Efficient Allocation of Resources
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EFFICIENT ALLOCATION OF RESOURCES[1]

By Tjalling C. Koopmans[2]

"For all these reasons then, and others perhaps too analytic to be verbally developed here . . . ."

Herman Melville, *Moby Dick*

INTRODUCTION

The problems of welfare economics can be broadly arranged in two groups: the first group is concerned with the most efficient allocation of resources in production, the second with the most desirable distribution of commodities or income. While these two problem areas can be distinguished, let us first ask ourselves whether they can be separated. Do they not hang together particularly through the effect of the reward for human effort on the quantity and quality of that effort?

Any theory of optimal choice must specify what aspects of reality are accepted as given facts, what aspects are regarded as subject to choice, and what is the objective guiding such choice. The famous socialist maxim, "from each according to his ability, to each according to his needs," seems to assume that human effort is available in given quantity once needs are met. However, such an assumption is not necessary to justify the separate study of allocative efficiency in production. We can study the potentialities of efficient production in the same way we study a demand curve or any other behavior schedule. That is, assuming that certain amounts of labor, land services, and other factors of production are available in given quantities, how can we characterize efficient modes of production according to some given criterion of efficiency? Such an inquiry can be useful even though considerations outside the sphere of production may further restrict the quantitative factor combinations that are indeed possible.

With this understanding, therefore, we can in the present paper concentrate entirely on the problem of efficient allocation in production. Much of the literature touching on this topic is concerned with the evaluation of alternative institutional or administrative forms of organizing production. This evaluation forms part of the grand debate on the merits of private or corporate enterprise versus a centrally directed economy—a debate touching upon the broad theme of the present

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[2] I am indebted to C. Christ, G. B. Dantzig, N. Georgescu-Roegen, C. Hildreth, J. Marschak, and S. Reiter for valuable comments and suggestions.

meetings[3]—and important insights about our topic can be gained from this debate. The famous article by Enrico Barone on "The Ministry of Production in the Collectivist State"[4] emphasized the idea that an economy under centralized direction, to be efficient, should in most of its operations satisfy the same formal conditions as are satisfied by the economic theorist's model of competitive society. This idea has been substantially accepted by all participants in the ensuing debate. The controversy was about methods of satisfying these conditions. Von Hayek[5] and Robbins[6] argued that it was impossible to impose these conditions by explicit calculation in one central office and by centralized administrative direction based on the results of such calculation. A number of writers, including Dickinson,[7] Lange,[8] and Lerner,[9] then took up the "socialist" side of the debate, incorporating in their model of a socialist economy the relevant theoretical consequences, but not the actual form, of competitive organization. These authors argued that, in order to attain the objectives of a socialist economy, centralized calculation is not necessary. Efficiency can also be achieved if all managers of individual plants or industries respond to a price system applicable to the whole economy, in a manner prescribed by the following rules: The manager of any plant should produce any output or output combination at minimum cost, and the manager of any plant or industry should arrange for production at such a level as to equate price and marginal cost.

This all too brief survey shows that, as the discussion went on, there was considerable adaptation of earlier notions of a socialist economy to the theorist's image of a competitive enterprise society. To remind us that the real world always offers a greater variety of problems than our attempts at theorizing have envisaged, a new contribution to the discussion has recently come from outside academic economics. M. K. Wood, a scientist and administrator, and G. B. Dantzig, a mathema-

[3] The meetings of the American Economic Association mentioned in footnote 1 were organized around the theme, "A Stocktaking of American Capitalism."

[4] Reprinted in *Collectivist Economic Planning*, F. von Hayek, ed., London: George Routledge and Sons, 1935; see pp. 245–290.

[5] F. von Hayek, "The Present State of the Debate," in *Collectivist Economic Planning*, *ibid.*, pp. 201–243, especially pp. 207–214.

[6] L. C. Robbins, *The Great Depression*, London: Macmillan and Co., 1934; see p. 151.

[7] M. D. Dickinson, "The Economic Basis of Socialism," *Political Quarterly*, September–December, 1930.

[8] O. Lange and F. M. Taylor, *On the Economic Theory of Socialism*, Minneapolis: The University of Minnesota Press, 1938.

[9] A. P. Lerner, *The Economics of Control*, New York: The Macmillan Co., 1946, 428 pp.

tician, both of the U. S. Department of the Air Force, were faced with the allocation problems of a widely ramified part of the military establishment. In these problems, the existing administrative structure provides no alternative to central direction of a complicated effort involving a large number of goods and services, all to be geared to one general objective: to maximize the military effectiveness or security obtained from given resources withdrawn from other uses, or, equivalently, to attain a required level of security at minimum cost in terms of resources withdrawn. In this situation, as they report in two articles in ECONOMETRICA,[10] they revert to the method discarded by all participants in the debate who came after Barone: the actual collection of relevant technical information in one center and the calculation of an allocation program to serve as the basis of a large number of detailed directives. They see in the development of electronic computers a new possibility for this method that was unforeseen in earlier phases of the discussion.

This interesting turn in the discussion shows, it seems to me, that the earlier discussions had been concerned too much with absolute institutional categories encompassing the entire economy. Even in the capitalistic enterprise economy there are many sectors where the guideposts of a competitive market are lacking and explicit analysis of the allocation problem is needed. Another example may be added to that discussed by Wood and Dantzig. In determining the best pattern of routing of empty railroad cars there are no market quotations placing differential prices on alternative geographic locations of cars. Present arrangements permit this complicated problem to be handled only by administrative direction.

A MODEL OF PRODUCTION

In most of the present paper we shall therefore set aside the question of the institutional arrangements under which allocative decisions are made. We wish to concentrate on the formal conditions for the efficient use of resources, so as to leave the door open for later application within the plant, or to the individual firm, to public enterprises or administrative organs, to an industry, or to the economy as a whole. The main departure from previous analyses in welfare economics is the adoption of

[10] Marshall K. Wood and George B. Dantzig, "Programming of Interdependent Activities; I. General Discussion," ECONOMETRICA, Vol. 17, July–October, 1949, pp. 193–199; and George B. Dantzig, "Programming of Interdependent Activities: II. Mathematical Model," *ibid.*, pp. 200–211. These articles have been reprinted, with some modifications, in *Activity Analysis of Production and Allocation*, Cowles Commission Monograph 13, Tjalling C. Koopmans, ed., New York: John Wiley and Sons, 1951, pp. 15–18, 19–32.

a different model of production.[11] In this model we shall not presuppose that marginal cost is necessarily known quantitatively to the manager of the individual production process, or even definable in terms of the technological data available to him. Actually, much substitution in production, between alternative factors as well as between alternative products or product-factor combinations, arises through shifts in the extent to which alternative discrete processes are used, rather than through continuous variation in factor combinations within the individual process. The purpose of the present model is to demonstrate that the possibility of such shifts is sufficient by itself to establish the concepts of marginal productivity and marginal cost, where applicable, and a more general concept where they are not applicable. For simplicity, we consider a static model only.

We shall employ two basic concepts,[12] the commodity and the activity. Each commodity is assumed homogeneous and perfectly divisible. An activity, if carried out at a unit level, consists in the transformation of given quantities of some commodities into given quantities of other commodities, per unit of time. It is assumed that the level of any activity can be any nonnegative multiple of the unit level, and that the commodity flows involved are the same multiple of those involved in the unit level of that activity. Negative flows represent inputs; positive flows, outputs. All commodity flows and activity levels are constant through time.

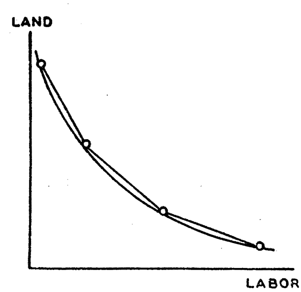
It will be clear that this model rules out indivisibilities as well as increasing or decreasing returns to scale, and cannot be used in the analysis of any problems in which these phenomena are important elements. Apart from this specialization, the flexibility of the model described needs to be emphasized. Cases where continuous substitution within one productive process is possible can be approximated as closely as desired, at least for purposes of theory, by introducing a larger number of activities. If wheat production depends continuously on the

[11] This model was first presented before the Madison meeting of the Econometric Society in August, 1948 [see "A Mathematical Model of Production" (abstract), ECONOMETRICA, Vol. 17, January, 1949, pp. 74–75]. It is closely related to the model of von Neumann [see "A Model of General Economic Equilibrium," *Review of Economic Studies*, Vol. 13(1), No. 33, 1945–46, pp. 1–9] and to that of Dantzig, *op. cit.*

[12] For illustrative applications of these concepts, see Marshall K. Wood and George B. Dantzig, *op. cit.*; C. Hildreth and S. Reiter, "On the Choice of a Crop Rotation Plan," Chapter XI in *Activity Analysis of Production and Allocation*, *op. cit.*, pp. 177–188, and Tjalling C. Koopmans and S. Reiter, "A Model of Transportation," *ibid.*, Chapter XIV, pp. 222–259; see also T. C. Koopmans, "Optimum Utilization of the Transportation System," *Proceedings of the International Statistical Conferences*, held in Washington, D. C., September 6–18, 1947, Vol. V (reprinted in Supplement to ECONOMETRICA, Vol. 17, July, 1949, pp. 136–146, and as Cowles Commission Paper, New Series, No. 34).

quantities of land and labor, one may select a sufficient number of quantitative ratios in which land and labor may be combined, and let each of these define a possible wheat-producing activity. With reference to a given isoquant curve in the land-labor-plane, this means selecting a number of points on the curve (see figure). Other points on the isoquant curve can then be approximated by combining two adjacent selected activities in suitable proportions.[13] A similar device can be applied if more than two factors are continuous substitutes. Thus, while the model can accommodate cases of continuous substitution, it is especially designed to include those cases where substitution consists in relative quantitative shifts between discrete alternatives such as we see so often in industry.

We shall classify the various goods and services into three categories, which may, of course, be defined differently in different applications of



the model. Primary goods are those which flow into production from nature, or from outside the sector of the economy considered, at a rate which, by assumption, cannot exceed given availability limitations. Final goods are those produced goods which are desired for purposes of consumption or delivery outside of the sector of the economy studied.[14] Intermediate goods are those produced goods which are not wanted in themselves, that is, for any purpose other than their use as inputs to

[13] For a given ratio of labor to land inputs (not corresponding to one of the points selected) the inputs of land and labor required to reach the isoquant level of output are slightly less in the continuous model than in the discrete model because in the continuous model labor can be applied uniformly in that ratio to all the land involved, whereas in the (artificially restricted) discrete model the labor is applied in different ratios to two sectors of the land. However, the difference in inputs between the two models can be made arbitrarily small by the selection of a sufficient number of basic points in the discrete model.

[14] To obtain a clear separation between primary and final goods, we shall regard direct consumption of goods available in nature, such as drinking water from a brook, as an activity converting the natural resource "water" (input coefficient -1) into the consumption-good "drinking water" (output coefficient $+1$). Similarly, leisure is the output of a recreation activity of which the input is that part of labor which is not used as input in other activities.

further activities of which the ultimate purpose is the production of final goods.

To any set of nonnegative levels of the activities corresponds what we shall call a (technologically) *possible* set of net output flows of all commodities. We shall call the set of activity levels, and also the corresponding set of net commodity flows, (economically) *attainable* if (*a*) the net flows of all final commodites are nonnegative (because we cannot draw directly on nature to fill a deficit), if (*b*) the net flows of all primary goods are nonpositive (because we have no use for the surplus) and stay within the availability limitations, and if (*c*) the net flows of all intermediate goods are zero[15] (because both considerations adduced in parentheses above apply).

We now come to the concept of productive efficiency upon which all subsequent analysis builds forth. An attainable set of commodity flows, as well as any set of activity levels giving rise to it, is called *efficient* if there is no other attainable set of commodity flows in which all flows are at least as large as the corresponding flows in the original set, while at least one is actually larger. As a synonym for "an efficient set of commodity flows" we also use the expression "an efficient point in the commodity space." In general, the notion of the set of all efficient points corresponds to the notion of a general transformation function, discussed by Lange[16] and others. However, in certain models, depending on the number of activities and the values of their technological coefficients, the efficient point set does not possess a sufficient number of dimensions to make the notion of a transformation function applicable.[17] The efficient point set therefore constitutes the more general concept of the two.

EFFICIENCY PRICES

We shall now give a number of conclusions that can be derived by mathematical analysis[18] from the model that has been formulated, while

[15] It might be thought that waste products are intermediate goods with positive net output. However, if their disposal is costless, we can maintain the zero net output condition by introducing a disposal activity with input coefficient -1, while all other coefficients are zero. If disposal ties up resources, additional negative coefficients of the disposal activity are called for, and the zero net output requirement is essential.

[16] Oscar Lange, "The Foundations of Welfare Economics," ECONOMETRICA, Vol. 10, July–October, 1942, pp. 215–228.

[17] This may come about because the number of activities considered is too small in relation to the number of final goods, or because a small number of activities jointly hold a position of technical superiority over all other activities for all conceivable compositions of demand.

[18] The type of mathematical analysis involved is rather different from that found in most mathematical discussions of production theory, largely because of

referring to another publication[19] for proofs and more detailed explanations. The first conclusion is that, where the efficient point set is indeed representable by a general transformation function, there are constant or decreasing returns in the output of any one final good relative to increases in the input limit (availability limit) on any one primary factor.

Further propositions introduce a price concept which is independent of the notion of a market. The foundations on which this price concept is erected consist only of the technological data (input-output coefficients of all activities) and the requirement of efficiency.

The *first price proposition* says that for each efficient set of commodity flows there exists an *associated set of prices* for all commodities, with the properties listed below. To formulate these properties we define the concept of the profitability of an activity as the aggregate value, at the prices in question, of the outputs associated with the unit level of that activity, minus the aggregate value of the corresponding inputs. The properties of the set of prices, associated with the efficient point in question, are the following:

(1) No activity has a positive profitability.

(2) Any activity carried out at a positive level to attain the efficient set of commodity flows has a zero profitability.

(3) The prices of all final goods are positive.

(4) The prices of all primary goods are nonnegative.

(5) The prices of all primary goods whose net input does not reach the availability limit are zero.[20]

It will be noted that no statement is made as to the signs of the prices of intermediate goods. Indeed, negative prices will arise for waste products, the disposal of which uses up positively priced goods.

The *second price proposition* states that the converse is also true. An attainable point with which a set of prices with the listed properties can be associated is an efficient point.

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the role played by linear inequalities such as arise from the nonnegative character of activity levels and from the availability limits on primary goods. The theory of convex sets, particularly convex polyhedral cones, is drawn upon. There are many points of contact with a model constructed by von Neumann (see "A Model of General Economic Equilibrium," *op. cit.*). An important difference is that in von Neumann's model the efficiency of allocation comes out at the end as a by-product of an analysis concerned mainly with an existence theorem, while in the present study efficiency of allocation is made the central theme of analysis.

[19] Tjalling C. Koopmans, "Analysis of Production as an Efficient Combination of Activities," in *Activity Analysis of Production and Allocation*, *op. cit.*, Chapter III, pp. 33–97.

[20] These goods are therefore properly called free goods with reference to the efficient point in question.

It should be mentioned that the set of prices associated with a given efficient set of commodity flows is not necessarily unique. In the case where a general transformation function exists, the set of prices will be unique almost everywhere on the hypersurface represented by that function. If the set of prices is unique, ratios of the prices of final and/or primary goods can be interpreted as marginal rates of substitution between these goods. The substitution in question arises from such variations in the activity levels as to leave all commodity flows constant except those of the two goods between which substitution is considered—while maintaining efficiency in the activity combination before and after variation. These rates of substitution are applicable to *finite* (as distinct from *infinitesimal*) variations in the commodity flows involved, but often only within finite limits of variation. This comes about because the hypersurface representing the transformation function consists of sections of hyperplanes joined at their intersections. In an efficient point located on one of these intersections, marginal rates of substitution are generally different for increases and for decreases in the net output of any one of the two commodities in question. In the special case of such an efficient point, more than one set of associated prices (to be precise, an infinity of sets of prices) satisfies the requirements stated.

It may be restated that the price concept established does not in any way presuppose the existence of a market or of exchanges of commodities between different owners. The price concept is found to be a mathematical consequence of an efficient choice of activity levels. In the important case in which the activities engaged in are sufficient in number and variety to lead to a unique solution of the condition (2) that their profitabilities be zero, the prices have already been interpreted as technological rates of substitution under efficient allocation. An additional interpretation, not thus limited in its applicability, is derived from the following *third price proposition*.

Let us consider a given efficient set of commodity flows, and let us add to the technology a number of exchange activities, defined by means of an arbitrarily selected set of prices, as follows: We shall imagine that, through contacts with another economy, any commodity can be exchanged for any other commodity at a price ratio computed from the selected set of prices. Then our proposition specifies the conditions under which, after the opportunity to engage in these exchange activities has been provided, the original set of commodity flows will still be efficient. This original set of flows remains efficient if and only if the prices selected to define the exchange activities are at the same time a set of prices associated with the original set of flows (in the sense of the first price proposition). In that case, and only in that case, there is no way of using the new opportunity of exchange, whether in combination with variations in the levels of the other (productive) activities or not, so as to increase the net "output" of any final good without decreasing the net "output" of some other final good. Because of this proposition a set of prices associated with an efficient point can also be called a corresponding set of *efficiency prices*.

DECENTRALIZATION OF DECISIONS

With the help of these formal propositions, we shall now consider, although still in a rather abstract fashion, some institutional arrangements under which efficient allocation may be attained. The above propositions, and in particular the profitability properties (1) and (2) of the efficiency prices, strongly suggest mechanisms that ensure efficiency by decentralization of decisions concerning activity levels, mechanisms which are similar to those referred to above. Let us assume, in the *first allocation model*, that a set of positive prices on final commodities is prescribed, either administratively by some central authority or as market prices reflecting a balance of preferences of consumers weighted by the income distribution. These may be called *steering prices* in that through their variation it may be possible to steer the allocation of resources to the production of alternative efficient sets of commodity flows. Let the level of each activity and the prices of primary and intermediate goods be determined by a bidding process governed by the following rules: Any activity yielding a negative profit is to be contracted. Any activity yielding a zero profit is to be maintained at a constant level. Any activity yielding a positive profit is to be expanded, if necessary by bidding up prices of its input commodities. Behavior according to these rules could either be induced by administrative authority, binding the action of managers of individual activities, or it could result from a competitive market structure, where each activity is engaged in by many independent entrepreneurs.[21] The second price proposition, then, implies that efficiency, once attained in such an administrative or market structure, is maintained.

It should be emphasized that, if an inefficient state of resource allocation prevails initially, it is not claimed that adherence by all concerned to the rules stated would lead the mechanism to an efficient point, or even close to such a point, in a stated time interval. To establish such a claim would require a dynamic analysis resting on a more precise dynamic specification of the rules in question (e.g., how much to expand a profitable activity, etc.). It is claimed only that adherence to the rules will perpetuate an efficient state once it has somehow come about.

[21] We are, of course, concerned only with the logical content of the rules, not with the question whether and how compliance by managers or entrepreneurs can be secured.

Thus, the idea of variation in the steering prices or in the technology to obtain other efficient points must also be understood in the sense of comparative statics, not as a change through time.

The rules that have been stated are not yet equivalent to those formulated by Lange, Lerner, and others. The latter rules apply to plant managers who control more than one activity, in our sense of the term, and who thus represent in one decision unit a combination of several, possibly many, of our fictitious activity managers. However, the Lange-Lerner model is closely approached in the following *second allocation model*: Define as a *process* a set of activities controlled by one manager. Define as the *composition* of a process the set of activity levels selected by its manager. Now, the attainment of efficiency in the economy as a whole has as prerequisites (A) the attainment of efficiency by each process manager *within the set of activities controlled by him*, and, in particular, (B) the selection of such an efficient activity composition by each process manager that an associated set of efficiency prices exists which is the same for all process managers. Efficiency for the economy as a whole, once attained, will be maintained if each process manager behaves according to the following rules: Choose only from those sets of activity levels that correspond to an efficient point within your process. If for all such points the profit on the entire process is negative, discontinue all activity. If you are in a point of nonnegative profit on the process, attempt to raise[22] your profit-at-the-given-prices by varying the composition of the process. If you are in a point of zero profit and there is no increase in profit possible by variation of activity levels, continue all activities at the same level. If your attempt to raise profit-at-given-prices leads to a rise in the prices of certain input commodities, determine your further action in the light of the new price situation.

The reader will have realized that behavior according to these rules presupposes a knowledge, on the part of each process manager, of the efficient point set that can be constructed on the basis of those activities involved in the process that he controls. Also, comparing the results of the above analysis with the Lange-Lerner discussions referred to, it appears that in one sense we have made more limited assumptions. We have ruled out both indivisibilities and increasing or decreasing returns to scale. As a result of that limitation we have obtained a stronger proposition. The rules on the allocation mechanisms stated have been found to be not only *necessary* but also *sufficient* for an efficient use of resources.

On the other hand, because of our simple assumptions we have not yet reached the problems arising from discrepancies between average

[22] If the profitability is initially positive, this can always be done by proportional increases in the levels of all activities in the process.

and marginal cost, problems which have received considerable emphasis in earlier discussions.

Another assumption, implicit in our model, has not yet been clearly brought out. We have considered only the flows of goods involved in any choice of activities, thus disregarding all *stocks* of goods and equipment whose presence may be necessary for these activities. This implies an assumption of capital saturation because no penalty has been placed on the use of capital-intensive activities. Physical depreciation of capital is taken into account, of course, as an input flow.

It is hoped that these limitations (some of which may perhaps be removed in further analysis) are justified because we have gone further than previous discussions in one particular direction. The production function of the individual plant, and the marginal cost information derivable from it, have usually been regarded as data, supposedly both meaningful and known to the plant manager. No such assumptions are made in our analysis, except in the second allocation model, which was designed to make a connection with previous discussions. In the essential part of our analysis, all that is presumed known to the individual manager or (where so assumed) to a central authority are the technical input-output coefficients characteristic of individual activities. It is perhaps indicative of the nature of our technology that the information supplied by engineers is most often of this type. The economist's concepts of a production function, marginal rates of substitution, and marginal cost, where applicable, are derived from these underlying data, and, where these concepts are not applicable, a somewhat more general analysis is found to remain applicable.

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