Systems & Signals 414 Prac4 12 May 2021

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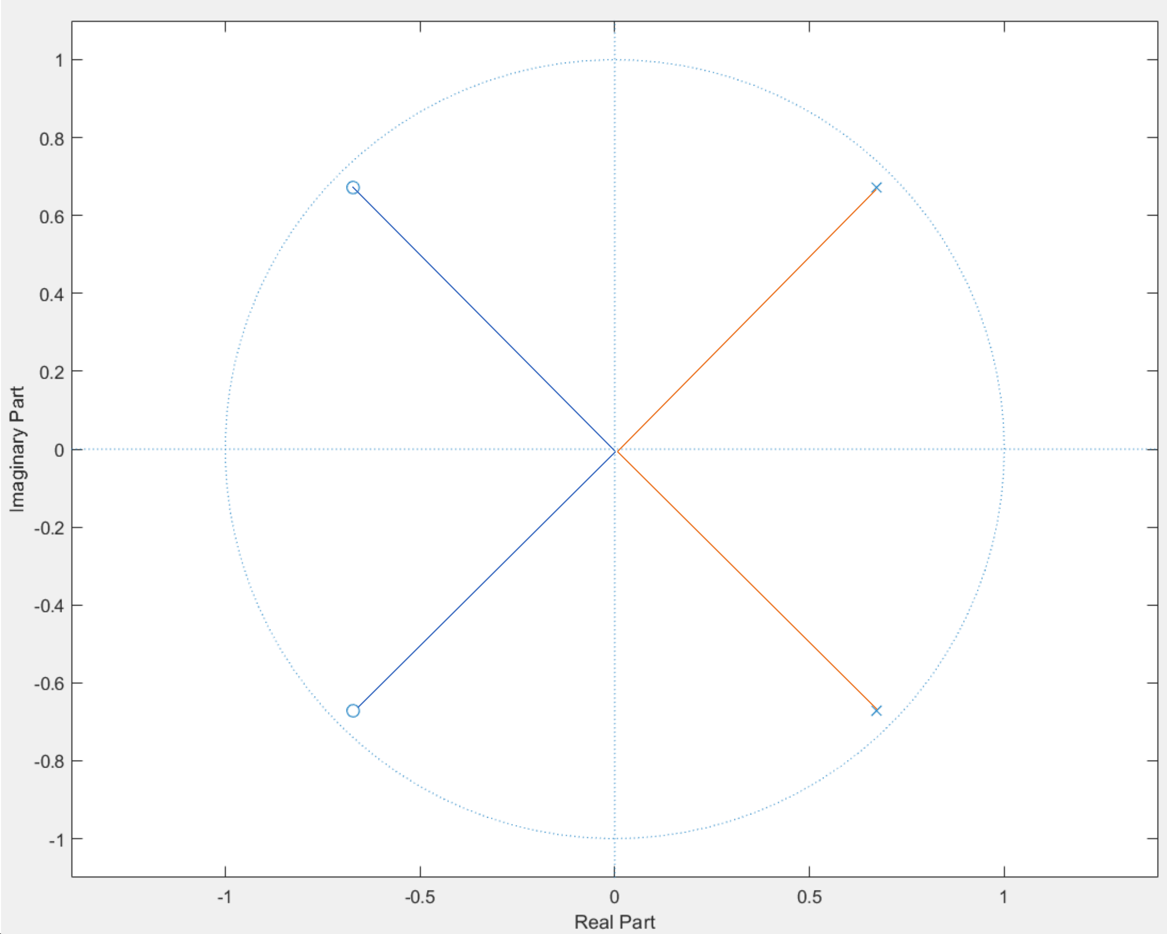
Q1.a) Z-Plane $z_{1,2} = -0.6718 \pm 0.6718i$ & $p_{1,2} = 0.6718 \pm 0.6718i$



$$H(z) = \frac{1 - 2\cos(\theta_1) r_1 z^{-1} + r_1^2 z^{-2}}{1 - 2\cos(\theta_2) r_2 z^{-1} + r_2^2 z^{-2}}$$

$$= \frac{(1 - r_1 e^{j\theta_1} z^{-1})(1 - r_1 e^{-j\theta_1} z^{-1})}{(1 - r_2 e^{j\theta_2} z^{-1})(1 - r_2 e^{-j\theta_2} z^{-1})}$$

poles : $1 - r_2 e^{j\theta_2} z^{-1} = 0$

$$z_{p_0} = r_2 e^{j\theta_2}$$
$$z_{p_1} = r_2 e^{-j\theta_2}$$

zeros : $1 - r_1 e^{j\theta_1} z^{-1} = 0$

$$z_{z_1} = r_1 e^{j\theta_1}$$
$$z_{z_1} = r_1 e^{-j\theta_1}$$

$\theta_1 = \frac{3\pi}{4}$ $\quad\theta_2 = \frac{\pi}{4}$ $\quad r_1 = 0.95 = r_2$

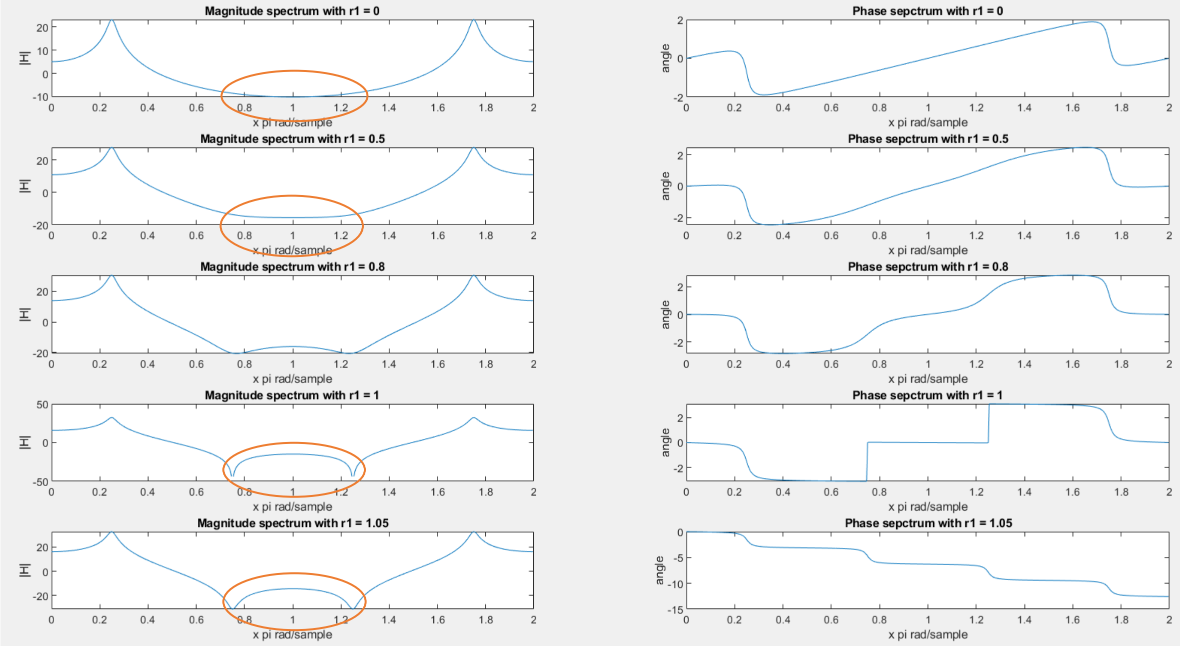
poles :

$$z_{p_0} = 0.95\left[\cos\left(\tfrac{\pi}{4}\right) + j\sin\left(\tfrac{\pi}{4}\right)\right]$$
$$= 0.95\left[\tfrac{1}{\sqrt{2}} + j\tfrac{1}{\sqrt{2}}\right]$$
$$= 0.6717 + j0.6717$$
$$z_{p_1} = 0.6717 - j0.6717$$

zeros

$$z_{z_0} = 0.95\left[\cos\left(\tfrac{3\pi}{4}\right) + j\sin\left(\tfrac{3\pi}{4}\right)\right]$$
$$= 0.95\left[-0.707 + j\tfrac{1}{\sqrt{2}}\right]$$
$$= -0.6717 + j0.6717$$
$$z_{z_1} = -0.6717 - j0.6717$$

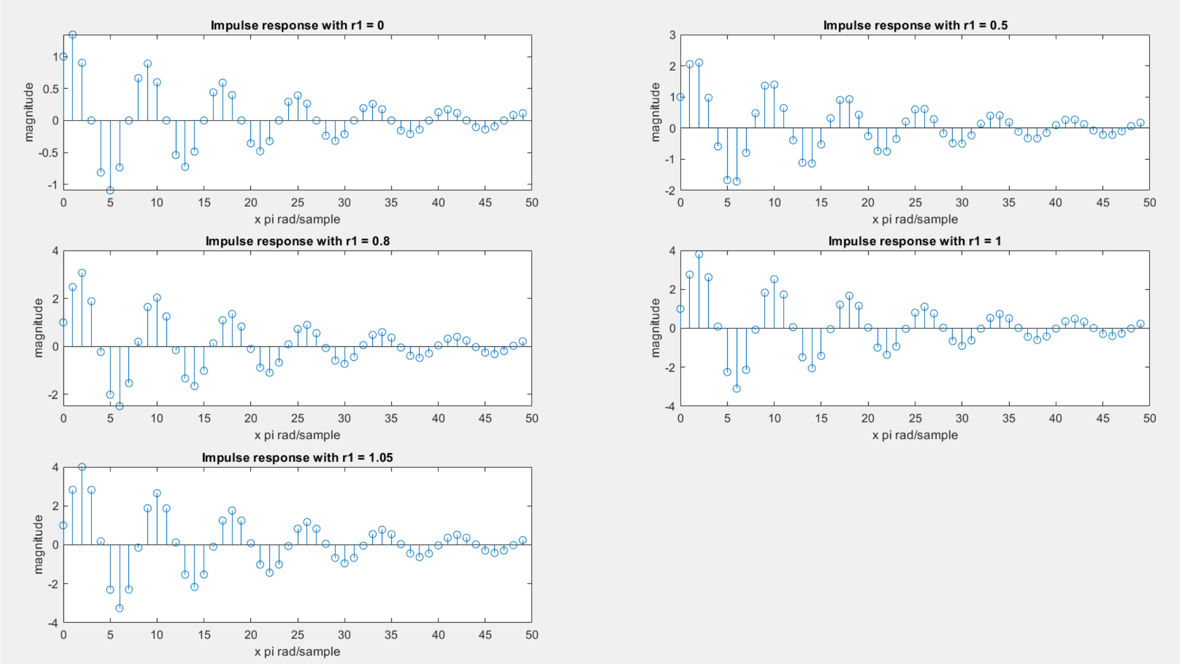
Q1.b) Frequency & Phase Response of H: varying R_1



Observations:

When we change the value of **R1** we are changing the distance of the zeros are from the unit circle but keeping the angle it makes with the real axis constant. The effect that has on the frequency response can be seen in the graphs above. At R1 = 0 there are two spikes arount 0.3 and 1.7 rad/sample and a steady decay to zero. As R1 increases the steady decay slows down and a small bump starts to apear between 0.7-1.3 rad/sample circled in orange.

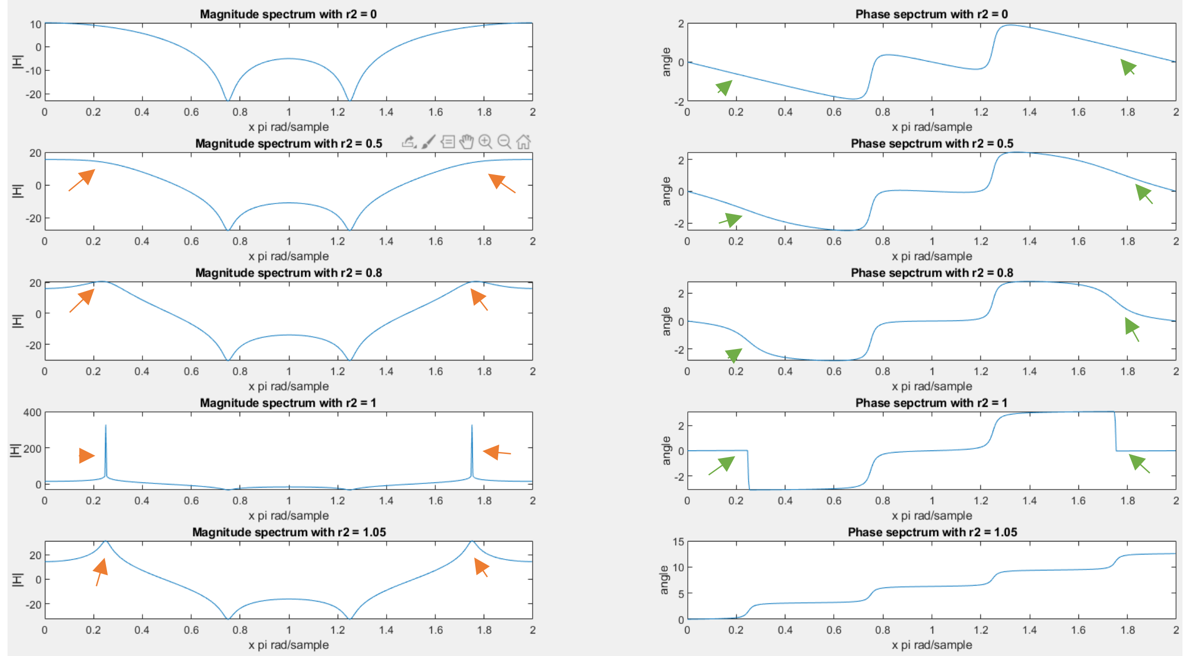
Impulse Response of H: varying R_1



Observations:

Only observation I can make is that the amplitude of the impulse response changes ever so slightly due to the phase shift of the signal.

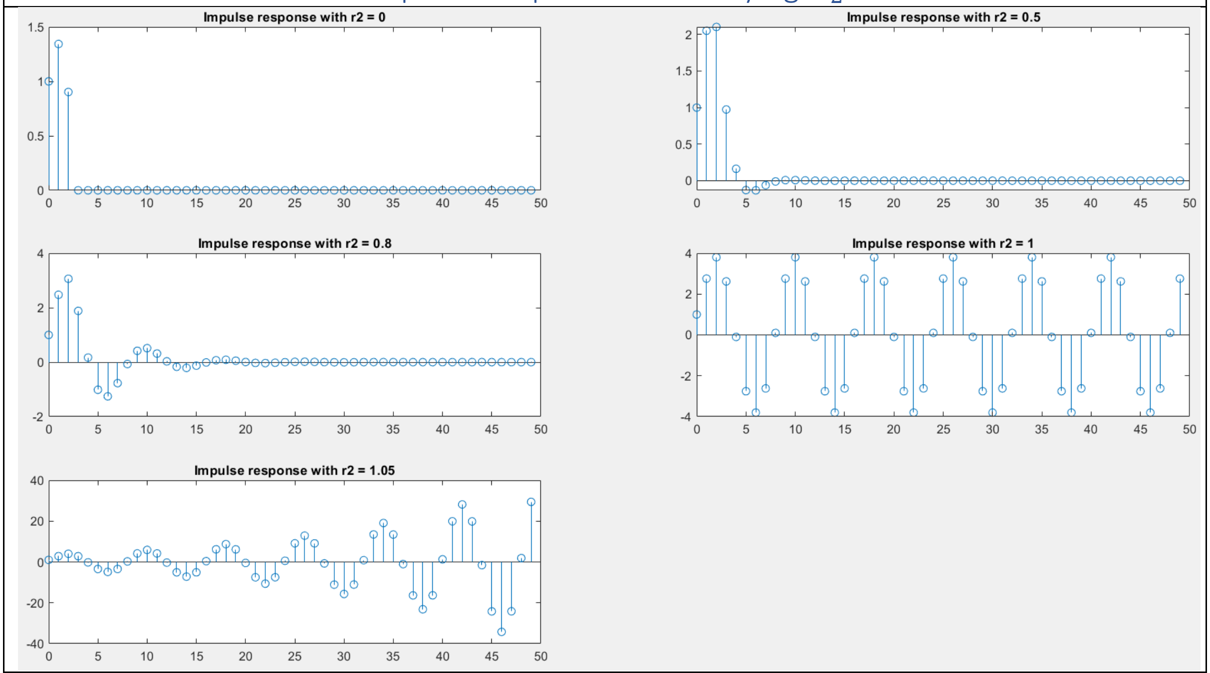
Frequency & Phase Response of H: varying R_2



Observation:

When we change the value of R2 we are changing the distance the poles are from the unit circle but keeping the angle it makes with the real axis constant. The effect that has on the frequency response can be seen in the graphs above. At R2 = 0 there is a fat lobes centered arount 0.3 and 1.7 rad/sample that decays to zero with a small bump between 0.7-1.3 rad/sample. As R2 increases the decay of the fat lobe becomes faster and a sharper spike starts to appear indicated by the orange arrow. As soon as the poles leave the unit circle the sharp spikes changes to small spikes on the fat lobes. The slope of the phase graph near the edges becomes steaper as indicated by the green arrows.

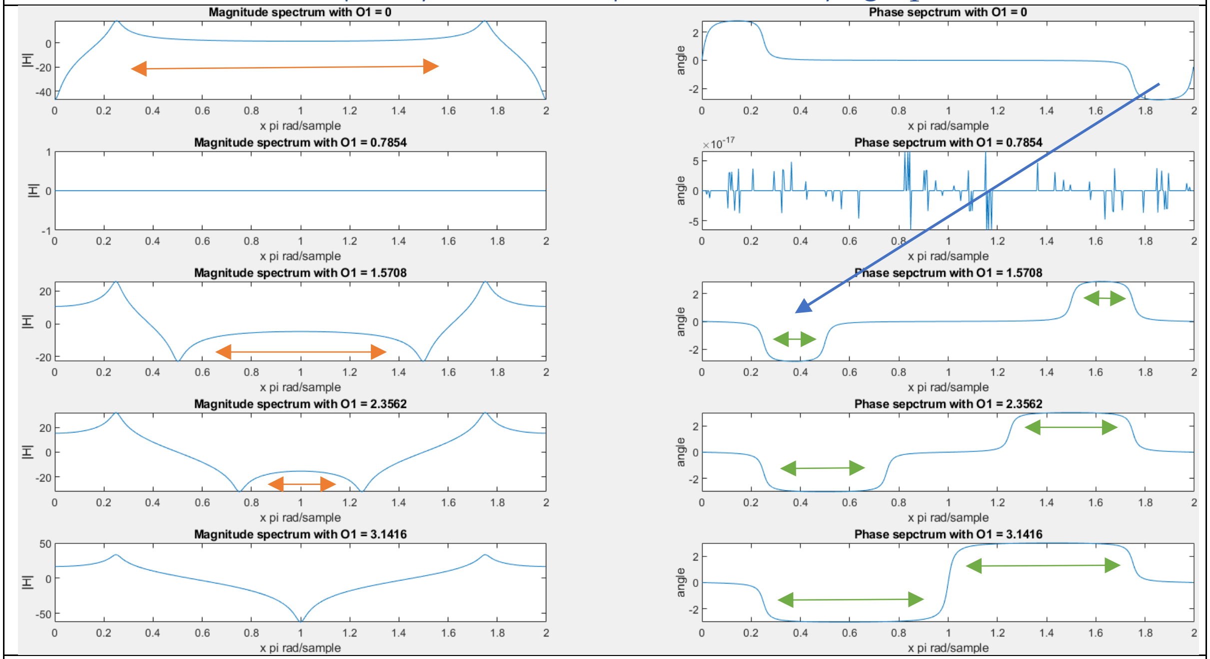
Impulse Response of H: varying R_2



Observation:

When the poles leave the unit circle the system becomes unstable. This can be seen in the graph where R2=1.05. When the poles are within the unit circle the system is stable on the impulse response dies out quickly. When R2=1 the system is marginally unstable for its impulse response won't die out, but it will not increase either. There is a minor phase shift between the graphs.

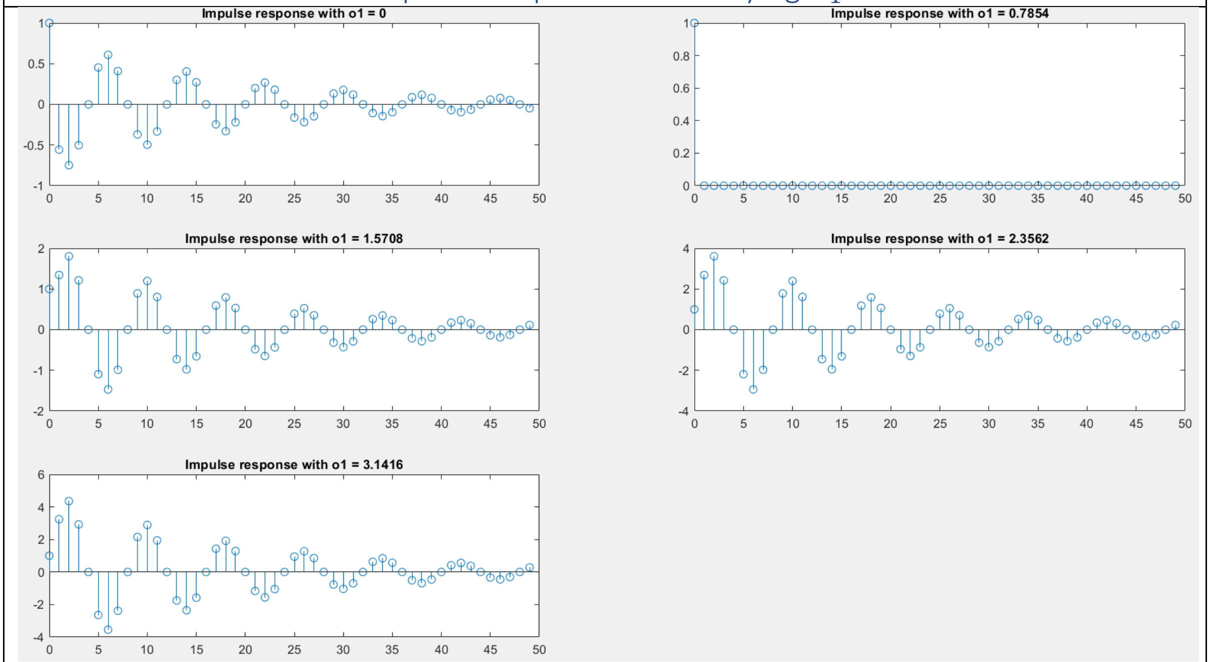
Frequency & Phase Response of H: varying θ_1



Observation:

When we change the value of θ_1 we are changing the angle the zeros makes with the real axis but keeping the distance to the origin constant. With $\theta_1 = 0$ the bump in the middle is very wide and morphs into the spikes on either side of it. As θ_1 increases the middle lobe becomes narrower and the decays from the spike peaks becomes faster. Also there is a 2π phase shift between θ_1= 0 and θ_1 = 0.5π.

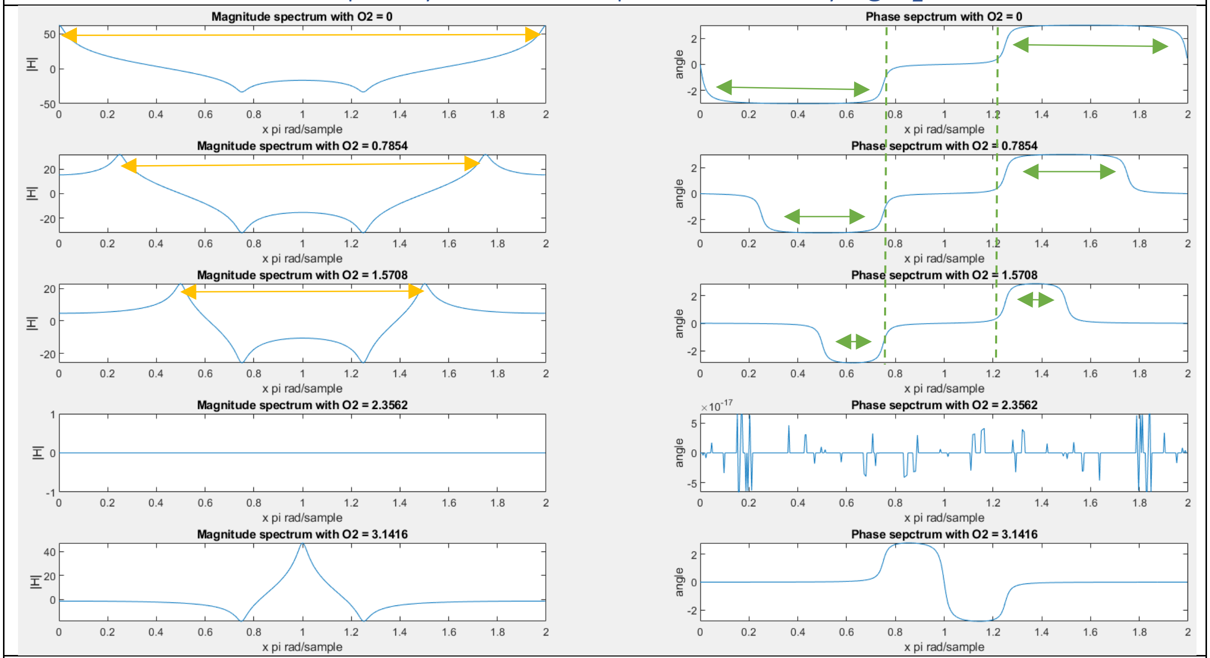
Impulse Response of H: varying θ_1



Observation:

The amplitude of the impulse response changes slightly as θ_1 increases due to the slight phase shift as theta changes. There is a 2π phase shift as θ_1 increaases.

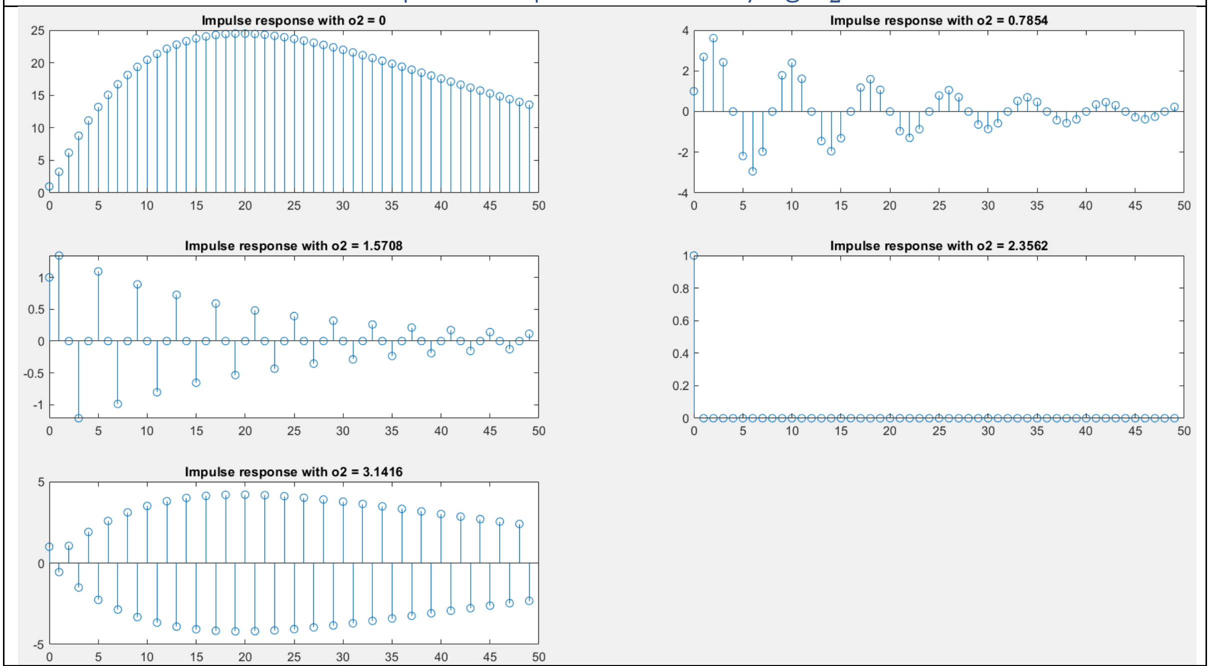
Frequency & Phase Response of H: varying θ_2



Observation:

When we change the value of $\theta_2$we are changing the angle the poles makes with the real axis but keeping the distance to the origin constant. With $\theta_2 = 0$ the tips of the spikes sit at the end of beginning and end of the graph indicated by the yellow arrow. As θ_2 increases the tips of the spikes move closer and closer to the middle until they merge. The green arrows show how the phase graph changes due to θ_2

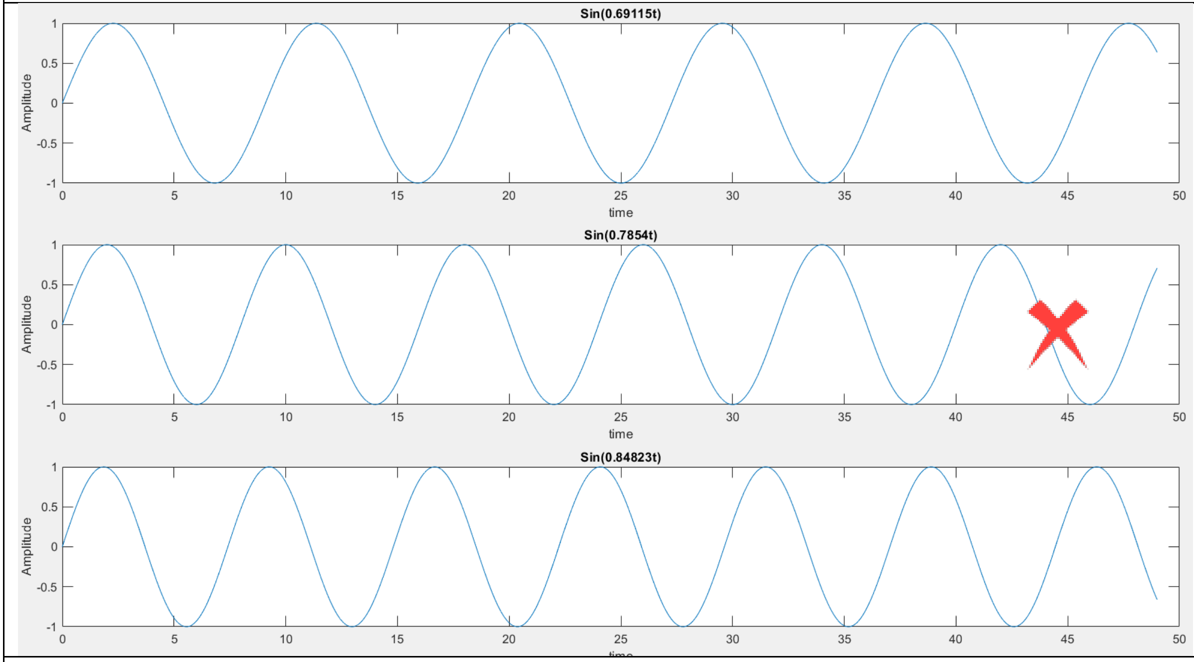
Impulse Response of H: varying θ_2



Observation:

It is also clear from the graph above the effect θ_2 has on the phase. All the impulse responses are stable for it does not shoot of to infinity. Except when θ_2 = π the signal keeps oscillating and is not stable.

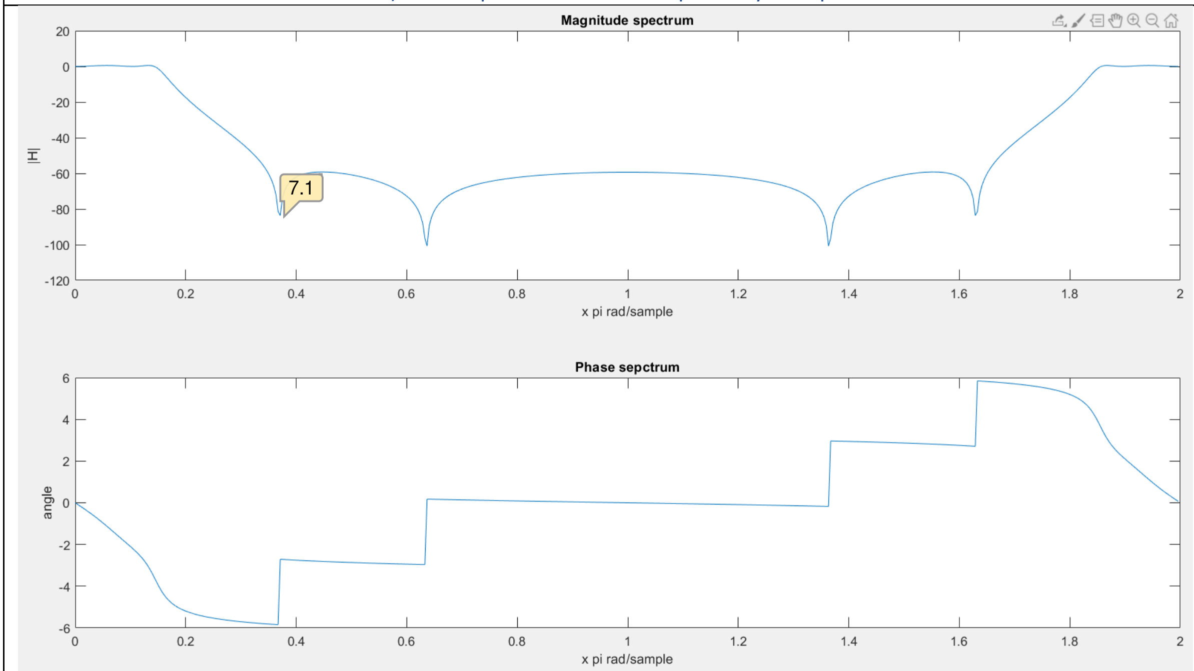
Sinusoidal response of H: varying ω

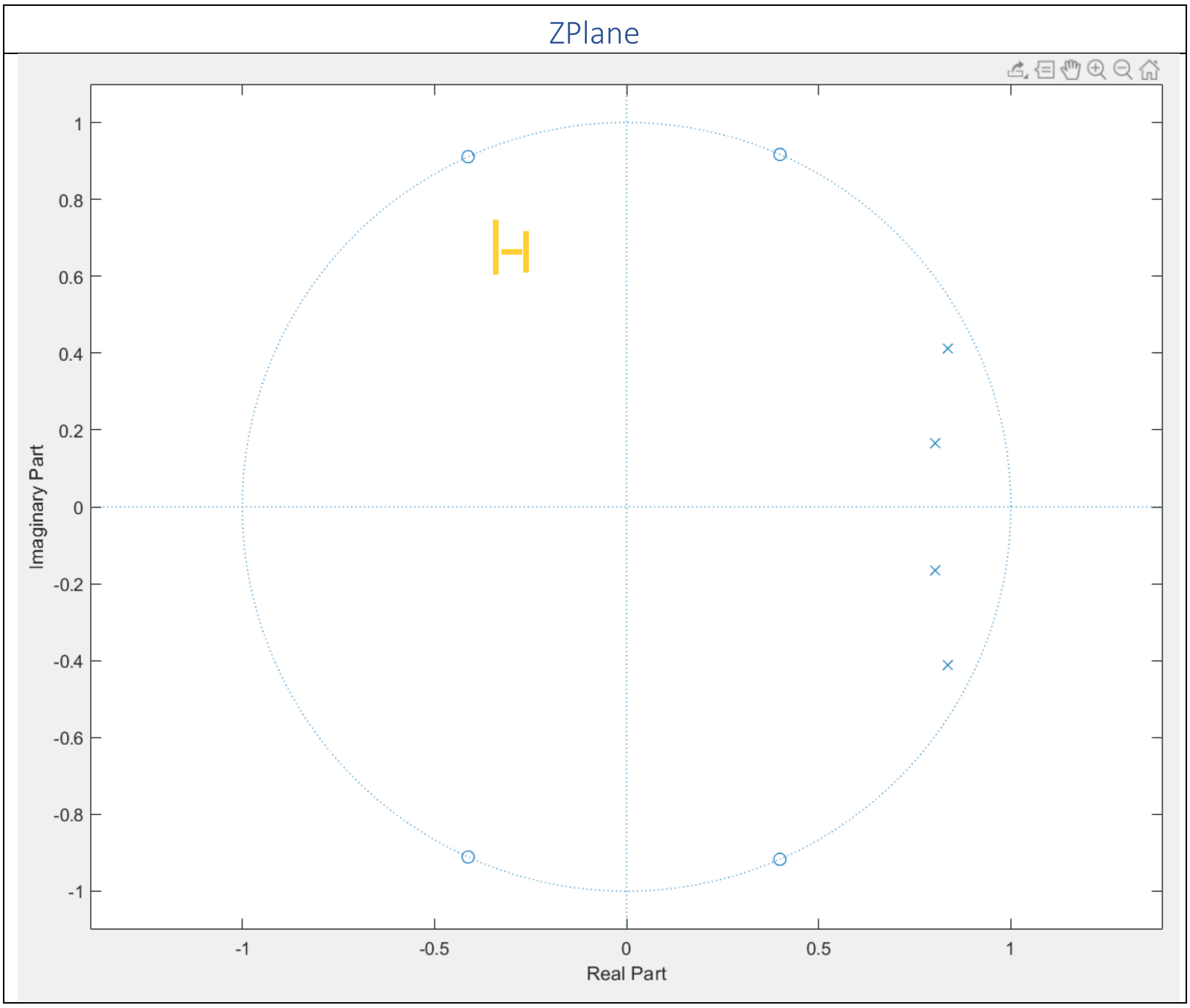


Observation:

No noticeable observation.

Q2.a.1) Bandpass Filter frequency response.




ZPlane
1
0.8
0.6
0.4
0.2
0
-0.2
-0.4
-0.6
-0.8
-1
Imaginary Part
-1
-0.5
0
0.5
1
Real Part

Index of comments

5.1 Amplitude changes have to do with the distance between the poles and zeros

6.1 Try to explain what the changes in the graph represent, rather than the changes themselves. In this case, the frequencies amplified by the transfer function get higher.

6.2 theta2 = pi will decay eventually if you plot more samples. It is oscillating at the highest frequency that can be represented by the sample rate.

7.1 The plot is correct but its not a bandpass filter, its a low pass filter. Remember the frequency response is mirrored around multiples of the sampling frequency. The plot shows the filter is letting through only low frequancies