1 Gauss-Seidel

```
A = [6 2 3 4 1;
     2 6 2 3 4;
     3 2 6 2 3;
     4 3 2 6 2;
     1 4 3 2 6;
];
b = [4;24;8;5;24];
ans = A\b
n = size(b,1);
x = zeros(n,1);
r = b - A*x;
eps = 1e-6;
while max(abs(r))>eps
    for i = 1:n
        sum1 = A(i,1:i-1)*x(1:i-1,1);
        sum2 = A(i,i+1:n)*x(i+1:n,1);
        x(i,1) = (b(i,1)-sum1-sum2)/A(i,i);
    end
    r = b - A*x;
end
x
```

```
ans =

    1.0000
    3.0000
   -1.0000
   -2.0000
    3.0000
```

```
x =

    1.0000
    3.0000
   -1.0000
   -2.0000
    3.0000
```

2 (a) $\frac{d}{dt}(1) = 0$

$\frac{d}{dt}(t) = 1$

$\frac{d}{dt}(t^2) = 2t$

$\frac{d}{dt}(t^3) = 3t^2$

$\frac{d}{dt}(x) = Ax$

$$A = \begin{bmatrix} 0 & 1 & 0 & 0 \\ 0 & 0 & 2 & 0 \\ 0 & 0 & 0 & 3 \\ 0 & 0 & 0 & 0 \end{bmatrix}$$

(b) $p(t) = 5t^2 - 12t^3 = \begin{bmatrix} 0 \\ 0 \\ 5 \\ -12 \end{bmatrix}$

$$\frac{d}{dt}p(t) = \begin{bmatrix} 0 & 1 & 0 & 0 \\ 0 & 0 & 2 & 0 \\ 0 & 0 & 0 & 3 \\ 0 & 0 & 0 & 0 \end{bmatrix} \begin{bmatrix} 0 \\ 0 \\ 5 \\ -12 \end{bmatrix} = \begin{bmatrix} 0 \\ 10 \\ -36 \\ 0 \end{bmatrix}$$

$$\frac{d^2}{dt^2}p(t) = \begin{bmatrix} 0 & 1 & 0 & 0 \\ 0 & 0 & 2 & 0 \\ 0 & 0 & 0 & 3 \\ 0 & 0 & 0 & 0 \end{bmatrix} \begin{bmatrix} 0 \\ 10 \\ -36 \\ 0 \end{bmatrix} = \begin{bmatrix} 10 \\ -72 \\ 0 \\ 0 \end{bmatrix}$$

$= -72t + 10$

3. $S: U \to V$. $T: V \to W$

$$s(u_j) = \sum_{i=1}^{m} a_{ij} v_i$$

$$T(v_k) = \sum_{i=1}^{p} b_{ik} w_i$$

$\therefore$

$$(T \circ S)(u_j) = T\left(\sum_{i=1}^{m} a_{ij} v_i\right)$$

$$= \sum_{i=1}^{m} a_{ij} T(v_i)$$

$$= \sum_{i=1}^{m} \sum_{k=1}^{p} a_{ij} b_{ki} w_k$$

Suppose tha matrix corresponding to $T \circ S$ is C.

then $$c_{kj} = \sum_{i=1}^{m} b_{ki} a_{ij}$$

that is, $C = BA$.

4 Interpolating Log

a

```
A = [1 1 1;
     1 2 4;
     1 3 9;
];
f = [log(1);log(2);log(3)];
A\f
```

```
ans =


   -0.9808
    1.1247
   -0.1438
```

b

$$L_{n,i}(x) = \prod_{j \neq i}^{n} \frac{x - x_j}{x_i - x_j}$$

Therefore,

$L_{2,0}(x) = \frac{(x-2)(x-3)}{2}$

$L_{2,1}(x) = -\frac{(x-1)(x-3)}{1}$

$L_{2,2}(x) = \frac{(x-1)(x-2)}{2}$

$$p(x_i) = \sum_{i=0}^{n} L_{n,i}(x) f_i$$

```
syms x
p = -(x-1)*(x-3)*log(2) + (x-1)*(x-2)*log(3)/2;
sym2poly(p)
```

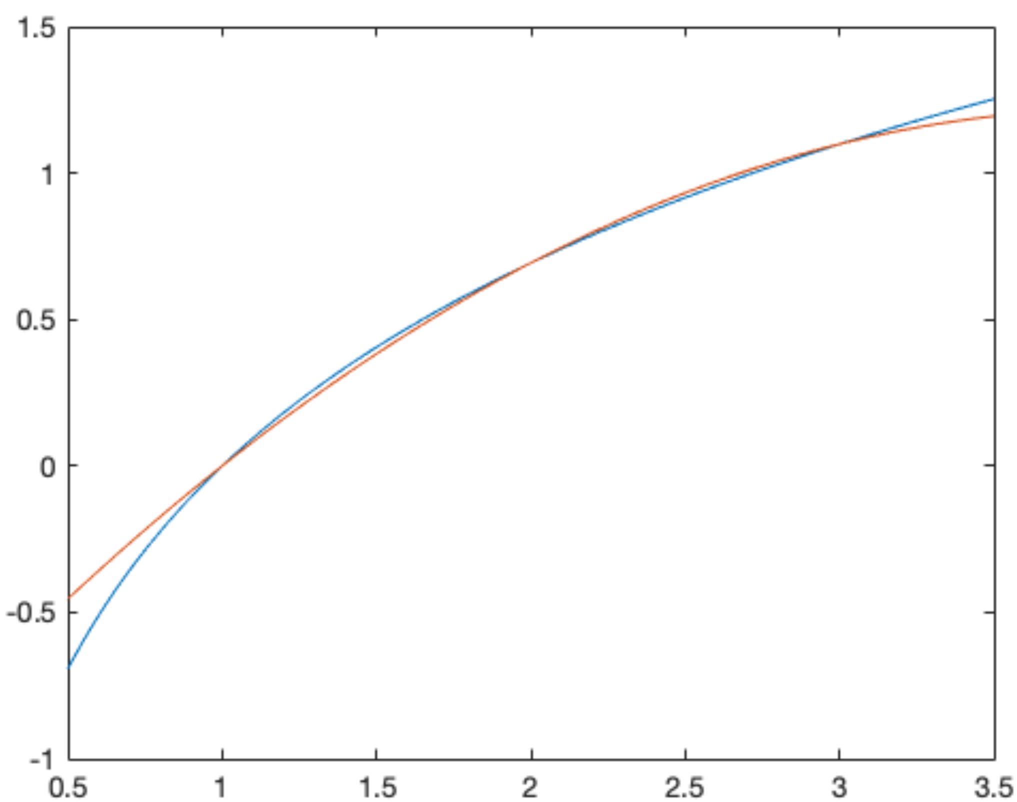
```
ans =


   -0.1438    1.1247   -0.9808
```

The results obtained by two methods are same.

C

```
x = linspace (0.5,3.5);
y1 = log(x);
y2 = -0.1438.*x.^2 + 1.1247.*x + -0.9808;
plot(x,y1)
hold on;
plot(x,y2)
```



5. $$c' = \underset{(2m+2)\times 1}{\begin{bmatrix} 1 \\ a_0 \\ \vdots \\ a_m \\ b_1 \\ \vdots \\ b_m \end{bmatrix}} = \underset{(2m+2)\times(2m+1)}{\underset{D}{\begin{bmatrix} 0 & \cdots & 0 \\ 1 & & \\ & \ddots & \\ & & 1 \end{bmatrix}}} \, c + \underset{(2m+2)\times 1}{\underset{e}{\left.\begin{bmatrix} 1 \\ 0 \\ \vdots \\ 0 \end{bmatrix}\right\}}} 2m+2$$

$$A = \begin{bmatrix} y_1 & \underbrace{0 & \cdots & 0}_{m+1} & y_1x_1 & y_1x_1^2 & \cdots & y_1x_1^m \\ y_2 & 0 & \cdots & 0 & y_2x_2 & y_2x_2^2 & \cdots & y_2x_2^m \\ \vdots & \vdots & & \vdots & \vdots & \vdots & & \vdots \\ y_{2m+1} & 0 & \cdots & 0 & y_{2m+1}x_{2m+1} & y_{2m+1}x_{2m+1}^2 & \cdots & y_{2m+1}x_{2m+1}^m \end{bmatrix}$$

$(2m+1) \times (2m+2)$

$$B = \begin{bmatrix} 0 & 1 & x_1 & x_1^2 & \cdots & x_1^m & 0 & \cdots & 0 \\ 0 & 1 & x_2 & x_2^2 & \cdots & x_2^m & 0 & \cdots & 0 \\ 0 & 1 & x_3 & x_3^2 & \cdots & x_3^m & 0 & \cdots & 0 \\ \vdots & \vdots & \vdots & \vdots & & \vdots & & & \\ 0 & 1 & x_{2m+1} & x_{2m+1}^2 & \cdots & x_{2m+1}^m & 0 & \cdots & 0 \end{bmatrix}$$

$(2m+1) \times (2m+2)$

Then, $Bc' = Ac'$

$B(Dc+e) = A(Dc+e)$

$(A-B)(Dc+e) = 0.$

Equivalent Form:

$$y_i = \frac{\sum_{j=0}^{m} a_j x_i^j}{1 + \sum_{j=1}^{m} b_j x_i^j} \,. \qquad \sum_{j=0}^{m} a_j x_i^j - y_i \sum_{j=1}^{m} b_j x_i^j = y_i$$

$$\therefore \begin{bmatrix} 1 & x_1 & x_1^2 & \cdots & x_1^m & -y_1x_1 & -y_1x_1^2 & \cdots & -y_1x_1^m \\ 1 & x_2 & x_2^2 & \cdots & x_2^m & -y_2x_2 & -y_2x_2^2 & \cdots & -y_2x_2^m \\ & & & & & \vdots & & & \\ 1 & x_{2m+1} & x_{2m+1}^2 & \cdots & x_{2m+1}^m & -y_{2m+1}x_{2m+1} & & \cdots & -y_{2m+1}x_{2m+1}^m \end{bmatrix} \begin{bmatrix} a_0 \\ a_1 \\ \vdots \\ a_m \\ b_1 \\ \vdots \\ b_m \end{bmatrix} = \begin{bmatrix} y_1 \\ y_2 \\ \vdots \\ y_{2m+1} \end{bmatrix}$$