

# An Interactive EM Propagation Simulation Based on FDTD Method

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**Abstract**—Measurement process like temperature sensing always faces the challenge in speed and accuracy. Recently, a Data Driven Fast Measurement(DDFM) method is proposed to solve the problem.

This project firstly gives a brief review of the DDFM method, and then focuses on applying the DDFM algorithm to a temperature measurement setup. Experiments applying DDFM and other methods, like Kalman filter are carried out to verify the theory and prove the efficiency of the method. Noise effect on the method, including measuring noise and quantisation noise are also discussed to give a guideline in improving the performance of DDFM.

*Index Terms*—FDTD,

## I. INTRODUCTION

MEASURING process could generally be seen as a dynamic system with the quantity of interest as input and the measured value as its outputs. Then the problem could be converted to estimating the system input with given output observations, and sometimes with system information. Current study shows two kinds of methods: 1) Model based method, which estimate the system parameter first and predict the value of interest based the model. 2) Model free method, which deduces the measurement directly by the observations. In this project, we mainly focus on applying DDFM, a model free method on a special thermo-sensing setup, in which the temperature is captured by a metal sensor.

In this context, the system  $S$  is simplified assuming the heat-transfer process following the Newton's Law of cooling:

$$\frac{dT}{dt} = k(T - T_0) \quad (1)$$

In which,  $k$  is a constant standing for heat transfer coefficient,  $T$  standing for the temperature of the sensor, while  $T_0$  standing for the target temperature.

Obviously, the system described by the differential eq. (1) is a first order system. In the context, the initial temperature and target temperature is known, at any time  $t$ , the temperature could be expressed as an exponential

$$T(t) = T_E + (T_0 - T_E)e^{kt} \quad (2)$$

Under the above assumption, it could be deduced that the dynamic system is a first order LTI system, which could also be expressed by state space  $S\{A, B, C, D\}$ .

$$\begin{aligned} \delta X &= AX + BU \\ Y &= CX + DU \end{aligned}$$

in which  $Y$  is the sensor output and  $U$  is the real value of interest.  $\delta$  stands for a unit delay. Rewriting the above state space expression to i/o form:

$$y(t) = Ce^{A(t-t_0)}x(t_0) + \int_{-\infty}^t W(t-\delta)u(\delta)d\delta \quad (3)$$

In this thermo-sensing case, eq.(3) could be adapted to

$$y(+\infty) = \bar{y} = G\bar{u} \quad (4)$$

where  $G$  stands for a calibration coefficient that is related to the sensor feature.

## II. METHODS

### A. System Augmentation

As it is introduced in the previous section, the research focus on the temperature sensing setup which could be seen as a SISO LTI system with constant input  $u$ . In this case, the system could be augmented in state space as following:

$$X_{aug} = \begin{bmatrix} x \\ u \end{bmatrix}, A_{aug} = \begin{bmatrix} A & B \\ 0 & I_m \end{bmatrix}, C_{aug} = [C \quad D]$$

Considering the output  $y$  is always infected with noise  $y_n$ ,  $y = y_0 + y_n$ . In this way, the problem could be solved in different ways. If the model is known, a Kalman filter could be used to give maximum likelihood estimation for the states. If the model is unknown, the problem could also be adapted in various way to be a system identification problem[2]. However, here we apply the DDFM, a model free method which bypass the system parameter estimation, and gives the prediction directly.

### B. DDFM method

The DDFM method is a subspace-type method[2]. Considering the augmented system in previous section again, it could in further be adapted to:

$$X_{aug} = \begin{bmatrix} x \\ 1 \end{bmatrix}, A_{aug} = \begin{bmatrix} A & B\bar{u} \\ 0 & 1 \end{bmatrix}, C_{aug} = [C \quad D\bar{u}] \quad (5)$$

Hankel Matrix of  $\Delta Y$

$$\mathcal{H}(\Delta y) = \begin{bmatrix} \Delta y(1) & \cdots & \Delta y(n_{max}) \\ \Delta y(2) & \cdots & \Delta y(n_{max} + 1) \\ \vdots & \ddots & \vdots \\ \Delta y(T - n_{max}) & \cdots & \Delta y(T) \end{bmatrix} \quad (6)$$

Then the output could be expressed as

$$y = G\bar{u} + y' \quad (7)$$

where  $\{y'\} = \text{subspace}\{\mathcal{H}(\Delta y)\}$ [2].

Then following the eq.(7), the system could be expressed as following.

$$\begin{bmatrix} G \\ \vdots \\ G \end{bmatrix} \mathcal{H}(\Delta y) \begin{bmatrix} \bar{u} \\ L \end{bmatrix} = \begin{bmatrix} y(n_{max} + 1) \\ y(n_{max} + 2) \\ \vdots \\ y(T) \end{bmatrix} \quad (8)$$

In this expression, output  $y$  is measurement acquired by the sensor, while calibration coefficient  $G$  is known by the sensor parameter. Solving eq.(8) by least square estimation, directly leads to the quantity of interest  $\bar{u}$ .

### C. Naive Method

As it is formalised, the dynamic measurement problem is in fact a input estimation problem for constant step input LTI system. According to eq.(4), with the input  $\bar{u}$  and observations  $y(t)$ , the estimation of input could be acquired by

$$\hat{u}(t) = G^{-1}y(t) \quad (9)$$

in which  $G$  is determined by the sensor settings. Ideally, when  $G$  is full rank and known, and system error, offset is corrected by calibration procedure.[2]  $\bar{u} = G^{-1}y(+\infty)$ . However, in the transient stage, it may take certain time for the estimation  $\hat{u}$  to approach  $\bar{u}$ .

## III. EXPERIMENTS AND ANALYSIS

### A. Experiment Settings

The experiment is conducted by using the LEGO Mindstorms EV3 Robot Kit. A temperature sensor in the kit is used to capture the data, and transfers it to the PC end. A MATLAB GUI program is written by the author to carry out the signal processing and demonstrating the result and the analysis. The measuring process targets the temperature of hot water held in a heat isolating cup, and lasts for 2 minute long. As the process is rather slow, the sampling rate is 0.5 Hz.

### B. Online processing

Recursive least square estimation is carried out for DDFM method to give online processing for the signal input. In this sense, Kalman filter is not applicable for online processing since the measuring process model is not given, otherwise online system identification has to be carried beforehand. The result of the method is shown in Fig.1. In which we could clearly find that the DDFM estimation is faster and more accurate than the naive method, it approximates the real input value in shorter time.

### C. Measuring Noise Influence

As it is shown in previous section, the measuring process is always affected by the noise  $y_n$ . To study the influence of the noise, firstly, LTI system is simulated by specifying system order, and state space parameters. Then, white gaussian noise with different variance is added to the simulation data,

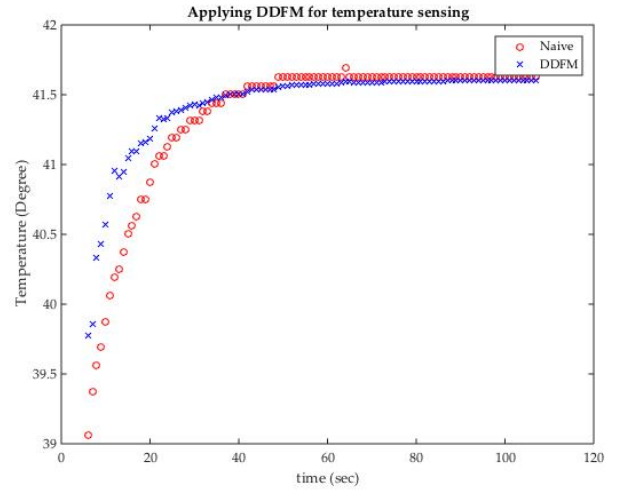


Fig. 1. Applying DDFM for temperature sensing

estimation error is collected to show the performance of the method.

The result is showed in the Fig.2, in which the black line shows the error of DDFM method estimation. It could be seen that the naive method is sensitive to the noise, while DDFM method perform some character of anti-noise in the signal processing, no significant increase in error with the increase of noise variance. The green line which indicates the performance of Kalman filter is the reference for best performance. Furthermore, according to the experiment, the method could also serve as de-noise filter in the data pre-processing stage in common use.

### D. Quantisation Noise Influence

Quantisation noise is unavoidable for digital measurement systems, and directly relates to the memory cost and computing speed. Experiment is carried out by simulating the LTI system as the measuring process and applying quantisation to the simulation data. Different quantisation levels are applied to test the method performance.

The result is showed in Fig.4, from which we could conclude that more quantisation levels would lead to better performance, but the improvement stopped at certain level. The quantisation would cause performance deterioration, especially in the transient stage.

Applying the MA(moving average) filter to the data, would cause a result showed in Fig.5, which indicates no enhance of performance is reached, on the contrary, it cause further negative effect for the performance.

Further, due to quantisation, a series of  $\Delta y$  may be zero even in transient stage. This may cause singular problem in the recursive least square estimation procedure. A possible solution is proposed to reset the recursion when detecting such behaviour, or by just adjust forgetting factor.

### E. Outlier Influence

TODO

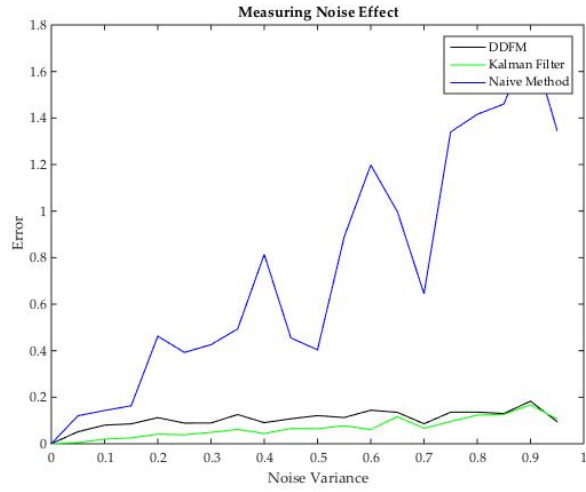


Fig. 2. Noise Effect Compare

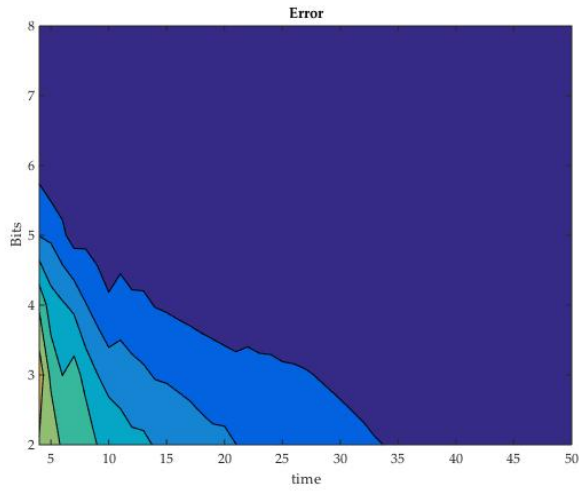


Fig. 3. Noise Effect Compare

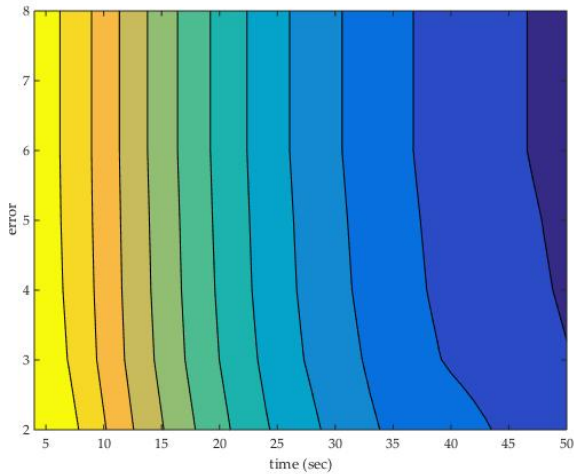


Fig. 4. Noise Effect Compare

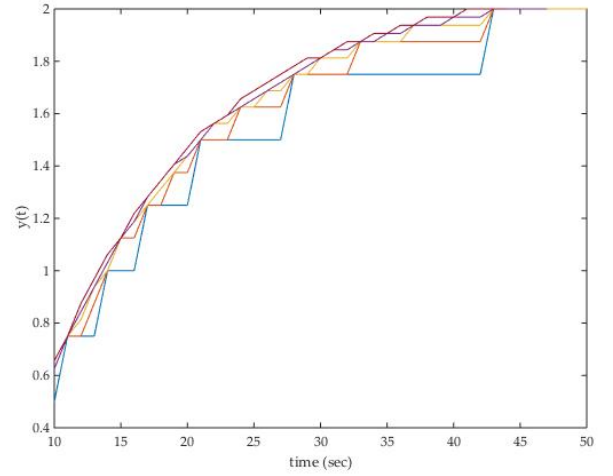


Fig. 5. Noise Effect Compare

#### IV. CONCLUSION

The conclusion goes here.

#### V. CONTRIBUTION SUMMERY

##### APPENDIX A

##### ENVIRONMENT INTRODUCTION

##### A. LEGO EV3

Lego Mindstorms EV3 is the third generation robotics kit in Lego's Mindstorms line. It is the successor to the second generation Lego Mindstorms NXT 2.0 kit. The LEGO EV3 main processor is TI Sitara AM1808(ARM926EJ-S core) with clock frequency 300 MHz, for memory, it equips 64 MB RAM with 16 MB Flash.[3]

##### B. MATLAB Interface

MATLAB®Support Package for LEGO®MINDSTORMS EV3 Hardware is a support package developed and supported by MATLAB®group to control LEGO®MINDSTORMS EV3 robots. The support package provides MATLAB functions to control the motors and interface with the hardware input sensors and output capabilities.

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