PROBLEM DESCRIPTION

Continuous-time system with disturbance:

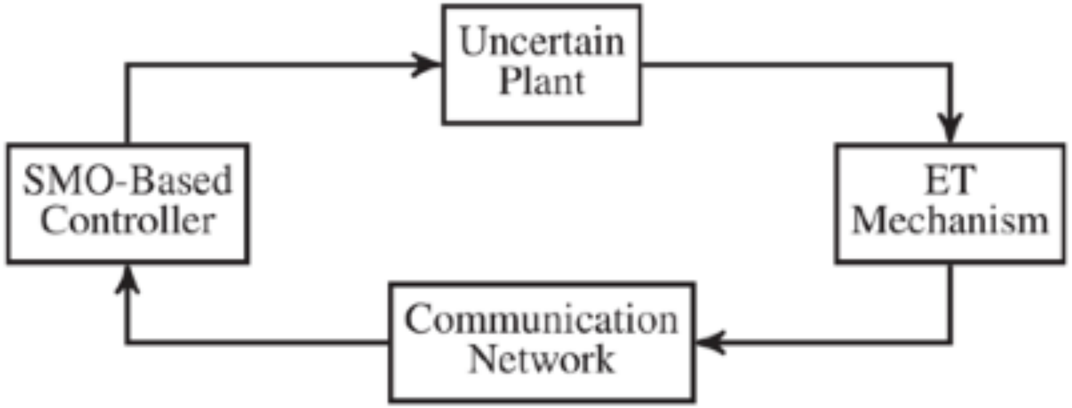
$$\dot{x} = Ax + B(u + d)$$
$$y = Cx$$

Assumption：

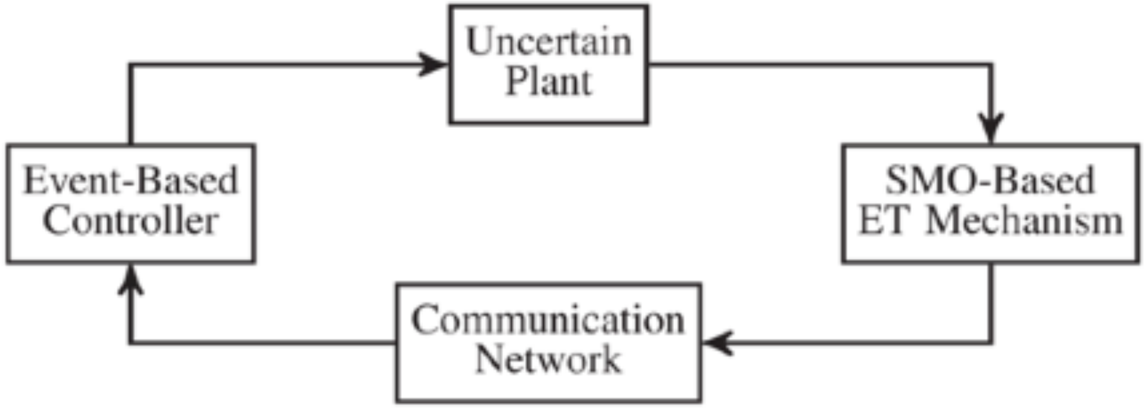
- $p > m$
- For some d_0 , $\|d(t)\| \leqslant d_0$ for all $t > 0$
- rank$(CB) = m$

Definition 1: Practical State Estimation

The observer $\dot{\hat{x}} = F(\hat{x}, y, u), \quad \hat{x}(0) \in \mathbb{R}^n$
is said to estimate the states practically if for any $\varepsilon > 0$, there exists a time $T \geq 0$ such that $\|x(t) - \hat{x}(t)\| \leqslant \varepsilon$ for all $t \geq T$.



(a) actuator side



(b) sensor side

PRELIMINARY RESULTS

Output matrix:

$$C = \begin{bmatrix} C_1 \\ C_2 \end{bmatrix} = \begin{bmatrix} C_{11} & C_{12} \\ C_{21} & C_{22} \end{bmatrix}$$

Assumption：

C_2 has full rank $\Rightarrow$ $C_2 B$ invertible

Let $V_1 \in \mathbb{R}^{n\times(n-m)}$ whose column space is $\mathcal{N}(C_2)$

$\Rightarrow$ rank$(V_1) = n - m$

$$\overline{V}_1 = (V_1^\top V_1)^{-1/2} V_1^\top \ , \ \overline{C}_2 = (C_2 C_2^\top)^{-1/2} C_2$$

$$V = \begin{bmatrix} \overline{V}_1 \\ \overline{C}_2 \end{bmatrix} \in \mathbb{R}^{n\times n}$$ orthogonal

Transformation 1:

$$x \mapsto Vx = \mathrm{col}(x_1, x_2)$$

$$\boxed{\begin{aligned} \dot{x}_1 &= \overline{A}_{11}x_1 + \overline{A}_{12}x_2 + \overline{B}_1(u+d) \\ \dot{x}_2 &= \overline{A}_{21}x_1 + \overline{A}_{22}x_2 + \overline{B}_2(u+d) \end{aligned} \qquad (2)}$$

where $x_1 \in \mathbb{R}^{n-m}$ and $x_2 \in \mathbb{R}^m$

$$\overline{A} = VAV^\top = \begin{bmatrix} \overline{V}_1 A \overline{V}_1^\top & \overline{V}_1 A \overline{C}_2^\top \\ \overline{C}_2 A \overline{V}_1^\top & \overline{C}_2 A \overline{C}_2^\top \end{bmatrix}$$

$$\overline{B} = VB = \begin{bmatrix} \overline{V}_1 B \\ \overline{C}_2 B \end{bmatrix} \quad \overline{C} = CV^\top = \begin{bmatrix} C_1 \overline{V}_1^\top & C_1 \overline{C}_2^\top \\ 0 & C_2 \overline{C}_2^\top \end{bmatrix}$$