Systems & Signals 414

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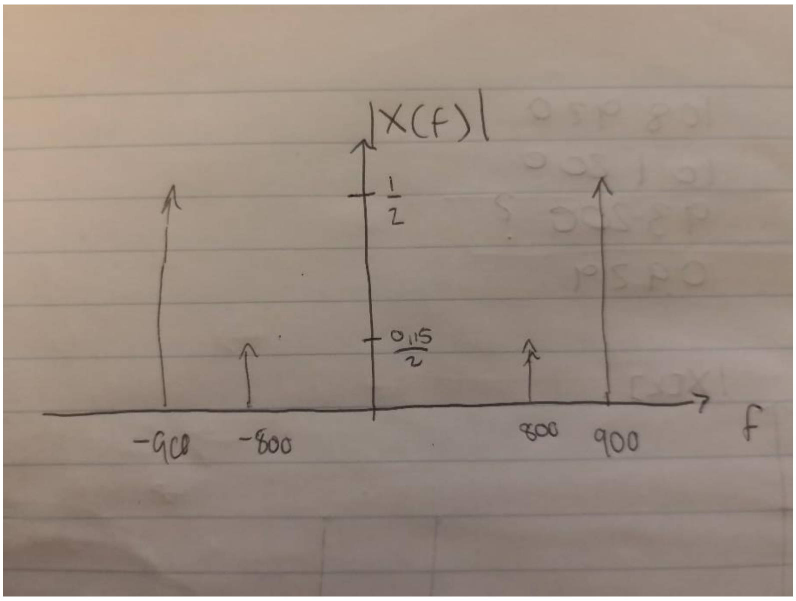
Aim: To explore the DFT with discrete time signal analyses.

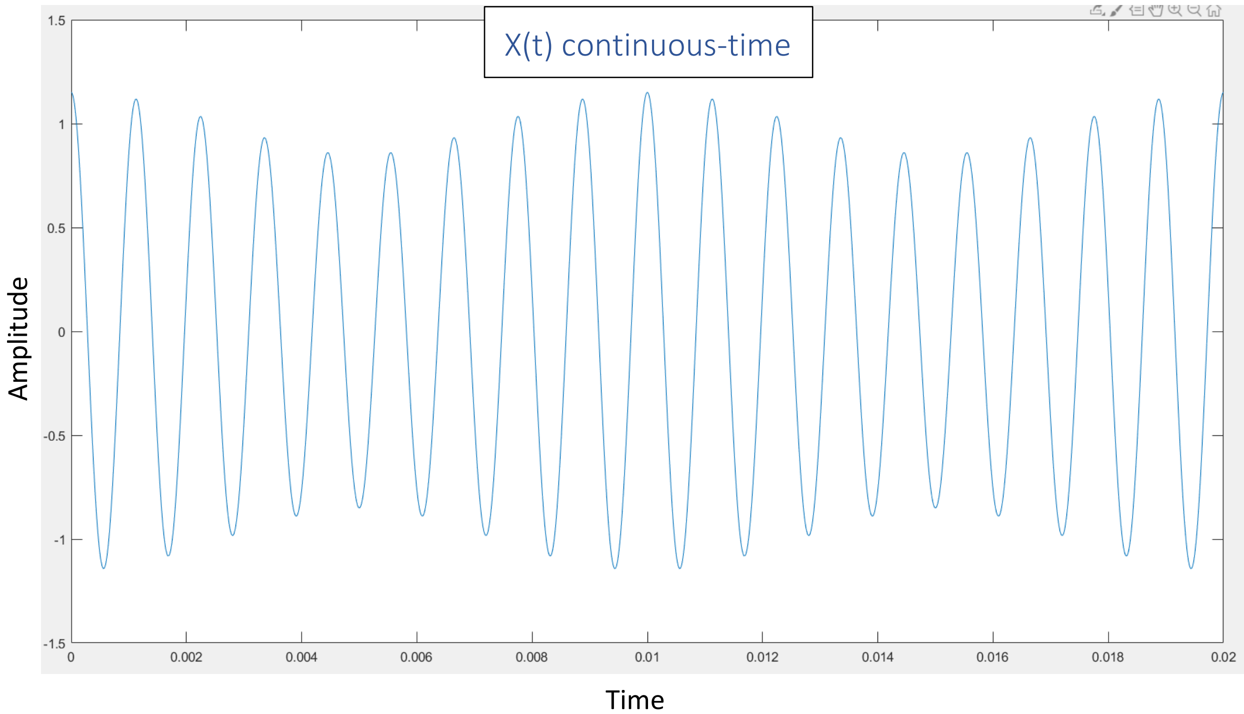
Given the following equation for a signal x(t).

$$x(t) = \cos(1800\pi t) + 0.15\cos(1600\pi t).$$

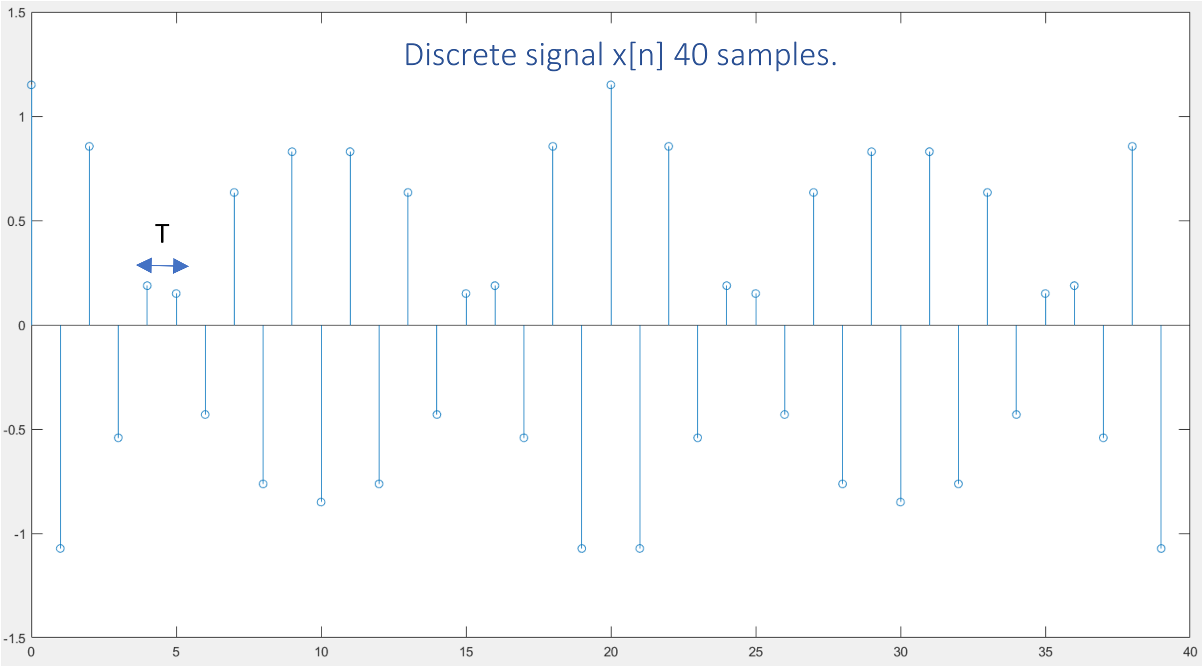
Q1.a) Sketch by hand.



Q1.b)

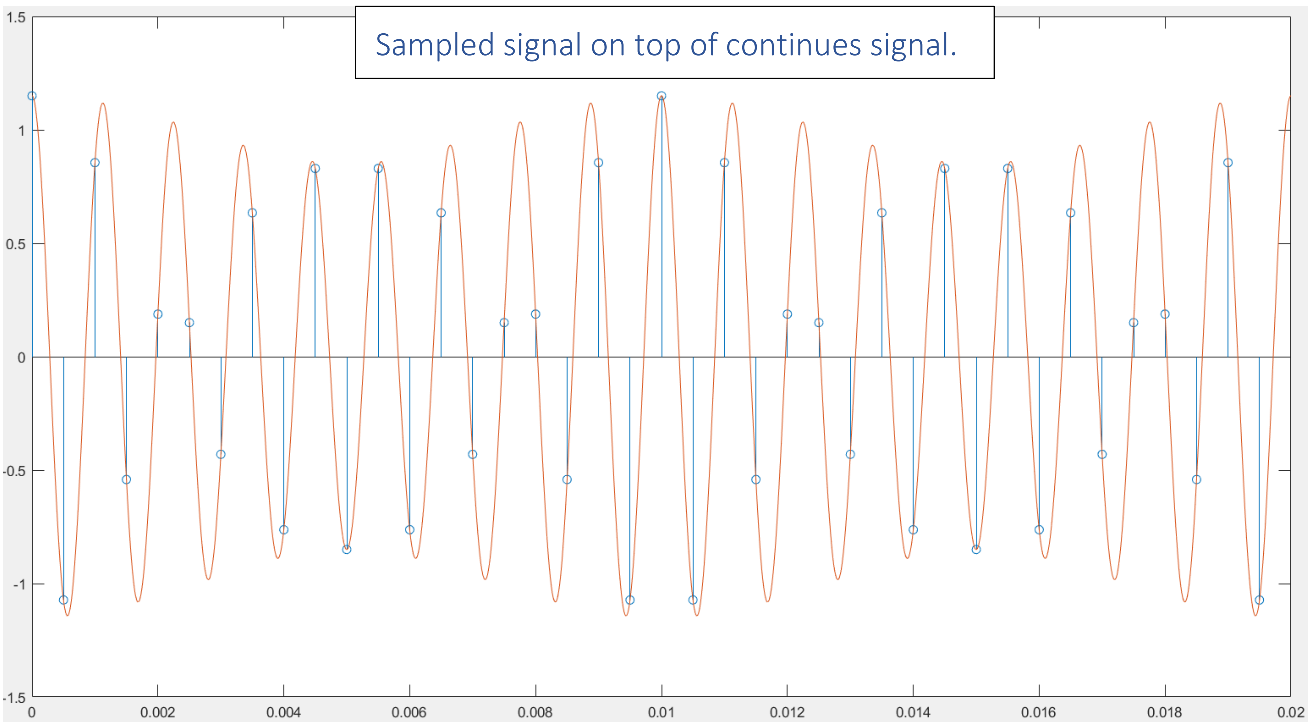


Q1.c) $f_s = 2000\ Hz, N = 40$

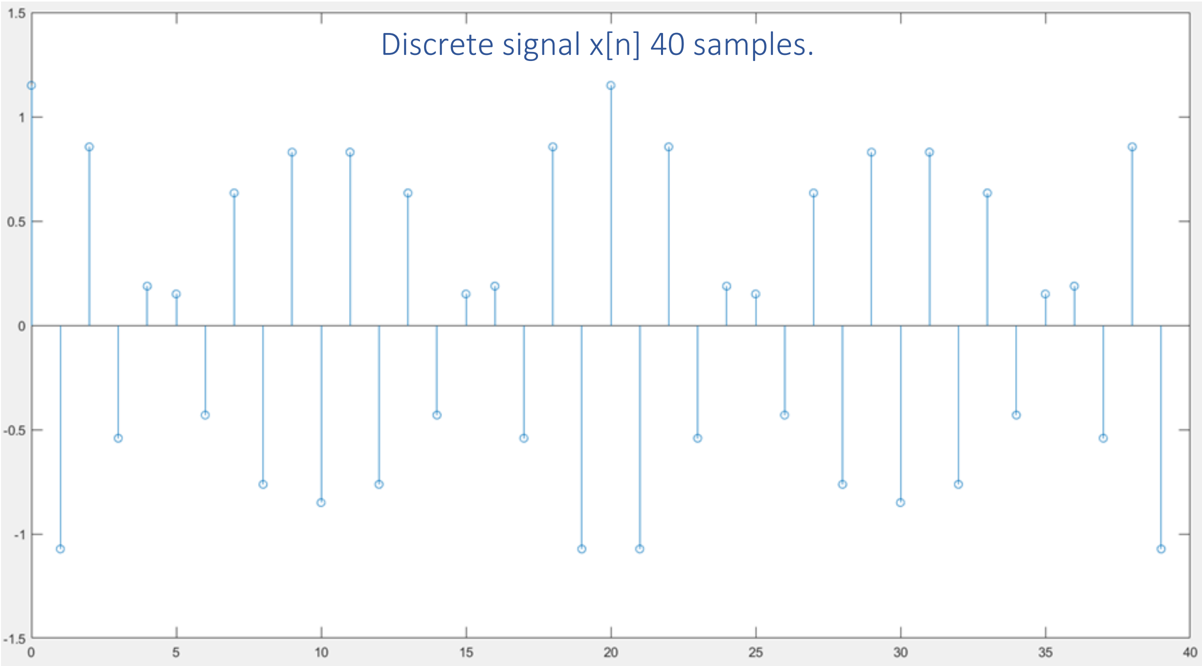


No aliasing occurs for the Nyquist criteria is met. The sampling frequency is at least twice as large as the highest frequency component in the signal. $f_c < \frac{1}{2T} = \frac{1}{2(0.0005)}$

Q1.d)



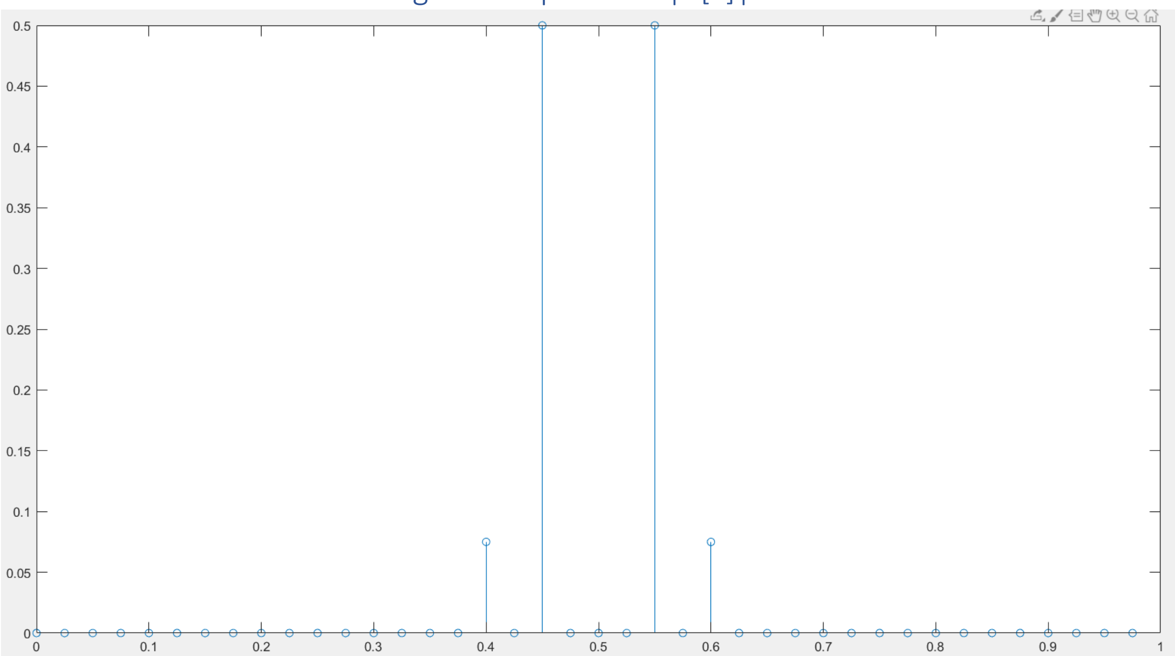
Q2.a)



N samples.

Q2.b)

Magnitude spectrum |X[k]|.



f_ω

f_ω → Cycles per sample

$$f_\omega = \frac{k}{N} = \frac{f}{f_s}$$

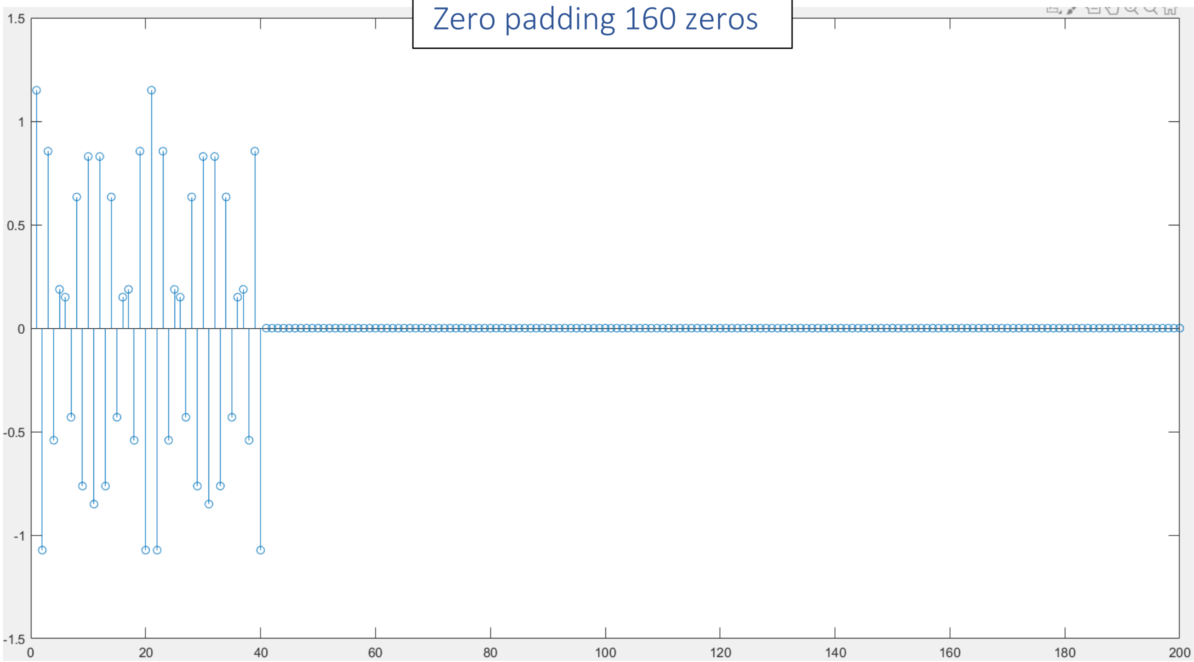
Q2.c) It is clear from the figure in Q2.b that there are frequency components at $f_\omega = 0.4$ & 0.45. Using the equation $f = f_\omega f_s$ the frequencies can be calculated as: Anything above 0.5 is above the Nyquist frequency.

$$f_\omega = 0.4 \rightarrow f = 0.4(2000) = 800\ Hz$$

$$f_\omega = 0.45 \rightarrow f = 0.45(2000) = 900\ Hz$$

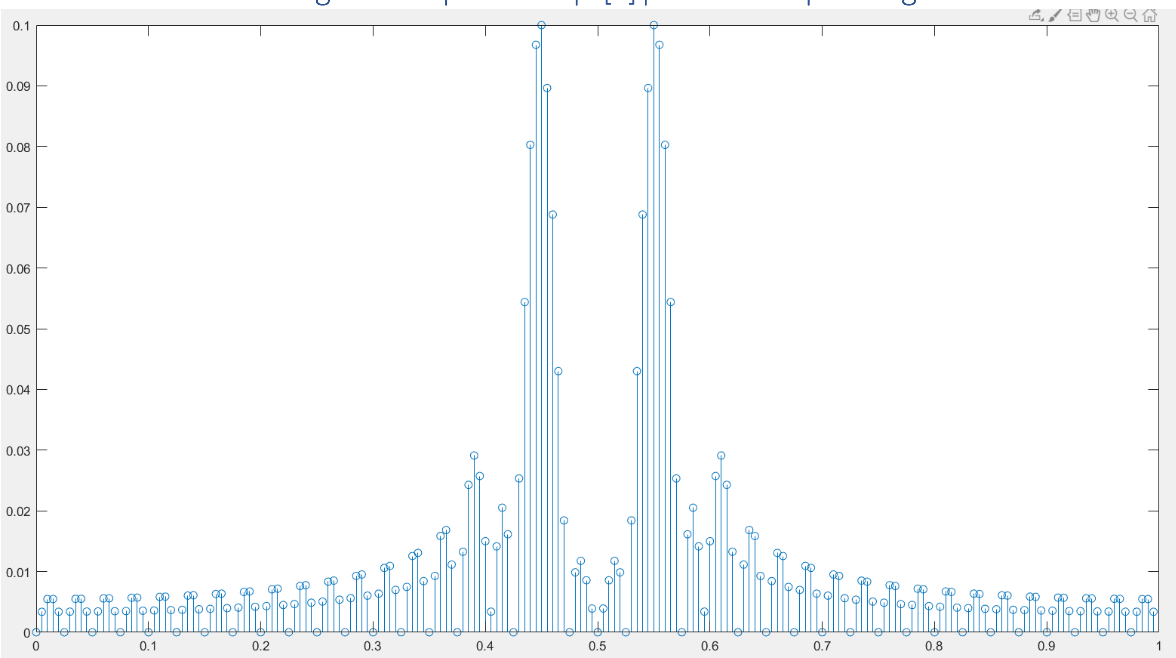
Q2.d)

Zero padding 160 zeros



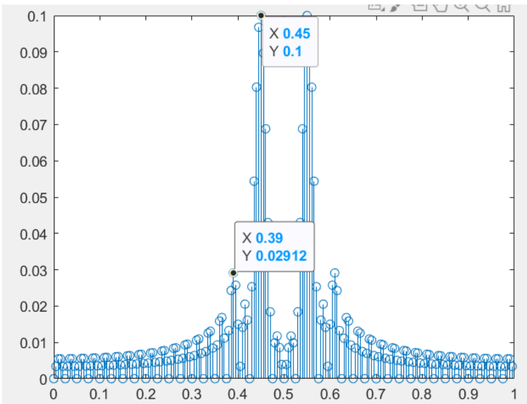
N samples

Magnitude spectrum |X[k]| after zero padding.

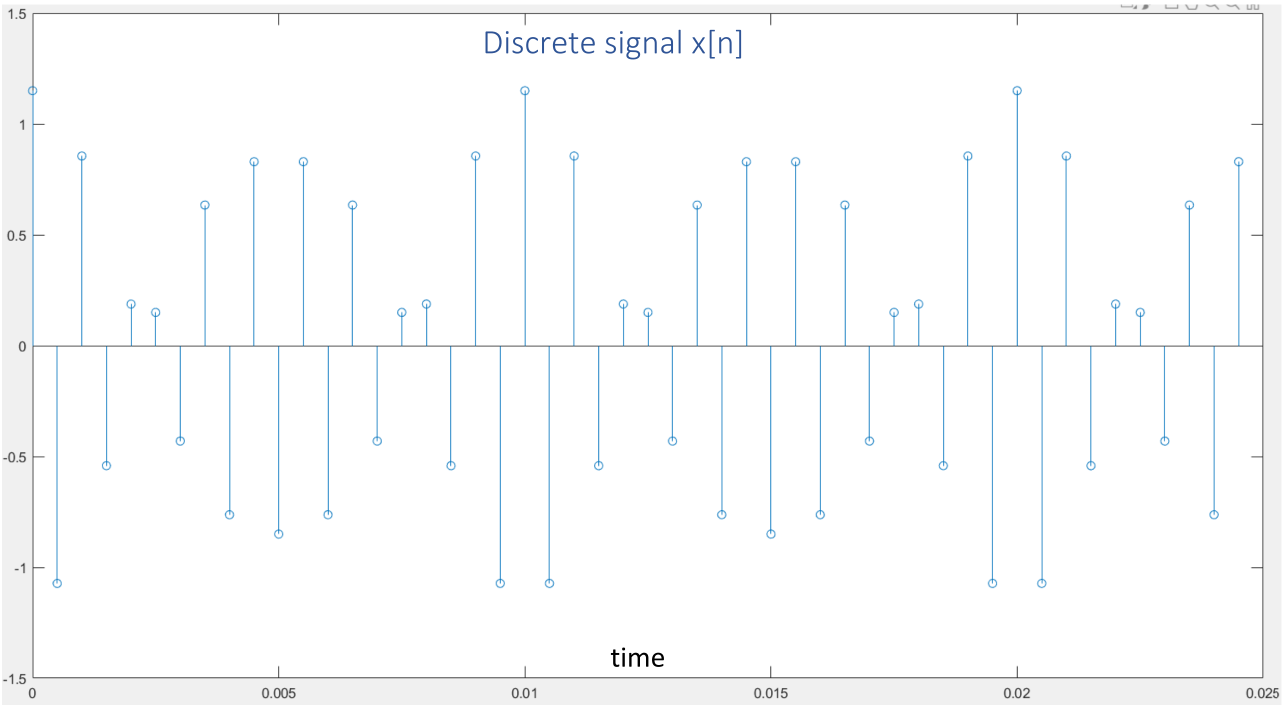


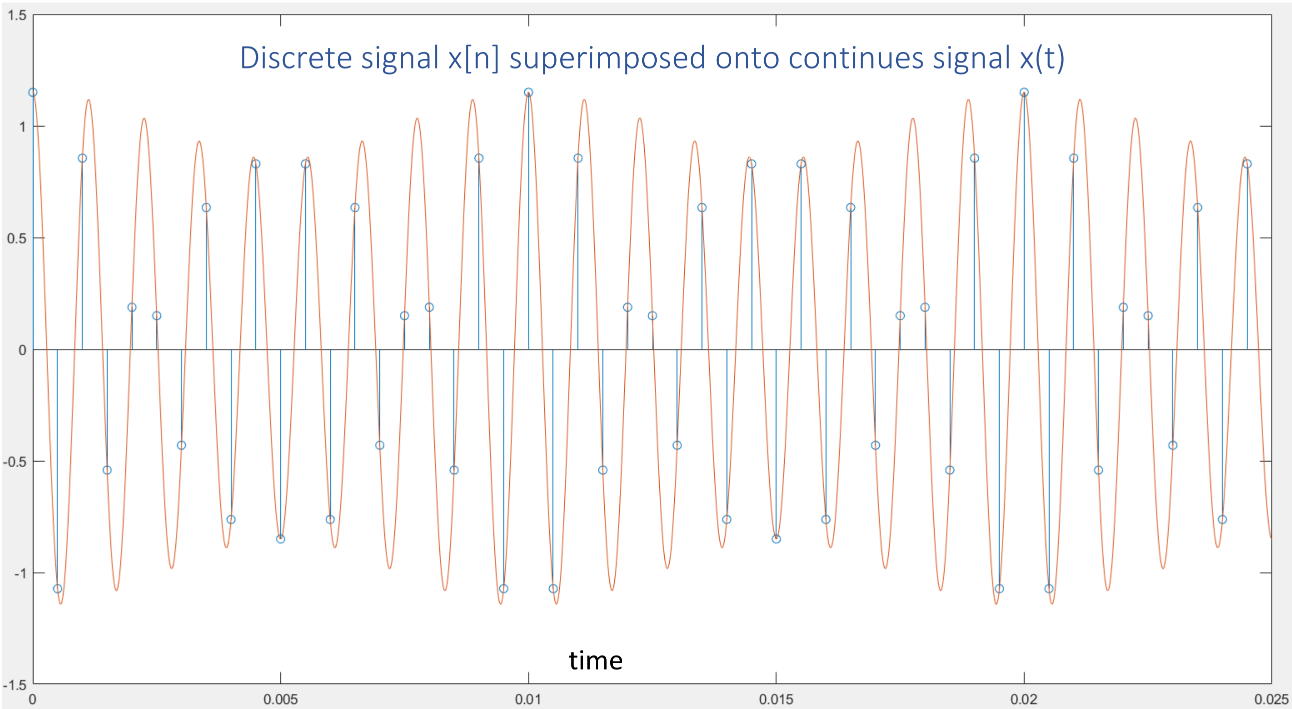
f_ω

The signal that we take the DFT on is a truncated signal because it is a discrete time signal. (Time limited) Appending zero to the end of the windowed signal does not add any signal information. The window itself has a frequency response. A square window will have a sinc function frequency response hence the frequency not being zero everywhere as before. The dominant signals are still that of the sinusoidal waves. $f_\omega = 0.39 \rightarrow 780Hz$ and $f_\omega = 0.45 \rightarrow 900Hz$



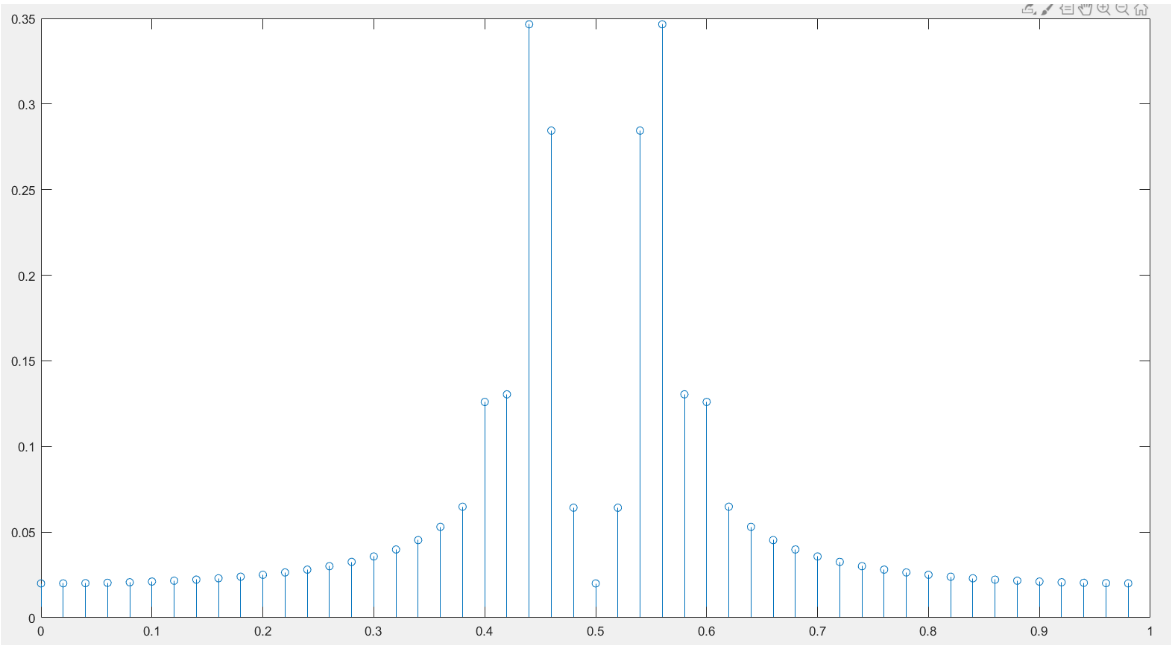
Q3.a)





Q3.b) Amplitude spectrum using DFT.

Magnitude spectrum |X[k]|



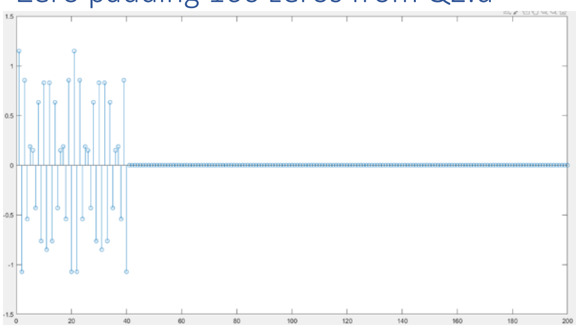
f_ω

Q3.c)

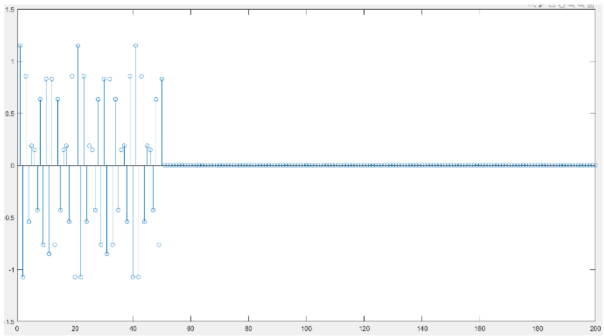
From the plot in Q3.b it is not clear where the frequency components sit exactly. The component at $f_\omega = 0.42$ (840Hz) is more dominant and the frequency component at $f_\omega = 0.445$ (890Hz) is most dominant. This results in an error in calculating the frequency components present in the signal. The reason is that there is not enough signal information yet.

Q3.d)

Zero padding 160 zeros from Q2.d



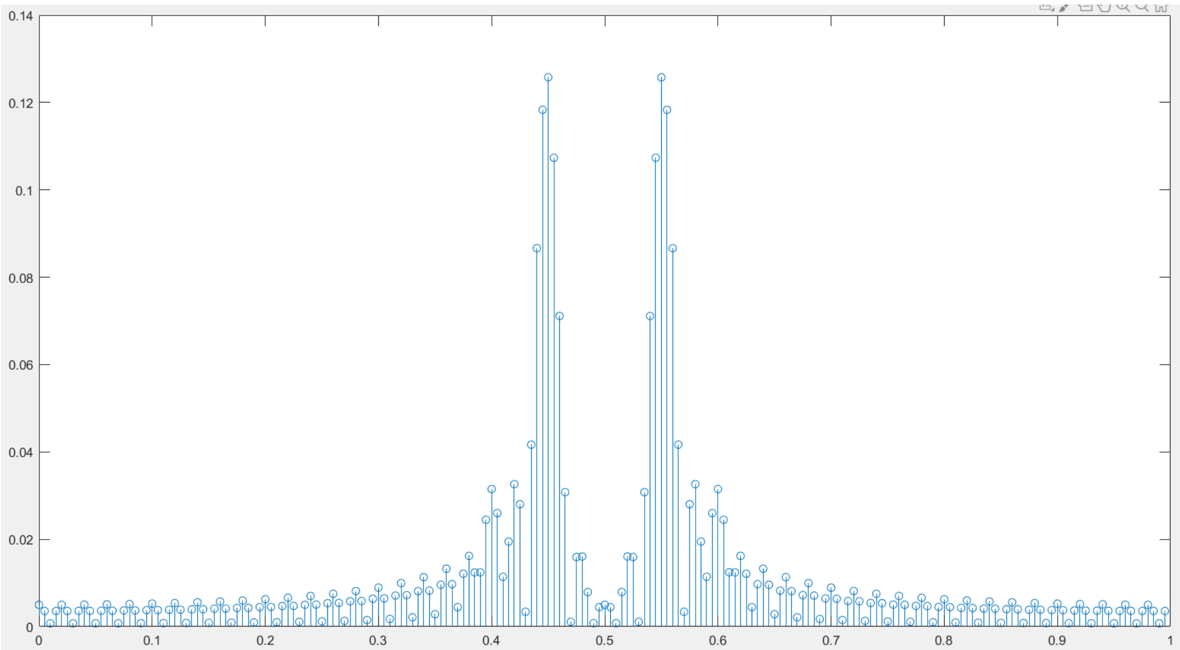
Zero padding 150 zeros from Q3.c



The DFT formula is:$X[k] = \sum_{n=0}^{N-1} x[n]\, e^{-j2\pi\frac{kn}{N}}$

The zero padded signal in Q3.c has fewer padded zero components than in Q2.d due to the signal itself being longer in duration. Therefore, the DFT will have more signal information to work from in Q3.c resulting in a better frequency spectrum as seen in the figure below. It is clear from the plot below that the frequency components of the signal are located at $f_\omega = 0.4$ (800Hz) and at $f_\omega = 0.45$ (900Hz)

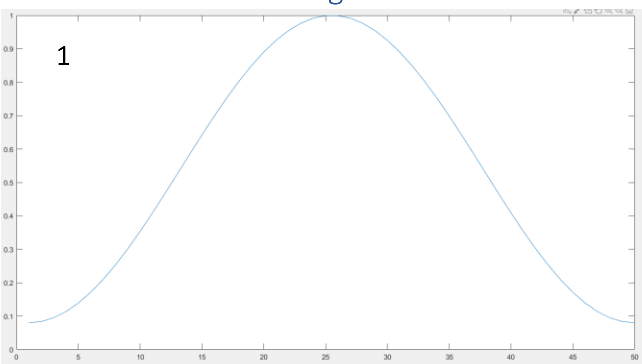
Magnitude spectrum |X[k]| after zero padding.



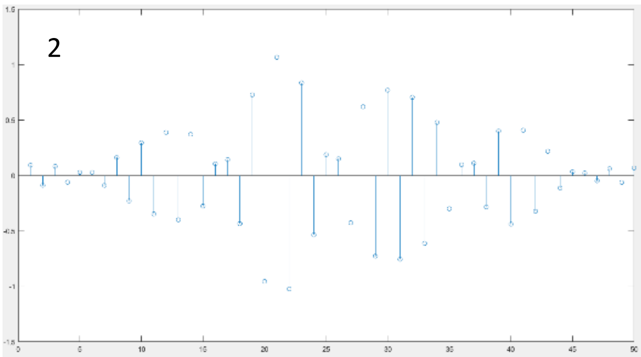
f_ω

Q3.e)

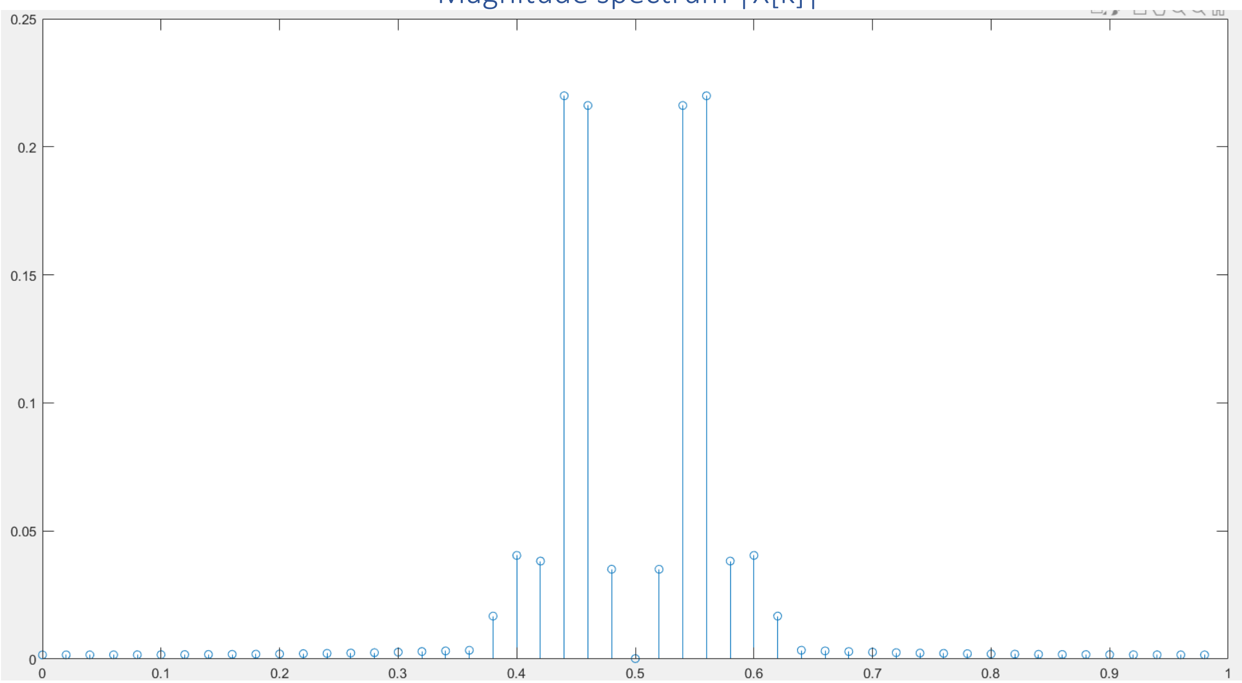
Hamming window.



Signal passed through Hamming window.



Magnitude spectrum |X[k]|

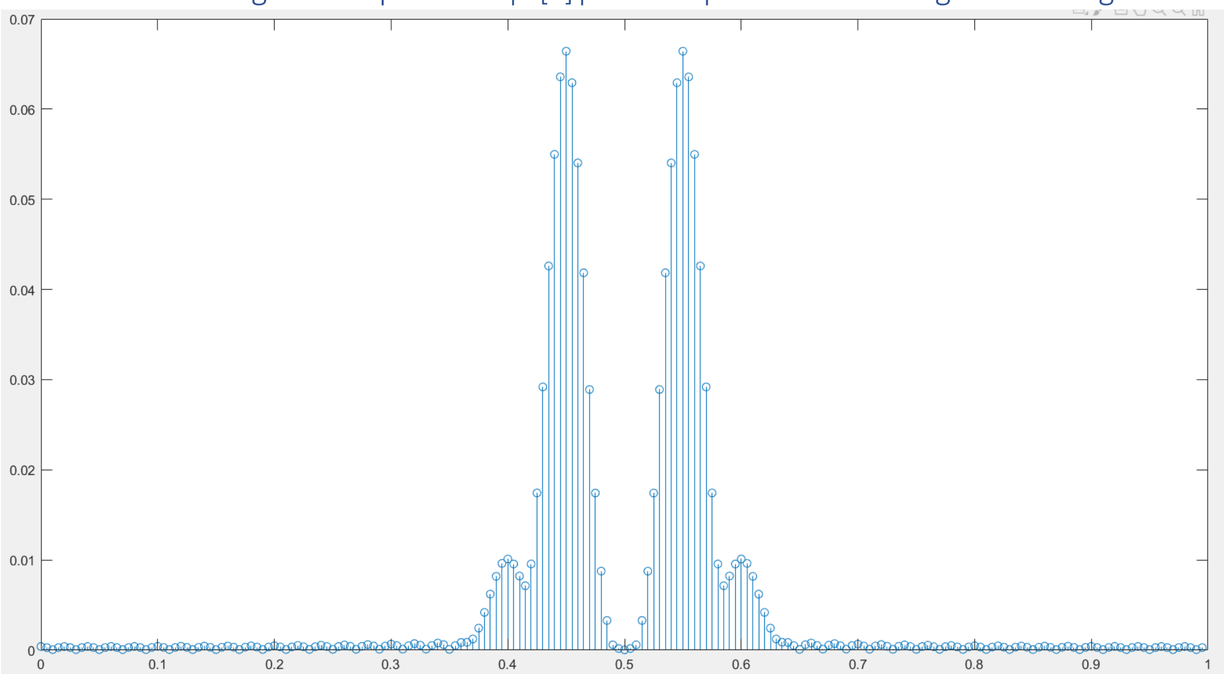


f_ω

The signal was already a truncated signal (Perfect square window) for it started at t=0 seconds and ended at t=0.025 seconds. Therefore, once we put the signal through a hamming window, we lost a bit of information (see figure 3.e.2) and therefore the resultant plot obtained through taking the DFT of the signal is less accurate than before.

Q3.f)

Magnitude spectrum |X[k]| of zero padded Hamming window signal.

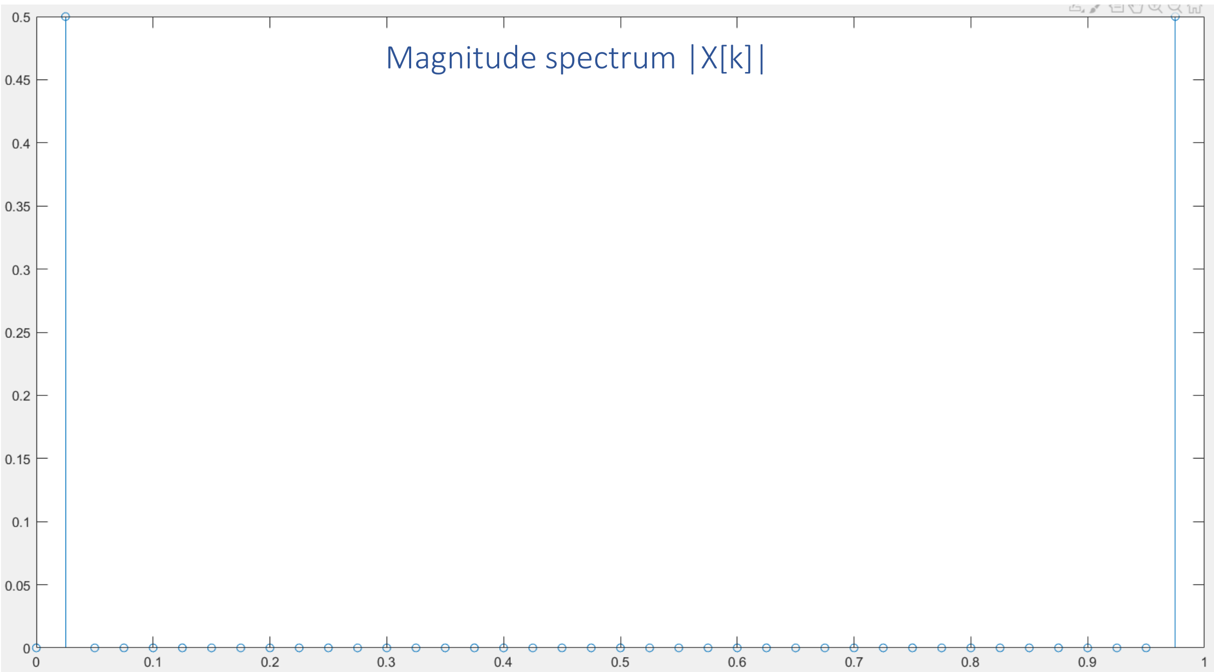


f_ω

Padding zeros at the end of the signal and then taking the DFT of the signal results in the plot above. Now that there are more samples for the DFT the magnitude spectrum more accurately represents the actual signal's magnitude spectrum. Even though no extra signal information was added. Just the number of samples has increased.

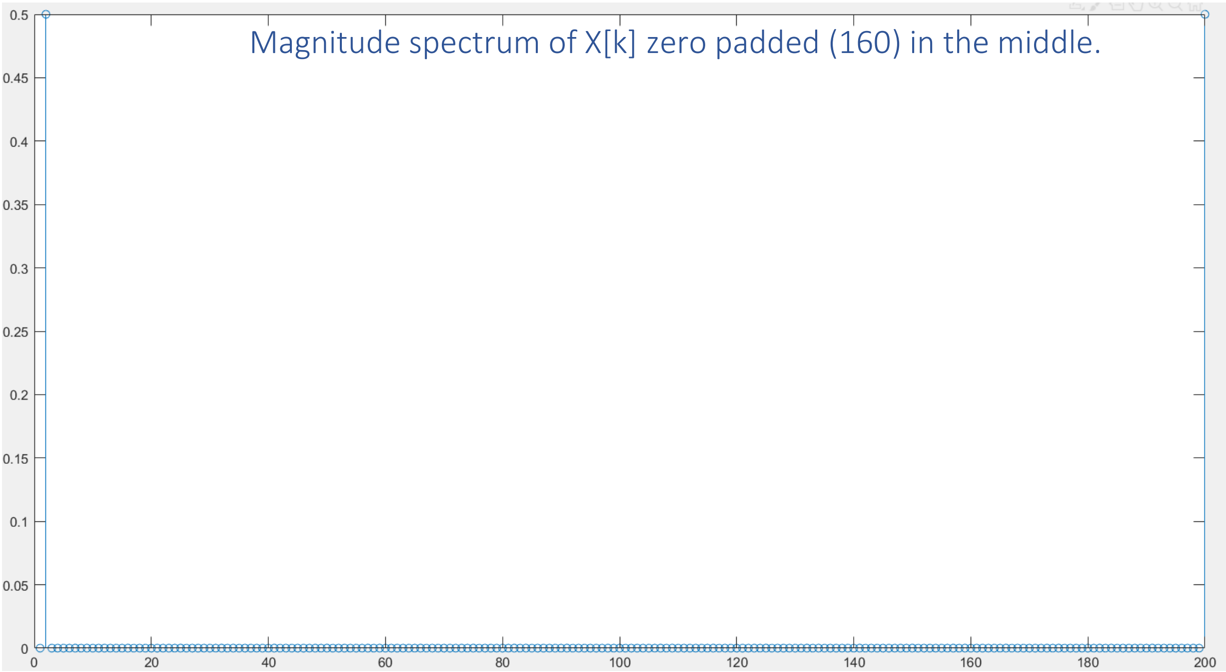
$f_\omega = 0.4$ (800Hz) and at $f_\omega = 0.45$ (900Hz) which matches the known signal exactly.

Q4.a) New signal: $x(t) = \cos(100\pi t)$



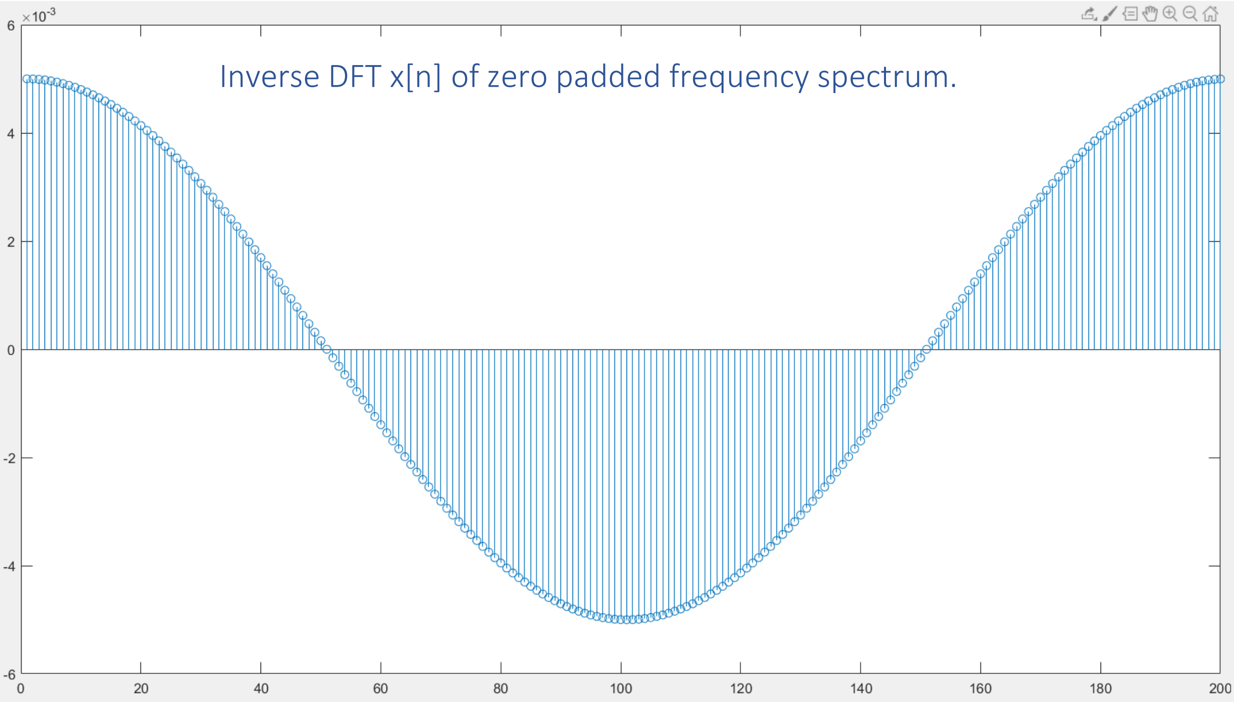
f_ω

Q4.b)



f_ω

Q4.c)

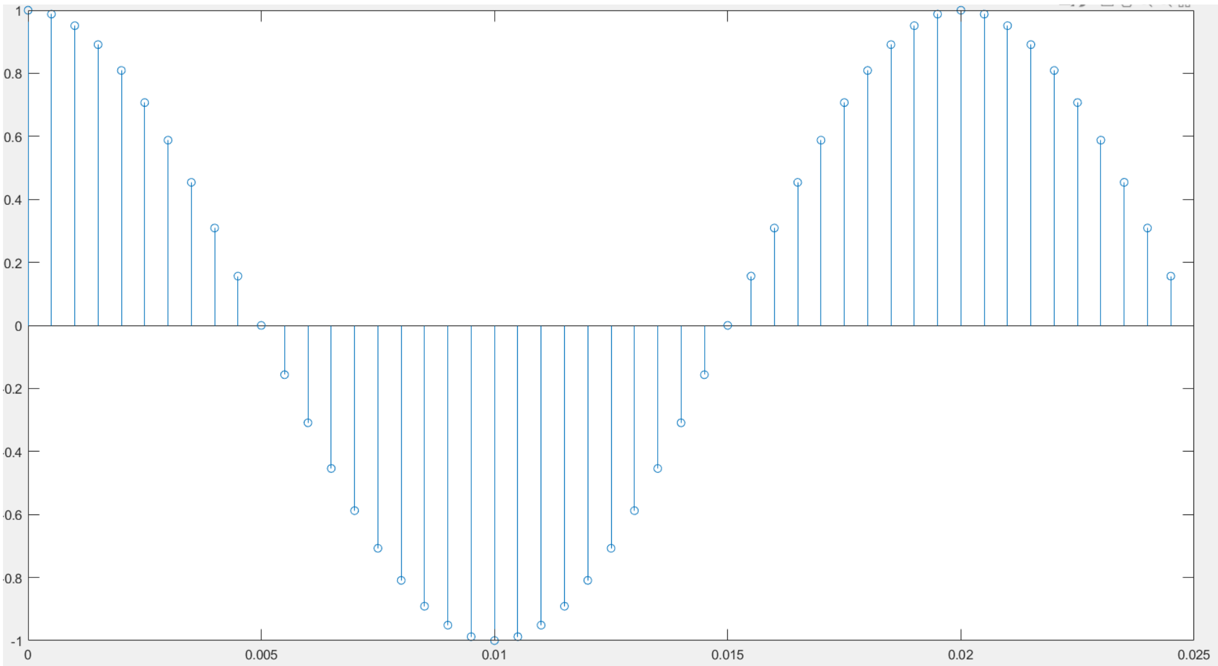


Time[n]

Above is the inverse DFT of the frequency response which is a reconstructed cosine signal. The signal has 200 samples instead of the original 40 samples. This is due to the 160 zero that were added to the frequency response. It is as if the signal were sampled faster.

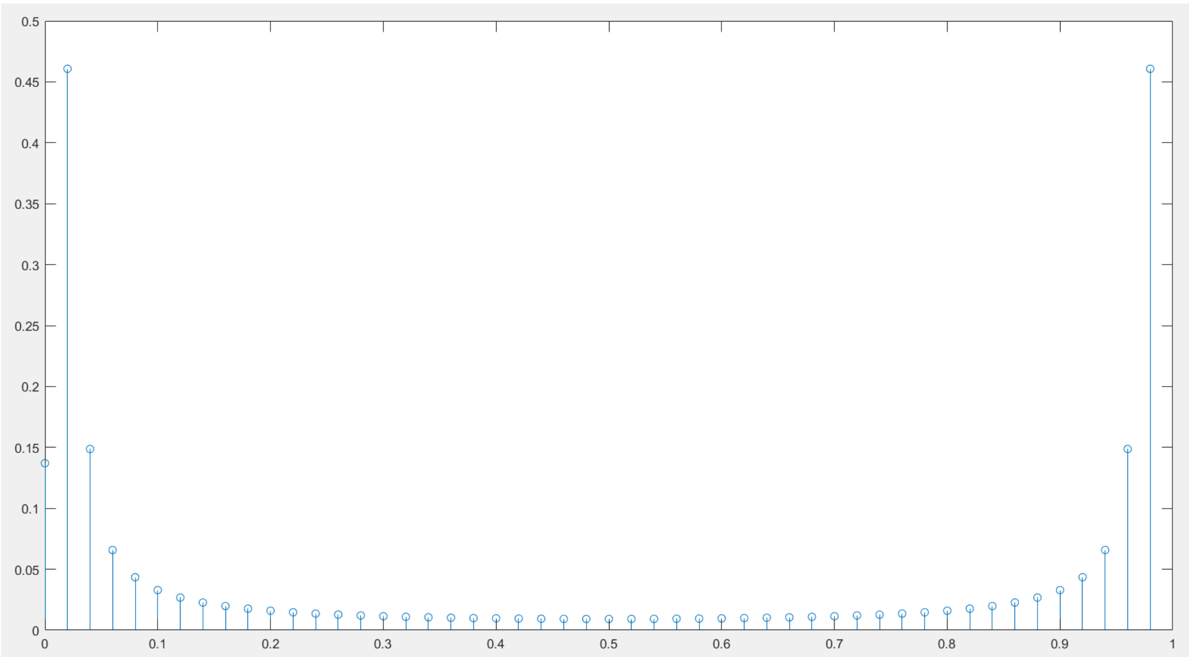
Q5)

x[n] for 50 samples sampled at 2000Hz.



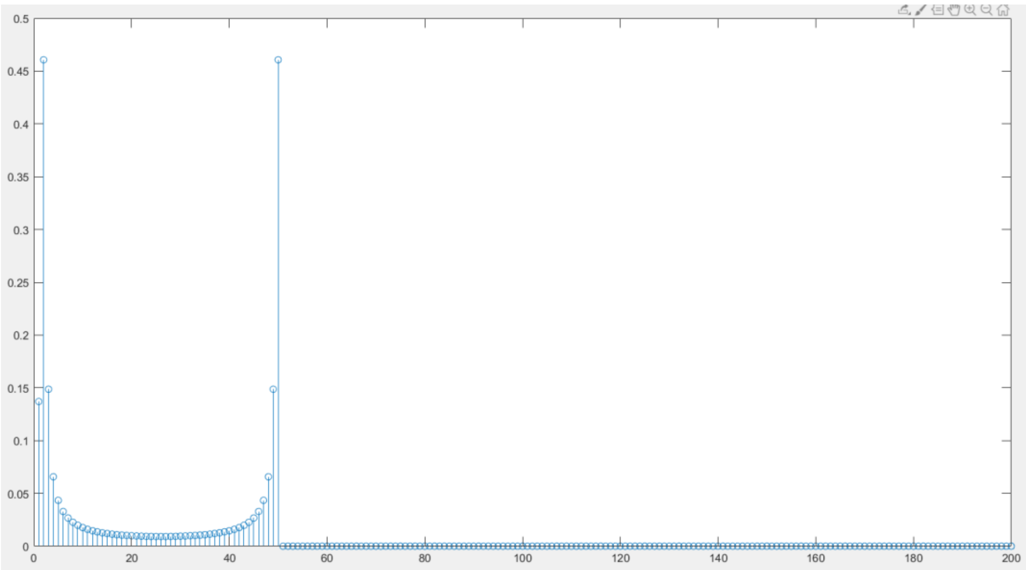
Time[n]

Magnitude spectrum of X[k] without zero padding.



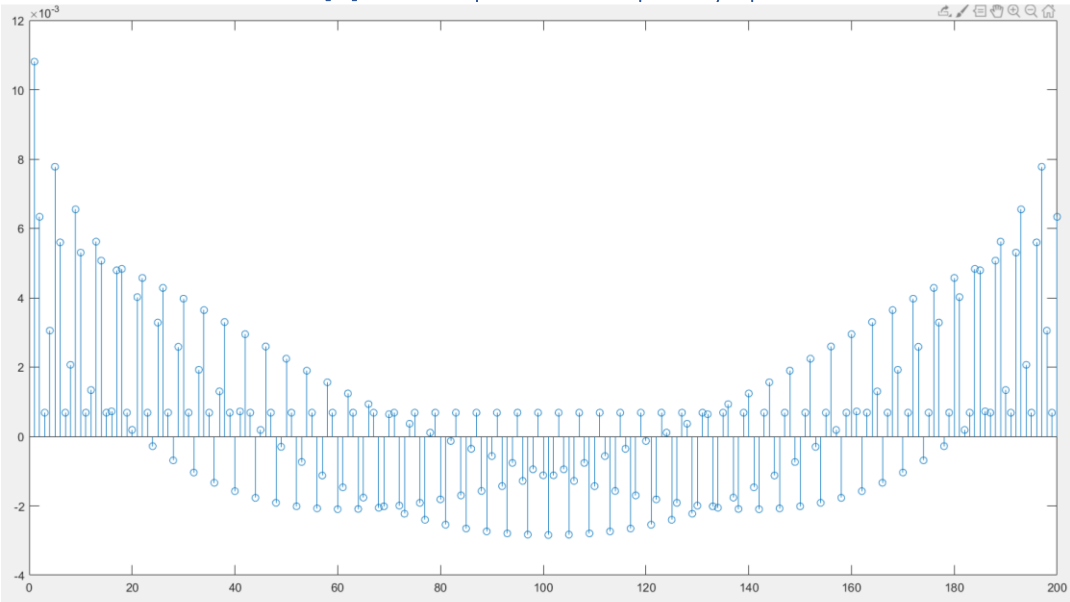
Frequency

Magnitude spectrum of X[k] zero padded at the back.



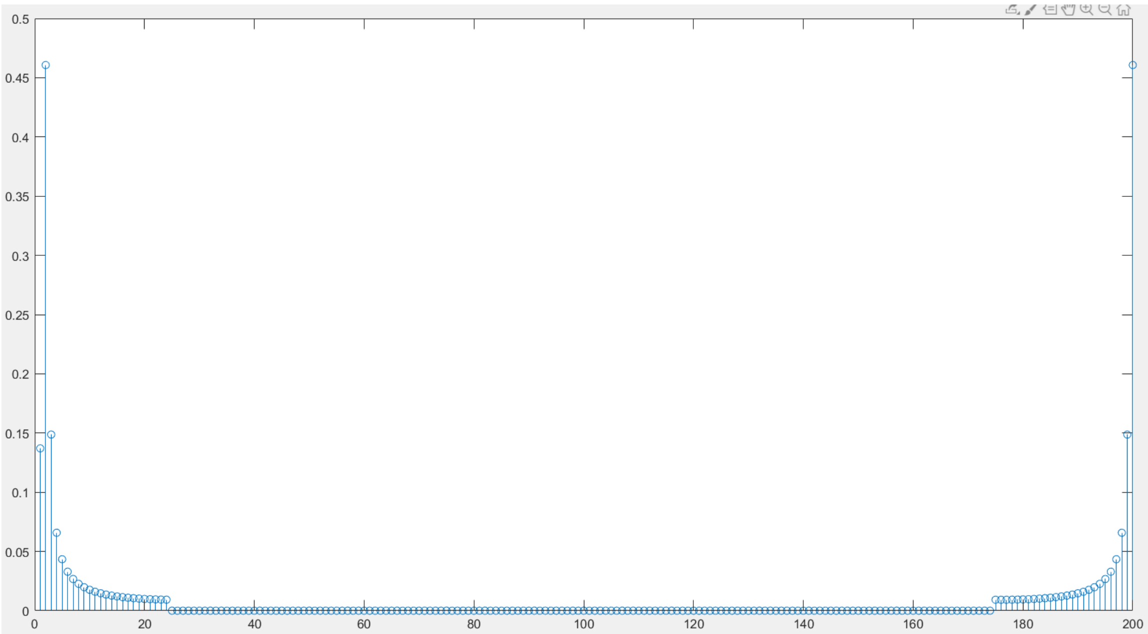
f_ω

Inverse DFT x[n] of zero padded frequency spectrum.



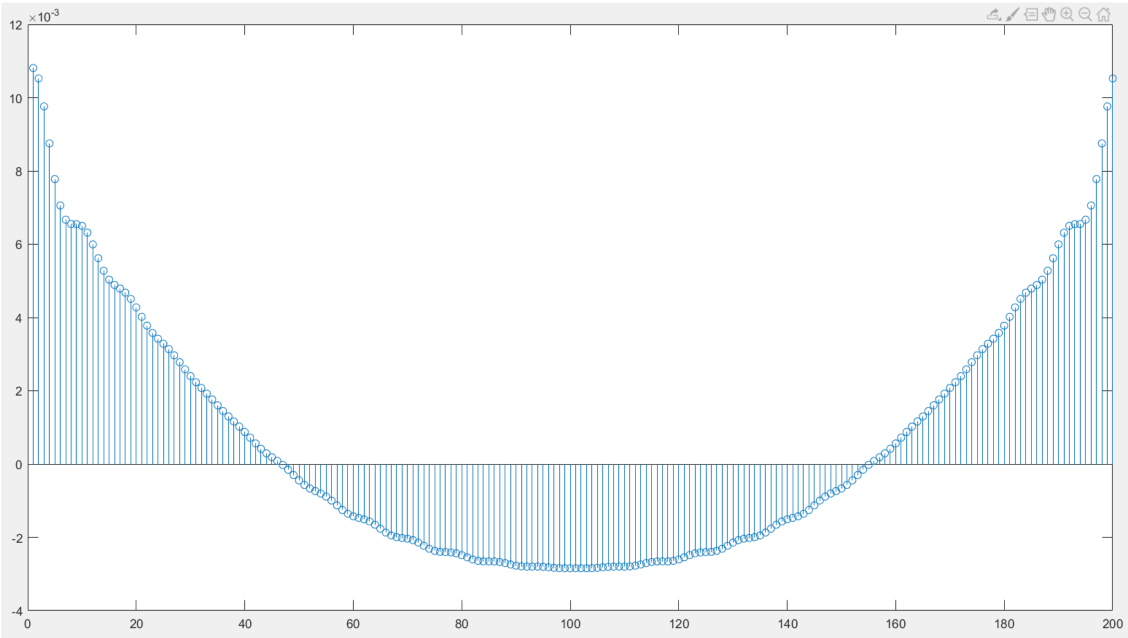
Time[n]

Magnitude spectrum of X[k] zero padded in the middle.



f_ω

Inverse DFT x[n] of zero padded frequency spectrum.



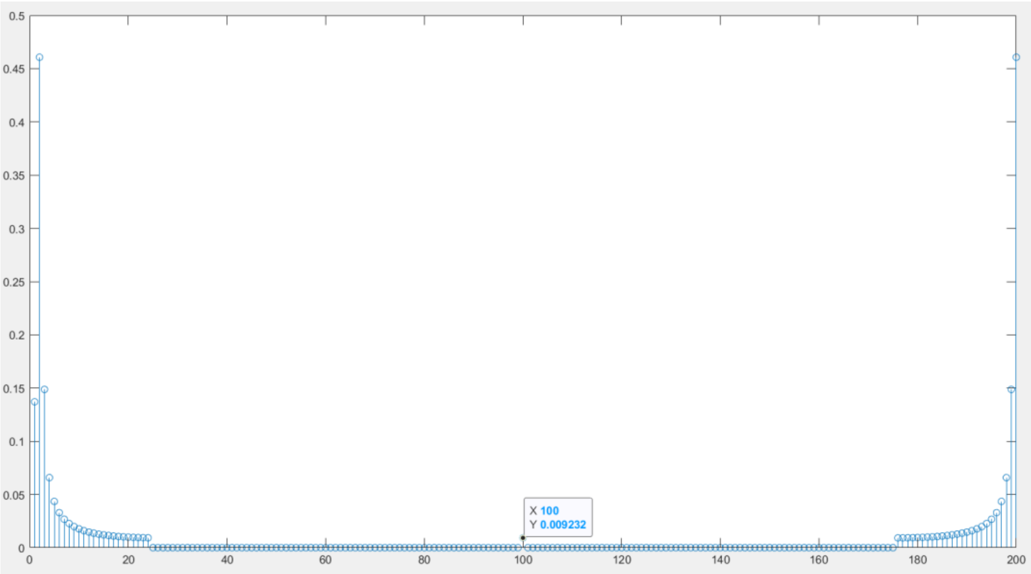
Time[n]

Padding the zeros at the end of the spectrum and then taking the IDFT results in a messy time domain signal.

Padding the zeros in the middle of the spectrum widens the gap between the two spikes of the since spectrum. This results in a sinusoidal signal but with the incorrect frequency. Also, the ends seem to not be smooth for the signal spectrum had unwanted frequencies around the centre frequency of the signal.

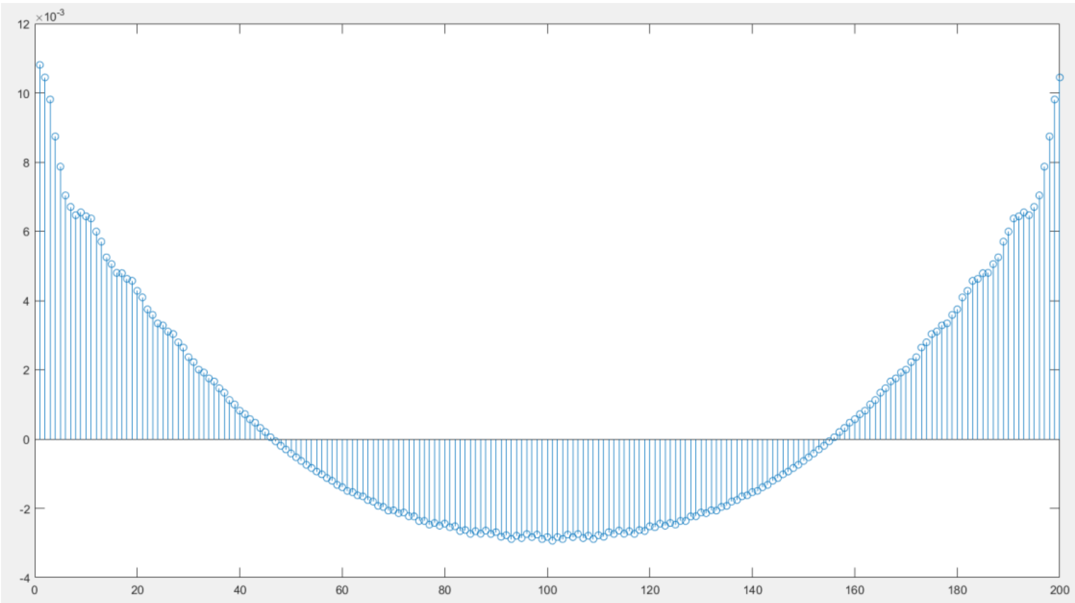
Padding 75 zeros before the 25th element of the spectrum and 75 zeros after the 25th element has the same effect as padding it in the middle, for in a pure sinusoidal wave the spectrum consists of zeros and scaled impulse at its centre frequency. Therefore the 25th element will basically be a zero in our case and therefore

Magnitude spectrum of X[k] zero padded around X[25].



f_ω

Inverse DFT x[n] of zero padded frequency spectrum.



Time[n]