Systems & Signals 414 29 May 2021

Walt Deyzel: 21750793

Task

Analyses of a transmitted signal from a radar and its reflected signal from an object.

1. **Derivation**

$$y[n] = a\,x[n-D] + w[n]$$

$$r_{xx}[i] = \sum_{-\infty}^{\infty} x[n]\,x[n-i]$$

$$r_{wx}[i] = \sum_{-\infty}^{\infty} w[n]\,x[n-i]$$

$$r_{yx}[i] = \sum_{n=-\infty}^{\infty} y[n]\,x[n-i]$$

$$= \sum_{n=-\infty}^{\infty} \left[a\,x[n-D] + w[n]\right] x[n-i]$$

$$= \sum_{n=-\infty}^{\infty} a\,x[n-D]\,x[n-i]$$

$$+ \sum_{n=-\infty}^{\infty} w[n]\,x[n-i]$$

$$\boxed{r_{yx}[i] = a * r_{xx}[i-D] + r_{wx}[i]}$$

2. From the derivation in Q1 we can calculate $\boldsymbol{r_{yx}[i]}$ as follow:

$$r_{yx}[i] = a * r_{xx}[i - D] + r_{wx}[i]$$

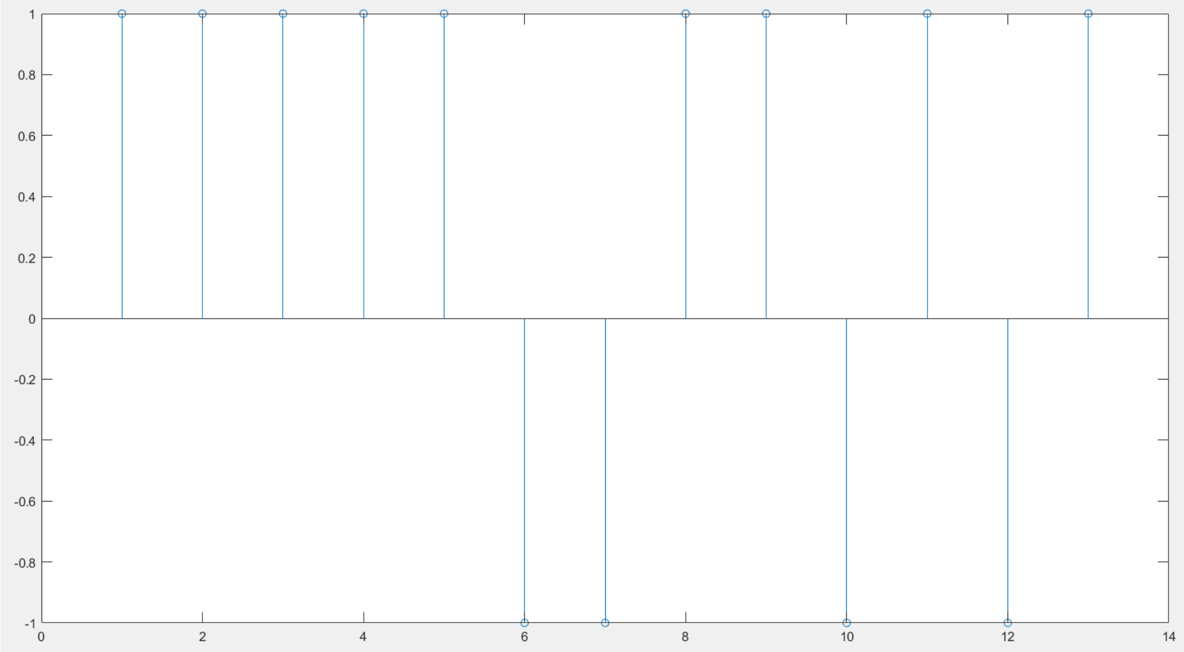
This shows that taking the cross correlation of **y** and **x** will capture a delay of amount **D** in the scaled autocorrelation of **x.** The cross-correlation of **w** and **x** will result in a noisy plot, for the white noise has nothing in common with the signal **x**. This implies the dominant term will be the scaled autocorrelation of **x**. Therefore, the choice of **x** is very important. Ideally, we are looking for a **x[n]** that has an autocorrelation of a single impulse or close to that. The better closer we can get to the ideal case the easier and more confidently we can determine the delay through the **cross-correlation of x and y**. This is evident in the following sections of the report.

3. Barker Sequence

$$x[n] = \{+1, +1, +1, +1, +1, -1, -1, +1, +1, -1, +1, -1, +1\},$$

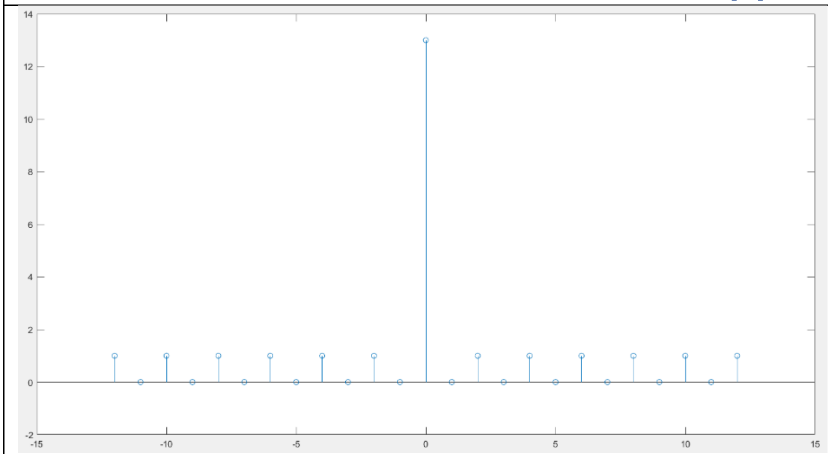
Ideal autocorrelation property

13 Point Barker Series.



n

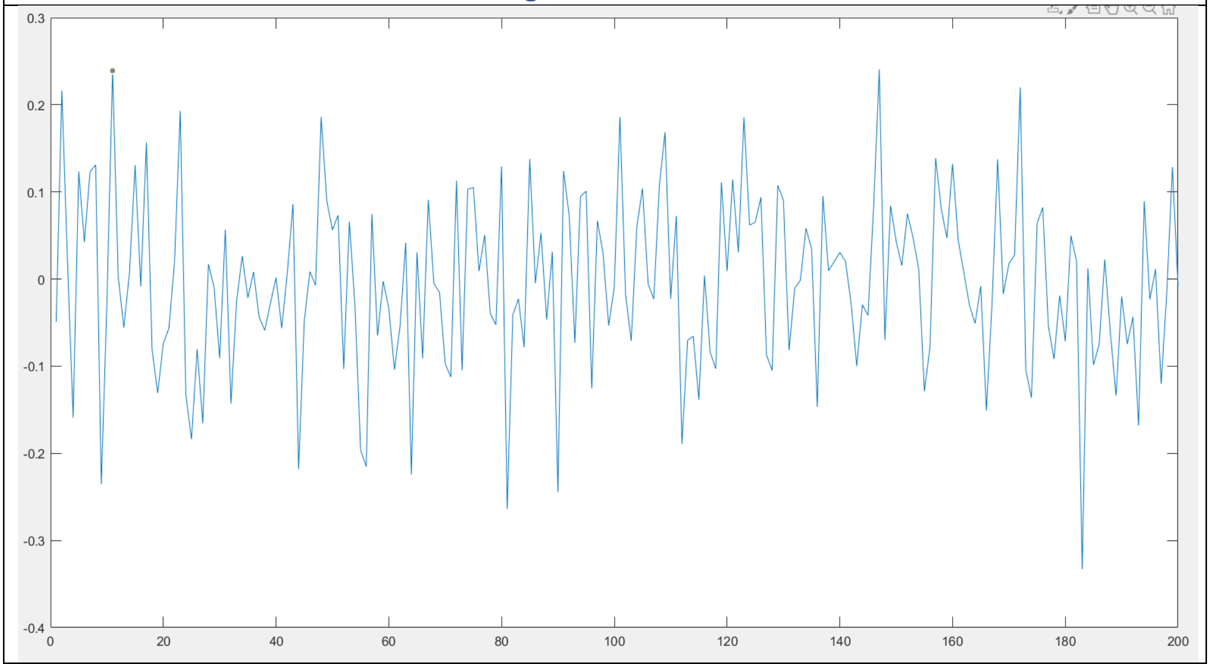
Auto correlation of x[n]



As seen in the Auto-correlation graph there is a peak at l = 0 and everywhere else the peaks are very small. Meaning the as soon as the signal is shifted even just by one sample the correlation becomes very weak. This makes the Barker series an ideal choice for x[n]. For when the correlation is strong you can be sure that the signals match.

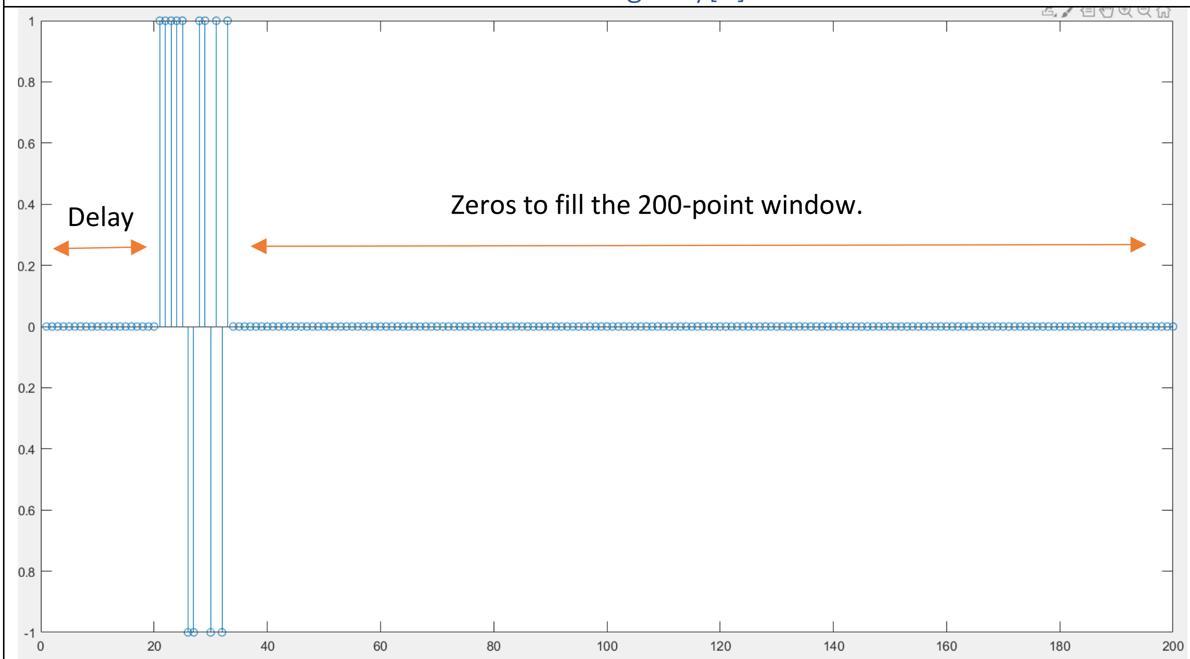
l

200 Point Gaussian Noise Signal with zero mean and 0.01 variance.



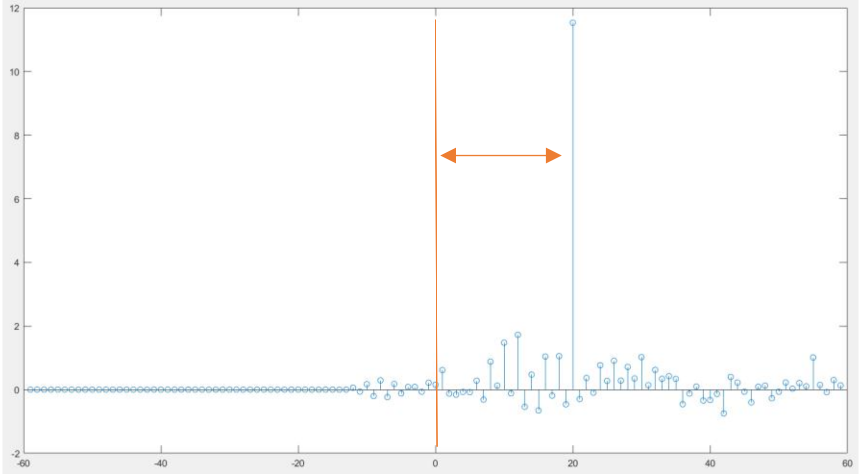
n

Reflected Signal y[n]



n

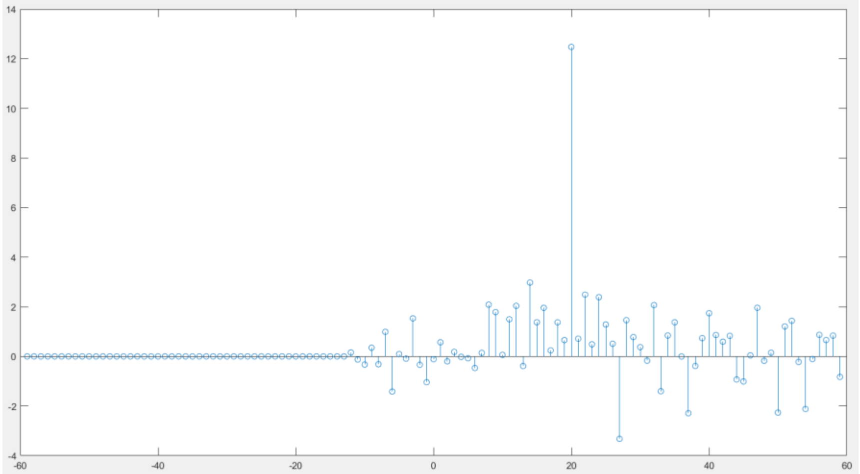
3.d) Cross between y[n] and x[n] (Noise var = 0.01)



The delay between the two signals are 20 samples. This is clear from the I index of the peak impulse in the graph on the left.

I

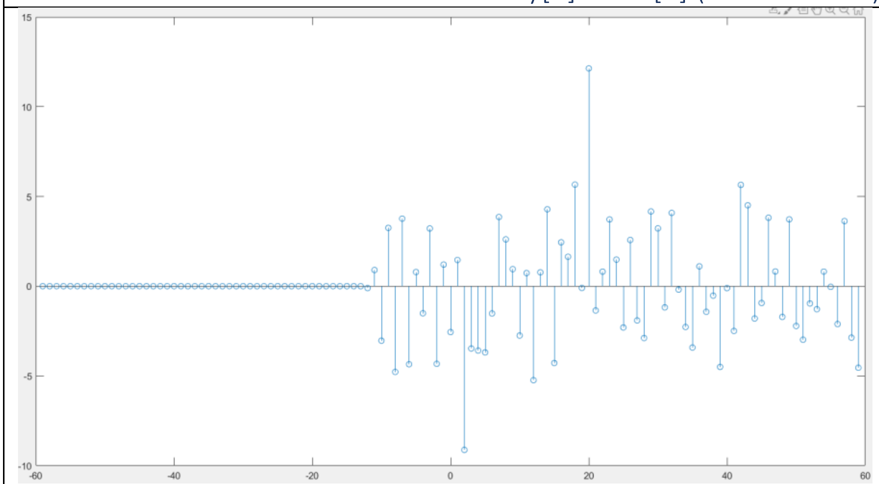
Cross between y[n] and x[n] (Noise var = 0.1)



The delay is still 20 samples. But the larger the noise variance the noisier the nondominant peaks become.

I

Cross between y[n] and x[n] (Noise var = 1)

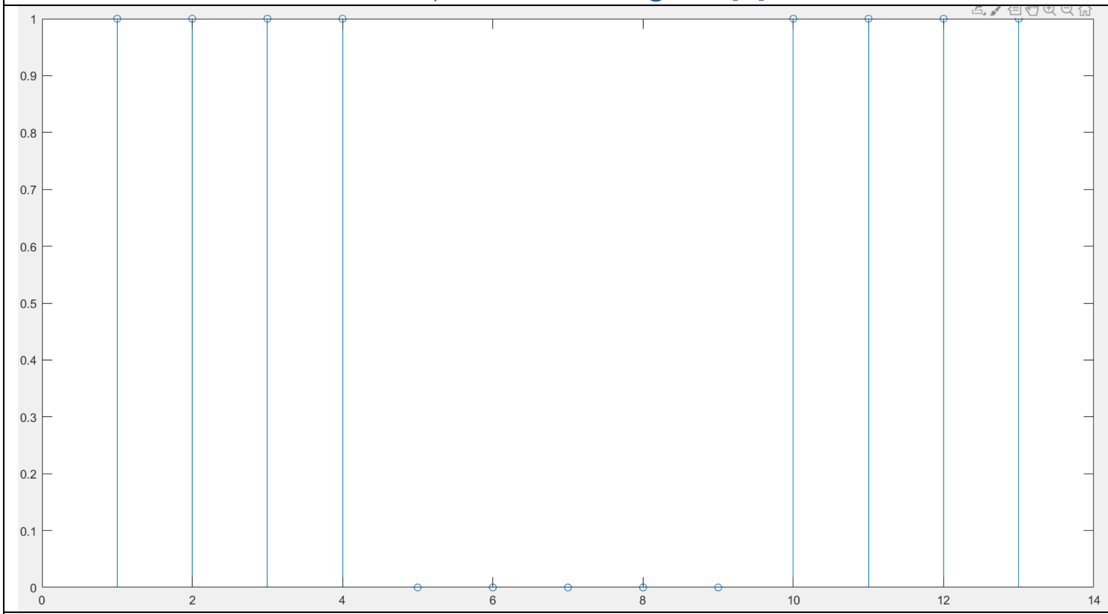
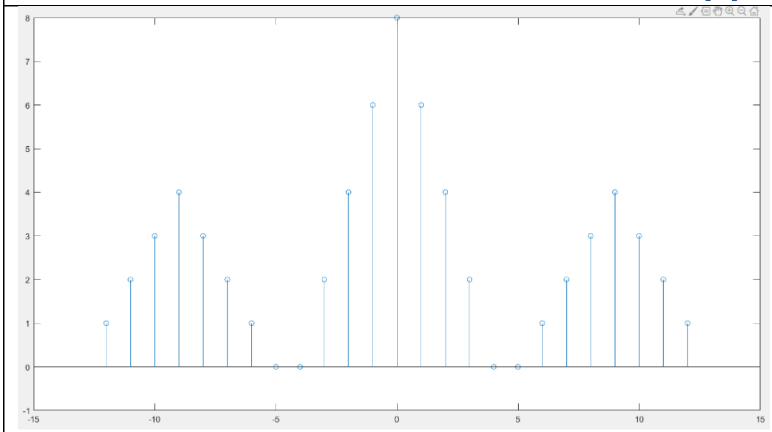


It becomes more difficult to confidently identify the delay between the two-signal the larger the noise variance becomes.

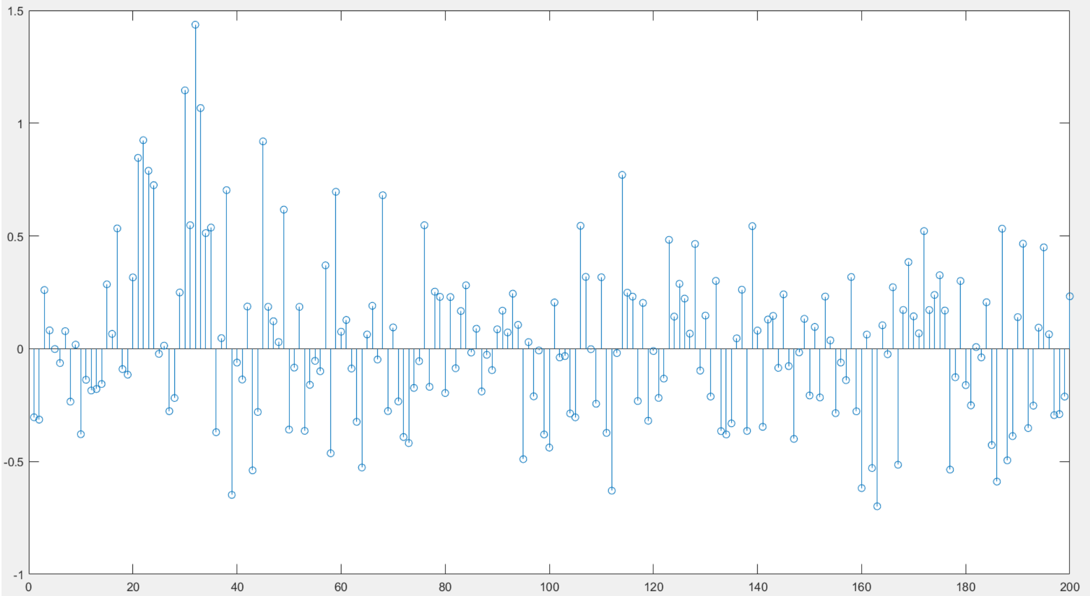
I

4.

$$x[n] = \{+1, +1, +1, +1, 0, 0, 0, 0, 0, +1, +1, +1, +1\}.$$

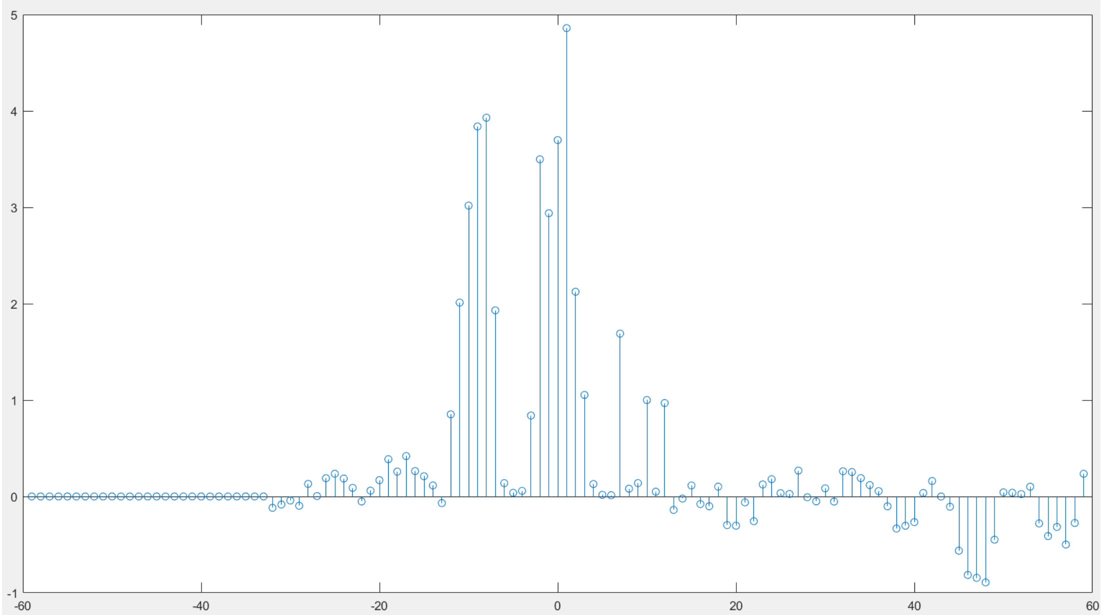
<table>
<tr><td colspan="2">4.a) Transmitted signal x[n]</td></tr>
<tr><td colspan="2">
</td></tr>
<tr><td colspan="2">n</td></tr>
<tr><td colspan="2">Auto correlation of x[n]</td></tr>
<tr><td>
</td><td>This is not an ideal option for x[n] for there are multiple peaks in the Auto-correlation graph. This means there are I index's you can shift your signal with that result in a reasonable strong correlation. The Barker Sequence only had one I index that resulted in a strong correlation.</td></tr>
<tr><td colspan="2">i</td></tr>
</table>

4.b) Reflected Signal y[n]



n

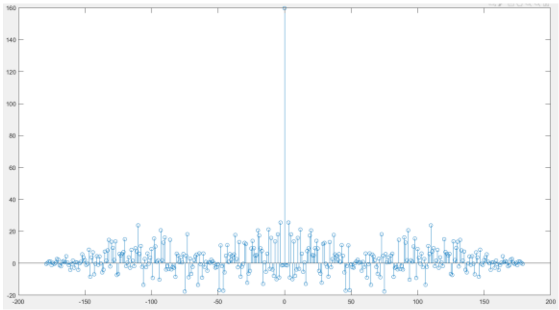
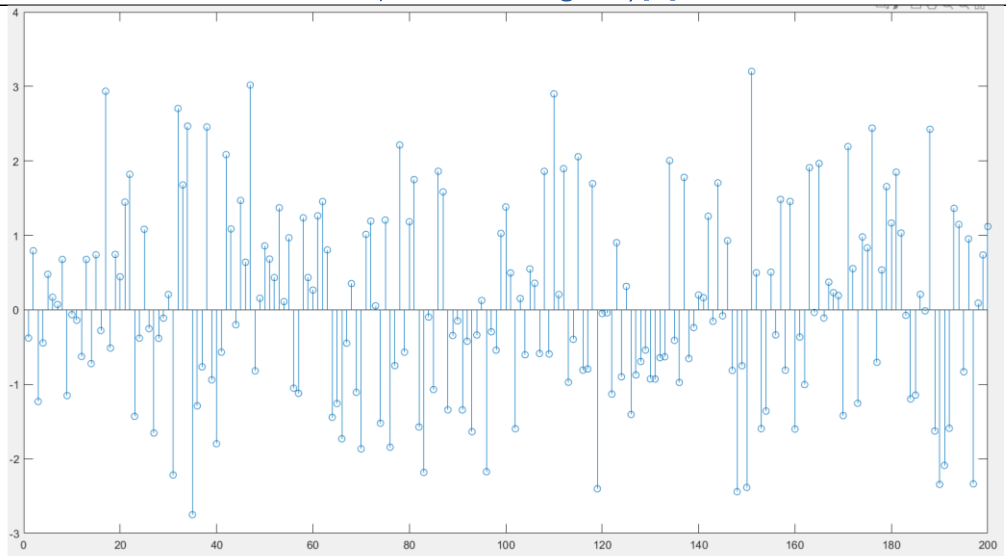
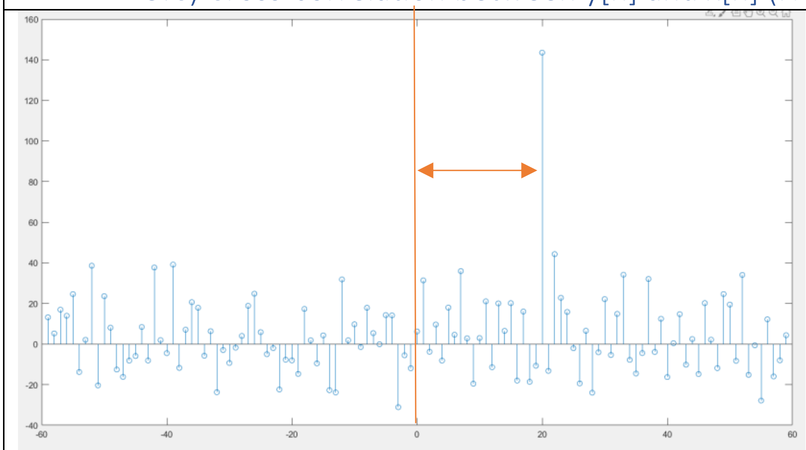
Cross-correlation between y[n] and x[n] (Noise var = 0.1)



l

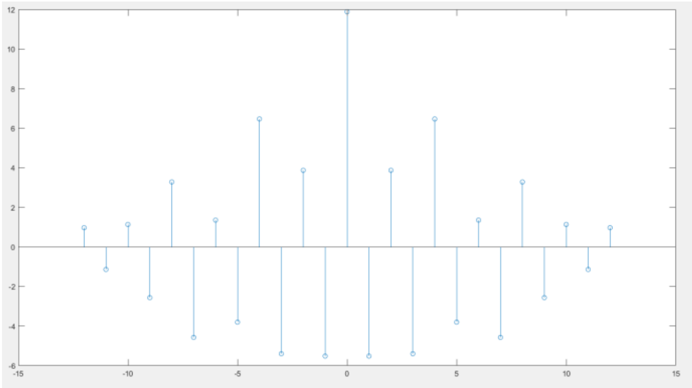
It is evident in the cross-correlation graph above that one cannot confidently identify the l'th index shift that equals the delay in the signal. We know the delay to be 20 but from the graph above the highest peak appears just past zero. This is due to the selection of x[n] as mentioned above.

5. **x[n]** consist of 200 samples of Gaussian white noise with variance $\sigma^2 = 1$

<table>
<tr><td colspan="2">5.a) Auto correlation of x[n]</td></tr>
<tr><td></td><td>The auto correlation for white noise (infinite samples) is zero. One can see that for the 200 samples it is basically zero. The more samples you add the closer the approximation will be to zero. (The peak appears at zero and everything around the peak will have a near zero amplitude). Making this an ideal signal to determine the delay.</td></tr>
<tr><td colspan="2">l</td></tr>
<tr><td colspan="2">5.b) Reflected Signal y[n]</td></tr>
<tr><td colspan="2"></td></tr>
<tr><td colspan="2">n</td></tr>
<tr><td colspan="2">5.C) Cross-correlation between y[n] and x[n] (Noise var = 1)</td></tr>
<tr><td></td><td>The two signals are delayed by 20 samples.</td></tr>
<tr><td colspan="2">l</td></tr>
</table>

6. x[n] consist of 13 samples of Gaussian white noise with variance $\sigma^2 = 1$

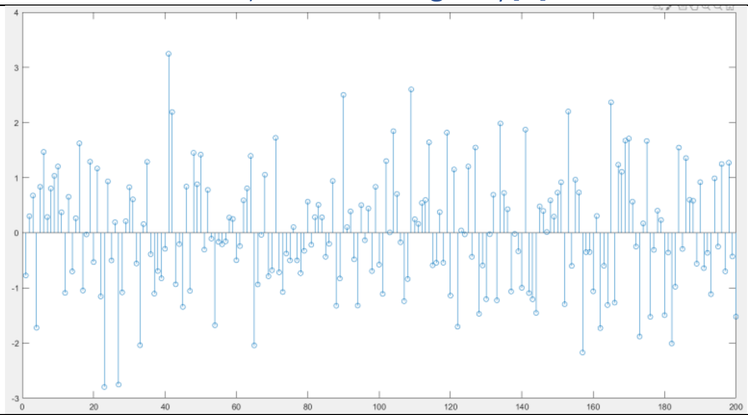
6.a) Auto correlation of x[n]



As mentioned in Q5 white noise (infinite samples) has a auto-correlation of zero. The fewer samples you have the less accurate the approximation will be. This is clear from the 13-sample white noise x[n] auto-correlation graph vs the 200-sample cross correlation graph. Making this signal not ideal for this purpose.

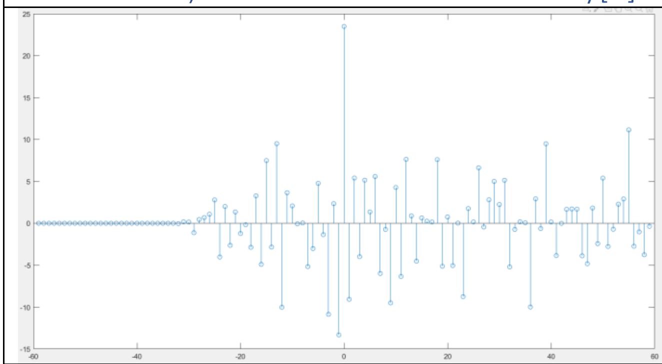
l

6.b) Reflected Signal y[n]



n

6.c) Cross-correlation between y[n] and x[n] (Noise var = 1)



Unable to tell what the delay is since x[n] is 20 samples of white noise. This is too little information and results in a cross correlation of 0.

l

The Barker sequence has the advantage over the gaussian method in that you only need 13 samples to produces a cross-correlation with a single impulse at l equal to zero. One can achieve the same result with the gaussian noise signal, but you will need a lot more samples to produce the same result as seen in figure 6.a. Also generating perfect white noise is very difficult. It is therefore faster to use the Barker sequence. If time is not important and you have access to white noise then you can use the latter method.