ME 332 Project 1

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Heat Transfer Project 1

Objective

Due to thermal expansion, a ring contained within a piston cylinder assembly is exposed to stress above its yield limit. To find the necessary dimensions of the ring, so that no yielding occurs, heat transfer relationships through the whole system must be identified. With relationships established the size of the ring can be adjusted until the stress is below the yield limit. Another benefit of this model is that any variable within the system may be varied to investigate its effects.

Assumptions:

- The wall and the top of the cylinder is grey body.
- The entire system is at steady state.
- The conduction occurring in each component is regarded as one dimensional.
- The heat transferred through the corner between the top of cylinder and the wall of cylinder is neglectable.
- The room temperature is 293K.
- Temperature used in thermal expansion is constant and equal to T_{avg}
- The black walls surrounding the piston have k=0.
- The thermal expansion in the piston only acts in the vertical direction.
- There is no convective or radiative heat transfer inside the piston (i.e. enclosed within the yellow).
- All parts of the piston (top and bottom plates, ring, and shaft) are all rigidly bonded, meaning no separation can occur.
- Top and bottom plates are infinitely stiff (i.e. no bending).
- The convective heat transfer condition is used for modeling the fin tip.

Design Constraints:

There are three constraints, Assumptions, processing power, and physical limitations.

The first constraints of this design is the assumptions made in this analysis. For a sufficient model a more complex analysis must be performed. For instance, the black wall surrounding the crank is ideally insulated. The system is at steady state, and none of the parts is moving It is assumed that all heat conducts in one direction, which may not represent the heat transfer as shape factor is ignored. A model with sufficient completeness may be orders of magnitude more expensive to simulate and will take much more time to complete, therefore this model is meant to be an inexpensive solution.The final constraint on our design is physical limitations. The goal is to reduce the stress in the ring by increasing its size, but if it needs to be bigger than the cylinder size to withstand the stresses, we will have to try a new approach, for example, reducing heat flow through ring by adding fins to the cylinder exterior.

Approach:

The correct approach begins with an energy balance of the cylinder. Heat has two paths to flow, up through the Cylinder head, and down through the piston. The heat flow through the top, walls and piston must be identified first. The software used for solving equations, conducting iterate method, making parametric table is EES.

1. To find Q through Top of Cylinder, Walls of Cylinder, and Piston
   a. To find Q through Top of Cylinder, Walls of Cylinder
      i. Use Resistance analogy for walls (Fig 5)
      ii. Set Q for conduction equal to Q of Convection and Radiation of walls (Eqn 2)
      iii. Iterate To acquire correct temperature of wall (T11) and h_r
      iv. Repeat for T12 (Fig 4)

b. To find Q through the Piston
   i. Use energy balance (Eqn 1)
2. To find T9 by modeling the fin with convective heat transfer condition
   a. Set equation of Q_base which is the convection heat transfer from the base to crank
      i. Find the resistance of convection from the bottom plate to the crank
      ii. Use resistance analogy to reach Q_base in the term of T9
   b. Set equation of Q_fin using convective heat transfer condition (Eqn 3)
   c. Use energy balance where Q through piston is equal to the Q through base and fin (Eqn 4)
3. To find T10 using the temperature distribution for the convective heat transfer condition (Eqn 3)
4. To find T2 to T8
   a. Draw the resistance diagram (Fig 6)
      i. Find resistance of contact and conduction for the plate, ring and shaft
      ii. Use resistance analogy to achieve T2 to T8 backward from T8 with the known value of T9 and Q through the plate (refers as Q_pistion) (Eqn 7)
5. Find change in length of the shaft and ring due to the thermal expansion (Eqn 5.)
6. Calculate the compressive stress within the components (Eqn 6)
   a. Note that the temperature used is the average temperature of bar.
7. Parametrically vary The size of the L3 until the both stresses are below yield, and SF_ring =1.1
8. With the new L3 achieved from step 7, recalculate the previous work from Step 1.

Solutions:

A. Q_Atmosphere = 464.4 [W]
B. Q_piston = q2 = 1401 [W]
C. See Appendix B: Detailed Results for detail
D. Sigma_Ring_Initial = 43.35 [MPa]
E. See Appendix B: Detailed Results for detail
F. L3 =0.024 [m] *for* FS_ring = 1.1

Conclusions:

In order for the Ring to have a factor of safety of 1.1, the length, L3, must be nearly doubled to withstand the thermal expansion induced stresses.

Regardless the exact numerical results achieved, the trend of temperatures, and application of the resistance analogy, makes physical sense. From the origin of heat (combustion), the temperature decreases in each direction. The decrease results from the resistance the heat flow faces on its dissipation to the free stream and crankcase temperature. The change of size of the ring has changed the system, but its effects are negligible. No temperatures within the system experienced significant change.

Appendix A: Schematics and Figures

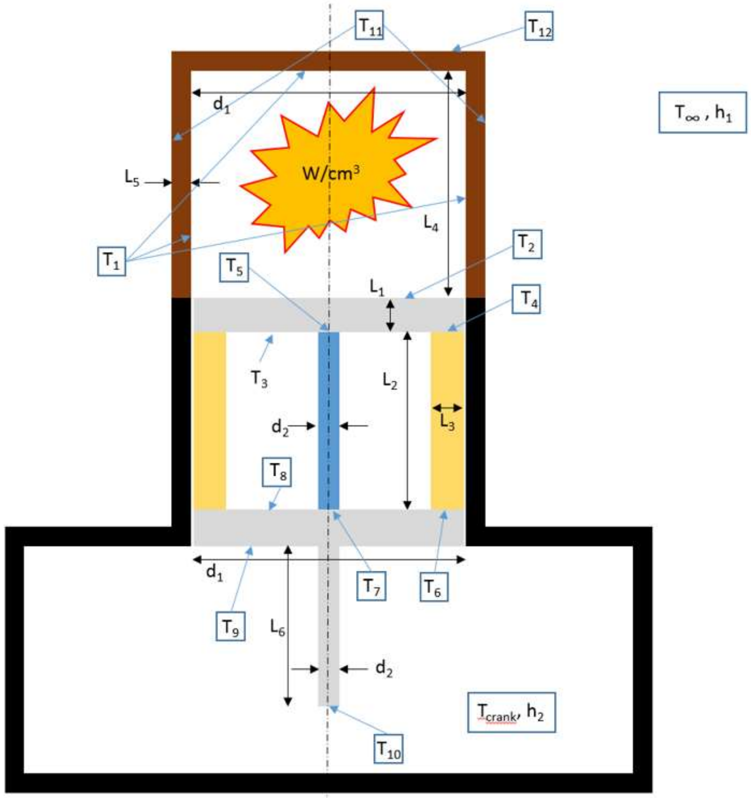

Fig 1. Schematic of Model

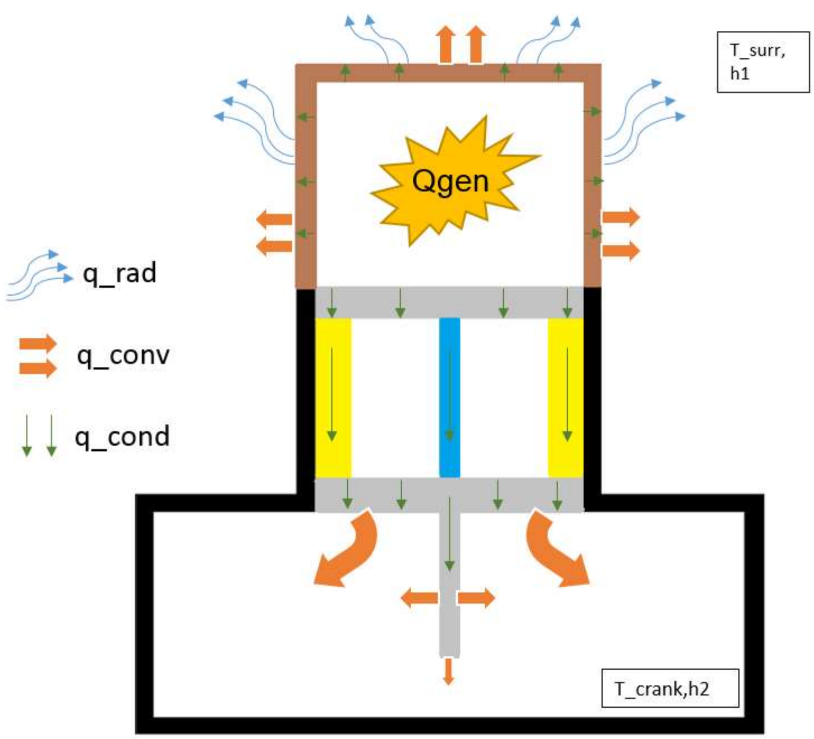

Fig 2. Schematic of Heat Transfer

Appendix B: Detailed Results

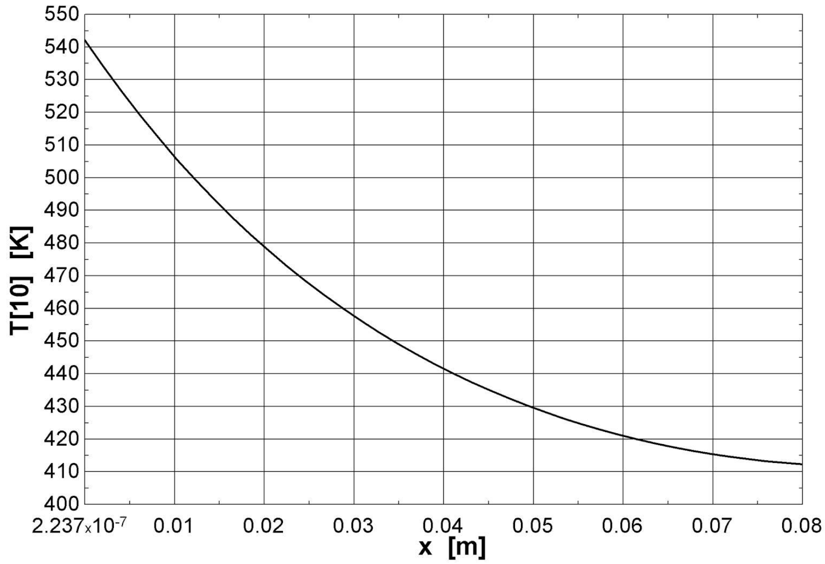

Fig 3. Plot of Temperature Distribution in Pin VS Distance from Base to Tip

Solutions			
Description	**Name**	**Value**	**Units**
Temp on top of top plate	T2	715.2	K
Temp on bottom of top plate	T3	692.4	k
Temp on top of ring	T4	655.9	k
Temp on top of shaft	T5	668.6	k
Temp on bottom of ring	T6	601.4	k
Temp on bottom shaft	T7	588.8	k
Temp on top of bottom plate	T8	565	k
Temp on bottom of bottom plate	T9	542.1	k
Temp on tip of fin	T10	412.3	k
Temp on sides of head	T11	748.2	k
Temp on top of head	T12	748.6	k
Initial stress in ring	σ1	43.35	MPa
Initial stress in shaft	σ2	216.2	MPa
Final stress in ring	σ3	30.44	MPa
Final stress in shaft	σ4	217.2	MPa
Final thickness of ring	L3	0.024	m
Heat transfer from cylinder to atmosphere	q1	464.4	W
Heat transfer from cylinder to piston	q2	1401	W

Table 1. Detailed Solution to all required parameter

Appendix C: Equations

$$Qin = Q_{top} + Q_{wall} + Q_{plate} \tag{1}$$

$$qcondtop = qradcovtop$$

$$qcondtop = \frac{T1 - T_{12}}{R_{top,cond}}$$

$$qradcovtop = \frac{T_{12} - Tinf}{R_{top,cov,rad}} \tag{2}$$

$$h_{r,top} = \varepsilon \cdot b \cdot (T_{12} + Tinf) \cdot (T_{12}{}^{2} + Tinf^{2})$$

$$Q_{fin} = M_b \cdot \left[\frac{\mathbf{sinh}(m \cdot L6) + \frac{h2}{m \cdot k2} \cdot \mathbf{cosh}(m \cdot L6)}{\mathbf{cosh}(m \cdot L6) + \frac{h2}{m \cdot k2} \cdot \mathbf{sinh}(m \cdot L6)} \right]$$

$$T_{10} = \left[\frac{\mathbf{cosh}(m \cdot (L6 - y)) + \frac{h2}{m \cdot k2} \cdot \mathbf{sinh}(m \cdot (L6 - y))}{\mathbf{cosh}(m \cdot L6) + \frac{h2}{m \cdot k2} \cdot \mathbf{sinh}(m \cdot L6)} \right] \tag{3}$$

$$M_b = \sqrt{h2 \cdot \pi \cdot D2 \cdot k2 \cdot A_{shaft}} \cdot (T_9 - T_{crank})$$

$$m = \sqrt{\frac{h2 \cdot \pi \cdot D2}{k2 \cdot A_{shaft}}}$$

$$Q_{plate} = Q_{fin} + \frac{T_9 - T_{crank}}{R_{plate,cov}} \tag{4}$$

$$\delta = CTE * \Delta T * l \tag{5}$$

Where

$$\Delta\delta = \delta_{ring} - \delta_{shaft}$$

$$\sigma_{ring} = \frac{\left(\frac{k_{shaft}}{k_{shaft}+k_{ring}}\right) * \Delta\delta * E}{l_{free}} \tag{6}$$

$$\sigma_{shaft} = \frac{\left(\frac{k_{ring}}{k_{shaft}+k_{ring}}\right) * \Delta\delta * E}{l_{free}}$$

Where

$$l_{free} = \delta + l$$

$$k = \frac{AE}{l}$$

$$Q = \frac{\Delta T}{R} \tag{7}$$

Appendix D: Resistance Diagrams:

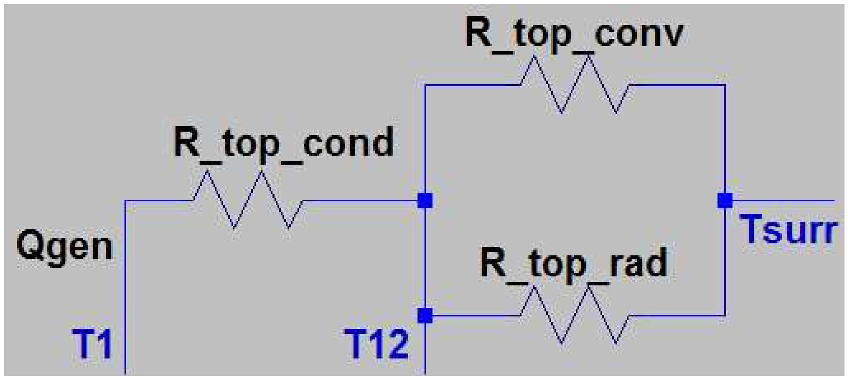

Fig 4. Resistance diagram of the Top of Cylinder

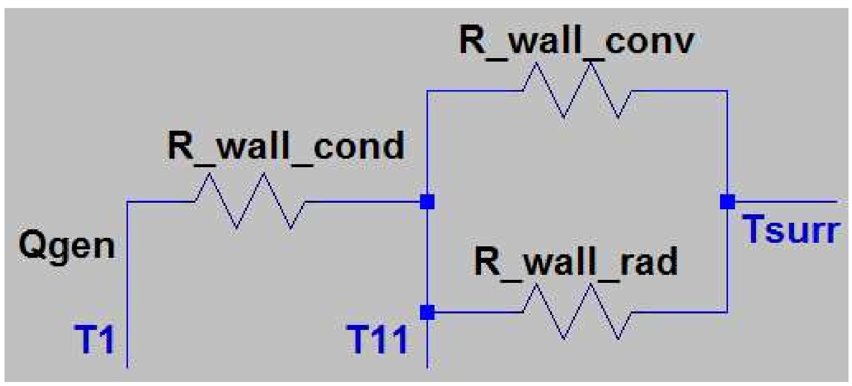

Fig 5. Resistance diagram of the Walls of Cylinder

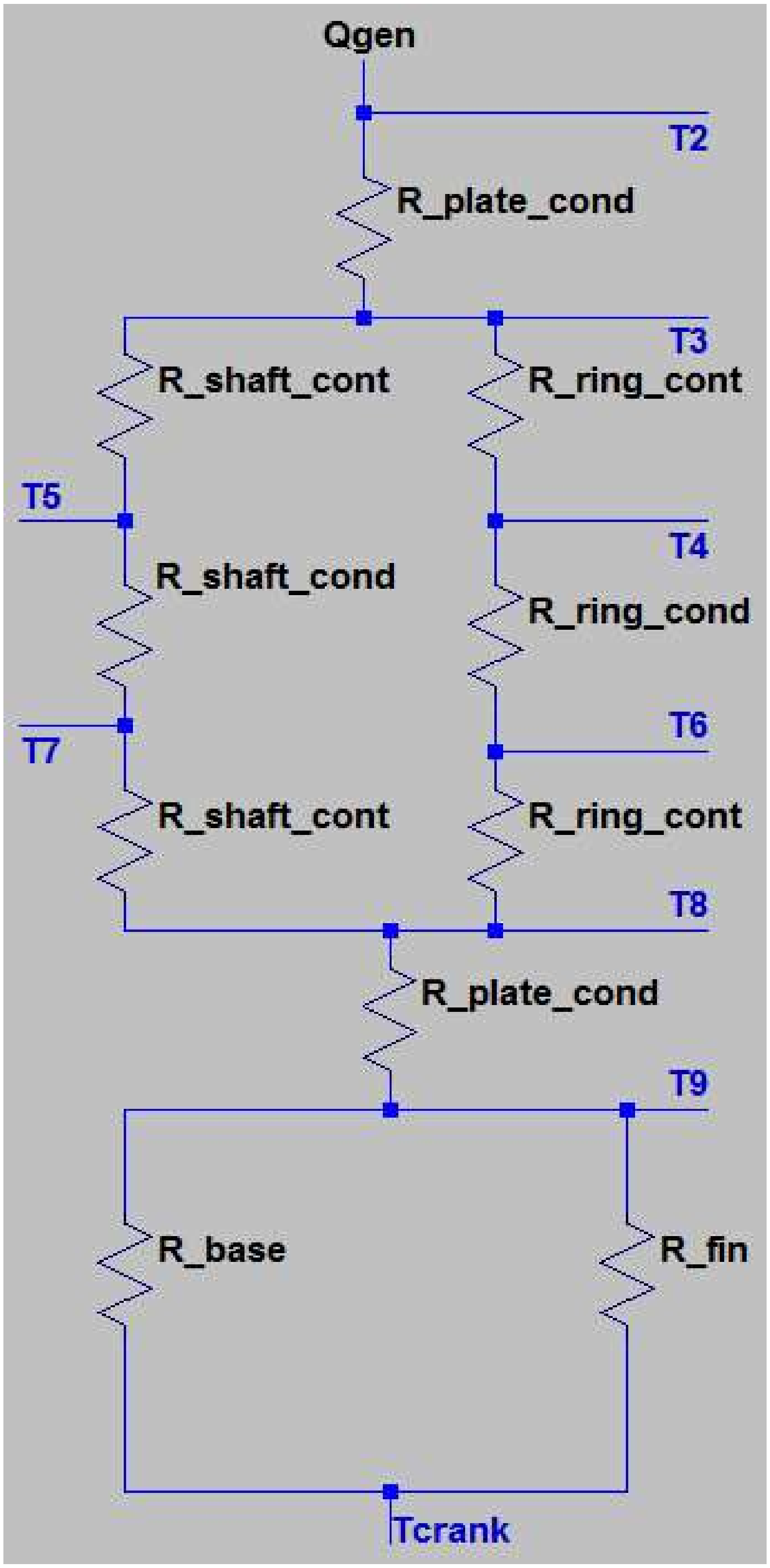

Fig 6. Resistance diagram of Piston

Appendix E: EES Code

```
"dimensions"
L1=.02[m]
L2=.09[m]
L3=0.015[m]
L4=.095[m]
L5=.03[m]
L6=.08[m]
D1=.1[m]
D2=.032[m]
ro=(D1+2*L5)/2
ri=D1/2

"Area and volumes"
S_A_plate=(pi/4)*D1^2
S_A_wall= pi*(D1+(2*L5))*L4
V_cyl = (pi/4)*D1^2*L4
A_ring = pi*((D1/2)^2-((D1/2)-L3)^2)
A_shaft= (pi/4)*D2^2

"knowns"
Tinf = 273[K]
T_crank=373[K]
T1=750[k]
Qgen=2.5*convert(w/cm^3,w/m^3)

"properties"
h1= 15[w/(m^2*K)]
h2= 750[w/(m^2*K)]
EPSILON=.07   "emissivity"
k1=177 [W/m*K]
k2=156[W/m*K]
k3= 401 [w/m*k]
k4= 13.4[w/m*k]
b= 5.67E-8 [W/(m^2*K^4)]          "boltzmann constant"

h_r_wall = EPSILON*b*(T[11]+Tinf) *((T[11])^2+(Tinf)^2)
h_r_top =  EPSILON*b*(T[12]+Tinf) *((T[12])^2+(Tinf)^2)

"resistances"
R1= 1.5E-4 [m^2*k/w]
R2=2E-3 [m^2*k/w]

R_wall_cond= (ln(ro/ri)/(2*pi*k1*L4))        "wall"
R_wall_cov_rad=(1/(h_r_wall*S_A_wall+h1*S_A_wall))     "wall"

R_top_cond = (L5/(k1*S_A_plate))          "TOP"
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```
R_top_cov_rad=(1/(h_r_top*S_A_plate+h1*S_A_plate))     "TOP"
R_plate_cond = L1/(k2*S_A_plate)          "plate"
R_shaft_contact = R2/ A_shaft   "plate"
R_shaft_cond = L2/(k4*A_shaft) "plate"
R_ring_contact = (R1)/(A_ring)    "ring"
R_ring_cond = L2/(k3*A_ring)      "ring"
R_plate_cov = 1/(h2*(S_A_plate-A_shaft))           "plate"

"fin equations"
M_b= sqrt((h2*(pi*d2)*k2*A_shaft))*(T[9]-T_crank)
m= sqrt((h2*pi*d2)/(k2*A_shaft))
y=.08[m]

T[10] =
((cosh(m*(L6-y))+(h2/(m*k2))*sinh(m*(L6-y)))/(cosh(m*L6)+(h2/(m*k2))*sinh(m*L6)))*(T[9]-T_crank)+T_crank

"Tfin =
((cosh(m*(L6-x))+(h2/(m*k2))*sinh(m*(L6-x)))/(cosh(m*L6)+(h2/(m*k2))*sinh(m*L6)))*(T[9]-T_crank)+T_crank"

Q_fin = M_b*(sinh(m*L6)+(h2/(m*k2))*cosh(m*L6))/(cosh(m*L6)+(h2/(m*k2)*sinh(m*L6)))

"energy balance equations"
Qin= Q_top+Q_wall+Q_plate
Qin = Qgen*V_cyl
Q_plate=Q_ring+Q_shaft
Q_atmosphere = Q_wall +Q_top
{Q_cylinder = Q_plate}

"solve for Q through Top, Walls, and bottom of cylinder, With this you get T11 and T12 Free!"
Q_top   = (T1-Tinf)/(R_top_cond+R_top_cov_rad)
Q_wall   = (T1-Tinf)/(R_wall_cond+R_wall_cov_rad)

qcondtop=qradcovtop     "q through top is uniform for conduction and radiation+convection"
qcondtop=(T1-T[12])/(R_top_cond)
qradcovtop=(T[12]-Tinf)/(R_top_cov_rad)
qcondwall=qradcovwall  "q through walls is same for conduction and radiation+convection"
qcondwall=(T1-T[11])/(R_wall_cond)
qradcovwall=(T[11]-Tinf)/(R_wall_cov_rad)

 "solve For T9"
Q_plate = Q_fin + (T[9]-T_crank)/R_plate_cov
 "solve for T8"
Q_plate = (T[8]-T[9])/R_plate_cond
 "solve for T3"
Q_ring= (T[3]-T[8])/(2*R_ring_contact + R_ring_cond)
Q_shaft= (T[3]-T[8])/(2*R_shaft_contact +  R_shaft_cond)
 "solve for T6"
Q_ring = (T[6]-T[8])/(R_ring_contact)
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```
"solve for T4"
Q_ring = (T[4]-T[6])/(R_ring_cond)
"solve for T7"
Q_shaft =  (T[7]-T[8])/(R_shaft_contact)
"solve for T5"
Q_shaft = (T[5]-T[7])/(R_shaft_cond)
"Solve for T2"
Q_plate = (T[2]-T[3])/R_plate_cond

"stresses"
{known/ properties}
CTE_shaft = 16E-6 [ m /(m*k)]
CTE_ring = 20.2E-6[m/(m*k)]

E_shaft = 193 *convert(GPa,Pa)
E_ring = 110 *convert(Gpa,Pa)

sigma_ring_yield= 33.3*convert(MPa,Pa)
sigma_shaft_yield = 250*convert(MPa,Pa)

FS_ring =  sigma_ring_yield/sigma_ring
FS_shaft =  sigma_shaft_yield/sigma_shaft

l_free_shaft = delta_shaft + L2
l_free_ring = delta_ring + L2

"temperature terms"
T_room = 293[k]            "ASSUMPTION"
DELTA_T_shaft= ((T[5]+T[7])/2)-T_room
DELTA_T_ring=((T[4]+T[6])/2)-T_room

"elongation terms"
delta_shaft= CTE_shaft *DELTA_T_shaft*L2
delta_ring = CTE_ring *DELTA_T_ring*L2
delta_DELTA = delta_ring- delta_shaft     "Constraint tying the deformations together"

"Stiffness terms"
k_ring = (A_ring*E_ring)/(L2)
k_shaft = (A_shaft*E_shaft)/(L2)

"stress terms"
sigma_ring = ((k_shaft/(k_shaft+k_ring))*delta_DELTA*E_ring)/l_free_ring
sigma_shaft =((k_ring/(k_shaft+k_ring))*delta_DELTA*E_shaft)/l_free_shaft
```

Appendix F: EES Output

Variables in Main

A_ring=0.00573 [m^2]	A_shaft=0.0008042 [m^2]
b=5.670E-08 [W/(m^2*K^4)]	CTE_ring=0.0000202 [m/(m*k)]
CTE_shaft=0.000016 [·m·/(m*k)]	D1=0.1 [m]
D2=0.032 [m]	delta_DELTA=0.0001269 [m]
delta_ring=0.0006103 [m]	delta_shaft=0.0004834 [m]
DELTA_T_ring=335.7 [k]	DELTA_T_shaft=335.7 [k]
EPSILON=0.07	E_ring=1.100E+11 [Pa]
E_shaft=1.930E+11 [Pa]	FS_ring=1.094
FS_shaft=1.151	h1=15 [w/(m^2*K)]
h2=750 [w/(m^2*K)]	h_r_top=2.574 [W/(m^2*k)]
h_r_wall=2.571 [w/(m^2*K)]	k1=177 [W/m*K]
k2=156 [W/m*K]	k3=401 [W/m*K]
k4=13.4 [W/m*K]	k_ring=7.004E+09 [N/m]
k_shaft=1.725E+09 [N/m]	L1=0.02 [m]
L2=0.09 [m]	L3=0.024 [m]
L4=0.095 [m]	L5=0.03 [m]
L6=0.08 [m]	l_free_ring=0.09061 [m]
l_free_shaft=0.09048 [m]	m=24.51 [1/m]
M_b=520.2 [w]	qcondtop=65.64 [w]
qcondwall=398.7 [w]	Qgen=2.500E+06 [w/m^3]
Qin=1865 [W]	qradcovtop=65.64 [w]
qradcovwall=398.7 [w]	Q_atmosphere=464.4 [w]
Q_fin=506.6 [w]	Q_plate=1401 [W]
Q_ring=1391 [w]	Q_shaft=9.557 [w]
Q_top=65.64 [W]	Q_wall=398.7 [W]
R1=0.00015 [m^2*k/w]	R2=0.002 [m^2*k/w]
ri=0.05 [m]	ro=0.08 [m]
R_plate_cond=0.01632 [k/w]	R_plate_cov=0.1891 [k/w]
R_ring_cond=0.03917 [k/w]	R_ring_contact=0.02618 [k/w]
R_shaft_cond=8.351 [k/w]	R_shaft_contact=2.487 [k/w]
R_top_cond=0.02158 [k/w]	R_top_cov_rad=7.245 [k/w]
R_wall_cond=0.004449 [k/w]	R_wall_cov_rad=1.192 [k/w]
sigma_ring=3.044E+07 [pa]	sigma_ring_yield=3.330E+07 [Pa]
sigma_shaft=2.172E+08 [pa]	sigma_shaft_yield=2.500E+08 [Pa]
S_A_plate=0.007854 [m^2]	S_A_wall=0.04775 [m^2]
T1=750 [K]	Tinf=273 [K]
T_crank=373 [K]	T_room=293 [K]
V_cyl=0.0007461 [m^3]	y=0.08 [m]