Aim: Understand the effect of poles and zeros on the responses of LTI systems, both in the time and frequency domain.

Task: Do the following assignment, using Matlab to obtain numerical results as required. Document your work in your practical book, indicating your methodology, theoretical results, numerical results and discussions. Graphs should have labelled axes and correct units indicated. If you struggle with Matlab, remember the `help` and `lookfor` functions.

1. Consider the LTI system (filter) with transfer function $H(z)$ given by

$$H(z) = \frac{1 - 2\cos(\theta_1)r_1 z^{-1} + r_1^2 z^{-2}}{1 - 2\cos(\theta_2)r_2 z^{-1} + r_2^2 z^{-2}},$$

with the following default parameter values:

$$\theta_1 = 3\pi/4, \quad \theta_2 = \pi/4, \quad r_1 = 0.95, \quad r_2 = 0.95.$$

(a) Algebraically (i.e. by hand) determine the locations of the poles and zeros of $H(z)$ in terms of θ_1, θ_2, r_1 and r_2. Sketch a pole-zero diagram for the default values. In Matlab you can use the `zplane` function to accomplish this.

(b) Keeping the other parameters at their default values, vary r_1 by letting it assume the following five values: $r_1 = \{0.0, 0.5, 0.8, 1.0, 1.05\}$.

i. Investigate the effect of this variation on both the magnitude and phase responses of the LTI system using Matlab's `freqz` function. Use linearly scaled axes for frequencies and phases, and decibels $(20\log_{10}(|A|))$ for amplitudes. Remember to use `log10` and not `log` in Matlab. The functions `abs`, `angle` and `unwrap` are useful for obtaining amplitudes and phases. NB: Plot more than one graph on a figure using Matlab's `subplot` function, otherwise you will end up with a lot of paper!

ii. Now investigate the effect on the impulse response of the system for the same values of r_1 using Matlab's `filter` function.

iii. Explain your observations in view of the locations of the poles and zeros of $H(z)$.

(c) Repeat the previous question but now vary r_2 over the same interval and keep $r_1 = 0.95$.

(d) Keeping the other parameters at their default values, vary θ_1 by letting it assume the following five values: $\theta_1 = \{0, \frac{\pi}{4}, \frac{\pi}{2}, \frac{3\pi}{4}, \pi\}$.

i. Investigate the effect of this variation on both the magnitude and phase responses of the LTI system using Matlab's `freqz` function.

ii. Now investigate the effect on the impulse response of the system for the same values of θ_1 using Matlab's `filter` function.

iii. Explain your observations in view of the locations of the poles and zeros of $H(z)$.

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(e) Repeat the previous question but now vary θ_2 over the same interval and keep $\theta_1 = 3\pi/4$.

(f) Now let $r_2 = 1.0$, while the other parameters take on their default values.

  i. Using Matlab's `filter` function, determine the output of the system when sinusoids with frequencies $\omega_1 = 0.22\pi$, $\omega_2 = 0.25\pi$ and $\omega_3 = 0.27\pi$ are applied to it (separately).

  ii. Explain your observations in view of the locations of the poles and zeros of $H(z)$.

2. Now consider an LTI system with transfer function $H(z)$ given by

$$H(z) = \frac{0.0038 + 0.0001z^{-1} + 0.0051z^{-2} + 0.0001z^{-3} + 0.0038z^{-4}}{1 - 3.2821z^{-1} + 4.2360z^{-2} - 2.5275z^{-3} + 0.5865z^{-4}}.$$

(a) Determine the system's magnitude and phase response using Matlab's `freqz` function.

(b) What type of filter is this system? Verify your answer by filtering a few sinusoids at appropriately chosen frequencies.

(c) Sketch the pole-zero diagram for this system using Matlab's `zplane` function. Can you explain the frequency response of the system from this plot? The functions `tf2zp`, `cart2pol` and `polar` are also useful in the analysis of poles and zeros.