In-class Activity

ME 570 - Prof. Tron

2023-10-17

Problem 1: Control Barrier Functions

Consider a filled-in sphere obstacle with center at the origin, $x_{\text{center}} = 0$ and radius $r = 2$. Let $h(x) = \|x - x_{\text{center}}\| - r$ be the signed distance, and $\mathcal{C} = \{x \in \mathbb{R}^2 : h(x) \geq 0\}$ be the free configuration space.

Question 1.1. Write the CBF constraint for h and the dynamics $\dot{x} = u$ (u is going to be the variable in the constraint), using $c_h = \frac{1}{4}$.

Question 1.2 (0.5 points). Compute the constraints on u, and draw the feasible set given by the CBF condition and the condition $\|u\|^2 \leq 1$ for the points x_A, x_B, x_C shown below.

x_A

x_B

x_C

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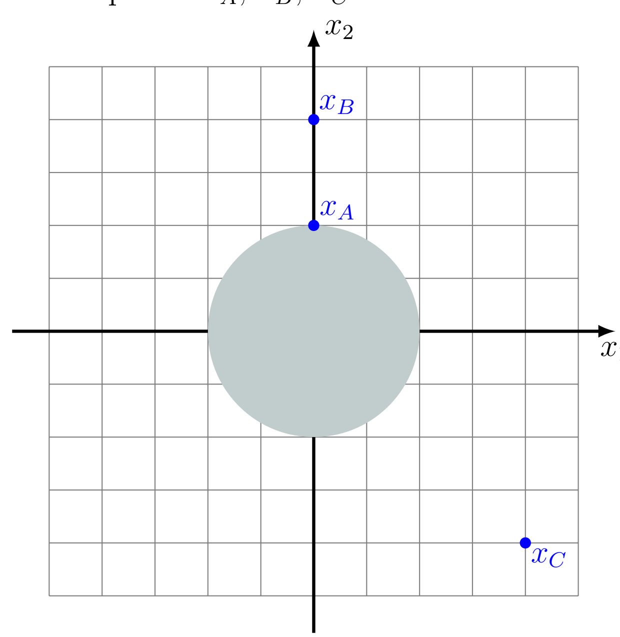

Note that the case for x_B coincides with the example of constrained optimization seen in the previous in-class activity.

Problem 2: Consensus protocol

Consider a multi-agent systems with modeled with the graph $G = (V, E)$, with $V = \{1, 2, 3\}$, $E = \{(1, 2), (2, 3)\}$. Consider the inter-agent potential $U_{ij} = \frac{1}{2}\|x_i - x_j\|^2$, where x_i is the state of the i-th agent.

Question 2.1. Write the total potential U of the system.

Question 2.2. Write the gradient of U for each node, that is $\nabla_{x_1} U$, $\nabla_{x_2} U$, $\nabla_{x_3} U$.

Question 2.3. What are the stationary point of U? Why?

Question 2.4. Explain why $\sum_i \nabla_{x_i} U = 0$ independently from the specific values of x_1, x_2, x_3.

Question 2.5. Using the above, and assuming that each node follows the update $\dot{x}_i = -\nabla_{x_i} U$, show that the average of the states is time-invariant, i.e., $\sum_i x_i(t) = \sum_i x_i(0)$ for all $t \geq 0$.

Question 2.6. Using the answers to Questions 2.3, 2.5, what is the asymptotic value for the states x_i, i.e., what is the value of $\lim_t x_i(t)$?