

\Rightarrow

$$(1) \quad [A, B] = AB - BA$$

$$(2) \quad [x, p]\psi = (xp - px)\psi = -i\hbar\left[x\frac{\partial}{\partial x}\psi - \frac{\partial}{\partial x}(x\psi)\right] = i\hbar\psi$$

$$(3) \quad \psi(x) = x\psi\psi(p) = p\psi$$

$$\begin{aligned} \hat{O} &= \\ &\sum_i \phi_i \phi_i \hat{O} \sum_j \phi_j \phi_j \\ &\sum_{ij} \phi_i \phi_i \hat{O} \phi_j \phi_j \\ &\sum_{ij} \phi_i \hat{O} \phi_j \phi_i \phi_j \\ &\sum_{ij} O_{ij} \phi_i \phi_j \\ &\psi\psi \\ &\mathbf{x} \frac{\mathbf{p}}{dt} \mathbf{F} = \frac{d\mathbf{p}}{dt} \end{aligned}$$

$$(4) \quad \frac{d\phi}{dt} = \frac{1}{i\hbar} \left[\frac{\mathbf{p}^2}{2m} + V(\mathbf{x}) \right] \phi$$

$$\begin{aligned} \mathbf{x} \frac{d\psi}{dt} &= \frac{d\psi}{dt} \mathbf{x} \psi + \psi \mathbf{x} \frac{d\psi}{dt} = -\frac{1}{i\hbar} \psi \left[\frac{\mathbf{p}^2}{2m} + V(\mathbf{x}) \right] \mathbf{x} \psi + \psi \mathbf{x} \frac{1}{i\hbar} \left[\frac{\mathbf{p}^2}{2m} + V(\mathbf{x}) \right] \psi = -\frac{1}{i\hbar} \left(\frac{\mathbf{p}^2}{2m} \mathbf{x} - \mathbf{x} \frac{\mathbf{p}^2}{2m} \right) \psi = -\frac{1}{2mi\hbar} \psi [\mathbf{p}^2, \mathbf{x}] \psi = -\frac{1}{2mi\hbar} \psi 2\mathbf{p}[\mathbf{p}, \mathbf{x}] \psi = \frac{1}{m} \phi \mathbf{p} \phi = \mathbf{p} \\ \mathbf{p} \frac{d\psi}{dt} &= \frac{d\psi}{dt} \mathbf{p} \psi + \psi \mathbf{p} \frac{d\psi}{dt} = -\frac{1}{i\hbar} \psi \left[\frac{\mathbf{p}^2}{2m} + V(\mathbf{x}) \right] \mathbf{p} \psi + \psi \mathbf{p} \frac{1}{i\hbar} \left[\frac{\mathbf{p}^2}{2m} + V(\mathbf{x}) \right] \psi = -\frac{1}{i\hbar} \psi [V(\mathbf{x}), \mathbf{p}] \phi = -\frac{1}{i\hbar} \phi i\hbar \frac{\partial V(\mathbf{x})}{\partial \mathbf{x}} \phi = \phi \left[-\frac{\partial V(\mathbf{x})}{\partial \mathbf{x}} \right] \phi = \mathbf{F} \\ \mathbf{x} \mathbf{p} \end{aligned}$$

$$\begin{aligned} \psi &= \\ &[V(x), -i\hbar \frac{\partial}{\partial \mathbf{x}}] \psi \\ &= -i\hbar V(x) \frac{\partial}{\partial \mathbf{x}} + \\ &= i\hbar \frac{\partial}{\partial \mathbf{x}} [V(\mathbf{x}) \psi] \\ &= -i\hbar V(x) \frac{\partial}{\partial \mathbf{x}} + \\ &= i\hbar \frac{\partial V(\mathbf{x})}{\partial \mathbf{x}} \psi + \\ &= i\hbar V(\mathbf{x}) \frac{\partial \psi}{\partial \mathbf{x}} \\ &= i\hbar \frac{\partial V(\mathbf{x})}{\partial \mathbf{x}} \psi \\ &= \{ H, \psi_i \} = \\ &= E_i \psi_i \\ &= \psi = \\ &= \sum_i c_i \psi_i \\ &= \psi H \psi = \\ &= \sum_i c_i^* \psi_i H \sum_j c_j \psi_j \\ &= \sum_{ij} c_i^* c_j E_j \delta_{ij} = \\ &= \sum_i c_i^* c_i E_i \geq \\ &= \sum_i c_i^* c_j E_{ij} = \\ &= E_{ij} \end{aligned}$$

$$(5) \quad H\psi_i = E_i\psi_i H = H^{(0)} + \lambda H^{(1)}$$

$$\begin{aligned} \{ \psi_i &= \\ \psi_i^{(0)} &+ \\ \lambda \psi_i^{(1)} & \\ E_i &= \\ E_i^{(0)} &+ \\ \lambda E_i^{(1)} & \\ \Rightarrow & \\ (H^{(0)} &+ \\ \lambda H^{(1)}) &(\psi_i^{(0)} + \\ \lambda \psi_i^{(0)}) &= \\ (E_i^{(0)} &+ \\ \lambda E_i^{(1)}) &(\psi_i^{(0)} + \\ \lambda \psi_i^{(1)}) & \\ \Rightarrow & \\ \{ (H^{(0)} &- \\ E_i^{(0)}) \psi_i^{(0)} &= \\ 0 & \\ (H^{(0)} &- \\ E_i^{(0)}) \psi_i^{(1)} &+ \\ (H^{(1)} &- \end{aligned}$$