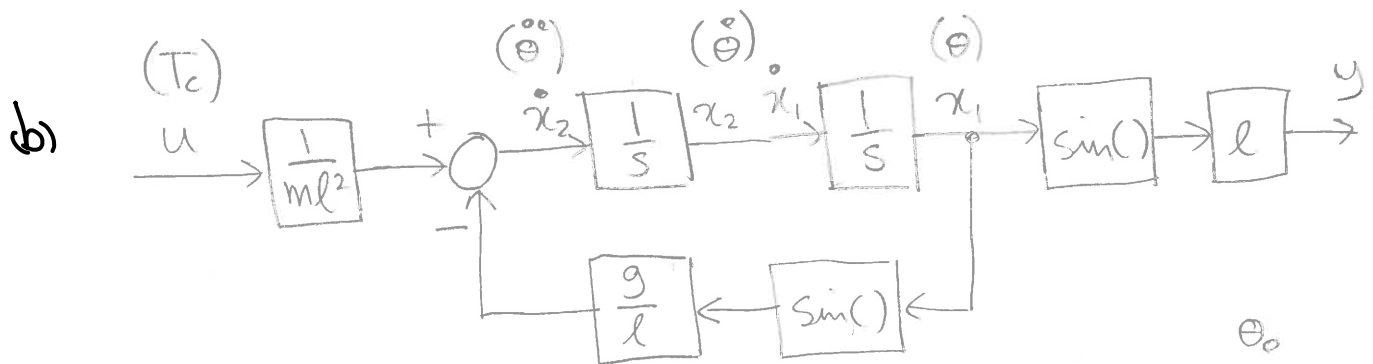


PRACTICAL 1 MEMO (CALCULATIONS)

Q1 (a) $\underline{x} = \begin{bmatrix} x_1 \\ x_2 \end{bmatrix} = \begin{bmatrix} \theta \\ \dot{\theta} \end{bmatrix}; u = T_c$

$$\dot{\underline{x}} = \begin{bmatrix} \dot{x}_1 \\ \dot{x}_2 \end{bmatrix} = \begin{bmatrix} x_2 \\ -\frac{g}{l} \sin x_1 + \frac{1}{ml^2} u \end{bmatrix} = \begin{bmatrix} f_1(x_1, x_2, u) \\ f_2(x_1, x_2, u) \end{bmatrix}$$

$$y = l \sin x_1 = g(x_1, x_2, u)$$



c)

$$\dot{\underline{x}}_0 = \begin{bmatrix} 0 \\ 0 \end{bmatrix} = \begin{bmatrix} x_2 \\ -\frac{g}{l} \sin x_1 + \frac{1}{ml^2} u_0 \end{bmatrix} \Rightarrow \underline{x}_0 = \begin{bmatrix} \overbrace{\arcsin\left(\frac{u_0}{mgl}\right)}^{\theta_0} \\ 0 \end{bmatrix}$$

d)

$$A = \left. \frac{\partial f}{\partial \underline{x}} \right|_{\underline{x}_0, u_0} = \begin{bmatrix} 0 & 1 \\ -\frac{g}{l} \cos x_1 & 0 \end{bmatrix}_{\underline{x}_0, u_0} = \begin{bmatrix} 0 & 1 \\ -\frac{g}{l} \cos \theta_0 & 0 \end{bmatrix}$$

$$\underline{b} = \left. \frac{\partial f}{\partial u} \right|_{\underline{x}_0, u_0} = \begin{bmatrix} 0 \\ \frac{1}{ml^2} \end{bmatrix} \quad \underline{c} = \left. \frac{\partial g}{\partial \underline{x}} \right|_{\underline{x}_0, u_0} = [l \cos \theta_0 \quad 0]$$

$$d = \frac{\partial g}{\partial u} = 0$$

$$\Rightarrow \begin{bmatrix} \delta \dot{x}_1 \\ \delta \dot{x}_2 \end{bmatrix} = \begin{bmatrix} 0 & 1 \\ -\frac{g}{l} \cos \theta_0 & 0 \end{bmatrix} \begin{bmatrix} \delta x_1 \\ \delta x_2 \end{bmatrix} + \begin{bmatrix} 0 \\ \frac{1}{ml^2} \end{bmatrix} \delta u$$

$$\delta y = [l \cos \theta_0 \quad 0] \begin{bmatrix} \delta x_1 \\ \delta x_2 \end{bmatrix}$$

Q2 SEE Prac1_setup.m AND Prac1_model.slx

Q3 $l=1, m=1, g=9,81, u_0=0 \Rightarrow \theta_0=0$

$$\Rightarrow A = \begin{bmatrix} 0 & 1 \\ -9,81 & 0 \end{bmatrix}, \underline{b} = \begin{bmatrix} 0 \\ 1 \end{bmatrix}, \underline{c} = [1 \ 0], d=0$$

SEE Prac1_model.slx FOR REST

RUN Prac1_model.slx

Q4 $\theta_0 = \arcsin\left(\frac{1}{9,81}\right) = 0,7946 \Rightarrow \underline{x}_0 = \begin{bmatrix} 0,7946 \\ 0 \end{bmatrix}$

$$A = \begin{bmatrix} 0 & 1 \\ -9,81 \cdot \cos 0,7946 & 0 \end{bmatrix} = \begin{bmatrix} 0 & 1 \\ -6,873 & 0 \end{bmatrix}$$

$$\underline{b} = \begin{bmatrix} 0 \\ 1 \end{bmatrix}, \underline{c} = [0,7006 \ 0], d=0$$

SEE Prac1_model.slx FOR REST

Q5 is similar to **Q3** and **Q4**, but system is now unstable and the output of both nonlinear and linearised systems will grow unbounded in response to a step input.

Screenshot of Simulink model and Scope output:

