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In-class Activity

ME 570 - Prof. Tron

2023-09-12

For questions tagged with the label **scan**, answer all the questions in the space provided; to submit, scan your work and submit to Gradescope. To scan your work, you can use the flatbed scanner in the ME department office, or an app on your phone such as Adobe Scan or Dropbox, available for both iOS and Android. Please make sure to upload the pages in the correct order, and that the scans have good contrast. If needed, you can download and print this document from Blackboard.

Problem 1: Tangents to parametric curves

Compute the analytical expression of the tangent to the given curve, draw the curve, and draw the tangents at the given points as arrows.

Question 1.1. Curve: $x(t) = \begin{bmatrix} t^2 - 3 \\ t \end{bmatrix}$. Compute the expression for the tangent $\dot{x}(t)$.

$$x = \begin{bmatrix} t^2 - 3 \\ t \end{bmatrix} \quad \frac{d}{dt}[x(t)] = \dot{x} = \begin{bmatrix} 2t \\ 1 \end{bmatrix}$$

Draw the curve and tangents.

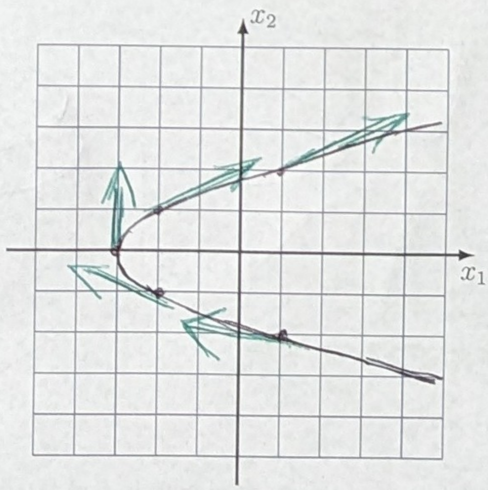
$t = 2$, $x(t) = \begin{bmatrix} 1 \\ 2 \end{bmatrix}$, $\dot{x}(t) = \begin{bmatrix} 4 \\ 1 \end{bmatrix}$

$t = 1$, $x(t) = \begin{bmatrix} -2 \\ 1 \end{bmatrix}$, $\dot{x}(t) = \begin{bmatrix} 2 \\ 1 \end{bmatrix}$

$t = 0$, $x(t) = \begin{bmatrix} -3 \\ 0 \end{bmatrix}$, $\dot{x}(t) = \begin{bmatrix} 0 \\ 1 \end{bmatrix}$

$t = -1$, $x(t) = \begin{bmatrix} -2 \\ -1 \end{bmatrix}$, $\dot{x}(t) = \begin{bmatrix} -2 \\ 1 \end{bmatrix}$

$t = -2$, $x(t) = \begin{bmatrix} 1 \\ -2 \end{bmatrix}$, $\dot{x}(t) = \begin{bmatrix} -4 \\ +1 \end{bmatrix}$



Green = TANGENT
Black = CURVE

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Question 1.2. Curve: $x(t) = \begin{bmatrix} \cos(t) \\ \sin(t) \end{bmatrix}$. Compute the expression for the tangent $\dot{x}(t)$.

$$x(t) = \begin{bmatrix} \cos t \\ \sin t \end{bmatrix} ; \frac{d}{dt}\left[x(t)\right] \Rightarrow \dot{x} = \begin{bmatrix} -\sin\theta \\ \cos\theta \end{bmatrix} \rightarrow \begin{bmatrix} -\sin(t) \\ \cos(t) \end{bmatrix}$$

Draw the curve and tangents.

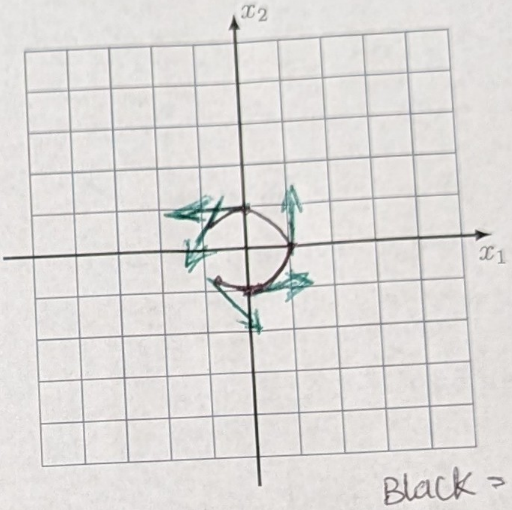
$t = \frac{3\pi}{4}$, $x(t) = \begin{bmatrix} -1/\sqrt{2} \\ 1/\sqrt{2} \end{bmatrix}$, $\dot{x}(t) = \begin{bmatrix} -1/\sqrt{2} \\ -1/\sqrt{2} \end{bmatrix}$

$t = \frac{\pi}{2}$, $x(t) = \begin{bmatrix} 0 \\ 1 \end{bmatrix}$, $\dot{x}(t) = \begin{bmatrix} -1 \\ 0 \end{bmatrix}$

$t = 0$, $x(t) = \begin{bmatrix} 1 \\ 0 \end{bmatrix}$, $\dot{x}(t) = \begin{bmatrix} 0 \\ 1 \end{bmatrix}$

$t = -\frac{\pi}{2}$, $x(t) = \begin{bmatrix} 0 \\ -1 \end{bmatrix}$, $\dot{x}(t) = \begin{bmatrix} 1 \\ 0 \end{bmatrix}$

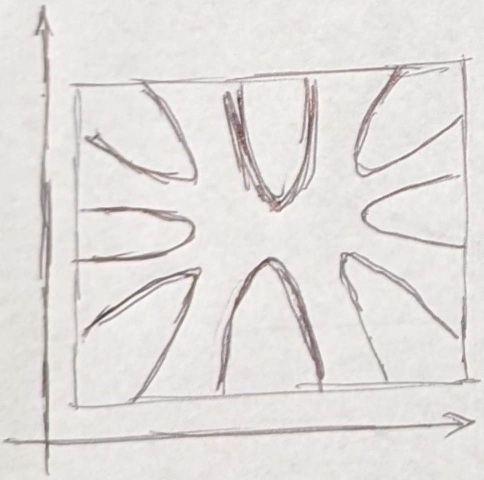
$t = -\frac{3\pi}{4}$, $x(t) = \begin{bmatrix} -1/\sqrt{2} \\ -1/\sqrt{2} \end{bmatrix}$, $\dot{x}(t) = \begin{bmatrix} 1/\sqrt{2} \\ -1/\sqrt{2} \end{bmatrix}$

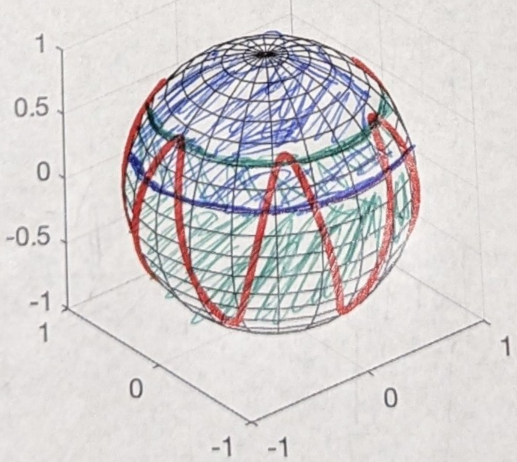


Black = CURVE
Green = Tangent.

Problem 2: Charts for the sphere

Question 2.1. Draw an atlas for the sphere (for best results, use different colors).





Next, consider the path shown in red. Draw the same curve on your charts.

Problem 3: Cartesian to polar coordinates

The following map $\phi : \mathbb{R}^2 \to \mathbb{R}^2$ defines a transformation from Polar coordinates to Cartesian coordinates:

$$\phi(r,\theta) = \begin{bmatrix} x \\ y \end{bmatrix} = \begin{bmatrix} r\cos\theta \\ r\sin\theta \end{bmatrix}. \tag{1}$$

Question 3.1. Compute the Jacobian of ϕ

$$\dot{x} = \frac{d}{dt}x = \frac{\partial x}{\partial r}\left(\frac{\partial r}{\partial t}\right) + \frac{\partial x}{\partial \theta}\left(\frac{\partial \theta}{\partial t}\right) = \cos\theta \cdot \dot{r} + (-r\sin\theta)\dot{\theta}$$

$$\dot{y} = \frac{d}{dt}y = \frac{\partial y}{\partial r}\left(\frac{\partial r}{\partial t}\right) + \frac{\partial y}{\partial \theta}\left(\frac{\partial \theta}{\partial t}\right) = \sin\theta \cdot \dot{r} + (r\cos\theta)\dot{\theta}$$

Question 3.2. Compute the determinant of the Jacobian.

$$\det\left[J(r,\theta)\right] = r\cos^2\theta + r\sin^2\theta$$
$$= r(\cos^2\theta + \sin^2\theta)$$
$$= r.$$

Question 3.3. Explain why the transformation (1) is a diffeomorphism, except at the origin.

Because it has a square Jacobian and is a non singular matrix.

continued →

$$\begin{bmatrix} \frac{\partial x}{\partial r} & \frac{\partial x}{\partial \theta} \\ \frac{\partial y}{\partial r} & \frac{\partial y}{\partial \theta} \end{bmatrix} = \begin{bmatrix} \cos\theta & -r\sin\theta \\ \sin\theta & r\cos\theta \end{bmatrix}$$

$$\begin{bmatrix} \dot{x} \\ \dot{y} \end{bmatrix} = J(r,\theta)\begin{bmatrix} \dot{r} \\ \dot{\theta} \end{bmatrix}$$