

Measurement of conductivity of potassium chloride solution and critical micelle concentration of sodium dodecyl sulfate

Rui Li

Qiushi science class (chemistry)
Chu Kochen Honor College

Zong Wei Huang

Qiushi science class (chemistry)
Chu Kochen Honor College

The infinite-dilute molar conductivity for potassium chloride solution as well as the critical micelle concentration of sodium dodecyl sulfate is obtained using Wheatstone bridge method. This measuring method is questioned according to the numerical analysis of the system using *Mathematica*.

Keywords: conductivity, potassium chloride, CMC, sodium dodecyl sulfate, Wheatstone bridge

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I. INTRODUCTION

Electrical resistivity is a fundamental property of a material that quantifies how strongly that material opposes the flow of electric current; while Electrical conductivity is defined as the reciprocal of electrical resistivity, and measures a material's ability to conduct an electric current. It is of great importance in electricity-involving activities, fuel cells being one of the examples.

Conductivity of a solution G follows the following equation:

$$G = \kappa \frac{A}{l} \quad (1)$$

where A stands for the cross sectional area of the solution towards the flow of electricity, and l for the length of it along the direction. κ is a colligative constant that is determined for a certain substance or a solution.

Molar conductivity of a solution is defined as the conductivity of a solution containing 1 mol of the solvent,

$$\Lambda_m = \kappa/c \quad (2)$$

For dilute solution of strong electrolytes, there is Kohlrausch law:

$$\Lambda_m = \Lambda_m^\infty - A\sqrt{c} \quad (3)$$

which reveals linear relation between Λ_m and c . Surfactants, however, presents a significant change of the slope A at around a density, which is called critical micelle concentration (CMC).

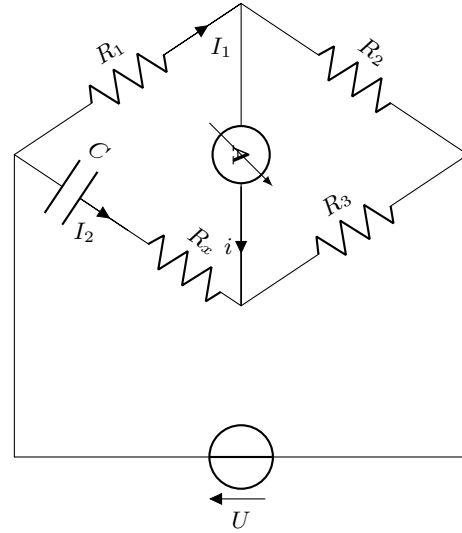


FIG. 1. Electric circuit of the experiment, where R_x stands for the electrical resistivity of the solution.

Wheatstone bridge is utilized to measure the electrical resistivity of the solution, thereby obtaining the molar conductivity of it. Alternating current is applied to avoid uneven concentration of solvents over the whole solution.

II. RESULTS AND ANALYSIS

The data from the experiment are shown in Table.I,II. The circuit adopted is represented by Fig.1 with the symbols given. Fig.2 shows graphic analysis of the data of the conductivity of potassium chloride solution, which shows strong inlinearity. The first data point contributes most to it, which might be attributed to contamination from the potassium chloride solution in high concentration. Excepting the first point, the line fitted is

$$\Lambda_m = 0.01992 - 0.04484\sqrt{c} \quad (4)$$

which leads to the conclusion that $\Lambda_m^\infty = 0.01992 \text{ S} \cdot \text{m}^2 \cdot \text{mol}^{-1}$, with cell constant $K_{\text{cell}} =$

TABLE I. Raw data of the measurement of conductivity of potassium chloride solution

c/M	R_1/Ω	R_2/Ω	R_3/Ω
0.001250	2500	2000	1725.50
	2500	3000	2611.00
	2500	4000	3478.00
0.002500	2500	4000	2161.70
	2500	5000	2652.50
	2500	3000	2611.00
0.005000	2500	5000	1413.40
	2500	6000	1700.70
	2500	7000	1952.50
0.01000	2500	7000	1127.40
	2500	8000	1290.80
	2500	9000	1410.30
0.02000	2500	2500	215.00
	2500	5000	431.21
	2500	10000	861.00

TABLE II. Raw data of the measurement of CMC of sodium dodecyl sulfate

c/M	R_1/Ω	R_2/Ω	R_3/Ω
0.005000	2500	2500	2124.60
	2500	3500	2975.80
	2500	4500	3836.90
0.007000	2500	2500	1655.50
	2500	3500	2307.40
	2500	5500	3701.00
0.007500	2500	2500	1624.30
	2500	3500	2259.30
	2500	4500	2910.80
0.008000	2500	2500	1565.00
	2500	3500	2259.30
	2500	4500	2762.00
0.008500	2500	2500	1506.20
	2500	3500	2098.00
	2500	4500	2710.25
0.009000	2500	2500	1413.13
	2500	3500	1970.73
	2500	4500	2503.75
0.01000	2500	3000	1541.45
	2500	4000	2060.05
	2500	5000	2560.30
0.01300	2500	3000	1309.80
	2500	3000	1325.00
	2500	4000	1766.50
	2500	5000	2177.50
0.01500	2500	5000	2020.00
	2500	6000	2400.20
	2500	8000	3217.40

59.53 m⁻¹.

Fig.3 and Fig.4 shows graphic analysis of the data of the conductivity of sodium dodecyl sulfate, with two linearly fitted lines from two separated datasets which have marked in the legends. Different slopes are conspicuous of two lines, nonetheless non-linearity is also apparent. The solutions for the intersection are 0.007660 M and 0.009804 M representing CMC, which present quite

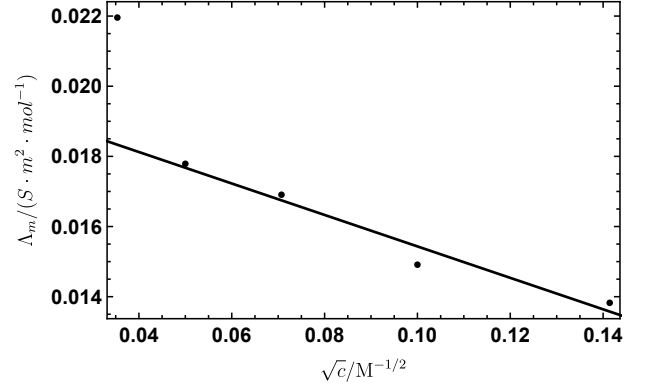


FIG. 2. $\Lambda_m - \sqrt{c}$ data plots with its linear model fit

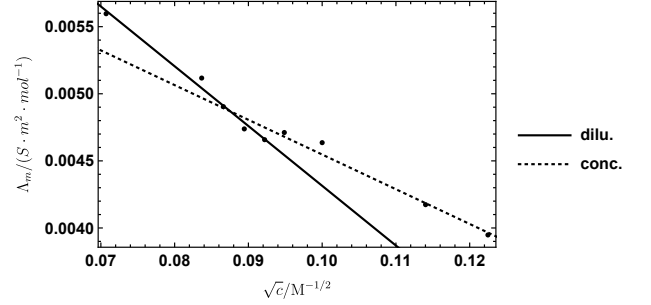


FIG. 3. $\Lambda_m - \sqrt{c}$ data plots with its linear model fits

different results.

there might exist a huge error in both experiments, which stems from the insensibility of the current detector in the electric bridge system. To further elaborate it, a differential equation system is deduced,

$$\begin{cases} I_1 R_1 + (I_1 - i) R_2 = U_0 \sin \omega t \\ I_2 R_x + (I_2 + i) R_3 = U_0 \sin \omega t - u \\ (I_1 - i) R_2 - i r - (I_2 + i) R_3 = 0 \\ I_2 = C \frac{du}{dt} \end{cases} \quad (5)$$

with each term represented in Fig.1. The solution is considered as a series connection of a theoretical conductor and resistance, regardless of the electrochemical reaction. Such an assumption does not lead to a much simpler form

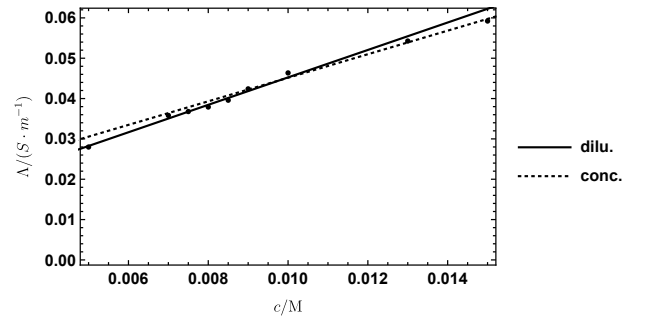


FIG. 4. $\Lambda - c$ data plots with its linear model fits

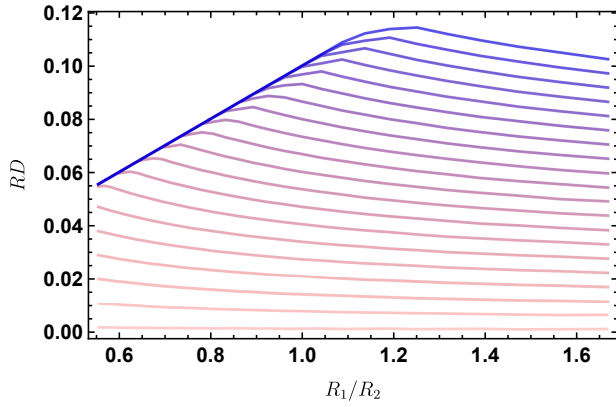


FIG. 5. The relative deviation of the R_x calculated from possible R_3 sets compared with true R_x value (1000 Ω) v.s. the amplifier (R_1/R_2), setting C to 1 F. Different lines represent different minimum detectable current of the current detector varying from $10^{-6} \sim 10^{-4}$ A, with each line representing minimum current 5×10^{-6} A higher than beneath.

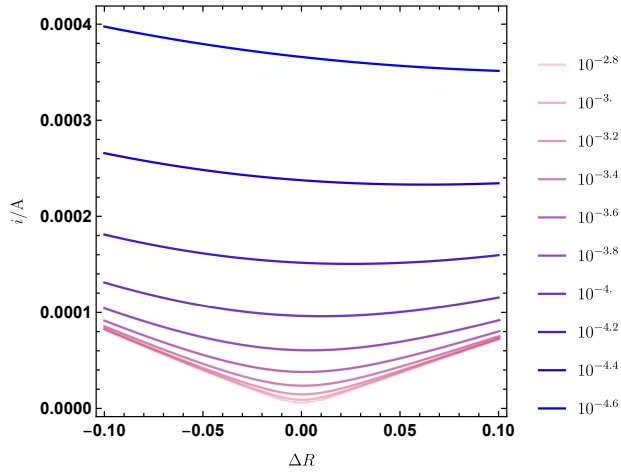


FIG. 6. The current that flows through the current detector v.s. the relative deviation of R_3 from R_x , with R_1/R_2 set to 1, at different values of C which have been listed in the legends with the unit F.

of the solution thus *Mathematica* is utilized to give analytical solution with numerical sets of data, thanks to its superior symbolic calculating system:

```
WheatstoneSolve =
DSolve[{I1[t] R1 + (I1[t] - i[t]) R2 == U0 Sin[w t],
I2[t] Rx + (I2[t] + i[t]) R3 ==
U0 Sin[w t] - u[t], (I1[t] - i[t]) R2 -
i[t] r - (I2[t] + i[t]) R3 == 0} /. {I2[t] ->
c D[u[t], t]}, {I1[t], u[t], i[t]}, t]
```

and is packaged as a function:

```
isolve = i[t] /. WheatstoneSolve;
relation[r1_, r2_, r3_, rx_, R_, u0_, q_, omega_] :=
FindMaximum[
isolve /. {w -> omega, U0 -> u0, R1 -> r1, R2 -> r2,
R3 -> r3,
```

```
Rx -> rx, r -> R, C[1] -> 0, c -> q}, {t, 0}][[1]]/
Sqrt[2];
```

Parameters are given as discrete sets of numerical data and manipulated by the packaged function, returning results:

```
testlist =
Table[Table[{2500, 2500 + 100 a, (2500 + 100 a)/2500*1000
+ b, 1000,
100, 8, q, 50}, {a, -10, 20}, {b, -100, 100, 0.2}], {q
0.1, 1,
0.1}]

relation @@@ Flatten[testlist, 2]

ratiosensitivitymatrix =
Table[Transpose[{Table[2500/(
2500 + 100 a), {a, -10,
20}], (Partition[
Length /@ (Select[#, # < q*10^-6 &] & /@
Partition[simulationdata, 1001]),
31][[1]]/10/1000)*
Table[2500/(2500 + 100 a), {a, -10, 20}]]], {q, 1,
100, 5}];

testlist2 =
Table[Table[{2500, 2500 + 100 a, (2500 + 100 a)/2500*1000
+ b, 1000,
100, 8, q/10^10, 50}, {a, -10, 20}, {b, -100, 100,
0.2}], {q, 0.1,
1, 0.2}]

simulationdata2 = relation @@@ Flatten[testlist2, 2]

ratiosensitivitymatrix2 =
Table[Table[
Transpose[{Table[2500/(
2500 + 100 a), {a, -10,
20}], (Partition[
Length /@ (Select[#, # < q*10^-6 &] & /@
Partition[simulationdata, 1001]), 31][[k
]]/10/1000)*
Table[2500/(2500 + 100 a), {a, -10, 20}]]], {q, 1,
100,
5}], {k, 1, 5}];

testlist3 =
Table[Table[{2500, 2500, 1000 + b, 1000, 100, 8, 10^(-q),
50}, {b, -100, 100, 0.2}], {q, 0, 10, 0.2}]

simulationdata3 = relation @@@ Flatten[testlist3, 1]
```

It needs to be stated that R_1 is set globally as 2500 Ω , R_x as 1000 Ω , and r , resistance of the current detector, as 100 Ω in the test data sets, U as 8 V, and R_2 changes on an 100 Ω basis around 2500 Ω , while R_3 changes on an 0.2 Ω basis, which simulates the manipulations during the experiment, around $\frac{R_1 R_2}{R_x}$, or 'what it should be'. The function returns the current that flows through the current detector, or to be more specific,

$$i = \frac{i_{\max}}{\sqrt{2}} \quad (6)$$

according to the sinusoidal behavior of alternative currents.

The corresponding result data is manipulated according to the principle that a current under a certain number is regarded as ‘undetectable’, and the corresponding R_3 sets are regarded as ‘optimized solutions’ for the circuit equally. This gives Fig.5, which highlights the dependence of the choice of R_1/R_2 , if the current detector is not sensitive enough. It also reveals the fact that increase of R_1/R_2 rather amplifies the error, if the sensitivity is relatively high, and it has the opposite trend in lower sensitivity of the current detector. In other words, it is only recommended to adopt lower R_1/R_2 , or larger amplifier of R_x towards R_3 , only if the current detector is not sensitive enough, at a level of 10^{-5} A.

Fig.6 reveals the dependence of the equivalent capacity of the solution, and that a solution having equivalent capacity larger than 10^{-3} F can completely ignore the effect of capacitor, and gives correct result of R_x , which is very hard to achieve. It means that system error might also arise from the capacity of the solution, even under alternative current. From the two figures the validity of

the method is questioned.

III. CONCLUSION

The infinite-dilute molar conductivity for potassium chloride solution is deduced as $\Lambda_m^\infty = 0.01992 \text{ S} \cdot \text{m}^2 \cdot \text{mol}^{-1}$, while the CMC of sodium dodecyl sulfate is given as 0.007660 M and 0.009804 M using two methods. The measuring method is questioned according to great deviation of R_x as well as inevitable system error that arises from the low capacity of the solution.

IV. ACKNOWLEDGEMENT

This report is greatly assisted by Zong Wei Huang, who provide numerous help in furnishing the theoretical model of the system.