SLIDING MODE OBSERVER

$$\widetilde{A}_1^\top P_1 + P_1 \widetilde{A}_1 = -Q_1$$

for any $Q_1 = Q_1^\top > 0$. With this, define the sets $\Omega_1(c_1) = \{z_1 \in \mathbb{R}^{n-m} : z_1^\top P_1 z_1 \leqslant c_1\}$ and $\Omega_2(c_2) = \{z_2 \in \mathbb{R}^m : \|z_2\| \leqslant c_2\}$ for any $c_1 > 0$ and $c_2 > 0$. Also, denote $\Omega(c_1, c_2) = \{z \in \mathbb{R}^n : z_1 \in \Omega_1(c_1)$ and $z_2 \in \Omega_2(c_2)\}$.

Define $\tilde{z}_1 := z_1 - \hat{z}_1$ and $\tilde{z}_2 := z_2 - \hat{z}_2$ as the estimation errors.

estimation error dynamics：

$$\dot{\tilde{z}}_1 = \widetilde{A}_1 \tilde{z}_1 \qquad \text{(7a)}$$

$$\dot{\tilde{z}}_2 = \widetilde{A}_{21} \tilde{z}_1 - K\mathrm{sign}(\tilde{z}_2) + d\,. \qquad \text{(7b)}$$

Theorem 1: Consider the system (3) and the observer (5). Let $c_1 > 0$ and $c_2 > 0$ be any scalars. Assume that the $(\widetilde{A}_{11}, \widetilde{C}_{11})$ pair is observable and $z(0), \hat{z}(0) \in \Omega(c_1, c_2)$. Then, there exists a gain $K > 0$ such that the estimation error $\tilde{z}(t) := z(t) - \hat{z}(t)$ is bounded for all $t \geqslant 0$, and $\tilde{z}(t) \to 0$ as $t \to +\infty$.

$$\dot{x} = Ax + B(u + d)$$
$$y = Cx$$

$$\dot{z}_1 = \widetilde{A}_{11} z_1 + \widetilde{A}_{12} z_2 \qquad \text{(3a)}$$

$$\dot{z}_2 = \widetilde{A}_{21} z_1 + \widetilde{A}_{22} z_2 + u + d \qquad \text{(3b)}$$

$$\dot{\hat{z}}_1 = \widetilde{A}_{11} \hat{z}_1 + \widetilde{A}_{12} \hat{z}_2 + L_1 \widetilde{C}_{11}(z_1 - \hat{z}_1) + \widetilde{A}_{12}(z_2 - \hat{z}_2) \qquad \text{(5a)}$$

$$\dot{\hat{z}}_2 = \widetilde{A}_{21} \hat{z}_1 + u + \widetilde{A}_{22} z_2 + K\mathrm{sign}(z_2 - \hat{z}_2) \qquad \text{(5b)}$$

EVENT-TRIGGERED SLIDING MODE OBSERVER

A. Actuator Side Implementation

$$t_{k+1}^{y} = \inf \left\{ t > t_k^y : \|e_y(t)\| \geqslant \sigma_a \alpha_a \right\}, \quad k \in \mathbb{Z}_{\geqslant 0}$$

$$t_0^y = 0 \qquad e_y(t) = \begin{bmatrix} \widetilde{C}_{11} \left(z_1(t_k^y) - z_1(t) \right) \\ z_2(t_k^y) - z_2(t) \end{bmatrix}$$

Event-based sliding mode observer:

$$\begin{aligned} \dot{\hat{z}}_1(t) &= \widetilde{A}_{11}\hat{z}_1(t) + \widetilde{A}_{12}\hat{z}_2(t) + L_1 \widetilde{C}_{11} \left(z_1(t_k^y) - \hat{z}_1(t) \right) \\ &\quad + \widetilde{A}_{12} \left(z_2(t_k^y) - \hat{z}_2(t) \right) \\ \dot{\hat{z}}_2(t) &= \widetilde{A}_{21}\hat{z}_1(t) + \widetilde{A}_{22} z_2(t_k^y) + K \mathrm{sign} \left(z_2(t_k^y) - \hat{z}_2(t) \right) \\ &\quad + u(t) \end{aligned} \tag{9}$$