Direct Digital Controller Design for a Rotor-Pendulum Model

- **Preparation**: See "What should I do in Week 12?" on SUNLearn.
- **Instructions**: You have to do this practical by yourself – no group work. Write your own practical report and hand it in on SUNLearn by Friday at 17:00. See SUNLearn for instructions.
- **Help**: Study the Matlab/Simulink documentation by clicking on the *Help* menu if you do not know how to use a certain function/block.

Direkte Digitale Beheerder-Ontwerp vir 'n Rotor-Pendulum-Model

- **Voorbereiding**: Sien "What should I do in Week 12?" op SUNLearn.
- **Instruksies**: Jy moet hierdie prakties self doen – geen groepwerk. Skryf jou eie praktiese verslag en handig dit in op SUNLearn teen Vrydag 17:00. Sien SUNLearn vir instruksies.
- **Hulp**: Lees die Matlab/Simulink-dokumentasie deur na die *Help*-kieslys te gaan indien jy nie weet hoe om 'n sekere funksie/blok te gebruik nie.

Background / *Agtergrond*

For Practical 5, we designed a discrete controller for a realistic rotor-pendulum simulation using *emulation*. For today's practical, we will design a discrete controller for the same plant using *direct digital design*, and then implement and test this controller.

In Prakties 5 het ons 'n diskrete beheerder ontwerp vir 'n realistiese rotor-pendulum simulasie deur *emulasie*. Vir vandag se prakties gaan ons 'n diskrete beheerder vir dieselfde aanleg ontwerp deur *direkte digitale ontwerp*, en die beheerder dan implementeer en toets.

Assignment / *Voorskrif*

For this practical, we will use the same realistic rotor-pendulum tutor simulation as for Practical 5. See the assignment for Practical 5 for a description of the plant and setup instructions. Use the same Simulink model `Prac5_setup.slx` and parameter file `Prac5_parameters.mat` you have received for Practical 5.

Vir hierdie prakties gaan ons dieselfde realistiese rotor-pendulum-tutor simulasie as vir Prakties 5 gebruik. Sien die voorskrif vir Prakties 5 vir 'n beskrywing van die aanleg en die opstellingsinstruksies. Gebruik dieselfde Simulink-model `Prac5_setup.slx` en parameter-lêer `Prac5_parameters.mat` wat jy ontvang het vir Prakties 5.

1 System Identification / *Stelselidentifikasie*

Problem statement: / *Probleemstelling:*

Set up a plant model of the rotor-pendulum.

Stel 'n aanlegmodel op van die rotor-pendulum.

Solution development: / *Ontwikkeling van oplossing:*

The plant dynamics can be described as a lightly-damped second-order system with a time delay, with the transfer function given by

Die aanleg-dinamika kan beskryf word as 'n liggedempte tweede-orde stelsel met 'n tydvertraging, met die oordragsfunksie gegee deur

$$G(s) = \frac{Y(s)}{U(s)} = \frac{Ce^{-T_d s}}{s^2 + 2\zeta\omega_n s + \omega_n^2}, \tag{1}$$

which is the same plant model used for Practical 5.

wat dieselfde aanlegmodel as vir Prakties 5 is.

Apply a step input with an amplitude of 10 V to the input of both the actual plant and plant model, and adjust the parameters of the plant model until the responses agree fairly well. You may reuse your identified parameters of Practical 5.

Wend 'n trapintree met 'n amplitude van 10 V aan op die intree van beide die werklike aanleg en die aanlegmodel, en verstel die parameters van die aanlegmodel totdat die gedrag redelik goed ooreenstem. Jy mag jou geïdentifiseerde parameters van Prakties 5 hergebruik.

Experiments and results: / *Eksperimente en resultate:*

Compare the step response of the plant model with that of the actual plant.

Vergelyk die trapweergawe van die aanlegmodel met die van die werklike aanleg.

Conclusions: / *Gevolgtrekkings:*

How accurate is the plant model? In which aspects do the response of the actual plant differ from that of the model?

Hoe akkuraat is die aanlegmodel? In watter aspekte verskil die gedrag van die werklike aanleg van die van die model?

2 Plant Discretisation / *Diskretisering van Aanleg*

Problem statement: / *Probleemstelling:*

Set up the discrete equivalent plant model.

Stel die diskrete ekwivalent aanlegmodel op.

Solution development: / *Ontwikkeling van oplossing:*

Find the discrete equivalent model of the plant and ZOH, $G_{\text{h0}}G(z)$, by hand using

Verkry die diskrete ekwivalente model van die aanleg en ZOH, $G_{\text{h0}}G(z)$, met die hand d.m.v.

$$G_{\text{h0}}G(z) = \left(1 - z^{-1}\right) \mathcal{Z}\left\{\frac{G(s)}{s}\right\}. \tag{2}$$

The sampling period is $T_s = 0.15$ s. To simplify matters, assume that the dead-time (time delay) in the plant is exactly one sampling period; the discrete equivalent plant model can now be calculated as

Die monsterperiode is $T_s = 0.15$ s. Om dinge te vereenvoudig, aanvaar dat die dooietyd (tydvertraging) in die aanleg presies een monsterperiode is; die diskrete ekwivalente aanlegmodel kan nou bereken word as

$$G_{\text{h0}}G(z) = \left(1 - z^{-1}\right) z^{-1} \mathcal{Z}\left\{\frac{G'(s)}{s}\right\}, \tag{3}$$

where $G'(s)$ is the plant model without dead-time, i.e.

waar $G'(s)$ die aanleg-model sonder dooietyd is, d.w.s.

$$G'(s) = \frac{C}{s^2 + 2\zeta\omega_n s + \omega_n^2}. \tag{4}$$

Verify your hand calculations by using the Matlab functions `tf` and `c2d` (the latter with argument `'zoh'`).

Bevestig jou handberekeninge deur die Matlab-funksies `tf` en `c2d` te gebruik (laasgenoemde met argument `'zoh'`).

Experiments and results: / *Eksperimente en resultate:*

Implement the discrete equivalent plant model, $G_{\text{h0}}G(z)$, in Simulink (use *Discrete Transfer Fcn* blocks in the *Discrete* folder). Apply the same step input as for the actual plant and the analogue plant model[1], and compare the output to that of the actual plant and the analogue plant model.

Implementeeer die diskrete ekwivalente aanlegmodel, $G_{\text{h0}}G(z)$, in Simulink (gebruik *Discrete Transfer Fcn*-blokke in die *Discrete*-kieslys). Wend dieselfde trapintree aan as vir die werklike aanleg en analoog-aanlegmodel[2], en vergelyk die uittree met dié van die werklike aanleg en analoog-aanlegmodel.

Conclusions: / *Gevolgtrekkings:*

How does the response of the discrete equivalent plant model compare with the analogue plant model as well as the actual plant? Explain any differences.

Hoe vergelyk die gedrag van die diskrete ekwivalente aanlegmodel met die van die analoog-aanlegmodel asook die werklike aanleg? Verduidelik enige verskille.

[1] I recommend that you set the *Sample time* parameter of all the step input blocks to be the same as $T_s = 0.15$ s – this will ensure that the step is applied at exactly the same time instant for both the continuous-time and discrete-time models.

[2] Ek stel voor jy stel die *Sample time* parameter van al die trapintree-blokke om dieselfde te wees as $T_s = 0.15$ s – dit sal verseker dat die trap aangewend word op presies dieselfde tydstip vir beide die kontinue-tyd en diskrete-tyd modelle.

3 Controller Design / *Beheerderontwerp*

Problem statement: / *Probleemstelling:*

Design a digital controller using the digital root locus.

Ontwerp 'n digital beheerder d.m.v. die digitale wortellokus.

Solution development: / *Ontwikkeling van oplossing:*

Use the discrete equivalent plant model $G_{\text{h0}}G(z)$ and z-plane root locus design to obtain (by hand) a discrete controller $D(z)$ that will satisfy the following specifications:

- Optimally damped (overshoot of 4.3%) with a 2% settling time of 2 s
- Zero steady state tracking error for constant reference angles

Hint: Cancel the ligthly-damped plant poles by controller zeros, and satisfy the zero steady state tracking error specification by placing one controller pole at $z = 1$[3].

Gebruik die diskrete ekwivalente aanleg-model $G_{\text{h0}}G(z)$ en z-vlak-wortellokus-ontwerp om 'n diskrete beheerder $D(z)$ te vind (met die hand) wat aan die volgende spesifikasies voldoen:

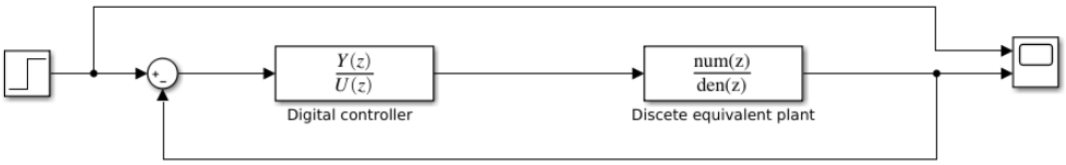
- Optimaal gedemp (oorskiet van 4.3%) met 'n 2%-wegsterftyd van 2 s
- Nul-gestadigde-toestand volgfout vir 'n konstante verwysings-hoek

Wenk: Kanselleer die liggedempte aanleg-pole deur beheerder-zeros, en bevredig die nul-gestadigde-toestand volgfout-spesifikasie deur 'n beheerder-pool by $z = 1$ te plaas[4].

Experiments and results: / *Eksperimente en resultate:*

Implement the closed-loop control system with *discrete equivalent* plant model $G_{\text{h0}}G(z)$ and controller $D(z)$ in Simulink as shown in the diagram below. Apply a step with amplitude 0.5 rad to the reference input. Measure the overshoot and 2% settling time, and observe the steady-state tracking error.

Implementeer die geslotelus-beheerstelsel met *diskrete ekwivalent* aanlegmodel $G_{\text{h0}}G(z)$ en beheerder $D(z)$ in Simulink soos getoon in die diagram hieronder. Wend 'n trap met amplitude 0.5 rad op die verwysings-intree aan. Meet die oorskiet en 2%-wegsterftyd, en let op die gestadigde-toestand volgfout.



Conclusions: / *Gevolgtrekkings:*

Does the closed-loop system satisfy the specifications? (Hint: What is the effect of a non-dominant pole on the step response?)[5]

Voldoen die geslotelusstelsel aan die vereistes? (Wenk: Wat is die effek van 'n nie-dominante pool op die trapweergawe?)[6]

[3]For the controller to be realisable, the number of controller poles should be equal or more than the number of controller zeros (i.e. ≥ 2). You therefore have to add another controller pole.

[4]Vir die beheerder om realiseerbaar te wees moet die getal beheerder-pole dieselfde of meer as die getal beheerder-zeros wees (d.w.s. ≥ 2). Jy moet dus nog 'n beheerder-pool byvoeg.

[5]If you have placed the dominant closed-loop poles correctly, you do not need to redesign the controller to satisfy the specification; however, make sure you understand the reason for the response.

[6]Indien jy die dominante gesloteluspole korrek geplaas het, hoef jy nie die beheerder te herontwerp om aan die vereistes te voldoen nie; maak egter seker jy verstaan die rede vir die gedrag.

4 Applying the Controller to the Analogue Plant Model / *Aanwending van die Beheerder op die Analoog Aanlegmodel*

Problem statement: / *Probleemstelling:*

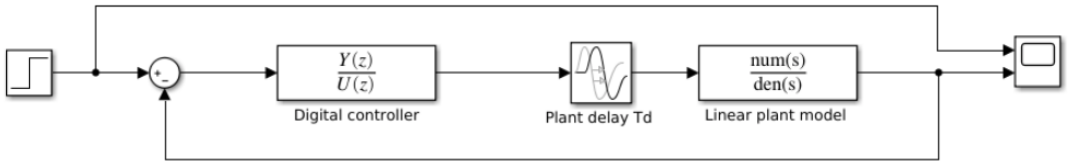
Apply the designed digital controller to the analogue plant model and analyse its behaviour.

Wend die ontwerpte digitale beheerder aan op die analoog aanlegmodel en analiseer die gedrag.

Experiments and results: / *Eksperimente en resultate:*

Implement the closed-loop control system with *analogue* plant model $G(s)$ and controller $D(z)$ in Simulink as shown in the diagram below. Apply a step with amplitude 0.5 rad to the reference input. Compare the step response with that of Question 3. Also compare the step response with that of Question 3 of Practical 5.

Implementeer die geslotelus-beheerstelsel met *analoog* aanlegmodel $G(s)$ en beheerder $D(z)$ in Simulink soos getoon in die diagram hieronder. Wend 'n trap met amplitude 0.5 rad op die verwysings-intree aan. Vergelyk die trapweergawe met dié van Vraag 3. Vergelyk die trapweergawe ook met dié van Vraag 3 van Prakties 5.



Conclusions: / *Gevolgtrekkings:*

What can you conclude from the comparison of the step response with that of Question 3 as well as that of Question 3 of Practical 5? Explain any differences.

Watter gevolgtrekking kan jy maak van die vergelyking van die trapweergawe met dié van Vraag 3 asook dié van Vraag 3 van Prakties 5? Verduidelik enige verskille.

5 Applying the Controller to the Actual Plant / *Aanwending van die Beheerder op die Werklike Aanleg*

Problem statement: / *Probleemstelling:*

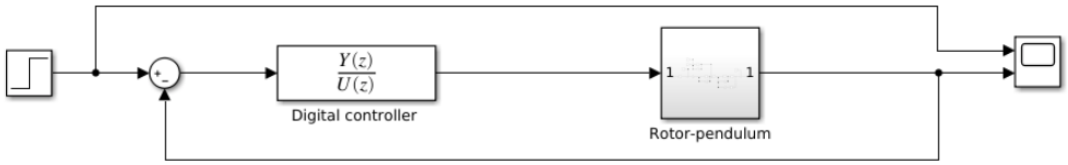
Apply the designed digital controller to the actual plant and analyse its behaviour.

Wend die ontwerpte digitale beheerder aan op die werklike aanleg en analiseer die gedrag.

Experiments and results: / *Eksperimente en resultate:*

Implement the closed-loop control system with *actual* plant and controller $D(z)$ in Simulink as shown in the diagram below. Apply a step with amplitude 0.5 rad to the reference input. Compare the step response with that of Question 5. Also compare the step response with that of Question 4 of Practical 5.

Implementeer die geslotelus-beheerstelsel met *werklike* aanleg en beheerder $D(z)$ in Simulink soos getoon in die diagram hieronder. Wend 'n trap met amplitude 0.5 rad op die verwysingsintree aan. Vergelyk die trapweergawe met dié van Vraag 5. Vergelyk die trapweergawe ook met dié van Vraag 4 van Prakties 5.



Conclusions: / *Gevolgtrekkings:*

What can you conclude from the comparison of the step response with that of Question 5 as well as that of Question 4 of Practical 5? Explain any differences.

Watter gevolgtrekking kan jy maak van die vergelyking van die trapweergawe met dié van Vraag 5 asook dié van Vraag 4 van Prakties 5? Verduidelik enige verskille.