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In-class Activity

ME 570 - Prof. Tron

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Problem 1: Gradients of attractive and repulsive potentials

In the next homework assignment we will use the following potentials.

Question 1.1. The expression for the attractive potential is

$$U_{\text{attr}}(x) = d^p(x, x_{\text{goal}}) = \|x - x_{\text{goal}}\|^p. \tag{1}$$

where p is a parameter to distinguish between conic ($p = 1$) and quadratic ($p = 2$) potentials. Write an expression for the gradient ∇U_{attr}.

when $p=2$

$$\nabla U_{attr} = 2 \cdot \|x - x_{goal}\|^{p} \nabla d(x, x_{goal}) = 2(x - x_{goal})$$

when $p=1$

$$\nabla U_{attr} = \frac{X - X_{goal}}{\|x - x_{goal}\|}$$

$$\therefore \boxed{\nabla U_{attr} = p\|x - x_{goal}\|^{p-1} \times \frac{X - X_{goal}}{\|X - X_{goal}\|}}$$

Question 1.2. The expression for the repulsive potential is

$$U_{\text{rep},i}(x) = \begin{cases} \frac{1}{2}\left(\frac{1}{d_i(x)} - \frac{1}{d_{\text{influence}}}\right)^2 & \text{if } 0 < d_i(x) < d_{\text{influence}}, \\ 0 & \text{if } d_i(x) > d_{\text{influence}}, \\ \infty & \text{otherwise}, \end{cases} \tag{2}$$

$\rightarrow 0$

Write an expression for the gradient ∇U_{rep}.

$$\nabla U_{rep} = 2 \times \frac{1}{2} \cdot \left(\frac{1}{di(X)} - \frac{1}{d_{inf}}\right) \nabla \left(\frac{1}{di(X)} - \frac{1}{d_{inf}}\right)$$

$$\nabla\left(\frac{1}{di(x)} - \frac{1}{d_{inf}}\right) = \nabla di^{-1}(x) - \nabla d_{inf} \rightarrow 0 \quad = -\frac{1}{di^2(X)} \nabla di(X)$$

$$\nabla U_{rep} = -\left(\frac{1}{di(X)} - \frac{1}{d_{inf}}\right) \cdot \frac{1}{di^2(X)} \nabla di(X)$$

Problem 2: Potential methods for a one-link manipulator

Consider a 1-link manipulator shown in Figure 1 The kinematic map for the end effector p_{eff} shown in the figure is given by

$$^{w}p_{\text{eff}} = \begin{bmatrix} r\cos\theta \\ r\sin\theta \end{bmatrix}, \tag{3}$$

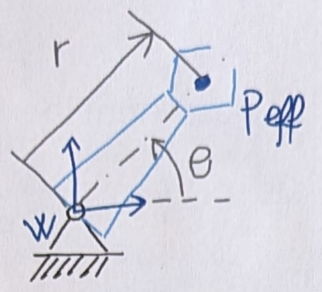

Figure 1: A 1-link manipulator

where r is a constant; note that (3) is the same as the definition of polar coordinates seen in a previous activity, except that r is not a variable coordinate but is constant.

Question 2.1. Write the Jacobian of (3) that maps $\dot{\theta}$ to ${}^{W}\dot{p}_{\text{eff}}$. What are its dimensions?

$$J = \frac{\partial\, {}^{W}p_{eff}}{\partial \theta} \; ; \quad {}^{W}p_{eff} = \begin{bmatrix} r\cos\theta \\ r\sin\theta \end{bmatrix} \quad \therefore J = \begin{bmatrix} -r\sin\theta \\ r\cos\theta \end{bmatrix}$$

↳ Dimmension are 2×1

Question 2.2. Define a potential function in the task space, $U(p_{\text{eff}}) = \|p_{\text{eff}} - p_{\text{goal}}\|$, where p_{goal} is an arbitrary fixed location. Imagine that the coordinate θ is given by a parametric curve $\theta(t)$. Compute $\frac{d}{dt}U$ using the chain rule and the Jacobian from Question 2.1.

$$\frac{d}{dt}U = \nabla U \cdot J \, \frac{d\theta(t)}{dt} \; ; \quad \nabla U = \frac{\partial U}{\partial x}, \frac{\partial U}{\partial y}$$

$$\therefore \frac{d}{dt}U = \nabla U \cdot \begin{bmatrix} -r\sin\theta \\ r\cos\theta \end{bmatrix} \cdot \dot{\theta}(t)$$

Question 2.3. Write a command u that a potential planner can use to control θ and drive the end effector toward p_{goal}.

command $u = -K * \nabla U$ where
u is the command to control angle θ
K is a positive constant that shows the strength of potential feild (gain)
& ∇U is gradient of the potential function $U(p_{eff})$ with ~~respect~~ respect to variable θ.