Version20230328

Results

- Given fraction only, this combo with the same volume as Apophis will break during the flyby.

Problem

- The dynamic moment of inertia $I_d = H^2/2T$ is always smaller than the median total inertia I_i.
- Is there some way to define the long axis for any polyhedron? This will be used to convert angular velocity to the LAM/SAM rotation.

1. Shape Model

1.1. UnitPoly

Two octahedrons construct this polyhedron combo. The most important feature of this shape model is normalization. The volume of this model can be normalized to 1 and costumed to any value. In this simulation, the volume is set as the Apophis volume $1.986 \times 10^7\ m^3$ and density is $\rho = 2000\ kg/m^3$.

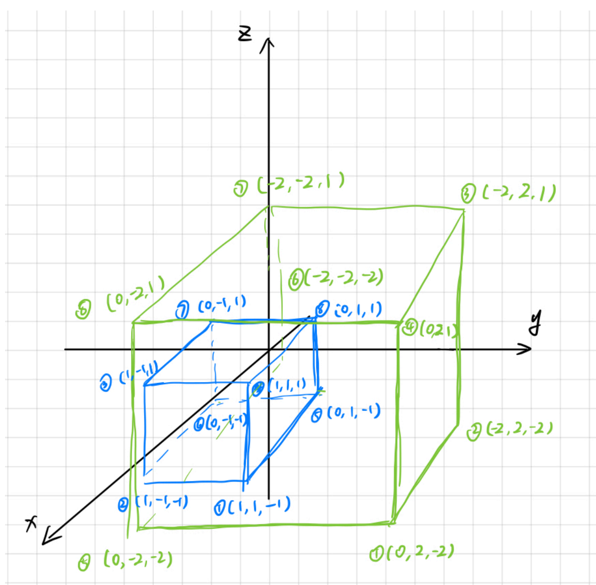

Fig.1

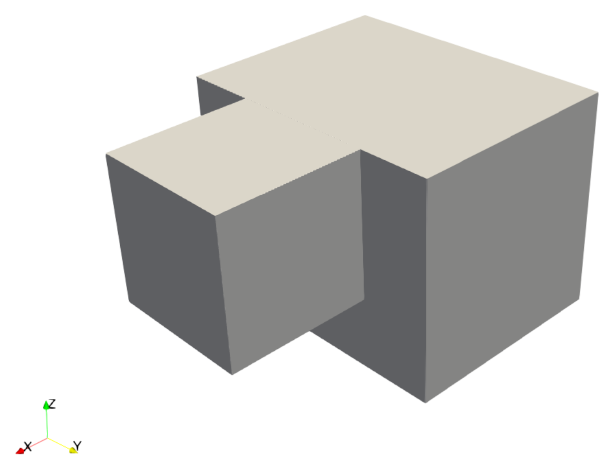

Fig.2

The fraction in the contact law is set as 1, and normal restitution is 1. The animation is shown in the following link.

https://user-images.githubusercontent.com/38872598/228975237-3d989f56-5bfe-4986-ba3a-aa6488f74c68.mp4

1.2. Conserved Quantities

$$T = \frac{1}{2M}\sum_{i=1}^{N-1}\sum_{j=i+1}^{N} m_i m_j \left(\boldsymbol{v}_{ij}\cdot\boldsymbol{v}_{ij}\right) + \frac{1}{2}\sum_{i=1}^{N} I_i|\boldsymbol{\omega}_i|^2$$

$$U = -\mathcal{G}\sum_{i=1}^{N-1}\sum_{j=i+1}^{N}\frac{m_i m_j}{|\boldsymbol{r}_{ij}|}$$

$$\boldsymbol{H} = \frac{1}{M}\sum_{i=1}^{N-1}\sum_{j=i+1}^{N} m_i m_j \left(\boldsymbol{r}_{ij}\times\boldsymbol{v}_{ij}\right) + \sum_{i=1}^{N} I_i\boldsymbol{\omega}_i$$

From these equations, we can compute the energy and angular momentum square,

$$E = T + U, H = \mathbf{H}\cdot\mathbf{H}$$

Without Earth perturbation, these two bodies have no relative motions, so fraction doesn't work in energy/momentum dissipation. We can also consider the states in terms of their effective spin rate $\omega_e = 2T/H$ and dynamic moment of inertia $I_d = H^2/2T$ where H and T are the rotational angular momentum magnitude and kinetic energy.

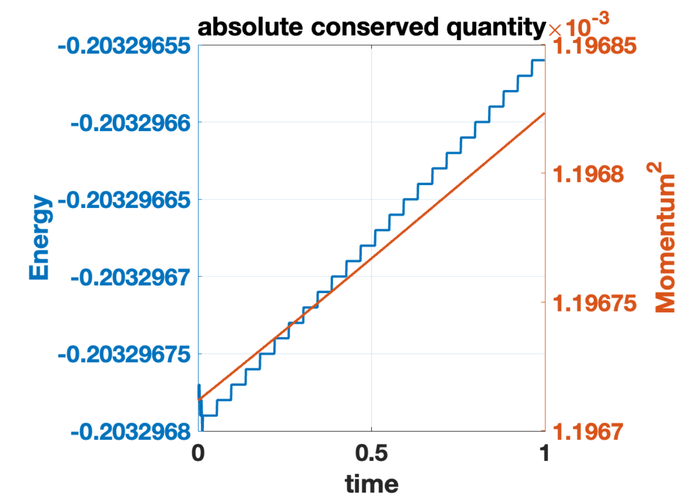

Fig.1

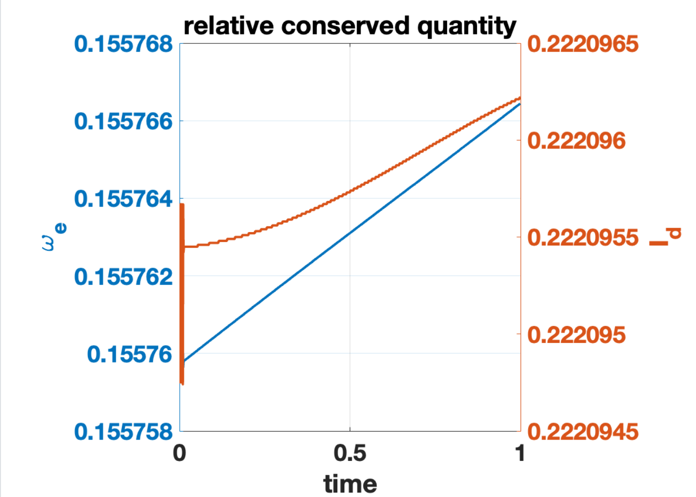

Fig.2

Here the integration arc is $dt = 1E-4$ and time span is from 0 to $1E4$. Note that the integration is without unit.

1.3. Scaled dynamic moment of inertia

Then, I tried to compute the scaled dynamic moment of inertia $\tilde{I}_d$. For short axis mode (SAM) rotation, we have $I_i < I_d < I_s$ and $\tilde{I}_d = (I_d - I_i)/(I_s - I_i)$. For the long axis mode (LAM) rotation, we have $I_l < I_d < I_i$ and $\tilde{I}_d = (I_d - I_i)/(I_s - I_l)$.

So $-1 < \tilde{I}_d < 1$ with the extremal values indicating uniform long/short axis rotation respectively and 0 indicating intermediate axis rotation or motion along the separatrix.

Here, I_l, I_i, I_s respectively mean the maximum inertia, median inertia, and minimum inertia.

$$I_l = max(diag(I_{total})),\ I_i = median(diag(I_{total})),\ I_s = min(diag(I_{total}))$$

The total inertia can be computed from every body's inertia addition $I_{total} = \sum I_i$.

But, the wired thing is the I_d always smaller than I_i.

2. Results

Given fraction only, this combo with the same volume as Apophis will break during the flyby. Animation is shown in the following link.

https://user-images.githubusercontent.com/38872598/228975271-22edc30c-2647-4148-a3fc-d7fbaddc1e5e.mp4

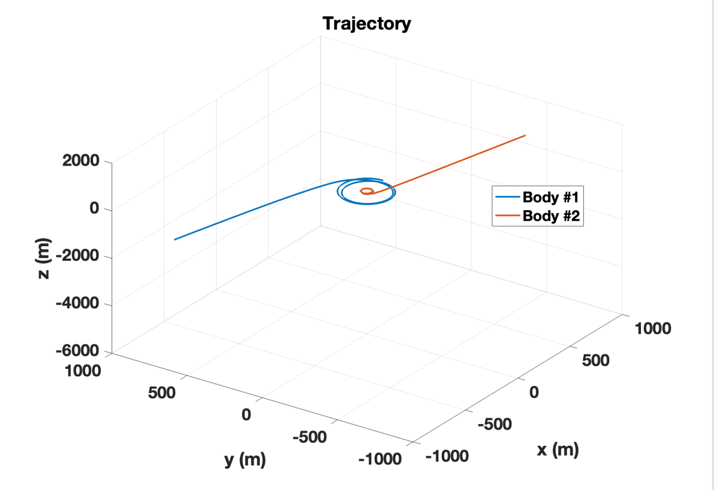

Fig.1

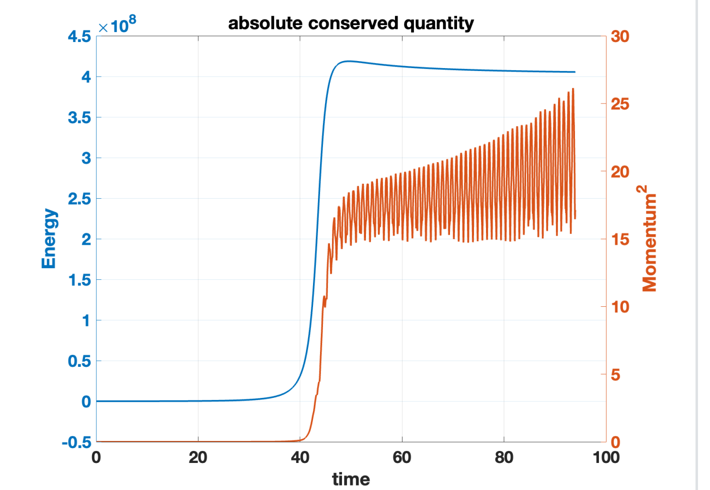

Fig.2

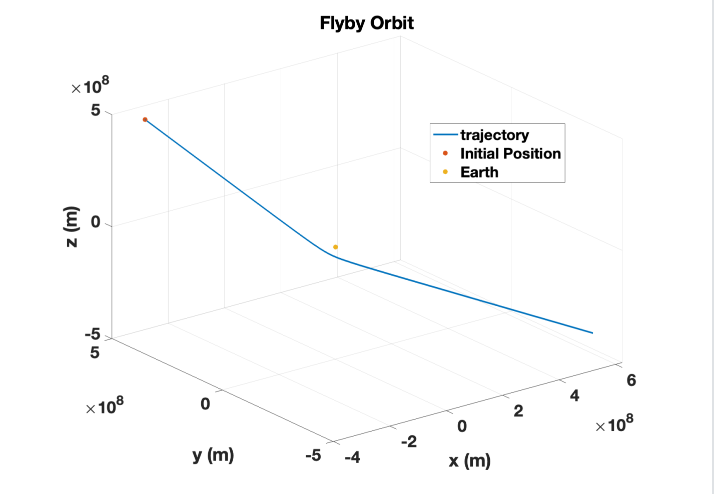

Fig.3

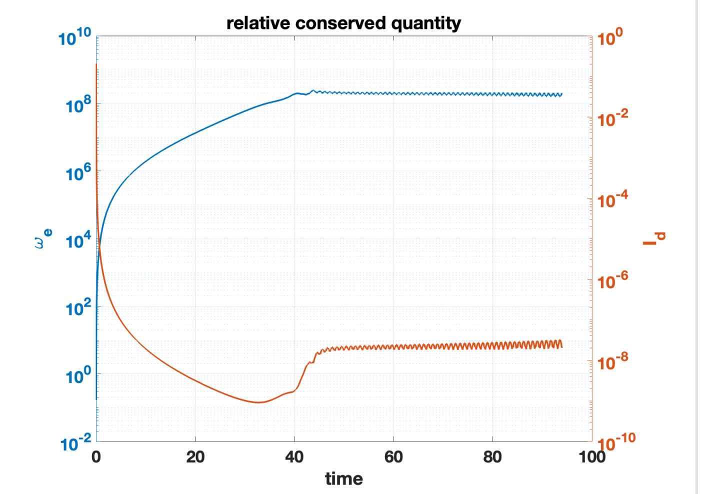

Fig.4