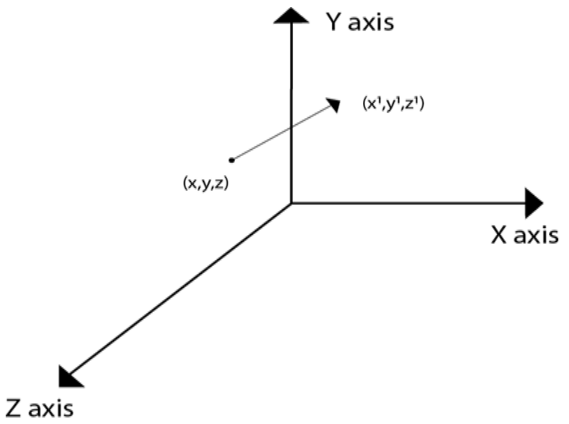
Three Dimensional Transformations

The geometric transformations play a vital role in generating images of three-Dimensional objects with the help of these transformations. The location of objects relative to others can be easily expressed. Sometimes viewpoint changes rapidly, or sometimes objects move in relation to each other. For this number of transformations can be carried out repeatedly.

3D Translation

It is the movement of an object from one position to another position. Translation is done using translation vectors. There are three vectors in 3D instead of two.

$x^1=x+T_x$
$y^1=y+T_y$
$z^1=z+T_z$

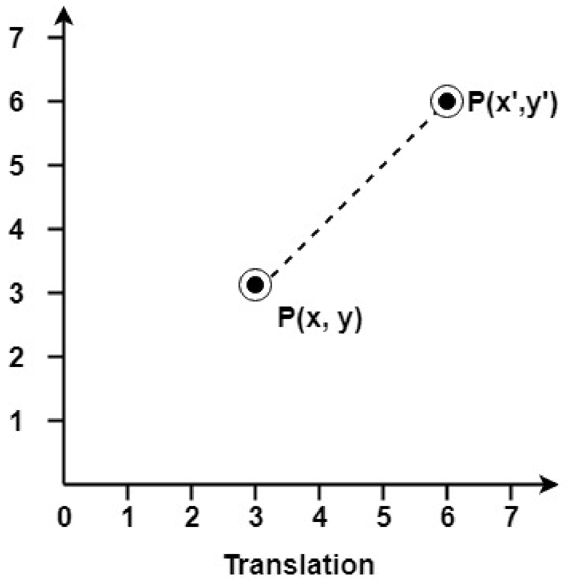


$$\begin{Bmatrix} 1 & 0 & 0 & 0 \\ 0 & 1 & 0 & 0 \\ 0 & 0 & 1 & 0 \\ T_x & T_y & T_z & 1 \end{Bmatrix} \text{ or } \begin{Bmatrix} 1 & 0 & 0 & T_x \\ 0 & 1 & 0 & T_y \\ 0 & 0 & 1 & T_z \\ 0 & 0 & 0 & 1 \end{Bmatrix}$$

2D Translation

It is the straight-line movement of an object from one position to another is called Translation. Here the object is positioned from one coordinate location to another.

$x_1=x+T_x$
$y_1=y+T_y$



$$\begin{vmatrix} 1 & 0 & 1 \\ 0 & 1 & 0 \\ t_x & t_y & 1 \end{vmatrix} \text{ Or } \begin{vmatrix} 1 & 0 & t_x \\ 0 & 1 & t_y \\ 0 & 0 & 1 \end{vmatrix}$$

Example: A point has coordinates in the x, y, z direction i.e., (5, 6, 7). The translation is done in the x-direction by 3 coordinate and y direction by 3 coordinates and in the z- direction by 2 coordinates. Shift the object. Find coordinates of the new position.

Solution: Co-ordinate of the point are (5, 6, 7)
Translation vector in x direction = 3
Translation vector in y direction = 3
Translation vector in z direction = 2

Multiply co-ordinates of point with translation matrix

$$(x^1 y^1 z^1) = (5,6,7,1) \begin{pmatrix} 1 & 0 & 0 & 0 \\ 0 & 1 & 0 & 0 \\ 0 & 0 & 1 & 0 \\ 3 & 3 & 2 & 1 \end{pmatrix}$$

= [5+0+0+3 0+6+0+3   0+0+7+2   0+0+0+1] = [8 9 9 1]

3D Scaling

Scaling is used to change the size of an object. The size can be increased or decreased. The scaling three factors are required Sx Sy and Sz.

$$\begin{Bmatrix} S_x & 0 & 0 & 0 \\ 0 & S_y & 0 & 0 \\ 0 & 0 & S_z & 0 \\ 0 & 0 & 0 & 1 \end{Bmatrix}$$

2D Scaling

It is used to alter or change the size of objects. The change is done using scaling factors. There are two scaling factors, i.e. Sx in x direction Sy in y-direction.

$$S = \begin{vmatrix} Sx & 0 & 0 \\ 0 & Sy & 0 \\ 0 & 0 & 1 \end{vmatrix}$$

2D Rotation:

It is a process of changing the angle of the object. Rotation can be clockwise or anticlockwise. For rotation, we have to specify the angle of rotation and rotation point. Rotation point is also called a pivot point. It is print about which object is rotated.

Matrix for rotation is an anticlockwise direction.

$$R = \begin{bmatrix} \cos\theta & \sin\theta \\ -\sin\theta & \cos\theta \end{bmatrix}$$

Matrix for homogeneous co-ordinate rotation (clockwise)

$$R = \begin{pmatrix} \cos\theta & -\sin\theta & 0 \\ \sin\theta & \cos\theta & 0 \\ 0 & 0 & 1 \end{pmatrix}$$

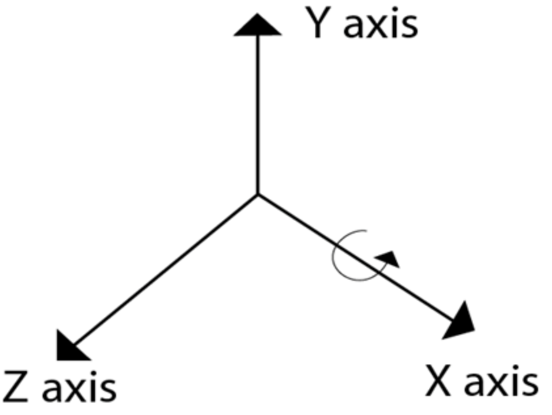
Matrix for homogeneous co-ordinate rotation (anticlockwise)

$$R = \begin{pmatrix} \cos\theta & \sin\theta & 0 \\ -\sin\theta & \cos\theta & 0 \\ 0 & 0 & 1 \end{pmatrix}$$

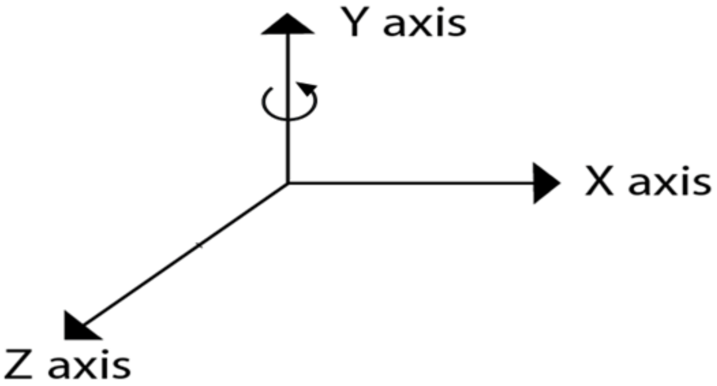
3D Rotation

It is moving of an object about an angle. Movement can be anticlockwise or clockwise. 3D rotation is complex as compared to the 2D rotation. For 2D we describe the angle of rotation, but for a 3D angle of rotation and axis of rotation are required. The axis can be either x or y or z.

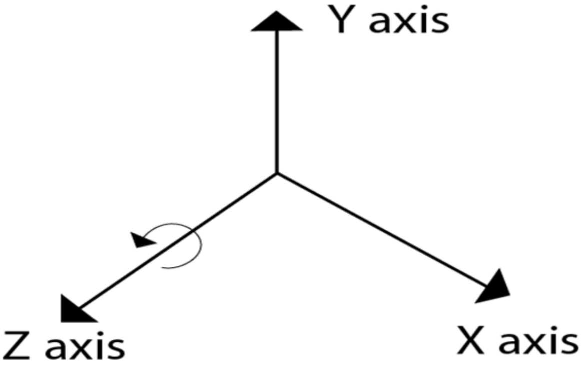
Following figures shows rotation about x, y, z- axis



rotation object X axis anticlockwise



rotation object Y axis anticlockwise



rotation object Z axis anticlockwise

Matrix for representing three-dimensional rotations about the Z axis

$$\begin{pmatrix} \cos\theta & -\sin\theta & 0 & 0 \\ \sin\theta & \cos\theta & 0 & 0 \\ 0 & 0 & 1 & 0 \\ 0 & 0 & 0 & 1 \end{pmatrix}$$

Matrix for representing three-dimensional rotations about the X axis

$$\begin{pmatrix} 1 & 0 & 0 & 0 \\ 0 & \cos\theta & -\sin\theta & 0 \\ 0 & \sin\theta & \cos\theta & 0 \\ 0 & 0 & 0 & 1 \end{pmatrix}$$

Matrix for representing three-dimensional rotations about the Y axis

$$\begin{pmatrix} \cos\theta & 0 & \sin\theta & 0 \\ 0 & 1 & 0 & 0 \\ -\sin\theta & 0 & \cos\theta & 0 \\ 0 & 0 & 0 & 1 \end{pmatrix}$$

2D Reflection:

It is a transformation which produces a mirror image of an object. The mirror image can be either about x-axis or y-axis. The object is rotated by180°.

1. Reflection about x-axis: The object can be reflected about x-axis with the help of the following matrix

$$\begin{bmatrix} 1 & 0 & 0 \\ 0 & -1 & 0 \\ 0 & 0 & 1 \end{bmatrix}$$

Reflection about y-axis:

$$\begin{bmatrix} -1 & 0 & 0 \\ 0 & 1 & 0 \\ 0 & 0 & 1 \end{bmatrix}$$

Reflection about line y=x:

$$\begin{bmatrix} 0 & 1 & 0 \\ 1 & 0 & 0 \\ 0 & 0 & 1 \end{bmatrix}$$

Reflection about an axis perpendicular to xy plane and passing through origin:

$$\begin{bmatrix} -1 & 0 & 0 \\ 0 & -1 & 0 \\ 0 & 0 & 1 \end{bmatrix}$$

3D Reflection

It is also called a mirror image of an object. For this reflection axis and reflection of plane is selected. Three-dimensional reflections are similar to two dimensions. Reflection is 180° about the given axis. For reflection, plane is selected (xy,xz or yz). Following matrices show reflection respect to all these three planes.

- Reflection about x-axis:-

x'=x   y'=-y   z'=-z

$$\begin{matrix} 1 & 0 & 0 & 0 \\ 0 & -1 & 0 & 0 \\ 0 & 0 & -1 & 0 \\ 0 & 0 & 0 & 1 \end{matrix}$$

Reflection about Y-axis:-

X'=-X   Y;=-Y   Z'=-Z

$$\begin{matrix} -1 & 0 & 0 & 0 \\ 0 & 1 & 0 & 0 \\ 0 & 0 & -1 & 0 \\ 0 & 0 & 0 & 1 \end{matrix}$$

- Reflection about z-axis:-

x'=-x   y'=-y   z'=z

$$\begin{matrix} -1 & 0 & 0 & 0 \\ 0 & -1 & 0 & 0 \\ 0 & 0 & 1 & 0 \\ 0 & 0 & 0 & 1 \end{matrix}$$

Reflection relative to XY plane

$$\begin{pmatrix} 1 & 0 & 0 & 0 \\ 0 & 1 & 0 & 0 \\ 0 & 0 & -1 & 0 \\ 0 & 0 & 0 & 1 \end{pmatrix}$$

Reflection relative to YZ plane

$$\begin{pmatrix} -1 & 0 & 0 & 0 \\ 0 & 1 & 0 & 0 \\ 0 & 0 & 1 & 0 \\ 0 & 0 & 0 & 1 \end{pmatrix}$$

Reflection relative to ZX plane

$$\begin{pmatrix} 1 & 0 & 0 & 0 \\ 0 & -1 & 0 & 0 \\ 0 & 0 & 1 & 0 \\ 0 & 0 & 0 & 1 \end{pmatrix}$$

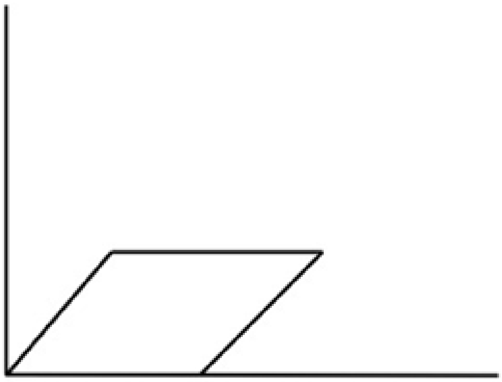
2D Shearing:

It is transformation which changes the shape of object. The sliding of layers of object occurs. The shear can be in one direction or in two directions.

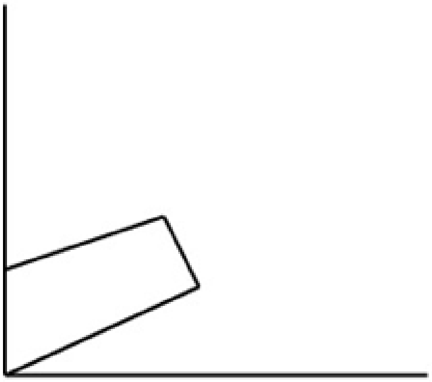
Shearing in the X-direction: In this horizontal shearing sliding of layers occur. The homogeneous matrix for shearing in the x-direction is shown below:

$$\begin{bmatrix} 1 & 0 & 0 \\ Sh_x & 1 & 0 \\ 0 & 0 & 1 \end{bmatrix}$$

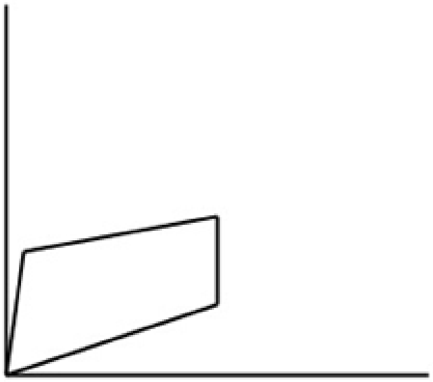
Original Object



Shear in X direction



Shear in Y direction



Shear in both directions

Shearing in the Y-direction: Here shearing is done by sliding along vertical or y-axis.

$$\begin{bmatrix} 1 & Sh_y & 0 \\ 0 & 1 & 0 \\ 0 & 0 & 1 \end{bmatrix}$$

Shearing in X-Y directions: Here layers will be slided in both x as well as y direction. The sliding will be in horizontal as well as vertical direction. The shape of the object will be distorted. The matrix of shear in both directions is given by:

$$\begin{bmatrix} 1 & Sh_y & 0 \\ Sh_x & 1 & 0 \\ 0 & 0 & 1 \end{bmatrix}$$

3D Shearing

It is change in the shape of the object. It is also called as deformation. Change can be in the x -direction or y -direction or both directions in case of 2D. If shear occurs in both directions, the object will be distorted. But in 3D shear can occur in three directions.

- In space, we divide shear transformation according to the direction of the surfaces xy,xz and yz. Values of Sx,Sy and Sz determine shear transformation sizes for all the directions.

A shear transformation about the xy plane :

$$A_{xy} = \begin{vmatrix} 1 & 0 & 0 & 0 \\ 0 & 1 & 0 & 0 \\ S_x & S_y & 0 & 0 \\ 0 & 0 & 0 & 1 \end{vmatrix}$$

A shear matrix about the xz plane :

$$A_{xz} = \begin{vmatrix} 1 & 0 & 0 & 0 \\ S_x & 1 & S_z & 0 \\ 0 & 1 & 1 & 0 \\ 0 & 0 & 0 & 1 \end{vmatrix}$$

A shear matrix about the yz plane :

$$A_{yz} = \begin{vmatrix} 1 & S_y & S_z & 0 \\ 0 & 1 & 0 & 0 \\ 0 & 0 & 1 & 0 \\ 0 & 0 & 0 & 1 \end{vmatrix}$$