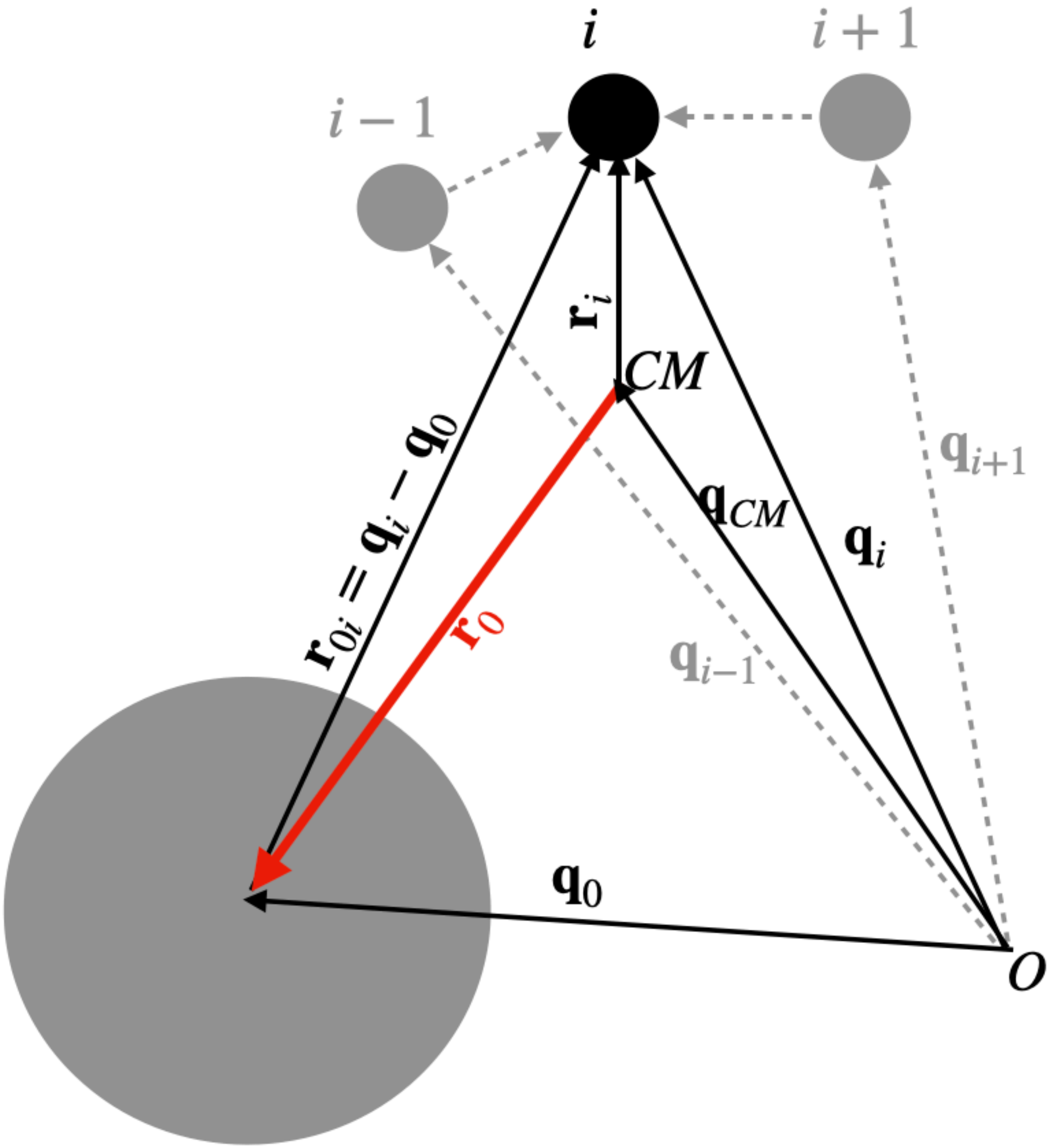
report20230318

Problem:

1. I am not sure the animation is right in showing the bodies' relative motion during the flyby. I am trying to verify my results.

2. Orbital Dynamics v2.0



This is an N-body problem, so the equation of motion is focused on one single body. At first, bodies that make up an asteroid are

$$m_i F^{ext} = m_i \ddot{\mathbf{q}}_i = -\sum_{\substack{j=1 \\ j\neq i}}^{n} \frac{G m_i m_j \left(\mathbf{q}_j - \mathbf{q}_i\right)}{\left\|\mathbf{q}_j - \mathbf{q}_i\right\|^3} = -\sum_{\substack{j=1 \\ j\neq i}}^{n} \frac{G m_i m_j \mathbf{r}_{ji}}{r_{ji}^3}$$

Then we added the planet perturbation

$$m_i F^{ext} + m_0 F^0 = m_i \ddot{\mathbf{q}}_i = -\sum_{\substack{j=1 \\ j \neq i}}^{n} \frac{G m_i m_j \mathbf{r}_{ji}}{r_{ji}^3} - \frac{G m_0 m_i \mathbf{r}_{0i}}{r_{0i}^3}$$

Transport the frame center to the asteroid's center mass

$\mathbf{q}_i = \mathbf{q}_{cm} + \mathbf{r}_i, \quad \mathbf{q}_{cm} = \frac{\sum_{i=1}^n m_i \mathbf{q}_i}{\sum_{i=1}^n m_i} = \frac{\sum_{i=1}^n m_i \mathbf{q}_i}{m_A}$

where $\mathbf{r_0}$ is the flyby orbit at the Apophis's center mass frame. Then the $i-th$ body's acceleration can be expressed as

$$m_i \ddot{\mathbf{r}}_i = m_i \ddot{\mathbf{q}}_i - m_i \ddot{\mathbf{q}}_{cm} = -\sum_{\substack{j=1 \\ j \neq i}}^{n} \frac{G m_i m_j \mathbf{r}_{ji}}{r_{ji}^3} - \frac{G m_0 m_i \mathbf{r}_{0i}}{r_{0i}^3} - m_i \ddot{\mathbf{q}}_{cm}$$

in which

$m_i \ddot{\mathbf{q}}_{cm} = \frac{G m_0 m_A \mathbf{r}_0}{r_0^3}$

$m_0 \ddot{\mathbf{q}}_0 = -\frac{G m_0 m_A \mathbf{r}_0}{r_0^3}$

The mass m_A is the asteroid mass.

3. Flyby Orbit

From the known knowledge, we have the Geocentric hyperbolic orbit elements σ_G. We need to transport it to the Apophis's center mass frame $\mathbf{r}_0$.

$\sigma_G \Rightarrow \mathbf{R}_G \Rightarrow \mathbf{r}_0$

The intermediate variable $\mathbf{R}_G$ is the asteroid's vector at the Earth Center Inertial Coordinates (ECI). We have the equation

$\mathbf{r_0} = -\mathbf{R}_G$

Considering the two body problems and hyperbolic orbit, we have

$r_0 = R_G = \frac{a(e^2-1)}{1+e\cos f}.$

Solve the two-body problem,

$m_i \ddot{\mathbf{q}}_{cm} = \frac{G m_0 m_A \mathbf{r}_0}{r_0^3}$

$m_0 \ddot{\mathbf{q}}_0 = -\frac{G m_0 m_A \mathbf{r}_0}{r_0^3}$

we can get the solution w.r.t $\mathbf{r}_0$.

$\ddot{\mathbf{r}}_0 = \ddot{\mathbf{q}}_0 - \ddot{\mathbf{q}}_{cm} = -\frac{G(m_0+m_A)\mathbf{r}_0}{r_0^3}$

Employing a normalized unit to speed up the integration

$[L]=R_E,\ [M]=m_0+m_A,\ [T]=\sqrt{[L]^3/G[M]},\ G=1$

So we have

$\ddot{\mathbf{r}}_0=-\frac{\mathbf{r}_0}{r_0^3}$

After integration, we will put the unit back in the main computation scheme.

Considering the higher-order terms in Earth's gravity by spherical harmonic expansion

$$U=\frac{Gm_0}{R_E}\sum_{n=0}^{\infty}\sum_{m=0}^{n}\left(\frac{R_E}{r}\right)^{n+1}P_{nm}(\sin\phi)\cdot\begin{bmatrix}\cos(m\lambda)\\ \sin(m\lambda)\end{bmatrix}\cdot\begin{bmatrix}C_{nm}\\ S_{nm}\end{bmatrix},$$

The 0th, 1st and 2nd orders are considered at first,

$U_0=\frac{Gm_0}{r},\ U_1=\frac{Gm_0}{r^3}\mathbf{r}\cdot\mathbf{r}_{cm},\ U_2=\frac{C_{20}}{r^3}\left(\frac{3}{2}\frac{z^2}{r^2}-\frac{1}{2}\right)+\frac{3C_{22}}{r^5}\left(x^2-y^2\right)$

in which,

$$\vec{r}_{cm}=\begin{bmatrix}R_EC_{11}\\ R_ES_{11}\\ R_EC_{10}\end{bmatrix},\ \frac{\mathbf{r}}{|r|}=\hat{\mathbf{r}}=\begin{bmatrix}\cos\phi\cos\lambda\\ \cos\phi\sin\lambda\\ \sin\phi\end{bmatrix}$$

Accelerations of these two terms are

$\frac{\partial U_0}{\partial\mathbf{r}}=-\frac{Gm_0\mathbf{r}}{r^3},\ \frac{\partial U_1}{\partial r}=\frac{Gm_0}{r^3}\left[I_{[3\times 3]}-3\hat{r}\hat{r}\right]\cdot r_{\mathrm{CM}}$

3.1. Flyby Orbit Implementation

The Apophis flyby orbit is a Geocentric hyperbolic orbit, Orbital elements are given

Geocentric hyperbolic orbit		
Periapsis radius	3.72×10^4	km
Eccentricity	4.229	
Inclination	47.8	deg
Argument of periapsis	-143.9	deg
Longitude of asc. node	1.8	deg

The true anomaly can be used to set the initial state of Apophis, and here it is $f=-100\ deg$.

Then the transformation from orbital elements to Position/Velocity can be employed here. With the initial state, an integration over **3 days** are computed based on the above equation of motion. The left one uses the normalized unit, and the right one is united km.

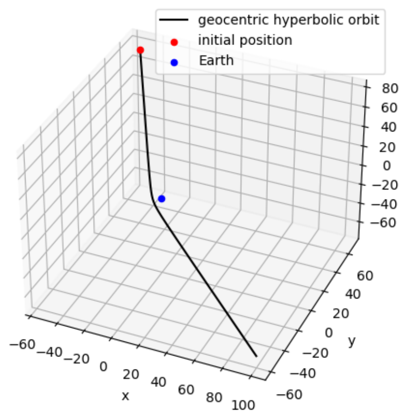


Fig.1

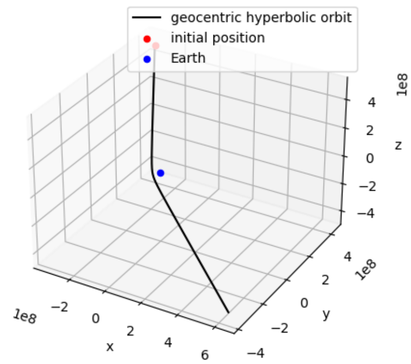
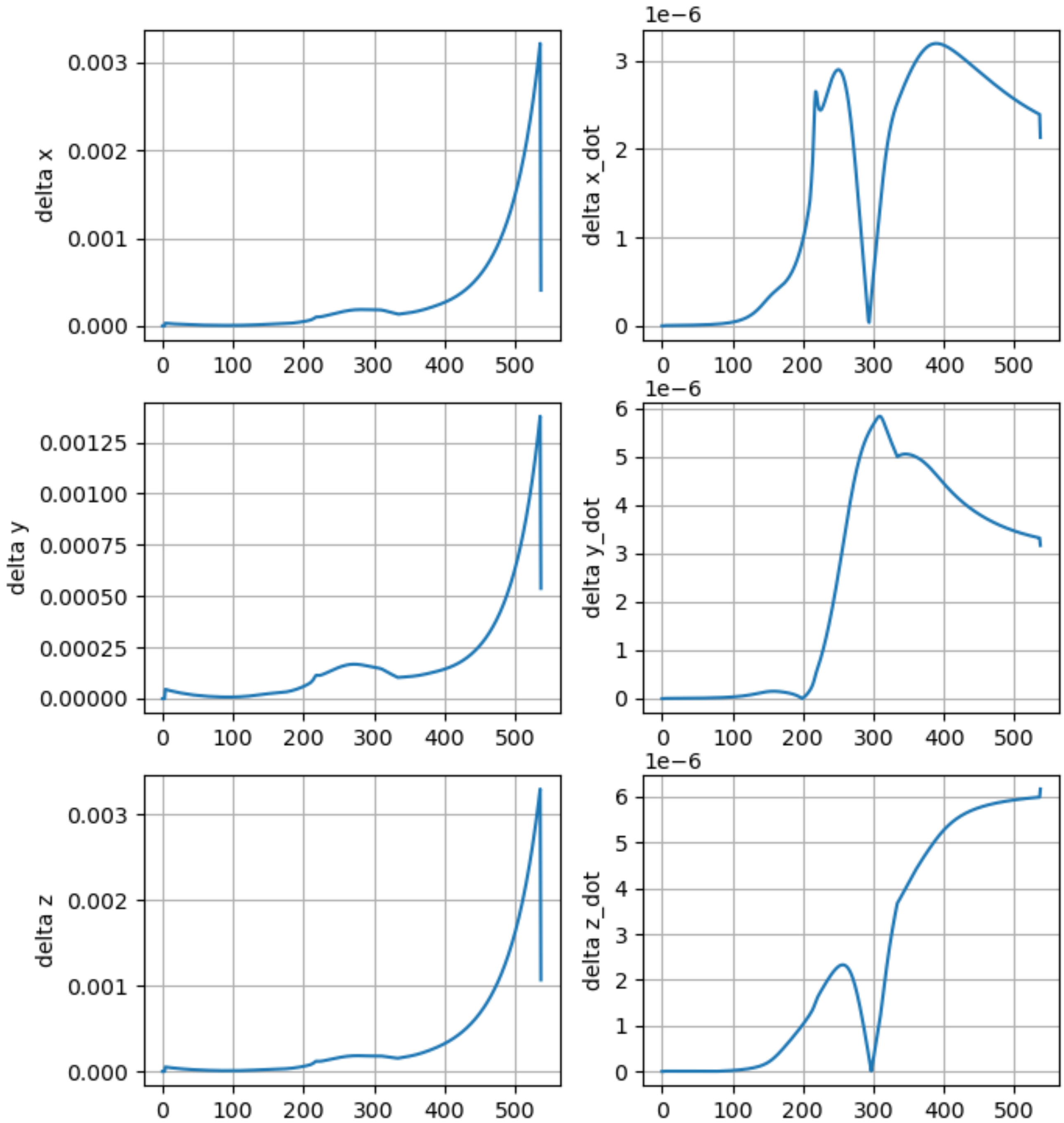


Fig.2

The comparison between the 0th-order and 2nd-order gravity is determined with the normalized unit.



delta x
0.003
0.002
0.001
0.000
0
100
200
300
400
500
1e−6
delta x_dot
3
2
1
0
delta y
0.00125
0.00100
0.00075
0.00050
0.00025
0.00000
1e−6
delta y_dot
6
5
4
delta z
1e−6
delta z_dot

4. Shape Model

4.1. Shape Model regular #1

This Shape model is made by two regular polygons generated from spheres with radii 'R' and 'r'. The two radii of the sphere can be solved from the known parameter 'volume' and 'largest extent.' For Apophis, the 'volume = 1.986E7 m^3', the 'largest extent = 410 m'. The 'largest extent' also can be substituted by the 'mean radius = 168 m'.

The initial positions are $[r_1, 0, 0]$ and $[-r_2, 0, 0]$, and these will be adjusted to the center of mass frame.

https://user-images.githubusercontent.com/38872598/225046066-256d5543-6817-4fb9-bac9-dd5a495a2acd.mp4

Then the shape model 1 plus the Earth's perturbation, but I'm not sure it's right.

https://user-images.githubusercontent.com/38872598/226085971-8ac1927a-9e38-4147-afe7-a60cd4bf3460.mp4