The research of the relation between the dielectric permittivity and the polarizability

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Introduction

Problem statement

Develop a theoretical model that describes the relationship between the **dielectric permittivity** of a substance and its **polarisability**.

Assemble the installation and conduct an experiment with a liquid with **non-polar** molecules and check whether the experimental data are described by the obtained law.

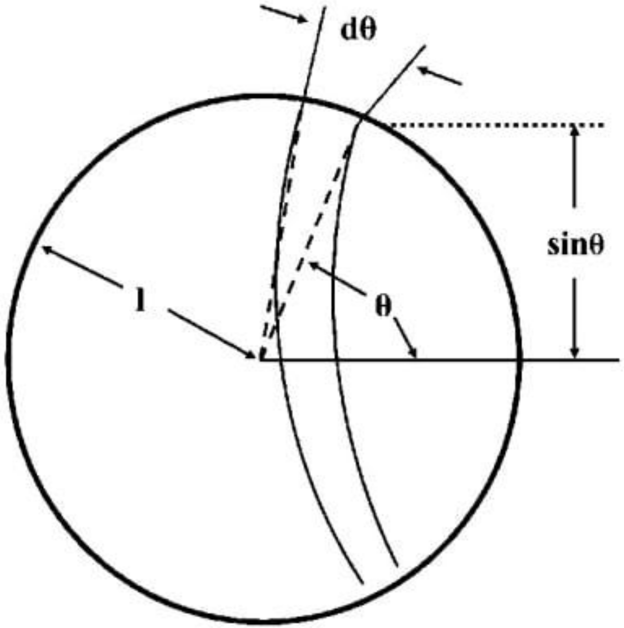
Relevance of the research

Knowledge of the properties of dielectric materials is still relevant today, despite the fact that this topic has been actively* developed since **1850**.

It was then that Ottaviano Fabrizio Mossotti discovered the equation that in the future became known as the Clausius-Mossotti Law.

* For example, articles [1], [2], [3], [4], [5] between 2019-2023 (see the "References" section)

Theoretical model



Consider the polarization density.

$$P = N(\alpha_e + \alpha_i + \frac{p^2}{3\epsilon T})\epsilon_0 E \quad (1)$$

It is determined by three factors:

- α_e - electron polarization e
- α_i - ionic polarizability
- N - number of molecules per unit volume
- E - electric field acting on the molecule

$$dS = 2\pi sin(\theta)d\theta$$

$$dN = Ae^{pEcos(\theta)/\epsilon T}sin(\theta)d\theta$$

$$A = \frac{N}{\int_0^{\pi} e^{pEcos(\theta)/\epsilon T}sin(\theta)d\theta}$$

$$dP = pcos(\theta)dN$$

$$P = \int_{\theta}^{\pi} dNpcos(\theta) = Ap\int_{\theta}^{\pi} e^{pEcos(\theta)/\epsilon T} sin(\theta)cos(\theta)d\theta$$

$$P = Np\frac{\int_{\theta}^{\pi} e^{pEcos(\theta)/\epsilon T} sin(\theta)cos(\theta)d\theta}{\int_{\theta}^{\pi} e^{pEcos(\theta)/\epsilon T} sin(\theta)d\theta}$$

By replacing of
$$b = pE/\epsilon T$$
$$x = cos\theta$$
$$dx = -sin(\theta)d\theta$$

The previous equation simplifying to the form below:

$$P = Np\frac{\int_{-1}^{1} e^{bx}xdx}{\int_{-1}^{1} e^{bx}dx}$$

Integrating by parts, we get:

$$P = Np(coth(b) - \frac{1}{b}) = NpL(b) \tag{2}$$

$L(b) = coth(b) - \dfrac{1}{b}$ — the Langevin function introduced by Langevin in 1905

At room temperature, for typical electric fields in laboratories, $pE \ll \epsilon T$

So, we get:

$$L(b) \approx \frac{b}{3} = \frac{pE}{3\epsilon T}$$

Via Eq. (2) we get relation for the average dipole moment below:

$$P = \frac{Np^2E}{3\epsilon T}$$

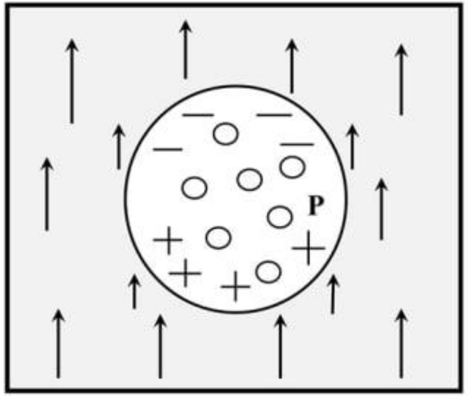


Fig. 2. Parallel plate capacitor

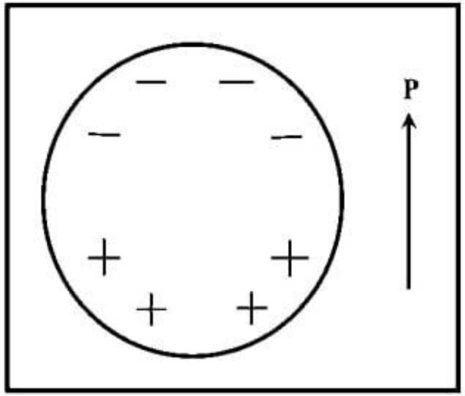


Fig. 3. Spherical cavity

The orientational polarizability per molecule:

$$\alpha_0 = \frac{P/N}{E\epsilon_0} = \frac{p^2}{3\epsilon\epsilon_0 T}$$

The electric susceptibility $\chi = \epsilon - 1$, also,

$$P = \epsilon_0 \chi E$$

We will get via substituting Eq. (1):

$$\chi = \epsilon - 1 = N(\alpha_e + \alpha_i + \frac{p^2}{3\epsilon T})$$

Figures 2 and 3 will help to understand the total electric field in a dielectric better:

$$E_{total} = E_1 + E_2 + E_3$$

A uniformly polarized dielectric sphere creates an electric field:

$$E_p = -\nabla\phi = -\frac{P}{3\epsilon_0}$$

The charge distribution is the same, so:

$$E_2 = -E_p = \frac{P}{3\epsilon_0}$$

It has been shown that

$$E_3 = 0$$

for materials whose atoms have a simple cubic lattice structure.

Then we will have:

$$E_{total} = E_1 + \frac{P}{3\epsilon_0}$$

We defined the polarizability of a molecule of the dielectric:

$$p_{total} = \epsilon_0 \alpha E_{total}$$

$$P = N p_{total} = \epsilon_0 N \alpha E_{total} = \epsilon_0 N \alpha (E + \frac{P}{3\epsilon_0}) \qquad P = \frac{N\alpha}{1 - N\alpha/3} \epsilon_0 E$$

Thus, we obtain a relation linking the macroscopic dielectric constant with the microscopic polarizability:

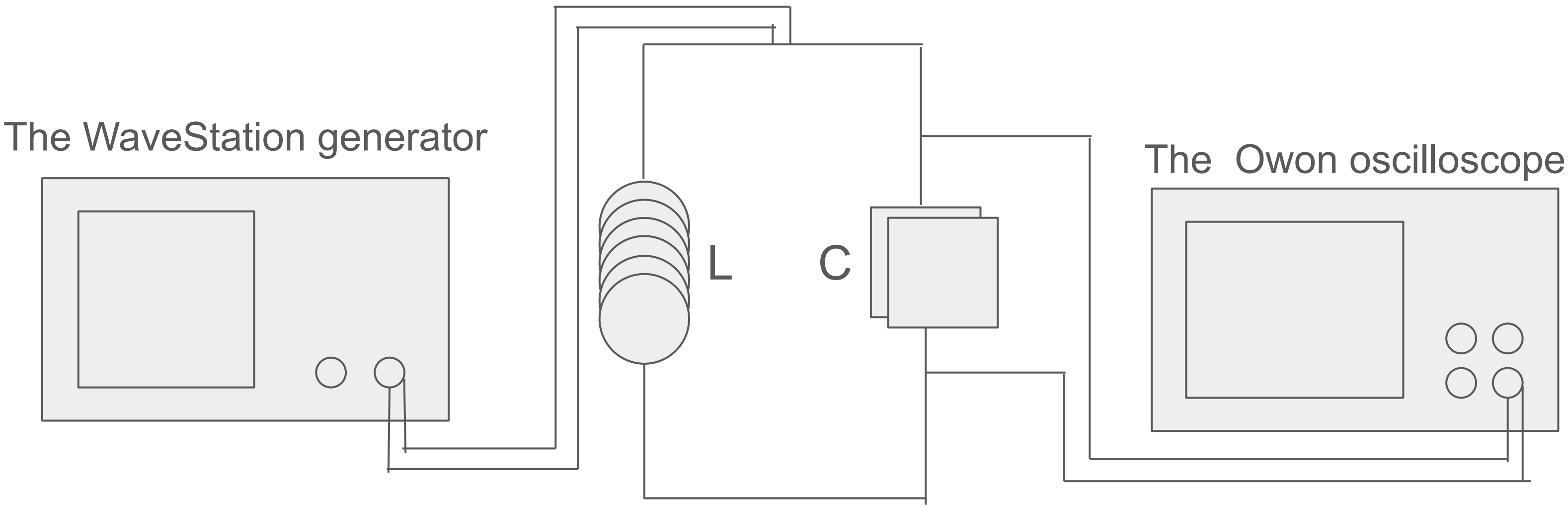
$$\chi = \epsilon - 1 = \frac{N\alpha}{1 - N\alpha/3} \longrightarrow \alpha = \frac{3}{N} (\frac{\epsilon - 1}{\epsilon + 2})$$

We will have in the GHS system for a binary mixture of substances:

$$\boxed{\frac{\epsilon - 1}{\epsilon + 2} = \frac{4\pi}{3} (N_1 \alpha_1 + N_2 \alpha_2)}$$

The Clausius-Mossotti equation

Experimental setup

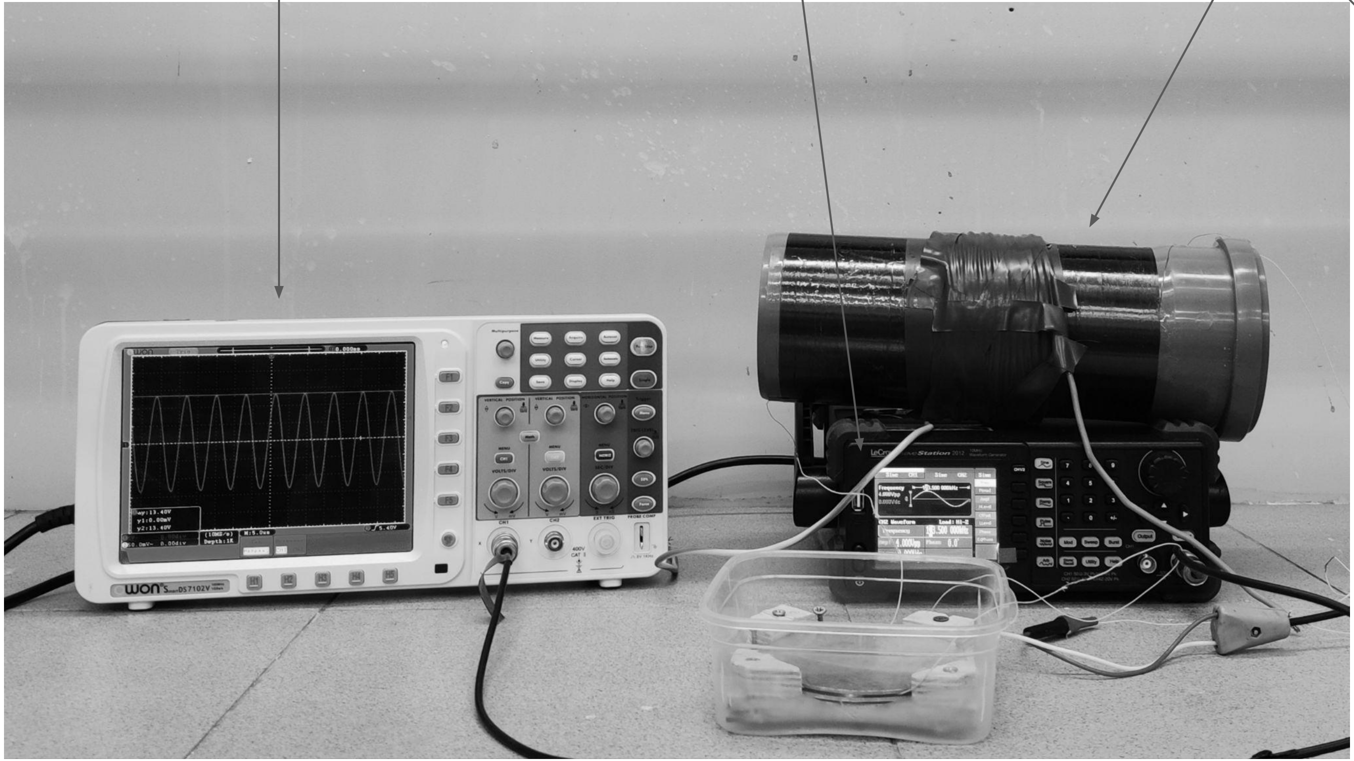

The WaveStation generator
L
C
The Owon oscilloscope

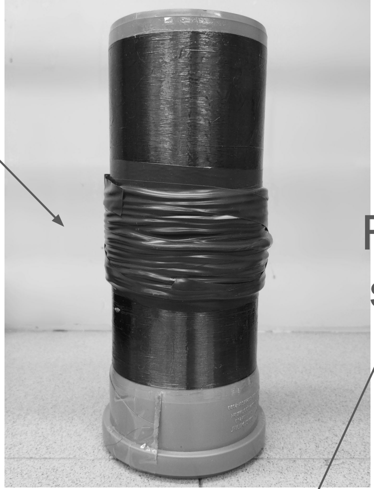
The oscilloscope

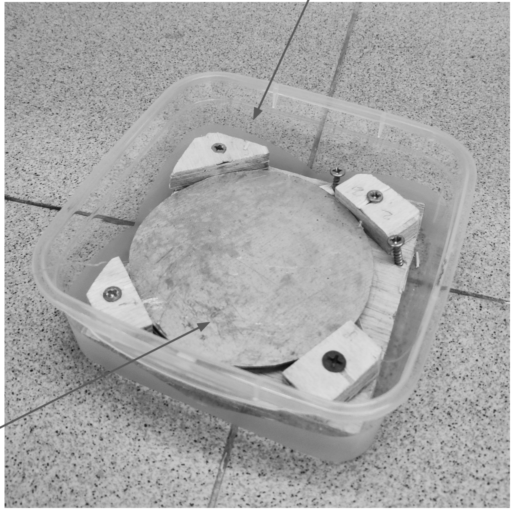
The generator

The coil

Fixing sides



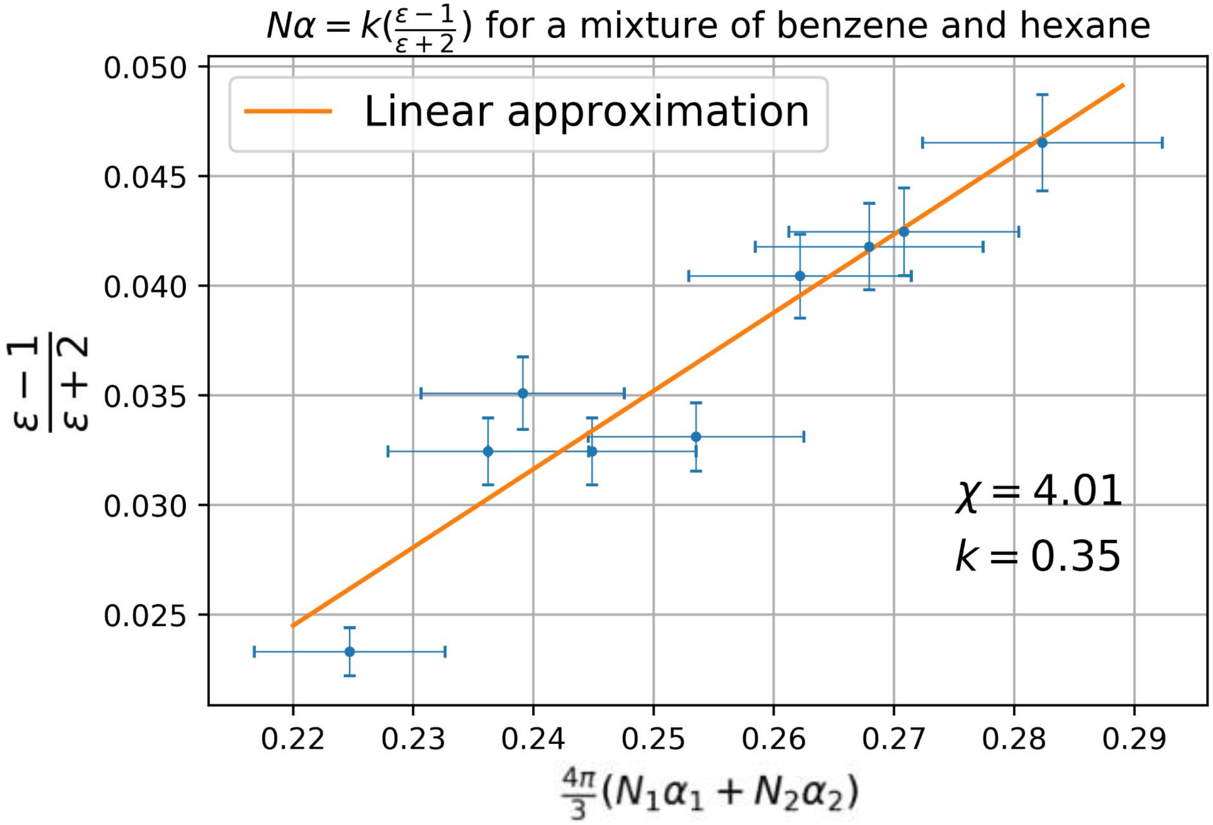




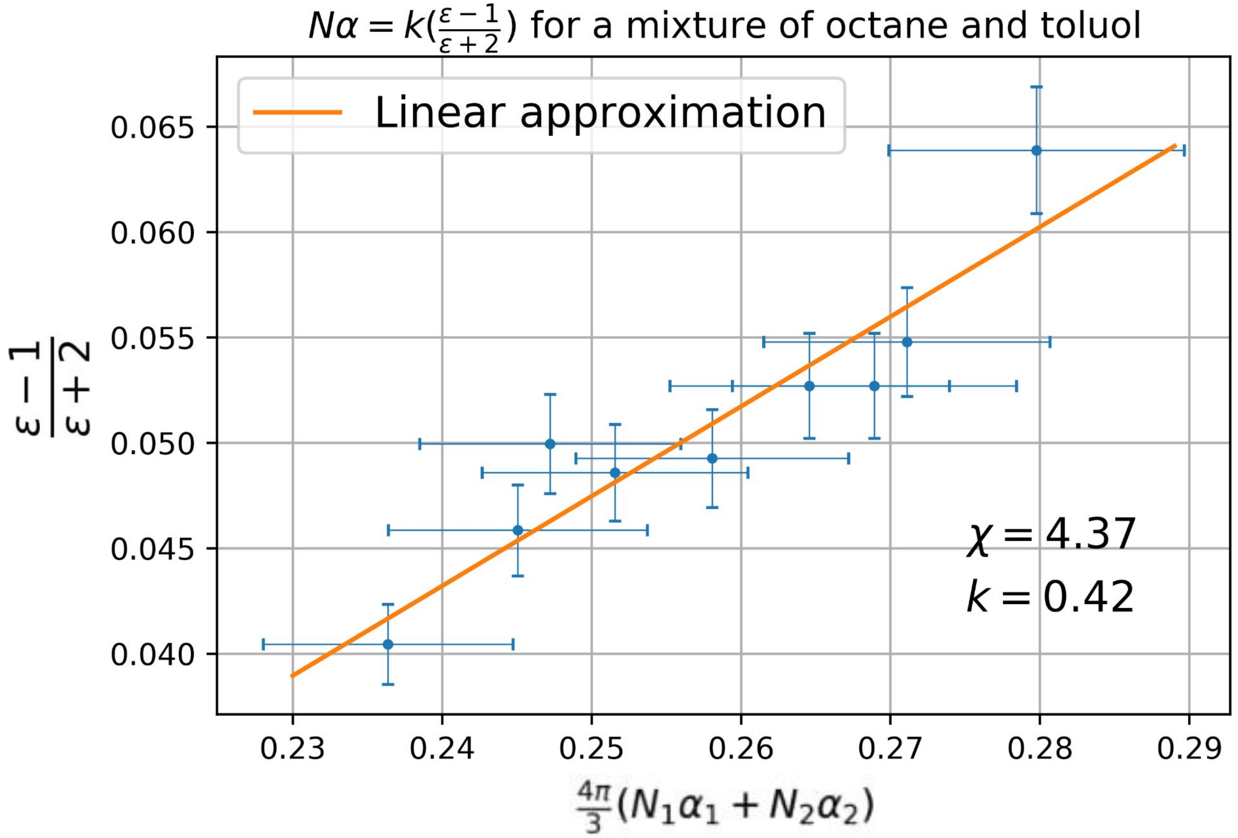
Capacitor plates

Experiment

A benzene and hexane mixture



An octane and toluol mixture



Results and analysis

Results:

- The dependence is **linear**, as expected from the theory
- **Similar results** were obtained for **two different** mixtures of liquids
- Experimental and theoretical data **converges in order**

$$k_1 = 0.35 \pm 0.08, k_2 = 0.42 \pm 0.08, k = 1$$

Analysis:

- Atoms have a **cubic lattice structure**
- The material under investigation is a **crystalline** substance

It is natural to assume that taking into account these corrections, we could see a **more** accurate **similarity** with the theory.

References

1. Ling Yuan Liao, Wen Hong Ruan, Ming Qiu Zhang, Li Weihao. Improving the Dimensional Stability of Polyphenylene Oxide without Reducing Its Dielectric Properties for High-Frequency Communication Applications. *Industrial & Engineering Chemistry Research* 2023, *62* (18) , 7007-7016
2. Yufeng Wang, Greg M. Swain, G. J. Blanchard. Charge-Induced Birefringence in a Room-Temperature Ionic Liquid. *The Journal of Physical Chemistry B* 2021, *125* (3) , 950-955
3. Yufeng Wang, Fatemeh Parvis, Md. Iqbal Hossain, Ke Ma, Romana Jarošová, Greg M. Swain, Gary J. Blanchard. Local and Long-Range Organization in Room Temperature Ionic Liquids. *Langmuir* 2021, *37* (2) , 605-615
4. M. J. Kishor Kumar, Jagannathan T. Kalathi. Interface Dominated Dielectric Response of PS-Fe3O4 Patchy Microspheres. *Langmuir* 2019, *35* (43) , 13923-13933
5. Hongfeng Ma, Said Bakhti, Anton Rudenko, Francis Vocanson, Daniel S. Slaughter, Nathalie Destouches, Tatiana E. Itina. Laser-Generated Ag Nanoparticles in Mesoporous TiO2 Films: Formation Processes and Modeling-Based Size Prediction. *The Journal of Physical Chemistry C* 2019, *123* (42) , 25898-25907

Thank you for your attention!