Stellenbosch University

Department of Electrical and Electronic Engineering

Systems and Signals 315

25 May 2020

Practical 4: Frequency Modulation

1 Aim

The purpose of Practical 4 is to design and build a FM modulator using a voltage-controlled oscillator (VCO). The theory regarding the spectrum of a tone-modulated FM signal will also be verified.

The following diagram illustrates the basic system that you will implement in this practical:

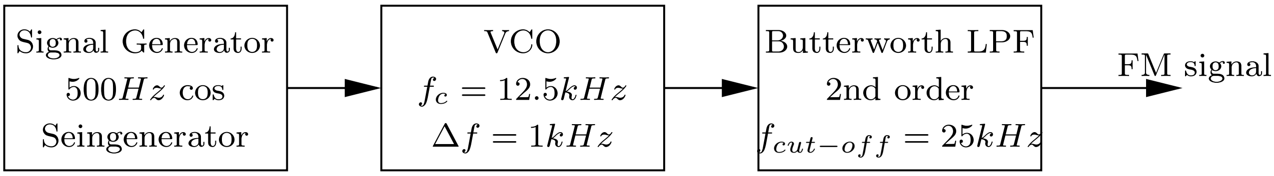


Figure 1: Overview of FM modulator

2 Instructions

- The practical will be performed synchronously using a live Microsoft Teams contact session during the scheduled practical time slot on Monday 14:00-17:00.
- Since we do not have physical access to the laboratories yet, the practical will involve building simulation models of electronic circuits and executing the models to obtain simulated results.
- The free LTSpice electronic simulation software will be used.
- Students who do not have access to a personal computer, or cannot install and run the LTspice software, must please notify the lecturers by completing the online form on SUNLearn.
- The lecturers will upload the practical instructions to SUNLearn in advance, and students are expected to attempt the practical before the scheduled practical session.

Live Microsoft Teams contact session

- Students must join the Systems and Signals 315 Microsoft Teams site and go to the "Practical 4 Chat (Praktika 4 Klets)" chat channel during the scheduled practical time slot on Monday 14:00-17:00.
- Students can ask for help on the "Practical 4 Chat (Praktika 4 Klets)" chat channel, and the lecturers and demies will then assist by starting temporary MS Teams meetings directly with individual students or groups of students.

Practical report

- Each student must complete their own practical report and submit an electronic pdf copy on SUNLearn. The practical report will be marked and will form part of your semester mark.
- The deadline for the report submission is Sunday 31 May at 23:59. Late submissions will receive a mark of zero. Failure to submit a practical report will result in an INCOMPLETE for the module.

Download and install LTspice electronic circuit simulator

Download and install the LTspice simulation software from SUNLearn for one of the following operating systems:

- Windows 7, 8, and 10 (LTspiceXVII.exe)
- Mac 10.9+ (LTspice.dmg)
- Windows XP (LTspiceIV.exe)

Join Microsoft Teams

- Join the Systems and Signals 315 Microsoft Teams site.
- A short guide is also provided on how to use Microsoft Teams.

3 Background

An FM signal has the form

$$\varphi_{\text{FM}}(t) = A_c \cos\left[\omega_c t + k_f \int_{-\infty}^{t} m(\alpha)\, d\alpha\right] \tag{5.5}$$

where $m(t)$ is the message signal, k_f is the deviation index, and ω_c is the centre frequency. The *instantaneous frequency* of an FM signal is the derivative of the cosine's angle:

$$\omega_i(t) = \omega_c + k_f m(t) \tag{5.4a}$$

3.1 FM bandwidth

The minimum frequency to which the instantaneous frequency can be pushed by the message signal (when $m(t)$ reaches its minimum voltage), is

$$\omega_{\min} = \omega_c - k_f m_p \tag{p. 212}$$

(where m_p is the peak value of the message signal; we assume zero DC in the message). Similarly, the maximum frequency to which the instantaneous frequency can be pushed, is

$$\omega_{\max} = \omega_c + k_f m_p \tag{p. 212}$$

From this we can derive the *peak frequency deviation* (in hertz) – the maximum difference between the centre frequency and the FM signal's instantaneous frequency:

$$\Delta f = \frac{\omega_{\max} - \omega_{\min}}{2 \cdot 2\pi} = k_f \frac{m_p}{2\pi} \tag{p. 212}$$

Even though this determines the complete range over which the FM signal's *instantaneous frequency* can vary, the FM signal's *bandwidth* is much larger – theoretically infinite (see eq. 5.9 in the textbook). The reason for this is that the FM signal's bandwidth also depends on the bandwidth of the modulating signal, $m(t)$. A reasonable estimate of an FM signal's *significant bandwidth* is given by Carson's rule:

$$B_{\text{FM}} \approx 2(\Delta f + B) \tag{5.13}$$

where B is the bandwidth of the modulating signal.

3.2 Tone modulation

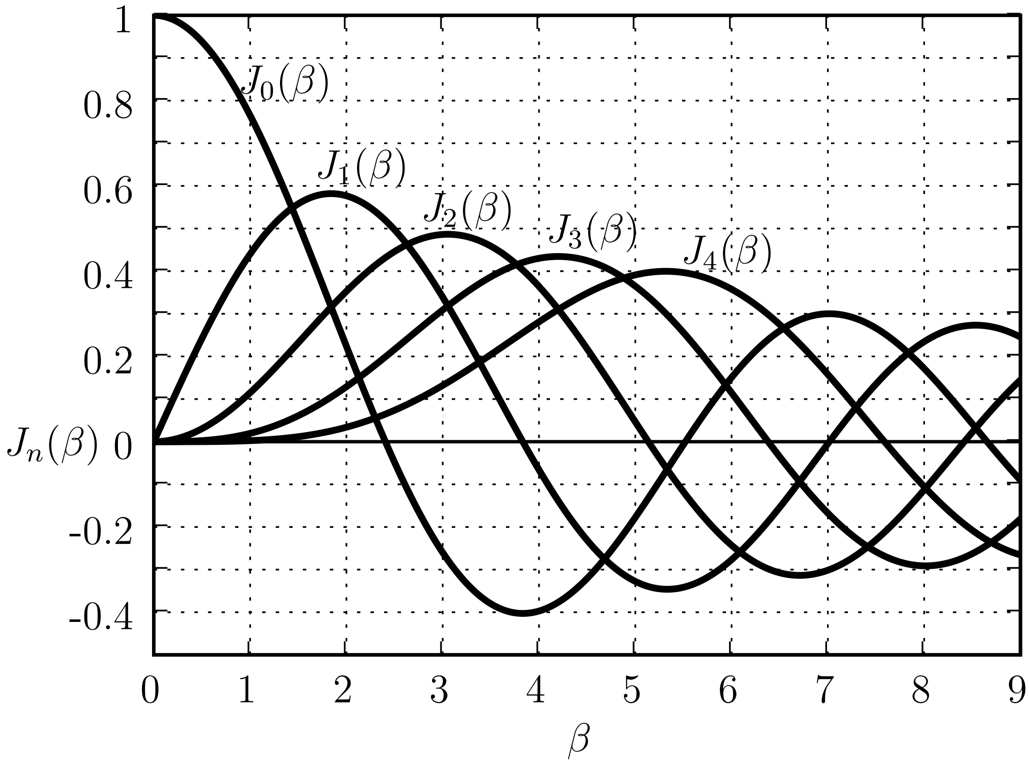
In general it is very difficult to predict the spectrum of an FM signal, except through simulation. One important exception is *tone modulation*, where the message signal is 'n pure sinusoid with constant frequency:

$$m(t) = \alpha \cos 2\pi f_m t \tag{p. 214}$$

Tone modulation is often used to test and calibrate FM transmitters. In this special case, the FM signal can be written as the following Fourier series:

$$\varphi_{\text{FM}}(t) = A_c \sum_{n=-\infty}^{\infty} J_n(\beta) \cos(\omega_c + n \cdot 2\pi f_m)t \tag{p. 215}$$

with $\beta = \Delta f / f_m$, and $J_n(\beta)$ read from the graph below (only the curves corresponding to the first few values of n are shown):



Also note that

$$J_{-n}(\beta) = (-1)^n J_n(\beta)$$

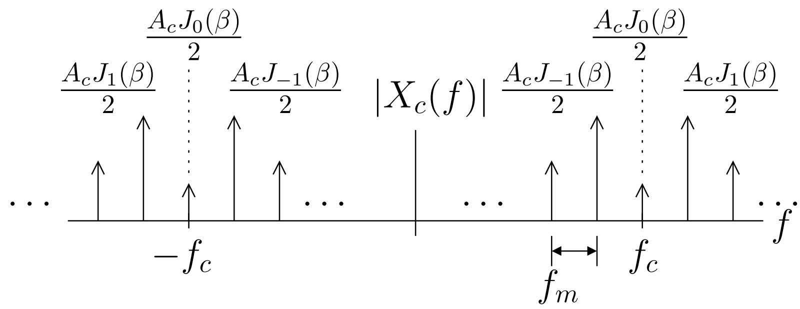
For example, let $f_m = 100$ Hz and $\Delta f = 200$ Hz. Then $\beta = 2$, and the following coefficients can be read from the graph:

$J_0(2)$	0.22
$J_1(2)$	0.58
$J_2(2)$	0.35
$J_3(2)$	0.13
etc.	

The symmetry of $J_n(\beta)$ can be used to derive the other coefficients:

$J_{-1}(2)$	-0.58
$J_{-2}(2)$	0.35
$J_{-3}(2)$	-0.13
etc.	

The resulting spectrum of the *tone modulated signal* consists of discrete impulses (like any periodic signal) – one for each term in the Fourier series, each with a weight equal to its corresponding $J_n(\beta)$ coefficient. The impulses are centred around ω_c ($n = 0$), and spaced by f_m. The graph below shows the spectrum for the signal in the current example:



It is only for tone modulation that the FM signal's spectrum consists of discrete lines. For more general modulating signals, the FM spectrum is continuous and difficult to predict.

3.3 Voltage-controlled oscillator

The circuit diagram of the FM modulator is shown below:

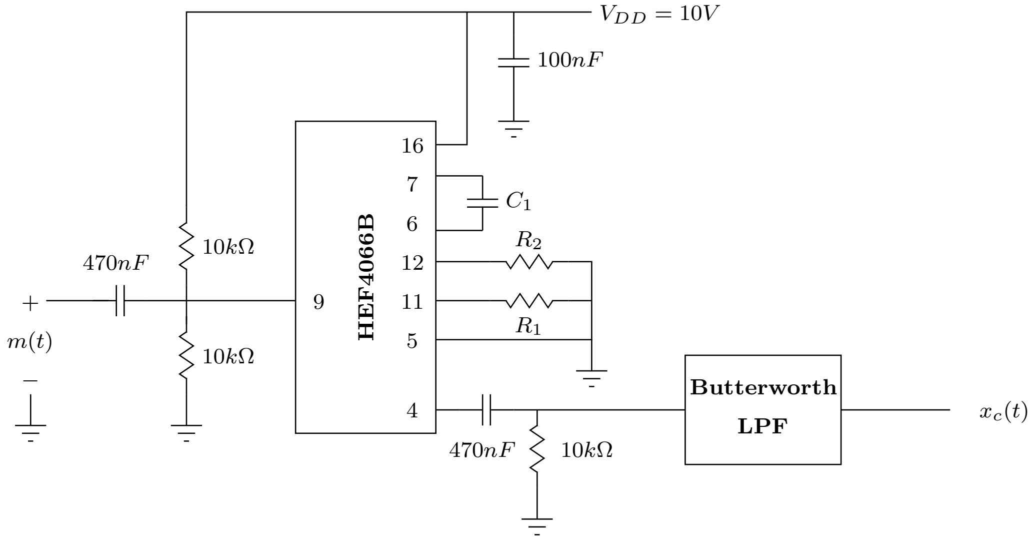


Figure 2: Circuit diagram for voltage-controlled oscillator

Frequency modulation is achieved in this system using a voltage controlled oscillator (VCO). The VCO is a subsystem on the HEF4046BP integrated circuit. The complete HEF4046B IC is a phase-locked loop circuit that consists of a linear (VCO) and two different phase comparators. The VCO requires one external capacitor (C_1) and one or two external resistors (R_1 and R_2). Resistor R_1 and capacitor C_1 determine the frequency range of the VCO. Resistor R_2 enables the VCO to have a frequency off-set if required. A LOW level at pin 5 enables the VCO, while a HIGH level turns off the VCO (to minimize stand-by power consumption).

For a given input voltage $V_{\mathrm{VCO_{IN}}}$, (pin 9 on the HEF4046B), the VCO generates a rectangular pulse train with frequency f_{VCO}, as shown in Fig. 3 (a more detailed representation can be found on Fig. 13, p. 13 of the datasheet of the HEF4046). 'n Supply voltage of $V_{DD} = 10V$ is recommended. The two 10kΩ resistors is used externally bias the input voltage in such a way that when no input message signal is connected, the effective voltage on pin 9 is equal to $\frac{V_{DD}}{2} = 5V$. The frequency generated by this (default) input voltage, is called the free-running frequency f_c of the VCO (the datasheet uses f_0). A DC-decoupled signal (such as the input signal in the diagram that is passed through a decoupling capacitor) causes the voltage on pin 9 to vary around the 5 V bias, thus making the free-running frequency the centre frequency of the frequency-modulated signal. V_{SS} must be connected to ground.

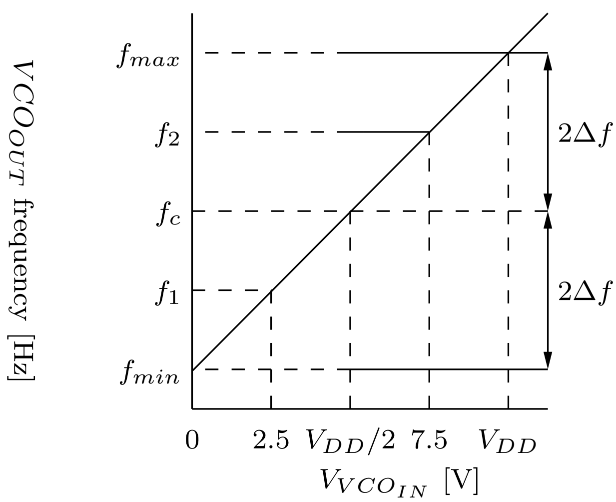


Figure 3: Relationship tussen the input voltage $V_{\mathrm{VCO_{IN}}}$ and output frequency $f_{\mathrm{VCO_{OUT}}}$ of the voltage-controlled oscillator.

For the correct functioning of the VCO, the free-running frequency and the frequency range of the VCO must be configured correctly. The resistor R_1 (which is connected from pin 11 to ground) and the capacitor C_1

(connected between pin 6 and pin 7) determines the free-running frequency f_c. The minimum output frequency of the VCO is determined by the resistor R_2 (which is connected from pin 12 to ground) and the capacitor C_1. The following procedure is recommended for the design of the VCO:

1. Determine the minimum output frequency $f_{\min}$ of the VCO (at input voltage $V_{\mathrm{VCO_{IN}}} = 0V$) as:

$$f_{\min} = f_c - 2\Delta f$$

   Choose resistor $R_2 = 100\mathrm{k}\Omega$ and determine a suitable value for capacitor C_1 by using Fig. 8 of the datasheet of the HEF4046B (on p. 11). Note that the supply voltage is $V_{DD} = 10V$.

2. Determine the ratio between the maximum and minimum output frequency of the VCO $\frac{f_{\max}}{f_{\min}}$. Use Fig. 9 of the datasheet of the HEF4046B se datasheet (on p. 12) to determine the ration of $\frac{R_2}{R_1}$. Use this ratio to determine a suitable value for the resistor R_1.

3.4 Two-pole active Butterworth low-pass filter

When the signal is output from the VCO, it is not yet ready for over-the-air transmission. Design a two-pole active Butterworth low-pass filter to condition the signal prior to transmission. The following circuit diagram can be used for the filter design: (remember the levelling capacitors)

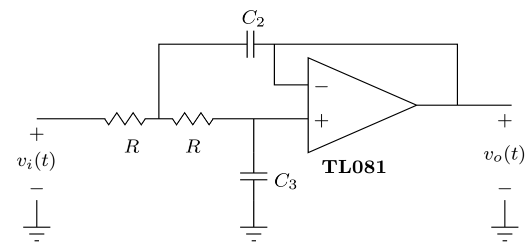


Figure 4: Circuit diagram for a two-pole active Butterworth low-pass filter

In this design, the cut-off frequency of the filter (in Hz) is given by the equation $f_{\mathrm{cut-off}} = \frac{1}{2\pi RC}$, with $C_2 = \sqrt{2}C$ and $C_3 = \frac{C}{\sqrt{2}}$. 25 kHz is a sensible cut-off frequency. If the cut-off frequency is too low, the higher signal frequencies will start to be attenuated. If the cut-off frequency is too high, to much of the square waveï¿½s harmonics will remain.

The TL081 pinout diagram can be found in the datasheet of the TL081.

Theoretical Preparation

The following preparation must be done before the practical.

Assume the message signal can be written as $m(t) = \alpha \cos(2\pi f_m t)$ with $f_m = 500$ Hz. The message signal is sent through a FM-modulator te generate the modulated signal $x_c(t)$ with a peak frequency deviation of $\Delta f = 1$ kHz and a carrier frequency of $f_c = 12.5$ kHz.

1. Calculate the approximate bandwidth of the FM-modulated signal by using Carson's rule.

2. Calculate and sketch the exact spectrum of the FM-modulated signal by using Bessel-functions. You only need to include sidebands which have significant amplitude according to Carson's rule.

3. Calculate the minimum and maximum frequency, f_1 and f_2, that the carrier signal will reach as a result of frequency modulation.

4. Study the given background of the voltage-controlled osscilator. Determine suitable values for the resistors R_1, R_2 and capacitor C_1 so that the VCO has a carrier frequency of $f_c = 12.5$ kHz and a peak frequency deviation of $\Delta f = 1$ kHz. Consider the following questions:

   - What is the frequency modulation constant k_f of the system in Hz/V in terms of the input voltage $V_{\mathrm{VCO_{IN}}}$, and the maximum and minimum VCO frequency $f_{\max}$ and $f_{\min}$?
   - What is a suitable maximum amplitude α for the input signal to achieve the specified peak frequency deviation of $\Delta f = 1$ kHz?

5. Study the given background of the two-pole active Butterworth low-pass filter. Determine suitable values for the two identical resistors R and the capacitors C_2 and C_3 so that the low-pass filter has a cut-off frequency of $f_{\mathrm{cut-off}} = 25$ kHz.

4 Execution of Practical

4.1 Voltage Controlled Oscillator

Construct the circuits above in LTSpice. LTSpice libraries have been provided for the TL081 op-amp and the CD4046 Phase-locked loop. **Note that V_{SS} must be connected to GND otherwise the circuit will not work.** The CD4046 model must be include using the `.inc CD4046Bg.sub` LTSpice directive.

Since the signal is very sensitive with regard to the specific values for the external components R_1, R_2 and C_1, you will need to tune the values of R_1 and R_2 to obtain the desired minimum and maximum frequencies f_1 and f_2.

Now connect a DC voltage of 0 V directly on pin 9 (i.e. not through the decoupling capacitor and without the external biasing network). You should see a square wave on the output, with a frequency equal to the system's minimum frequency. With $R_2 = 100k$ adjust C_1 until you achieved your calculated $f_{\min}$ frequency.

Now disconnect the DC voltage that was directly connected on pin 9 and connect the external biasing network. When no input is connected you should see a square wave on the output, with a frequency equal to the systemâs free-running frequency. Adjust R_1 until the free-running frequency is exactly 12.5 kHz. This is now the centre frequency of the FM modulator. Measure the voltage at pin 9; this is the DC input voltage that is directly converted into the centre frequency (it should be about 5). Now connect a DC voltage of 5 V directly on pin 9 (i.e. not through the decoupling capacitor and without the external biasing network). Vary the input voltage carefully around 5 V and note how the output frequency varies. Verify that the minimum output frequency $f_{\min}$ at $V_{\text{VCO}_{\text{IN}}} = 0V$ and the maximum output frequency $f_{\max}$ at $V_{\text{VCO}_{\text{IN}}} = V_{DD} = 10V$ agrees with the design specifications. If the frequencies differ, resistor R_2 can be adjusted (while keeping $V_{\text{VCO}_{\text{IN}}} = 0V$) to adjust the minimum output frequency. The maximum output frequency can be adjusted using resistor R_1 (while keeping $V_{\text{VCO}_{\text{IN}}} = 10V$).

Task 1: Measure the Voltage-Frequency of the VCO

Sketch a graph of the theoretical voltage-frequency curve as predicted Fig. 3, compared to the measured response. Connect a 500 Hz sine wave (without any DC offset) as input signal (this time via the decoupling capacitor and external bias network). Use a peak voltage for the sine wave that ensures that the correct frequency deviation will be achieved on the output. Note how the modulated output signal's instantaneous frequency now varies with the input signal's voltage (plot both the input signal and the output and view them simultaneously).

Task 2: Measure the peak frequency deviation and the FM spectrum

Use the cursor-function of LTSpice to measure the output signal's period at the maximum and minimum output frequency. Is the correct frequency deviation achieved? Remember that you can adjust the amplitude of the input signal to change the frequency deviation of the FM signal to achieve the correct frequency deviation.

Use the FFT function of LTSpice to view the spectrum of the output. Verify that the spectrum around 12.5 kHz agrees with the theoretically predicted spectrum of a tone-modulated signal. You can measure the amplitude and frequency of the peaks in the spectrum by usin the cursor-function of LTSpice. Remember that LTSpice measures the magnitude in the DFT as decibel.

Task 3: Comparison of Theoretical and Measured FM Spectrum

Compare the measured spectrum of the DFT with the theoretical predicted spectrum. Why do you not see impulses in the measured spectrum as predicted by the theory? Give an explanation by referring to the properties of the DFT.

4.2 Two-pole active Butterworth low-pass filter

LTSpice libraries have been provided for the TL081 op-amp and the CD4046 Phase-locked loop and test the filter. Verify that the measured cut-off frequency meets the design specifications.

Task 4: Low-pass filtered FM Spectrum

Inspect the output of the low-pass filter. Is there any variation in signal amplitude? What causes this variation? Is the signal perfectly sinusoidal? Why not? How would you change the filter design to reduce these non-idealities? Explain why it is necessary to send the output of the VCO through a low-pass filter before it is ready for transmission.

4.3 Write and submit a report

Reports should include design decisions, calculations, results, interpretation of the results and general conclusions. Your report should at least cover the following items:

- Introduction and description of the experimental setup.
- Theoretical preparation including the prediction of the bandwidth and spectrum of the tone-modulated FM signal.
- Design and calculation of VCO circuit and low-pass filter resistor and capacitor values.
- Comparison of measured results with the theoretical predictions.
- Conclusion.