

EVENT-TRIGGERED SLIDING MODE OBSERVER

A. Actuator Side Implementation

$$\begin{aligned}\dot{\hat{z}}_1(t) = & \widetilde{A}_{11}\hat{z}_1(t) + \widetilde{A}_{12}\hat{z}_2(t) + L_1\widetilde{C}_{11}\left(z_1(t_k^y) - \hat{z}_1(t)\right) \\ & + \widetilde{A}_{12}\left(z_2(t_k^y) - \hat{z}_2(t)\right)\end{aligned}\quad (9)$$

$$\begin{aligned}\dot{\hat{z}}_2(t) = & \widetilde{A}_{21}\hat{z}_1(t) + \widetilde{A}_{22}z_2(t_k^y) + K\text{sign}\left(z_2(t_k^y) - \hat{z}_2(t)\right) \\ & + u(t)\end{aligned}$$

Let $e_1(t) = z_1(t_k^y) - z_1(t)$ and $e_2(t) = z_2(t_k^y) - z_2(t)$

$$\begin{aligned}\dot{\hat{z}}_1(t) = & \widetilde{A}_{11}\hat{z}_1(t) + \widetilde{A}_{12}\hat{z}_2(t) + L_1\widetilde{C}_{11}\tilde{z}_1(t) + \widetilde{A}_{12}\tilde{z}_2(t) \\ & + L_1\widetilde{C}_{11}e_1(t) + \widetilde{A}_{12}e_2(t) \\ \dot{\hat{z}}_2(t) = & \widetilde{A}_{21}\hat{z}_1(t) + \widetilde{A}_{22}z_2(t) + K\text{sign}\left(\tilde{z}_2(t) + e_2(t)\right) \\ & + u(t) + \widetilde{A}_{22}e_2(t).\end{aligned}\quad (10)$$

$$\dot{z}_1 = \widetilde{A}_{11}z_1 + \widetilde{A}_{12}z_2 \quad (3a)$$

$$\dot{z}_2 = \widetilde{A}_{21}z_1 + \widetilde{A}_{22}z_2 + u + d \quad (3b)$$

Dynamics of Estimation Error:

$$\dot{\tilde{z}}_1(t) = \widetilde{A}_1\tilde{z}_1(t) - \widetilde{L}_1e_y(t)$$

$$\begin{aligned}\dot{\tilde{z}}_2(t) = & \widetilde{A}_{21}\tilde{z}_1(t) - \widetilde{A}_{22}e_2(t) - K\text{sign}\left(\tilde{z}_2(t) + e_2(t)\right) \\ & + d(t)\end{aligned}$$

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A. Actuator Side Implementation

Theorem 2: Consider the plant (3) and the observer (10) with the event-triggering mechanism (8). Let $\varepsilon_a > 0$ be any scalar. Suppose that the $(\tilde{A}_{11}, \tilde{C}_{11})$ pair is observable. Choose $c_1 > \lambda_{\min}^2(P_1)\varepsilon_a^2/(16\lambda_{\max}(P_1))$ and $c_2 > \varepsilon_a/4$. Assume that $z(0), \hat{z}(0) \in \Omega(c_1, c_2)$, and $z(t)$ is bounded for all $t \geq 0$. Moreover, the triggering parameter satisfies the following inequality:

$$\alpha_a < \min \left\{ \frac{\varepsilon_a}{2}, \sqrt{\frac{\lambda_{\min}(P_1)}{\lambda_{\max}(P_1)}} \frac{\lambda_{\min}(Q_1)}{\|P_1 \tilde{L}_1\|} \frac{\varepsilon_a}{4\sqrt{2}} \right\}. \quad (12)$$

Then, there exist $K > 0$, $\tau_a > 0$, and $T_a \geq 0$ such that

- 1) the estimation error $\tilde{z}(t) := z(t) - \hat{z}(t)$ is bounded for all $t \geq 0$, and moreover, it holds that

$$\|\tilde{z}(t)\| \leq \varepsilon_a, \quad \forall t \geq T_a$$

- 2) $t_{k+1}^y - t_k^y \geq \tau_a$ for all $k \in \mathbb{Z}_{\geq 0}$.

$$\dot{z}_1 = \tilde{A}_{11}z_1 + \tilde{A}_{12}z_2 \quad (3a)$$

$$\dot{z}_2 = \tilde{A}_{21}z_1 + \tilde{A}_{22}z_2 + u + d \quad (3b)$$

$$\begin{aligned} \dot{\hat{z}}_1(t) &= \tilde{A}_{11}\hat{z}_1(t) + \tilde{A}_{12}\hat{z}_2(t) + L_1 \tilde{C}_{11}\tilde{z}_1(t) + \tilde{A}_{12}\tilde{z}_2(t) \\ &\quad + L_1 \tilde{C}_{11}e_1(t) + \tilde{A}_{12}e_2(t) \\ \dot{\hat{z}}_2(t) &= \tilde{A}_{21}\hat{z}_1(t) + \tilde{A}_{22}\hat{z}_2(t) + K \text{sign}(\tilde{z}_2(t) + e_2(t)) \\ &\quad + u(t) + \tilde{A}_{22}e_2(t). \end{aligned}$$