GAM

Geometry and Measurement
Day 2

Quiz: From the last lesson

- 1. What are the four undefined terms?
- 2. What are the four parts of an axiomatic system?
- 3. Using variables write out the reflexive, symmetric, and transitive postulates.
- 4. Complementary angles are two angles whose__________________.
- 5. Supplementary angles are two angles whose__________________.

Review

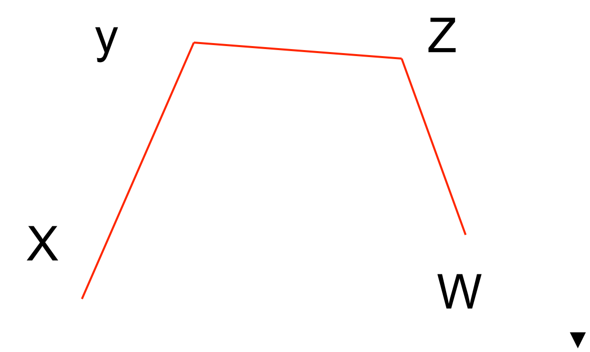
- 1. Set, point, line, plane
- 2. Undefined terms, Definitions,
  Axioms/Postulates, Theorems
- 3. Reflexive: $x = x$
  Symmetric: if $x = y$, then $y = x$
  Transitive: if $x = y$ and $y = z$, then $x = z$
- 4. Sum = 90^0
- 5. Sum = 180^0

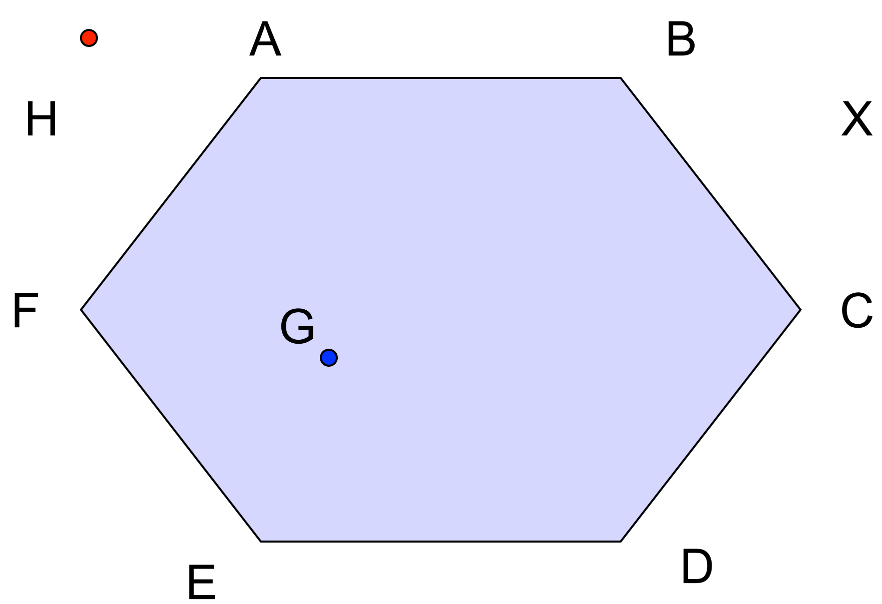
Triangles

Chapter 2

Six new Undefined Terms

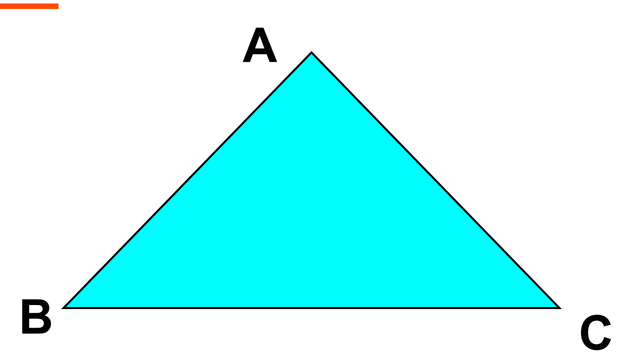
- Sides, closed, included, opposite, interior, exterior





Definition of a Triangle

- Let A, B, and C be three points not on the same line. The figure formed by the three segments $\overline{AB}$, $\overline{BC}$ and $\overline{AC}$ is called a triangle, denoted ΔABC.
- The three segments are sides and the three points are vertices.



Triangles classified by angles

1. An acute triangle has three acute angles.
2. A right triangle is a triangle with one right angle. The side opposite the right angle is the hypotenuse and the other two sides are the legs.
3. An obtuse triangle is a triangle in which one angle is obtuse (measures more then 90 degrees.)
4. An equiangular triangle is a triangle in which all three angles are equal in measure.

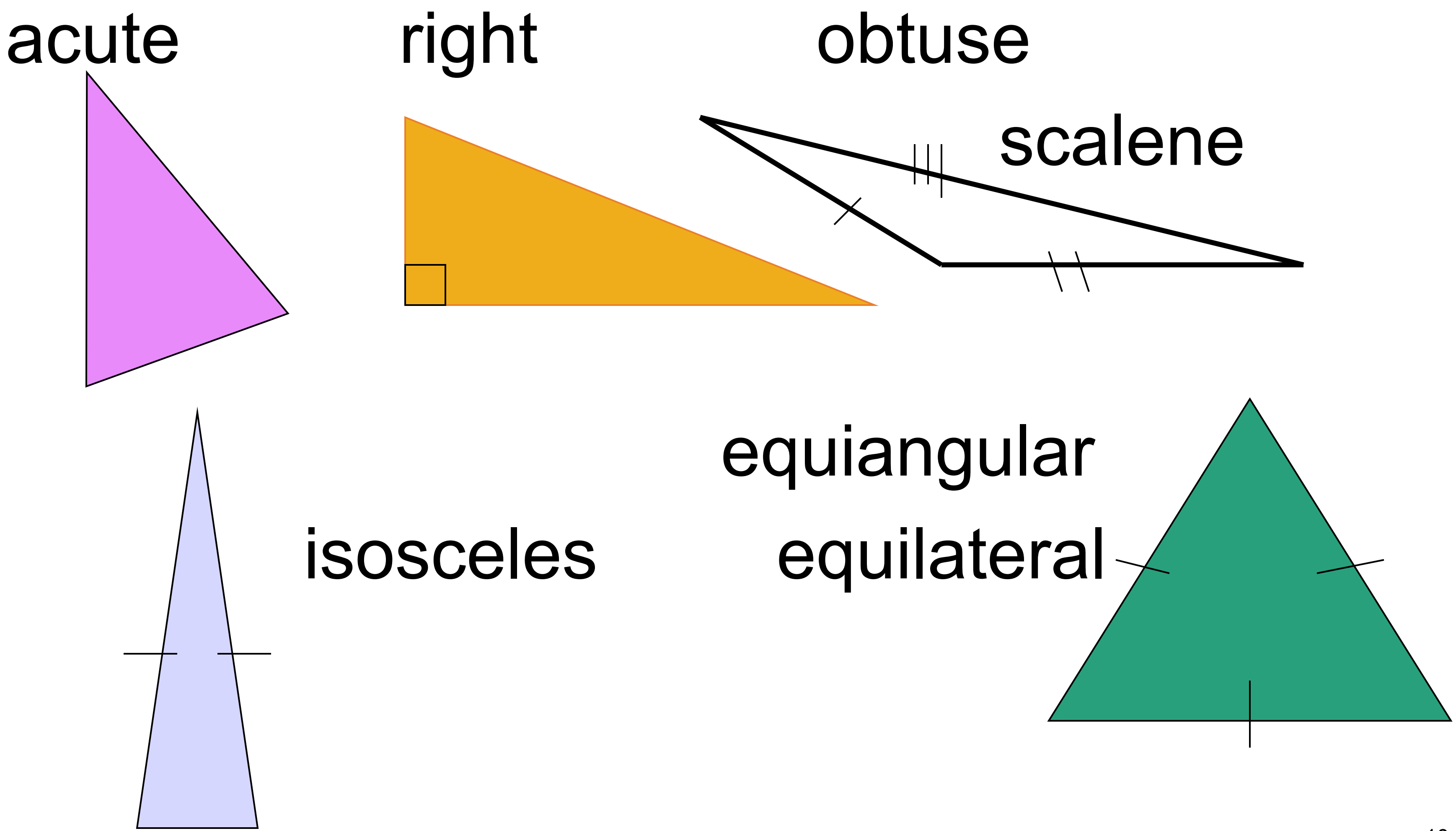
Triangles classified by sides

1. A scalene triangle is a triangle in which no two sides are equal in length.
2. An isosceles triangle is a triangle in which two sides are equal in length. The third side is its base.
3. An equilateral triangle is a triangle in which all three sides are equal in length.

Isosceles Triangles

- An isosceles triangle is a triangle with two equal sides
- The third side is called the base of the isosceles triangle
- The angle included between the equal sides is the vertex angle
- The other two angles are called base angles

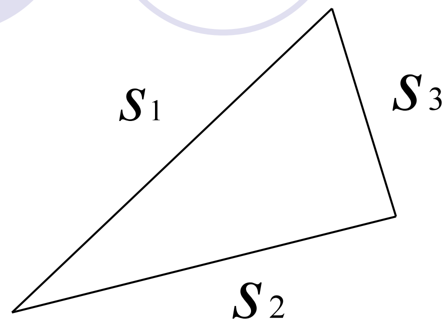
Triangles


acute
right
obtuse
scalene
equiangular
isosceles
equilateral

Perimeter and interior and exterior angles

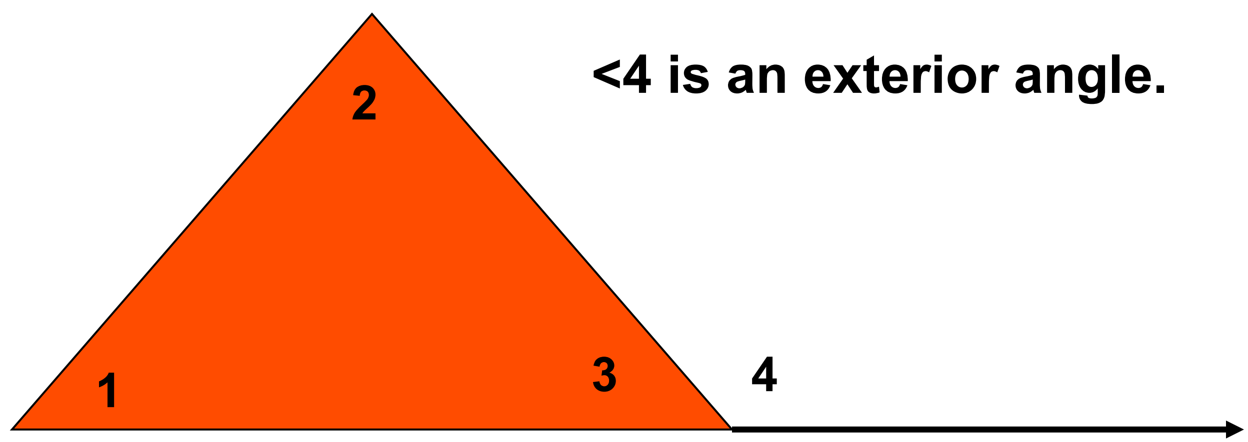
- Perimeter is the sum of the lengths of a shape's sides.
- An interior angle of a triangle is an angle formed by two sides of the triangle such that the angle is on the inside of the triangle.
- An exterior angle of a triangle is an angle formed by a side of the triangle and an extension of another side. Both these sides have a common endpoint. The angle lies on the outside of the triangle.

$$p = side_1 + side_2 + side_3 = S_1 + S_2 + S_3$$



$< 1, < 2, and < 3$ **are interior <s**

<4 is an exterior angle.



<1 and <2 are remote interior <s relative to <4.

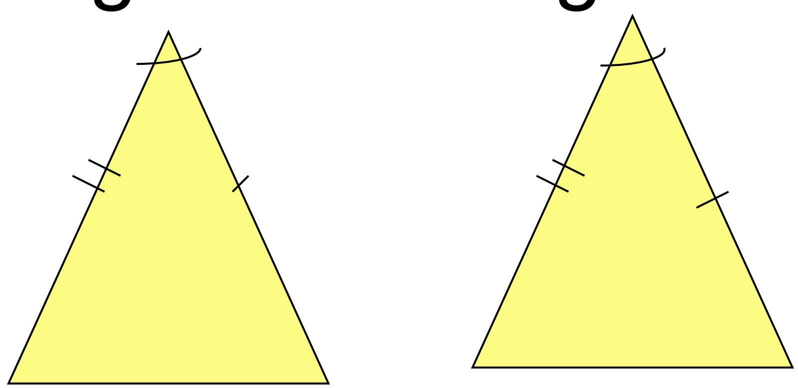
<3 is an adjacent interior < with respect to <4

Congruence

- Congruent segments are two segments with the same measure.
- Congruent angles are two angles with the same measure.
- * If all **six parts** of one triangle are congruent to the corresponding parts of another triangle, the two triangles are congruent.

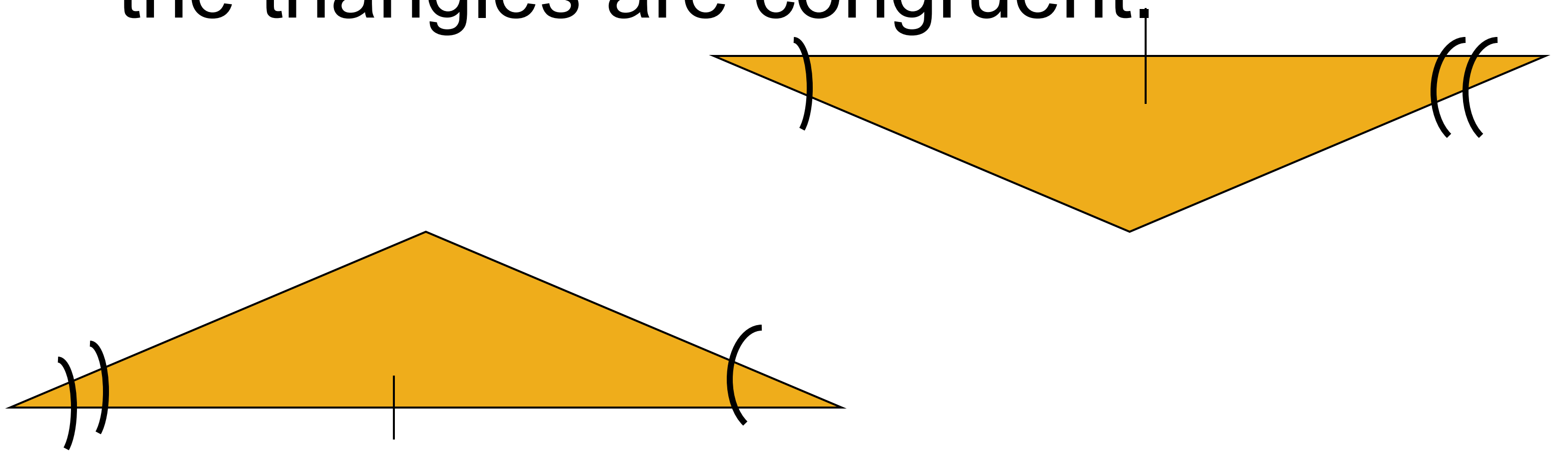
SAS postulate

- Post. 2.1 If two sides and the included angle of one triangle are congruent to two corresponding sides and the included angle of a second triangle, then the triangles are congruent.



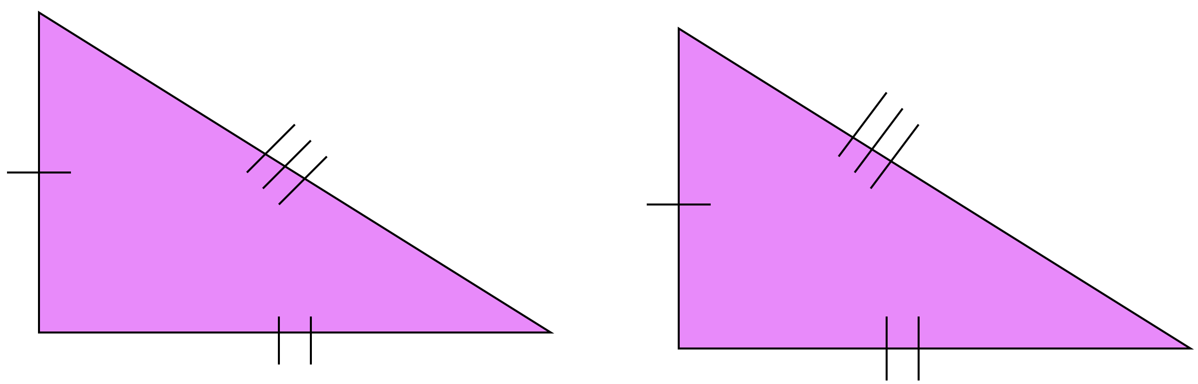
ASA postulate

- Post 2.2 If two angles and the included side of one triangle are congruent to the corresponding two angles and the included side of a second triangle, then the triangles are congruent.



SSS Postulate

- Post. 2.3 If three sides of one triangle are congruent to the corresponding three sides of a second triangle, then the triangles are congruent.

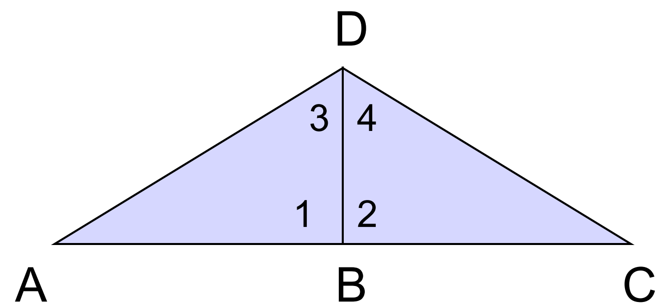


Postulates to prove two triangles congruence

- SAS
- ASA
- SSS

Complete the proof

- Given: $< 1 \cong < 2, < 3 \cong < 4,$
- Prove: $\Delta ABD \cong \Delta CBD$



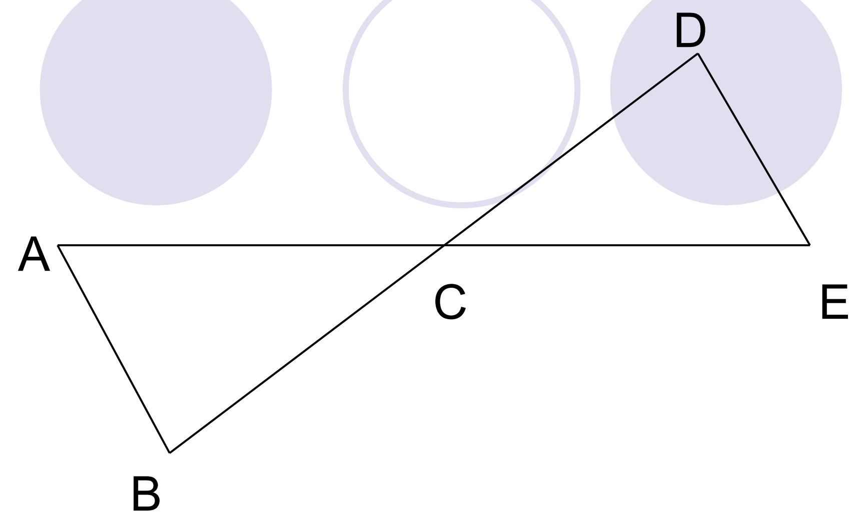
Statements	Reasons
1. <1 ≅ <2	1.
2.	2. Given
3.	3. Reflexive
4. ΔABD ≅ ΔCBD	4.

Complete the proof

Given: C is the midpoint of $\overline{AE}$

$\angle E \cong \angle A$

Prove: $\Delta ABC \cong \Delta EDC$

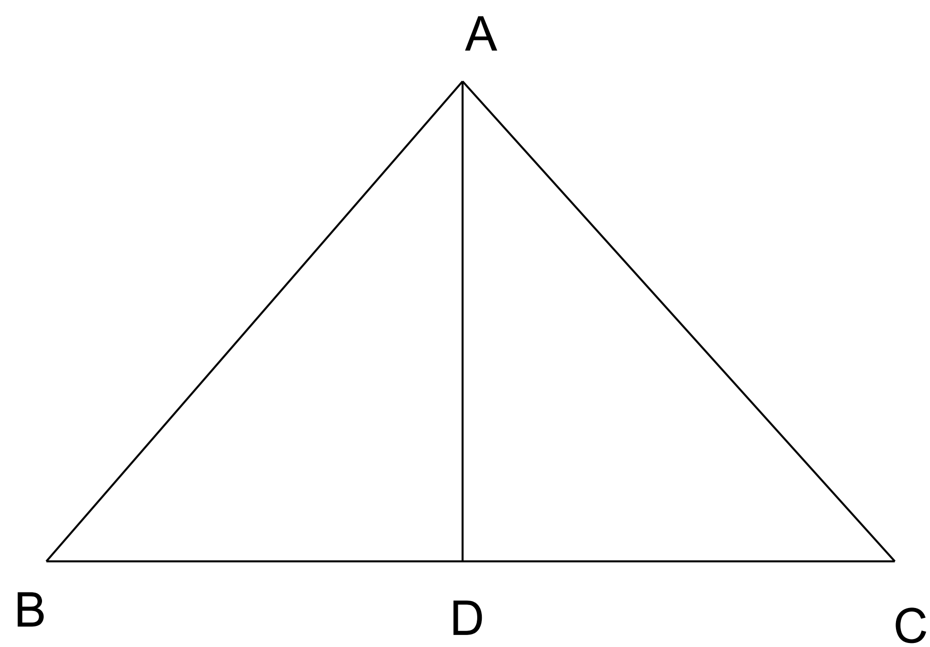


Assume that

$AB = AC$

$\overline{AD}$ bisects $\overline{BC}$

Prove that $\Delta ABD \cong \Delta ACD$



Proofs Involving Congruence

Def- CPCTC: Corresponding Parts of Congruent Triangles are Congruent.

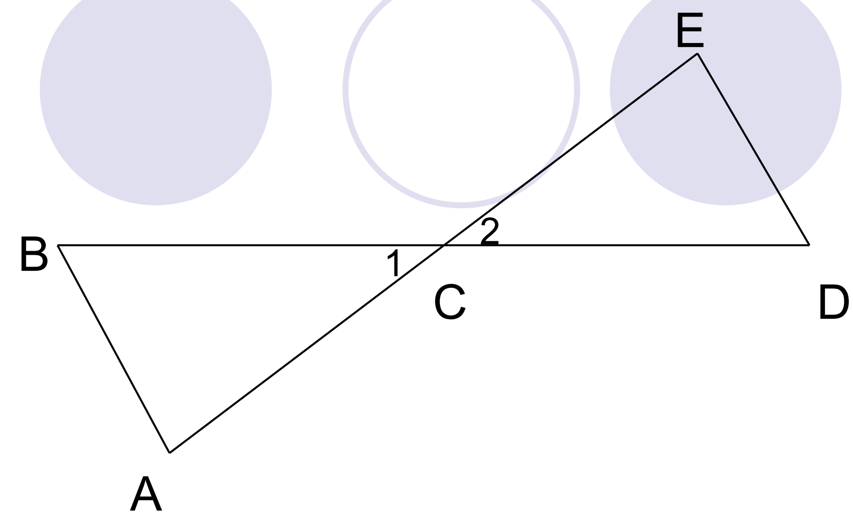
- If two triangles are congruent, the six pairs of corresponding parts are congruent.
- CPCTC is used after you have proved two triangles are congruent

Complete the proof

Given: <A ≅ <E

$\overline{AC} \cong \overline{EC}$

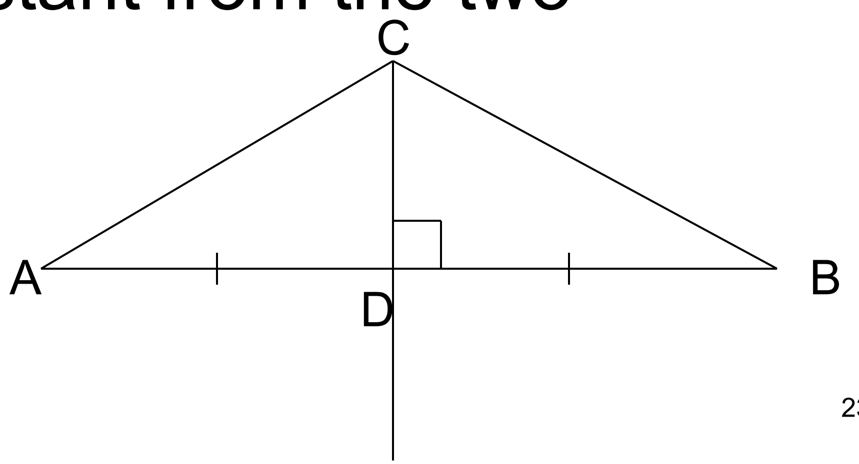
Prove: <B ≅ <D



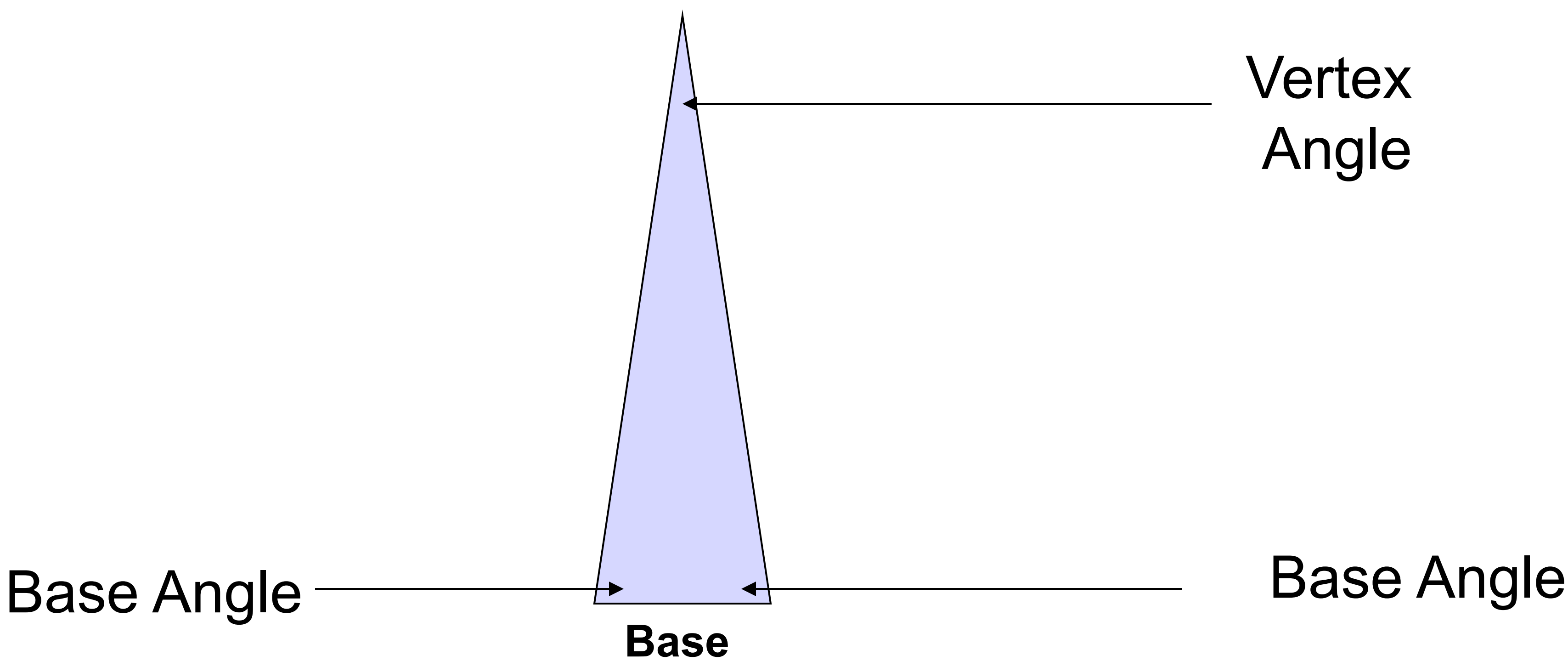
Theorems

- Th. 2.1 Transitive Law for Congruent Triangles-
  If $\Delta ABC \cong \Delta DEF$ and $\Delta DEF \cong \Delta GHI$,
  then $\Delta ABC \cong \Delta GHI$.

- Th.2.3 Every point on the perpendicular bisector of a segment is equidistant from the two endpoints.



Angles of an Isosceles Triangle

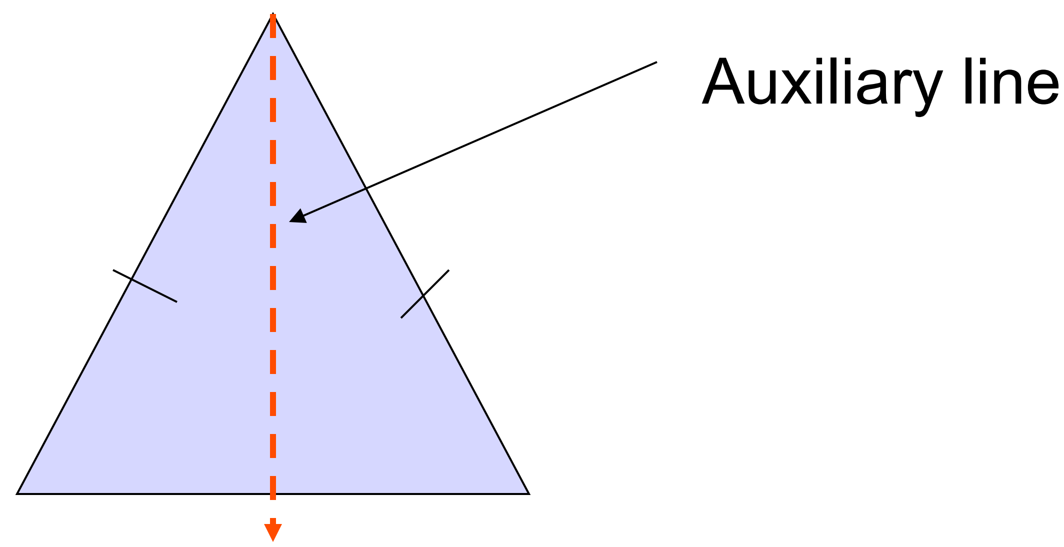


Auxiliary Lines or Segments

- Some proofs require the use of auxiliary lines or segments.
- Often these lines are drawn in a figure using a dashed line because they are not part of the original diagram.

Isosceles Triangles

- Th. 2.5 If two sides of a triangle are congruent, then the angles opposite them are also congruent.



Theorems

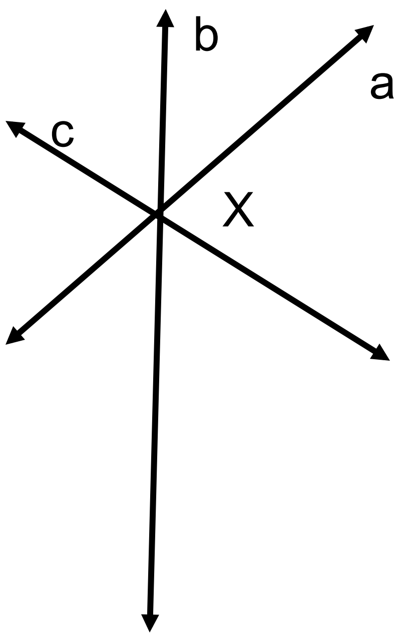
- Cor. 2.6 If a triangle is equilateral, then it is equiangular.
- Th. 2.7 If two angles of a triangle are congruent, then the sides opposite these angles are also congruent.

(This is the converse of Th. 2.5. A converse of a statement interchanges the hypothesis and the conclusion.)

What is the converse of Cor. 2.6?

Concurrent lines

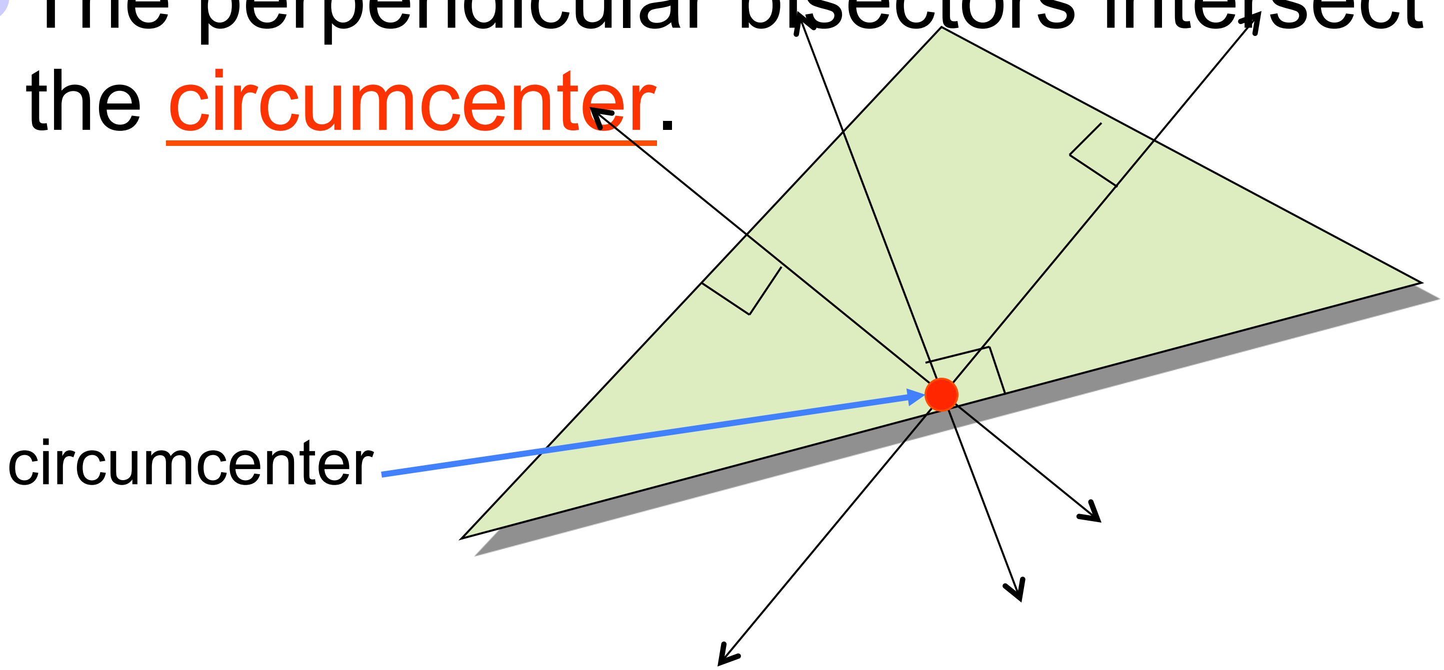
- Def.-Two or more lines are concurrent if they intersect in one and only one point.



Lines a, b, and c intersect at point X so they are concurrent.

Triangles and perpendicular bisectors

- Th. 2.9 The perpendicular bisectors of the sides of a triangle are concurrent.
- The perpendicular bisectors intersect at the circumcenter.



Triangles and Medians

- A median of a triangle is the segment joining a vertex to the midpoint of the side opposite that vertex.
- The centroid of a triangle is the point of intersection of the medians of a triangle.

- Th. 2.10 The medians of a triangle are concurrent and meet at a point that is two thirds the distance from the vertex to the midpoint of the opposite side.

Triangles and altitudes

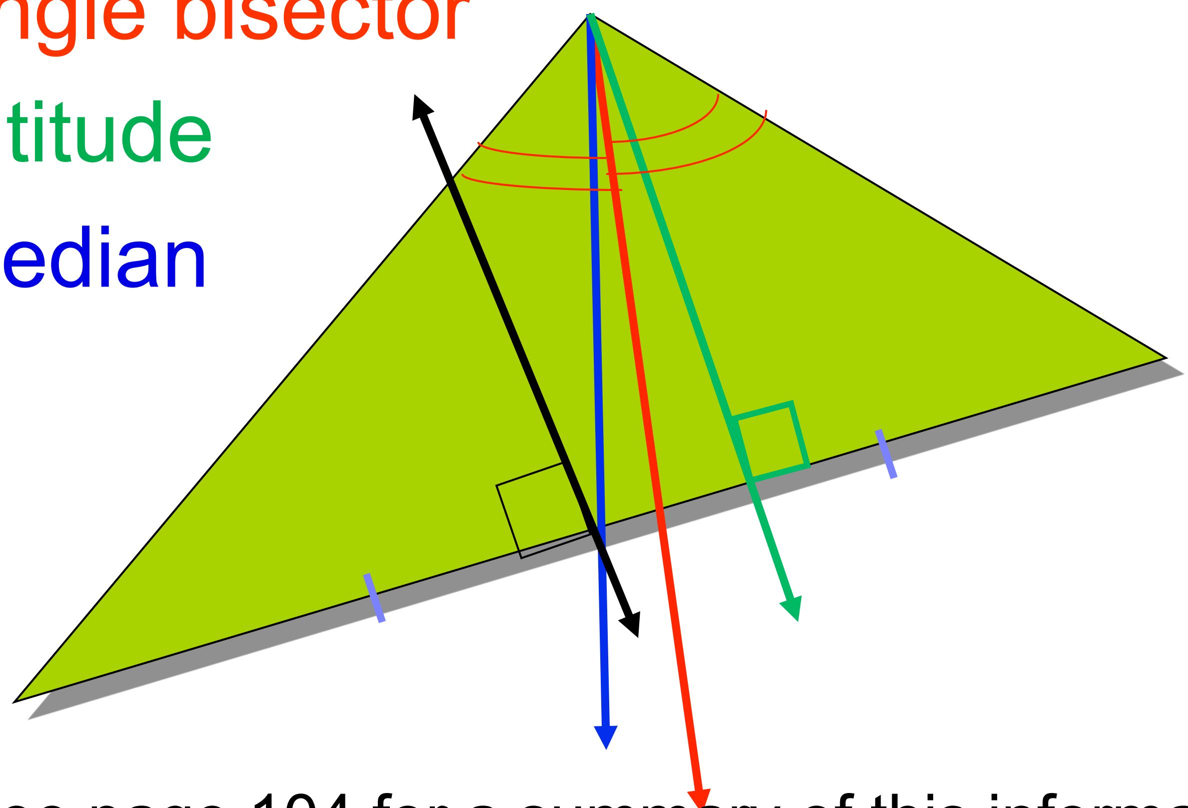
- An altitude of a triangle is a line segment from a vertex *perpendicular* to the side opposite that vertex (possibly extended.)
- The orthocenter of a triangle is the point of intersection of the three altitudes of a triangle.
- Th. 2.11 The altitudes of a triangle are concurrent.

Triangles and angle bisectors

- An angle bisector of a triangle is the line segment (or ray) that separates the given angle into two congruent adjacent angles.
- The incenter of a triangle is the point of intersection of the three angle bisectors of the triangle.
- Th. 2.12 The bisectors of the angles of a triangle are concurrent and meet at a point equidistant from the sides of the triangle.

Altitudes, angle bisectors, & medians

- Perpendicular bisector
- Angle bisector
- Altitude
- Median



See page 104 for a summary of this information.

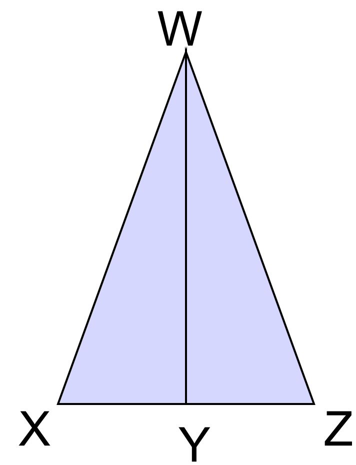
Proving Right Triangles Congruent

- Th. 2.13 LA (Leg Angle)- If a leg and acute angle of one right triangle are congruent, to a leg and the corresponding acute angle of anther right triangle, then the two right triangles are congruent.

- Th.2.14 LL (Leg-Leg)- If the two legs of one right triangle are congruent to the two legs of another right triangle, then the two right triangles are congruent.

Prove

- Given: $\overline{WY} \perp \overline{XZ}$, $\overline{XY} \cong \overline{YZ}$
- Prove: $\Delta XYW \cong \Delta ZYW$



Beginning of Chapter 3
Parallel Lines and Polygons

Chapter 3

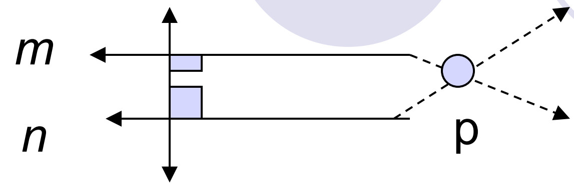
3.1 Indirect proofs and the Parallel Postulate

- Indirect proof: Given the conditional statement $P \rightarrow Q$, if you assume that Q is *false* and this leads to a contradiction, then you are forced to conclude that Q is true.
- Indirect proof require the ability to form the negation of a statement.
- Example: Line *m* is perpendicular to line *n.*
- *Thus:* Line m is not perpendicular to line n.
- See page 129 for an example.

3.1 Parallel lines

- Two lines in the same plane that do not intersect are called parallel lines.
- Post. 3.1 For a given line $\overleftrightarrow{AB}$ and a point P not on $\overleftrightarrow{AB}$, one and only one line through P is parallel to $\overleftrightarrow{AB}$.

Th. 3.1 If two lines in a plane are both perpendicular to a third line, then they are parallel.



Given: Lines l, m and n with $m \perp l$ and $l \perp n$

Prove: m||n

Proof

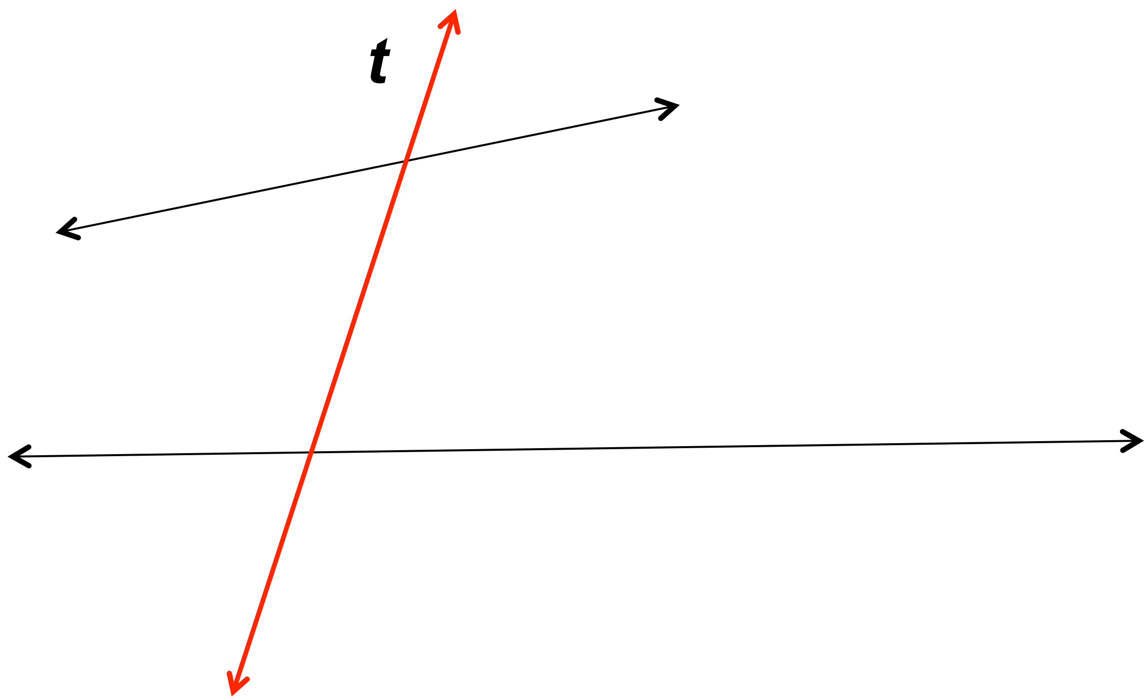
1. Assume $m \nparallel n$	1. Assumption we wish to show incorrect
2. m and n intersect at some point, p	2. Def of || lines
3. $m \perp l$ and $l \perp n$	3. Given

This is a contradiction of Post 1.18 “there exists one and only one line perpendicular to a given line passing through a point not on the line.”

Thus, the assumption is incorrect so m||n

3.2 Transversals and Angles

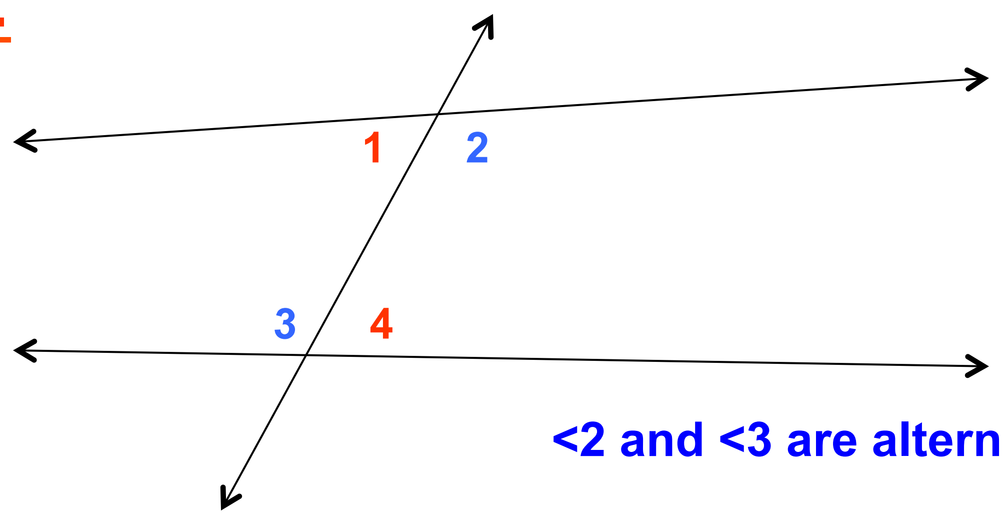
- A transversal is a line that intersects two or more distinct lines in different points.



Angles formed by a transversal

- Suppose two lines are cut by a transversal

The nonadjacent angles on the opposite sides of the transversal but on the interior of the two lines are called <u>alternate interior angles.</u>



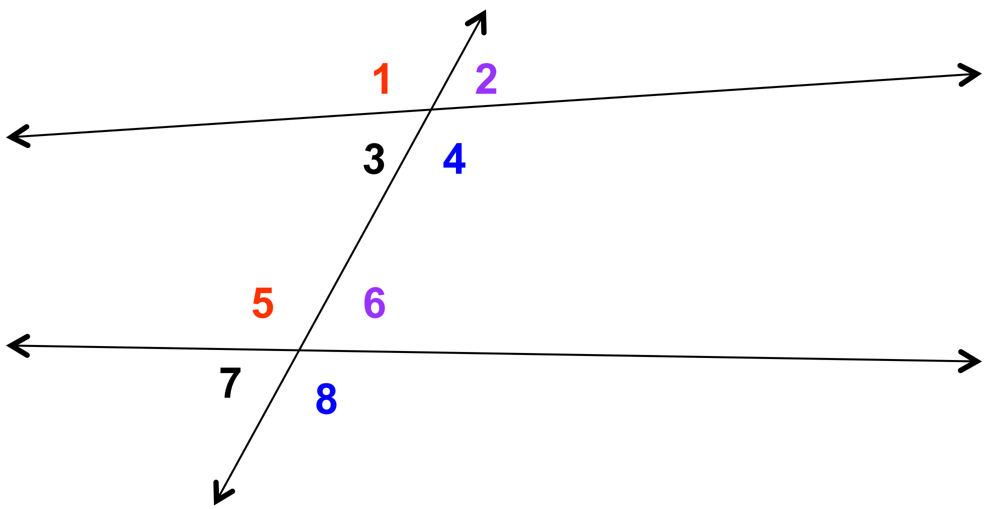
<2 and <3 are alternate interior <s.

<1 and 4 are alternate interior <s.

Angles formed by a transversal

- Suppose two lines are cut by a transversal.

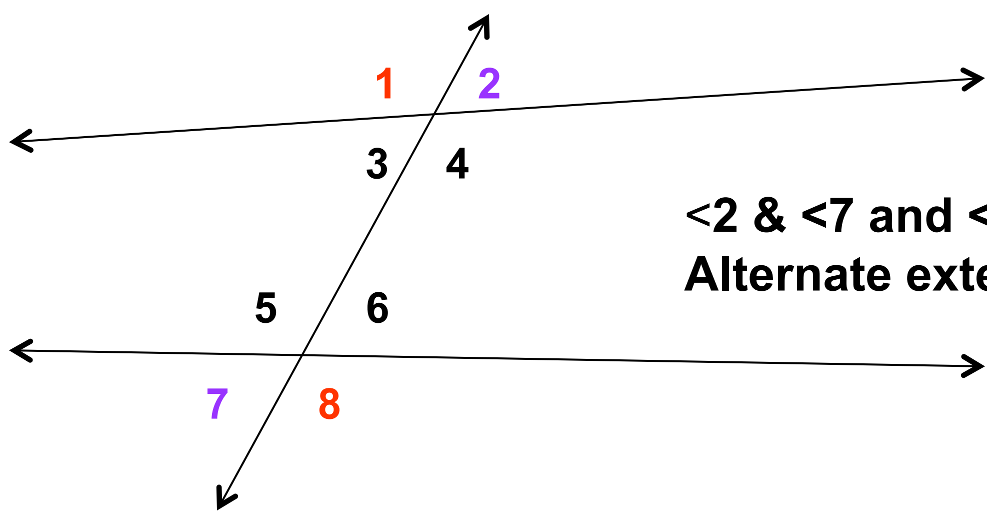
The nonadjacent angles on the same side of the transversal and in the same corresponding positions with respect to the two lines are called corresponding angles.(<1&<5, <2&<6, <3&<7, <4&<8)



Angles Formed by a transversal

- Suppose two lines are cut by a transversal.

The nonadjacent angles on the opposite sides of the transversal and on the exterior of the two lines are called alternate exterior angles.



<2 & <7 and <1 & <8 are Alternate exterior angles.

Obj.-To prove lines parallel

- Th. 3.2 If two lines are cut by a transversal and a pair of alternate interior angles are congruent, then the lines are parallel.

Give: $m<1 = m<2$

Prove: L1 || L2

Proof: ★

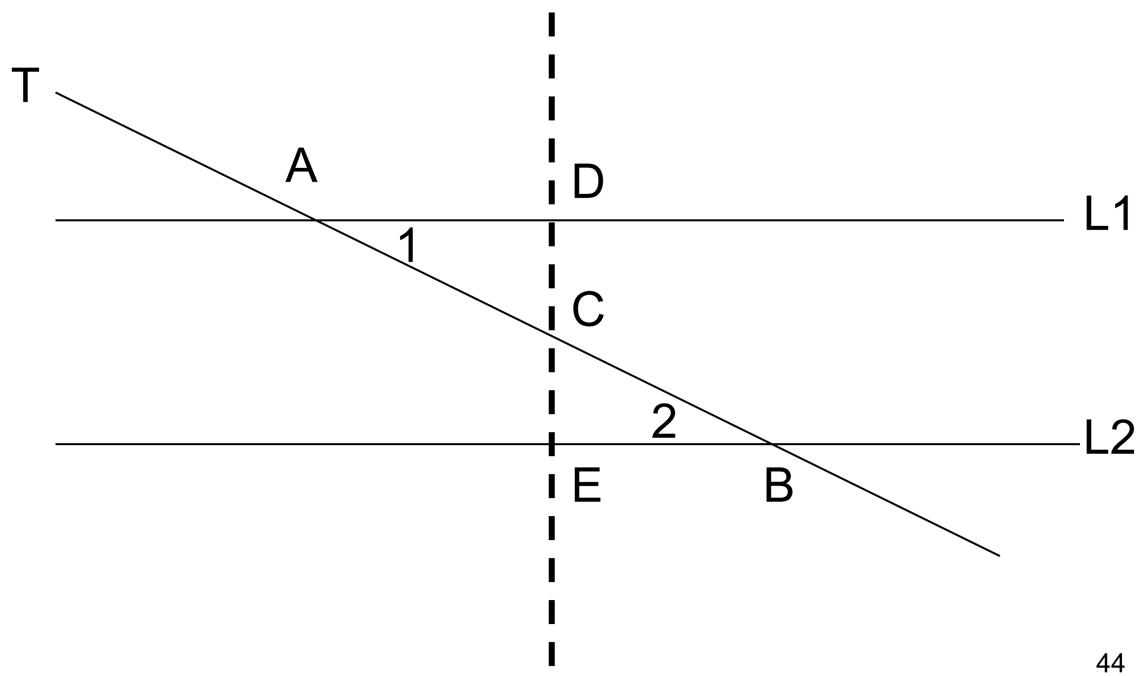
Let C be the mid point of $\overline{AB}$

Let $\overline{DE}$ pass through point c and $\overline{CD} \perp \overline{AD}$

$\Delta ADC \cong \Delta BEC$ ASA

$<BEC \cong < ADC = 90^0$

=> L1 || L2



Obj.-To prove lines parallel

- Th. 3.4 If two lines are cut by a transversal and a pair of alternate exterior angles are congruent, then the lines are parallel.

To prove lines parallel use the following

- Th. 3.5 If two lines are cut by a transversal and two interior angles on the same side of the transversal are supplementary, then the lines are parallel.

Theorems about parallel lines

- Th. 3.6 If two parallel lines are cut by a transversal, then all pairs of alternate interior angles are congruent.

- Th. 3.7 If two lines are parallel and a third is perpendicular to one of them , then it is also perpendicular to the other.

Theorems about parallel lines

- Th. 3.8 If two parallel lines are cut by a transversal, then all pairs of corresponding angles are congruent.

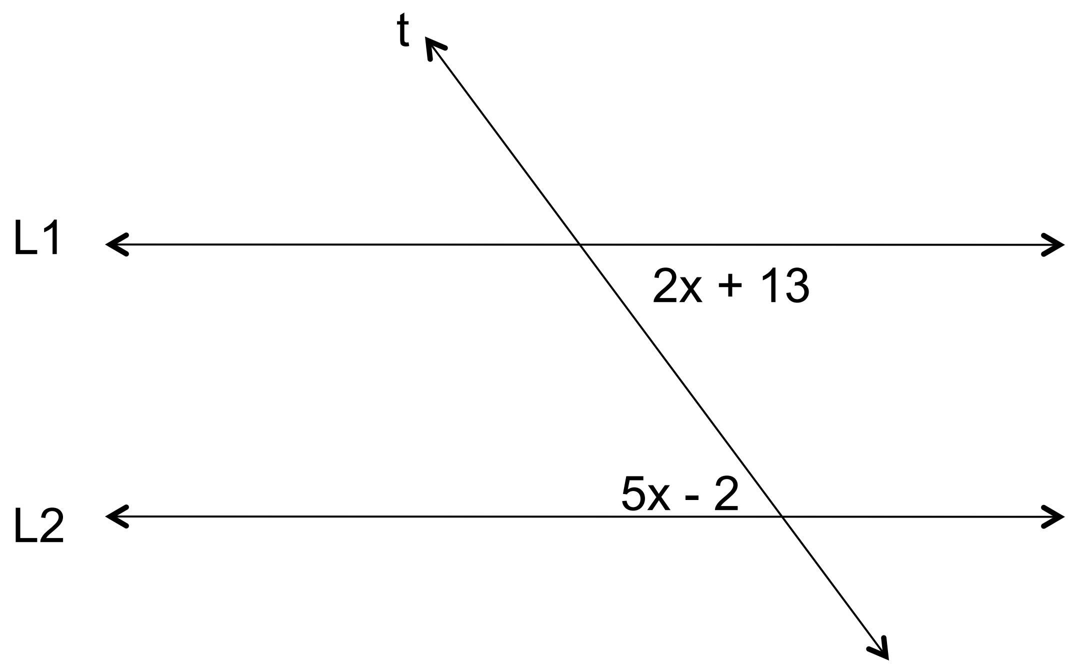
More theorems about parallel lines

- Th.3.9 If two parallel lines are cut by a transversal, then all pairs of alternate exterior angles are congruent.

- Th. 3.10 If two parallel lines are cut by a transversal, then all pairs of interior angles on the same side of the transversal are supplementary.

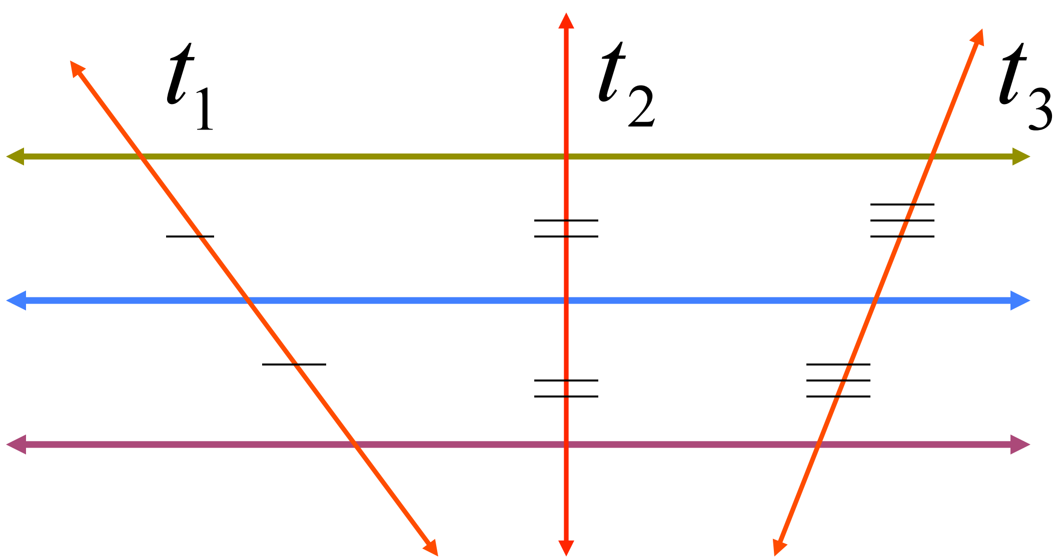
Example

L1 // L2, t is a transversal. What is x?



Transversal

- Th. 4.23 If three or more parallel lines intercept congruent segments on one transversal, then they intercept congruent segments on all transversals.

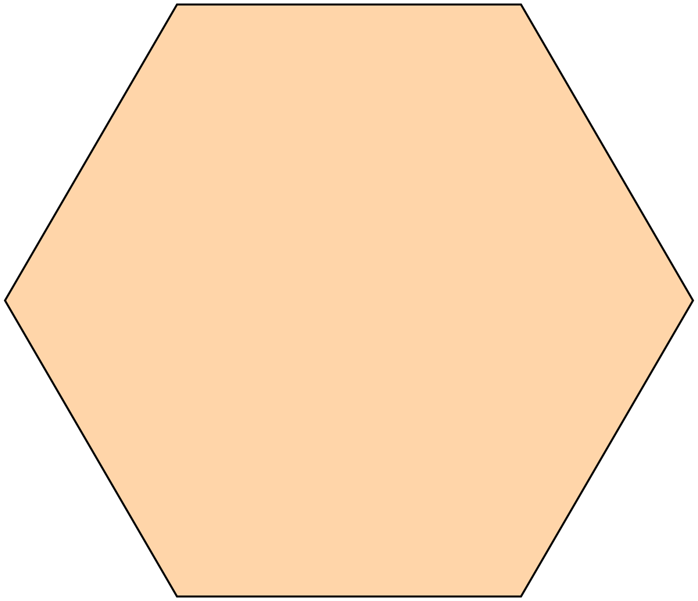


3.3 Polygons and Angles

- A polygon is a closed figure in a plane.
  - **It has n segments (where n≥3) called sides that intersect only at their endpoints.**
  - **Each endpoint is called the vertex of the polygon.**
  - **No two consecutive sides are on the same line.**

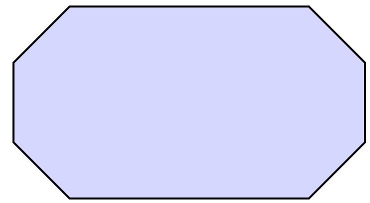
Regular Polygon

- A polygon is a regular polygon if all its sides are congruent and all its angles are congruent.

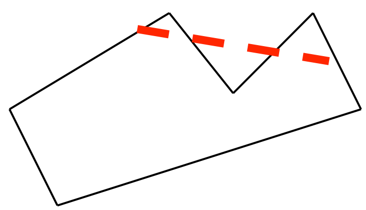


Polygons (cont.)

- The angles of a <u>convex</u> polygon measure between 0˚ and 180 ˚.



- A polygon is <u>concave</u> if a line segment joining two points in the polygon may include points not in the interior to the polygon.

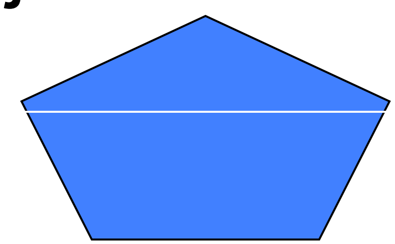


Polygons are named by the number of sides they have.

Number of sides	Polygon
3	Triangle
4	Quadrilateral
5	Pentagon
6	Hexagon
7	Heptagon
8	Octagon
9	Nonagon
10	Decagon
N	N-gon

More facts on polygons

- Post. 3.2 A polygon has the same number of angles as sides.
- Def- A <u>diagonal</u> of a polygon is a segment that joins two nonadjacent vertices.



- Def- The <u>perimeter</u> of a polygon is the sum of the lengths of its sides.

Angles and Triangles

- Th. 3.11 The sum of the interior angles of a triangle is 180˚.

- Cor. 3.12 Any triangle can have at most one right angle or at most one obtuse angle.

Angles and Triangles

- Cor. 3.13 If two angles of one triangle are congruent, respectively, to two angles of another triangle, then the third angles are also congruent.

Exterior Angles

- Cor. 3.14 The measure of an exterior angle of a triangle is equal to the sum of the measures of the nonadjacent interior angles.

Angles and polygons

- Th. 3.15 The sum of the measures of the angles of a polygon with n sides is given by the formula

  $S = (n-2)180^\circ$.

- Cor. 3.16 The measure of each angle of a regular polygon with n sides is given by the formula

$$a = \frac{(n-2)180^\circ}{n}$$

Example

- The sum of the measures of the angles of a polygon is 1080 degrees. Find the number of sides the polygon has.

Last two angle theorems

- Th. 3.17 The sum of the measures of the exterior angles of a polygon, one at each vertex, is 360˚.

- Cor. 3.18 The measure of each exterior angle of a regular polygon with n sides is determined with the formula

$$e = \frac{360°}{n}$$

3.4 More congruence triangle theorems

- Th. 3.19 AAS (angle-angle-side): If two angles and any side of one triangle are congruent to the corresponding two angles and side of another triangle, then the two triangles are congruent
- AAS ➔ ASA

More congruence triangle theorems★

- Th. 3.20 HA (Hypotenuse Angle) If the hypotenuse and an acute angle of one right triangle are congruent to the hypotenuse and acute angle of another triangle, then the two right triangles are congruent.
- Th. 3.21 HL (Hypotenuse Leg) – If the hypotenuse and a leg of one right triangle are congruent to the hypotenuse and leg of another right triangle, the two right triangles are congruent.

Solve the following

- Find the sum of the interior angle measures of a pentagon.
- Find the measure of each interior angle of a regular pentagon.
- Find the measure of each exterior angle of a regular pentagon.

Review

- Today you learned
  - Triangles
  - Properties of triangles
  - Congruence of triangles
  - Parallel lines and properties
  - Properties of polygons
  - Interior angle measures of polygons
  - Exterior angle measures of polygons

Test 1

- Test 1 (next class) covers
  - Chapter 1.1 – 1.5
  - Chapter 2.1 – 2.5
  - Chapter 3.1 – 3.4

Assignments

- Study the following chapters for test 1
  - Chapter 1.1 to 1.5
  - Chapter 2.1 to 2.5
  - Chapter 3.1 to 3.4

Assignments

- Read the following chapters for next class
  - Read Chapter 4
  - Read Chapter 5.1

End of Lesson 2