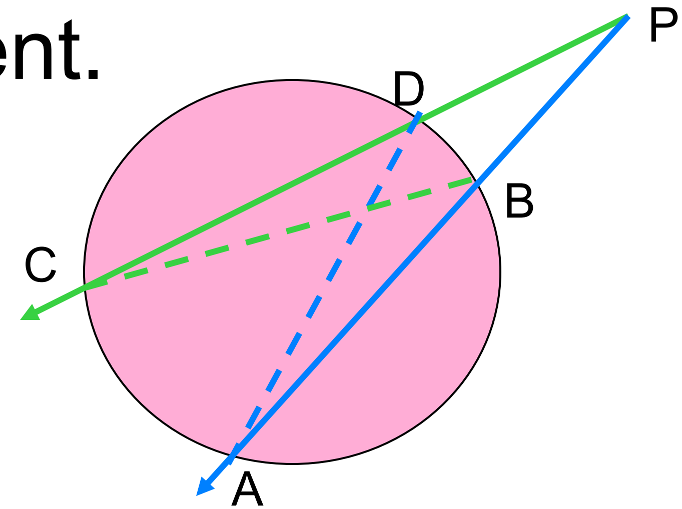
$\boxed{m\angle APB = \frac{1}{2}(m\overset{\frown}{AB} - m\overset{\frown}{CD})}$



Last Secant Theorem

- Th. 6.15 If two secants are drawn to a circle from an external point, the product of the lengths of one secant segment and its external segment is equal to the product of the lengths of the other secant segment and its external segment. (p.294)

(PA)(PB)=(PC)(PD)



Test 2

- Test 2 (next class) covers
  - Chapter 4 (omit kite)
  - Chapter 5 (5.1 – 5.4)
  - Chapter 6 (6.1 – 6.2)

Assignment

- Study the following chapters for test 2
  - Chapter 4 (omit kite)
  - Chapter 5 (5.1 – 5.4)
  - Chapter 6 (6.1 – 6.2)

Assignment

- Read the following chapters for next class
  - Read Chapter 7.1 - 7.3
  - Read Chapter 8.1- 8.2, 8.4 - 8.5