In-class Activity

ME 570 - Prof. Tron

2023-10-02

Problem 1: Gradients of attractive and repulsive potentials

In the next homework assignment we will use the following potentials.

Question 1.1. The expression for the attractive potential is

$$U_{\text{attr}}(x) = d^p(x, x_{\text{goal}}) = \|x - x_{\text{goal}}\|^p. \tag{1}$$

where p is a parameter to distinguish between conic ($p = 1$) and quadratic ($p = 2$) potentials. Write an expression for the gradient ∇U_{attr}.

Question 1.2. The expression for the repulsive potential is

$$U_{\text{rep},i}(x) = \begin{cases} \frac{1}{2}\left(\frac{1}{d_i(x)} - \frac{1}{d_{\text{influence}}}\right)^2 & \text{if } 0 < d_i(x) < d_{\text{influence}}, \\ 0 & \text{if } d_i(x) > d_{\text{influence}}, \\ \infty & \text{otherwise,} \end{cases} \tag{2}$$

Write an expression for the gradient ∇U_{rep}.

Problem 2: Potential methods for a one-link manipulator

Consider a 1-link manipulator shown in Figure 1 The kinematic map for the end effector p_{eff} shown in the figure is given by

$$^{\mathcal{W}}p_{\text{eff}} = \begin{bmatrix} r\cos\theta \\ r\sin\theta \end{bmatrix}, \tag{3}$$

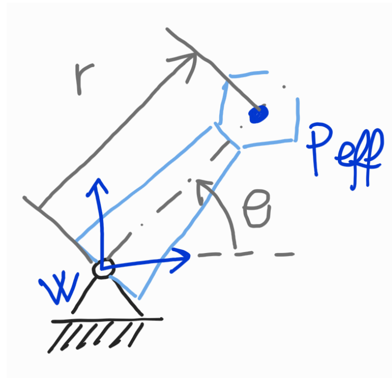

Figure 1: A 1-link manipulator

where r is a constant; note that (3) is the same as the definition of polar coordinates seen in a previous activity, except that r is not a variable coordinate but is constant.

Question 2.1. Write the Jacobian of (3) that maps $\dot{\theta}$ to ${}^{W}\dot{p}_{\text{eff}}$. What are its dimensions?

Question 2.2. Define a potential function in the task space, $U(p_{\text{eff}}) = \|p_{\text{eff}} - p_{\text{goal}}\|$, where p_{goal} is an arbitrary fixed location. Imagine that the coordinate θ is given by a parametric curve $\theta(t)$. Compute $\frac{\mathrm{d}}{\mathrm{d}t}U$ using the chain rule and the Jacobian from Question 2.1.

Question 2.3. Write a command u that a potential planner can use to control θ and drive the end effector toward p_{goal}.