Control Systems Practical 5 **14 October 2020**

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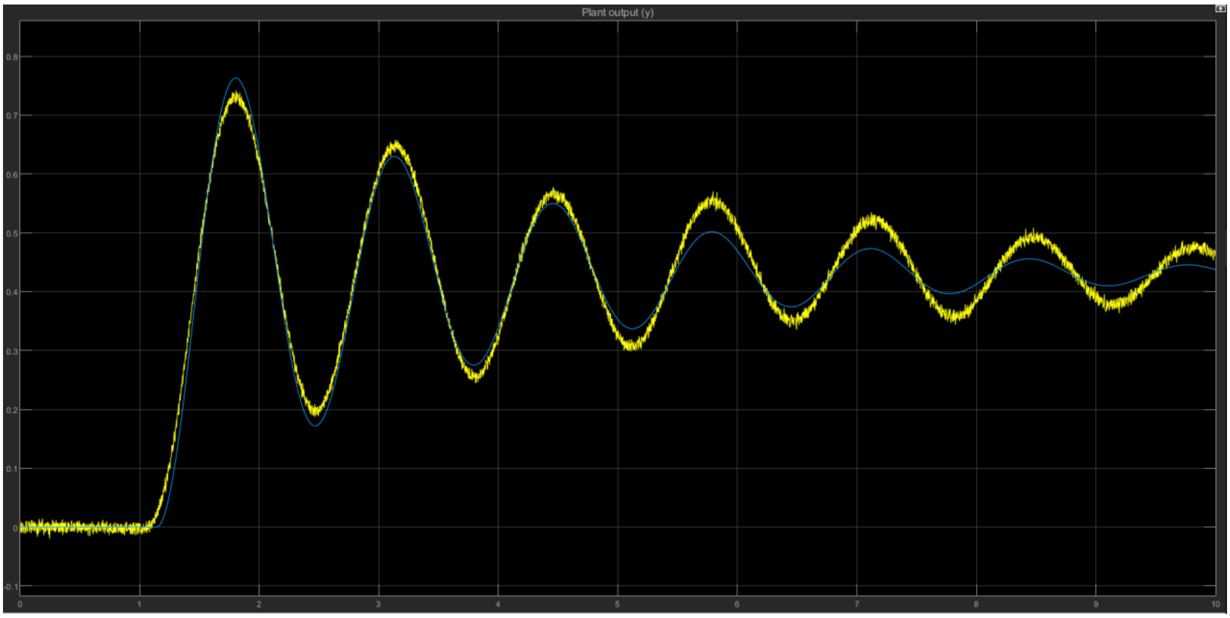
Question 1:

The plant model is described by the following transfer function.

$$G(s) = \frac{Ce^{-T_d S}}{S^2 + 2\omega_n \delta S + \omega_n^2}$$

I found the following values represented the actual plant the best. $C = 0.97$, $\omega_n = 4.75\ rad/s$, $\delta = 0.81$, $T_d = 0.14\ s$

Plan model with response after a step input of 0.5 rad is applied.



As you can see the plant model matches the response of the actual plant very well. It is not a perfect match as evident in the oscillations not matching up 100%. In the beginning when the pendulum moves from the hanging position to 30°, the oscillation peaks and the linearised model overshoots the plant response very slightly. The equilibrium position for both plant and model are the same for a step input. After the third oscillation the plant and plant model differ more, however it is a more stable state relative to when it started oscillating and therefore it is not of concern.

Question 2: Analog controller

Optimally damped $\rightarrow \zeta = 0,707$

overshoot = $\rightarrow 0,043$

$t_{2\%} \rightarrow 2s$

zero steady state tracking error.

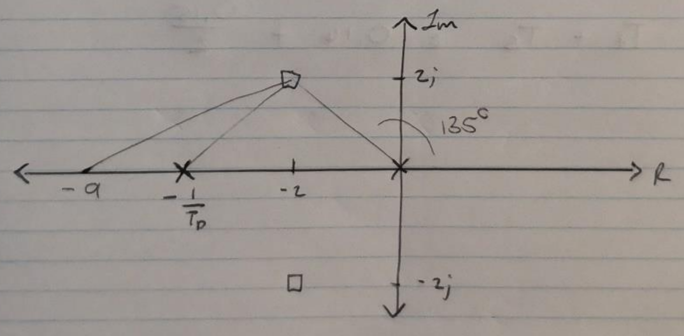
$$t_{2\%} = \frac{4}{\zeta w_n} = 2 \rightarrow w_n = 2.826 \text{ rad/s}$$

$$M_p = e^{-\frac{\pi\zeta}{\sqrt{1-\zeta^2}}} = 0,043$$

$$s^2 + 2\zeta w_n s + w_n^2 = s^2 + 4s + 7.986$$

$$s = \frac{-4 \pm \sqrt{4^2 - 4(7.986)}}{2}$$

$$s = -2 \pm 2j$$



$$G'(s) = \left(\frac{\frac{1}{T_D}}{s+\frac{1}{T_D}}\right)\left(\frac{C}{s^2+2\zeta w_n s+w_n^2}\right)$$

$$D_a = K_D \frac{s^2+2\zeta w_n s + w_n^2}{s(s+a)}$$

Angle Condition

$$135° + \tan^{-1}\left(\frac{2}{a-2}\right) + \tan^{-1}\left(\frac{2}{\frac{1}{T_D}-2}\right) = 180° + 360°n$$

$$135° + \tan^{-1}\left(\frac{2}{a-2}\right) + 37.03° = 180°$$

$$\tan^{-1}\left(\frac{2}{a-2}\right) = 7.9696$$

$$a = 16.82$$

Magnitude Condition

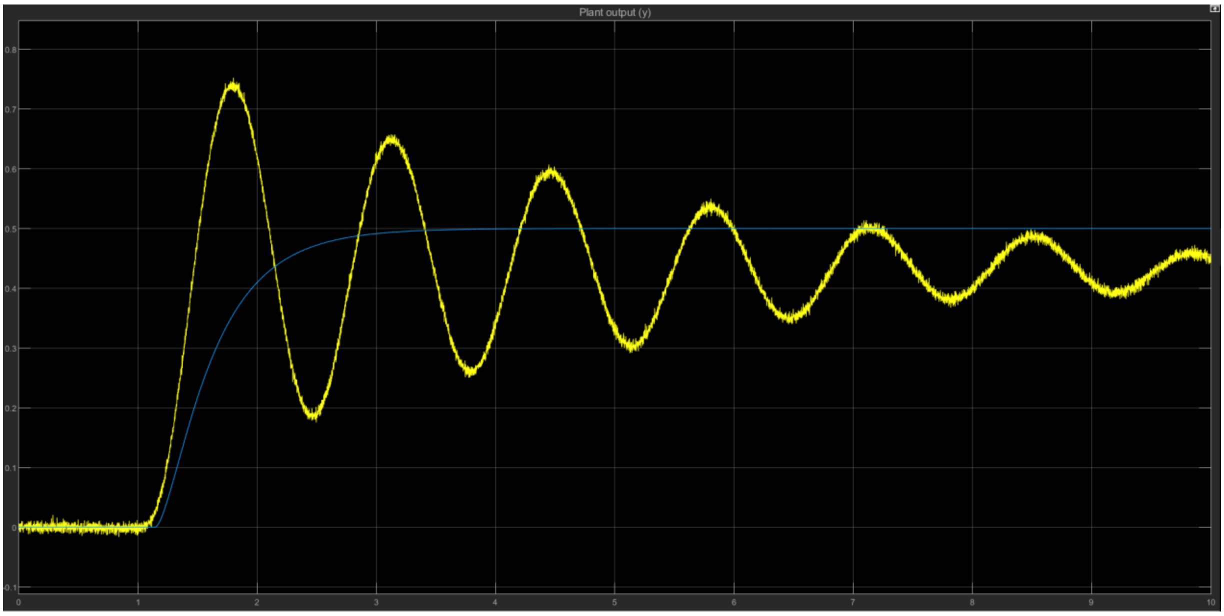
$$\left| D_a(s)\, G'(s) \right| = 1$$

$$\left| K_D \frac{\frac{1}{T_D} C}{s(s+a)(s+\frac{1}{T_D})} \right|_{s=-2+2j} = 1$$

$$K_D = 30.2$$

$$T_D = T_d + T_s = 0.14 + \frac{0.15}{2}$$

Analog controller attached to the plant model.



The controller is designed to be optimally damped with an overshoot of 4.3%. The gain of the controller is calculated using the gain condition to be 30 and the controller pole is situated at a = - 18. The step response of 0.5 rad with the controller attached to the plant model can be seen above.

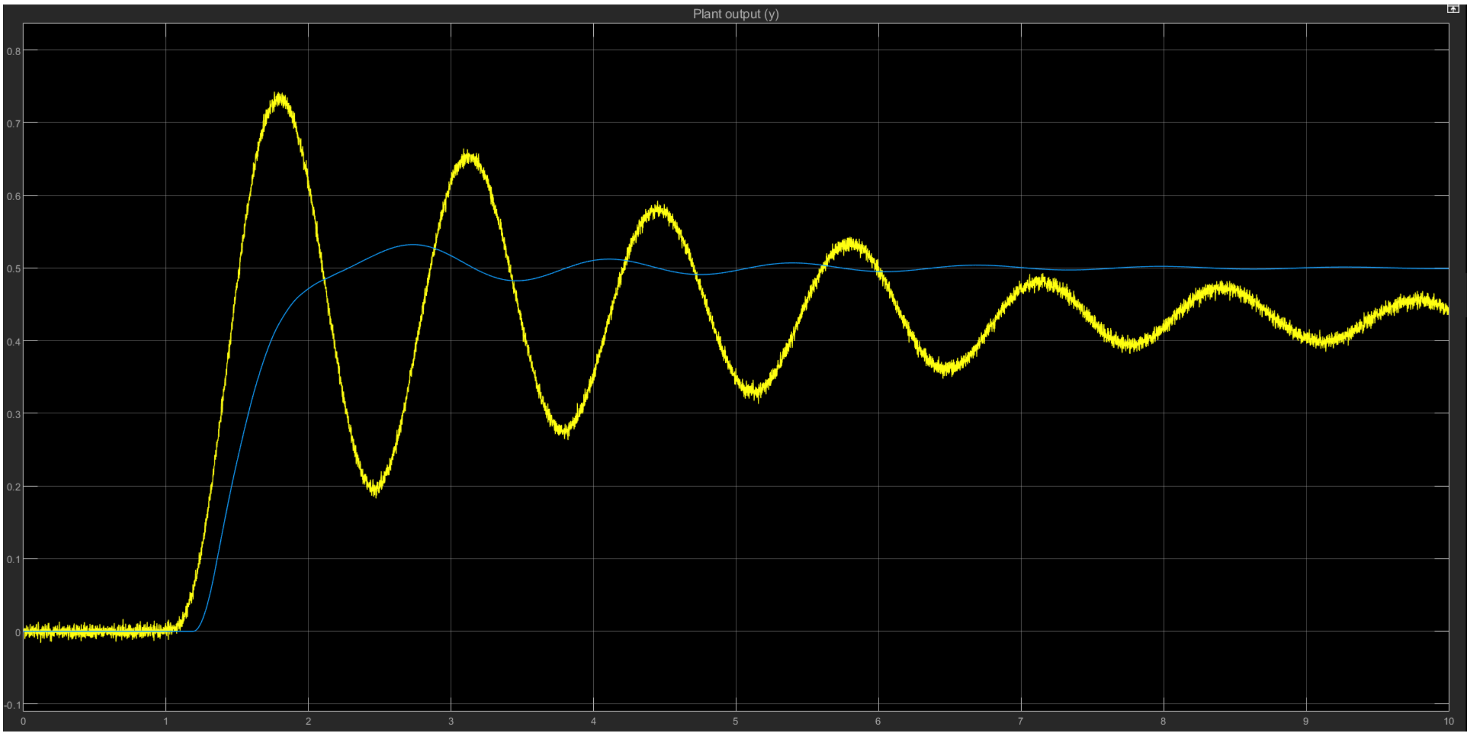
It is important to remember that the controller is designed with a sampling period of 0.15s in mind. The response of the plant is overdamped as seen due to the sampling period being continues.

Question 3

Using the MATLAB command $D_h = c2d(D, 0.15, 'tustin')$ I could convert the controller transfer function to the discrete transfer function sampled at 0.15_s . The resulting transfer function is:

$$Z(s) = \frac{16Z^2 - 23.6Z + 14.45}{Z^2 - 0.9Z - 0.1}$$

Discrete controller attached to the plant model.



The step response of the discrete controller is shown above. The overshoot is measured to be 3.5%. This is not 4.3% but is close enough. The settling time is measured to be 2.06 seconds which is also close enough to 2 seconds as per the practical requirements. The steady state tracking error of the controller and plant model is zero.

The response is not overdamped as before because here the discrete controller samples at 0.15s. Because the incoming signal is not continuous the controller zeros do not match up perfectly with the plant poles. This means it does not cancel out perfectly that is why we se more oscillations.

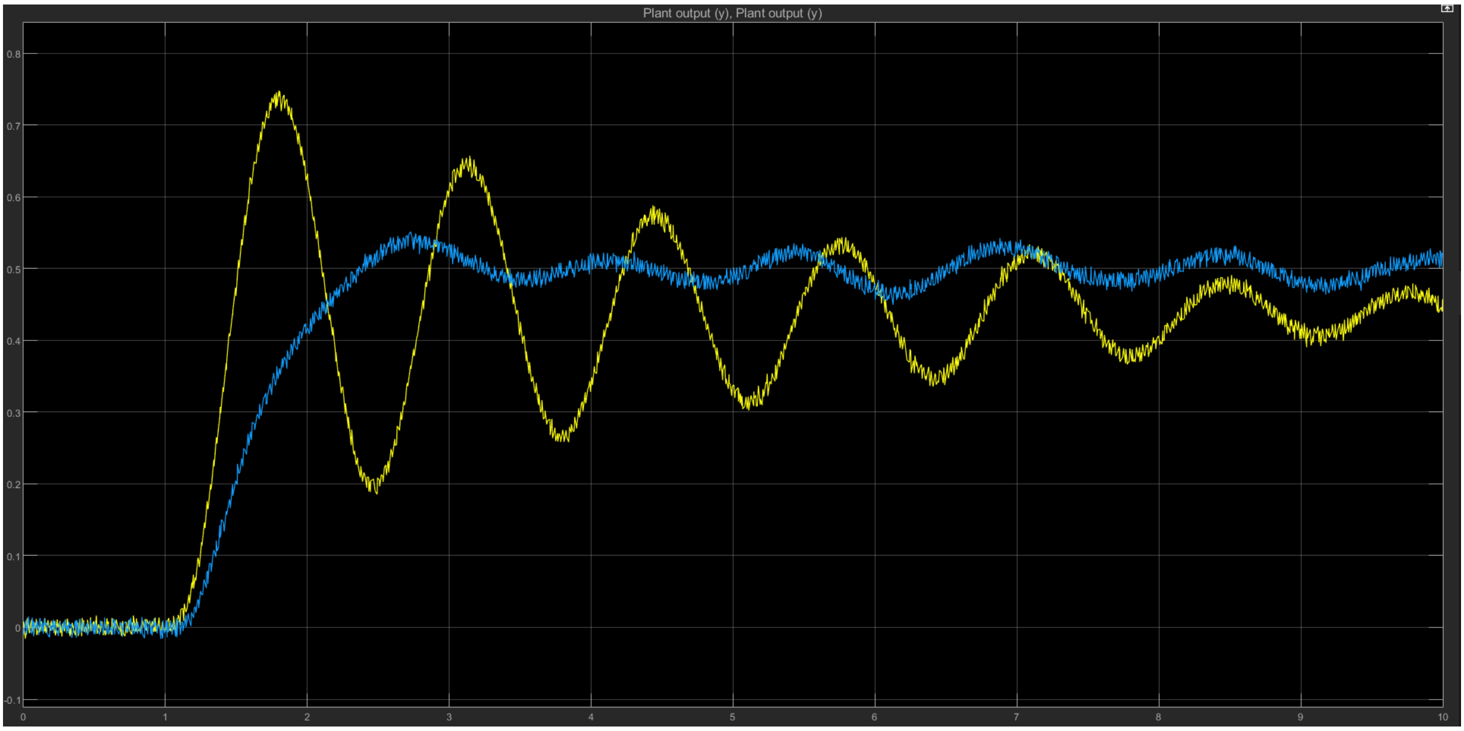
Question 5

After I designed the discrete controller and tested it with the plant model, I attached it to the actual plant and plotted the plant response with a unit step input with amplitude 0.5 radians. The result is shown below. The overshoot of plant response is measured to be 4.3% but the settling time is 2.09 seconds. This close to the desired overshoot and settling time but I had to do a fair amount of tweaking to a value and K value of the controller. My final values are for $a = 17, K = 27$.

$$Z(s) = \frac{14Z^2 - 20.7Z + 12.68}{Z^2 - 0.87Z - 0.12}$$

Again, due to the sampling (not a continues signal) of the discrete controller the controller zeroes and the plant poles do not overlap perfectly and does not cancel out completely.

Plant and controller response for a step input of 0.5 radians



Appendix

```
Td = 0.14;
wn = 4.75;
zeta =0.081;
C = 0.97;

TD = Td + 0.15/2;
zw2 = 2*zeta*wn;

a = 17;
K = 27;

s2 = zw2+1/TD;
s = wn^2+zw2*1/TD;
l = wn^2*1/TD;

D = K*tf([1  zw2 wn^2],[1 a 0]);
G = tf(C,[1 zw2, wn^2]);
delay = tf(1/TD,[1 1/TD]);

Dz = c2d(D, 0.15, 'tustin');
[num,den] = tfdata(Dz,'v');

A = rad2deg(1/tan(45));
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