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In-class Activity

ME 570 - Prof. Tron

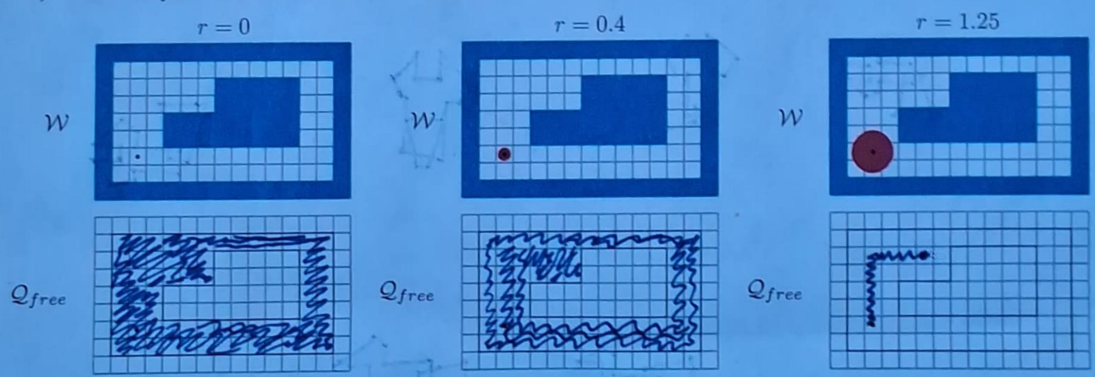
2023-09-06

For questions tagged with the label **scan**, answer all the questions in the space provided; to submit, scan your work and submit to Gradescope. To scan your work, you can use the flatbed scanner in the ME department office, or an app on your phone such as Adobe Scan or Dropbox, available for both iOS and Android. Please make sure to upload the pages in the correct order, and that the scans have good contrast. If needed, you can download and print this document from Blackboard.

Problem 1: Free configuration space for a 2-D robot

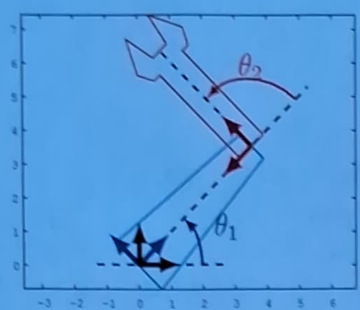
Consider a disk-like robot with radius r in a plane (i.e., the workspace is $\mathbb{R}^2$). As generalized coordinates x, we use the coordinates of the center of the disk, so that we can represent the configuration space with another 2-D plane. Let $R(x) = \{p \in \mathbb{R}^2 : d(x,p) \leq r\}$ be the set of points occupied by the robot when in position x. For each one of the cases below:

1) Draw, in the top images, two configurations of the robot, one in collision with some obstacle (mark with an x), the other not (mark with an o).
2) Draw $\mathcal{Q}_{free}$ as a subset of the plane.

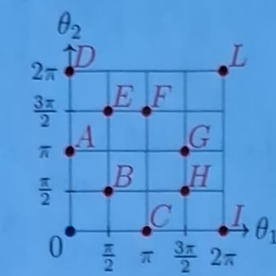


Problem 2: From generalized coordinates to configurations

Consider the relative angle choice of coordinates for the manipulator (Fig. 1a). Represent the configuration space as the $[0, 2\pi] \times [0, 2\pi]$ square. Use the space in Fig. 2 to draw the manipulator configurations for the red points marked in Fig. 1b (the blue point is given as an examples).

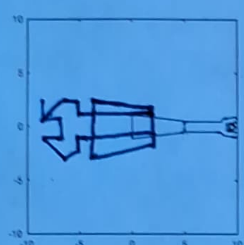


(a) Convention used for coordinates.



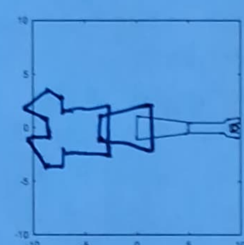
(b) Configuration space

Figure 1: Requested onfigurations

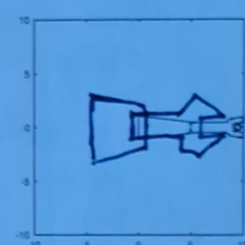


(a) Configuration *A*

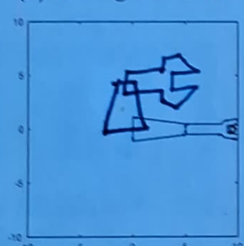
(b) Configuration *B*



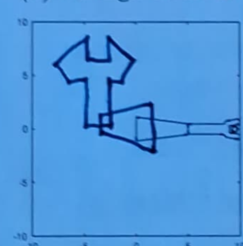
(c) Configuration *C*



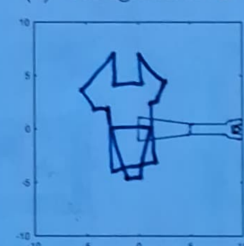
(d) Configuration *D*



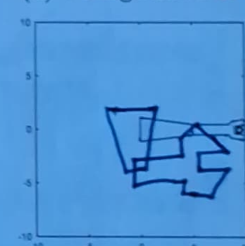
(e) Configuration *E*



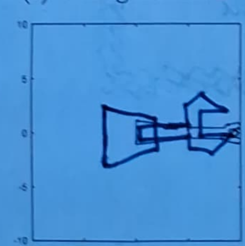
(f) Configuration *F*



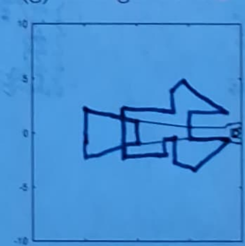
(g) Configuration *G*



(h) Configuration *H*



(i) Configuration *I*



(j) Configuration *L*

Figure 2: Output of the kinematic map (the configuration $(0, 0)$ is drawn in blue for reference)

Question 2.1. Which sets of coordinates correspond to the same configuration?

D, I, L and contifiguration (0,0)

Question 2.2. Which sets of coordinates correspond to the same end effector position?

D, I, L and Configuration (0,0)

Question 2.3. Do the 2-D coordinates of the end effector represent a valid choice of coordinates for the configuration space of the manipulator?

No they donot as for a given 2D coordinates of a end effector has multiple configuration of manipulatore. There is no such unique configuration if we use 2D coordinates of end effector.

Problem 3: Parametric curves

For each one of the questions below, perform the following steps:

1) Compute the values of the parametric curves $x(t)$ for the given values.
2) Draw the corresponding points on the provided axes.
3) Draw the curve for the interval spanning from the minimum to the maximum of the given values of t.
4) Add an arrow indicating the direction for the curve with t increasing.

In the axes, it is assumed that each square represents one unit.

Question 3.1. A line $x_A(t) = \begin{bmatrix} a_1 t + b_1 \\ a_2 t + b_2 \end{bmatrix}$, with parameters $a_1 = 2, b_1 = -2, a_2 = 0, b_2 = 1$

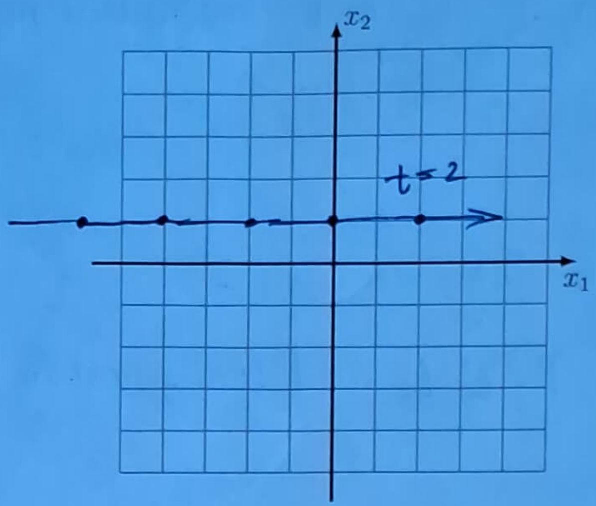
$t = 2,\ x_A(t) = \begin{bmatrix} 2 \\ 1 \end{bmatrix}$

$t = 1,\ x_A(t) = \begin{bmatrix} 0 \\ 1 \end{bmatrix}$

$t = 0,\ x_A(t) = \begin{bmatrix} -2 \\ 1 \end{bmatrix}$

$t = -1,\ x_A(t) = \begin{bmatrix} -4 \\ 1 \end{bmatrix}$

$t = -2,\ x_A(t) = \begin{bmatrix} 6 \\ 1 \end{bmatrix}$



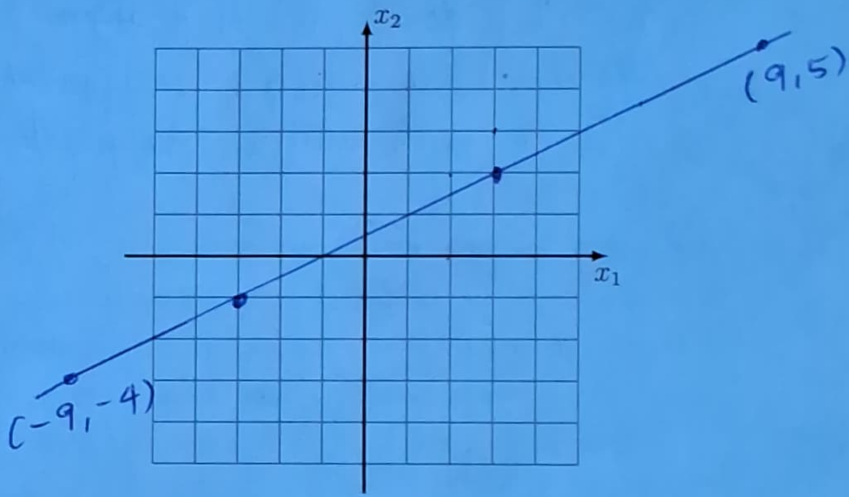
Question 3.2. A line $x_B(t) = (1-t)x_0 + t x_1$, with parameters $x_0 = \begin{bmatrix} -3 \\ -1 \end{bmatrix}$, $x_1 = \begin{bmatrix} 3 \\ 2 \end{bmatrix}$.

$t = 2,\ x_B(t) = \begin{bmatrix} 9 \\ 5 \end{bmatrix}$

$t = 1,\ x_B(t) = \begin{bmatrix} 3 \\ 2 \end{bmatrix}$

$t = 0,\ x_B(t) = \begin{bmatrix} -3 \\ -1 \end{bmatrix}$

$t = -1,\ x_B(t) = \begin{bmatrix} -9 \\ -4 \end{bmatrix}$



Question 3.3. Where do x_A and x_B intersect? For which values of t?

$= x(t_a) = x_b(t_b);\quad x_a(t_a) = \begin{bmatrix} 2t_a - 2 \\ 1 \end{bmatrix} \quad x_b(t_b) = \begin{bmatrix} -3+3t_b \\ -1+t_b \end{bmatrix} + \begin{bmatrix} 3t_b \\ 2t_b \end{bmatrix} = \begin{bmatrix} -3+6t_b \\ -1+3t_b \end{bmatrix}$

$\Rightarrow \begin{bmatrix} 2t_a - 2 \\ 1 \end{bmatrix} = \begin{bmatrix} -3+6t_b \\ -1+3t_b \end{bmatrix} \Rightarrow 1 = -1 + 3t_b ;\ \therefore\ t_b = \frac{2}{3} = \boxed{0.666}$

$\Rightarrow 2t_a - 2 = -3 + 6\left(\frac{2}{3}\right) \Rightarrow 2t_a - 2 = -3 + 4$

$t_a = \frac{3}{2} = \boxed{1.5}$

Question 3.4. The curve $x(t) = \begin{bmatrix} t^2 - 3 \\ t \end{bmatrix}$

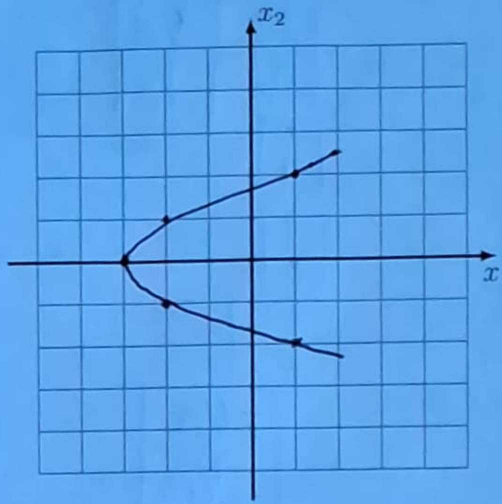
$t = 2,\ x(t) = \begin{bmatrix} 1 \\ 2 \end{bmatrix}$

$t = 1,\ x(t) = \begin{bmatrix} -2 \\ 1 \end{bmatrix}$

$t = 0,\ x(t) = \begin{bmatrix} -3 \\ 0 \end{bmatrix}$

$t = -1,\ x(t) = \begin{bmatrix} -2 \\ -1 \end{bmatrix}$

$t = -2,\ x(t) = \begin{bmatrix} 1 \\ -2 \end{bmatrix}$



Question 3.5. The curve $x(t) = \begin{bmatrix} 0 & 1 \\ 1 & 0 \end{bmatrix} \begin{bmatrix} t^2 - 3 \\ t \end{bmatrix}$

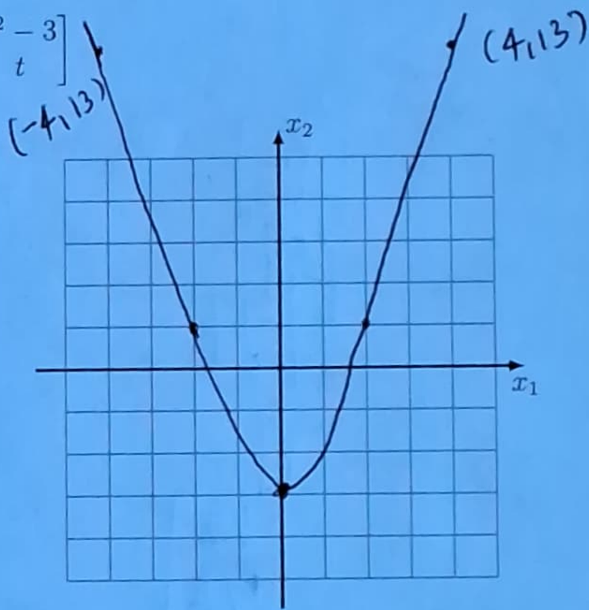
$t = 4,\ x(t) = \begin{bmatrix} 4 \\ 13 \end{bmatrix}$

$t = 2,\ x(t) = \begin{bmatrix} 2 \\ 1 \end{bmatrix}$

$t = 0,\ x(t) = \begin{bmatrix} 0 \\ -3 \end{bmatrix}$

$t = -2,\ x(t) = \begin{bmatrix} -2 \\ 1 \end{bmatrix}$

$t = -4,\ x(t) = \begin{bmatrix} -4 \\ 13 \end{bmatrix}$



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Question 3.6. $x(t) = \begin{bmatrix} \cos(t) \\ \sin(t) \end{bmatrix}$

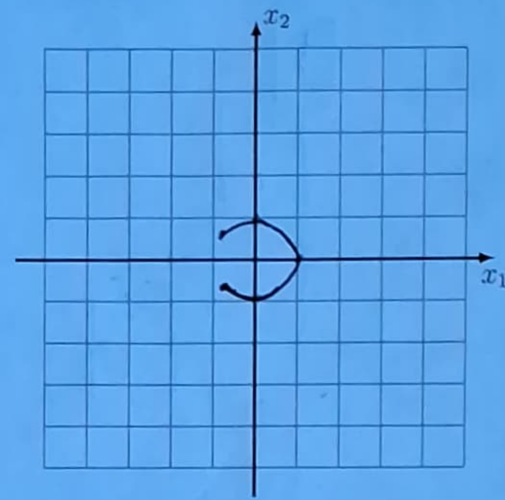
$t = \frac{3\pi}{4}$, $x(t) = \cancel{\begin{bmatrix} 0 \\ -1 \end{bmatrix}} \begin{bmatrix} -1/\sqrt{2} \\ 1/\sqrt{2} \end{bmatrix}$

$t = \frac{\pi}{2}$, $x(t) = \begin{bmatrix} 0 \\ 1 \end{bmatrix}$

$t = 0$, $x(t) = \begin{bmatrix} 1 \\ 0 \end{bmatrix}$

$t = -\frac{\pi}{2}$, $x(t) = \begin{bmatrix} 0 \\ -1 \end{bmatrix}$

$t = -\frac{3\pi}{4}$, $x(t) = \begin{bmatrix} -1/\sqrt{2} \\ -1/\sqrt{2} \end{bmatrix}$



Question 3.7. $x(t) = \begin{bmatrix} \cos(t-\pi) \\ \sin(t-\pi) \end{bmatrix}$

$t = \frac{3\pi}{4}$, $x(t) = \begin{bmatrix} 1/\sqrt{2} \\ -1/\sqrt{2} \end{bmatrix}$

$t = \frac{\pi}{2}$, $x(t) = \begin{bmatrix} 0 \\ -1 \end{bmatrix}$

$t = 0$, $x(t) = \begin{bmatrix} -1 \\ 0 \end{bmatrix}$

$t = -\frac{\pi}{2}$, $x(t) = \begin{bmatrix} 0 \\ 1 \end{bmatrix}$

$t = -\frac{3\pi}{4}$, $x(t) = \begin{bmatrix} 1/\sqrt{2} \\ +1/\sqrt{2} \end{bmatrix}$