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Techniques for suppressing grating lobes

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Abstract

English

The English abstract.

Afrikaans

Die Afrikaanse uittreksel.

Contents

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Nomenclature

Variables and functions

$E(A)$ Electric field intencity.

Acronyms and abbreviations

GA	Genetic Algorithm.
SLL	Side-Lobe Level.
GLL	Grating-Lobe Level
HPBW	Half Power Bandwidth.
FT	Fourier Transform.
GL	Grating Lobes.

Chapter 1

Introduction

Antennas are designed to radiate electromagnetic energy and is used for long distance communication and/or transmitting of data. [1] In the most basic sense an antenna can be modeled as a point source radiating in all directions at a particular frequency. These point source antennas are knows as omnidirectional antennas. Arranging the antennas in a linear or planar array you can direct the radiation energy in a particular direction. An antenna array also increase the power gain due to he superposition of each antenna's radiation patter. To steer the antennas to an desired direction the antennas can be moved mechanically, however mechanical steering is very slow and limits potential applications that can be exploited.
A solution to the problem is to steer the radiating beam of the antenna array digitally through tweaking digital parameters. This allows the radiating beam of the linear antenna array to be steered much faster, almost instantaneously, due to having no moving parts. However this methods introduces grating lobes when an antenna is steered beyond a critical angle. This is undesirable because it results in power loss, an inefficient antenna array and interference with the transmitted or received data from non-desirable angles.

1.1. Problem statement

This report investigates the effect the parameters (element spacing, phase difference and amplitude taper) have on grating lobes and methods to suppress the grating lobes.

1.2. Objective

The aim is to find a fixed element spacing and the resulting phase shift and amplitude tapering that allow for a wider steering range. This must be achieved while keeping the half power beam width (HPBW) as narrow as possible and maintaining a superior side lobe level (SLL) reduction compared to a uniformly spaced antenna array. The solutions of the investigation of this report results in an improved antenna arrangement and parameter values that yield improved directivity of the radiation energy over a wider steering range for the antenna array, increasing broadcasting efficiency. This means information can be transmitted faster and further than before.

1.3. Summary of work

The paper will look at what grating lobes are and what causes grating lobes. A genetic algorithm (GA) is used to optimise the spacing of the antennas to suppress grating lobes. This is important because after a spacing is selected the antennas will remain fixed and only the phase and amplitude of the signal entering the antenna can be altered. After different element spacing parameters were obtained each is tested for different steering angles to determine which performed best over a wider range. The solution is then compared to existing methods such as uniform antenna array and keeping the HPBW as small as possible.

1.4. Scope

This report did not investigate any antenna arrangements that are not linear such as planer antenna arrays. This report does not investigate the feed system needed to implement the phase shift of amplitude taper for a physical antenna array. Nor does the report describe how to build an antenna or antenna array. It only covers the theory to design an antenna array.

The report is set out as follows. Chapter 2 covers interesting related research that has been done as an attempt to solve the problem statement. 1.1. Chapter 3 discusses how the Genetic Algorithm is used to solve the problem. In chapter 4 the findings are documented and compared to current solutions and the uniform array.

Chapter 2

Literature Review

This chapter presents possible solutions to suppress grating lobes as given in literature. It will focus on the influence element spacing, phase shift and amplitude taper has on grating lobes. The focus will be on finding the best non-uniform spacing for the array to increase steering direction to avoid or suppress grating lobes if they appear.

Cheng [1] showed that the configuration, element spacing, amplitude taper and phase shift of the antenna elements affect the radiation pattern. This is also clear from inspection of the AF 3.1

In 1960 King et al. [2] presented a computational technique to design a non-uniformly spaced linear array. The designer can select a SLL and design the spacing accordingly. However it was limited to only a certain number of Amplitude levels. Unz. [3] was the first to explore the possibility of a non-uniform distribution of array elements when Unz presented a matrix relationship between the radiating pattern and array elements. The idea was however just stated in his short paper and never fully realised.

Harrington [4] explored the possibility of reducing SLL with non-uniform element spacing. The idea he got from King et al. [2] who found certain non-uniform spacing that produced low SLL and out perform uniformly spaced elements. Harrington [4] managed to suppress the SLL without increasing the beam width of the main beam but stated : "It appears unlikely that a simple optimization procedure can be devised for nonuniform arrays because of the nonlinear character of the basic equations with respect to element position." - Harrington [4] But suggested that more work should be done on the topic.

Ridwan et al. [5] deployed a GA to design the optimal excitation amplitudes for a ULA and compared it to the Dolph-Chebyshev and Binomial amplitude distributions. [5]. Ridwan et al. produced amplitude tapering for a given set of antennas that outperformed a uniformly tapered LAA. Ridwan et al. did not explore the use of a GA for the spacing of the elements.

KhalilpourJafar et al. [6] in a recent publication publicized a novel algorithm to suppress grating lobes in a phased-array when the separation between elements was around one wave length apart. KhalilpourJafar et al. [6] showed significant SLL and GLL reduction for an eleven element phased array with up to 180 degree scan angle using randomly spaced elements that creates nulls to cancel out grating lobes. It is important to note that randomly spaced array and non-symmetrically spaced arrays were found. This will be discussed later.

Short coming from literature is that although GA is used together with Dolph-Chebyshev etc the beam is only steered straight forward or normal to the antenna array. Thus optimizing it for an antenna with no steering. The area of beam steering together with non-uniformly spaced array leaves room for a lot of novel work to be done.

2.1. Array Geometry

From the study of other work done on the topic of the suppression of grating lobes it is clear that the spacing of the antenna elements is most important. The paper will consider the uniform array distribution, the non-uniform array distribution that is symmetrical around the mean and a random array distribution.

2.1.1. Uniformly spaced antenna array

From the work done by Matsepe [7] it was shows that uniform arrays will have no grating lobes if the spacing between elements are uniform and less or equal to 0.5λ. The antenna array can be designed for a desired angle which results in a spacing greater than 0.5λ with no grating lobes but still smaller than 1λ. When the distance decreases the constructive and deconstructive areas move further apart. Decreasing the distance between LAA elements result in suppressing grating lobes, for the distance between them move further apart, but increases HPBW. [7] Grating lobes are not suppressed using non-uniform Amplitudes such as binomial, Dolph-Chebyshev and Taylor. They suppress side lobes. Grating lobes bring side lobes. [7]

2.1.2. Symmetrical non-uniformly spaced antenna array

The report will investigate the performance of an antenna array symmetrically spaced round the center point of the antenna arrays with spacing ranging from $0 - 2\lambda$. The spacing will be determined by the Genetic optimization algorithm. More about the GA in section 2.4.

2.1.3. Randomly spaced antenna array

Investigation will also be done on a antenna array spaced randomly with spacing ranging from $0 - 2\lambda$. It will also be using the GA to determine the parameters. Key difference between non-uniformly symmetrically and randomly spaced is that the antennas will not be symmetrical about the center point of the antennas. The idea is that this random distribution might break the periodicity of the AF and result in the suppressing in grating lobes for spacing larger than 1λ

2.2. Amplitude Taper

Different amplitude distributions were recommended in literature where the binomial- and Dolph-Chebyshev distributions being the most common. It is mentioned by Matsepe [7] that changing the amplitude influences the SLL but has not effect on the grating lobes. The results in 4 supports this claim. However [6] showed that if you reduce the grating lobes to SLL you can suppress it further with amplitude tapering.

2.3. Grating Lobes

Grating lobes are unwanted lobes that appear when you steer the main beam beyond a critical angle that is determined by the the array formation. These grating lobes are unwanted because they waste energy and cause interference from radiating stations at directions not in the intended direction. [8]

Grating lobes are as a result of the periodic nature of the AF as is clear from the Fourier Transform properties 3.1.

2.4. Genetic Algorithm

Genetic Algorithm (GA) is an optimization algorithm that uses the concept of evolution and natural selection to solve complex problems. It is a very good approximation tool and does not always yield the best possible solution but can produce a very good solutions relatively quickly. It uses crossover and mutation techniques to evolve a population of random solution to produce the best approximation.

A population of size N of random solutions is created. Each solution is passed through a fitness or error function to determine how good the solution is. Once all N solutions have a fitness score a new population of size N is created and filled via a selection process that allows for cross over and mutation of solution to occur.

The algorithm can be applied even if the fitness function is discontinuous, non differentiable, stochastic or nonlinear.

2.5. Antenna types

There are many different types of antennas for different applications. Omnidirectional antennas such as the monopole and dipole antennas can be made directional if placed in an array. There are directional antennas such as the Rhombic,Yagi and Horn antennas but with them comes the added complexity of antenna design and limitations during beam steering. Since the most popular antenna used in antenna arrays are that of the half-wave dipole antenna this paper will use it during the calculations and simulation. This decision is based on the popularity of the half-wave dipole antenna, the simplicity of its design and to keep the focus on the spacing between the antenna and not on the type of antenna used.

2.6. scope

This paper will not investigate antenna design. It will not investigate 2D antenna arrays. The information gained from this paper can be used to develop 2D antenna array.

Chapter 3

Design and Simulation

To understand how grating lobes appear and how the parameters, element spacing, amplitude taper and phase affect the position and amplitude of the grating lobes the array factor needs to be investigated. 3.1

3.1. Array Factor

The general Array Factor equation for the N element linear antenna-array far field radiation pattern is 3.1 .

$$\sum_{m=0}^{N-1} I_m \exp\left(j\gamma_m\right) \tag{3.1}$$

$$\gamma_m = \beta d_m \cos\theta + \alpha_m \tag{3.2}$$

Each antenna can be modeled as a point source with amplitude I_m with some phase shift due to the relative distance between the elements and the angle of transmitting or receiving signal. 3.3

$$i_m = I_m \exp\left(jkd_m \cos\theta\right) \tag{3.3}$$

The propagation constant $\beta = 2\pi/\lambda$. All the antennas operator at the same frequency. To achieve the maximum gain in the steering direction γ_m should equal to zero. This is achieved by setting

$$\alpha_m = -kd_m \cos\theta_0 \tag{3.4}$$

where

$$k = \frac{2\pi f}{c} = \frac{2\pi}{\lambda} \tag{3.5}$$

and d_m is the distance of the m_{th} antenna from the left most antenna. The cosine function with amplitude one is always between the bands -1 and 1 eq. 3.6.

$$|\cos\theta| \leq 1 \tag{3.6}$$

Solving the angles of eq. 3.6 yields $0 \leq \theta \leq \pi$ or $-\pi \leq \theta \leq 0$. Only the region $0 \leq \theta \leq \pi$ will be considered since the two domains are symmetrical and it is known as the visible region. The pattern is periodic due to the periodic nature of the cosine function.

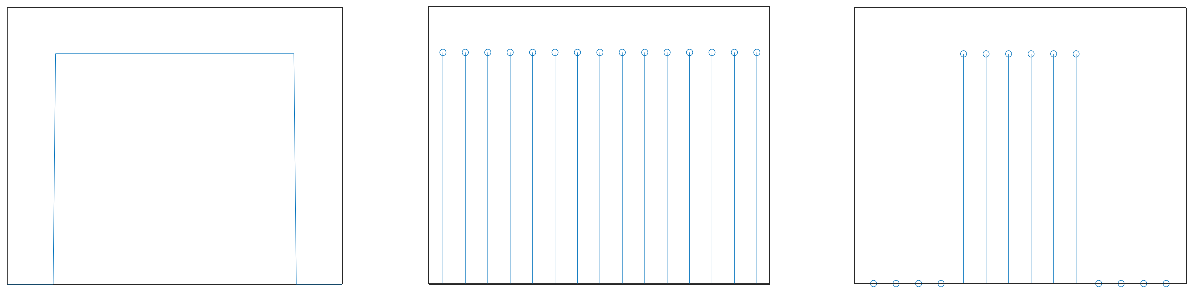
Combing eq. 3.1, 3.4 and 3.5 the following equation can be derived 3.7.

$$\cos\theta - \cos\theta_0 = q\frac{\lambda}{d_{avg}} \tag{3.7}$$

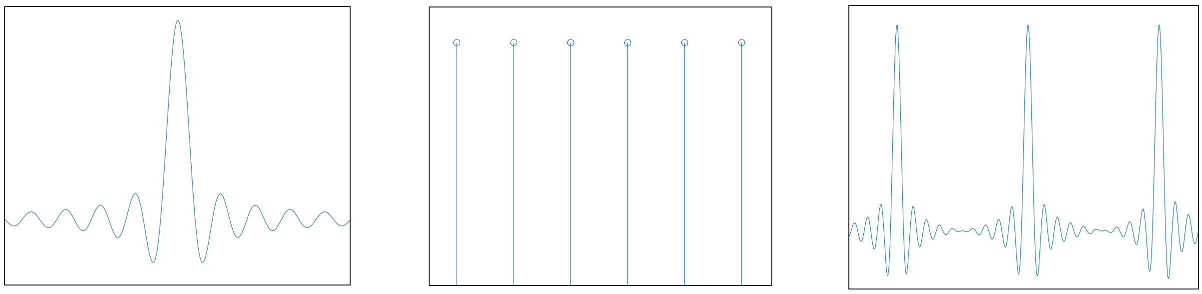
This is an important equation to take note of for it shows that the array factor equation 3.1 is periodic. There will be many main lobes but only $q = 1$ is considered the main lobe while every other q value is considered a grating lobe. Equation 3.7 therefore can predict the location of the grating lobe by solving for θ for a desired steering direction θ_0. It also shows that the only factor that influences the frequency at which the function repeats itself is determined by the average spacing distance d_{avg}. Therefore one can adjust the frequency at which the antennas operate to increase the separation of GL or one can decrease the average spacing between the antenna elements. Only the antenna spacing will be considered in this report.

3.2. Fourier Transform and the AF

The AF eq. 3.1 is the sum of N antenna elements shifted by some distance d_m. This is also the equation of the discrete Fourier transform. Which means the AF is equal to the Fourier transform of the elements.
As shown in eq. 3.3 each antenna element can be modeled as a point source with a frequency shift. Let us assume that each element is uniformly excited and has no additional phase shift and the spacing is uniform. The antenna array can then be seen as a sampled rectangular window function. From our Fourier theory we know that a rectangular window function transforms into a sinc function when the Fourier transform is applied. The main beam of the sinc function being the main beam of the AF and the ripples of the sinc function being the side lobes that come with the main beam. However when we sample the rectangular window function we get a impulse train equalling that of the N antennas. The Fourier transform of a sampled rectangular window function is a sinc function convoluted with a Dirac comb function. These extra main beams are the GL we want to suppress. See the graphical representation 3.1



(a) Rectangular Window. **(b)** Dirac Comb. **(c)** Sampled Window.



(d) FT: Sinc function. **(e)** FT: Dirac Comb. **(f)** Convolution.

Figure 3.1: $X°$ beam steering from $90°$

Plot 3.1a is the rectangular window with a width equal to N. The sample function can be modeled as a Dirac comb function with $T = \frac{\lambda}{d}$ multiplied by a rectangular window. From our FT understanding the AF will equal a sinc function convoluted with a Dirac comb function eq. 3.8.

$$AF = rect(N) * \sum_{-\infty}^{\infty} \delta(x - md) \tag{3.8}$$

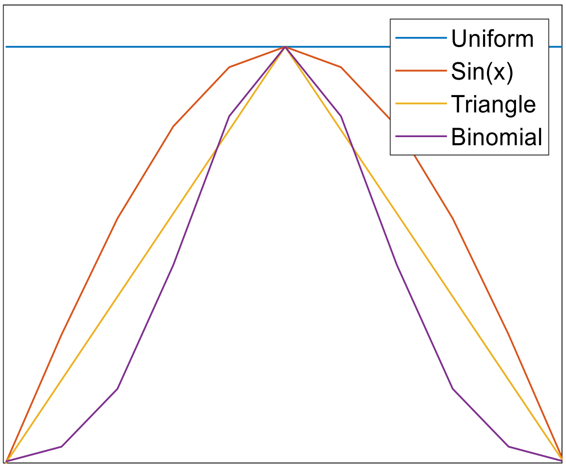
We know that the FT of such a rectangular window is the sinc function. [9] In our case it will be in the visible domain which is between $0°$ to $180°$.

$$AF = \frac{\sin \frac{\pi N}{\lambda}}{\frac{\pi N}{\lambda}} * \sum_{-\infty}^{\infty} \delta(u - \frac{m\lambda}{d}) \tag{3.9}$$

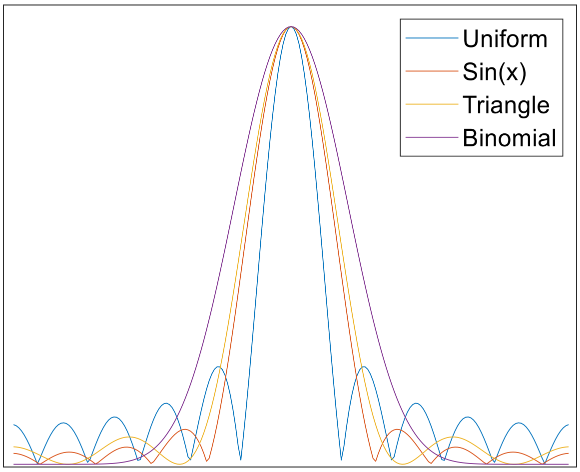
This periodicity is the cause of grating lobes. The first sinc beam being the main beam and every consecutive sinc beam being a grating lobes. The position of the m_{th} beam is represented by $m\frac{\lambda}{d}$. This again shows that decreasing the d will increase the distance between the grating lobes. Taking the absolute of the graph and wrapping it around a point yield a polar plot.

It is known from the Nyquist criteria in Digital Signal sampling [9] that the $f_S \geq 2B$. The sampling frequency myst be at least twice as large as the highest frequency you want to sample. Since the antenna array can be seen as as a sampled rectangular window with different excitation levels the sample frequency must be $d \leq 0.5\lambda$ to avoid aliasing. From digital signal sampling we know that this holds true even if we sample at a random interval the $f_{avg} \geq 2B$ to avoid aliasing. Indicating that a randomly spaced array would need to have a average spacing of 0.5λ.

From our FT understanding it know that changing the window shape has an effect on the ripples of the sinc function or the side lobes. [9] [8]. This is illustrated in fig. **??**



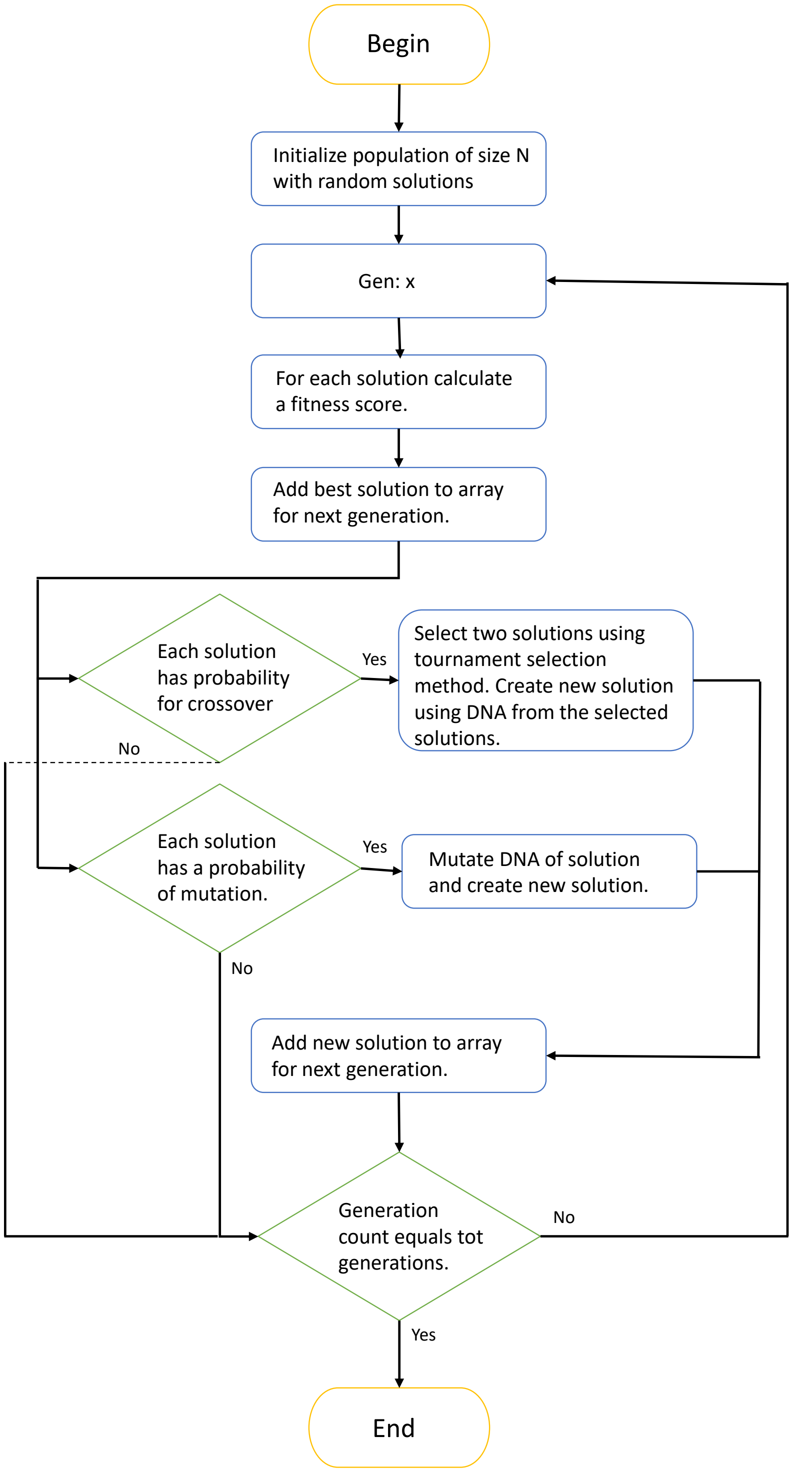
(a) Window shape.



(b) Fourier Transform of window.

3.3. Genetic Algorithm

The GA will be structured as follows.



(a) GA flow chart

3.3.1. Fitness score

The most important part of the genetic algorithm is the fitness function. It described how good a solution is. In the case of the antenna array is takes the element spacing, phase shift and amplitude of each antenna as input parameters and calculates a fitness score of the LAA.

Many fitness functions were considered however the two that produced the best results and were used interchangeably during simulations are eq 3.12.

The first fitness function calculates the ratio of the average gain within the main beam at a desired angle θ_T given a error margin to average gain outside the main beam with the same error margin. The GA then tries to minimize eq. 3.12

$$inner = \frac{1}{N_o} \sum_{\theta_T - q}^{\theta_T + q} AF(\theta) \tag{3.10}$$

$$outer = \frac{1}{N_p} (\sum_{0}^{\theta_T - q} AF(\theta) + \sum_{\theta_T + q}^{\pi} AF(\theta)) \tag{3.11}$$

$$fitness = \frac{outer}{inner} \tag{3.12}$$

The second fitness function is a tweaked version of the directivity equation. 3.13 which calculates the ratio between the max gain at a desired angle θ_T to the average gain over the entire visible space.

$$D_o = \frac{\pi |AF_{max}|^2}{\int_{\pi}^{0} AF} \tag{3.13}$$

$$fitness = \frac{\sum_0^{\pi} AF(\theta)}{AF(\theta_T)^2} \tag{3.14}$$

The GA tries to minimize the fitness function as far as possible. The numerator in both fitness functions are wasted energy and should be zero. The denominator is the part we want to maximize. Maximizing the denominator or minimizing the error/numerator would minimize the fitness function. It is the It is unlikely that the GA will find the absolute minimal value but as seen in the results section it found really good results that could compete or even out perform the uniform linear array. These fitness scores are then used during the selection process.

3.3.2. Selection

Two popular selection techniques were considered namely tournament selection and roulette selection. Roulette selection scales the probability of a random solution being selected by the fitness score achieved relative to all the other fitness scores. This means the better the solution the more likely it will be picked for cross over. Limiting the diversity in the population. Tournament selection is therefore implemented due it being less biased towards the best solution. It follows the following procedure.Four solutions are picked at random from the population and their fitness scores are compared. The top two solutions are then considered for crossover.

3.3.3. Crossover

During every selection process there is a probability that crossover can occur or a crossover rate. There is a chance that no crossover occurs and then the selected solution is immediately copied over to the new generation. If crossover occurs half the DNA from the two selected solution are combined to produce a new potentially better solution and is then added to the new generation.

3.3.4. Mutation

During every selection process there is a probability that mutation can occur or a mutation rate. During this process the selected solution will undergo a random change to its DNA. The reason for a mutation rate is to help the algorithm not get stuck on a local minimum/maximum but introduce some randomness to nudge it out of it. The solution is then added to the new generation.

These steps are then repeated until the indicated generation runs are finished or when a solution is found.

Chapter 4

Results

4.1. Amplitude Taper

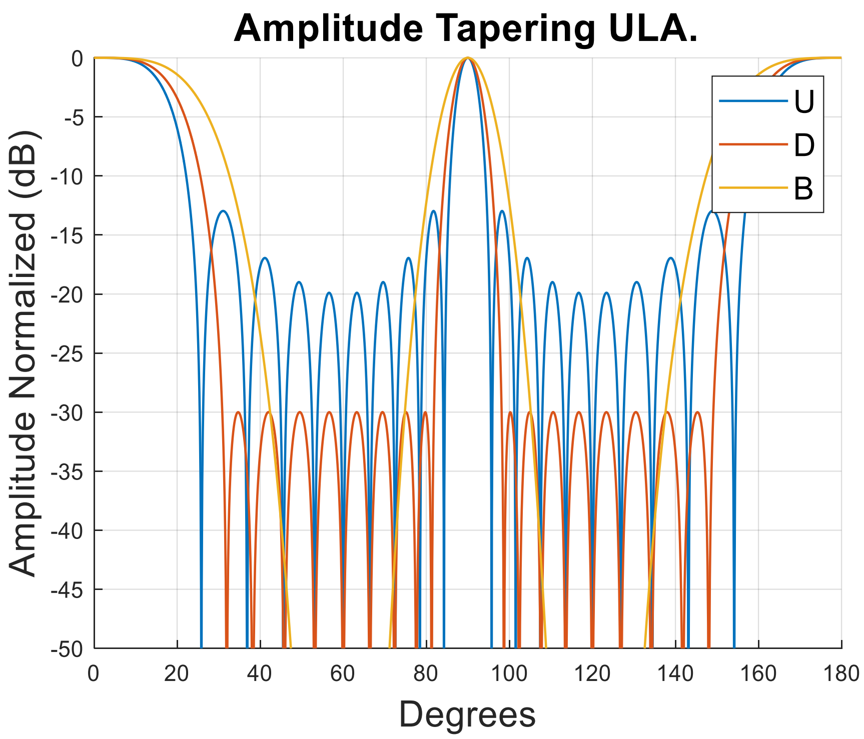


Figure 4.1: Uniform vs Dolph-Chebyshev vs Binomial Amplitude taper.

4.2. Uniformly spaced array

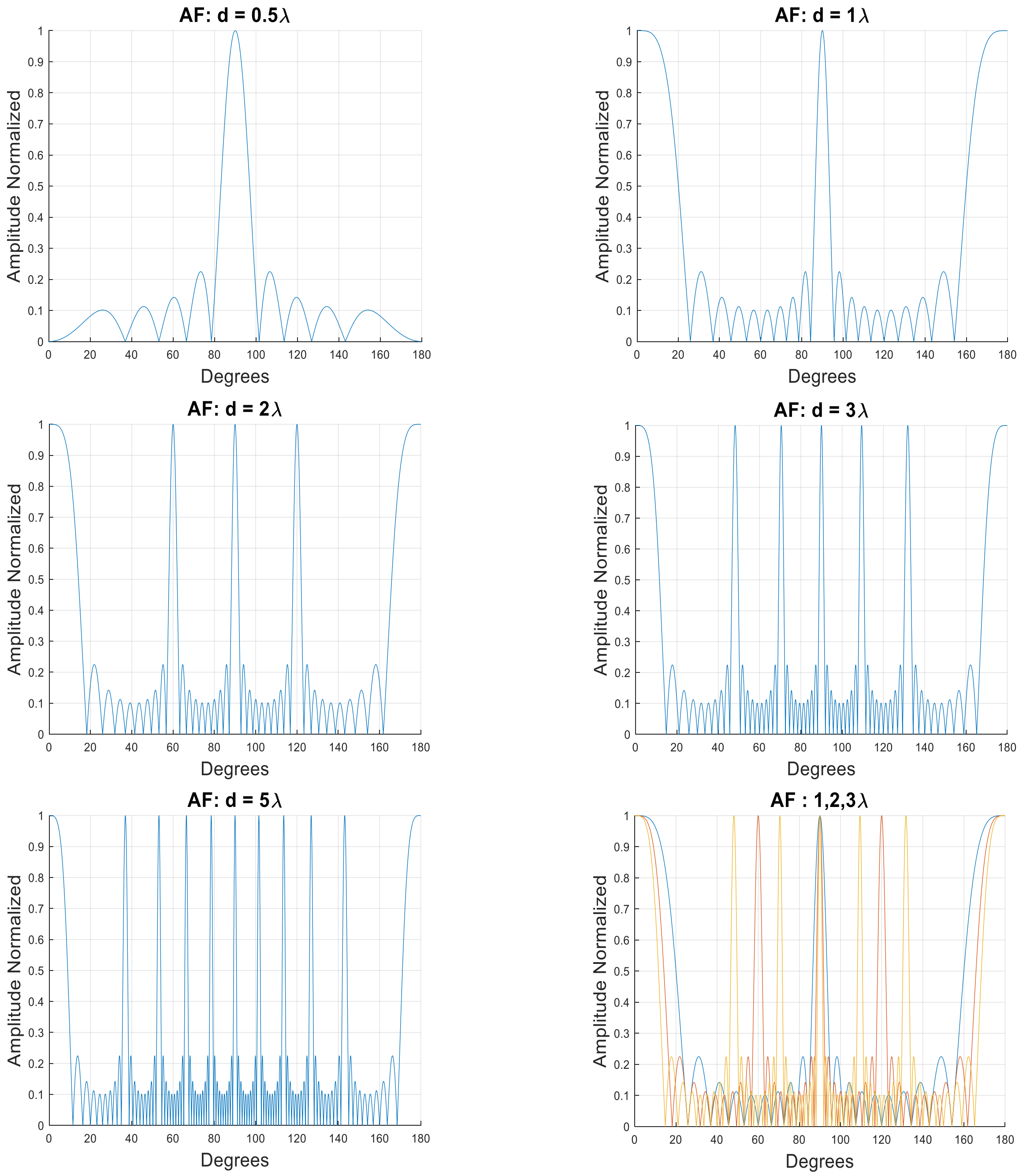


Figure 4.2: AF for Uniform element spacing Antenna Arrays.

The results obtained in is for a uniformly spaced and amplitude antenna arrangement. It is clear from that as the distance between the antenna increase beyond 1λ grating lobes start to appear. The position of the grating lobes are predicted by $\arccos(k * \frac{\lambda}{d})$

Uniformly Spaced	GL $k = 1$	Predicted	GL $k = 2$	Predicted	GL $k = 3$	Predicted
0.5λ	NONE	-	NONE	-	NONE	-
1λ	$0°$	$0°$	NONE	-	NONE	-
2λ	$60°$	$60°$	$0°$	$0°$	NONE	-
3λ	$70.54°$	$70.528°$	$48.18°$	$48.189°$	$0°$	$0°$
5λ	$70.46°$	$78.463°$	$66.42°$	$66.421°$	$53.13°$	$53.13°$

Table 4.1: Mathematical predictions vs AF calculation. Only considering $0 - 90°$ since symetry around main beam at $90°$

4.3. Symmetrical non-uniformly spaced array

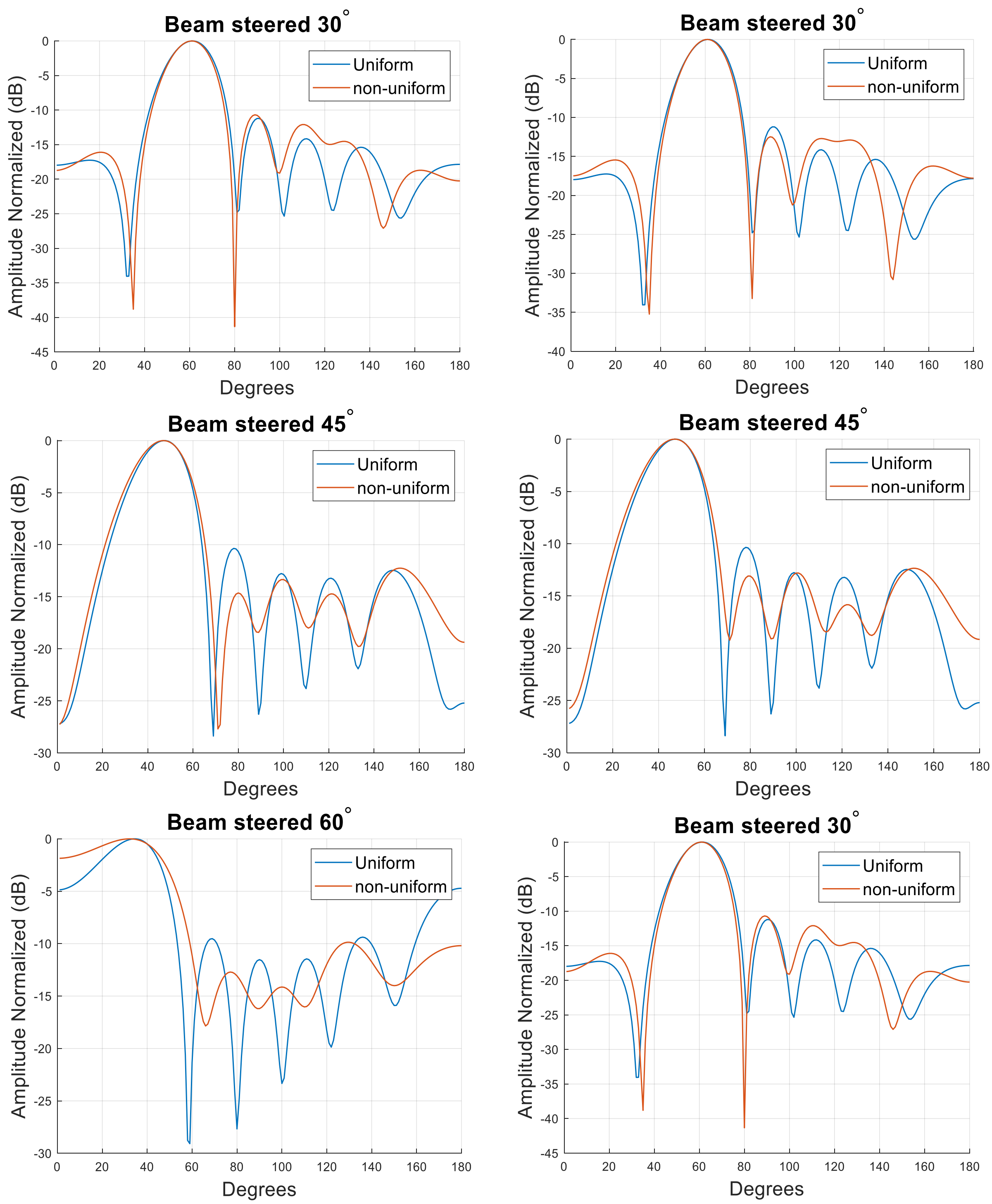


Figure 4.3: X° beam steering from 90°

45 deg there is a reduction in the first side lobe of -10.41 to -14.63 = 4.22 db with the highest side lobe being at -12.26 db compared to -10.41 of the uniform array. After optimization the highest side lobe sits at -12.36 db compared -10.41

Direction	Inter Element Spacing λ	Phase Shift $X°$
30°	0.492 0.577 0.472 0.577 0, 492	245 136.3 53.8 311.5 210.3 123.9
45°	0.434 0.5 0.501 0.5 0.434	176.5 51.7 276.4 1623 45 274.9

Table 4.2: Element Spacing and Phase shift.

60 deg beam steer. Uniform only at 35 deg. Highest side love -9.868 vs -9.379 db HPBW much wider.

Chapter 5

Summary and Conclusion

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Appendix A

Project Planning Schedule

This is an appendix.

Appendix B

Outcomes Compliance

This is another appendix.

Appendix C

More Results