A data envelopment analysis approach to measure the mutual fund performance

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Abstract

In this paper we present a model which can be used to evaluate the performance of mutual funds. This model applies an operational research methodology, called data envelopment analysis (DEA), which allows to measure the relative efficiency of decision making units. This approach allows to define mutual fund performance indexes that can take into account several inputs and thus consider different risk measures and, above all, the investment costs (subscription costs and redemption fees). Moreover, the DEA approach can naturally envisage other output indicators, in addition to the mean return considered by the traditional indexes. Therefore, a generalized version of the DEA mutual fund performance indexes is defined, too, which includes among the outputs a stochastic dominance indicator that reflects both the investors' preference structure and the time occurrence of the returns. In addition, the procedure allows to identify, for each mutual fund, a composite portfolio which can be considered as a particular benchmark. The performance indexes proposed are tested on empirical data.

Keywords: Finance; Data envelopment analysis; Mutual fund performance indexes

1. Introduction

It is a common practice to evaluate the performance of mutual funds on the ground of the past returns, even if nothing ensures that the same return paths will arise again in the future.

To measure the mutual funds performance, some numerical indexes have been devised in the literature and are widely used in the practice: let us just remind the well-known reward-to-volatility ratio (Sharpe, 1966) and reward-to-variability ratio (Treynor, 1965). These indexes are ratios between the expected excess return of the portfolio and a risk indicator; in this way they take into account both expected return and risk and synthesize them in a unique numerical value. Nevertheless, they do not consider the subscription and redemption costs

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required by the investment which contribute to determine the overall return on the investment.

To take into account also the initial and final investment costs we can use a performance measurement methodology that allows to evaluate the efficiency of a decision making unit in the presence of several inputs and outputs.

A technique with these characteristics, which can be used to this aim, is data envelopment analysis (DEA), proposed by Charnes, Cooper and Rhodes in 1978. In effect, DEA has originally been devised to measure the relative efficiency of public sector activities and non-profit organizations, such as, for example, educational institutions and health services. However, afterwards this methodology has been applied also to many profit oriented companies, especially bank branches. For a bibliography of various applications of the DEA approach see for example Charnes, Cooper, Lewin and Seiford (1994). The DEA efficiency measure is computed by solving a fractional linear programming model that can be converted into an equivalent linear program and thus can be easily solved.

The main purpose of this contribution is to use the DEA methodology in order to define mutual fund performance indexes that can take into account several inputs, in particular different risk measures and the investment costs. Moreover, the DEA approach can naturally consider other output indicators, in addition to the mean return considered by the traditional indexes. Therefore, a generalized version of the DEA mutual fund performance indexes is defined, too, which includes among the outputs a stochastic dominance indicator that reflects both the investors' preference structure and the time occurrence of the returns.

One of the single output DEA indexes proposed is similar to the DPEI index proposed by Murthi, Choi and Desai (1997) who first used DEA to take into account the investment costs in defining a mutual fund performance index. Moreover, the Sharpe, Treynor, and reward-to-half-variance indexes can be obtained as special cases in the class of the DEA performance indexes proposed.

In addition, the DEA technique allows to identify, for each inefficient fund, a corresponding efficient set of funds (the peer group) which represent a composite portfolio that can be seen as a particular benchmark and characterizes the portfolio style.

Finally, an empirical application based on data from the Italian financial market has been carried out in order to test the applicability and the properties of the DEA indexes proposed and compare the results with those obtained with the traditional performance indexes.

The structure of the paper is as follows. In Section 2 we briefly describe the DEA model that will be used, namely the input-oriented CCR model. In Section 3 we define a class of multiple inputs-single output DEA performance indexes for mutual fund portfolios, whereas in Section 4 we introduce a generalization to a multiple inputs-multiple outputs portfolio performance measure. Section 5 analyzes the dependence of the performance measures from the investment horizon while Section 6 describes how to build composite portfolios to be used as benchmarks. Section 7 presents the empirical analysis carried out on the Italian financial market. Finally, some concluding remarks are presented in Section 8.

2. The data envelopment analysis

The data envelopment analysis (Charnes, Cooper and Rhodes, 1978) is a widely used optimization based technique that allows to measure the relative performance of decision making units which are characterized by a multiple objectives and/or multiple inputs structure.

DEA gives a measure of efficiency which is essentially defined as a ratio of weighted outputs to weighted inputs. The computation of a weighted ratio requires a set of weights to be defined and this can be not easy; Charnes, Cooper and Rhodes's idea is to define the efficiency measure by assigning to each unit the most favourable weights. On the one hand, this means that the weights will generally not be the same for the different units. On the other hand, if a unit turns out to be inefficient, compared to the other ones, when the most favourable weights are chosen, we cannot say that it depends on the choice of the weights!

Let us define:

$j = 1, 2, \ldots, n$	decision making units
$r = 1, 2, \ldots, t$	outputs
$i = 1, 2, \ldots, m$	inputs
y_{rj}	amount of output r for unit j
x_{ij}	amount of input i for unit j
u_r	weight assigned to output r
v_i	weight assigned to input i

The DEA efficiency measure for a decision making unit j is given by a ratio of weighted sum of outputs to weighted sum of inputs

$$h = \frac{\sum_{r=1}^{t} u_r y_{rj}}{\sum_{i=1}^{m} v_i x_{ij}}. \tag{2.1}$$

The weights in the ratio (2.1) are chosen in such a way as the efficiency measure h have an upper bound, usually chosen equal to 1, which will be reached only by the most efficient units. For each decision making unit the most favourable weights are chosen; they are computed by maximizing the efficiency ratio of the unit considered, subject to the constraints that the efficiency ratios of all units, computed with the same weights, have an upper bound of 1.

Formally, to compute the DEA efficiency measure for a target decision making unit $j_0 \in \{ 1, 2, \ldots, n \}$ we have to solve the following fractional linear programming problem

$$\max_{\{v_i, u_r\}} \quad h_0 = \frac{\sum_{r=1}^{t} u_r y_{rj_0}}{\sum_{i=1}^{m} v_i x_{ij_0}} \tag{2.2}$$

subject to

$$\begin{aligned} &\frac{\sum_{r=1}^{t} u_r y_{rj}}{\sum_{i=1}^{m} v_i x_{ij}} \le 1 \qquad && j = 1, \ldots, n \\ &u_r \ge \varepsilon && r = 1, \ldots, t \\ &v_i \ge \varepsilon && i = 1, \ldots, m \end{aligned} \tag{2.3}$$

where ε is a convenient small positive number that prevents the weights from vanishing (formally, ε should be seen as a non-archimedean constant; on this subject see Charnes, Cooper, Lewin and Seiford, 1994).

The optimal objective function value (2.2) represents the efficiency measure assigned to the target unit j_0 considered. To find the efficiency measures of other decision making units we have to solve similar problems, targeted on each unit in turn.

As we have seen, an efficiency measure equal to 1 characterizes the efficient units: at least with the most favourable weights, these units cannot be dominated by the other ones in the set. We thus obtain a Pareto efficiency measure in which the efficient units lie on the efficient frontier (see Charnes, Cooper, Lewin and Seiford, 1994).

Unlike multiple criteria approaches, in DEA modelling the weights which allow to aggregate the inputs and outputs do not reflect the preference structure of the decision maker. Instead, being obtained by solving optimization problems which change with the decision making units, the weights are peculiar to each unit. Nevertheless, there exist some formal analogies between the two modelling, as pointed out by Joro, Korhonen and Wallenius (1998); furthermore, the two approaches can be combined as made, for example, by Post and Spronk (1999); for a discussion of the ways of using DEA as a tool for multiple criteria decision making see Bouyssou (1999).

The fractional problem (2.2)-(2.3) can be conveniently converted into an equivalent linear programming problem; by letting $\sum_{i=1}^{m} v_i x_{ij_0} = 1$ we obtain the so called input-oriented CCR (Charnes, Cooper and Rhodes) linear model

$$\max \quad \sum_{r=1}^{t} u_r y_{rj_0} \tag{2.4}$$

subject to

$$\begin{aligned} & \sum_{i=1}^{m} v_i x_{ij_0} = 1 \\ & \sum_{r=1}^{t} u_r y_{rj} - \sum_{i=1}^{m} v_i x_{ij} \leq 0 \qquad\qquad j = 1, \ldots, n \\ & -u_r \leq -\varepsilon \qquad\qquad r = 1, \ldots, t \\ & -v_i \leq -\varepsilon \qquad\qquad i = 1, \ldots, m. \end{aligned} \tag{2.5}$$

This problem has $t+m$ variables (the weights u_r and v_i which have to be chosen so as to maximize the efficiency of the targeted unit j_0) and $n+t+m+1$ constraints.

The dual problem will also be used afterwards

$$\min \quad z_0 - \varepsilon \sum_{r=1}^{t} s_r^+ - \varepsilon \sum_{i=1}^{m} s_i^- \tag{2.6}$$

subject to

$$
\begin{aligned}
& x_{ij_0} z_0 - s_i^- - \sum_{j=1}^{n} x_{ij}\lambda_j = 0 \qquad && i = 1, \dots, m \\
& - s_r^+ + \sum_{j=1}^{n} y_{rj}\lambda_j = y_{rj_0} \qquad && r = 1, \dots, t \\
& \lambda_j \geq 0 \qquad && j = 1, \dots, n \\
& s_i^- \geq 0 \qquad && i = 1, \dots, m \\
& s_r^+ \geq 0 \qquad && r = 1, \dots, t \\
& z_0 \quad \text{unconstrained.}
\end{aligned}
\tag{2.7}
$$

It can be seen that the CCR model gives a piecewise linear production surface which, in economic terms, represents a production frontier: in fact, it gives the maximum output empirically obtainable from a decision making unit given its level of inputs; from another point of view, it gives the minimum amount of input required to achieve the given output levels.

DEA model (2.2)-(2.3) is the first, simplest and still most used DEA technique. Nevertheless, in the meantime a number of extensions and variants have been proposed in the literature to better cope with special purposes; for a review see for example Charnes, Cooper, Lewin and Seiford (1994).

3. A DEA performance measure for mutual fund portfolios

In order to completely rank a set of investment funds, some numerical indexes have been proposed in the literature that evaluate the performance of mutual funds by taking into account both expected return and risk, synthesizing them in a unique numerical value.

Among these indexes we find, for example, the reward-to-volatility ratio (Sharpe, 1966), the reward-to-half-variance index (Ang and Chua, 1979), the reward-to-variability ratio (Treynor, 1965) and Jensen's measure (Jensen, 1968). The former indexes are ratios between the expected excess return (an output) and a different risk indicator (an input):

$$I_{j,Sharpe} = \frac{E(R_j) - \delta}{\sigma_j} \tag{3.1}$$

$$I_{j,half-var} = \frac{E(R_j) - \delta}{\sqrt{HV_j}} \tag{3.2}$$

$$I_{j,Treynor} = \frac{E(R_j) - \delta}{\beta_j} \tag{3.3}$$

where $E(R_j) - \delta$ denotes the (expected) excess return of portfolio j, i.e. the difference between the portfolio expected return $E(R_j)$ and the riskless rate of return δ, $\sigma_j = \sqrt{Var(R_j)}$ is the standard deviation of the return, HV_j

denotes the half-variance risk indicator $HV_j = E\left(\min\left[R_j - E(R_j), 0\right]\right)^2$; $\beta_j = Cov(R_j, R_m)/Var(R_m)$ represents the ratio of the covariance between the portfolio return R_j and the market portfolio return R_m to the variance of the market portfolio return. Note that the β coefficient is relevant as a measure of risk when the investors' portfolios are well diversified. Jensen's index, instead, measures the portfolio performance through the intercept $I_{j,Jensen}$ of the following C.A.P.M. (capital asset pricing model) regression line

$$E(R_j) - \delta = I_{j,Jensen} + \beta(E(R_m) - \delta). \tag{3.4}$$

These indicators do allow to compare and rank a set of portfolios, by suggesting to choose the ones with the higher index value, but are based on strong assumptions on the market behaviour and investors' preferences. Moreover, they do not take into account the subscription and redemption costs required by an investment in mutual fund portfolios.

In the previous section we have seen that DEA has been proposed to measure the efficiency of decision making units in the presence of a multiple input-multiple output situation; therefore, DEA could allow to take initial and final investment costs into consideration. A first mutual funds efficiency measure which applies the DEA methodology is the DPEI index developed by Murthi, Choi and Desai (1997).

A first natural way to define a DEA measure for evaluating the performance of mutual fund portfolios is a direct generalization of the performance indexes (3.1)–(3.3). In fact, we may build a DEA model which considers as output a return measure (such as the expected return or the expected excess return) and as inputs both some risk measures and the subscription and redemption costs.

Let us consider a set of n mutual funds $j = 1, 2\ldots, n$ and choose as output o_j the expected return $E(R_j)$ or the expected excess return $E(R_j) - \delta$; note that the first choice would allow to reduce the occurrence of negative values in the output while the second one is suggested by the traditional performance indicators (3.1)–(3.3). Moreover, let us take, for each fund j, k subscription and/or redemption costs $c_{1j}, \ldots, c_{kj}$ and h risk measures $q_{1j}, \ldots, q_{hj}$ as inputs; in our empirical analysis we have used the risk measures $\sigma_j, \sqrt{HV_j}, \beta_j$ (with $h \leq 3$) but nothing prevents from choosing other risk measures.

The first DEA performance index of mutual fund investments that we propose, I_{DEA-1}, involves the solution of the following fractional linear programming problem

$$\max_{\{u, v_i, w_i\}} \quad \frac{u o_{j_0}}{\sum_{i=1}^{h} v_i q_{i j_0} + \sum_{i=1}^{k} w_i c_{i j_0}} \tag{3.5}$$

subject to

$$\begin{aligned}
&\frac{u o_j}{\sum_{i=1}^{h} v_i q_{ij} + \sum_{i=1}^{k} w_i c_{ij}} \leq 1 \qquad && j = 1, \ldots, n \\
&u \geq \varepsilon && \\
&v_i \geq \varepsilon \qquad && i = 1, \ldots, h \\
&w_i \geq \varepsilon \qquad && i = 1, \ldots, k.
\end{aligned} \tag{3.6}$$

If we let an asterisk denote the optimal values of the variables, the I_{DEA-1} index for fund j_0 is computed as

$$I_{j_0,DEA-1} = \frac{u^* o_{j_0}}{\sum_{i=1}^{h} v_i^* q_{ij_0} + \sum_{i=1}^{k} w_i^* c_{ij_0}} \tag{3.7}$$

which makes evident that there exist manifest analogies with the traditional performance indicators (3.1)–(3.3). In effect, it can be shown that the I_{DEA-1} index generalizes the traditional Sharpe, Treynor and reward-to-half-variance indexes.

Let us prove this assertion for the Sharpe ratio (3.1); more precisely, we show that if we choose the excess return as output and the standard deviation of the return as the only input, thus omitting the entrance and exit investment costs (i.e., $h = 1$, $k = 0$), then the I_{DEA-1} efficiency index (3.7) coincides with the Sharpe ratio multiplied by a normalization constant which scales it off in the interval [0,1]. With these choices the linear programming version of the fractional problem (3.5)–(3.6) can be written as follows

$$\max \quad u[E(R_{j_0}) - \delta] \tag{3.8}$$

subject to

$$\begin{aligned} & v\sigma_{j_0} = 1 \\ & u[E(R_j) - \delta] - v\sigma_j \leq 0 \qquad\qquad j = 1, \ldots, n \\ & u \geq \varepsilon \\ & v \geq \varepsilon. \end{aligned} \tag{3.9}$$

From the equality constraint in system (3.9) we have $v^* = 1/\sigma_{j_0}$; by substituting this value for v in the other constraints we get the inequalities:

$$u \leq \left[\sigma_{j_0} \frac{E(R_j) - \delta}{\sigma_j}\right]^{-1} \qquad\qquad j = 1, \ldots, n. \tag{3.10}$$

Therefore the value of u which maximizes the objective function value while satisfying constraints (3.9) is

$$u^* = \left[\sigma_{j_0} \max_j \frac{E(R_j) - \delta}{\sigma_j}\right]^{-1} \tag{3.11}$$

and thus

$$I_{j_0,DEA-1} = \left[\max_j \frac{E(R_j) - \delta}{\sigma_j}\right]^{-1} \frac{E(R_{j_0}) - \delta}{\sigma_{j_0}} = \frac{I_{j_0,Sharpe}}{\max_j I_{j,Sharpe}}. \tag{3.12}$$

Analogously, by taking as risk measure either the root of the half-variance or the β coefficient we get the reward-to-half-variance and the Treynor indexes (3.2),(3.3), respectively.

The first DEA measure that has been proposed to evaluate mutual fund performance is the DPEI index developed by Murthi, Choi and Desai (1997), which considers the mutual fund return as output and the standard deviation and transaction costs as inputs. In a sense, I_{DEA-1} can be seen as a generalization of DPEI which allows to consider different risk measures.

Moreover, we may observe that there exist slight differences with regard to the transaction costs which DPEI and I_{DEA-1} take into consideration. In fact, DPEI includes also the operational expenses, management fees, markets and administrative expenses, and it considers as an additional input a trading turnover ratio. On the contrary, I_{DEA-1} takes into account only the subscription costs and redemption fees that directly burden the investors but not the other expenses that have already been deducted from the fund quotations and therefore from the net return of the portfolio.

4. A multiple output portfolio performance measure

As we have seen, the DEA performance measurement technique can take into account not only many inputs but also many outputs. A natural generalization of the I_{DEA-1} index, therefore, could consider some other output indicators besides the mean portfolio return. These additional output indicators could shed light on different aspects of the portfolios returns; for example, useful features that could be taken into consideration are the stochastic dominance relations between the mutual fund returns. In fact, the utility theory suggests that a portfolio which is dominated by other ones should be given a “bad” score.

The stochastic dominance decision models are presently employed in various areas of economics, finance and statistics. These techniques allow the partial ranking of random variables by assuming that investors act according to the expected utility paradigm, that is they prefer those alternatives that maximize their expected utility. The assumptions underlying the stochastic dominance rules concern the signs of successive derivatives of the investors’ utility function. In effect, the complete knowledge of this function is not required: it only has to belong to a wide class of utility functions, which depends on the order of stochastic dominance; these classes describe some basic properties of the individual preferences. For a review on stochastic dominance criteria see for example Kroll and Levy (1980) and Levy (1992).

Let X and Y be two random variables which could represent, for example, the returns of two portfolios. Moreover, let $U = U^{(0)}$ denote the utility function of the decision maker; for the sake of simplicity, let us assume $U \in C^{\infty}$. The n-th order stochastic dominance criterion requires that the investors’ utility functions belong to the set

$$\mathcal{U}_n = \{\, U \in \mathcal{U}_{n-1} : (-1)^n U^{(n)} \leq 0 \,\}. \tag{4.1}$$

The random variable X is said to dominate the random variable Y according to the n-th order stochastic dominance criterion if and only if

$$E(U(X)) \geq E(U(Y)) \qquad \forall\, U \in \mathcal{U}_n \tag{4.2}$$

and there exists $U^* \in \mathcal{U}_n$ such that $E(U^*(X)) > E(U^*(Y))$.

For $n = 1$ we have the first order stochastic dominance criterion which requires that the investors' utility functions belong to the set $\mathcal{U}_1$ of nondecreasing utility functions; this is a minimal hypothesis required from any utility function and reflects the principle of non satiety. For $n = 2$ the second order stochastic dominance criterion requires that the utility function is both non decreasing and concave which means that the marginal utility is non increasing and the preferences exhibit risk aversion.

The stochastic dominance rules are progressively weaker with respect to the order of the dominance: if a prospect X dominates a prospect Y by the n-th order stochastic dominance, then the dominance relation holds for any order greater than n, too. Obviously, the more restrictive is the class of utility functions considered (the higher is the order of the dominance rule), the higher will be the number of dominance relations observed. However, more restrictions on the utility functions imply that the efficient set is suitable for a smaller group of decision makers and may involve a loss in generality. Moreover, while the economic meaning of the lowest stochastic dominance orders is clear, the economic rationale of the highest orders rules is not evident.

Another dominance rule is related to the widely accepted hypothesis of decreasing absolute risk aversion (DARA). In fact, since de Finetti (1952), Arrow (1965) and Pratt (1964), it is widely accepted that investors' utility functions exhibit decreasing absolute risk aversion. If we define $R = -U''/U'$ whenever this quantity exists, the DARA hypothesis implies $R' \leq 0$. Let us define the set

$$\mathcal{U}_{DARA} = \{U \in \mathcal{U}_2 : U^{'} \neq 0,\ R^{'} \leq 0\}. \tag{4.3}$$

X is said to dominate Y according to the DARA dominance rule if and only if

$$E[U(X)] \geq E[U(Y)] \qquad \forall\, U \in \mathcal{U}_{DARA} \tag{4.4}$$

and there exists $U^* \in \mathcal{U}_{DARA}$ such that $E[U^*(X)] > E[U^*(Y)]$.

As the set $\mathcal{U}_{DARA}$ is a proper subset of $\mathcal{U}_3$, the third order stochastic dominance implies the DARA dominance, so that the DARA efficient set turns out to be a subset of the third order efficient set; moreover, the DARA and the third order criteria are equivalent when $E(X) = E(Y)$ (see Vickson, 1975).

Another property of the investors' utility functions which can be used to define a dominance relation is prudence, which entails a positive third derivative and characterizes the so called precautionary saving (Kimball, 1990). For measuring the degree of prudence Kimball introduces the absolute prudence index $\varphi = -U^{'''}/U^{''}$ and shows that decreasing absolute prudence is a necessary and sufficient condition that guarantees that the saving of wealthier people is less sensitive to the risk associated to future incomes. Let

$$\mathcal{U}_{DAP} = \{U \in \mathcal{U}_3 : U^{''} \neq 0,\ \varphi^{'} \leq 0\}; \tag{4.5}$$

then we can say that X dominates Y according to the DAP dominance rule if and only if

$$E[U(X)] \geq E[U(Y)] \qquad \forall\, U \in \mathcal{U}_{DAP} \tag{4.6}$$

and there exists $U^* \in \mathcal{U}_{DAP}$ such that $E[U^*(X)] > E[U^*(Y)]$.

The decreasing absolute prudence (DAP) hypothesis entails that the fourth derivatives of the agents' utility functions are negative, a feature called temperance. Therefore, we can give an economic meaning to the first four orders of stochastic dominance.

Moreover, DAP can be used in conjunction with DARA giving standard risk aversion (SRA), which guarantees on the one hand that introducing a zero-mean background risk to wealth makes people less willing to accept another independent risk and on the other hand that an increase in the risk of the returns distribution of an asset reduces the demand for this asset. Standard risk aversion can be used to define another dominance relation. Let us define

$$\mathcal{U}_{SRA} = \{U \in \mathcal{U}_3 : U^{'} \neq 0,\ U^{''} \neq 0,\ R^{'} \leq 0,\ \varphi^{'} \leq 0\}; \tag{4.7}$$

then we can say that X dominates Y according to the SRA dominance rule if and only if

$$E[U(X)] \geq E[U(Y)] \qquad \forall\, U \in \mathcal{U}_{SRA} \tag{4.8}$$

and there exists $U^* \in \mathcal{U}_{SRA}$ such that $E[U^*(X)] > E[U^*(Y)]$.

If we consider a set of alternative portfolios, the stochastic dominance rules can be used in order to determine an efficient set containing only the non dominated alternatives. There exist many empirical applications in portfolio analysis that uses stochastic dominance criteria to determine an efficient set of portfolios; see for example Tehranian (1980), Jean and Helms (1988), Cardin, Decima and Pianca (1992), Basso and Pianca (1997).

By testing the stochastic dominance relationships among a set of mutual funds on consecutive time periods it is possible, to some extent, to take into account the time occurrence of the mutual fund returns. In particular, the financial press often publishes the past returns of mutual funds in the last periods; hence, it could be preferred a fund which is not dominated by other funds in the last periods.

Let us consider the past returns of mutual funds over a convenient time period and let us divide this period into subperiods (usually of equal length). We can build a stochastic dominance indicator which reflects both the investors' preference structure and the time occurrence of the returns assigning a higher score to the mutual funds which are not dominated by other funds in the higher number of subperiods. This can easily be done by assigning to fund j, for $j = 1, 2, \ldots, n$, the relative number of subperiods in which it is not dominated by other funds

$$d_j = \frac{\text{number of non dominated subperiods for fund } j}{\text{total number of subperiods}}. \tag{4.9}$$

A two outputs DEA portfolio performance measure, I_{DEA-2}, may therefore be defined as the optimal value of the objective function of the following fractional linear programming problem

$$\max_{\{u_r, v_i, w_i\}} \qquad \frac{u_1 o_{j_0} + u_2 d_{j_0}}{\sum_{i=1}^{h} v_i q_{ij_0} + \sum_{i=1}^{k} w_i c_{ij_0}} \tag{4.10}$$

subject to

$$
\begin{aligned}
&\frac{u_1 o_j + u_2 d_j}{\sum_{i=1}^{h} v_i q_{ij} + \sum_{i=1}^{k} w_i c_{ij}} \leq 1 && j = 1, \ldots, n \\
&u_r \geq \varepsilon && r = 1, 2 \\
&v_i \geq \varepsilon && i = 1, \ldots, h \\
&w_i \geq \varepsilon && i = 1, \ldots, k.
\end{aligned}
\tag{4.11}
$$

Of course, the stochastic dominance indicator (4.9) can be defined using any stochastic dominance criteria. In the empirical applications which will be presented later on we have used the DARA rule which is based on a widely accepted hypothesis, is more selective that the first three orders of stochastic dominance and can be computationally tested using a convenient dynamic programming algorithm originally proposed by Vickson (1975) (for the details on the application of the algorithm see Basso and Pianca, 1997). Moreover, the computational experiences show that the DARA criterion is generally the most efficient (Basso and Pianca, 1997).

5. The DEA performance measure and the investment horizon

It is known that the traditional indexes (3.1)–(3.4) are sensitive to the assumed investment horizon. More precisely, we may get different results according to the data frequency, i.e. the time interval (week, month, year...) to which the portfolio returns refer. In particular, Levy (1972) shows that the Sharpe index (3.1) depends on the investment horizon assumed. Hence, a systematic bias could be observed if the index is computed by assuming a holding period which is different from the one chosen by the investors. Levhari and Levy (1977) indicate that a similar bias exists also for the Treynor and Jensen performance measures (3.3)–(3.4)[1].

Nevertheless, if the instantaneous (logarithmic) rates of return are used instead of the compounded rates of return, the effect of a change in the investment horizon can easily be determined.

Let us first analyze the effect of a change of the investment period on the Sharpe index (3.1). A very useful property of the instantaneous rates of return ensures that the overall rate of return in a time period is the sum of the rates of return observed in each subperiod; for example, an annual rate of return is the sum of the monthly rates of return observed during the year. Let us assume stationarity and independence of the returns over time and let us compare the Sharpe measures obtained for the same fund j using two different investment horizons of length L_1 and $L_2 = L \cdot L_1$, respectively. Using the interval of length L_2, both the expected instantaneous rate of return and the instantaneous riskless rate have values which are equal to the ones computed for the interval of length L_1 multiplied by the proportionality factor L. Moreover, the volatility of the instantaneous rate of

[1] We thank an anonymous referee for having pointed out the importance of the length of the investment horizon and suggested these two references.

return related to the length L_2 is equal to the volatility computed with reference to L_1 multiplied by $\sqrt{L}$. Therefore, if we denote by $I_{j,Sharpe}(L_1)$ and $I_{j,Sharpe}(L_2)$ the Sharpe indexes related to the horizons of length L_1 and L_2, respectively, we have

$$I_{j,Sharpe}(L_2) = \sqrt{L} \cdot I_{j,Sharpe}(L_1). \tag{5.1}$$

Hence, a change in the investment horizon entails that the Sharpe index is multiplied by the factor $\sqrt{L}$ but that does not modifies the relative ranking of the portfolios. A similar relationship holds for the reward-to-half-variance index.

As for as the β coefficient is concerned, it is easy to see that it is not affected by changes in the investment period (more precisely, the β is a ratio between a covariance and a variance which are both proportional to the length of the investment period). As a consequence, the Treynor indexes computed over the time horizons of length L_1 and L_2, $I_{j,Treynor}(L_1)$ and $I_{j,Treynor}(L_2)$, are related by the relationship

$$I_{j,Treynor}(L_2) = L \cdot I_{j,Treynor}(L_1). \tag{5.2}$$

An analogous formula holds for the Jensen measure: $I_{j,Jensen}(L_2) = L \cdot I_{j,Jensen}(L_1)$, where $I_{j,Jensen}(L_1)$ and $I_{j,Jensen}(L_2)$ denote the Jensen measures related to L_1 and L_2, respectively.

Let us now turn to the effect of a change of the investment horizon on the DEA efficiency measures I_{DEA-1} and I_{DEA-2}. If we consider the instantaneous rates of return once more, we may see the effects of such a change on the expected rate of return and on the volatility of the returns as a change of the measurement units used. Now, a *units invariance theorem* concerning the DEA efficiency measures (see Cooper, Seiford and Tone, 2000) states that the optimal values of max h_0 in (2.2) are independent of the units in which the inputs and outputs are measured provided these units are the same for every decision making unit.

Therefore, using logarithmic returns and under the assumption of stationarity and independence of the returns over time, I_{DEA-1} is invariant with respect to the investment horizon chosen.

On the other hand, the I_{DEA-2} measure may depend on the investment horizon indirectly, as the stochastic dominance indicator (4.9) may depend on the subdivision into subperiods, which should naturally be related to the holding period assumed.

6. Benchmark portfolios

The measures of the relative efficiency of the decision making units represent only one kind of information resulting from the DEA methodology. In effect, the DEA approach can also suggest to the inefficient units a “virtual unit” that they could imitate in order to improve their efficiency.

With regard to this, it is known (see for example Boussofiane, Dyson and Thanassoulis, 1991) that for each inefficient unit the solution of the input-oriented CCR dual model (2.6)–(2.7) permits to identify a set of corresponding efficient units, called *peer units*, which are efficient with the inefficient unit's weights. The peer units are associated with the (strictly) positive basic multipliers in the optimal solution λ_j^*, that is the non null optimal dual variables. Therefore, for each inefficient unit j_0 it is possible to build a composite unit with output $\sum_{j=1}^n \lambda_j^* y_{rj}$, $r = 1, \ldots, t$, and input $\sum_{j=1}^n \lambda_j^* x_{ij}$, $i = 1, \ldots, m$, that outperforms unit j_0 and lies on the efficient frontier.

From a financial point of view, this composite unit could be considered as a benchmark for the inefficient fund j_0. Fund j_0 could improve its performance by trying to imitate the behaviour of the efficient composite unit. This (efficient) benchmark portfolio has an input/output orientation (a particular "style") which is similar to that of the (inefficient) fund j_0; the knowledge of this benchmark composite unit can therefore be useful in studying the style of the portfolio management. For the importance of analyzing the management style of an asset portfolio see Sharpe (1992).

The DEA performance indexes I_{DEA-1} and I_{DEA-2} are defined using the mutual funds rate of return; hence, the decision making units (the mutual funds) are implicitly scaled in terms of the same amount of invested capital. Hence it is worth computing the normalized multipliers

$$l_j = \frac{\lambda_j^*}{\sum_{k=1}^n \lambda_k^*} \tag{6.1}$$

which indicate the relative composition of the benchmark portfolio.

7. An empirical analysis

We have tested the DEA performance indexes for mutual fund investments I_{DEA-1} and I_{DEA-2} on empirical data of the Italian financial market. We have considered the weekly logarithmic returns of 47 mutual funds, for which homogeneous information are available, and those of the Milan stock exchange Mibtel index (closing price); in some experiments we have also considered the instantaneous rate of return δ of a riskless asset, the 12 months Italian Treasury bill (B.O.T.), measured on a weekly base. The data regard the Monday net prices in the period 01/01/1997 to 30/06/1999. The mutual funds analyzed belong to different classes (stocks funds, balanced funds and bonds funds), have different total capital and refer to different management companies.

In the empirical application we have used as first output indicator the expected return (in order to limit the presence of negative values among the outputs) and as second output the stochastic dominance indicator (4.9) defined using the DARA criterion. To this aim the overall time period has been divided into half-yearly subperiods over which the DARA dominance has been tested; d_j in (4.9) is computed as the number of half-years in which fund j is not dominated by any other fund, divided by the total number of half-years (namely 5). Of course, a different subdivision of the time period considered could have been chosen, according to the investment horizon assumed for the investors.

Among the inputs, we have considered as risk measures the portfolio standard deviation, the square root of the half-variance and the β coefficient; actually, the β coefficient has been used as an additional measure of risk, together with one of the first two risk indicators, as it takes into account the portfolio diversification. The computation of the β coefficient has been carried out using the Mibtel index as market portfolio. Moreover, with regard to the initial and final costs we have considered the per cent subscription costs per different amounts of initial investment (5 000, 25 000 and 50 000 Euros) and the per cent redemption costs per length of investment period (1, 2 and 3 years).

We point out that the empirical analysis has been carried out only on a few dozens of mutual funds, while Murthi, Choi and Desai, 1997, made a wider empirical work by considering up to 2083 mutual funds. On the one hand, this limits the generality of the results since our analysis of the market is far from being complete; on the other hand, however, this enables us to investigate in detail the results of the ranking obtained with the different performance measures.

In the algorithmic applications, the DEA efficiency measure may depend on the value which has been chosen for the small real constant ε which bounds the weights from below and formally would be a non-Archimedean constant. The optimization of such problems, involving non-Archimedean infinitesimal, can be computed using a convenient two-stage process; on this subject, see for example Charnes, Cooper, Lewin and Seiford (1994).

Table 1 shows the results obtained with the DEA approach by comparing the stocks, balanced and bonds funds separately, using as risk measures both the standard deviation and the β coefficient of the returns. The results refer both to I_{DEA-1} and I_{DEA-2} performance measures and have been computed with a subscription cost and a redemption cost (for 5 000 Euros of invested capital and a 1 year investment period). The relative ranking is given in italics.

Note that using I_{DEA-2}, i.e. using two output indicators instead of one, the number of efficient funds (those with $I_{DEA-2} = 1$) increases; in fact, adding one more output we have one more indicator with respect to which some funds can be considered as efficient. On the contrary, by including more subscription and redemption costs the efficient funds do not change. In both cases, the relative ranking undergoes slight changes.

This is confirmed by the first part of table 2 which reports the correlation coefficients between the efficiency measures obtained using different inputs and outputs. The results shown in table 2 refer to the following trials: I_{DEA-1} and I_{DEA-2}, one and all subscription and redemption costs, σ or $\sqrt{HV}$ as the first risk measure, with and without the β coefficient as a second risk measure. We have made all the cross trials for each class of funds and have reported the average values of the correlation coefficients thus obtained; moreover, we have computed the weighted averages on all classes (weighting with the number of funds in each class).

From this table it can be seen that the differences observed using one or all subscription and redemption costs, as well as those observed changing the first risk measure, are negligible. Somewhat more significant are the differences

Table 1. DEA performance measures for the different classes of funds, with one and two outputs, and for one and all subscription and redemption costs. The standard deviation and the β of the returns are used simultaneously as risk measures. The relative ranking is given in italics.

	I_{DEA-1}				I_{DEA-2}			
Fund	one cost		all costs		one cost		all costs	
Stocks funds								
1. Arca 27	1.000	*1*	1.000	*1*	1.000	*1*	1.000	*1*
2. Azimut Borse Int.	0.853	*18*	0.853	*18*	0.853	*18*	0.853	*18*
3. Azimut Europa	0.848	*20*	0.848	*20*	0.848	*20*	0.848	*20*
4. Centrale Global	0.959	*8*	0.959	*8*	1.000	*1*	1.000	*1*
5. Centrale Italia	1.000	*1*	1.000	*1*	1.000	*1*	1.000	*1*
6. Epta Azioni Italia	0.929	*10*	0.929	*11*	0.929	*10*	0.929	*11*
7. EptaInternational	0.766	*24*	0.766	*24*	0.766	*24*	0.766	*24*
8. Fideuram Azione	0.858	*17*	0.858	*17*	0.858	*17*	0.858	*17*
9. Fondicri Int.	1.000	*1*	1.000	*1*	1.000	*1*	1.000	*1*
10. Fondicri Sel. Italia	0.918	*12*	0.951	*9*	0.918	*12*	0.951	*9*
11. Genercomit Azioni It.	0.993	*7*	0.993	*7*	1.000	*1*	1.000	*1*
12. Genercomit Int.	0.894	*13*	0.894	*14*	0.915	*13*	0.918	*12*
13. Gesticredit Borsit	0.886	*15*	0.907	*13*	0.886	*15*	0.907	*14*
14. Gesticredit Euro Az.	0.829	*21*	0.829	*21*	0.829	*21*	0.829	*21*
15. Imi Europe	0.851	*19*	0.851	*19*	0.851	*19*	0.851	*19*
16. Imi Italy	0.883	*16*	0.883	*16*	0.883	*16*	0.883	*16*
17. Investire Azion.	1.000	*1*	1.000	*1*	1.000	*1*	1.000	*1*
18. Investire Europa	0.784	*23*	0.784	*23*	0.784	*23*	0.784	*23*
19. Investire Intern.	1.000	*1*	1.000	*1*	1.000	*1*	1.000	*1*
20. Oasi Azionario Italia	1.000	*1*	1.000	*1*	1.000	*1*	1.000	*1*
21. Prime Global	0.931	*9*	0.931	*10*	0.931	*9*	0.931	*10*
22. Sanpaolo H. Europe	0.918	*11*	0.918	*12*	0.918	*11*	0.918	*13*
23. Sanpaolo H. Intern.	0.821	*22*	0.821	*22*	0.821	*22*	0.821	*22*
24. Mibtel index	0.891	*14*	0.891	*15*	0.895	*14*	0.895	*15*
Balanced funds								
1. Arca BB	1.000	*1*	1.000	*1*	1.000	*1*	1.000	*1*
2. Arca TE	1.000	*1*	1.000	*1*	1.000	*1*	1.000	*1*
3. Azimut Bil.	0.875	*7*	0.875	*7*	0.875	*7*	0.875	*7*
4. Eptacapital	0.877	*6*	0.877	*6*	0.880	*6*	0.880	*6*
5. Fideuram Performance	0.999	*4*	0.999	*4*	1.000	*1*	1.000	*1*
6. Fondo Centrale	0.863	*8*	0.863	*8*	0.863	*8*	0.863	*8*
7. Genercomit	0.988	*5*	0.988	*5*	0.988	*5*	0.988	*5*
8. Genercomit Espansione	0.671	*10*	0.671	*10*	0.671	*10*	0.671	*10*
9. Investire Bil.	1.000	*1*	1.000	*1*	1.000	*1*	1.000	*1*
10. Mibtel index	0.724	*9*	0.724	*9*	0.724	*9*	0.724	*9*
Bonds funds								
1. Arca Bond	0.824	*6*	0.824	*6*	0.824	*6*	0.824	*6*
2. Azimut Rend. Int.	0.183	*11*	0.201	*15*	0.183	*11*	0.200	*15*
3. Bpb Tiepolo	0.892	*5*	0.892	*5*	1.000	*1*	1.000	*1*
4. Centrale Tasso Fisso	0.149	*14*	0.208	*13*	0.149	*14*	0.208	*13*
5. Epta 92	0.182	*12*	0.215	*12*	0.182	*12*	0.215	*12*
6. Eptabond	0.296	*9*	0.429	*9*	0.296	*9*	0.429	*9*
7. Fideuram Security	1.000	*1*	1.000	*1*	1.000	*1*	1.000	*1*
8. Genercomit Obb. Estere	0.522	*8*	0.552	*8*	0.527	*8*	0.557	*8*
9. Imi Bond	0.140	*15*	0.203	*14*	0.140	*15*	0.203	*14*
10. Investire Bond	0.629	*7*	0.660	*7*	0.629	*7*	0.660	*7*
11. Oasi Bond Risk	1.000	*1*	1.000	*1*	1.000	*1*	1.000	*1*
12. Oasi Btp Risk	1.000	*1*	1.000	*1*	1.000	*1*	1.000	*1*
13. Prime Reddito Italia	0.164	*13*	0.273	*11*	0.164	*13*	0.273	*11*
14. Primebond	0.190	*10*	0.293	*10*	0.190	*10*	0.293	*10*
15. Sanpaolo H. Bonds	0.102	*16*	0.127	*16*	0.102	*16*	0.127	*16*
16. Riskless asset	1.000	*1*	1.000	*1*	1.000	*1*	1.000	*1*

Table 2. Correlation coefficients of I_{DEA-1} and I_{DEA-2} indexes with respect to the traditional indexes and to different inputs and outputs choices, for the different classes of funds.

Correlations between DEA indexes with different inputs and outputs

	I_{DEA-1}/I_{DEA-2}	One/all costs	$\sigma/\sqrt{HV}$	With/without β
Stocks funds	0.969	0.995	0.996	0.922
Balanced funds	0.979	1.000	0.997	0.975
Bonds funds	0.990	0.996	0.997	0.974
Weighted averages	0.978	0.996	0.996	0.949

Correlations between DEA and traditional indexes

	DEA/Sharpe	DEA/reward-to-HV	DEA/Treynor	DEA/Jensen
Stock funds: I_{DEA-1}	0.696	0.657	0.473	0.624
Stock funds: I_{DEA-2}	0.681	0.680	0.500	0.630
Balanced funds: I_{DEA-1}	0.823	0.919	0.777	0.896
Balanced funds: I_{DEA-2}	0.823	0.919	0.776	0.895
Bonds funds: I_{DEA-1}	0.715	0.449	0.755	0.796
Bonds funds: I_{DEA-2}	0.714	0.449	0.754	0.795

Table 3. DEA performance measures for stocks funds when the alternatives includes the riskless asset (12 months B.O.T.), with one and two outputs, and for one and all subscription and redemption costs. The standard deviation and the β of the returns are used simultaneously as risk measures. The relative ranking is given in italics.

	I_{DEA-1}				I_{DEA-2}			
Fund	one cost		all costs		one cost		all costs	
Stocks funds								
1. Arca 27	1.000	*1*	1.000	*1*	1.000	*1*	1.000	*1*
2. Azimut Borse Int.	0.430	*12*	0.460	*16*	0.430	*12*	0.460	*16*
3. Azimut Europa	0.466	*10*	0.497	*14*	0.466	*10*	0.497	*14*
4. Centrale Global	0.277	*23*	0.370	*22*	0.277	*23*	0.370	*22*
5. Centrale Italia	0.402	*13*	0.516	*13*	0.402	*13*	0.516	*13*
6. Epta Azioni Italia	0.524	*9*	0.589	*9*	0.524	*9*	0.589	*9*
7. EptaInternational	0.366	*16*	0.420	*18*	0.366	*16*	0.420	*18*
8. Fideuram Azione	0.242	*25*	0.369	*23*	0.242	*25*	0.369	*23*
9. Fondicri Int.	0.303	*20*	0.480	*15*	0.303	*20*	0.480	*15*
10. Fondicri Sel. Italia	0.392	*14*	0.573	*10*	0.392	*14*	0.573	*10*
11. Genercomit Azioni It.	0.993	*6*	0.993	*6*	1.000	*1*	1.000	*1*
12. Genercomit Int.	0.277	*22*	0.435	*17*	0.277	*22*	0.435	*17*
13. Gesticredit Borsit	0.373	*15*	0.518	*12*	0.373	*15*	0.518	*12*
14. Gesticredit Euro Az.	0.334	*17*	0.417	*19*	0.334	*17*	0.417	*19*
15. Imi Europe	0.246	*24*	0.328	*25*	0.246	*24*	0.328	*25*
16. Imi Italy	0.310	*18*	0.404	*20*	0.310	*18*	0.404	*20*
17. Investire Azion.	1.000	*1*	1.000	*1*	1.000	*1*	1.000	*1*
18. Investire Europa	0.784	*7*	0.784	*7*	0.784	*7*	0.784	*7*
19. Investire Intern.	1.000	*1*	1.000	*1*	1.000	*1*	1.000	*1*
20. Oasi Azionario Italia	1.000	*1*	1.000	*1*	1.000	*1*	1.000	*1*
21. Prime Global	0.461	*11*	0.533	*11*	0.461	*11*	0.533	*11*
22. Sanpaolo H. Europe	0.308	*19*	0.375	*21*	0.308	*19*	0.375	*21*
23. Sanpaolo H. Intern.	0.284	*21*	0.345	*24*	0.284	*21*	0.345	*24*
24. Mibtel index	0.714	*8*	0.732	*8*	0.713	*8*	0.732	*8*
25. Riskless asset	1.000	*1*	1.000	*1*	1.000	*1*	1.000	*1*
Correlation of results with and without riskless asset	0.499		0.563		0.465		0.528	

Table 4. Relative composition of the benchmark portfolios in the DEA model with two outputs and eight inputs: the standard deviation and the β of the returns, all the subscription and redemption costs.

Fund	standardized multipliers			
Stocks funds				
1. Arca 27*	$l_1 = 1.000$			
2. Azimut Borse Int.	$l_1 = 0.577$	$l_5 = 0.148$	$l_9 = 0.276$	
3. Azimut Europa	$l_1 = 0.494$	$l_5 = 0.408$	$l_{20} = 0.098$	
4. Centrale Global*	$l_4 = 1.000$			
5. Centrale Italia*	$l_5 = 1.000$			
6. Epta Azioni Italia	$l_5 = 0.476$	$l_{20} = 0.524$		
7. EptaInternational	$l_1 = 0.502$	$l_5 = 0.253$	$l_9 = 0.246$	
8. Fideuram Azione	$l_5 = 0.071$	$l_9 = 0.929$		
9. Fondicri Int.*	$l_9 = 1.000$			
10. Fondicri Sel. Italia	$l_5 = 0.668$	$l_{20} = 0.332$		
11. Genercomit Azioni It.*	$l_{11} = 1.000$			
12. Genercomit Int.	$l_1 = 0.087$	$l_4 = 0.125$	$l_9 = 0.788$	
13. Gesticredit Borsit	$l_5 = 0.772$	$l_{20} = 0.228$		
14. Gesticredit Euro Az.	$l_1 = 0.215$	$l_5 = 0.472$	$l_9 = 0.313$	
15. Imi Europe	$l_5 = 0.447$	$l_9 = 0.553$		
16. Imi Italy	$l_5 = 1.000$			
17. Investire Azion.*	$l_{17} = 1.000$			
18. Investire Europa	$l_{17} = 0.425$	$l_{19} = 0.575$		
19. Investire Intern.*	$l_{19} = 1.000$			
20. Oasi Azionario Italia*	$l_{20} = 1.000$			
21. Prime Global	$l_1 = 0.567$	$l_5 = 0.114$	$l_9 = 0.319$	
22. Sanpaolo H. Europe	$l_5 = 0.381$	$l_9 = 0.619$		
23. Sanpaolo H. Intern.	$l_1 = 0.068$	$l_5 = 0.263$	$l_9 = 0.669$	
24. Mibtel index	$l_1 = 0.021$	$l_5 = 0.145$	$l_{11} = 0.217$	$l_{20} = 0.617$
Balanced funds				
1. Arca BB*	$l_1 = 1.000$			
2. Arca TE*	$l_2 = 1.000$			
3. Azimut Bil.	$l_1 = 1.000$			
4. Eptacapital	$l_1 = 0.966$	$l_2 = 0.034$		
5. Fideuram Performance*	$l_5 = 1.000$			
6. Fondo Centrale	$l_1 = 0.454$	$l_2 = 0.546$		
7. Genercomit	$l_1 = 1.000$			
8. Genercomit Espansione	$l_1 = 0.126$	$l_2 = 0.874$		
9. Investire Bil.*	$l_9 = 1.000$			
10. Mibtel index	$l_1 = 0.895$	$l_9 = 0.105$		
Bonds funds				
1. Arca Bond	$l_{11} = 0.383$	$l_{12} = 0.617$		
2. Azimut Rend. Int.	$l_{11} = 0.154$	$l_{16} = 0.846$		
3. Bpb Tiepolo*	$l_3 = 1.000$			
4. Centrale Tasso Fisso	$l_{12} = 0.048$	$l_{16} = 0.952$		
5. Epta 92	$l_{12} = 0.157$	$l_{16} = 0.843$		
6. Eptabond	$l_{12} = 0.146$	$l_{16} = 0.854$		
7. Fideuram Security*	$l_7 = 1.000$			
8. Genercomit Obb. Estere	$l_3 = 0.112$	$l_7 = 0.445$	$l_{12} = 0.443$	
9. Imi Bond	$l_{12} = 0.073$	$l_{16} = 0.927$		
10. Investire Bond	$l_7 = 0.426$	$l_{12} = 0.574$		
11. Oasi Bond Risk*	$l_{11} = 1.000$			
12. Oasi Btp Risk*	$l_{12} = 1.000$			
13. Prime Reddito Italia	$l_{12} = 0.122$	$l_{16} = 0.878$		
14. Primebond	$l_{11} = 0.064$	$l_{12} = 0.161$	$l_{16} = 0.775$	
15. Sanpaolo H. Bonds	$l_{12} = 0.066$	$l_{16} = 0.934$		
16. Riskless asset*	$l_{16} = 1.000$			

observed when we add one more output indicator (first column in table 2) and the β coefficient (last column).

The second part of table 2 reports the value of the correlation coefficients between the results obtained with the DEA approach and the traditional Sharpe, reward-to-half-variance, Treynor and Jensen indexes. The DEA results regard both the I_{DEA-1} and I_{DEA-2} indexes and have been obtained using as inputs all subscription and redemption costs, the standard deviation of the returns and the β coefficient. As can be observed, the value of the correlation coefficient between the DEA and the traditional indexes ranges from 0.449 and 0.919. We have seen in Section 3 that Sharpe, reward-to-half-variance and Treynor indexes can be obtained as DEA efficiency measures with the excess return as output and without investment costs; therefore, we can notice that the inclusion of the subscription and redemption costs does alter the performance results. In effect, it has to be pointed out that in some cases the traditional indexes could not be computed as, being the excess returns of some funds negative, the ratios (3.1)–(3.3) had no meaning.

If we compare the correlation results obtained with the I_{DEA-1} and I_{DEA-2} indexes to the results obtained for the DPEI index by Murthi, Choi and Desai (1997), we may see that the correlations of the DEA performance measures with the Sharpe index have fairly similar values. On the contrary, the correlations of the DEA measures with the Jensen index that we obtain are somewhat higher that the ones obtained by Murthi, Choi and Desai. This is not surprising, as we allow for a risk measure derived in the C.A.P.M. framework (the β coefficient) to be taken into account in our performance measures.

In tables 1 and 2 the performance measures for the stocks and balanced funds have been evaluated by comparing the funds in each class, including also the Stock Exchange Mibtel index. However, when the aim of the analysis is to provide a tool which may help investors to choose a convenient investment, the inclusion of a riskless asset in the reference set may be meaningful. For comparative purposes, Table 3 presents the results obtained for the stocks funds including in the reference set the 12 months B.O.T.. As can be seen, this inclusion changes the efficiency measures and the relative ranking substantially. In fact, the correlation coefficients between the results obtained with and without a riskless asset have values around 0.5.

It is interesting to observe that, unlike the traditional mutual funds performance indexes, the value of which don't change when the set of mutual funds to be compared is modified, the values of the DEA efficiency indexes are affected by the funds included in the reference set. With regard to this, it has to be remarked that the DEA measure is a relative efficiency measure which is constrained to be not greater than 1 for all the decision making units considered; this is the reason why the DEA measure depends on the choice of the set of the decision making units to be compared. On the other hand, if the traditional indexes are normalized as in (3.12), their value, too, will change with this set.

Table 4 reports the peer groups and the relative composition (normalized multipliers (6.1)) of the benchmark portfolios for the different classes of funds in the I_{DEA-2} model with both standard deviation and β as risk measures and all the subscription and redemption costs. We may observe that the efficient funds have no need to define a benchmark portfolio while they often enter in the benchmark portfolios of the other funds.

8. Concluding remarks

In this paper we propose to use the DEA methodology in order to evaluate the performance of mutual funds. The DEA performance indexes for mutual funds proposed represent a generalization of various traditional numerical indexes and permit to take into account several inputs as well as several outputs.

Two classes of DEA indexes are proposed. The first one generalizes the traditional measures by including in the evaluation different risk indicators and subscription and redemption costs that burden the fund investment. The second class of indexes considers a multiple inputs-multiple outputs structure. In this way it is possible to monitor not only the mean return but also other features such as stochastic dominance and the time lay-out.

The computational procedure, in addition, allows to identify, for each inefficient fund, a corresponding efficient set of funds. Such a fund portfolio can be seen as a particular benchmark and characterizes a portfolio style.

Some results obtained by testing the DEA performance index on the Italian financial market indicate the importance of the subscription and redemption costs in determining the fund ranking.

The results suggest that the DEA methodology for evaluating the mutual funds performance may usefully complement the traditional indexes. The DEA approach, indeed, provides some additional information that may be useful in carrying out a careful comparative analysis.

In addition, we point out that recently the market of ethical finance has grown considerably, and the ethical funds, or socially responsible funds, are now widespread in many developed countries. An interesting subject for future research on performance measures of mutual funds is given by the study of new performance measures which are more apt to analyze the ethical contents. In our opinion, the DEA approach is one of the most promising techniques which deserves to be explored to this aim for its ability to take into account conflicting objectives such as the return on the investment and the pursuit of social objectives.

Another interesting direction for future research could be given by the analysis of other, and perhaps better, ways of incorporating stochastic dominance into the framework of the DEA performance measures for mutual fund portfolios.

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