

Midterm Report

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The following linear models and classification models, respectively, were generated for MATH341 midterm project:

The following classification models were also generated:

```
In [55]: cbind(c("Full MLR"),c("Subset Selection"),c("MLR Ridge"),c("MLR Lasso"),c("kNN Re  
cbind(c("Logistic Regression"),c("kNN Classifier"),c("LDA"),c("QDA"),c("Naive Bay
```

Full MLR Subset Selection MLR Ridge MLR Lasso kNN Regression

Logistic Regression kNN Classifier LDA QDA Naive Bayes

Loading in Libraries

```
In [2]: library(readr)
library(glmnet)
library(ggplot2)
library(boot)
library(caret)
library(leaps)
library(class)
library(FNN)
library(Metrics)
library(pROC)
library(tidyverse)
library(e1071)
library(naivebayes)
library(MASS)
options(warn=-1) #To Suppress Warnings
```

Loading Train and Test Data

```
In [3]: setwd("C:/Users/whwal/Desktop/Machine Learning/Midterm Project")
train <- read_csv("StudentDataTrain.csv")
test <- read_csv("StudentDataTest.csv")
train = na.omit(train)
test = na.omit(test)
train = train[,-c(1,2)]
test = test[,-c(1,2)]
```

Parsed with column specification:

```
cols(
  Race_Ethc_Visa = col_character(),
  Gender = col_character(),
  HSGPA = col_double(),
  SAT_Total = col_double(),
  Entry_Term = col_double(),
  Term.GPA = col_double(),
  Persistence.NextYear = col_double(),
  N.RegisteredCourse = col_double(),
  N.Ws = col_double(),
  N.DFs = col_double(),
  N.As = col_double(),
  N.PassedCourse = col_double(),
  N.CourseTaken = col_double(),
  Perc.PassedEnrolledCourse = col_double(),
  Perc.Pass = col_double(),
  Perc.Withd = col_double(),
  N.GraduateCourse = col_double(),
  FullTimeStudent = col_double()
)
```

Parsed with column specification:

```
cols(
  Race_Ethc_Visa = col_character(),
  Gender = col_character(),
  HSGPA = col_double(),
  SAT_Total = col_double(),
  Entry_Term = col_double(),
  Term.GPA = col_double(),
  Persistence.NextYear = col_double(),
  N.RegisteredCourse = col_double(),
  N.Ws = col_double(),
  N.DFs = col_double(),
  N.As = col_double(),
  N.PassedCourse = col_double(),
  N.CourseTaken = col_double(),
  Perc.PassedEnrolledCourse = col_double(),
  Perc.Pass = col_double(),
  Perc.Withd = col_double(),
  N.GraduateCourse = col_double(),
  FullTimeStudent = col_double()
)
```

Data Exploration/Cleaning

In [4]: `cor(train)`

	HSGPA	SAT_Total	Entry_Term	Term.GPA	Persistence.Nex
HSGPA	1.000000000	-0.003758312	-0.103645005	0.060280036	0.174561339
SAT_Total	-0.003758312	1.000000000	-0.038816754	-0.022627828	-0.007603759
Entry_Term	-0.103645005	-0.038816754	1.000000000	0.004810567	-0.111316139
Term.GPA	0.060280036	-0.022627828	0.004810567	1.000000000	0.477542392
Persistence.NextYear	0.174561339	-0.007603759	-0.111316139	0.477542392	1.000000000
N.RegisteredCourse	0.017466136	-0.001550347	-0.001491140	-0.001990355	0.003985445
N.Ws	0.015323271	0.020477721	-0.184489208	-0.010872277	0.020733271
N.DFs	-0.028319672	-0.024496385	0.226961904	0.007662234	-0.070613672
N.As	0.005033852	-0.013697361	-0.120398872	0.019541574	0.07820852
N.PassedCourse	0.030230332	0.003755455	-0.055260637	-0.001629591	0.040145455
N.CourseTaken	0.011234555	-0.011955572	0.090962118	0.003299299	-0.006085572
Perc.PassedEnrolledCourse	0.017840237	0.004104826	-0.083494649	0.002810394	0.052317826
Perc.Pass	0.030307725	0.022810338	-0.226454081	-0.001818066	0.065834081
Perc.Withd	0.007988513	0.025873840	-0.188673739	-0.005920765	0.011234555
N.GraduateCourse	0.016572219	-0.020778679	0.003716367	0.010332220	-0.003299299
FullTimeStudent	0.003711931	-0.002089033	0.077900207	0.014200012	-0.000560207

In this step, we identify any variables that have significant (>0.4) correlation with each other, in order to reduce redundancies.

```
In [5]: train = train[,-c(6,7,8,10,11,12)]
test = test[,-c(6,7,8,10,11,12)]
cor(train)
colnames(train) #Left with 10 predictors
```

	HSGPA	SAT_Total	Entry_Term	Term.GPA	Persistence.NextYear
HSGPA	1.000000000	-0.003758312	-0.103645005	0.060280036	0.1745613391
SAT_Total	-0.003758312	1.000000000	-0.038816754	-0.022627828	-0.0076037587
Entry_Term	-0.103645005	-0.038816754	1.000000000	0.004810567	-0.1113161389
Term.GPA	0.060280036	-0.022627828	0.004810567	1.000000000	0.4775423915
Persistence.NextYear	0.174561339	-0.007603759	-0.111316139	0.477542392	1.0000000000
N.As	0.005033852	-0.013697361	-0.120398872	0.019541574	0.0782088497
Perc.Pass	0.030307725	0.022810338	-0.226454081	-0.001818066	0.0658342385
Perc.Withd	0.007988513	0.025873840	-0.188673739	-0.005920765	0.0112324292
N.GraduateCourse	0.016572219	-0.020778679	0.003716367	0.010332220	-0.0032990788
FullTimeStudent	0.003711931	-0.002089033	0.077900207	0.014200012	-0.0005602344

'HSGPA' 'SAT_Total' 'Entry_Term' 'Term.GPA' 'Persistence.NextYear' 'N.As' 'Perc.Pass'
'Perc.Withd' 'N.GraduateCourse' 'FullTimeStudent'

Regression Models

Multi-Linear Regression

We perform Multi-Linear Regression with the `train()` function from the "caret" package. With it we are able to easily utilize 5-fold Cross Validation.

```
In [6]: train.control = trainControl(method = "cv", number = 5)
lm.model.fit = train(Term.GPA~., data = test, trControl = train.control, method = "lm")
lm.predict.train = predict(lm.model.fit, train)
lm.predict.test = predict(lm.model.fit, test)
lm.train.MSE = mean((lm.predict.train - train$Term.GPA)^2)
lm.test.PMSE = mean((lm.predict.test - test$Term.GPA)^2)
RSS.lm = sum((lm.predict.test - test$Term.GPA)^2)
TSS.lm = sum((test$Term.GPA - mean(test$Term.GPA))^2)
lm.Rsqr = 1 - (RSS.lm/TSS.lm)
```

Ridge Regression

We fit a ridge regression model to the training data. Here, we choose an optimal value of lambda

through cross validation. By default, `glmnet()` performs 10-fold CV, but we change it to 5 using the `nfolds` argument.

```
In [7]: set.seed(1)
train.mat = model.matrix(Term.GPA ~ ., data = train)
test.mat = model.matrix(Term.GPA ~ ., data = test)

grid = 10 ^ seq(4, -2, length = 100)
ridge.model = glmnet(train.mat, train$Term.GPA, alpha = 0, lambda = grid, thresh
cv.ridge = cv.glmnet(train.mat, train$Term.GPA, alpha = 0, lambda = grid, thresh
bestlambda.ridge = cv.ridge$lambda.min
bestlambda.ridge #This value is 0.01
```

0.01

Now that we have acquired an optimal lambda, we can now test the model through train and test predictions:

```
In [8]: ridge.predict.train = predict(ridge.model, s = bestlambda.ridge, newx = train.mat)
ridge.predict.test = predict(ridge.model, s = bestlambda.ridge, newx = test.mat)
ridge.train.MSE = mean((ridge.predict.train - train$Term.GPA)^2)
ridge.test.PMSE = mean((ridge.predict.test - test$Term.GPA)^2)
predict(ridge.model, s = bestlambda.ridge, type = "coefficients")
RSS.ridge = sum((ridge.predict.test - test$Term.GPA)^2)
TSS.ridge = sum((test$Term.GPA - mean(test$Term.GPA))^2)
ridge.Rsqr = 1 - (RSS.ridge/TSS.ridge)
```

11 x 1 sparse Matrix of class "dgCMatrix"

	1
(Intercept)	-1.502994e+01
(Intercept)	.
HSGPA	-1.504636e-03
SAT_Total	-8.326111e-05
Entry_Term	7.718919e-03
Persistence.NextYear	1.219804e+00
N.As	-1.237038e-02
Perc.Pass	-5.504550e-02
Perc.Withd	-1.822924e-03
N.GraduateCourse	1.362237e-02
FullTimeStudent	2.387999e-02

Lasso Regression

We fit a lasso regression model to the training data. Like in ridge regression, we use 5-fold CV to obtain a lambda that will minimize MSE.

```
In [9]: lasso.model = glmnet(train.mat, train$Term.GPA, alpha = 1, lambda = grid, thresh
cv.lasso = cv.glmnet(train.mat, train$Term.GPA, alpha = 1, lambda = grid, thresh
bestlambda.lasso = cv.lasso$lambda.min
bestlambda.lasso

lasso.predict.train = predict(lasso.model, s = bestlambda.lasso, newx = train.mat
lasso.predict.test = predict(lasso.model, s = bestlambda.lasso, newx = test.mat)
lasso.train.MSE = mean((lasso.predict.train - train$Term.GPA)^2)
lasso.test.PMSE = mean((lasso.predict.test - test$Term.GPA)^2)
predict(lasso.model, s = bestlambda.lasso, type = "coefficients")
RSS.lasso = sum((lasso.predict.test - test$Term.GPA)^2)
TSS.lasso = sum((test$Term.GPA - mean(test$Term.GPA))^2)
lasso.Rsqr = 1 - (RSS.lasso/TSS.lasso)
```

0.01

11 x 1 sparse Matrix of class "dgCMatrix"

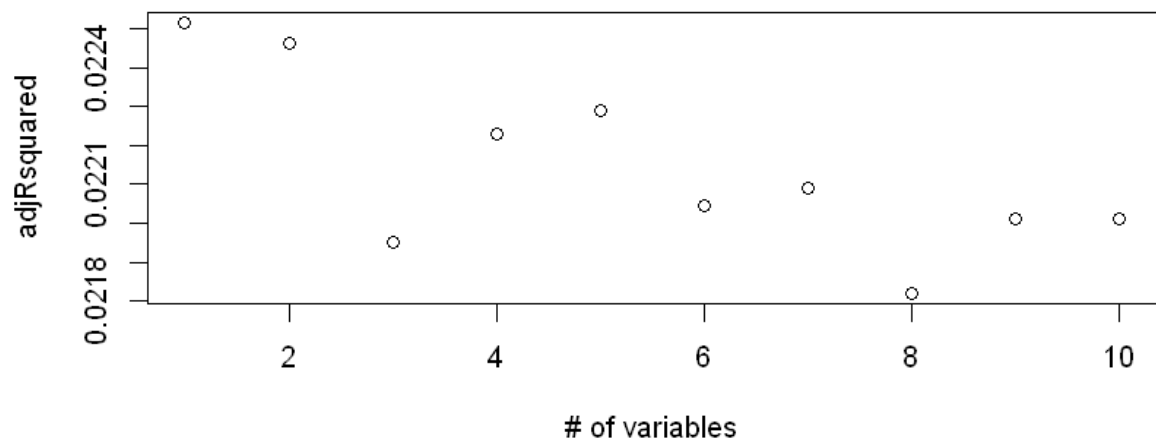
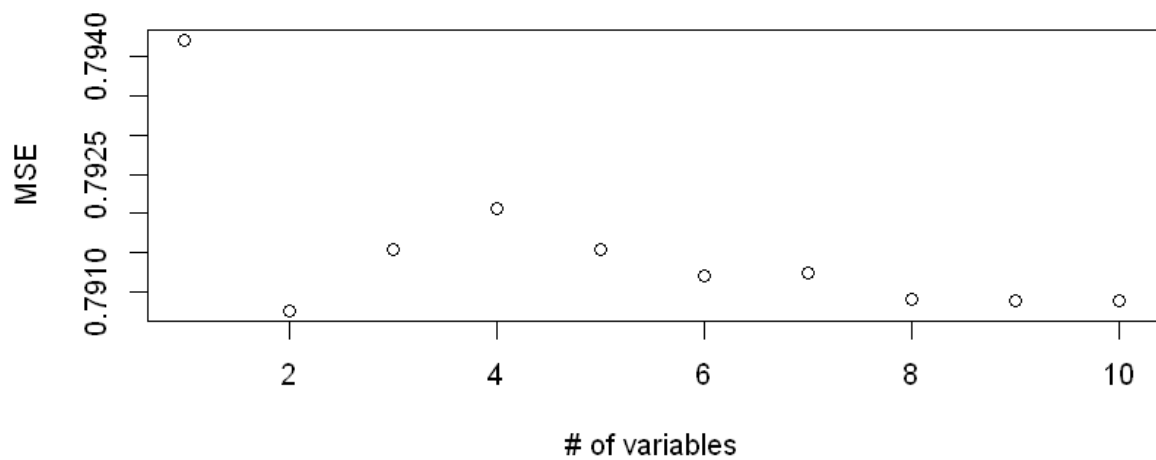
	1
(Intercept)	-1.330540e+01
(Intercept)	.
HSGPA	-6.391154e-04
SAT_Total	-3.621187e-05
Entry_Term	6.863673e-03
Persistence.NextYear	1.197693e+00
N.As	.
Perc.Pass	-4.021311e-02
Perc.Withd	.
N.GraduateCourse	2.356422e-03
FullTimeStudent	1.697602e-03

Best Subset Selection (Backwards Selection)

We proceed with conducting best subset selection on the training set, specifically backwards selection to iteratively remove least contributing predictors. We train the model using the "leapBackward" method and using the train() function from caret, performing 5-fold CV. We then extract a relationship between MSE, adjusted R squared and # of variables.

```
In [10]: set.seed(123)
train.control <- trainControl(method = "cv", number = 5)

step.model <- train(Term.GPA ~., data = train,
                    method = "leapBackward",
                    tuneGrid = data.frame(nvmax = 1:10),
                    trControl = train.control
                    )
step.adj.rsq = step.model$results$RsquaredSD
step.MSE = (step.model$results$RMSE)^2
par(mfrow=c(2,1))
plot(step.model$results$nvmax, step.MSE, xlab = "# of variables", ylab = "MSE")
plot(step.model$results$nvmax, step.adj.rsq, xlab = "# of variables", ylab = "adj
```



From the graphs, we can assert that the best model has 2 variables, since that is the point where the highest R squared and MSE coincide. We can check this:

```
In [11]: step.model$bestTune
```

nvmax	
2	2

```
In [12]: coef(step.model$finalModel, 2)
```

(Intercept)	-18.4944056625729
Entry_Term	0.00921905774701395
Persistence.NextYear	1.22128425825323

Hence, backwards selection creates a model with only 2 variables from the original 10.

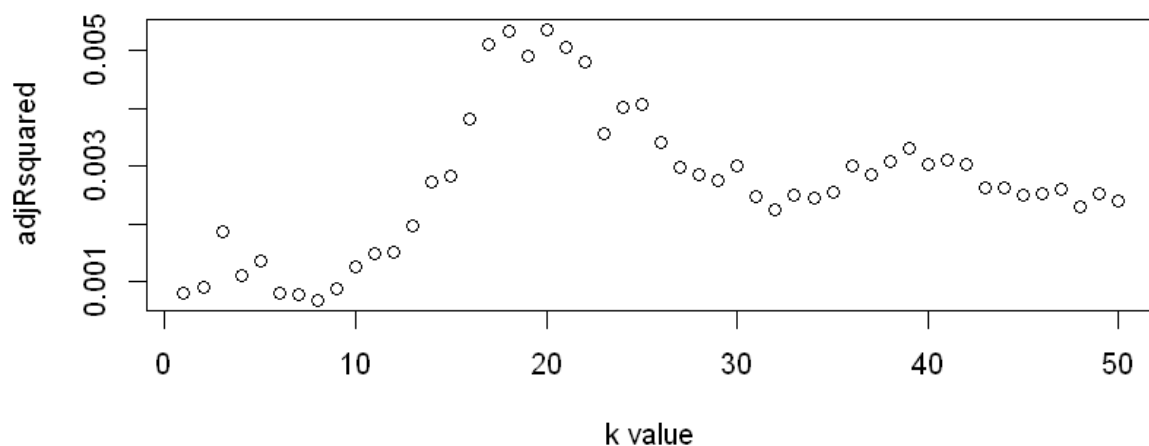
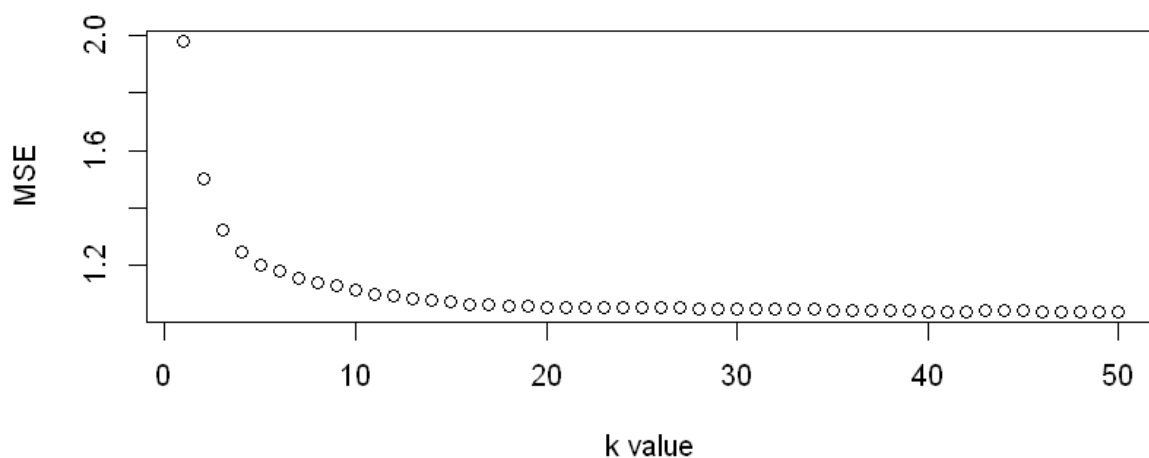
We can test the model using our test data:

```
In [23]: step.model.train = lm(Term.GPA~Entry_Term + Persistence.NextYear, data = train)
step.model.test = lm(Term.GPA~Entry_Term + Persistence.NextYear, data = test)
step.pred.train = predict(step.model, data = train)
step.pred.test = predict(step.model, data = test)
step.train.MSE = mean((step.pred.train - train$Term.GPA)^2)
step.test.PMSE = mean((step.pred.test - test$Term.GPA)^2)
RSS.step = sum((step.pred.test - test$Term.GPA)^2)
TSS.step = sum((test$Term.GPA - mean(test$Term.GPA))^2)
step.Rsqr = 1 - (RSS.step/TSS.step)
```

kNN Regression

Here we will train a kNN regression model again using the train() function in caret. In order to output results for different values of k, we will set tuneGrid equal to a vector 1:50 to test MSE's for values of k ranging from 1 to 50. This is how we will find the best k Nearest Neighbors for our estimation.


```
In [14]: trcontrol.knn = trainControl(method = "cv",
                                     number = 5)
knn.model = train(Term.GPA~.,
                  method = "knn",
                  tuneGrid = expand.grid(k = 1:50),
                  trControl = trcontrol.knn,
                  data = train)
knn.MSE = (knn.model$results$RMSE)^2
knn.adjrsqr = knn.model$results$RsquaredSD
par(mfrow=c(2,1))
plot(knn.model$results$k,knn.MSE, xlab = "k value", ylab = "MSE")
plot(knn.model$results$k, knn.adjrsqr, xlab = "k value", ylab = "adjRsquared")
```



We notice a consistent decrease in MSE as we increase k, but we also notice R squared is highest at k = 20, so that is our optimum k value for this model. We can proceed to model testing.

```
In [15]: knn.pred.train = predict(knn.model, train)
knn.pred.test = predict(knn.model, test)
knn.MSE.train = mean((knn.pred.train - train$Term.GPA)^2)
knn.PMSE.test = mean((knn.pred.test - test$Term.GPA)^2)
RSS.knn = sum((knn.pred.test - test$Term.GPA)^2)
TSS.knn = sum((test$Term.GPA - mean(test$Term.GPA))^2)
knn.Rsqr = 1 - (RSS.knn/TSS.knn)
```

Multi-Regression Results

```
In [25]: trainMSE = c(lm.train.MSE,ridge.train.MSE,lasso.train.MSE,step.train.MSE,knn.MSE)
testMSE = c(lm.test.PMSE,ridge.test.PMSE,lasso.test.PMSE,step.test.PMSE,knn.PMSE)
ResultsMLR = cbind(trainMSE, testMSE)
row.names(ResultsMLR) = c("MLR", "Ridge", "Lasso", "Step", "kNN")
ResultsMLR
```

	trainMSE	testMSE
MLR	0.8408511	0.8002639
Ridge	0.7892362	0.8163820
Lasso	0.7899882	0.8134883
Step	0.7906234	1.2494092
kNN	0.9959190	1.0163150

Best value of k nearest neighbors was 20, and lambda was 0.01 for Lasso and Ridge.

Unfortunately, linear techniques did not perform well at all. Unfortunately since my R squared value for kNN is negative, I have made a big mistake in the calculations. It seems, however, that the full linear model is the best performing.

Classification Models

Data Exploration

We analyze the correlation matrix:

In [30]: `cor(train)`

	HSGPA	SAT_Total	Entry_Term	Term.GPA	Persistence.NextYear
HSGPA	1.000000000	-0.003758312	-0.103645005	0.060280036	0.1745613391
SAT_Total	-0.003758312	1.000000000	-0.038816754	-0.022627828	-0.0076037587
Entry_Term	-0.103645005	-0.038816754	1.000000000	0.004810567	-0.1113161389
Term.GPA	0.060280036	-0.022627828	0.004810567	1.000000000	0.4775423915
Persistence.NextYear	0.174561339	-0.007603759	-0.111316139	0.477542392	1.0000000000
N.As	0.005033852	-0.013697361	-0.120398872	0.019541574	0.0782088497
Perc.Pass	0.030307725	0.022810338	-0.226454081	-0.001818066	0.0658342385
Perc.Withd	0.007988513	0.025873840	-0.188673739	-0.005920765	0.0112324292
N.GraduateCourse	0.016572219	-0.020778679	0.003716367	0.010332220	-0.0032990788
FullTimeStudent	0.003711931	-0.002089033	0.077900207	0.014200012	-0.0005602344

The Best predictors of Persistence Next Year is Term.GPA, Entry_Term, and HSGPA. Hence we will proceed to explore these predictors.

Logistic Regression

We fit a logistic regression model to the training data using caret with 5-fold CV:

```

In [32]: log.fit = train(Persistence.NextYear~Term.GPA+Entry_Term+HSGPA, method = "glm",
                        ,data = train)
glm.predict.train = predict(log.fit, train)
glm.predict.train[glm.predict.train>.5]="Persistent"
glm.predict.train[glm.predict.train<=.5]="Dropped"

train.Persistence = train$Persistence.NextYear

train.Persistence[train.Persistence>.5]="Persistent"
train.Persistence[train.Persistence<=.5]="Dropped"

table(glm.predict.train, train.Persistence)

log.Err.train = mean(glm.predict.train != train.Persistence)
log.Acc.train = mean(glm.predict.train == train.Persistence)
log.Prec.train = 561/(561+254)

glm.predict.test = predict(log.fit, test)
glm.predict.test[glm.predict.test>.5]="Persistent" #1
glm.predict.test[glm.predict.test<=.5]="Dropped" #0

test.Persistence = test$Persistence.NextYear
test.Persistence[test.Persistence>.5]="Persistent"
test.Persistence[test.Persistence<=.5]="Dropped"

table(glm.predict.test, test.Persistence)

log.Err.test = mean(glm.predict.test != test.Persistence)
log.Acc.test = mean(glm.predict.test == test.Persistence)
log.Prec.test = 17/(17+7)
log.sens = 1294/(1294 + 7)
log.spec = 17/(17 + 127)

```

	train.Persistence	
glm.predict.train	Dropped	Persistent
Dropped	318	97
Persistent	849	4493

	test.Persistence	
glm.predict.test	Dropped	Persistent
Dropped	17	7
Persistent	127	1294

With Logistic Regression, we are able to make a ROC graph to plot Sensitivity and Specificity Trade-off:

```
In [33]: par(pty = "s")
glm.roc = roc(test$Persistence.NextYear, predict(log.fit, test), plot = TRUE, per
glm.roc

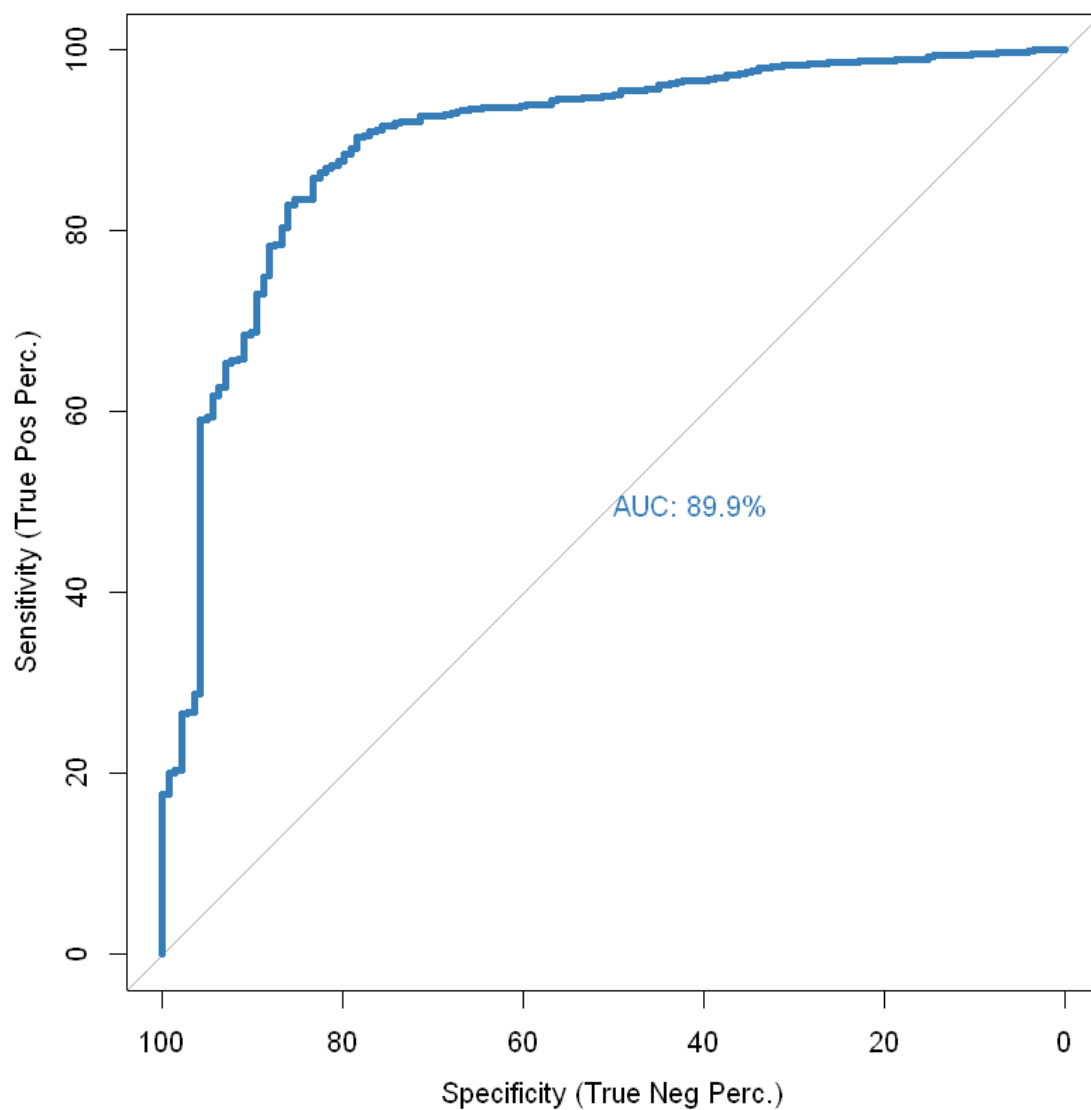
glm.roc.info = roc(test$Persistence.NextYear, predict(log.fit, test), legacy.axes
glm.roc.df = data.frame(tpp = glm.roc.info$sensitivities*100,
                        fpp = (1 - glm.roc.info$specificities)*100,
                        thresholds = glm.roc.info$thresholds)
```

Setting levels: control = 0, case = 1
Setting direction: controls < cases

Call:
roc.default(response = test\$Persistence.NextYear, predictor = predict(log.fit,
test), percent = TRUE, plot = TRUE, xlab = "Specificity (True Neg Perc.)",
ylab = "Sensitivity (True Pos Perc.)", col = "#377eb8", lwd = 4, print.auc
= TRUE)

Data: predict(log.fit, test) in 144 controls (test\$Persistence.NextYear 0) < 13
01 cases (test\$Persistence.NextYear 1).
Area under the curve: 89.91%

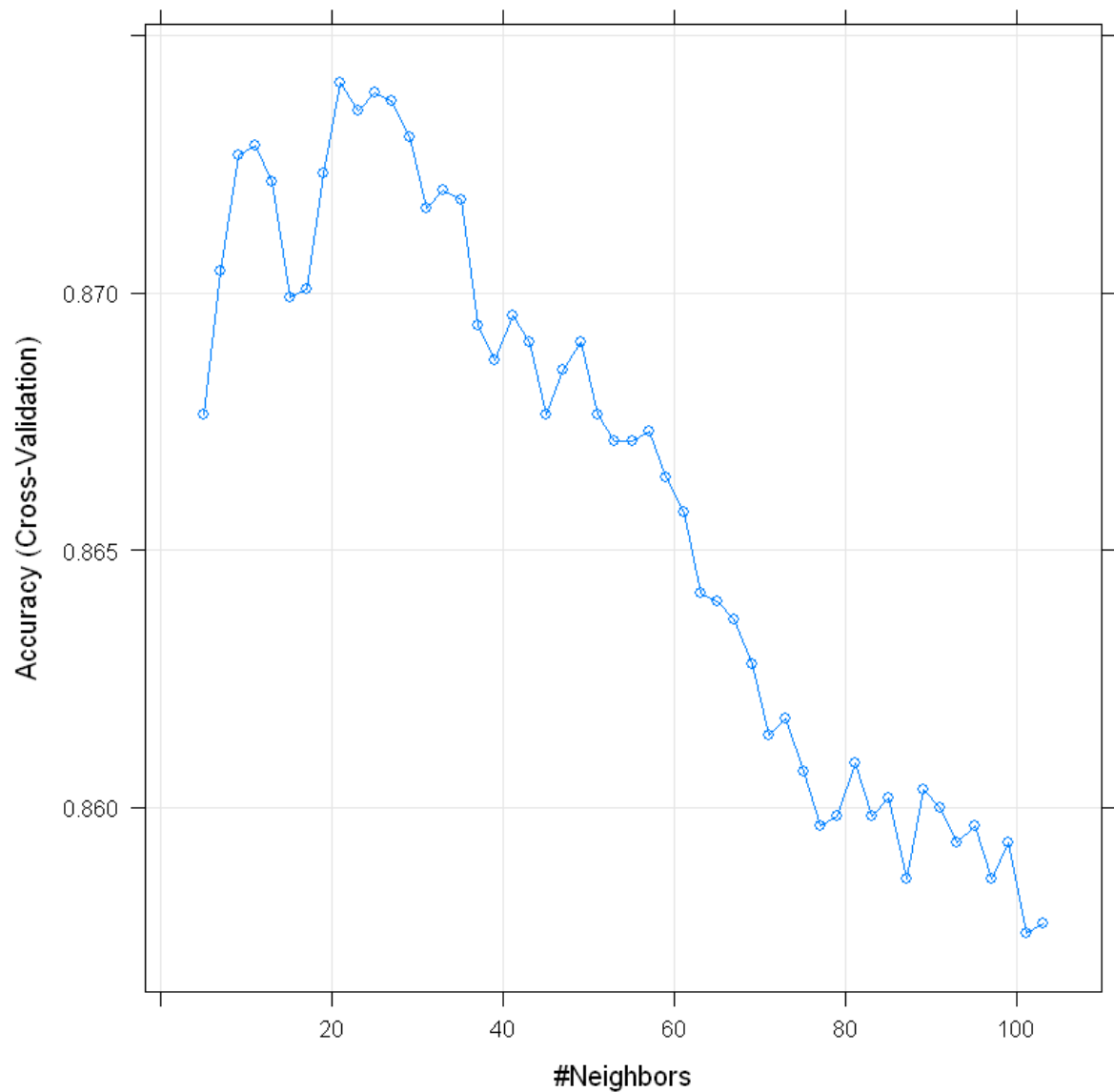
Setting levels: control = 0, case = 1
Setting direction: controls < cases



kNN Classifier

We use `train()` to train a kNN classifier using our test data. Again, we iterate through values of `k` neighbors from 1 to 50:

```
In [34]: set.seed(123)
knn.fit.class = train(as.factor(Persistence.NextYear)~Term.GPA+Entry_Term+HSGPA,
  method = "knn",
  tuneLength = 50,
  trControl = train.control,
  preProcess = c("center", "scale"),
  data = train,
  metric = "Accuracy")
plot(knn.fit.class)
```



We observe the optimal k nearest neighbors to be $k = 21$. We can now make predictions on this model using the test set:

```
In [44]: knn.predict.train = predict(knn.fit.class,train)
knn.predict.test = predict(knn.fit.class,test)
table(knn.predict.train,train.Persistence)
table(knn.predict.test,test.Persistence)
knn.Err.train = mean(knn.predict.train != train$Persistence.NextYear)
knn.Acc.train = mean(knn.predict.train == train$Persistence.NextYear)
knn.Prec.train = 723/(723+226)
knn.Err.test = mean(knn.predict.test != test$Persistence.NextYear)
knn.Acc.test = mean(knn.predict.test == test$Persistence.NextYear)
knn.Prec.test = 113/(113+62)
knn.sens = 1239/(1239+62)
knn.spec = 113/(113+31)
```

	train.Persistence	
knn.predict.train	Dropped	Persistent
0	718	229
1	449	4361

	test.Persistence	
knn.predict.test	Dropped	Persistent
0	113	62
1	31	1239

Naive Bayes Classifier

We will use `train()` to train a naive bayes classifier using 5-fold CV.


```
In [36]: nb.fit = train(as.factor(Persistence.NextYear)~Term.GPA+Entry_Term+HSGPA, method
          trControl = train.control,
          data = train,
          metric = "Accuracy")
nb.predict.train = predict(nb.fit,train)
nb.predict.test = predict(nb.fit,test)
table(nb.predict.train, train.Persistence)
nb.Err.train = mean(nb.predict.train != train$Persistence.NextYear)
nb.Acc.train = mean(nb.predict.train == train$Persistence.NextYear)
nb.Prec.train = 568/(568+136)
nb.predict.test = predict(nb.fit, test)
table(nb.predict.test, test.Persistence)
nb.Err.test = mean(nb.predict.test != test$Persistence.NextYear)
nb.Acc.test = mean(nb.predict.test == test$Persistence.NextYear)
nb.Prec.test = 94/(94+25)
nb.sens = 1276/(1276 + 25)
nb.spec = 94/(94 + 50)
```

	train.Persistence	
nb.predict.train	Dropped	Persistent
0	568	136
1	599	4454

	test.Persistence	
nb.predict.test	Dropped	Persistent
0	94	25
1	50	1276

LDA

Here we perform linear discriminant analysis and train the model using train() and 5-fold CV.

```
In [37]: lda.fit = train(as.factor(Persistence.NextYear)~Term.GPA+HSGPA+Entry_Term,method
              data = train)
lda.predict.train = predict(lda.fit,train)
lda.predict.test = predict(lda.fit,test)
table(lda.predict.train, train.Persistence)
lda.Err.train = mean(lda.predict.train != train$Persistence.NextYear)
lda.Acc.train = mean(lda.predict.train == train$Persistence.NextYear)
lda.Prec.train = 550/(568+258)
lda.predict.test = predict(lda.fit, test)
table(lda.predict.test, test.Persistence)
lda.Err.test = mean(lda.predict.test != test$Persistence.NextYear)
lda.Acc.test = mean(lda.predict.test == test$Persistence.NextYear)
lda.Prec.test = 45/(45+22)
lda.sens = 1279/(1279 + 22)
lda.spec = 45/(45 + 99)
```

```
              train.Persistence
lda.predict.train Dropped Persistent
              0      550      258
              1      617      4332
```

```
              test.Persistence
lda.predict.test Dropped Persistent
              0      45      22
              1      99      1279
```

Here we perform quantitative descriptive analysis and train the model using train() and 5-fold CV

```
In [38]: qda.fit = train(as.factor(Persistence.NextYear)~Term.GPA+HSGPA+Entry_Term,method
              data = train)
qda.predict.train = predict(qda.fit,train)
qda.predict.test = predict(qda.fit,test)
table(qda.predict.train, train.Persistence)
qda.Err.train = mean(qda.predict.train != train$Persistence.NextYear)
qda.Acc.train = mean(qda.predict.train == train$Persistence.NextYear)
qda.Prec.train = 603/(603+312)
qda.predict.test = predict(qda.fit, test)
table(lda.predict.test, test.Persistence)
qda.Err.test = mean(qda.predict.test != test$Persistence.NextYear)
qda.Acc.test = mean(qda.predict.test == test$Persistence.NextYear)
qda.Prec.test = 45/(45+22)
qda.sens = 1279/(1279 + 22)
qda.spec = 45/(45 + 99)
```

```
              train.Persistence
qda.predict.train Dropped Persistent
              0      603      312
              1      564      4278
```

```
              test.Persistence
lda.predict.test Dropped Persistent
              0      45      22
              1      99      1279
```

Classification Results

```
In [49]: train.err = c(log.Err.train,knn.Err.train,nb.Err.train,lda.Err.train,qda.Err.train)
test.err = c(log.Err.test,knn.Err.test,nb.Err.test,lda.Err.test,qda.Err.test)
train.prec = c(log.Prec.train,knn.Prec.train,nb.Prec.train,lda.Prec.train,qda.Prec.train)
test.prec = c(log.Prec.test,knn.Prec.test,nb.Prec.test,lda.Prec.test,qda.Prec.test)
train.acc = c(log.Acc.train,knn.Acc.train,nb.Acc.train,lda.Acc.train,qda.Acc.train)
test.acc = c(log.Acc.test,knn.Acc.test,nb.Acc.test,lda.Acc.test,qda.Acc.test)
sensitivity = c(log.sens,knn.sens,nb.sens,lda.sens,qda.sens)
specificity = c(log.spec,knn.spec,nb.spec,lda.spec,qda.spec)
results.class = cbind(train.err,test.err,train.prec,test.prec,train.acc,test.acc,
row.names(results.class) = c("Logistic Reg","kNN","Naive Bayes","LDA","QDA")
results.class
```

	train.err	test.err	train.prec	test.prec	train.acc	test.acc	sensitivity	specificity
Logistic Reg	0.1643217	0.09273356	0.6883436	0.7083333	0.8356783	0.9072664	0.9946195	0.1180556
kNN	0.1177697	0.06435986	0.7618546	0.6457143	0.8822303	0.9356401	0.9523444	0.7847222
Naive Bayes	0.1276707	0.05190311	0.8068182	0.7899160	0.8723293	0.9480969	0.9807840	0.6527778
LDA	0.1519889	0.08373702	0.6658596	0.6716418	0.8480111	0.9162630	0.9830899	0.3125000
QDA	0.1521626	0.08719723	0.6590164	0.6716418	0.8478374	0.9128028	0.9830899	0.3125000

From the Results table, it suffices to say that the superior model in this case is kNN with $k = 21$. While Naive Bayes comes is also a healthy model, kNN has better sensitivity to specificity trade off.