

# PRELIMINARY RESULTS

Transformation 2: (nonsingular)

$$z = U \text{col}(x_1, x_2) = \text{col}(z_1, z_2)$$

$$\text{Where } U = \begin{bmatrix} I_{n-m} & -\bar{B}_1 \bar{B}_2^{-1} \\ 0 & \bar{B}_2^{-1} \end{bmatrix}$$

$$\dot{z}_1 = \widetilde{A}_{11} z_1 + \widetilde{A}_{12} z_2 \quad (3a)$$

$$\dot{z}_2 = \widetilde{A}_{21} z_1 + \widetilde{A}_{22} z_2 + u + d \quad (3b)$$

$$y = \widetilde{C} z = \begin{bmatrix} y_1 \\ y_2 \end{bmatrix} = \begin{bmatrix} \widetilde{C}_1 \\ \widetilde{C}_2 \end{bmatrix} z = \begin{bmatrix} \widetilde{C}_{11} & \widetilde{C}_{12} \\ 0 & \widetilde{C}_{22} \end{bmatrix} \begin{bmatrix} z_1 \\ z_2 \end{bmatrix}. \quad (3c)$$

System and input matrix:

$$\widetilde{A} = \begin{bmatrix} \widetilde{A}_{11} & \widetilde{A}_{12} \\ \widetilde{A}_{21} & \widetilde{A}_{22} \end{bmatrix} \quad \text{and} \quad \widetilde{B} = \begin{bmatrix} 0 \\ I_m \end{bmatrix}$$

*Lemma 1:*

If the pair  $(\widetilde{A}_{11}, \widetilde{C}_{11})$  is observable,  
then the pair  $(\widetilde{A}, \widetilde{C})$  is observable.

$$C_2 V_1 = C_2 A V_1 = 0$$

The pair  $(\widetilde{A}, \widetilde{C})$  is observable iff  
the pair  $(\widetilde{A}_{11}, \widetilde{C}_{11})$  is observable

$$y = \widetilde{C}z = \begin{bmatrix} y_1 \\ y_2 \end{bmatrix} = \begin{bmatrix} \widetilde{C}_1 \\ \widetilde{C}_2 \end{bmatrix} z = \begin{bmatrix} \widetilde{C}_{11} & \widetilde{C}_{12} \\ 0 & \widetilde{C}_{22} \end{bmatrix} \begin{bmatrix} z_1 \\ z_2 \end{bmatrix}. \quad (3c)$$

A new output function:

$$\widetilde{C}_{11}z_1 = y_1 - \widetilde{C}_{12}\widetilde{C}_{22}^{-1}y_2$$

Observer:

$$\dot{\hat{z}}_1 = \widetilde{A}_{11}\hat{z}_1 + \widetilde{A}_{12}\hat{z}_2 + L_1\widetilde{C}_{11}(z_1 - \hat{z}_1) + \widetilde{A}_{12}(z_2 - \hat{z}_2) \quad (5a)$$

$$\dot{\hat{z}}_2 = \widetilde{A}_{21}\hat{z}_1 + u + \widetilde{A}_{22}z_2 + K\text{sign}(z_2 - \hat{z}_2) \quad (5b)$$

where  $L_1 \in \mathbb{R}^{(n-m) \times (p-m)}$  is the linear observer gain,  $K > 0$  is some scalar and sign function is given by  $\text{sign}(\gamma) = [\text{sign}(\gamma_1) \text{sign}(\gamma_2) \cdots \text{sign}(\gamma_m)]^\top$  for any vector  $\gamma = [\gamma_1 \gamma_2 \cdots \gamma_m]^\top$ . Here,  $\hat{z} = \text{col}(\hat{z}_1, \hat{z}_2)$  denotes the estimate of  $z = \text{col}(z_1, z_2)$ .

*Lemma 1:*

If the pair  $(\widetilde{A}_{11}, \widetilde{C}_{11})$  is observable,  
then the pair  $(\widetilde{A}, \widetilde{C})$  is observable.



- $L_1$  exists
- The eigenvalues of  $\widetilde{A}_1 := \widetilde{A}_{11} - L_1\widetilde{C}_{11}$  are stable

*Lyapunov equation:*

$$\widetilde{A}_1^\top P_1 + P_1 \widetilde{A}_1 = -Q_1$$