Report 20230301

- [x] Debug the shape generation
- [x] Flyby physical model
- [ ] Debug flyby code
- [ ] Implement the Shape Segmentation Algorithm

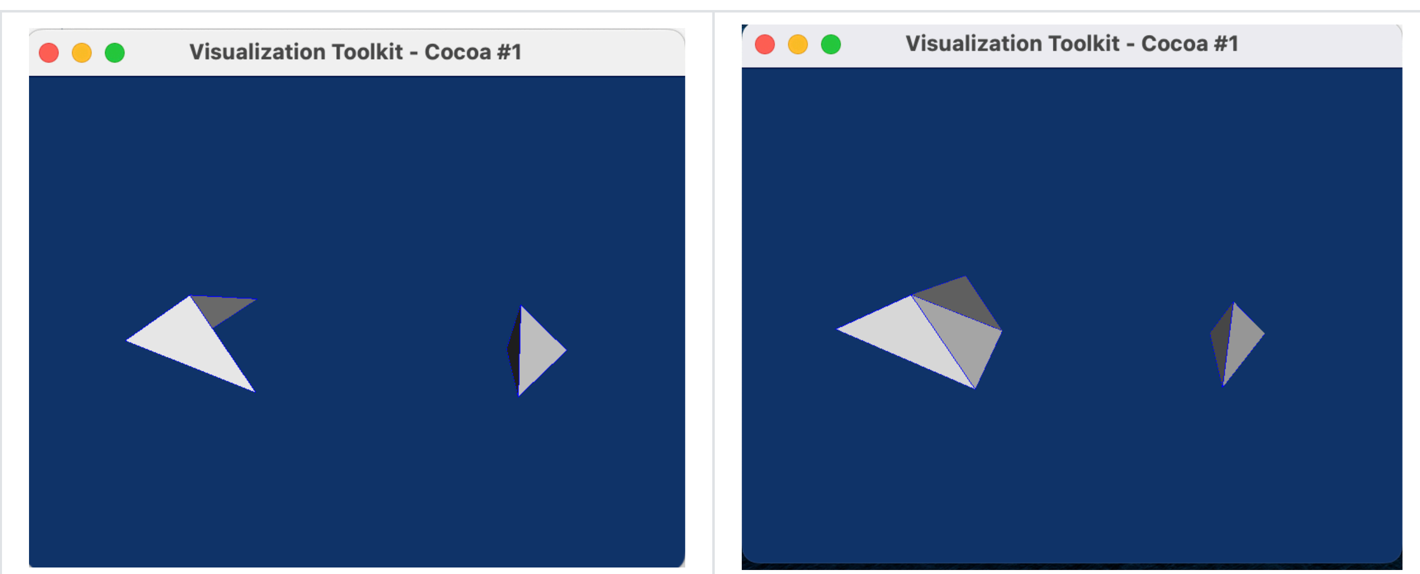
- The first-order acceleration in Conor2023 Eq.3 and Takahashi2013 Eq.16 is different. Is there any difference in their model? Where is Conor's equation from?

$$\begin{aligned} \boldsymbol{a}_{\text{tide}} &= -\frac{\mu_e}{R^3}[\boldsymbol{U} - \hat{\boldsymbol{R}}\hat{\boldsymbol{R}}] \cdot \boldsymbol{r} \\ \frac{\partial U_1}{\partial r} &= \frac{GM^*}{r^3}\left[1_{[3\times 3]} - 3\hat{r}\hat{r}\right] \cdot r_{\text{CM}} \end{aligned} \tag{1}$$

- How to track the polyhedron combo's altitude?
- When constructing a polyhedron combination, is it necessary to ensure the quality and volume parameters are consistent with Apophis?

version20230214

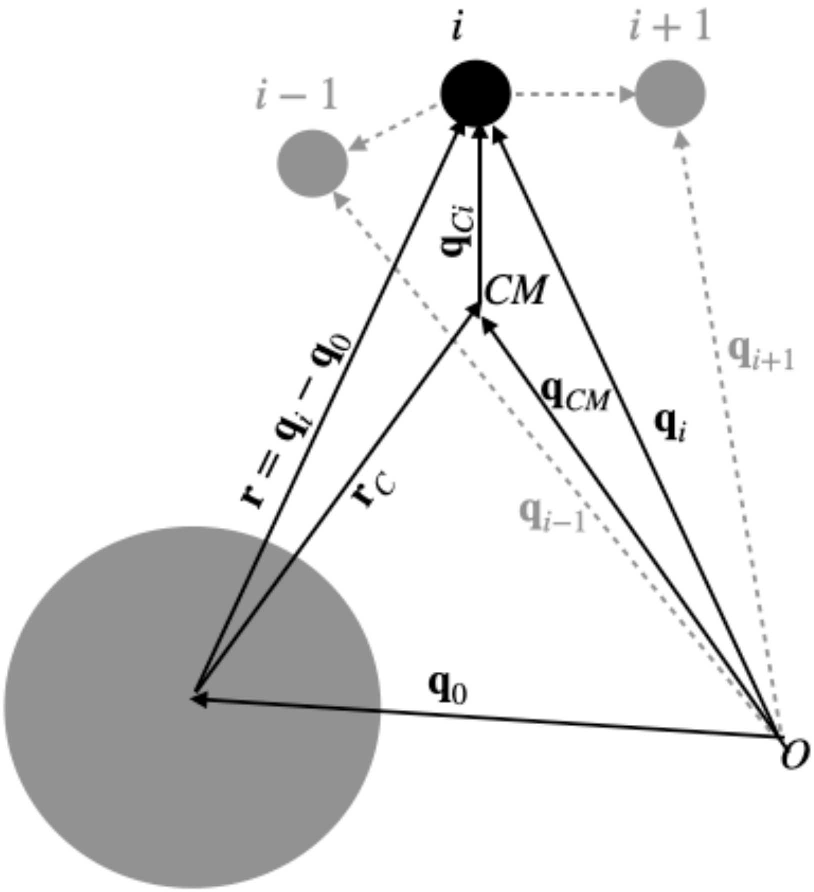
In this version, we can create any polyhedron we want from 'gen_sample.py', whether convex or concave. But the concave polyhedron will meet errors in 'Computation.py', such as 'Error DiscreteGeometry::build_HE_Hdl: Humm contour not closed impossible. Error: impossible to create the HE structure'



The left is the concave polyhedron, and the right one is the convex polyhedron. For the convex configuration, an animation can be generated, shown in following https://user-images.githubusercontent.com/38872598/221667287-256bf48b-62ab-4483-93cd-5e270db99a5a.mp4

version20230227

Orbital Dynamics



This is an N-body problem, so the equation of motion is focused on one single body. At first, bodies that make up an asteroid are

$$m_i F^{ext} = m_i \ddot{\mathbf{q}}_i = \sum_{\substack{j=1 \\ i \neq i}}^{n} \frac{G m_i m_j \left(\mathbf{q}_j - \mathbf{q}_i\right)}{\left\|\mathbf{q}_j - \mathbf{q}_i\right\|^3} \tag{2}$$

Then we added the planet perturbation

$$m_i F^{ext} + m_0 F^0 = m_i \ddot{\mathbf{q}}_i + m_0 \ddot{\mathbf{q}}_0 = \sum_{\substack{j=1 \\ i \neq i}}^{n} \frac{G m_i m_j \left(\mathbf{q}_j - \mathbf{q}_i\right)}{\left\|\mathbf{q}_j - \mathbf{q}_i\right\|^3} - m_i \frac{\partial U_0}{\partial \mathbf{r}} \tag{3}$$

in which $\mathbf{R}$ is the distance between body i and planet.

$$U_0 = \frac{G M_E}{R_E} \sum_{n=0}^{\infty} \sum_{m=0}^{n} \left(\frac{R_E}{r}\right)^{n+1} P_{nm}(\sin \phi) \cdot \left[\cos(m\lambda) C_{nm} + \sin(m\lambda) S_{nm}\right] \tag{4}$$

where G is the gravitational constant, M_E and R_E are the reference mass and reference radius, $P_{n!n}$ is the associated Legendre function of degree n and order m, ϕ and λ are the latitude and longitude of the spherical body in the body frame. The first-degree potential is expressed as

$$U^1 = \frac{GM_E}{r^3}\boldsymbol{r} \cdot \boldsymbol{r}_{\mathrm{CM}} \tag{5}$$

in which

$$\boldsymbol{r}_{CM} = [R_E C_{11}, R_E S_{11}, R_E C_{10}]^T \tag{6}$$

So the Planet's perturbation force is

$$\frac{\partial U^1}{\partial \boldsymbol{r}} = \frac{GM_E}{r^3}\left[1_{[3\times 3]} - 3\hat{r}\hat{r}\right] \cdot \boldsymbol{r}_{\mathrm{CM}} \tag{7}$$

From the center of mass of the Asteroid, we have the vector equation

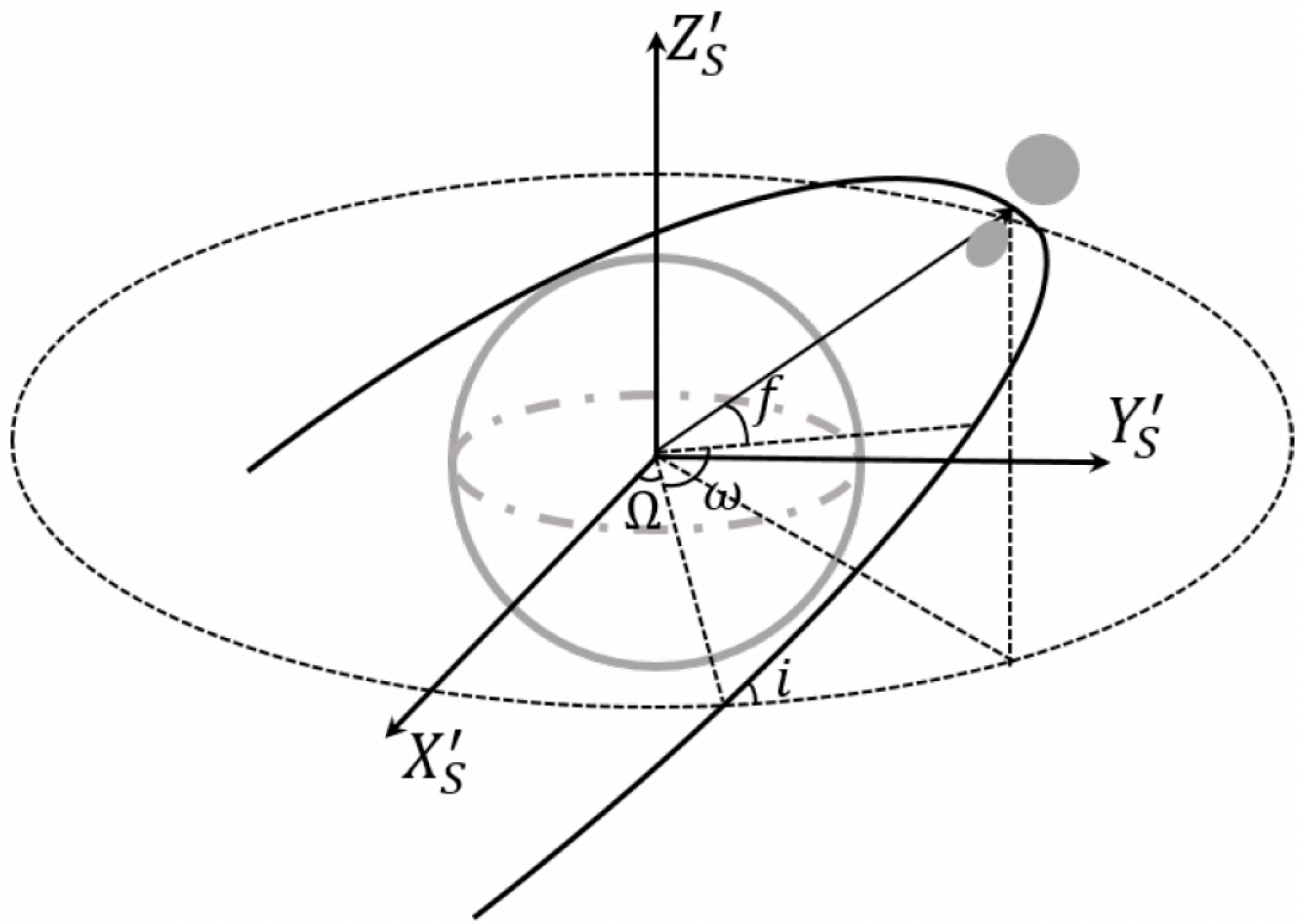
$$\mathbf{r} = \mathbf{r}_C + \mathbf{q}_{Ci} = \mathbf{r}_C + \mathbf{q}_i - \mathbf{q}_{CM} \tag{8}$$

where

$$\mathbf{q}_{CM} = \sum_{i=1}^{n} m_i \mathbf{q}_i / \sum_{i=1}^{n} m_i = \sum_{i=1}^{n} m_i \mathbf{q}_i / M_A \tag{9}$$

Using the Apophis orbital elements, we can model its flyby orbit $\mathbf{r}_C$ as a parabolic orbit,

$$\ddot{\mathbf{r}}_C = -M_A \frac{\partial U^1}{\partial \mathbf{r}_C} \tag{10}$$



Rotational Dynamics