Specular Delay Retracing Problem

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Application and Context:

For the global navigation satellite system (GNSS), pseudorandom code is used to roughly determine the range between satellites and the receiver. When a signal is received, a locally generated pseudorandom code is correlated with the incoming signal. Due to the autocorrelated character of the pseudorandom code, the correlation power can only be accumulated when the local code is shifted till near alignment with the incoming signal. For the legacy GPS L1 C/A signal, it will form a triangular-shape autocorrelation function (ACF) when the estimated delay $\hat{\tau}$ is within $\pm$ 1 chip (~ 293 m) of the actual delay, see Figure 1.

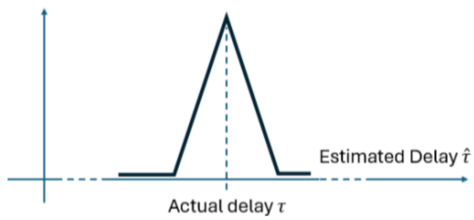

Fig. 1 Autocorrelation function

To track the actual delay, receivers usually generate 3 or more local replicas with different offsets relative to the estimated delay, see Figure 2. Discriminator functions such as early minus late power (EML, see (1)) can make use of the triangular geometry to determine the current delay estimation error. Fitting methods can also be used to determine both the amplitude and delay since the triangular shape is known.

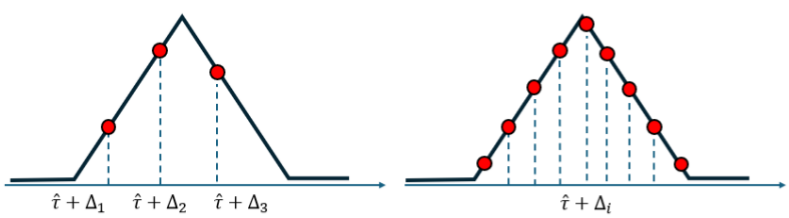

Fig. 2 Correlation values

$$\Delta\tau = \frac{1}{2} \cdot \frac{E^2 - L^2}{E^2 + L^2} \tag{1}$$

However, for GNSS-reflectometry (GNSS-R) applications, where the signal is reflected on the earth's surface before it is received, the autocorrelation function is no longer triangular due to the scattering effect during reflection, see Figure 3. The scattered signal is attenuated and delayed relative to the specular reflection. The ACF under scattering can be seen as the convolution of specular ACF and an attenuation function, see Figure 4. The attenuation function shows the power attenuation when the scattering delay drifts away from the specular delay, and is usually determined by reflection surface roughness, material, receiver-transmitter geometry and motion. To simplify the model, we assume the attenuation function is a power exponential function in (2) in this study.

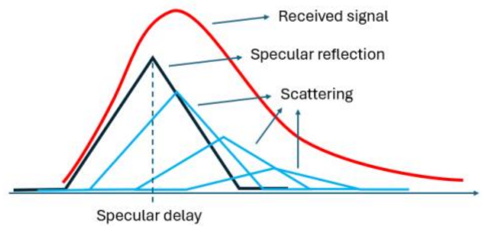

Fig. 3 Autocorrelation under scattering

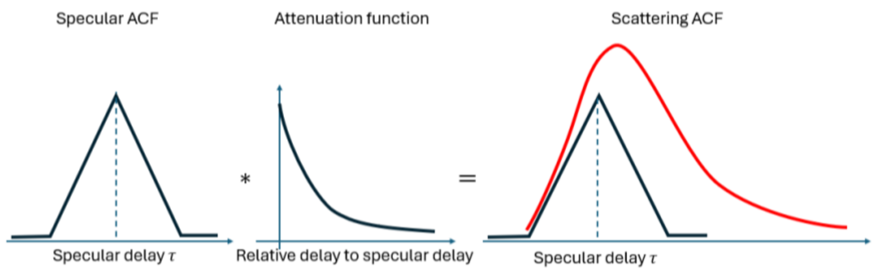

Fig. 4 Scattering ACF

$$g(\tilde{\tau}) = \sqrt{\frac{\alpha \cdot 2^{\frac{1}{\alpha}}}{\mathrm{a} \cdot \Gamma\left(\frac{1}{\alpha}\right)} e^{-\left(\frac{\tilde{\tau}}{a}\right)^{\alpha}}} \qquad \left(\int_0^{+\infty} g^2(\tilde{\tau})\, d\tilde{\tau} = 1\right) \tag{2}$$

In GNSS-R applications, retracing the specular delay using pseudorandom code is the key point for absolute altimetry retrieval. Furthermore, the retrieval of the attenuation function can be used to estimate sea surface roughness, which is correlated with ocean wind speed.

The challenges in this study and relevant techniques are as follows:

1. Parameters in the attenuation function make the system highly non-linear and unstable, a particle filter is desired.
2. In the state, we not only estimate specular delay and amplitude, but also the scattering ACF model. This is suitable for Rao-Blackwellized PF where the states are separated into easy and tough states. Here, the specular delay and its derivative is linear and therefore easy states. But the amplitude and scattering ACF model is highly non-linear and tough states.
3. The correlation noises are spatially correlated, forming red noises in the measurements.
4. Reflection signals are usually weak and don't last long, the system might not even converge during such a short time and large measurement noise. We probably need to add backward filtering to make sure the entire segment has been well-filtered.
5. State amplitude should always be a positive value. In strong signal condition, for instance A=10, a process noise with std=1 is reasonable. However, in weak signal condition, A=1, it can cause the amplitude to flip to the negative side and cause the system to deviate. So the process noise distribution can't be designed as a symmetry pdf such as normal distribution. This can be done in a PF where the importance pdf q can simply be cut off below 0.

Initial Problem Formulation

The state variables are:

$$\boldsymbol{x} = [\tau, A, a, \alpha]^T$$

Where τ and A is the specular delay and amplitude, see Figure 4. a and α are parameters in the attenuation function, see (2). The measurement equation is:

$$\boldsymbol{y}_i = A \cdot R(\tau - \hat{\tau} - \Delta_i) * g(\tilde{\tau}) + \varepsilon_i, \qquad R(\Delta\tau) = \begin{cases} 1 - |\Delta\tau|, & |\Delta\tau| < 1 \\ 0, & otherwise \end{cases}$$

Where $\hat{\tau}$ is the specular delay guess and is known, Δ_i is different offsets relative to the specular delay guess. The (i, j) entry of the measurement noise covariances matrix is:

$$cov(\varepsilon_i, \varepsilon_j) = \frac{1}{4T} \cdot R(\Delta_i - \Delta_j)$$

Where T is the coherent integration time and is fixed at 20 ms in this study. The dynamics of τ and A is

$$\tau_{k+1} \sim N\left(\tau_k, \frac{T^3}{3} \cdot 2e - 8\right);\ A = \sqrt{\frac{10^{\frac{C/N_0}{10}}}{2}};\ C/N_{0_{k+1}} \sim N(C/N_{0_k}, T \cdot 1e - 3)$$

To simply the model further, we assume that a and α are constants and don't have dynamics.

Objectives

Level 1:

(a). Assume attenuation function is powered exponential with fixed parameters, signal quality is good ($C/N_0 >$ 30 dB-Hz). Apply sufficient amount of correlation values and long signal duration, see if the states converge to the truth. (b). Compare results with filter that uses perfect triangular peak as the ACF.

Level 2:

(a). Lower signal quality to ~ 25 dB-Hz and duration to ~ 100sec, which is the typical level for GNSS-R on ocean surface, use backward filtering to make sure the entire segments are well-filtered. (b). Limit the number of correlation values to only 3 and compares the results to large number of correlation values.

Level 3:

(a). Add changes to the powered exponential parameters and see if the states are able to follow the changes. (b). Change the attenuation function of the measurements to other classical functions such as Matern and spherical, while still using powered exponential as the model. See how this affect the results. (c). What if we use Matern or spherical as the model?

Task and Milestone Roadmap

Week 1: Implement the dynamics and measurement equations. Compare the nominal measurements with different signal qualities from 25 dB-Hz to 40 dB-Hz, and evaluate the difference among correlation values.

Week 2: Code up Rao-Blackwellized PF and investigate whether we need backward filtering. Compare results with filter that uses perfect triangular peak as the ACF.

Week 3: Test various parameters, such as Lower signal quality to ~ 25 dB-Hz, duration to ~ 100sec, and Limit the number of correlation values to only 3.

Week 4: Writing project reports and try the challenge objectives. Add changes to the powered exponential parameters and Change the attenuation function of the measurements to other classical functions.