Numerical Project Pt. 2:
Finite differences and numerical algorithms for the heat equation.

Walter Ullon.

March 2, 2016

Results and discussion.

For part 2 of the numerical project we experiment with several finite difference approximation schemes (explicit, implicit and Crank Nicolson) and explore their relative strengths and weaknesses with regards to implementation.

For instance, the explicit scheme, while easiest to implement, restricts us in terms of the allowable spatial and time steps that would result in stable behavior of the solution to the heat equation. Following the vonNeumann "stability criterion", any ratio $\frac{dt}{dx^2} > 1/2$ results in the solution blowing up. This implies that the explicit scheme is conditionally stable. Figure 1 below illustrates the behavior.

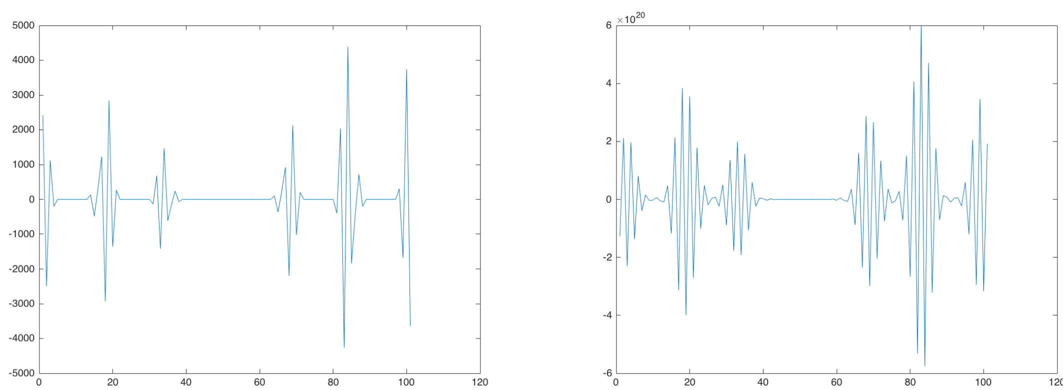

Figure 1: Instability of the explicit scheme for $dt = 0.001, dx = 0.01$ at different time steps. Ratio $\frac{dt}{dx^2} = 10$.

The implicit and the Crank Nicolson schemes, however, are unconditionally stable, since dt, dx can be chosen without having to satisfy the vonNeumann criterion. Figure 2 below illustrates the results. Please note the incorrect local oscillations present in the Crank Nicolson scheme discussed in Olver's text.

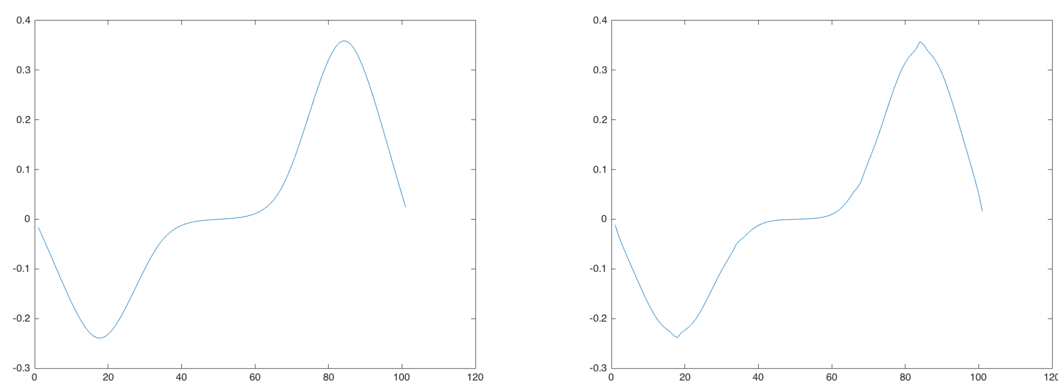

Figure 2: Left: implicit scheme. Right: Crank Nicolson. $dt = 0.001, dx = 0.01$

Regardless of the scheme chosen (provided conditions are met), we can see decay in the solution of the heat equation.