Aim: Understand the use of the DFT in discrete-time signal analysis.

Task: Do the following assignment using Matlab. Document the task in your practical book, indicating your methodology, theoretical results, numerical results and discussions. Graphs should have labelled axes with the correct units indicated. If you struggle with Matlab, remember the `help` and `lookfor` functions. *Hint:* plot "continuous-time" signals and spectra with Matlab's `plot` function, en discrete-time signals and spectra with `stem`.

Consider the following continuous-time signal $x(t)$:

$$x(t) = \cos(1800\pi t) + 0.15\cos(1600\pi t).$$

1. Sampling the continuous-time signal.
   (a) What frequency components are present in $x(t)$? Sketch (by hand) the magnitude spectrum $|X(f)|$ of $x(t)$, where the frequency axis is labelled in cycles/second (Hz).
   (b) Plot $x(t)$ for $0 \leq t < 0.02$, where t is time in seconds, so that you can see the shape of the continuous-time signal.
   (c) The discrete-time signal $x[n]$ is obtained by sampling $x(t)$ at a sampling frequency of $f_s = 2$ kHz. Obtain the 40 samples of $x[n]$ for $n = 0 \ldots 39$ and plot these on a graph where the horizontal axis shows discrete time n (samples). Does aliasing occur during sampling?
   (d) Again plot $x(t)$ for $0 \leq t < 0.02$, but now also superimpose the 40 samples obtained in the previous question on this graph so that you are able to see the sampling instants clearly. *Hint:* Follow the same procedure as in Task 4 of Practical 1.

2. Obtain 40 samples of $x[n]$ for $n = 0 \ldots 39$.
   (a) Plot these samples (same as in Question 1.(c) above).
   (b) Use the Matlab function `fft` to calculate the DFT of $x[n]$. Plot the amplitude (magnitude) spectrum $|X[k]|$, using the Matlab function `abs`. Label your axes correctly!
   (c) Estimate the frequencies present in $x[n]$ directly from this amplitude spectrum. State the frequencies in cycles/sample, and then determine the corresponding frequencies in Hz using the (known) sampling frequency.
   (d) Zero-pad $x[n]$ by appending 160 zeros to the 40 samples you have taken. Now determine and plot the amplitude spectrum using the DFT. Explain what you see.

3. Obtain 50 samples of $x[n]$ for $n = 0 \ldots 49$. That is, keep the sampling rate $f_s = 2$ kHz but sample the signal up to $t = 0.025$ s.
   (a) Plot these samples.
   (b) Determine and plot the amplitude spectrum using the DFT.
   (c) Estimate the frequencies present in $x[n]$ directly from this amplitude spectrum. Why is it more difficult than before?
   (d) Zero-pad $x[n]$ by appending 150 zeros to the 50 samples you have taken. Now determine and plot the amplitude spectrum using the DFT. Are you better able to determine the frequencies present in $x[n]$?

(e) Apply a Hamming window to the 50 samples of $x[n]$ (use Matlab's `hamming` function — careful, it returns a column vector while your signal is probably stored as a row vector). Determine and plot the amplitude spectrum of this windowed signal. Explain the changes that have occurred.

(f) Zero-pad the Hamming-windowed 50 samples by appending 150 zeros. Again, determine and plot the amplitude spectrum using the DFT. Are you better able to determine the frequencies present in $x[n]$? Explain why (not).

4. Now consider the continuous-time function

$$x(t) = \cos(100\pi t).$$

Obtain 40 samples ($n = 0 \ldots 39$) of the discrete-time signal $x[n]$ by sampling $x(t)$ at a sampling frequency of $f_s = 2$ kHz.

(a) Determine the DFT $X[k]$ of these 40 samples, and plot its magnitude.

(b) Zero-pad the DFT by inserting 160 zeros into the **middle** (i.e. between samples 20 and 21) of $X[k]$. In other words, zero-pad the spectrum, and **not** the time signal! Plot the magnitude of this new sequence.

(c) Determine the IDFT of the above zero-padded DFT, using Matlab's `ifft` function. Plot the result. Note that there may be a (very small!) imaginary component after taking the IDFT due to round-off errors. Remove this by means of Matlab's `real` function. Explain what you see.

5. Repeat the previous question but now use 50 samples of $x[n]$ and insert 150 zeros during zero-padding of $X[k]$. Experiment with inserting the zeros into the DFT spectrum between samples $X[24]$ and $X[25]$, between samples $X[25]$ and $X[26]$, and half the zeros before sample $X[25]$ and half after it. Explain your observations.