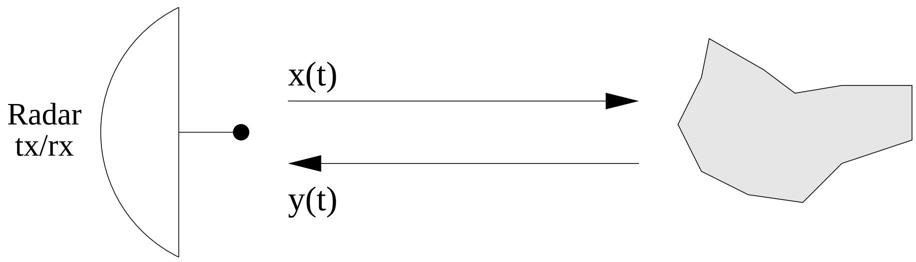
Aim: Investigate the use of correlation in the detection of a signal corrupted by noise.

Task: Do the following assignment, using Matlab to obtain numerical results as required. Document your work in your practical book, indicating your methodology, theoretical results, numerical results and discussions. Graphs should have labelled axes and correct units indicated. If you struggle with Matlab, remember the `help` and `lookfor` functions.

Determining a signal delay by means of correlation

Consider a radar system used to determine the distance between its transmitting/receiving antenna and a particular object.



The radar transmits a signal $x(t)$, which is reflected by the object and received by the radar as $y(t)$. Now assume that this reflected signal $y(t)$ is a delayed version of the transmitted signal $x(t)$ with additive noise $w(t)$, i.e.

$$y(t) = a \cdot x(t - t_D) + w(t).$$

The distance between the radar and the object may be deduced from the time delay t_D. To proceed, let us assume that $x(t)$ and $y(t)$ have been sampled with a sampling period T, without any aliasing. This results in discrete-time signals $x[n] = x(nT)$ and

$$y[n] = y(nT) = a \cdot x(nT - DT) + w(nT) = a \cdot x[n - D] + w[n],$$

where it has been assumed that $t_D = DT$ with D an integer.

Please turn over...

Assignment:

1. Derive an algebraic expression for $r_{yx}[i]$ in terms of $r_{xx}[i]$ and $r_{wx}[i]$. Do this by hand, i.e. pen and paper.

2. Using this result, explain how you could determine D from an estimate of $r_{yx}[i]$. Under what condition(s) will this scheme be effective?

3. Let $x[n]$ be the 13-point **Barker sequence**:

$$x[n] = \{+1, +1, +1, +1, +1, -1, -1, +1, +1, -1, +1, -1, +1\},$$

   and let $w[n]$ be zero-mean Gaussian white noise with a variance $\sigma_w^2 = 0.01$ (this can be generated using Matlab's `randn` function with appropriate scaling).

   (a) Why is this an appropriate choice of $x[n]$?
   (Hint: check out $r_{xx}[i]$ using Matlab's `xcorr` function.)

   (b) Take $a = 0.9$ and $D = 20$, and now calculate and plot $y[n]$ for $0 \leq n \leq 199$.

   (c) From your result in (b), use Matlab's `xcorr` function to determine $r_{yx}[i]$. Plot[1] the result for $-59 \leq i \leq 59$ and identify the delay D.

   (d) Repeat (b) and (c) for $\sigma_w^2 = 0.1$ and for $\sigma_w^2 = 1.0$. What is the significance of σ_w and how does it affect the identification of D?

4. Next, let $x[n]$ instead be the following 13-point sequence:

$$x[n] = \{+1, +1, +1, +1, 0, 0, 0, 0, 0, +1, +1, +1, +1\}.$$

   (a) Is this a suitable choice for $x[n]$? Why (not)?

   (b) Repeat steps (b) and (c) of question 3 with $w[n]$ taken as zero-mean Gaussian white noise with a variance $\sigma_w^2 = 0.1$. Can you identify the delay D?

5. Now let $x[n]$ itself consist of 200 samples of Gaussian white noise with variance $\sigma_x^2 = 1.0$.

   (a) Is this an appropriate choice for $x[n]$? Why (not)?

   (b) Repeat steps (b) and (c) of question 3 with $w[n]$ taken as zero-mean Gaussian white noise with a variance $\sigma_w^2 = 1.0$. Can you identify the delay D from the plot of $r_{yx}[i]$?

6. Finally, let $x[n]$ consist of 13 samples of Gaussian white noise with variance $\sigma_x^2 = 1.0$. Repeat steps (b) and (c) of question 3 with $w[n]$ taken as zero-mean white noise with a variance $\sigma_w^2 = 1.0$. How do you results compare with those you obtained from the previous question? Explain your observations. Comment on the advantages and disadvantages of using the Barker sequence over the noise sequences. Under which circumstances would one or the other be a better choice?

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[1]Note that the result of the `xcorr` function includes both negative and positive lags. To obtain the correct horizontal axis labelling do `[c,i]=xcorr(y,x)` followed by `stem(i,c)`. See `help xcorr` for further details.