Report20230720

1. The flyby animation is wrong, I am not sure it provides a correct computation results. But the output figures seem good.
2. By shortening the integration time interval, the global momentum error will decrease.

Shape Model

In version 4 code (main_gen_sample_v4.py, main_computation_v4.py), I used the two-body configuration (Apophis_N2_v2000.obj) to create a test case. The restitution is zero, and the fraction is 0.5.

Torque-free case

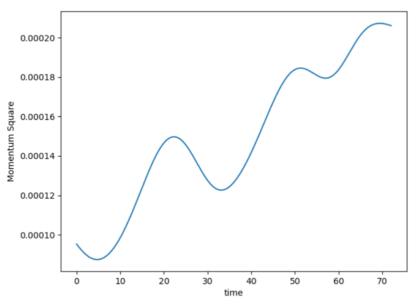


Fig.1 Angular Momentum Square

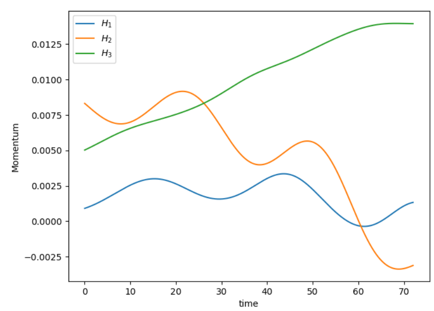


Fig.2 Angular Momentum components

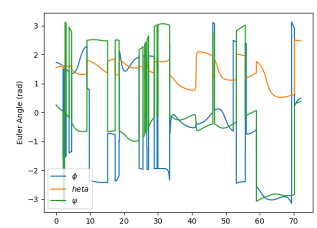


Fig.3 Euler Angle

There are some errors during the simulation. The calculation (for example, mass, coordination, etc) always has only eight effective figures, which come from the underlying C language code, and I can't change that. But I can change the propagation time interval to shrink errors.

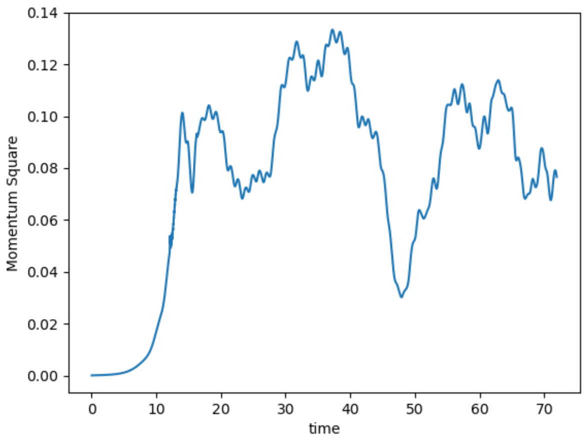


Fig.4 Time Interval 1E-1

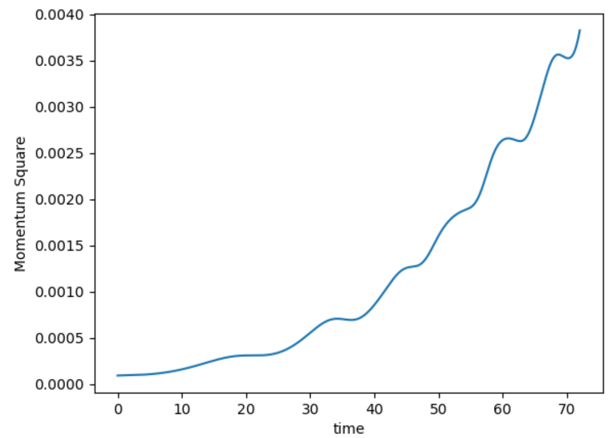


Fig.5 Time Interval 1E-2

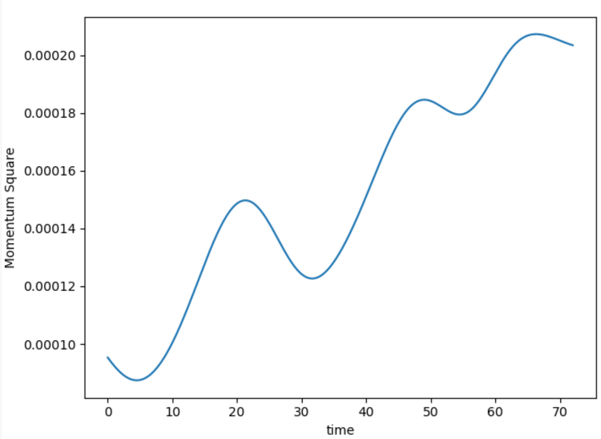


Fig.7 Time Interval 1E-3

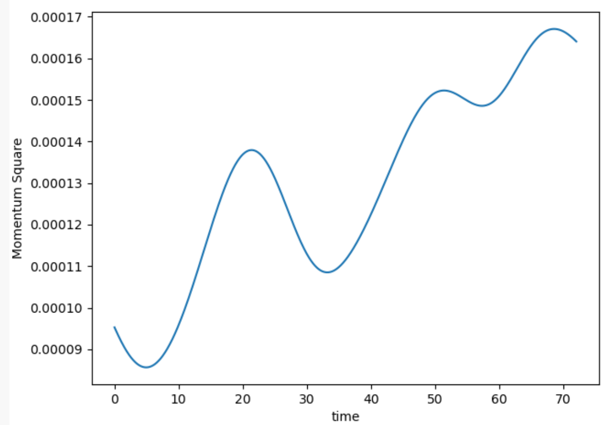


Fig.8 Time Interval 1E-4

Smaller time intervals spend longer time on computation but higher precision. These results show the best time interval is $dt = 1E - 3$. Then an animation provides more insight

Video 1. Torque-free animation

Flyby case

I simulated a 3-day flyby case, by inversely integrating position/velocity at C/A to get the initial state. The closest Approach at Apr 13 2029 21:46 (38012 km radius) is

$$\begin{aligned} Pos_{C/A} &= [-1.918E+04, 3.225E+04, 6.007E+03]\ km \\ Vel_{C/A} &= [6.332E+00, 3.405E+00, 1.844E+00]\ km/s \end{aligned} \tag{1}$$

and the units are

$$[L] = 168\ m,\ [M] = 2.650/G\ kg,\ [T] = \sqrt{[L]^3/G[M]} \tag{2}$$

And non-dimensional time interval is $dt = 1E-3$.

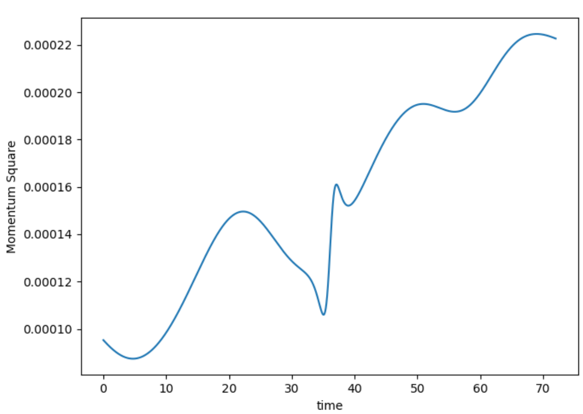


Fig.9 Flyby Angular Momentum Square

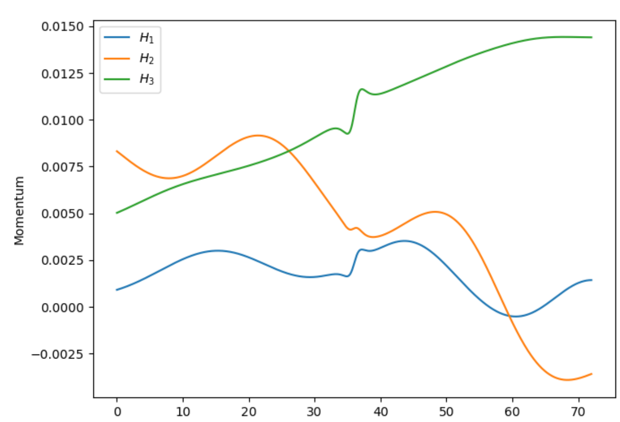
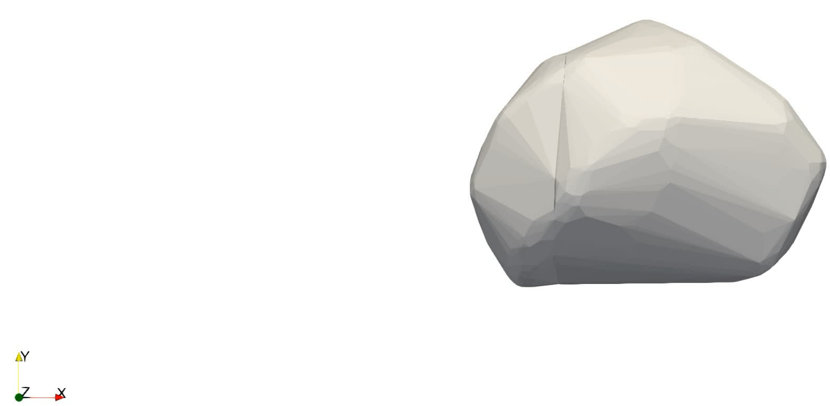


Fig.10 Flyby Angular Momentum Components

Video 2. Flyby animation with 0.5 fraction

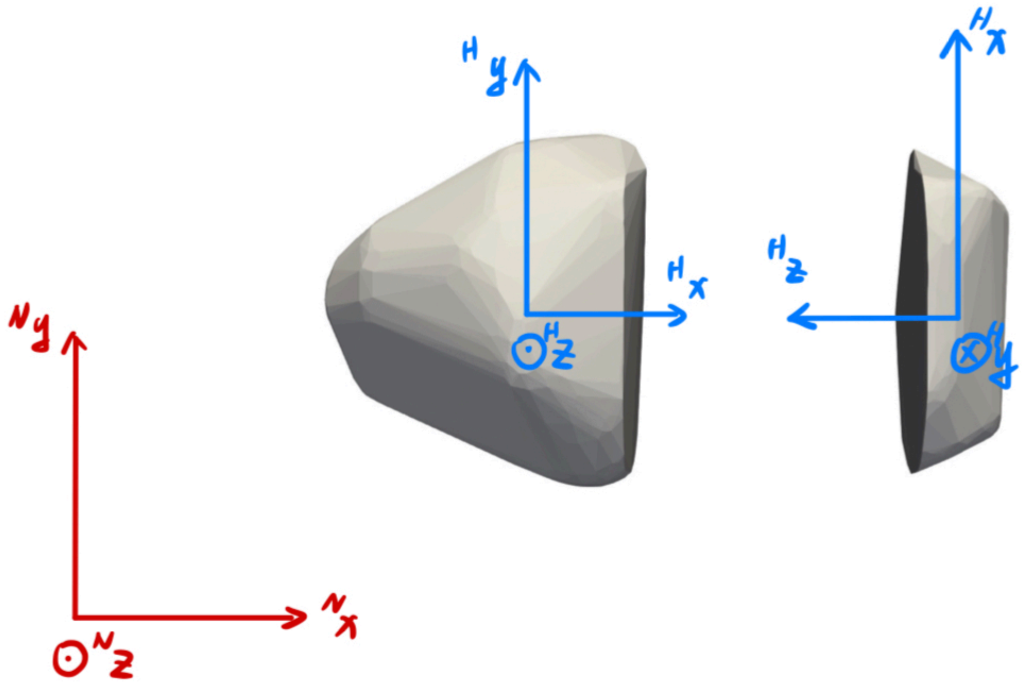


Video 3. Flyby animation with zero fraction

Spin Rate

In version 3 code (main_gen_sample_v3.py, main_computation_v3.py), I used the two-body configuration (Apophis_N2_v2000.obj) to create a test case. The density is 10, and the total volume normalizes the polyhedron avatars. And a tricky thing is the initial spin rate setting.

Three frames should be introduced first: body-fixed ($\mathcal{B}$), body-inertial ($\mathcal{H}$), and inertial frames ($\mathcal{N}$). The body-fixed and body-inertial frames have the same origin, but the body-fixed frame is fixed on the polyhedron body and the axes are aligned with the principal inertia axis.



Therefore, when we call the function of setting the initial velocity

```
vel_ome = [vel[0],vel[1],vel[2],omega[0],omega[1],omega[2]]
poly.imposeInitValue(component=[1,2,3,4,5,6], value=vel_ome)
```

the ω input here means ${}^{\mathcal{B}}\omega_{\mathcal{B}/\mathcal{H}}$. If I set $\omega =^{\mathcal{B}} \omega_{\mathcal{B}/\mathcal{H}} = [0, 0, 2\pi]$, we have

Video 4. Fixed position and body-fixed spin rate animation

We can get the DCM from the moment of inertia. The moment of inertia of polyhedron can be computed ${}^{\mathcal{N}}[I]$, from this, the DCM is the stacked Eigenvectors.

$$[BN]^{\mathcal{N}}[I][NB] = {}^{\mathcal{B}}[I] \tag{3}$$

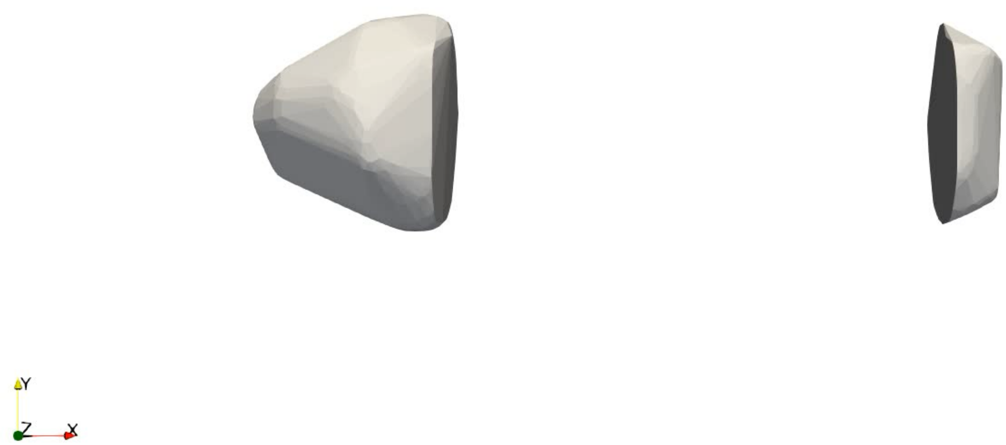
If I set the ${}^{\mathcal{N}}\omega_{\mathcal{B}/\mathcal{H}} = [0, 0, 2\pi], \omega = [BN]^{\mathcal{N}}\omega_{\mathcal{B}/\mathcal{H}}$, we have

Video 5. Fixed position and inertial spin rate animation

The initial velocity can be obtained from

$$vel_0 = {}^{\mathcal{N}}\omega_{\mathcal{B}/\mathcal{H}} \times r_i \tag{4}$$

where r_i is the body distance between center of mass and body-i.



Video 6. Inertial rate animation

Total Moment of Inertia

$$^{\mathcal{N}}I_{total} = {}^{\mathcal{N}}I_i + V_i \begin{bmatrix} d_2^2 + d_3^2 & -d_1 d_2 & -d_1 d_3 \\ -d_1 d_2 & d_1^2 + d_3^2 & -d_2 d_3 \\ -d_1 d_3 & -d_2 d_3 & d_1^2 + d_2^2 \end{bmatrix} \tag{5}$$

where $d = [d_1, d_2, d_3]$ is the center of mass of i-th body w.r.t total center of mass.

$$^{\mathcal{N}}[I_{total}] = [NB]^{\mathcal{B}}[I_{total}][NB]^T \tag{6}$$

```
I_diag, P = np.linalg.eigh(inertia_total)
P[:, 2] = np.cross(P[:, 0], P[:, 1])
```

Here

$$P = [NB] \tag{7}$$