PRELIMINARY RESULTS

Transformation 2: (nonsingular)

$$z = U\text{col}(x_1, x_2) = \text{col}(z_1, z_2)$$

Where $U = \begin{bmatrix} I_{n-m} & -\overline{B}_1\overline{B}_2^{-1} \\ 0 & \overline{B}_2^{-1} \end{bmatrix}$

$$\dot{z}_1 = \widetilde{A}_{11}z_1 + \widetilde{A}_{12}z_2 \qquad (3a)$$

$$\dot{z}_2 = \widetilde{A}_{21}z_1 + \widetilde{A}_{22}z_2 + u + d \qquad (3b)$$

$$y = \widetilde{C}z = \begin{bmatrix} y_1 \\ y_2 \end{bmatrix} = \begin{bmatrix} \widetilde{C}_1 \\ \widetilde{C}_2 \end{bmatrix} z = \begin{bmatrix} \widetilde{C}_{11} & \widetilde{C}_{12} \\ 0 & \widetilde{C}_{22} \end{bmatrix} \begin{bmatrix} z_1 \\ z_2 \end{bmatrix}. \qquad (3c)$$

System and input matrix:

$$\widetilde{A} = \begin{bmatrix} \widetilde{A}_{11} & \widetilde{A}_{12} \\ \widetilde{A}_{21} & \widetilde{A}_{22} \end{bmatrix} \quad \text{and} \quad \widetilde{B} = \begin{bmatrix} 0 \\ I_m \end{bmatrix}$$

Lemma 1:

If the pair $(\widetilde{A}_{11}, \widetilde{C}_{11})$ is observable,

then the pair $(\widetilde{A}, \widetilde{C})$ is observable.

$$C_2V_1 = C_2AV_1 = 0$$

The pair $(\widetilde{A}, \widetilde{C})$ is observable iff

the pair $(\widetilde{A}_{11}, \widetilde{C}_{11})$ is observable

$$y = \widetilde{C}z = \begin{bmatrix} y_1 \\ y_2 \end{bmatrix} = \begin{bmatrix} \widetilde{C}_1 \\ \widetilde{C}_2 \end{bmatrix} z = \begin{bmatrix} \widetilde{C}_{11} & \widetilde{C}_{12} \\ 0 & \widetilde{C}_{22} \end{bmatrix} \begin{bmatrix} z_1 \\ z_2 \end{bmatrix}. \quad (3c)$$

A new output function:

$$\widetilde{C}_{11} z_1 = y_1 - \widetilde{C}_{12} \widetilde{C}_{22}^{-1} y_2$$

Observer:

$$\dot{\hat{z}}_1 = \widetilde{A}_{11}\hat{z}_1 + \widetilde{A}_{12}\hat{z}_2 + L_1 \widetilde{C}_{11}(z_1 - \hat{z}_1) + \widetilde{A}_{12}(z_2 - \hat{z}_2) \quad (5a)$$

$$\dot{\hat{z}}_2 = \widetilde{A}_{21}\hat{z}_1 + u + \widetilde{A}_{22} z_2 + K \mathrm{sign}(z_2 - \hat{z}_2) \quad (5b)$$

where $L_1 \in \mathbb{R}^{(n-m)\times(p-m)}$ is the linear observer gain, $K > 0$ is some scalar and sign function is given by $\mathrm{sign}(\gamma) = [\ \mathrm{sign}(\gamma_1) \quad \mathrm{sign}(\gamma_2) \quad \cdots \quad \mathrm{sign}(\gamma_m)\ ]^\top$ for any vector $\gamma = [\ \gamma_1 \quad \gamma_2 \quad \cdots \quad \gamma_m\ ]^\top$. Here, $\hat{z} = \mathrm{col}(\hat{z}_1, \hat{z}_2)$ denotes the estimate of $z = \mathrm{col}(z_1, z_2)$.

Lemma 1:

If the pair $(\widetilde{A}_{11}, \widetilde{C}_{11})$ is observable, then the pair $(\widetilde{A}, \widetilde{C})$ is observable.

$\downarrow$

- L_1 exists
- The eigenvalues of $\widetilde{A}_1 := \widetilde{A}_{11} - L_1 \widetilde{C}_{11}$ are stable

Lyapunov equation:

$$\widetilde{A}_1^\top P_1 + P_1 \widetilde{A}_1 = -Q_1$$