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Techniques for Suppressing Grating Lobes

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Abstract

English

Two methods of beam steering are possible: namely mechanical steering and phased arrays. Mechanical steering physically rotates the antenna which is a slow process. Phased arrays supply a phase shift to each antenna's signal to digitally steer the beam to the target direction. This allows for instantaneous steering abilities but introduces grating lobes when steered beyond a critical angle. This work investigates grating lobes and how they can be suppressed by altering digital parameters and the antenna spacing. A Genetic Algorithm is designed and deployed to optimize the spacing of a linear antenna array (LAA). The performance of the LAA is then compared to that of the uniform LAA spaced half a wavelength apart. It is shown that the obtained solution suppresses the grating lobes that appear in the uniform LAA and has lower sidelobe levels but a wider half-power beamwidth.

Afrikaans

Twee metodes bestaan om die stralingspatroon van 'n antenna-skikking te stuur: naamlik meganiese stuur en gefaseerde stuur. Meganiese stuur draai die antenna-skikking in die teiken rigting. Dit is 'n stadige proses. Gefaseerde stuur verskaf 'n faseverskuiwing in die sein van elk van die antenna om die stralingspatroon digitaal na die teikenrigting te stuur. Dit is dus moontlik om die posisie van die stralingspatroon onmiddellike te verander. Die nadeel van die metode is dat sekondêre lobbe ontstaan wanneer die stralingspatroon verder as 'n kritieke hoek gestuur word. Hierdie verslag ondersoek die voorwaardes vir die sekondêre lobbe en hoe dit onderdruk kan word deur net die digitale waardes en die posisie van die antennas te verander. 'n Algoritme wat die optimale spasiëring van 'n antenna-skikking vind is ontwerp. Dit is bewys dat die resulterende antenna-skikking geen sekondêre lobbe het nie. Die sylobvlakke is ook verlaag, maar die halfkrag-bandwydte is verbreër.

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Nomenclature

Variables and functions

I	Current excitation of antenna element
k	Wave number
c_0	Speed of light in a vacuum
λ	Wavelength of antenna signal
f	Operating frequency of antenna
$\Delta\gamma$	Phase shift
θ_T	Target Direction
α_m	Phase shift for maximum beam steering

Acronyms and abbreviations

LAA	Linear Antenna Array
ULA	Uniform Linear Array
ULAA	Uniform Linear Antenna Array
GA	Genetic Algorithm
SLL	Side-Lobe Level
GLL	Grating-Lobe Level
HPBW	Half Power Beamwidth
FT	Fourier Transform
GL	Grating Lobes
AF	Array Factor
DFT	Discrete Fourier Transform

Chapter 1

Introduction

Antennas are designed to radiate Electromagnetic energy. It is used for long-distance communication and/or transmitting of data [2]. In the simplest form, an antenna can be modelled as a point source radiating in all directions at a particular frequency. These point source antennas are known as omnidirectional antennas Figure 1.1. The antennas can be arranged in a linear array (straight line) or a planar array (surface) with the distance between the antennas a fraction of the signal wavelength.

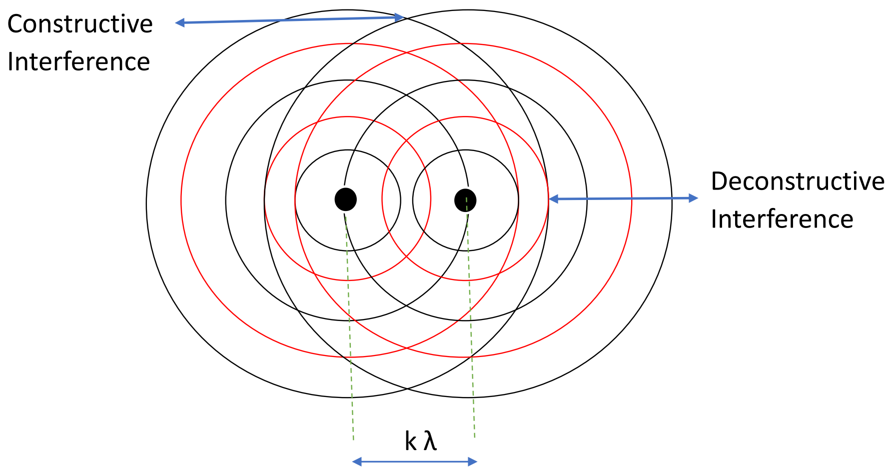


Figure 1.1: Superposition of point source antenna radiation pattern.

Arranging the antennas in an array improves the overall power gain of the net radiation pattern which improves its directivity. This is a result of the principle of superposition because each antenna contributes to the overall radiation pattern Figure 4.3. It means that the radiation energy is more concentrated in the target direction due to constructive interference. This result makes the transmission of data over a larger distance possible. To steer the beam of the antennas in the desired direction the antennas can be moved mechanically. The National Radiation Astronomy Observatory antenna array in Figure 1.2a is 25 meters in diameter and weighs 230 tons [3], making mechanical steering difficult and slow. South Africa has a large radio telescope array called MeerKat Figure 1.2b with a height of 19.5 meters and weighs 40 tons. Due to the sheer size of the antennas the moment of inertia is large which means more energy is needed to control the rotation of the antenna. This limits potential applications that require fast rotation of an antenna such as radar to track missiles.

(a) National Radio Astronomy Observatory (NRAO) 80.47 km west of Socorro, New Mexico [3]

(b) South Africa's radio telescope array "MeerKAT" located in the Northern Cape. [4]

(c) Eglin air force base phased array radar for tracking space objects, [5]

A solution to the problem is to steer the radiating beam of the antenna array digitally through altering digital parameters such as the phase shift and amplitude distribution of the signal entering each antenna in the antenna array. This is known as a phased array. This allows the radiating beam of the antenna array to be steered much faster, almost instantaneously, due to not having any moving parts. In Colorado USA, an antenna phased array is used to track multiple space missiles at the same time by steering the beam over the entire scanning range very quickly, see Figure 1.2c.

This method introduces grating lobes (GL) when the beam is steered beyond a critical angle (CA). This means that an antenna array that uses non-mechanical beam steering has a limited range that the main beam can be steered to before GL's appear. These lobes are undesirable because it results in power loss decreasing the efficiency of the antenna array.

1.1. Problem Statement

In this project, an investigation is made on how the position, phase shift and amplitude distributions of each antenna element affect grating lobes and how these parameters should be optimized to suppress the grating lobes.

1.2. Objective

The aim is to find a fixed element spacing and the resulting phase shift and amplitude distribution that allow for the suppression of grating lobes. This would result in a wider steering range and more efficient antenna. This must be achieved while keeping the half-power beamwidth (HPBW) as narrow as possible and maintaining a superior sidelobe level (SLL) and grating lobe level (GLL) compared to a uniformly spaced linear antenna array of half a wavelength.

1.3. Scope

This report will investigate what grating lobes are and how the parameters: element spacing, phase shift and amplitude distribution affect grating lobes. A Genetic Algorithm (GA) is implemented to optimise the parameters of the antenna array to suppress grating lobes. After a linear antenna array (LAA) spacing is selected the antennas will remain fixed and only the phase and amplitude of the signal entering the antenna can be altered to steer the beam. Different element spacings were obtained and tested for different steering angles to determine which out performs the uniform antenna array of 0.5λ.

This report did not investigate any antenna arrangements that are not linear such as planer antenna arrays. It will not investigate the feed system needed to implement the phase shift or amplitude distribution for a physical antenna array. Nor does the report describe how to build an antenna or antenna array. This report will only cover the theory to design an antenna array.

The report is set out as follows: Chapter 2 covers interesting related research that has been done as an attempt to solve the problem statement. Chapter 3 discusses antenna array fundamentals and how the Genetic Algorithm is designed to solve the problem. In Chapter 4 the findings are documented and compared to the uniformly spaced LAA spaced 0.5λ apart. The conclusion is drawn in the Chapter 5 together with recommendations for future work.

Chapter 2

Literature Review

This chapter presents possible solutions to suppress grating lobes as given in the literature. It will focus on the influence non-uniform element spacing, phase shift and non-uniform amplitude distribution have on grating lobes. The focus will be on finding the best set of parameters for the array to increase steering direction to avoid or suppress grating lobes if they appear.

Cheng [2] showed that the configuration, element spacing, amplitude distribution and phase shift of the antenna elements affect the radiation pattern. The radiation pattern is also known as the array factor (AF), equation (3.2).

In 1960 King et al. [6] presented a computational technique to design a non-uniformly spaced LAA. The designer can design the spacing for a desired SLL. However, only a certain number of amplitude distributions could then be implemented. Unz. [7] was the first to mention the possibility of a non-uniform distribution of antenna elements when Unz presented a matrix relationship between the radiating pattern and antenna elements. The idea was however just stated in his short paper and never fully realised.

Harrington [8] was inspired by King et al. [6] and explored the possibility of reducing SLL with non-uniform element spacing. King et al. [6] found certain non-uniform spacing that produced low SLL and outperform uniformly spaced elements. Harrington [8] managed to suppress the SLL without increasing the HPBW of the main beam but stated: ”It appears unlikely that a simple optimization procedure can be devised for non-uniform arrays because of the nonlinear character of the basic equations with respect to element position.” - Harrington [8] But suggested that more work should be done on the topic.

Ridwan et al. [9] deployed a GA to design the optimal amplitudes distribution for a uniform LAA and compared it to the Dolph-Chebyshev and Binomial amplitude distributions. [9]. Ridwan et al. produced amplitude distribution for a given set of antennas that outperformed a uniform LAA but did not explore the use of a GA for the spacing of the elements or beam steering.
KhalilpourJafar et al. [10] in a recent publication publicized a novel algorithm to suppress

grating lobes in a phased array when the separation between elements was around one wavelength apart. KhalilpourJafar et al. [10] showed significant SLL and GLL reduction for an eleven element phased array with up to 180° scan angle using randomly spaced elements that creates nulls to cancel out grating lobes.
The shortcoming from the literature is that although GA is used to optimize the amplitude distribution for a LAA it was not fully exploited for optimizing the spacing of a LAA. Therefore, using the GA for beam steering and optimizing the amplitude distribution and LAA element spacing leaves room for a lot of novel work to be done.

2.1. Grating Lobes

Grating lobes are unwanted (secondary main) lobes that appear when you steer the main beam beyond a critical angle that is determined by the array formation. These grating lobes are unwanted because they are wasted energy and cause interference from radiating stations in non-intended directions. [11]

2.2. Array Geometry

From the study of other work done on the topic of the suppression of grating lobes it is clear that the spacing of the antenna elements is most important. The uniform array distribution and the non-uniform array distribution that is symmetrical around the mean will be considered.

2.2.1. Uniformly Spaced Linear Antenna Array

From the work done by Matsepe [12] it was shows that uniform LAA will have no grating lobes if the spacing between elements are uniform and less or equal to half a wavelength. However, beam steering is only possible to a certain angle that is know as the critical angle (CA). Steering beyond this CA the SLL become greater than $-3\ dB$ with respect to the peak of the main beam and can thus be considered a grating lobe.

The antenna array can be designed for a desired angle where the element spacing can be greater than 0.5λ with no grating lobes, however the steering range is limited. Decreasing the distance between LAA elements result in suppressing grating lobes, for the distance between grating lobes move further apart. But it is noted that as the distance between antenna elements become smaller the HPBW becomes wider. [12] Grating lobes are not affected using non-uniform Amplitude distributions such as the Binomial, Dolph-Chebyshev or Taylor distributions. These amplitude distributions suppress side lobes and can increase the HPBW. It is important to note that grating lobes introduce side lobes. [12]

2.2.2. Symmetrical Non-Uniformly Spaced Linear Antenna Array

The report will investigate the performance of an antenna array symmetrically spaced round the center point (mean) of the LAA with spacing ranging from 0λ to 2λ. The idea is that by choosing a particular arrangement in antenna elements the grating lobes can be suppressed. The antenna element spacing will be determined by the GA.

2.3. Amplitude Distribution

Different amplitude distributions were recommended in literature where the Binomial- and Dolph-Chebyshev distributions being the most common. It is mentioned by Matsepe [12] that changing the amplitude distribution affects the SLL but has not effect on the grating lobes. The results in Chapter 4 supports this claim. However, KhalilpourJafar et al. [10] showed that if you reduce the grating lobes to the SLL you can suppress it further with an appropriate amplitude distribution.

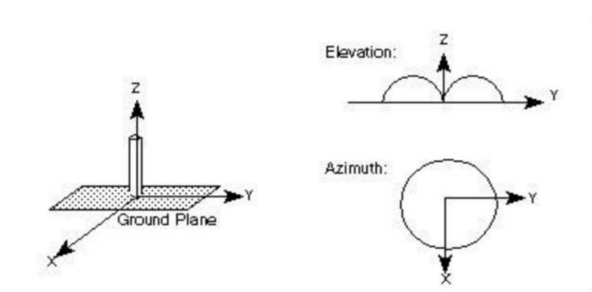
2.4. Genetic Algorithm

A Genetic Algorithm (GA) is an optimization algorithm that uses the concept of evolution and natural selection to solve complex problems [13]. It is a very good approximation tool and does not always yield the best possible solution but can produce very good solutions relatively quickly (solutions who's errors are minimized). It uses crossover and mutation techniques to evolve a population of random solutions to produce the best approximation [13].

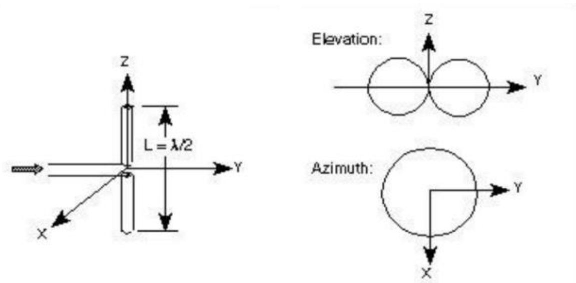
A population of size N of random solutions (also known as genomes) are created. Each solution is passed through a fitness or error function to determine how good the solution is. Once all N solutions have a fitness score the solutions are added to the next generation by either going through crossover or mutation process. The new generation's solutions are then passed through the fitness or error function and the mentioned process is repeated until the maximum number of generations are reached.

2.5. Antenna Types

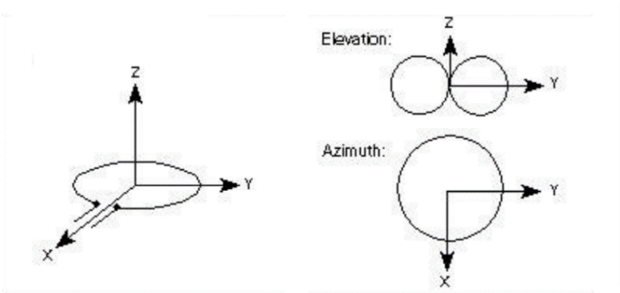
There are many different types of antennas Figure 2.1 for different applications. Omnidirectional antennas such as the monopole Figure 2.1a and dipole Figure 2.1b antennas can be made directional if placed in an array. There are directional antennas such as the Rhombic Figure 2.1d, Yagi Figure 2.1f and Horn Figure 2.1e antennas but with these comes the added complexity of antenna design and limitations during beam steering. Since the most popular antenna used in antenna arrays is that of the half-wave dipole antenna Figure 2.1b this project will use it during the calculations and simulation. This decision is based on the popularity of the half-wave dipole antenna, the simplicity of its design and keeping the focus on the spacing between the antenna and not on the type of antenna used.



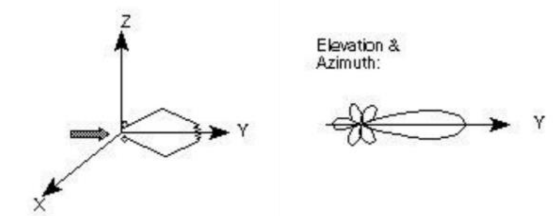
(a) Monopole Antenna



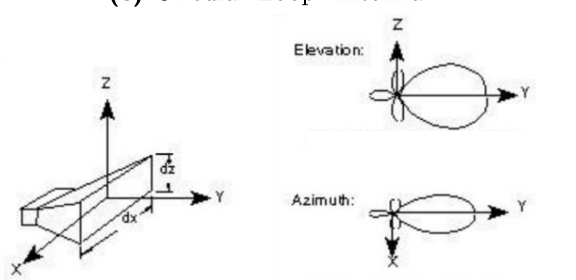
(b) Dipole Antenna



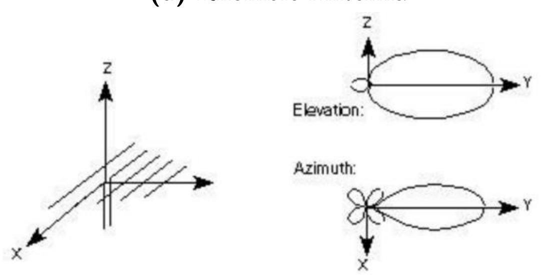
(c) Circular Loop Antenna



(d) Rhombic Antenna



(e) Horn Antenna



(f) Yagi Antenna

Figure 2.1: Antenna Types and Antenna Patterns [1]

Chapter 3

Design and Simulation

In this chapter, we will discuss the mathematical representation of the radiation pattern of an LAA also known as the Array Factor (AF) and how there is a relationship between the AF and the Fourier transform (FT). Once we understand the mathematics of the AF for an LAA we will design a GA to optimize the parameters: element spacing, phase shift and amplitude distribution to steer the beam off-boresight and while suppressing grating lobes.

3.1. Array Factor

The generalized AF is equation (3.1)

$$AF(\theta, \phi) = \sum_{m=0}^{N-1} I_m \exp\left(j[k\hat{r} \cdot \mathbf{r}\,'_{\mathbf{m}}]\right) \tag{3.1}$$

Where $\hat{r} = \sin\theta\cos\phi\ \hat{\mathbf{x}} + \sin\theta\sin\phi\ \hat{\mathbf{y}} + \cos\theta\ \hat{\mathbf{z}}$. The LAA is spaced along the z axis resulting in $\mathbf{r}\,'_{\mathbf{m}} = md_m\ \hat{\mathbf{z}}$ yielding $\hat{r} \cdot \mathbf{r}\,'_{\mathbf{m}} = kd_m \cos\theta$

The Array Factor for a N element LAA far field radiation pattern is equation (3.2).

$$\sum_{m=0}^{N-1} I_m \exp\left(j\gamma_m\right) \tag{3.2}$$

$$\gamma_m = kd_m \cos\theta + \alpha_m \tag{3.3}$$

Where k is known as the wavenumber which measures the number of wavelengths in a cycle. Each antenna can be modeled as a point source with amplitude I_m and phase shift α_m positioned a distance d_m away from the first antenna in the array with equation (3.4).

$$i_m = I_m \exp\left(jkd_m \cos\theta\right) \tag{3.4}$$

To achieve the maximum gain in the boresight direction γ_m should equal to zero. This is achieved by equation (3.5).

$$\alpha_m = -kd_m \cos\theta_T \tag{3.5}$$

All the antennas operate at the same operating frequency and to keep the calculations simple an operating frequency of 1 GHz was selected to match the speed of light at 300 m/s so that the parameters can cancel out in equation (3.6).

$$k = \frac{2\pi f}{c} = \frac{2\pi}{\lambda} \tag{3.6}$$

The cosine function in equation (3.5) with amplitude equal to 1 will always be between the bands -1 and 1 as in equation (3.7).

$$|\cos\theta| \leq 1 \tag{3.7}$$

Solving the angles of equation (3.7) yields $[0 \leq \theta \leq \pi]$ or $[-\pi \leq \theta \leq 0]$. Only the region $[0 \leq \theta \leq \pi]$ will be considered since the two domains are symmetrical. This domain is known as the visible region.

Combining equation (3.2), (3.5) and (3.6) equation (3.8) can be derived.

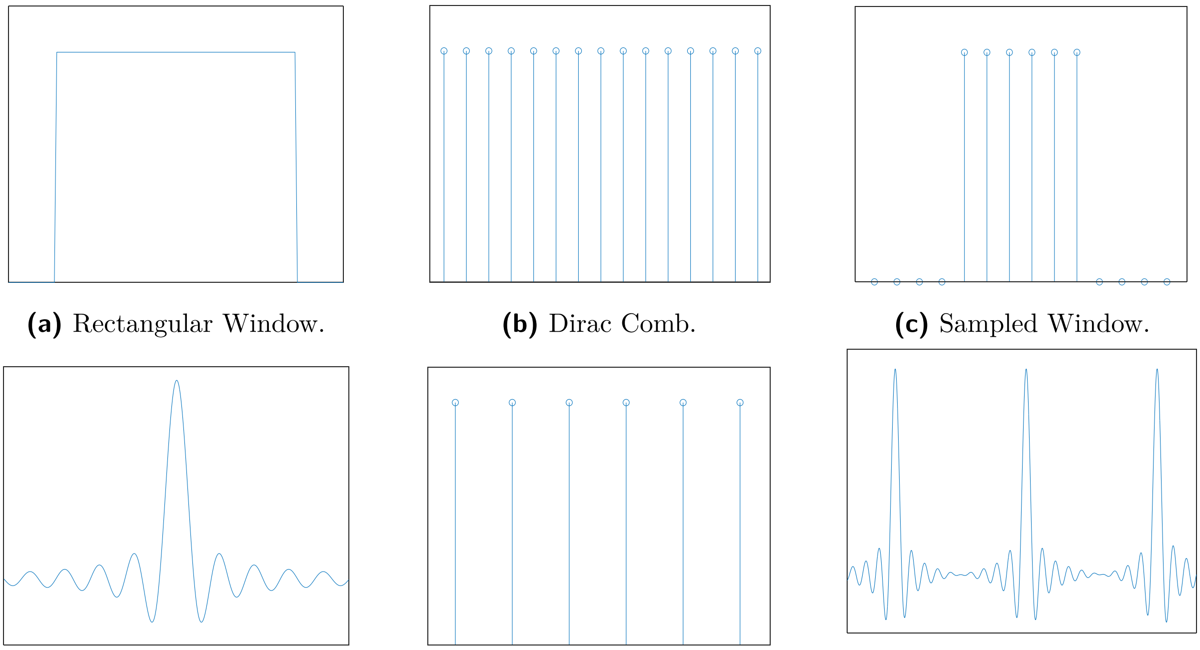
$$\cos\theta - \cos\theta_0 = q\frac{\lambda}{d_{avg}} \tag{3.8}$$

This is an important equation because it shows that the AF equation (3.2) is periodic. There will be many main lobes but only $q = 1$ is considered the main lobe while every other q value that yield a real solution is considered a grating lobe. The equation can predict the location of the grating lobe by solving for θ for the desired steering direction θ_T. It also shows that the only factor that influences how frequently the function repeats itself is determined by the average spacing distance d_{avg}. In other words it is clear from equation (3.8) that if the spacing d_{avg} is decreased the distance between the grating lobes will increase and if the spacing d_{avg} is increased the distance between the grating lobes will decrease.

3.2. Fourier Transform and Array Factor

The AF equation (3.2) is the sum of N antenna elements shifted by a distance d_m from the first antenna element. This is similar to the equation of the Discrete Fourier Transform (DFT). This means the AF is equal to the Fourier transform of the antenna elements.

As shown in equation (3.4) each antenna element can be modelled as a point source with a frequency shift. Let us assume that each element is uniformly excited and that the spacing between the antennas are uniform. The LAA can then be seen as a sampled rectangular window function as in Figure 3.1c. From our Fourier theory, it is known that a rectangular window function transforms into a Sinc function, Figure 3.1d, when the Fourier transform is applied [14]. The main beam of the sinc function is the main beam of the AF and the ripples of the Sinc function are the side lobes that come with the main beam. However, when the rectangular window function is sampled the result is N impulses equalling that of the N antennas as in Figure 3.1c. The Fourier transform of a sampled rectangular window function is a Sinc function convolved with a Dirac comb function as in Figure 3.1f. There are many Sinc functions but only the ones that fall within the visible region will be of concern. These extra main beams are the undesirable GL that needs to suppressed. See the graphical representation Figure 3.1



(a) Rectangular Window. **(b)** Dirac Comb. **(c)** Sampled Window.

(d) FT: Sinc function. **(e)** FT: Dirac Comb. **(f)** Sinc convolved with Dirac Comb.

Figure 3.1: Visual representation of the Fourier transform of a LAA.

Figure 3.1a shows the rectangular window with a width equal to N. The sample function can be modelled as a Dirac comb function with the period $T = \frac{\lambda}{d}$ multiplied by a rectangular window. From our FT understanding, the AF will be a Sinc function convolved with a Dirac comb function equation (3.9).

$$AF = rect(N) * \sum_{-\infty}^{\infty} \delta(x - md) \tag{3.9}$$

In our case it will be in the visible domain which is between 0° to 180°.

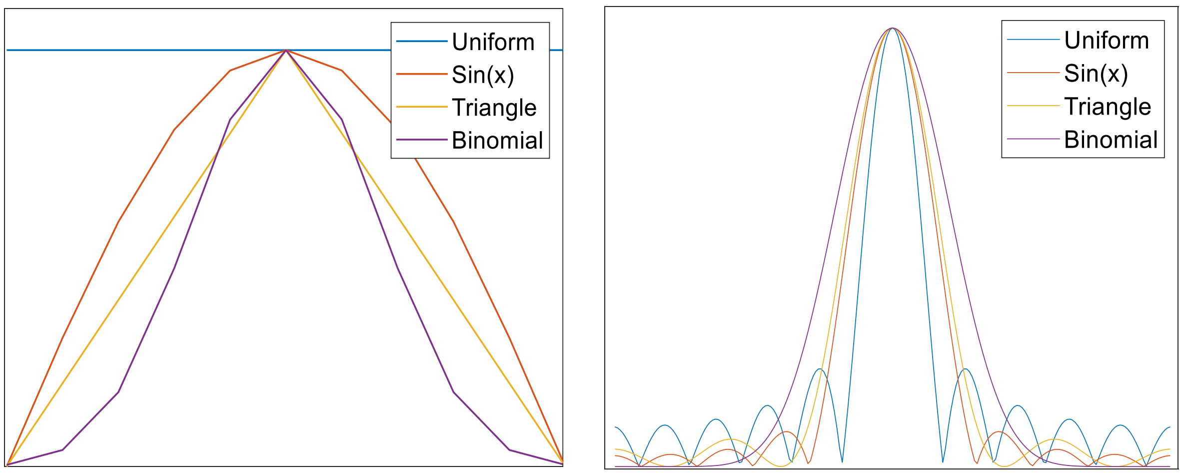
$$AF = \frac{\sin \frac{\pi N}{\lambda}}{\frac{\pi N}{\lambda}} * \sum_{0^\circ}^{180^\circ} \delta(u - \frac{m\lambda}{d}) \tag{3.10}$$

Where $u = \cos\theta$. This periodicity of impulses is the cause of grating lobes. The first sinc beam being the main beam and every consecutive sinc beam being a grating lobe.

The position of the m_{th} beam or grating lobe is represented by $m\frac{\lambda}{d}$. This again shows that decreasing the d will increase the distance between the grating lobes.

The Nyquist criteria states that the sampling frequency must be at least twice as large as the highest frequency you want to sample to be able to fully reconstruct the signal without aliasing [14]. Since the antenna array can be seen as a sampled rectangular window with different amplitude distributions the sampling frequency must be $d \leq 0.5\lambda$ to avoid aliasing [14]. From digital signal sampling, this holds true even if we sample at a random interval i.e. the $f_{avg} \geq 2B$ to avoid aliasing. This indicates that a randomly spaced array would need to have an average spacing of 0.5λ if grating lobes are to be suppressed.

From our FT theory understanding, it is know that changing the window shape, Figure 3.2a has an effect on the width and ripples of the Sinc function (Figure 3.2b) or the side lobes [11] [14].

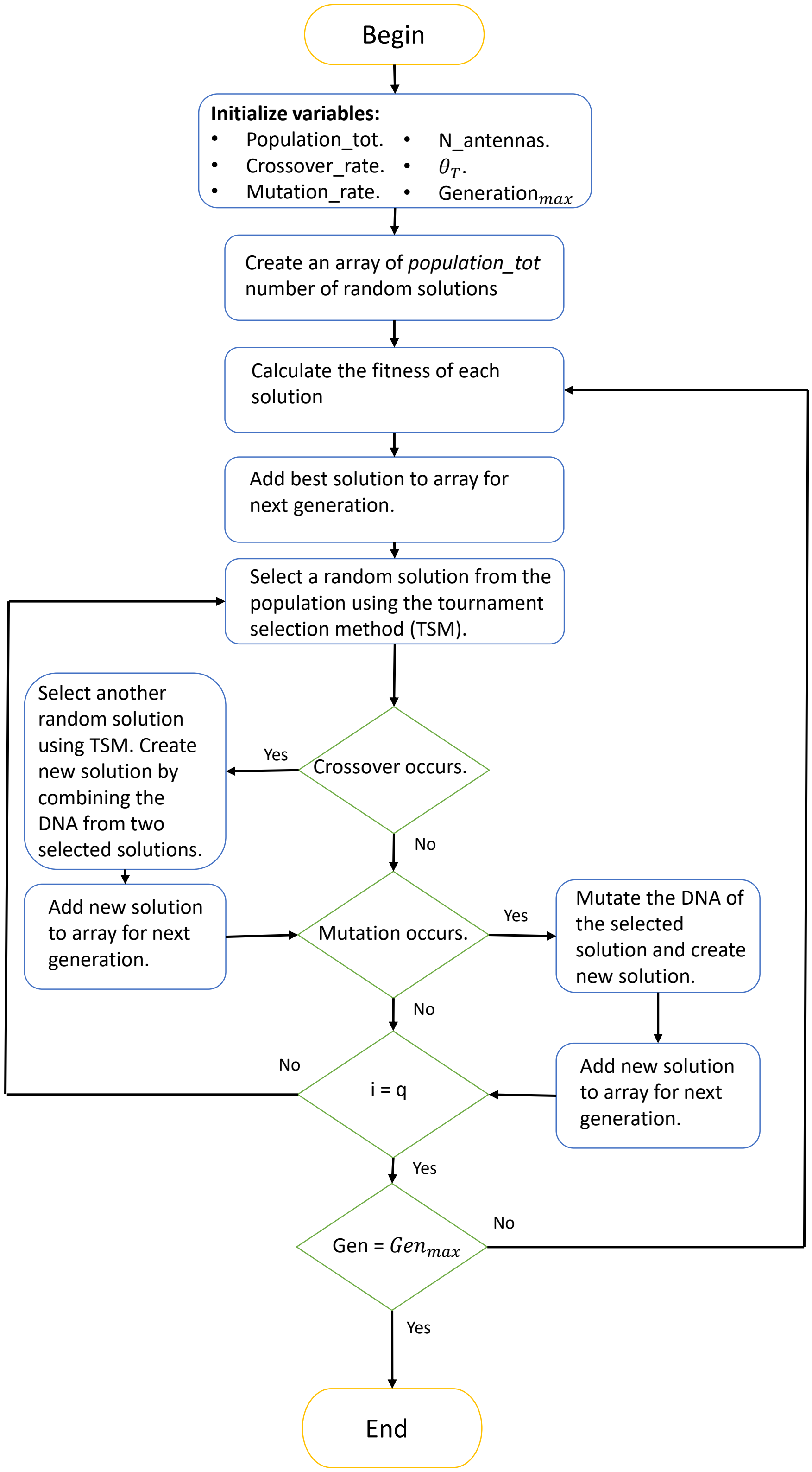


(a) Window shape

(b) Fourier Transform of window.

3.3. Genetic Algorithm

The GA will be structured as follows.



(a) GA flow chart

The fist step of the GA is to initialize the different parameters for the number of antennas, *N*. The number of random solutions the algorithm will work with is known as the population or *population_tot* . Each of the solutions in the population will be given a fitness score. This score is the error of the solution which means that a lower score indicates that the solution is better relative to the other solutions in the population. The GA will aim to minimize the fitness score until it reaches a absolute minimum or equilibrium. The best solution of the current generation is automatically added to the array for the next generation. This is also known as elitism [13]. During each generation the solutions will have a probability to undergo crossover (*crossover_rate*) and mutation (*mutation_rate*) before getting added to the next generation. Crossover allow the best solutions to share DNA (parts of their solution) to produce a potentially better solution for the next generation. Mutation is responsible to add some randomness to the solutions to help the algorithm get out of local minimums and reach the global minimum [13]. This process is repeated until the current generation reaches the maximum number of generation decided by the user.

3.3.1. Fitness score

The most important part of the GA is the fitness function. It describes how good a solution is. In the case of the antenna array is takes the element spacing, phase shift and amplitude distribution of each antenna as input parameters to calculate the AF. It then uses the AF to calculate a fitness score for the LAA.

Equation (3.11) calculates the ratio between the maximum gain at a desired target direction θ_T to the average gain over the entire visible space. This is also known as the directivity equation. It is used to give an indication how well an antenna is performing.

$$D_o = \frac{\pi |AF_{max}|^2}{\int_0^{\pi} AF(\theta)\ d\theta} \tag{3.11}$$

Removing the constant π and taking the inverse of the directivity over the visible space the following fitness function is derived equation (3.12).

$$fitness = \frac{\sum_0^{\pi} AF(\theta)}{AF(\theta_T)^2} \tag{3.12}$$

The GA tries to minimize the fitness function as far as possible but it is not guaranteed that the GA will find the absolute minimum value. The numerator in the fitness function is wasted energy and should ideally be zero. The denominator is the gain to be maximized. Maximizing the denominator or minimizing the numerator/error would minimize the fitness score. These fitness scores are then used during the selection process.

3.3.2. Selection

Two popular selection techniques were considered namely tournament selection and roulette selection [13]. Roulette selection scales the probability of a solution being selected at random by its fitness score relative to all the other solutions fitness scores. This means the better the solution is relative to the population the more likely it is that it will be selected from the population for crossover and/or mutation. This creates a bias towards the better solutions and limits the diversity in the overall population increasing the risk of getting caught in a local minimum. Tournament selection selects four random solution from the population with equal probability and then selects the the two solutions with the best fitness score from the four for crossover and/or mutation. Tournament selection is therefore implemented due to it being less biased towards the best solution.

3.3.3. Genome

Each member in the population is known as a genome and describes a potential solution for the main beam a LAA of size $N = 6$ steered to a desired direction. The solution of the genome is known as its DNA. Each genome has a fitness score, an array of size $N - 1$ describing the spacing between the antenna elements and two arrays of size N describing the phase shift of each antenna element and the amplitude distribution of each antenna element.

3.3.4. Crossover

Crossover is a method that uses known solutions, (two parent solutions) to produce a new solution (new genome) by combining the DNA of the parent solutions [13]. During every selection process, there is a probability that crossover can occur or a crossover rate. When no crossover occurs the selected solution from the population is immediately copied over to the next generation.

When combining the DNA of the two parent arrays as illustrated in Figure 3.4 one must introduce a weighting factor. The weights for α and β can be selected freely but $\alpha + \beta = 1$. This ensures that the new solution that is generated is a valid solution that falls within the expected range. For the inter element spacing the value must fall between 0.1λ and 1λ. The phase shift must fall between $0°$ and $360°$. The amplitude distribution must fall between 0 and 1. If an α and β value of 0.5 is chosen the resultant solution is simply the midpoint of each array index of the two parents.

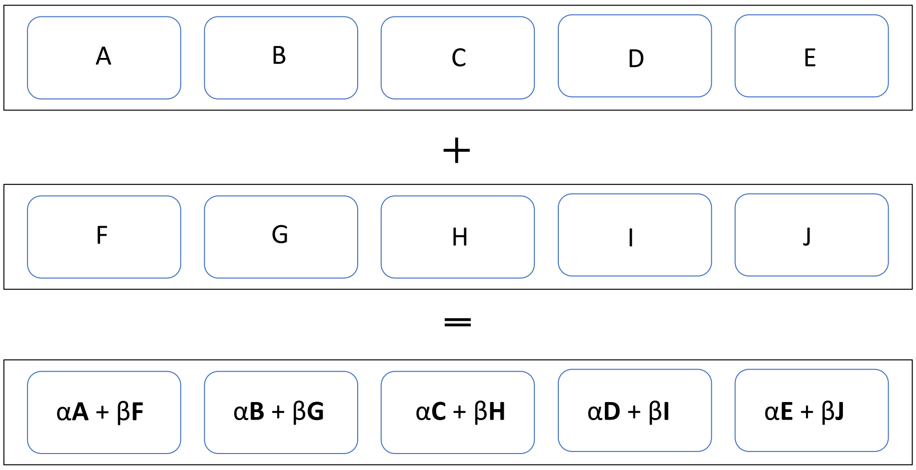


Figure 3.4: Crossover process

3.3.5. Mutation

During every selection process, there is a probability that a mutation can occur or a mutation rate. During this process, the selected solution will undergo a random change to its DNA. Mutation is illustrated in Figure 3.5 using a binary array of length $N = 5$. The fourth index is randomly selected and the value is changed from a 1 to a 0. The reason for mutation is to help the algorithm not get caught on a local minimum/maximum solution. This is achieved by introducing some randomness to the solution [13]. The new solution is then added to the next generation.

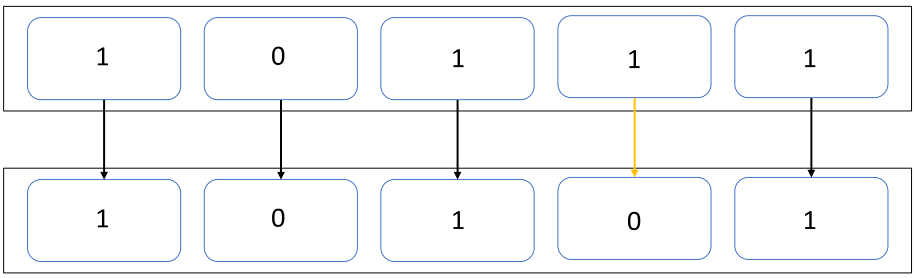


Figure 3.5: Mutation process

3.4. Chapter Summary

In this chapter, we investigated the radiation pattern of an LAA and expressed it mathematically with the AF. Using the array factor and our knowledge of the FT we could prove that it's periodic and that this fact gives rise to GL's. Using the AF and the directivity function of antennas we could design a GA to optimize the spacing of the antennas and digital parameters to steer and suppress grating lobes.

Chapter 4

Results

It was decided in an earlier section to use a half-wave dipole antenna. Table 4.1 shows the parameters used to model the dipole antenna in a MATLAB package called Antenna Toolkit.

Variable	Value
frequency	$3GHz$
c_0	$3\text{x}10^8 m/s$
dipole radius	$0.001m$
λ	$1m$

Table 4.1: Antenna Setup

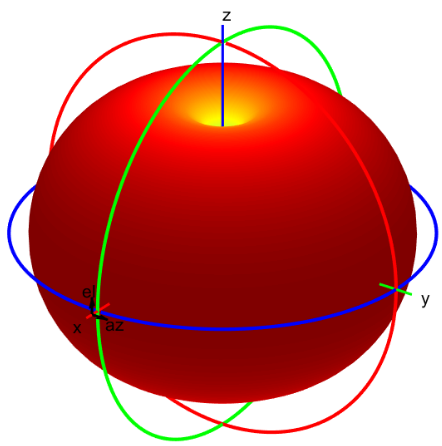


Figure 4.1: Half-wave dipole antenna radiation pattern.

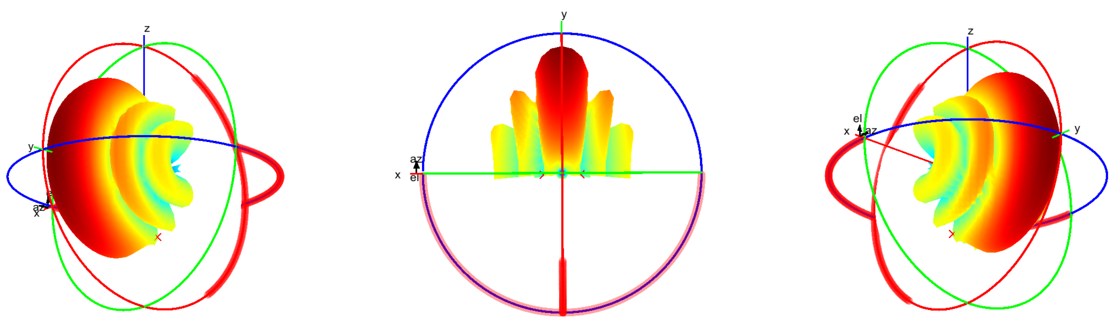


Figure 4.2: 3D radiation pattern for 6 element LAA.

4.1. Uniform Linear Antenna Array Results.

To design a non-uniform LAA we first need to understand how well the uniform LAA performs. The performance of the uniform LAA will be used as a baseline to measure how well the GA performs. The GA can then be utilised to determine the best non-uniformly spaced LAA and the corresponding phase shift and amplitude distribution over the entire visible space. The best results are documented in this section.

4.1.1. Amplitude Distribution

From our research in the literature review Chapter 2 it was stated that the amplitude distribution only affects the SLL and not the grating lobes. When the Fourier transform of different window functions in Figure 3.2b are compared it is clear that the SLL is reduced while the HPBW is increased. Using the GA to optimize the amplitude distribution for a LAA spaced 1λ apart it is shown that the SLL were reduced by 21.21 dB (159.59%) compared to the uniform distribution with only a $2°$ increase in the HPBW.

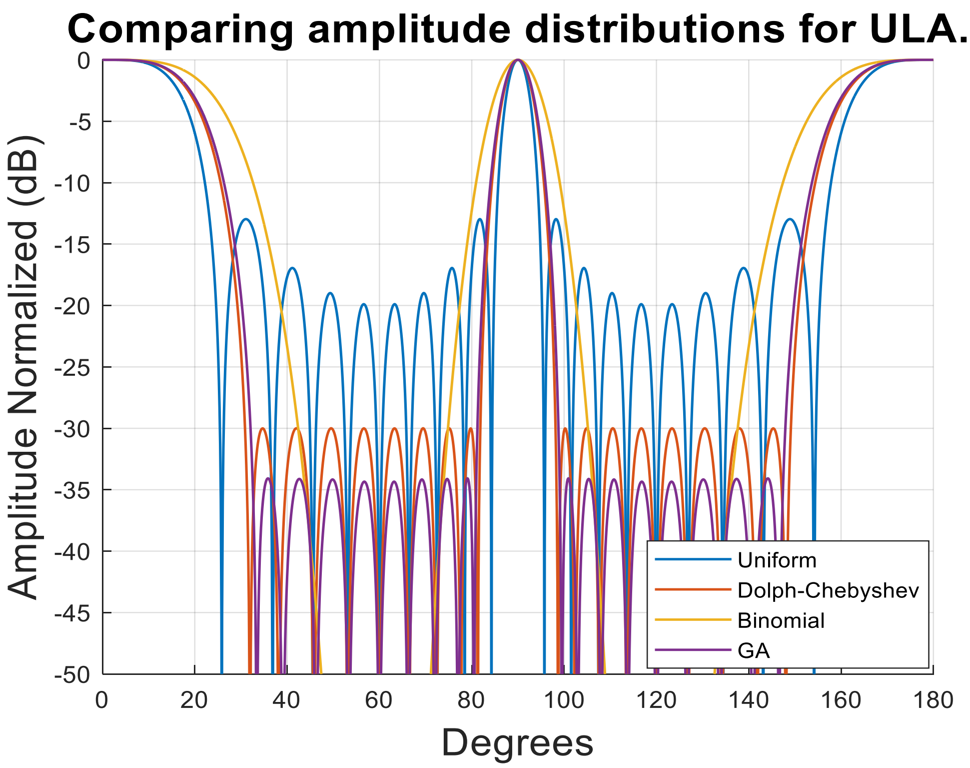


Figure 4.3: Uniform vs Dolph-Chebyshev vs Binomial Amplitude vs GA amplitude distribution.

4.1.2. Uniform Linear Antenna Array Beam Steering

Equation (4.1) which forms part of the AF, equation (3.2), is used to steer the radiation pattern of a uniform LAA space 0.5λ apart to a target direction θ_T.

$$\Delta\gamma = \frac{360^\circ d \sin(\theta_T)}{\lambda} \tag{4.1}$$

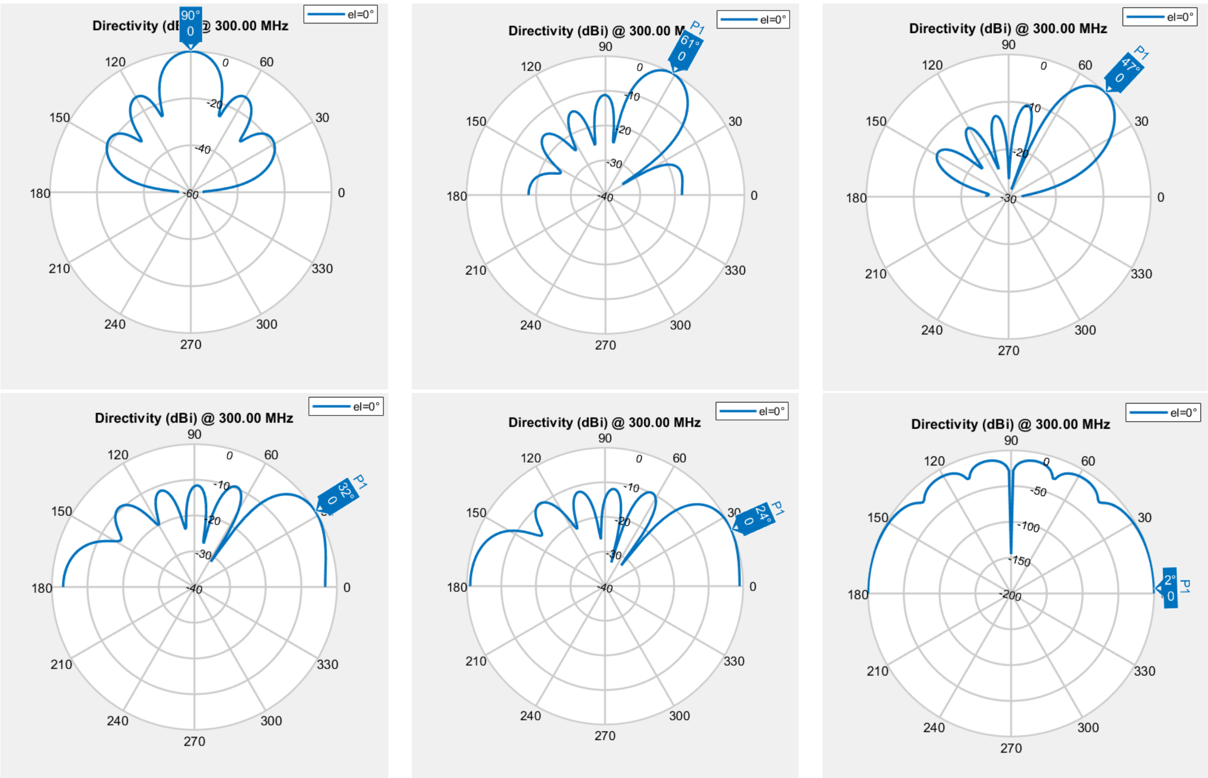


Figure 4.4: Beam steering (with equation (4.1)) with a uniform LAA of 6 antennas elements spaced 0.5λ apart.

θ_T	HPBW	SLL dB	Directivity	Measured	θ_{err}	$\Delta\delta$
90°	18°	−13.29	4.935	90°	0°	0°
60°	20°	−11.21	4.064	61°	1°	90°
45°	23°	−10.35	3.52	47°	2°	127.28°
40°	25°	−9.913	3.09	43°	3°	115.7°
30°	32°	−4.714	2.412	34°	4°	169.14°
27°	36°	−3.191	2.293	33°	6°	95.38°
20°	+40°	−1	2.144	24°	4°	155.88°
10°	$+\circ$	0	2.094	0°	-	177.27°

Table 4.2: The performance of a Uniform LAA spaced 0.5λ apart steered using equation (4.1).

The results obtained for steering a 6 antenna LAA with uniform amplitude distribution spaced uniformly 0.5λ apart to different target directions using equation (4.1) is shown in Figure 4.4 and Table 4.2.

As the beam is steered from boresight to 63° from boresight the HPBW increases from 18° to 36° while the SLL shifts from $-13.29\ dB$ to $-3.191\ dB$. When the beam is steered beyond 27° the side lobe increases beyond $-3.191\ dB$ and turns into a grating lobe. This means the LAA has a steering range ±63° from boresight.

When the theoretical steering target direction is compared to the measured direction achieved during simulation a small steering error occurs. This means when equation (4.1) is used to steer the beam to 27° the beam is measured to be at 33°. This θ_{err} becomes larger (0° to 6°) the further away from boresight the beam is steered. This means the actual steering range achieved by the LAA is $90° - 33° = 57°$ symmetrically around boresight.

These results will be the baseline for the rest of the simulations to either match or improve upon.

4.1.3. GA Optimizes Phase Shift for Uniform LAA.

The purpose of this experiment is to test if the fitness equation (3.12) can be used to effectively steer the radiation pattern. The steering performance of the GA is then compared to the performance of the know equation (4.1). This equation is only applicable to a uniform LAA and would not work for a non-uniform LAA. Therefore, it is important that the GA with the chosen fitness function can be used during the optimization process.

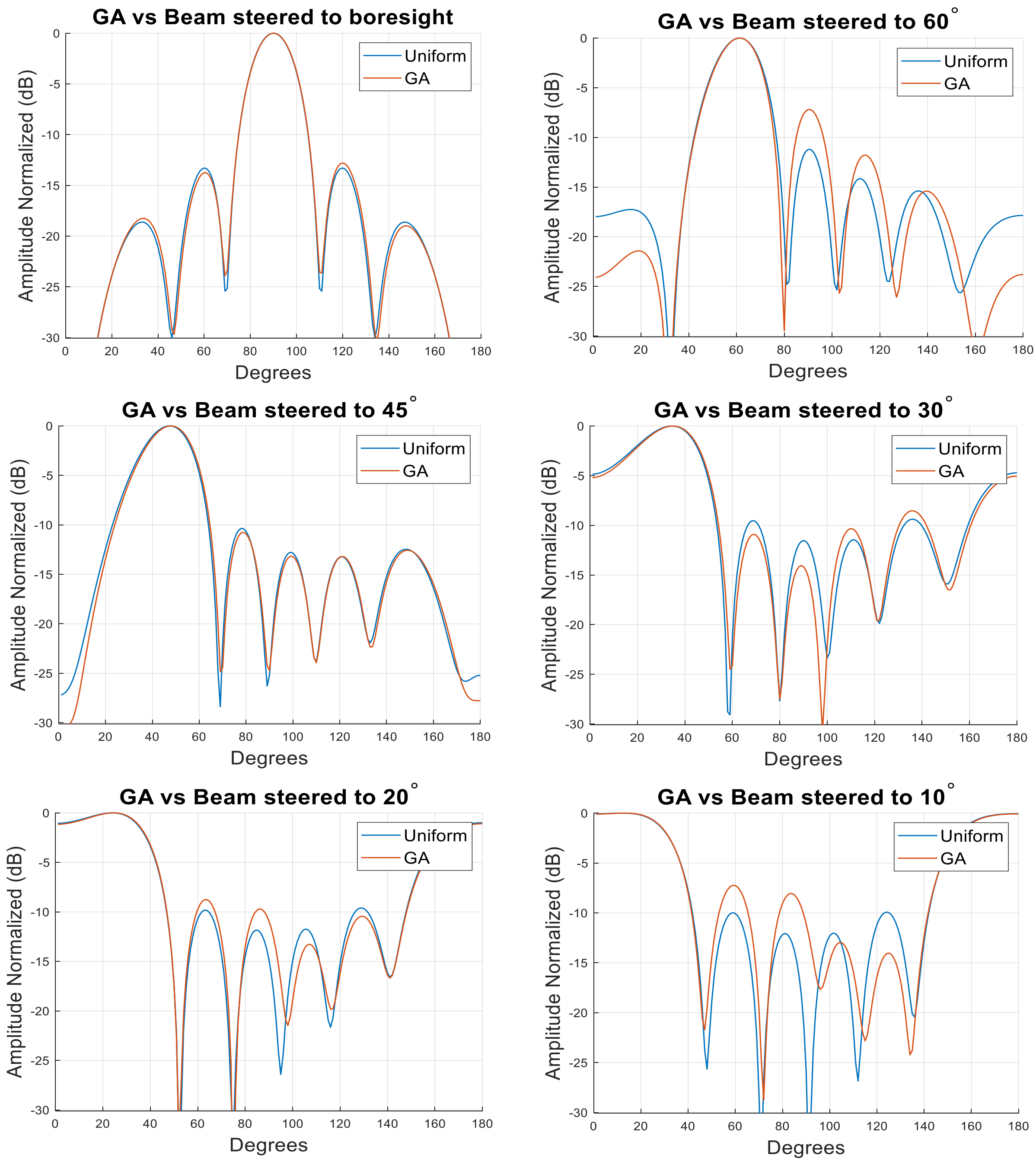


Figure 4.5: The AF for Uniform LAA spaced 0.5λ apart steered using the GA.

θ_T	**HPBW**	**SLL**	**fit**	**GA fit**	Δ_{fit}	**Measured**	**Calculated**	θ_{err}
90°	18°	−12.79	6.406	6.36	−0.046	90°	90°	0°
60°	20°	−7.195	7.931	7.36	−0.571	61°	60°	+1°
45°	23°	−10.76	8.525	8.404	−0.121	48°	45°	+2°
30°	32°	−4.714	12.374	12.33	−0.044	34°	30°	+4°
20°	+39.5°	−1.097	13.566	13.574	0.008	24°	20°	+4°
10°	+∘	0	13.866	13.997	0.131	13°	10°	+3°

Table 4.3: The performance of a uniform LAA spaced 0.5λ apart steered using the GA.

The GA code can be viewed in Section C.1. The parameters used in the GA for beam steering was determined in Section C.2 and documented in Table C.1 and C.2. When steering the radiation pattern of the LAA using the GA to different target directions between boresight and 80° from boresight the GA matches the steering algorithm equation (4.1) almost exactly, see Figure 4.5.

The GA aim is to minimize equation (3.12) which reduces the error of a particular solution at a desired target direction. Applying the fitness function to both the solution determined by the steering equation and the solution determined by the GA yield fitness scores as seen in Table 4.3 in the fit and $GA\ fit$ column. The biggest difference in fitness score is $\frac{0.571}{7.931} = 7.2\%$ which is also an outlier (more about this result in the next paragraph) with the second biggest being $\frac{0.121}{8.525} = 1.4\%$. The GA algorithm managed to obtain a phase shift for each antenna in the LAA that produced a fitter solution than that of the steering algorithm, meaning the fitness score is lower for the GA solution than that of the steering equation (3.12).

Something interesting happens at 60° when the GA produces a solution that is 7.2% fitter than that of the uniform LAA. The SLL is clearly higher for the GA solution than for the uniform LAA however, the fitness score is 0.571 points better for the solution of the GA. This means the fitness function converges to a lower value than that of the solution of the steering equation for a uniform LAA even though the SLL are higher. This is as a result of the fitness function and its limitations rather than the performance of the GA. It indicates that the fitness function needs to be adjusted by adding more constraints to prioritise suppressing the highest SLL first before suppressing the average SLL. For our application the fitness function performs well enough within a reasonable error margin of well below 10% and will be used to steer the beam for a non-uniform LAA.

4.2. Uniformly Spaced Linear Antenna Array

The only parameter that has an affect on the grating lobes is the spacing between the antenna elements of the LAA. To understand how the element spacing influences the grating lobes the radiation pattern is simulated for different uniform spacing ranging from 0.5λ to 5λ.

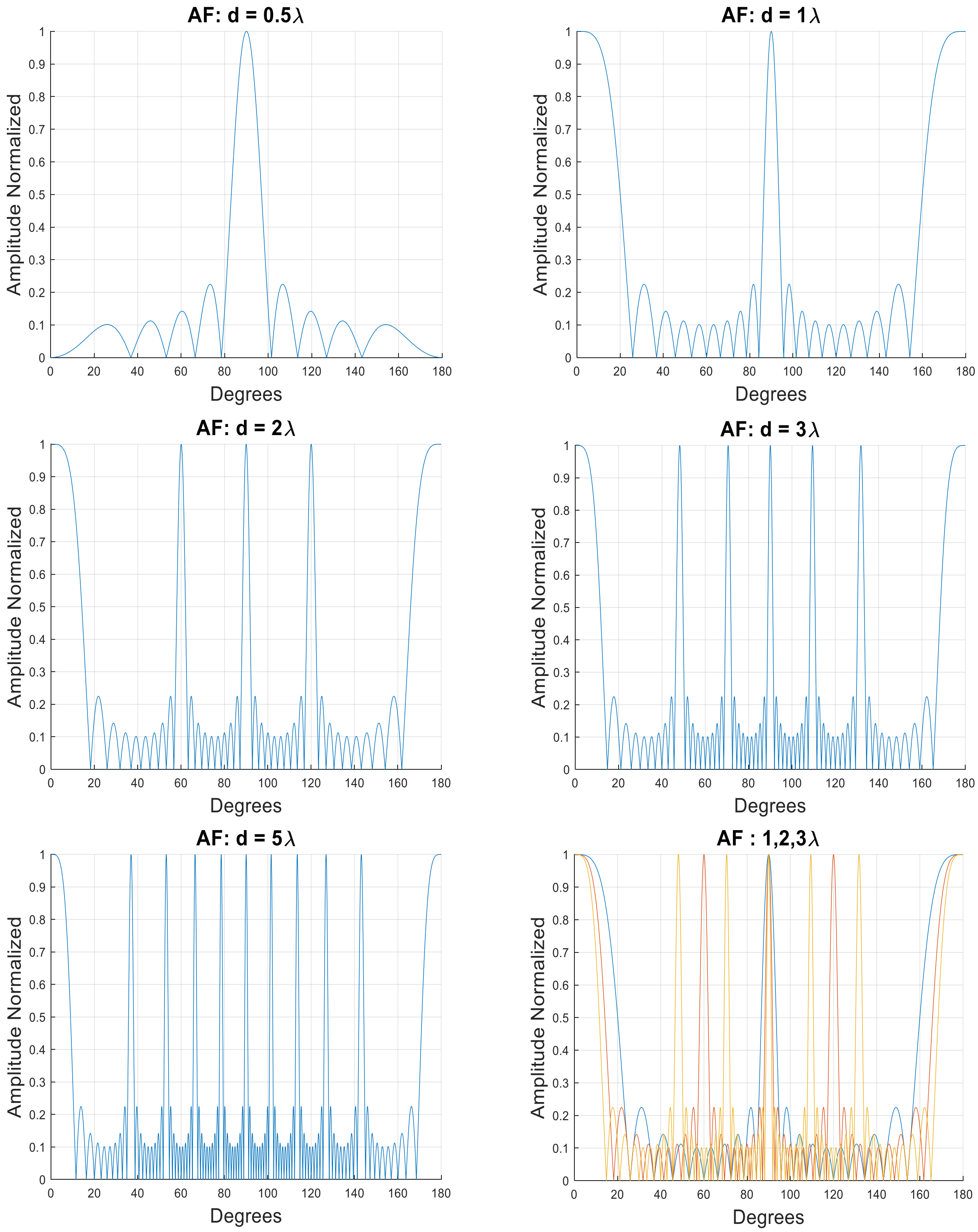


Figure 4.6: The AF for a uniform LAA spaced from 0.5λ to 5λ apart.

Uniformly Spaced	GL $k=1$	Predict	GL $k=2$	Predict	GL $k=3$	Predict
0.5λ	NONE	-	NONE	-	NONE	-
1λ	$0°$	$0°$	NONE	-	NONE	-
2λ	$60°$	$60°$	$0°$	$0°$	NONE	-
3λ	$70.54°$	$70.528°$	$48.18°$	$48.189°$	$0°$	$0°$
5λ	$70.46°$	$78.463°$	$66.42°$	$66.421°$	$53.13°$	$53.13°$

Table 4.4: Mathematical predictions vs AF calculation for the location of grating lobes.

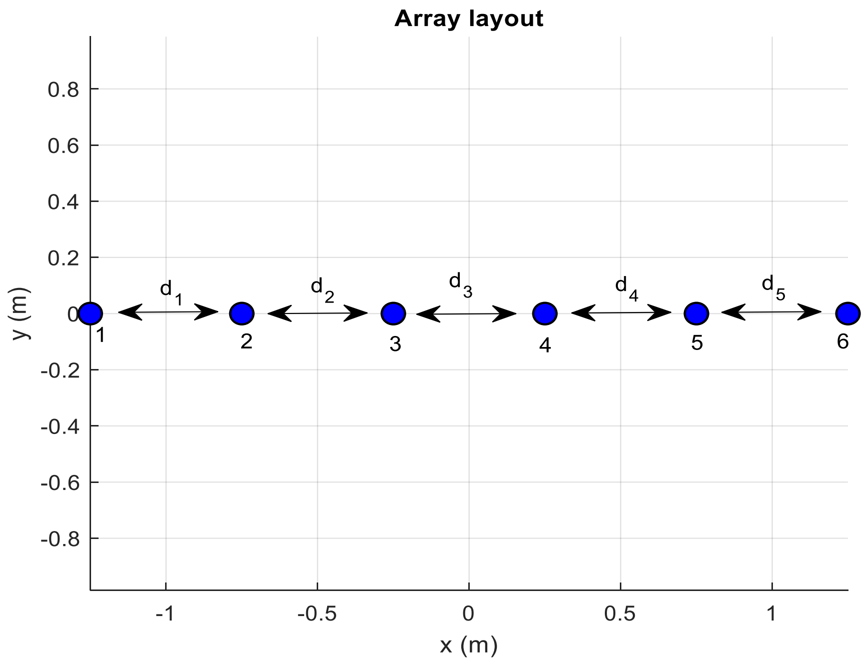


Figure 4.7: Six antennas separated by a constant value $d = x\lambda$

The position of the grating lobes are predicted by the equation $-1 \leq \arccos(k\frac{\lambda}{d}) \leq 1$ for all the value of k resulting in a real solution. The solution must be real for only real solution will appear in the visible space $0° \leq \theta \leq 180°$.

The results for the predicted values compared to the simulated values for different uniform LAA's are in Table 4.4 and is accurate to within 1%. This means that it can be mathematically predicted where the grating lobes will appear for a given uniform LAA and the affect the spacing has on the grating lobes.

A key takeaway from Figure 4.6 is that as the spacing between a uniform LAA increases the spacing between the grating lobes becomes smaller and visa versa. Another observation that can be made from Figure 4.6 is that as the spacing between the antenna elements increase the HPBW becomes smaller. This means there is a trade off between the HPBW and the spacing between grating lobes.

The following Figures 4.8, 4.10 and 4.12 show the 3-dimensional radiation pattern of the LAA with different uniform spacings.

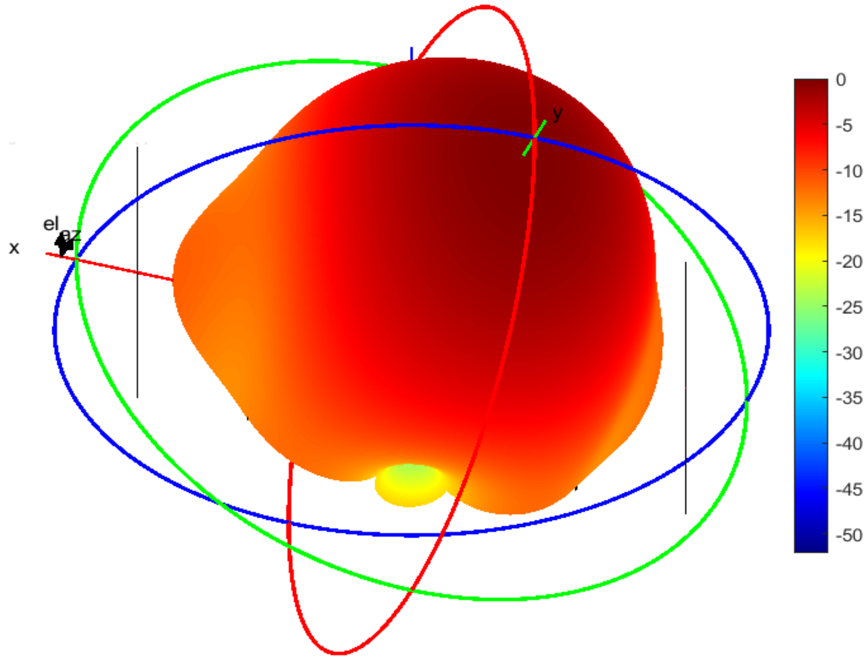


Figure 4.8: Radiation pattern of a ULAA with 6 elements spaced 0.17λ apart.

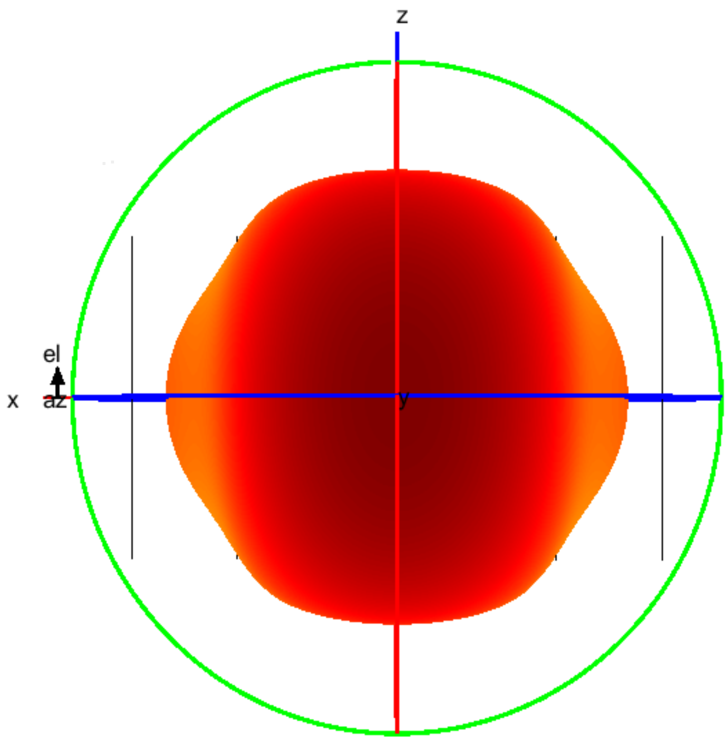


Figure 4.9: Radiation pattern of a ULAA with 6 elements spaced 0.17λ apart viewed perpendicular to the antenna axis.

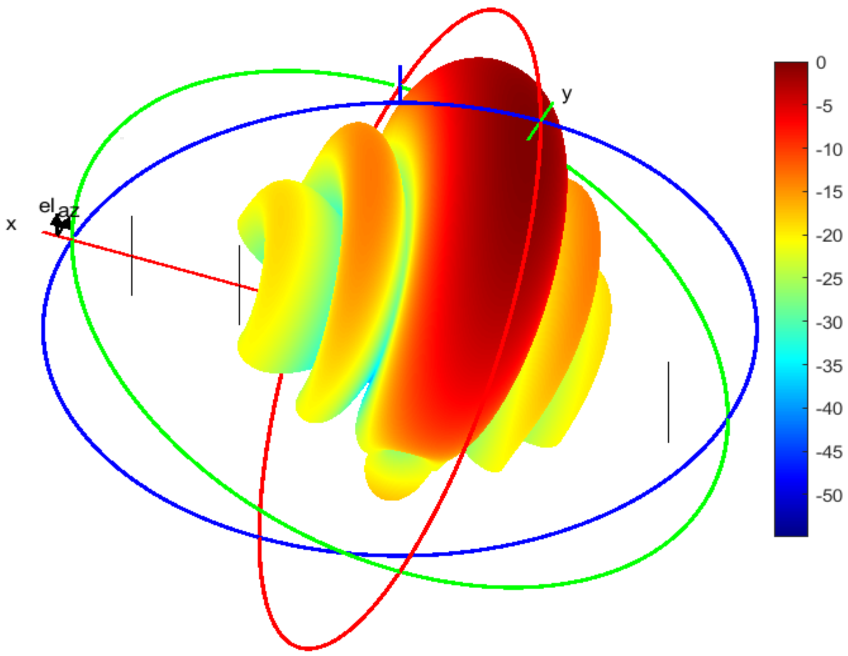


Figure 4.10: Radiation pattern of a ULAA with 6 elements spaced 0.5λ apart.

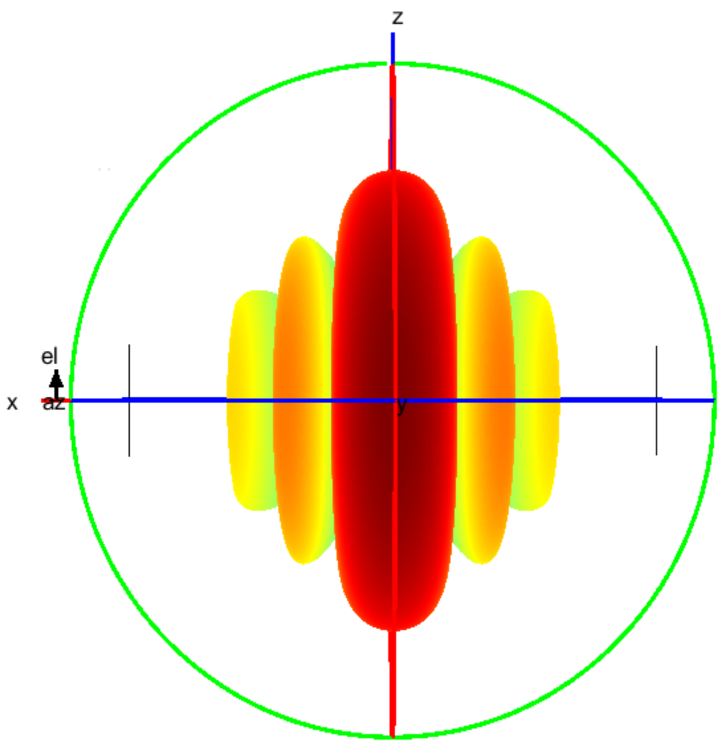


Figure 4.11: Radiation pattern of a ULAA with 6 elements spaced 0.5λ apart viewed perpendicular to the antenna axis.

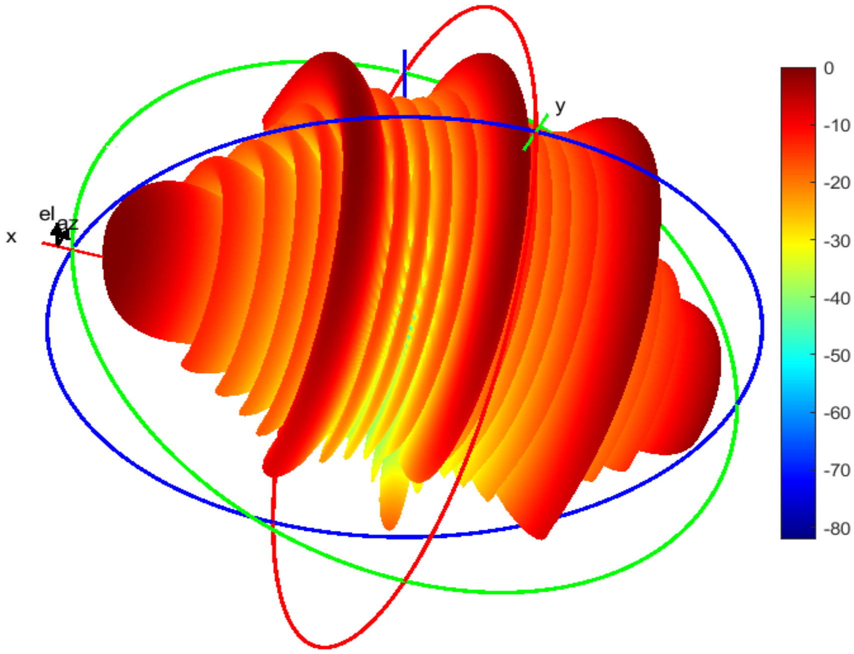


Figure 4.12: Radiation pattern of a ULAA with 6 elements spaced 2λ apart.

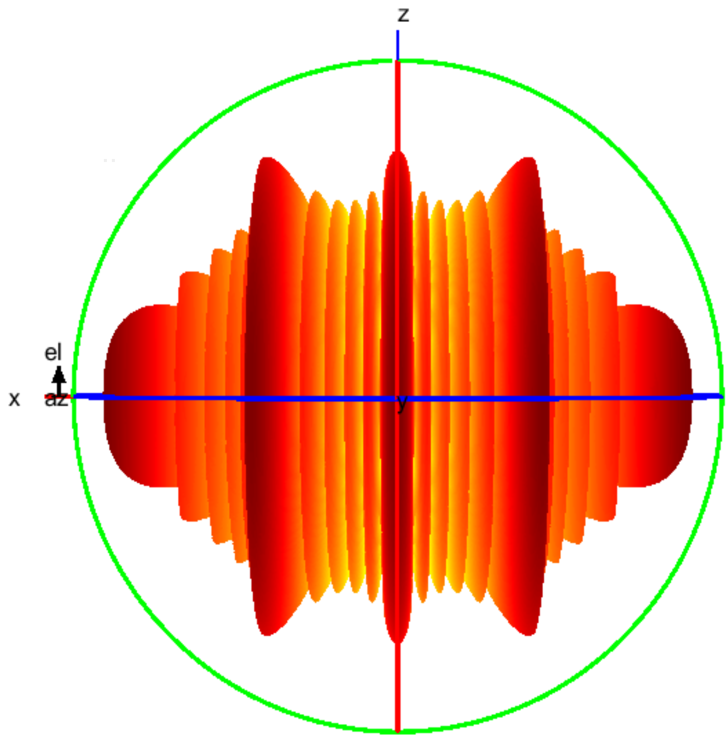


Figure 4.13: Radiation pattern of a ULAA with 6 elements spaced 2λ apart viewed perpendicular to the antenna axis.

In Figure 4.8 the LAA is spaced 0.17λ apart and has no grating lobes and no side lobes but has a very large HPBW as predicted. When the LAA is spaced 0.5λ apart as in Figure 4.10 the uniform LAA (the current best solution) has some side lobes and no grating lobes. Only when the LAA is spaced 2λ apart as in Figure 4.12 we see two grating lobes appear that are positioned symmetrical around the main beam.

The main beam, side lobes and grating lobes appear as rings around the x axis. This is due to the fact that the antenna array is 1-dimensional and that the dipole antennas radiate in all directions equally. Resulting in constructive wave in the shape of a ring around the x axis. By taking a slice of the x, y plane of the 3D plots the following graphs in Figure 4.6 can be generated.

4.3. Symmetrical Non-Uniformly Spaced Linear Antenna Array

Three solutions were obtained from the GA that suppress the grating lobe that appears when a uniform LAA spaced 0.5λ apart is steered beyond the CA. The element spacing can be seen in Table 4.5. The performance of the three solutions (S_1 = solution 1, S_2 = solution 2, S_3 = solution 3) is compared to the performance of the uniform LAA spaced 0.5λ in Figure 4.14 and documented in Table 4.6.

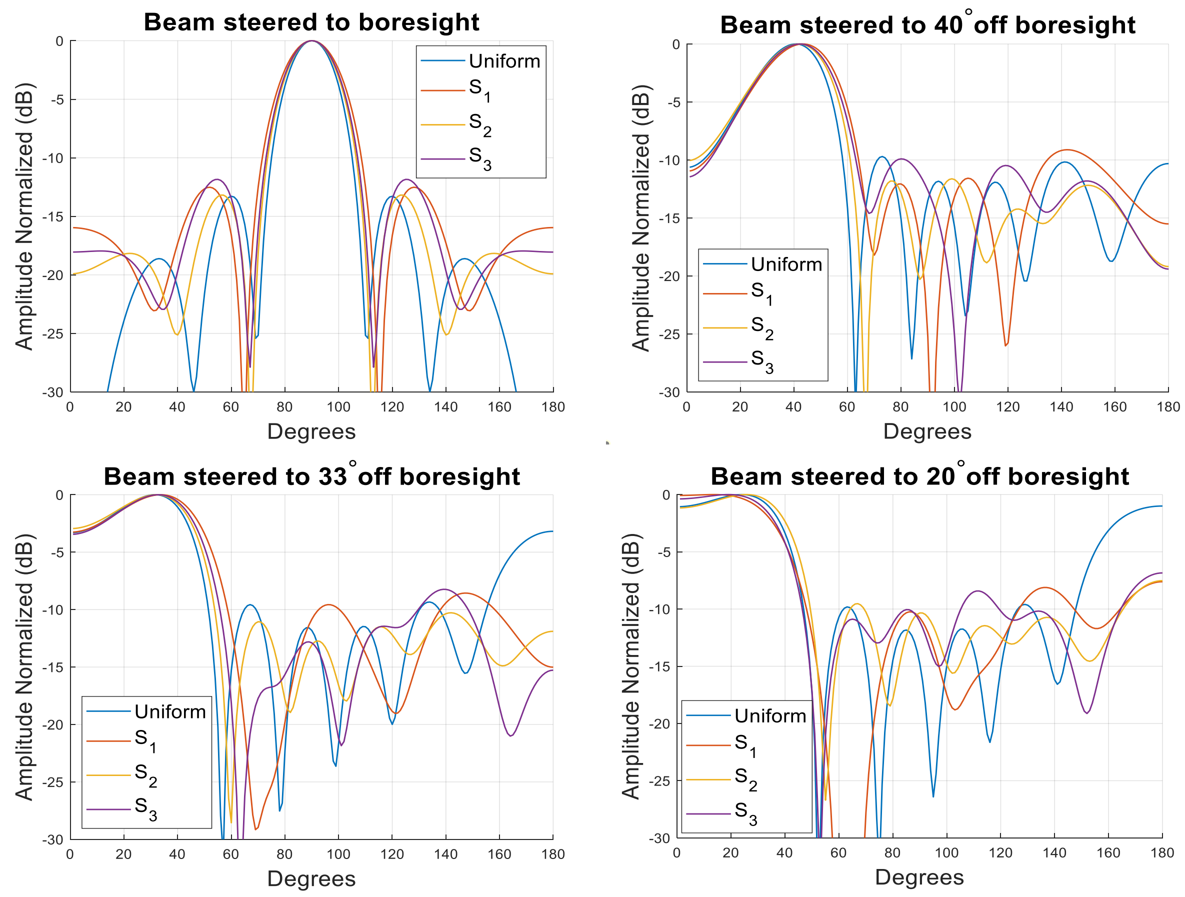


Figure 4.14: Steering beam from boresight to different target directions.

Solution	Inter Element Spacing (λ)	d_{avg}
ULA	[0.5 0.5 0.5 0.5 0.5]	0.5
S_1	[0.3 0.475 0.52 0.475 0.3]	0.414
S_2	[0.425 0.475 0.5 0.475 0.425]	0.460
S_3	[0.35 0.48 0.56 0.48 0.35]	0.444

Table 4.5: Element Spacing that yield best grating lobe suppression.

θ_T	Solution	HPBW	SLL	Δ_{HPBW}	Δ_{SLL} (dB)	$\Delta_{Directivity}$	$\Delta_{\theta_{err}}$
90°	S_1	22°	−12.52	+4°	+0.77	−1.187	0°
90°	S_2	20°	−13.19	+2°	+0.1	−0.719	0°
90°	S_3	20.5°	−11.84	+2.5°	+1.45	−1.024	0°
40°	S_1	29.5°	−9.134	+4°	+0.568	−0.2527	1.5°
40°	S_2	28°	−11.62	+2.5°	−1.918	−0.0075	1°
40°	S_3	28.5°	−9.907	+3°	−0.205	−0.1675	1°
33°	S_1	43°	−8.563	+7°	−5.372	+0.0749	1°
33°	S_2	46°	−10.31	+10°	−7.119	+0.2146	1.2°
33°	S_3	39.5°	−8.236	+3.5°	−5.045	+0.1326	1.1°
20°	S_1	37°	−7.613	−2.5°	−6.613	+0.3198	0°
20°	S_2	41°	−7.523	+1.5°	−6.523	+0.2284	0°
20°	S_3	37.5°	−6.846	−2°	−5.846	+0.2307	0°

Table 4.6: Performance of non-uniformly spaced solutions.

An observation that can be made when looking at the solutions in Table 4.5 is that the average spacing is always less than 0.5λ as predicted in Chapter 3. Another observation is that the spacing between element 3 and 4 (the middle inter element spacing) is always the largest and the subsequent inter element spacing becomes smaller. The corresponding phase shifts for each solution can be seen in Table C.3.

When comparing the solutions in Figure 4.14 the first observation is that the HPBW is slightly wider for the three solutions obtained by the GA compared to the uniform LAA at all target directions. When the main beam is directed at boresight (90°) the solutions obtained have a HPBW 2° to 4° (11.11% to 22.22%) wider with SLL 0.1 dB to 1.45 dB (0.75% to 10.91%) higher than the uniform LAA. With the SLL being higher and the HPBW being wider the directivity of the solutions are $S_1 = -1.187$, $S_2 = -0.719$ and $S_3 = -1.024$ less directive than the uniform LAA. The GA solutions are $S_1 = -24.05\%$, $S_2 = -14.57\%$ and $S_3 = -20.75\%$ less directive than the uniform LAA.

When the beam is steered to the CA of 33° or 57° from boresight the solutions obtained by the GA are superior to that of the uniform LAA. At the CA the HPBW ranges from 39.5° to 46° which is 3.5° to 10° (9.72% to 27.78%) wider than the uniform LAA. The SLL of the uniform LAA is $-3.191\ dB$. The GA solution with the worst SLL is $S_3 = -8.236\ dB$. This solution results in a $5.045\ dB$, (158.1%) reduction in the growing grating lobe level. With the superior SLL the overall directivity is only slightly improved for the GA solutions ($S_1 = +0.075$, $S_2 = +0.215$ and $S_3 = +0.133$) due to the increase in the width of the HPBW. The GA solutions are $S_1 = +3.27\%$, $S_2 = +9.36\%$ and $S_3 = +5.78\%$ more directive than the uniform LAA.

When the beam is steered beyond the CA the uniform LAA officially has a grating lobe however, the solutions obtained by the GA steered to 20° or 70° from boresight do not have any grating lobes. The highest SLL at 20° is $S_3 = -6.846\ dB$. This solution results in a $5.846\ dB$, (584.6%) suppression of the grating lobe of the uniform LAA. With the superior SLL the overall directivity is improved for the GA solutions ($S_1 = +0.320$, $S_2 = +0.228$ and $S_3 = +0.231$). The GA solutions are $S_1 = +14.92\%$, $S_2 = +10.65\%$ and $S_3 = +10.76\%$ more directive than the uniform LAA.

4.4. Chapter Summary

This chapter investigated and measured the performance of a uniform LAA spaced 0.5λ apart with uniform amplitude distribution. A GA was deployed to optimize the amplitude distribution of the signals entering the LAA and compared it to known solutions such Dolph-Chebyshev and Binomial distributions. The GA obtained a amplitude distribution with superior SLL reduction compared to the other solutions with only a slight increase in the HPBW compared to the uniform distribution.

The GA was then used to steer a uniform LAA spaced 0.5λ apart with uniform amplitude distribution to different target directions. The results were compared to the steering equation and it was shown that it matched the performance of the steering equation to within 2% accuracy. Proving that the GA could be used to steer the beam of a antenna array to any target direction.

A GA was then implemented to optimize the spacing of a LAA when steered to the CA (of the uniform LAA) and beyond. Three different antenna spacing that suppressed grating lobes were obtained. These solutions outperformed the uniform LAA in steering range and SLL reduction but had a wider HPBW.

Chapter 5

Summary and Conclusion

5.1. Summary

For a uniform, Linear Antenna Array (LAA) spaced half a wavelength apart, the Critical Angle (CA) at which the lowest Side Lobe Level (SLL) becomes larger than $-3\ dB$, which qualifies the side lobe as a grating lobes is 33° or 57° from boresight. This means the steering range of a uniform LAA is 114° (63.33%) of the visible space (180°).

A Genetic Algorithm was designed and deployed to optimize the amplitude distribution and phase shift independently. It was then combined to optimize the antenna spacing. It is shown that the amplitude distribution has no effect on the grating lobes but instead affect the half-power beamwidth (HPBW) and SLL. The phase shift only steers the position of the main beam. Due to the grating lobe being a secondary main beam the phase shift affect the position of the grating lobe.

A Genetic Algorithm was designed and deployed to optimize the spacing between a 6 element LAA to suppress the grating lobe that appears at the CA of the uniform LAA. Three solutions were obtained and documented in this report that at the mentioned CA has no grating lobes. The highest SLL of one of the solutions at the CA is 5.045 dB below that of the uniform LAA. This is a 158.1% reduction in the grating lobe level. The HPBW of these solutions ranges between 3.5° to 10° (9% to 27%) wider than the uniform LAA. This can be undesirable depending on the specifications of the intended application. There is a clear trade-off between the SLL and HPBW.

To achieve a narrow HPBW, a wider spacing between antenna elements is required however, this will increase the SLL and decrease the range at which the main beam can be steered at. A wider spacing decreases the distance between grating lobes. To achieve a low SLL, a narrower spaced LAA should be considered but this will increase the HPBW. To ensure grating lobes are suppressed it is shown that the average spacing between antennas should not be greater than half a wavelength although a single spacing between two antennas can be greater than half a wavelength.

5.2. Conclusion

The conclusion drawn from the investigation of techniques to suppress grating lobe formed by a LAA is that grating lobes are as a result of the periodic nature of the Array Factor. This periodicity is as a result of the inter element spacing of the antennas in a antenna array.

It is therefore concluded that a non-uniformly spaced LAA can suppress the grating lobes of the uniform LAA when steered to and beyond the CA. Depending on the specifications and intended use the steering range can vary between 114° to 180° (63.33% to 100%) of the visible space. That is an increase of more than 36.67% in steering capabilities compared to the uniform LAA. The grating lobe is suppressed 158.1% at the CA but at the cost of a 9% to 27% increase of the HPBW.

5.3. Future Work

In this project the GA was used to optimize the inter element spacing for a target direction of 33°. This yielded good results around the target direction but weaker results around smaller steering angles. This is because no information is obtained or taken into consideration for smaller steering angles. It would be beneficial to deploy the optimization algorithm for multiple steering angles to obtain a fitness score that captures more information for each set of spacings.

Another interesting recommendation for future work is to investigate planer antenna arrays. Expanding the LAA to 2-dimensions open up interesting techniques such as density tapering, different rotation designs and sub-arrays that potentially suppress grating lobes. From the knowledge gained from studying the LAA a planer antenna array will be more directive and allow for beam steering in an entire hemisphere.

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Appendix A

Project Planning Schedule

Date	**Activity**
8 Aug - 15 Aug	Background
16 Aug - 23 Aug	Related work and shortcomings
24 Aug - 27 Aug	Type of antennas
28 Aug - 4 Sept	Formation of antennas
5 Sept - 13 Sept	Factors for grating lobes
14 Sept - 17 Sept	Beam steering
12 Sept - 15 Oct	Matlab Antenna Array Toolkit
23 Sept - 15 Oct	Genetic Algorithm
20 Aug - 8 Nov	Document findings in report
13 Oct - 27 Oct	First draft submission
28 Oct - 5 Nov	Improvements after feedback
15 Nov	Final draft submission
19 Nov - 25 Nov	Oral examination
30 Nov	Present poster at open day

Table A.1: Project Planning.

Appendix B

Outcomes Compliance

Elo	Chapter(s)	Description
1. Problem solving	3, 4	The parameters that create and influence grating lobes for a antenna array were investigated and optimized to suppress grating lobes.
2. Application of scientific and engineering knowledge	1, 2	Knowledge of antenna theory was used to analyse the LAA and to design an algorithm optimize the parameters of a LAA to suppress the grating lobes.
3. Engineering design	3, 4	I designed and implement a genetic algorithm that optimizes the parameters of a arbitrary length LAA to suppress grating lobes together with steering the main beam to any desired direction.
4. Investigations, experiments and data analysis	3, 4	Simulated results were obtained, documented, analysed, interpreted and explained.
5. Engineering methods, skills and tools, including information technology	3, 4	MATLAB, MATLAB's antenna designer toolkit and a genetic algorithm developed in the python programming language was used to solve the problem statement.
6. Professional and technical communication	1, 2, 3, 4, 5	This written report showcases how I can effectively communicate technical information to the reader.
8. Individual work	1, 2, 3, 4, 5	All the research that went into studying and understanding the literature together with applying the knowledge and developing the genetic algorithm was done individually.
9. Independent learning ability	2, 3, 4, 5	All the research that went into studying and understanding the literature together with learning how to use the software to develop an algorithm to solve the problem statement was done individually.

Table B.1: Outcome and Compliance.

Appendix C

More Results

C.1. Genetic Algorithm (Python).

```python
import numpy as np
from functions import *
class EvoAmp:
    #Initiate parameters
    def __init__(self, population_tot, crossover_rate, mutation_rate, generations, N, d, target):
        self.population_tot = population_tot
        self.crossover_rate = crossover_rate
        self.mutation_rate = mutation_rate
        self.generations = generations
        self.N = N
        self.target = target

        # Spacing between antennas
        self.pop_dis = np.ndarray(shape=(self.population_tot, self.N-1))

        # Phase Shift of the signal of each antenna element
        self.pop_pha = np.random.uniform(size = (self.population_tot, self.N), low = 0, high = 2*np.pi*0) # randians

        # Amplitude of the signal of each antenna element.
        self.pop_amp = np.random.uniform(size=(self.population_tot, self.N), low = 0, high=1)

        # Population with same spacing and phase -> Only optimizing amplitude
        for i in range(self.population_tot):
            self.pop_dis[i, :] = d

        # Fitness scores
        self.pop_fit = np.zeros(self.population_tot, dtype = np.float)

        self.evolve()

    def results(self):
        # Return the best solution
        q = np.argmin(self.pop_fit)
        return self.pop_amp[q]

    def evolve(self):
        # probabilities for cross over. To avoid calculationg random numbers every itteration.
        crossovers = np.random.uniform(size=(self.generations), low = 0, high=1)
        mutations = np.random.uniform(size=(self.generations), low = 0, high=1)

        for i in range(self.generations):
            # Calculate the fitness score of each solution
            for dna in range(self.population_tot):
                score = fitness(self.pop_dis[dna], self.pop_pha[dna], self.pop_amp[dna], self.target)
                self.pop_fit[dna] = score;
            # Index of fittest solution
            index = np.argmin(self.pop_fit)
            # Copy fitteset solution over to next genertation (first and last index of array)
            self.pop_amp[0] = self.pop_amp[index]
            self.pop_amp[-1] = self.pop_amp[index]

            # Stop simulating when the generation matches the maximun no of generations.
            if i == self.generations-1:
                print('DONE')
                break
            # Itterations per generations
            for _ in range(1):

                if crossovers[i] < self.crossover_rate:
                    # crossover occurs (Two parent solutions)
                    s_1 = selection(self.pop_fit)
                    s_2 = selection(self.pop_fit)
                    self.pop_amp[s_1] = (self.pop_amp[s_1] + self.pop_amp[s_2])/2

                if mutations[i] < self.mutation_rate:
                    # mutation occurs
                    s_1 = selection(self.pop_fit)
                    self.pop_amp[s_1][np.random.randint(low=0, high=self.N)] =  np.random.uniform(low = 0, high=1)
```

Figure C.1: GA that optimizes the amplitude of the signal entering a LAA of size N to any desired direction.

```python
import numpy as np
from evolution import selection, fitness
from functions import *

class EvoPhase:
    #Initiate parameters
    def __init__(self, population_tot, crossover_rate, mutation_rate, generations, N, d, target):
        self.population_tot = population_tot
        self.crossover_rate = crossover_rate
        self.mutation_rate = mutation_rate
        self.generations = generations
        self.N = N
        self.target = target

        # Spacing between antennas
        self.pop_dis = np.ndarray(shape=(self.population_tot, self.N-1))

        # Phase Shift of the signal of each antenna element
        self.pop_pha = np.random.uniform(size = (self.population_tot, self.N), low = 0, high = 2*np.pi) # randians

        # Amplitude of the signal of each antenna element.
        self.pop_amp = np.ndarray(shape=(self.population_tot, self.N))
        # Uinform amplitude
        amp = np.array([1, 1, 1, 1, 1, 1])

        # Population with same spacing and amplitude -> Only optimizing phase
        for i in range(self.population_tot):
            self.pop_dis[i, :] = d
            self.pop_amp[i, :] = amp

        # Fitness scores
        self.pop_fit = np.zeros(self.population_tot, dtype = np.float)

        self.evolve()

    def results(self):
        # Return the best solution
        q = np.argmin(self.pop_fit)
        return self.pop_pha[q]

    def evolve(self):
        # probabilities for crossover. To avoid calculationg random numbers every itteration.
        crossovers = np.random.uniform(size=(self.generations), low = 0, high=1)
        mutations = np.random.uniform(size=(self.generations), low = 0, high=1)

        for i in range(self.generations):
            tot = 0
            # Calculate the fitness score of each solution
            for dna in range(self.population_tot):
                score = fitness(self.pop_dis[dna], self.pop_pha[dna], self.pop_amp[dna], self.target)
                self.pop_fit[dna] = score;
                tot += score
            # Index of fittest solution
            index = np.argmin(self.pop_fit)
            # Copy fitteset solution over to next genertation (first and last index of array)
            self.pop_pha[0] = self.pop_pha[index]
            self.pop_pha[-1] = self.pop_pha[index]

            # Stop simulating when the generation matches the maximun no of generations.
            if i == self.generations-1:
                print('DONE')
                break

            # Itterations per generations
            for _ in range(1):
                if crossovers[i] < self.crossover_rate:
                    # crossover occurs (Two parent solutions)
                    s_1 = selection(self.pop_fit)
                    s_2 = selection(self.pop_fit)
                    # Ofspring
                    self.pop_pha[s_1] = np.divide(self.pop_pha[s_1] + self.pop_pha[s_2],2)

                if mutations[i] < self.mutation_rate:
                    # mutation occurs
                    s_1 = selection(self.pop_fit)
                    self.pop_pha[s_1][np.random.randint(low=0, high=self.N)] =  np.random.uniform(low = 0, high=2*np.pi)

```

Figure C.2: GA that only optimizes the phase for a LAA of size N to any desired direction.

```python
import numpy as np
import matplotlib.pyplot as plt
from functions import *
from evophase import *

global LOW, HIGH
LOW = 0.3
HIGH = 0.6

class Evolution:
    #Initiate parameters
    def __init__(self, population_tot, crossover_rate, mutation_rate, generations, N, target):
        self.population_tot = population_tot
        self.crossover_rate = crossover_rate
        self.mutation_rate = mutation_rate
        self.generations = generations
        self.N = N
        self.target = target

        # Spacing between antennas
        self.pop_dis = np.ndarray(shape=(self.population_tot, self.N-1))

        # Non-uniform symetrically spaced LAA
        for i in range(self.population_tot):
            a = np.random.uniform(low = LOW, high=HIGH)
            b = np.random.uniform(low = LOW, high=HIGH)
            self.pop_dis[i][4] = a;
            self.pop_dis[i][0] = a
            self.pop_dis[i][1] = b
            self.pop_dis[i][3] = b

            self.pop_dis[i][2] = np.random.uniform(low = LOW, high=HIGH)

        # Phase Shift of the signal of each antenna element.
        self.pop_pha = np.random.uniform(size = (self.population_tot, self.N), low = 0, high = 2*np.pi) # randians

        # Amplitude of the signal of each antenna element.
        self.pop_amp = np.random.uniform(size=(self.population_tot, self.N), low = 1, high=1)

        # Fitness scores
        self.pop_fit = np.zeros(self.population_tot, dtype = np.float)

        self.evolve()

    def results(self):
        # Return the best solution
        q = np.argmin(self.pop_fit)
        return self.pop_dis[q], self.pop_pha[q], self.pop_amp[q]

    def evolve(self):
        # probabilities for crossover. To avoid calculationg random numbers every itteration.
        crossovers = np.random.uniform(size=(self.generations), low = 0, high=1)
        mutations = np.random.uniform(size=(self.generations), low = 0, high=1)
        # Probabilities for which parameter to mutate or to undergo crossover
        choice = np.random.uniform(size=(self.generations), low = 0, high=1)

        for i in range(self.generations):

            for dna in range(self.population_tot):
                score = fitness(self.pop_dis[dna], self.pop_pha[dna], self.pop_amp[dna], self.target)
                self.pop_fit[dna] = score;
            # Index of fittest solution
            index = np.argmin(self.pop_fit)
            # Copy fitteset solution over to next genertation (first and last index of array)
            self.pop_dis[0] = self.pop_dis[index]
            self.pop_pha[0] = self.pop_pha[index]
            self.pop_dis[-1] = self.pop_dis[index]
            self.pop_pha[-1] = self.pop_pha[index]

            # Optimize the phase shift for the fittest solution.
            evo = EvoPhase(5, 0.5, 1, 2000, self.N, self.pop_dis[0], self.target)
            self.pop_pha[0] = evo.results()

            # Stop simulating when the generation matches the maximun no of generations.
            if i == self.generations-1:
                print('Finished')
                break

            # Itterations per generations
            for _ in range(1):
                if crossovers[i] < self.crossover_rate:
                    # crossover occurs (Two parent solutions)
                    s_1 = selection(self.pop_fit)
                    s_2 = selection(self.pop_fit)

                    # Only share spacing DNA
                    if choice[i] <= 0.5:
                        self.pop_dis[s_1] = (self.pop_dis[s_1] + self.pop_dis[s_2] )/2
                    # Only share phase DNA
                    elif choice[i] > 0.5:
                        self.pop_pha[s_1] = (self.pop_pha[s_1] + self.pop_pha[s_2] )/2

                if mutations[i] < self.mutation_rate:
                    # mutation occurs
                    s_1 = selection(self.pop_fit)
                    # Only mutate spacing DNA
                    if choice[-i] <= 0.5:
                        rs = np.random.uniform(low = LOW, high=HIGH)
                        a = np.random.randint(low=0, high=self.N-1)
                        # SYMETRICAL SPACING
                        self.pop_dis[s_1][a] =  rs
                        #self.pop_dis[s_1][(self.N-2 - a)] =  rs
                    # Only mutate phase DNA
                    elif choice[-i] > 0.5:
                        self.pop_pha[s_1][np.random.randint(low=0, high=self.N)] =  np.random.uniform(low = 0, high=2*np.pi)
```

Figure C.3: GA that optimizes the antenna element spacing and the phase shift for a LAA of size N to any desired direction.

C.2. Genetic Algorithm Parameters

The crossover and mutation rate is altered to determine which parameters will yield the smallest error in the fastest time when the GA is used to steer the beam of a uniform LAA to 45° off boresight. The population size is held constant at 100 and the performance is measured over 1000 generations.

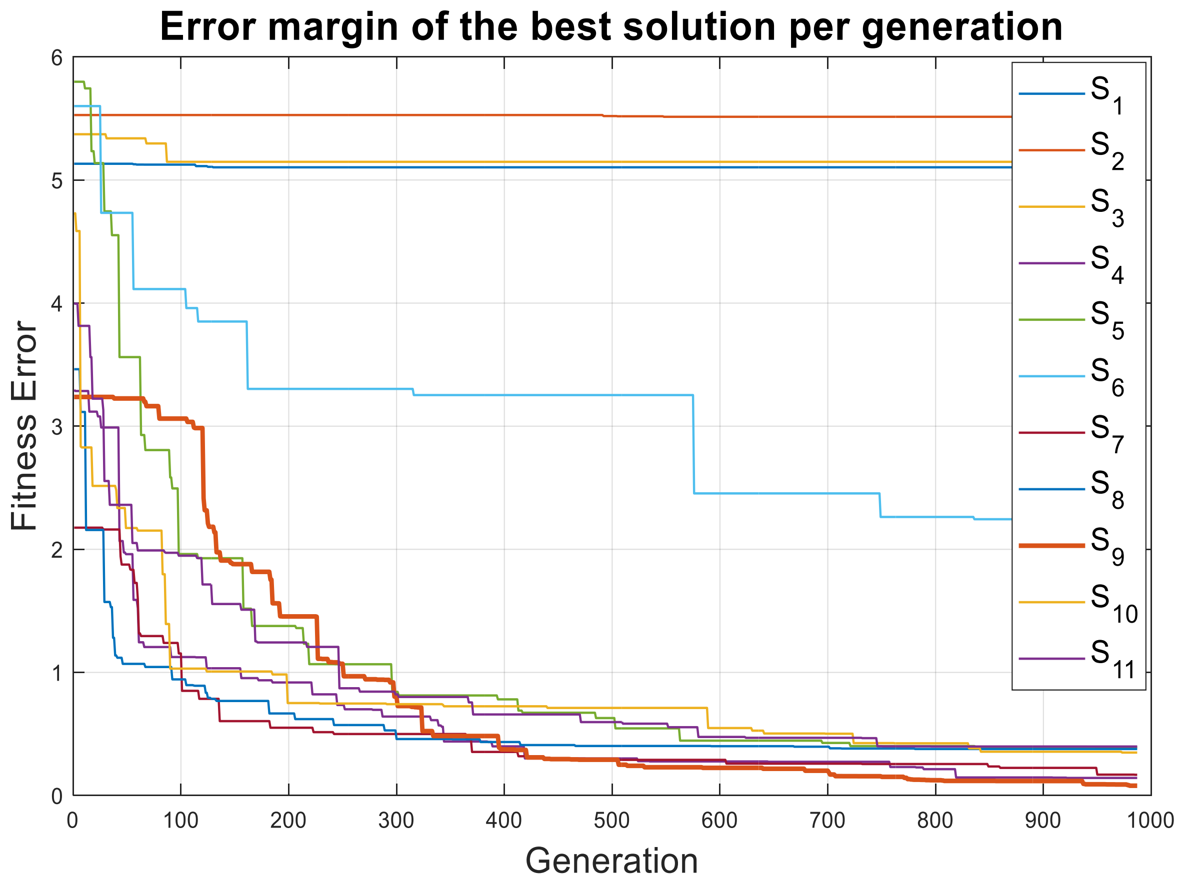

Figure C.4: Parameter performance for GA

From Figure (C.4) and Table (C.1) it is clear that the crossover and mutation rate has a influence on how many generations are needed to achieve the smallest error and the time it take to complete 1000 generations. When the program starts off all the solutions are random and as a result have a large error. For each generation the program aims to reach a solution with a smaller error. This results in a graph that decays exponentially as the generations continues. If the error decays too quickly the solution might get stuck in a local minimum (This is also evident when the graph stays constant for long periods of time see S_6 in Figure (C.4)). When both the crossover and mutations rate are very high the amount of computations done by the computer increases leading to longer execution times. The best result (smallest error of 0.0796) is achieved with S_9 with a crossover rate of 50% and a mutation rate of 5%. This result does not decay too quickly (it

Solution	Crossover rate	Mutation Rate	Error	Time (sec)
S_1	1.0	0.0	5.102	33.58
S_2	0.5	0.0	5.513	27.07
S_3	0.05	0.0	5.147	26.32
S_4	0.0	1.0	0.142	27.971
S_5	0.0	0.5	0.391	32.12
S_6	0.0	0.05	2.040	29.34
S_7	0.5	0.5	0.168	30.74
S_8	0.5	0.25	0.378	25.94
S_9	0.5	0.05	0.0796	26.57
S_{10}	0.25	0.5	0.348	30.80
S_{11}	0.05	0.5	0.398	29.54

Table C.1: Performance of different parameters for GA for 1000 generations.

improves for the entire 1000 generations) and has a execution time of 26.57 seconds. This means to steer the beam of a LAA to a desired direction it will take on average 26.57 seconds.

Experimenting with different parameter values the following values in table (C.2) consistently out perform the best solution S_9 in Figure (C.4). With a crossover rate of 0% and a mutation rate of 100% with a population size of 2 the GA consistently reaches an error margin less than that of S_9 (0.0796). This is a 83% reduction in error and a 98.9% reduction in execution time. This suggests that there does not exists local minima and that improving the phase of each element one at a time is enough to steer the beam. All the steering solutions in Chapter 4 is simulated with these parameters. This deduction will be assumed to hold even during non-uniformly spaced LAA.

Solution	Crossover rate	Mutation Rate	Error	Time (sec)
S_1	0.0	1.0	0.0178	0.272
S_2	0.0	1.0	0.0132	0.272
S_3	0.0	1.0	0.040	0.272

Table C.2: Performance of different parameters for GA for 1000 generations.

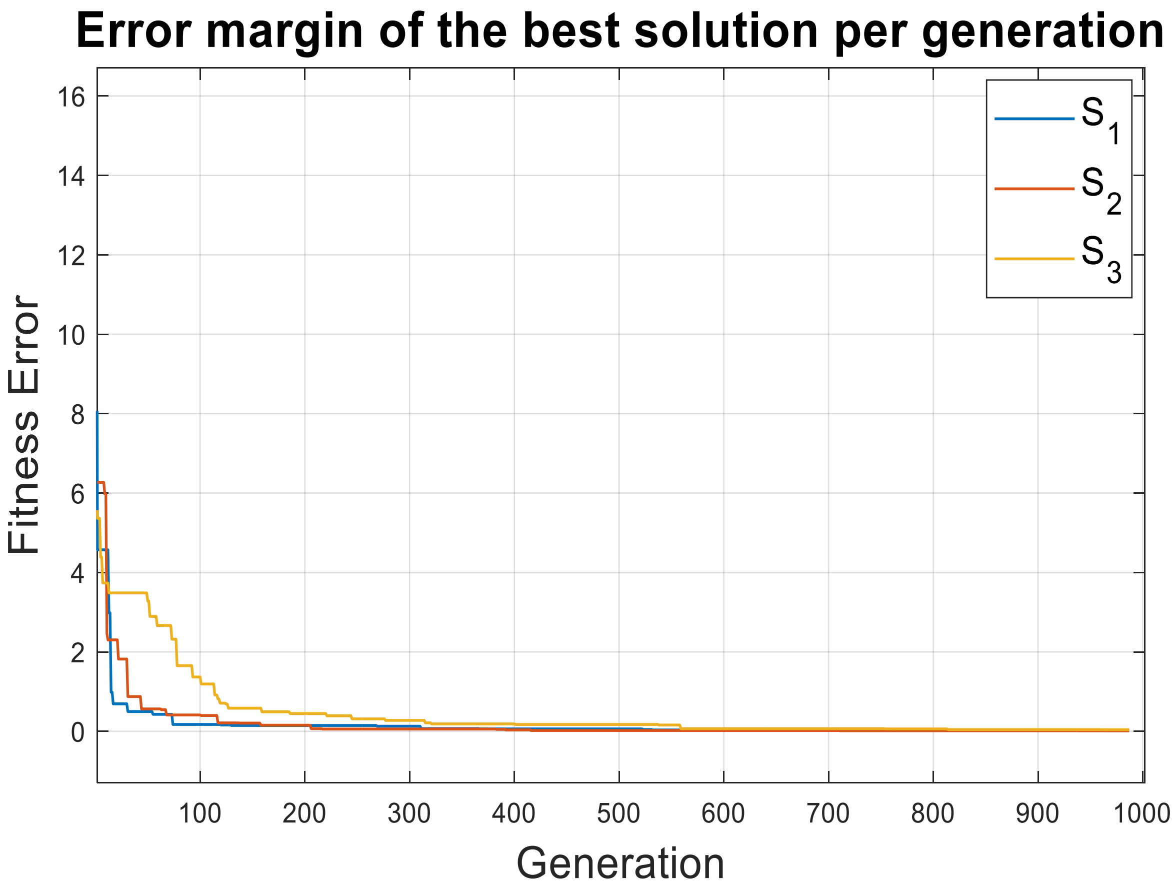

Figure C.5: Parameter performance for GA

C.3. Steering

θ_T	**Solution 1**	**Solution 2**	**Solution 3**
40°	[5, 240, 110, 340, 200, 125]	[288, 162, 25, 255, 120, 354]	[220, 90, 310, 160, 60, 290]
33°	[230, 105, 315, 193, 40, 277]	[250, 104, 316, 162, 15, 231]	[138, 20, 211, 59, 275, 133]
20°	[250, 75, 270, 125, 320, 188]	[214, 70, 270, 109, 309, 164]	[142, 10, 200, 25, 230, 85]

Table C.3: The corresponding phase shift (in degrees) for the inter element spacing solutions in Table 4.5 in Section 4.3