Treynor on Institutional Investing
By Jack L. Treynor

CHAPTER 10

How to Rate Management of Investment Funds

The performance of mutual, trust, and pension funds can be quantitatively compared despite market fluctuations and different risk policies.

Investment management has become an important industry in the United States. The responsibilities of investment managers are enormous, and their potential rewards are great. In order to reward management for good performance in this field, however, it is necessary to be able to recognize it. Unfortunately, pension funds, trust funds, and mutual funds all share one serious problem: To the extent that they are heavily invested in common stocks, the return achieved in any one period is subject to wide fluctuations which are beyond the control of investment management. The result has been that, although many believe the quality of investment management is important, no one has devised a satisfactory way to measure its impact on performance.

In this article we shall look at a new way to rate the performance of a fund's investment managers. The comprehensiveness of this rating is a question for the reader to decide for himself, depending on how he thinks about the "quality" of investment management. Most readers are likely to agree, however, that at least one dimension—and a critical one—of the quality of the investment management is analyzed by this new method.

ANALYZING RISK

It is almost ironic that the presence of market risk should pose such a serious problem. The assets controlled by investment managers are remarkably liquid. To a degree almost unmatched in other enterprises, the investment manager is free to act independently of the investment decisions of his predecessors. Furthermore, although there are varying institutional restrictions placed on the investment manager's decisions, by and large he competes directly with other investment managers, buying and selling securities in the same market. If it were not for the problems created by market risk,

therefore, performance comparisons in the investment management industry would be more meaningful than in many other industries.

Actually, of course, there is more than one kind of risk in a diversified fund. There is a risk produced by general market fluctuations—the volatility of the stock market. There is also a risk resulting from fluctuations in the particular securities held by the fund. In any event, here are important practical consequences of either or both of these risks:

1. The effect of management on the rate of return on investments made in any one period is usually swamped by fluctuations in the general market. Depending on whether, during the period in question, the general market is rising or falling, the more volatile funds (stock funds) will look better or worse than the less volatile funds (balanced funds). As the Wharton Report points out, the difficulty is not solved by averaging return over a number of periods.[1] For any sample interval of reasonable length, average return is still dominated by market trends.
2. Measures of average return make no allowance for investors' aversions to risk. The importance of fluctuations in one or a few stocks from the investor's point of view is apparent when one considers that, after all, if this kind of risk were not important, investors would not diversify. It is sometimes argued that because the importance attached to risk varies from investor to investor, no absolute measure of fund performance is possible.

Overcoming Difficulties

In order to have any practical value, a measure of management performance in handling a trust fund invested in equities or in handling pension or mutual funds must deal effectively with both problems. It should tend to remain constant so long as management performance is constant—even in the face of severe market fluctuations. Also, it should take into account the aversion of individual shareholders or beneficiaries to investment risk. The method to be described here overcomes both difficulties.

This article has three parts. The first describes a simple graphical method for capturing the essence of what is permanent and distinctive about the performance of a fund, including the effects of fund management. The second develops a concept of fund performance which takes investment risk into account. The third develops a measure for rating fund-management performance which can be applied directly, using the graphical technique developed in the first part. For the statistician, Appendix 10.1 details certain relationships used.

THE CHARACTERISTIC LINE

The first main step to obtaining a satisfactory performance measure is to relate the expected rate of return of a trust, pension, or mutual fund to the rate of return of a suitable market average. The device for accomplishing this is the *characteristic line*. Let us examine its nature and significance.

Application to Funds

If the rate of return—taking into account both market appreciation and dividends—is plotted for a fund invested substantially in common stocks, wide swings from period to period are often evident. It is not generally known, however, that most managed

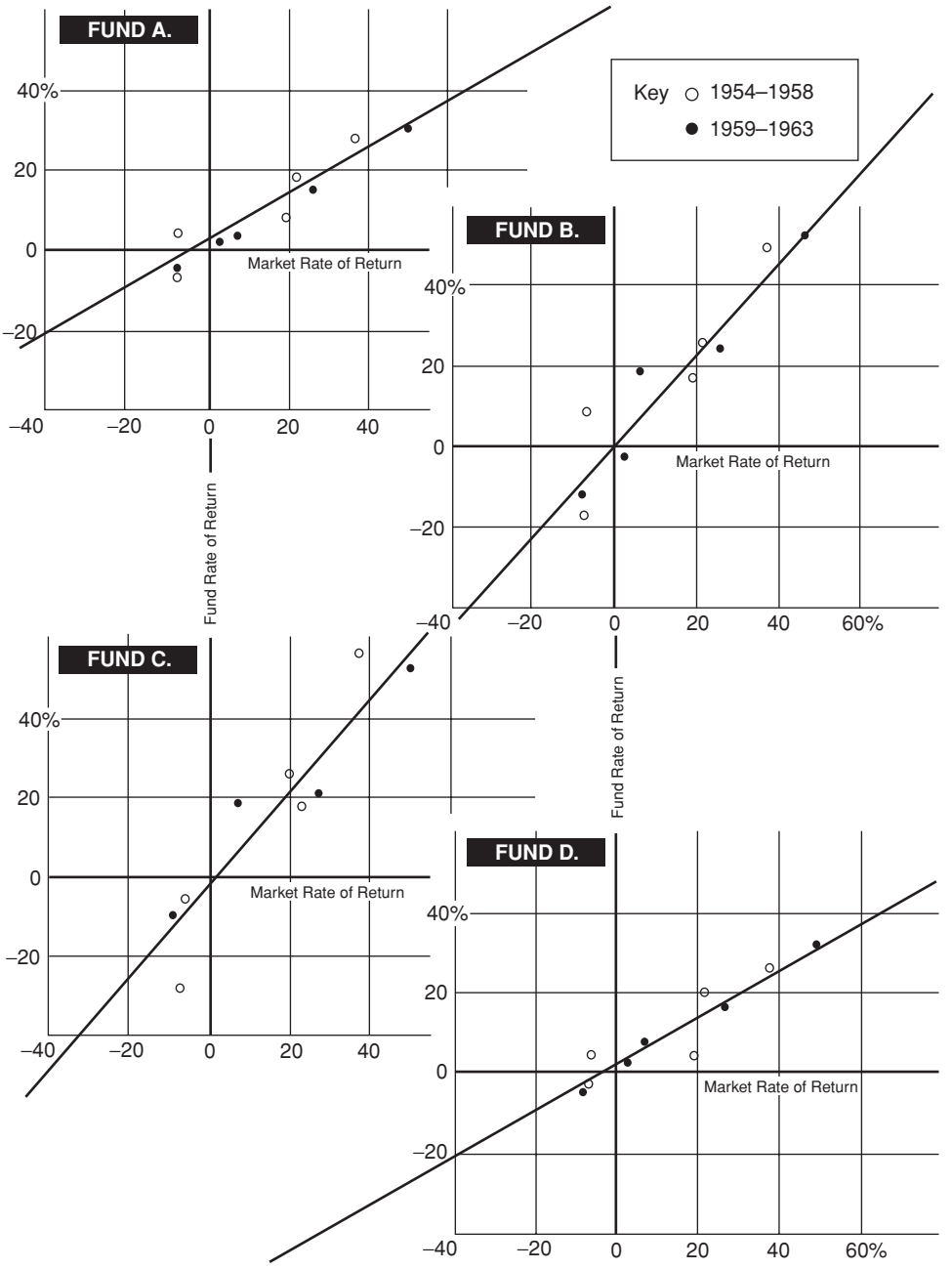

EXHIBIT 10.1 Characteristic Lines

funds actually demonstrate a remarkably stable performance pattern over time when viewed in terms of the simple graphical device that I call the characteristic line.

Exhibit 10.1 summarizes the performance history of four actual managed funds:

- The horizontal and vertical axes in these figures are measured in terms of percent rate of return. (For both individual funds and market averages, rate of return is

computed by dividing the sum of dividends, interest, and market appreciation on the funds available at the beginning of the year by the value of the funds available at the beginning of the year. Any increase in asset value during the year due to infusion of new funds is eliminated, as is any reduction due to distributions to beneficiaries or shareholders. Rates of return defined in this way are obviously approximations, because the value of funds available for investment typically fluctuates more or less continuously throughout the year.)

- The horizontal axis measures the corresponding rate of return recorded for a general *market* average (the Dow-Jones Industrial Average); the vertical axis shows the rate of return for the *fund*.
- Each point represents a year in the ten-year interval ending January 1, 1963. The open points represent the five years in the latter half of the ten-year interval; the filled points, the years in the former half.

Although the funds exhibited wide swings in rate of return over the ten-year interval, the rate of return in each year fell into a straight-line pattern, which remained virtually fixed throughout the ten-year interval. This line—the characteristic line—can be fitted by eye or by statistical methods. The significant thing about it is that it tends to be stationary over time, despite wide fluctuations in short-term rate of return.

Information Revealed

The characteristic line contains information about both expected rate of return and risk. The slope of the line measures volatility. Thus, a steep slope means that the actual rate of return for the fund in question is relatively sensitive to fluctuations in the general stock market; a gentle slope indicates that the fund in question is relatively insensitive to market fluctuations.

The slope angle of the characteristic line obviously provides a more refined measure of a fund's volatility than the usual categories of "balanced fund," "stock fund," or "growth fund." The range of volatilities observed in actual practice is enormous. Among mutual funds, for example, I have found that volatilities range from roughly one-third to about two. A volatility of two means that a one-percent increase (or decrease) in the rate of return demonstrated by the Dow-Jones Average is accompanied, on the average, by a two-percent increase (or decrease) in the rate of return demonstrated by the particular fund in question.

For any individual investor who is risk-averse, the observed differences in volatility are surely large enough to be worth measuring. The differences also disclose important contrasts in management policy.

What Deviations Mean

As users of the characteristic-line method will discover, the plotted points in a typical chart will not all lie on the characteristic line. What this means is that not all of the risk in the fund in question is explained by fluctuations in the general market level.

As pointed out earlier, one can consider that investment risk in a diversified fund is the sum of responses to (1) general market fluctuations and (2) fluctuations peculiar to the particular securities held by the fund. If a fund is properly diversified, the latter

risk, which tends to be causally unrelated one security from another, tends to average out. The former risk, being common to all common stocks in greater or lesser degree, does not tend to average out.

If the management of a fund attempts to maintain a constant degree of volatility, then the slope of the characteristic line will tend to measure that volatility. If there are excessive deviations from the characteristic line, we have a strong indication that:

- Either the fund is not efficiently diversified to minimize risk unrelated to the general market (in which case the owner or beneficiary incurs additional risk without any compensating prospects of additional return).
- Or, perhaps inadvertently or perhaps as a matter of deliberate policy, management has altered the volatility of the fund. By increasing fund volatility when it is optimistic and decreasing volatility when it is pessimistic, management can speculate for the fund beneficiaries on fluctuations in the general market.

The appropriateness of such action is an interesting question but outside the scope of this article. It is worth noting, though, that in a sample I have taken of 54 American mutual funds, four out of five demonstrate fairly clear-cut characteristic-line patterns, with correlation coefficients equal to or exceeding 90 percent.

Possibly this pattern indicates wide agreement that causing fund volatility to vary greatly leaves the individual owner unable to rely on a stable estimate of the risk in the portion of his personal portfolio represented by the fund in question. His ability to strike what for him is the optimal overall portfolio balance between expected return and risk is then impaired. But if, in retrospect, fund management has speculated successfully with the volatility of a fund, it is conceivable that beneficiaries may consider the disadvantage more than offset by the improved rate of return.

Suppose the characteristic line itself shifts. This may happen when fund volatility remains constant but fund performance varies widely from year to year. A sweeping change in the personnel constituting fund management, for example, might be accompanied by a sudden shift in fund performance.

Comparing Performance

The characteristic line also contains information about management's ability to obtain a consistently higher return than the competition's. If, for example, two trust or mutual funds demonstrate precisely the same volatility, their respective characteristic lines would have the same slope, but one line would be consistently higher than the other (unless they coincide). For instance, suppose a certain fund had exactly the same slope as Fund A in Exhibit. 10.1. If its characteristic line were plotted on the chart, it would run parallel to Fund A's but higher or lower. The fund with the higher line would demonstrate consistently higher performance—in good years and bad.

Although the problem of comparing performances of fund management is obviously not so simple when the slopes differ, the characteristic line does contain, as we shall see presently, the information necessary to make such comparisons.

Implications for Control

The characteristic line has implications for management control, too. No matter how widely the rate of return for a fund may fluctuate, management performance is unchanged so long as the actual rate of return continues to lie on the characteristic line. One can establish control limits on either side of the line; points falling within these limits are assumed to represent a continuation of past management performance, while points falling outside the limits require special scrutiny. Without the characteristic line it is virtually impossible to tell whether the rate of return demonstrated in a given year represents a real change in the quality of fund management. With it, early detection of important changes becomes possible.

In summary, therefore, the graphical method provides a simple test of:

1. The extent to which a fund has adhered, purposely or not, to a single characteristic line.
2. The degree of volatility associated with the fund.
3. The success of fund management in maintaining a high rate of return under a variety of market conditions.

PERFORMANCE MEASURE

We turn now to a second line. This one deals not with an individual fund but with a *portfolio* containing a certain fund. The purpose of the line is to relate the expected return of a portfolio containing the fund to the portfolio owner's risk preferences. This line can be called the *portfolio-possibility* line. We shall see that the slope of this line is a measure of fund performance which transcends differences in investors' attitudes toward risk.

Risk Preference

Whether the performance pattern of a given fund rates high or low should depend on whether individual investors choose it in preference to the pattern demonstrated by other funds. During the last few years we have witnessed the rapid development of a theory of rational choice among portfolios.[2] The theory is too complex to be reviewed here in detail, but certain fragments of it provide the basis for a concept of fund-management performance.

It is interesting to note that when one talks about the historical performance pattern of a fund, he is looking at the past; but when he considers the preferences of individual investors and their choices among funds, he is talking about their appraisal of the future. We shall continue to talk about the performance of funds in terms of historical performance patterns, even though actual investor choices among funds are necessarily based on expectations regarding future performance patterns. The implication is that a good historical performance pattern is one that, if continued into the future, would cause investors to prefer it to others.

Economists sometimes study the investor's choice among possible portfolios in terms of a risk-return diagram (like the one shown in Exhibit 10.2):

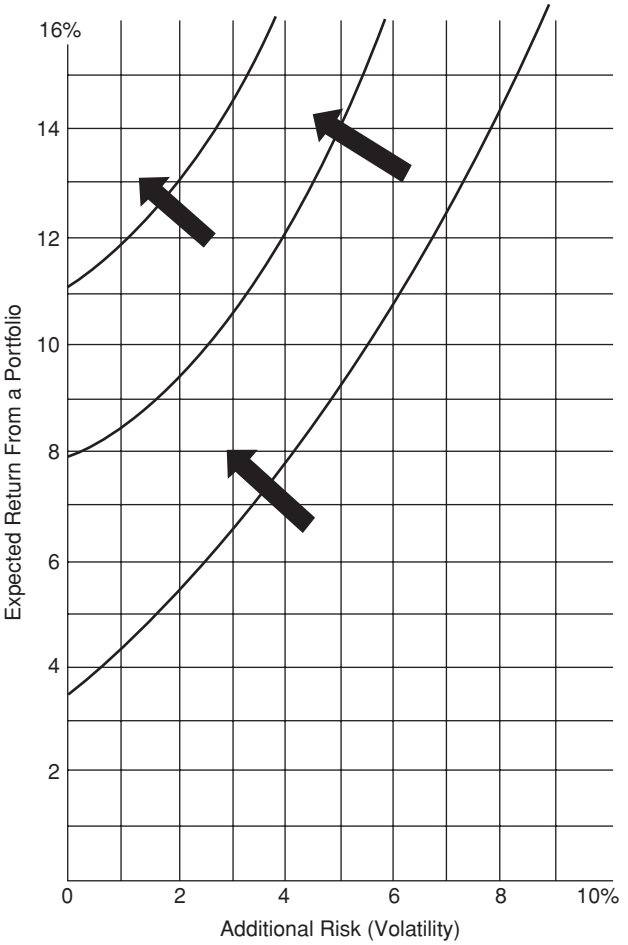

EXHIBIT 10.2 Investors' Indifference Curves

- The vertical axis in the exhibit measures the return that the investor would expect to get, on the average, from a given portfolio. The horizontal axis is some appropriate measure of risk.

  (As a technical note for those interested in detail, let me add that it is traditional to measure the respective axes in terms of *expected rate of return,* where the rate is a weighted mean of possible future outcomes, and *standard error,* where standard error is a statistical measure of potential variability around the expected performance. Under certain assumptions regarding the nature of investment uncertainty, expected return and standard error completely characterize a given portfolio. These assumptions seem to fit actual stock-market experience fairly well. When the performance pattern of a mutual fund is clustered closely around the characteristic line, the slope of this line, which is our graphical measure of risk, is statistically an excellent measure of the standard error.)
- The rate of return is for a standard time period—perhaps a month, quarter, or year—per dollar of the individual investor's initial capital.

- The curved lines in the diagram are called indifference curves for the reason that the investor is indifferent to portfolio choices lying on a particular indifference curve; that is, he would just as soon have, say, 5 percent more return at $4\frac{1}{2}$ percent more risk as 8 percent more return at $6\frac{1}{2}$ percent more risk, and so on (see the curve at the right of Exhibit 10.2).[3]
- There is a useful analogy between the investor's relative preference, as shown by indifference curves, and relative heights, as shown by contour lines on a topographical map—that is, lines along which elevation is constant. The arrows in the figure show the direction in which one moves to go from less to more desirable portfolios (or, to complete the topographical analogy, uphill).

Portfolio Choices

What kinds of portfolio choices are available to the investor? The assets he can include in his portfolio consist of two fundamentally different kinds:

- Money-fixed claims, such as checking deposits; savings deposits; government, municipal, and corporate bonds.
- Equity assets, including equity in personal business and partnerships and corporate common stocks.

The investor who holds money-fixed claims is subject to the risk of changes in both the interest rate and price level. Although both risks are real, in American financial history they have been small compared to the risk entailed in owning equities. The relative insignificance of market risk in money-fixed claims is reflected in the narrow range of net returns available in such claims. We shall simplify slightly and represent all assets of this type by a single point on the vertical axis of the risk-return diagram (point B in Exhibit 10.3).

If the investor wants to raise the expected rate of return of his overall portfolio above the rate offered by money-fixed claims, he must undertake some equity risk. On the risk-return diagram in Exhibit 10.3, the investor has available to him the opportunity to invest in shares in a particular balanced or growth fund, Fund A, as well as the opportunity to invest in money-fixed claims, B. If he is free to vary the investment in each outlet more or less continuously, then the locus of portfolio combinations available to him is the straight line—the portfolio-possibility line—joining points A and B. The combination which is best for him will lie at point D along the line which is farthest "uphill" as indicated by the "contour lines" on his indifference map. The preferred combinations for other investors will differ, depending on the precise shape of their indifference curves.

Now consider a second investment, Fund C (top right of Exhibit 10.3). The line BC is the locus of possible portfolios made available to our investor by the existence of this investment. As in the case of locus BA, there will, in general, be a single point, E, along BC, which is the farthest "uphill" for the investor.

The significant fact is that, although the location of the points of optimum balance along lines BC and BA will differ from one investor to another, the optimum point D along line BA will always be superior for a given investor to the optimum point E along line BC. For every possible level of risk an investor might choose, the return on a combined portfolio containing Fund A is greater than the return on a portfolio containing Fund C, which provides the same level of risk. This ensures that, whatever

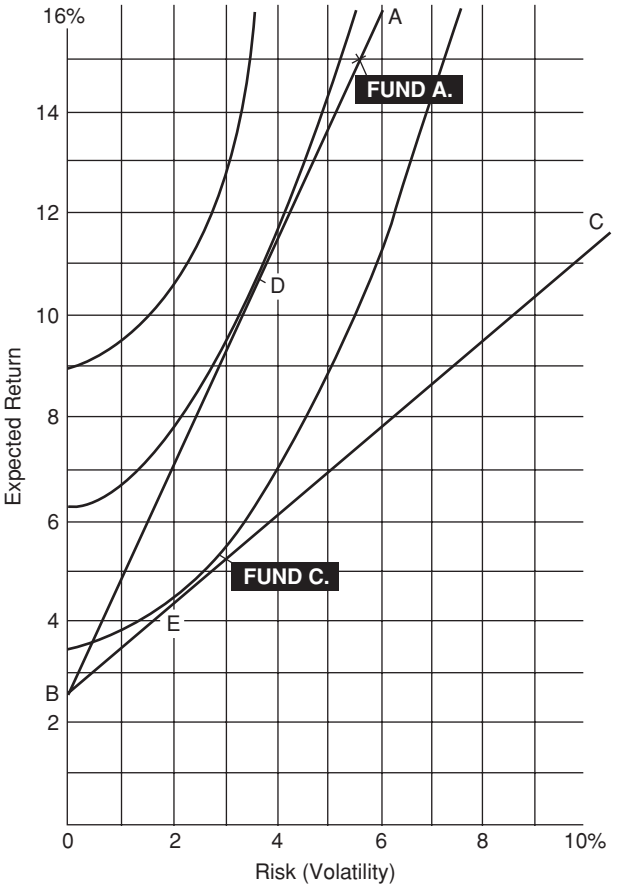

EXHIBIT 10.3 Risk-Return Diagram for an Investor

the optimum point along line BC may be for a particular investor, the point on BA directly above it (that is, with the same risk) will have a greater expected return. This will be true for every investor who is risk averse, quite independently of the precise shape of his indifference curve.

But if, for every risk-averse investor, line BA is superior to line BC, then, in terms of the portfolio possibilities this line makes available to investors, Fund A is absolutely superior to Fund C. Now it is apparent from Exhibit 10.3 that lines BA and BC differ only in slope. Line BA, which is superior to line BC, slopes upward more sharply, showing that the rate of gain from shifting the investor's portfolio in the direction of greater risk is greater for Fund A than for Fund C. *The steepness of the portfolio-possibility line associated with a given fund is thus a direct measure of the desirability of the fund to the risk-averse investor.* The force of the preceding argument is not diminished by the fact that many investment funds contain money-fixed claims as well as equities.

Pension and Trust Funds

All very well for mutual funds, you may say. After all, the investor in mutual funds is free to adjust the fraction of his portfolio invested in each one pretty much as he

pleases. But what about cases involving pension funds and trust funds, in which the individual beneficiary has no freedom whatever to alter the fraction of his total assets that are managed by the fund? To answer this question, let us consider an illustration: Suppose a man has a certain fraction of his assets invested in a pension fund. Suppose further that the management performance of the pension fund (measured in terms of the slope of the portfolio-possibility line) ranks just equal to the performance of a certain mutual fund. A certain segment of the portfolio-possibility line for the mutual fund will be unavailable to the investor if part of his funds are irrevocably committed to the pension fund, since he is not free to convert all his assets to money-fixed claims. Within the range of the portfolio-possibility line available to him, however, he can achieve the same portfolio behavior with part of his capital committed to the pension fund as he could achieve if he were free to compose the risky portion of his portfolio entirely from the mutual fund in question. If his attitude toward portfolio risk leads him to choose a portfolio in this range, then he will be indifferent as to a choice of a pension fund or a mutual fund with an equal performance ranking. If, on the other hand, his choice lies outside this range, then the pension fund is less useful to him than a mutual fund with a similarly sloped portfolio-possibility line.

Quantitative Measure

The performance demonstrated by a fund can be measured by the tangent of the slope angle, symbolized by the figure $\propto$. (For instance, the slope angle for Fund C in Exhibit 10.3 would be the difference between the slope of line BC and a horizontal line going through B; the slope angle for Fund A, which is larger, is the difference between BA and a horizontal.)

The formula for tangent α follows directly from the geometry of Exhibit 10.3. As detailed in Appendix 10.1, it is:

$$\text{tangent} \propto = \frac{\mu - \mu^*}{\sigma}$$

where μ equals the expected fund rate of return at a particular market rate of return, μ^* is measured from a horizontal line through a point that would represent a fund consisting only of fixed-income securities, and σ is the symbol for volatility (which can serve as an approximate measure of investment risk as plotted on the horizontal axis of Exhibit 10.3).

RATING MANAGEMENT

We are now ready to begin with the practical application of the concepts previously described. We will see how performance ratings can be read directly from the characteristic line.

Relative Ranking

In order to plot a fund, and the associated portfolio-possibility line, on a risk-return chart of the type discussed in the last section, one needs both an expected rate of

return and an appropriate measure of risk. A measure of risk is provided by the slope of the characteristic line. The characteristic line also enables management to estimate the expected rate of return. In order to obtain a value for the expected rate of return for the fund, however, it is necessary to assume a rate-of-return value for the market. Depending on the choice of market rate of return, expected return for the fund—hence the slope of the opportunity locus—will vary. The effect of changing the assumed market rate is illustrated in Exhibits 10.4 and 10.5 as follows:

- Exhibit 10.4 portrays a sample of characteristic lines for 20 actual managed funds based on rate-of-return data for the years 1953 through 1962. By making specific assumptions about the market rate of return, the characteristic lines for these funds can be transformed into points on the risk-return charts shown in Parts A and B of Exhibit 10.5. (The term "volatility" on the horizontal axes of these charts, as indicated before, refers to the amount of risk in the fund due to fluctuations in the general market.)
- Part A of Exhibit 10.5 was plotted by assuming a market return of 10 percent. (The characteristic line for each fund is inspected to determine its pattern of return when the market's return is 10 percent, and this pattern is converted to a point reflecting risk and return.) Given this assumption, the funds in question can easily be ranked visually; by drawing straight lines from Point Q to these points, one can obtain the portfolio-possibility lines for the funds in question. The problem is, of course, that the market-return assumption is arbitrary and other returns depend on it.
- Part B results when a market rate of return of 30 percent is assumed instead. Although the risk values for the individual funds are unchanged, the expected rates of return are affected, and a new set of portfolio-possibility lines results.

Inspection shows that the ranking of the funds is unchanged in Parts A and B of Exhibit 10.5. For example, the highest- and lowest-ranking funds in Part A are, respectively, the highest- and lowest-ranking funds in Part B, despite the fact that the two diagrams are based on widely differing assumptions about the expected rate of return for the general market. This illustrates what is actually a quite general result: although the absolute position of funds on a risk-return chart (and their corresponding portfolio-possibility lines) may vary with the level of market rate of return assumed, *the ranking of funds with respect to each other does not.*

Numerical Measure

What is desired, therefore, is a number that will measure the relative ranking of a fund—preferably without being affected by changes in the absolute level of rate of return of the kind illustrated by Parts A and B of Exhibit 10.5. It happens that there is a number that has these properties: It is the level of rate of return for the general market at which the fund in question will produce the same return as that produced by a fund consisting solely of riskless investment. As Exhibit 10.6 shows, its value can be read directly from the characteristic line:

A horizontal line is drawn so as to intersect the vertical axis at a point representing the rate of return available on money-fixed claims. In Exhibit 10.6 the horizontal line is drawn at 4 percent. (The choice of rate within the range of $3\frac{1}{2}$ percent to

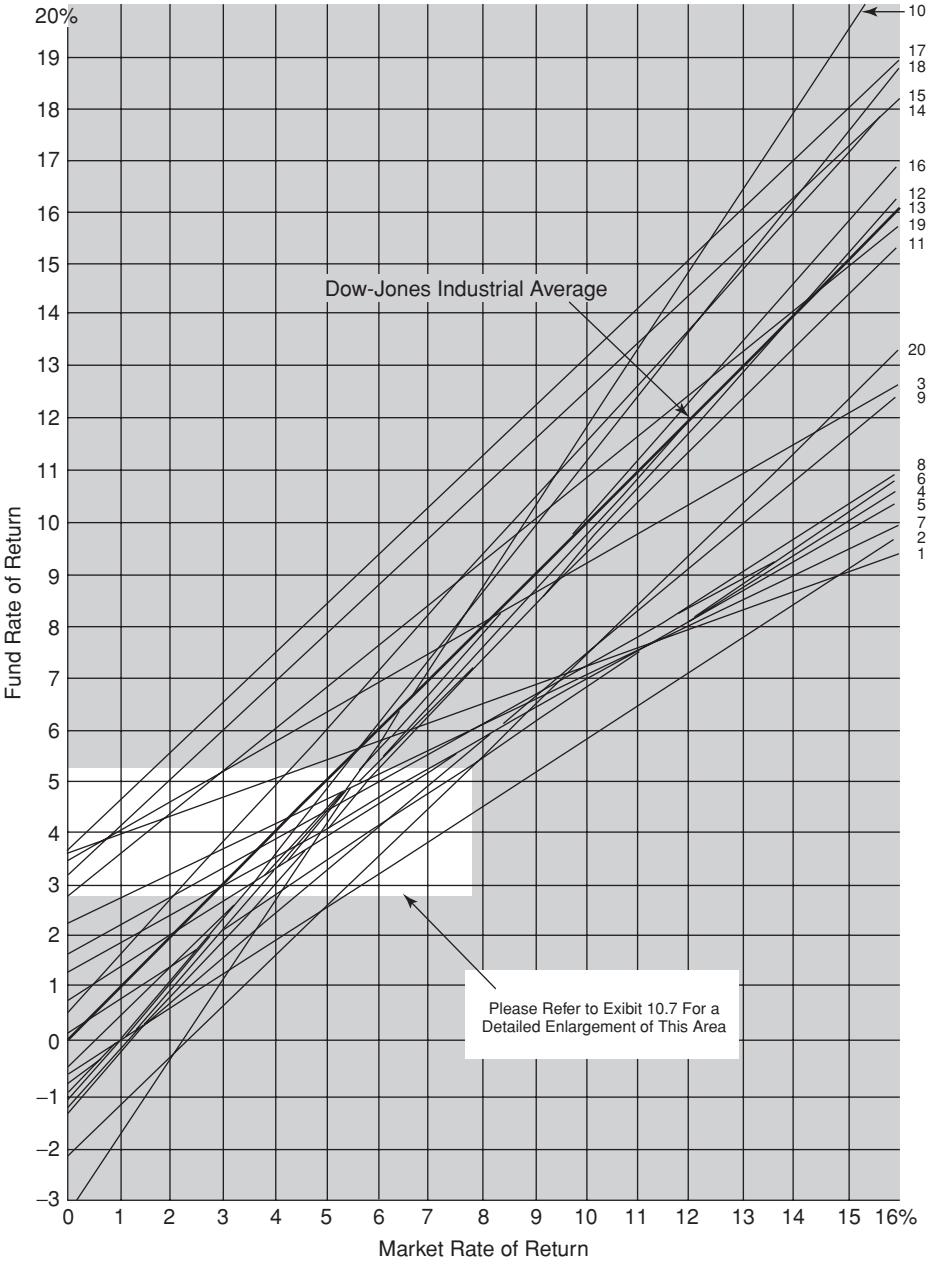

EXHIBIT 10.4 Comparison of 20 Managed Funds

5 percent is somewhat arbitrary, but not especially critical as regards its effect on performance ratings.) The point at which the horizontal line intersects the characteristic line determines the rating of the fund, which is read off the horizontal axis as a percentage. The lower this percentage, the higher the rating of the pension, trust, or mutual fund. For those interested in a formal proof that the number just defined

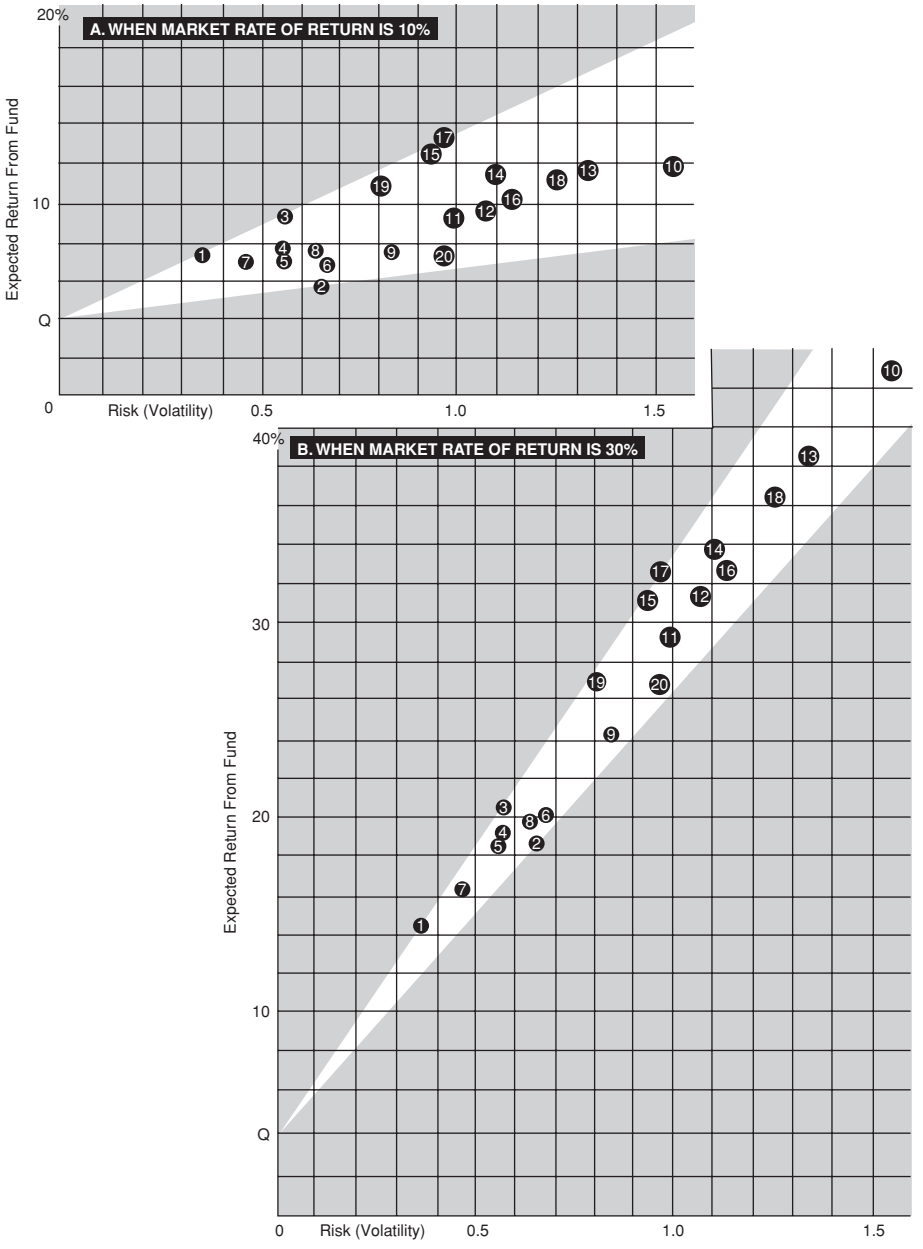

EXHIBIT 10.5 Fund Rankings under Different Market Conditions

will have the special properties desired, Appendix 10.1 sets forth the steps in the reasoning.

In order to demonstrate the practical significance of the rating technique, let us refer back to Exhibit 10.4. Each of the performance ratings of the 20 funds whose characteristic lines are shown in this chart could be read directly from the figure

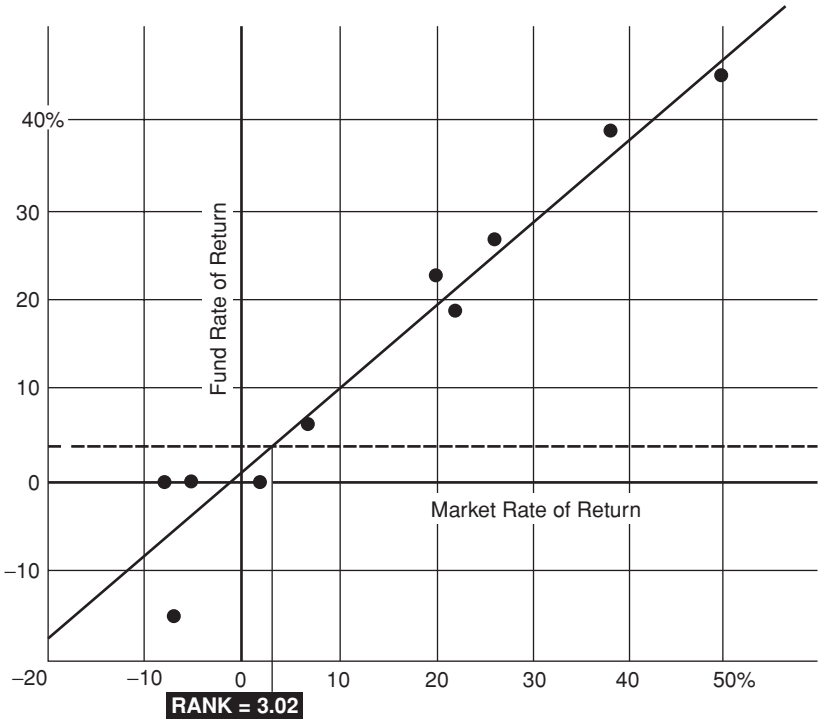

EXHIBIT 10.6 Ranking Number of a Fund

if a horizontal line corresponding to the rate of return on a riskless portfolio (here 4 percent) were added. The performance rating for each fund could be determined by the value of market rate of return at which its characteristic line intersects the horizontal 4 percent line. Now see Exhibit 10.7. The characteristic lines are the same as the ones in Exhibit 10.4, but a 4 percent horizontal has been added, and the area of intersection with it has been expanded for ease in reading. Note that the performance ratings for the 20 funds (read off the horizontal axis) range from less than 1 percent to more than 7 percent.

Differences Important? Is the difference between the best and worst rated fund in Exhibit 10.7 large enough to be significant to an investor? Let us consider an illustration:

Suppose that an investor specifies his portfolio volatility should be equal to one. The amount of "riskless" investment or borrowing which he undertakes will depend on the volatility of the fund. Let us say that Fund XYZ has a volatility of two. Since the desired portfolio volatility is one, then the portfolio must be blended of equal parts (in terms of dollars invested) of the fund and riskless investment. If, for example, the beneficiary's capital is initially worth $10,000, then, since a 1 percent reduction in the market rate of return will be accompanied on the average by a 2 percent reduction in the rate of return on $5,000 invested in the fund, the effective reduction in *portfolio* rate of return is 1 percent since:

$$\frac{.02 \times \$5,000}{\$10,000} = .01$$

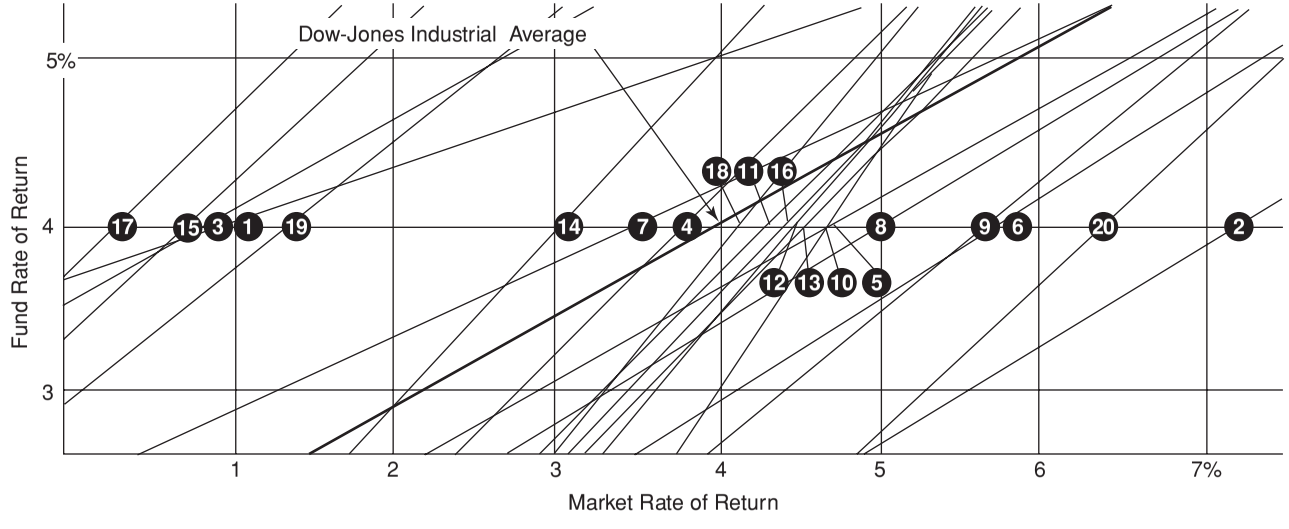

EXHIBIT 10.7 Performance of Funds

Now assume that the fund in question has a volatility of three-quarters. If the investor's desired portfolio volatility is one, he must invest an amount exceeding his own capital. If his capital is again $10,000, and he borrows $3,333 and invests that sum in Fund XYZ, then a one-percent reduction in the market rate of return will be accompanied on the average by a .0075 percent reduction in the rate of return on $13,333 invested in the fund. The effective reduction in *portfolio* rate of return is then 1 percent because:

$$\frac{.0075 \times \$13,333}{\$10,000} = .01$$

In both cases the portfolios have a volatility equal to one—the value specified—but the differing fund volatilities necessitate quite different investment strategies.

Is the significance of rating differences for a sample of funds influenced by market conditions? It is to a certain extent. We have already seen that one cannot employ characteristic-line data to obtain an expected rate of return for a fund without first assuming a value for the market rate of return. It is consequently not possible to make categorical statements about the spread in expected portfolio performance between the best and worst managed funds which results when an investor specifies a certain level of portfolio volatility. It is nevertheless possible to get a rough idea of the significance of the spread in performance ratings observed in a sample by making different assumptions about the market. If we take the extreme cases in the sample of 20 funds already described, for instance, we find these differences in investment return:

Expected market rate of return	10%	30%
Return of highest-ranked fund	13.6%	33.4%
Return of lowest-ranked fund	6.6%	26.6%

These figures suggest the following conclusions about differences in ratings:

1. In the range of normal market rate of return, the difference in portfolio rate of return between funds ranked high and low is substantial.
2. The difference seems relatively less important, the higher the performance of the general market is. Hence the consequences of rating differences for portfolio performance will be relatively more significant in a normal market than during the bull market of recent history.

CONCLUSION

In this article we have seen that there is a good way of cutting through the confusion of facts and figures in the marketplace to compare the performance of individual trust, pension, and mutual funds. The new method described is surely not a perfect answer to the needs of fund managers and investment analysts, for it requires the making of certain assumptions about fund performance with which not everyone will completely agree (e.g., the desirability of a fund's holding to a consistent investment policy). But the method goes at least part of the way, I believe, to providing answers that have long eluded executives in the investment business.

We have seen that, consistent with any specified level of the market rate of return, there is associated with each fund a range of combinations of expected portfolio return and risk. The slope of the portfolio-possibility line measures the rate at which the individual investor increases the expected rate of return of his portfolio as his burden of portfolio risk increases. A comparison of slopes among funds provides a means of rating funds which transcends variations in individual investors' attitudes toward risk. Although the slopes vary just as the market rate of return varies, it can be proved that the ranking of the funds represented remains unchanged. The relative rankings can be read directly from the characteristic lines of funds to be compared.

Differences in ranking based on the characteristic lines can be quite significant for individual investors, even though they take varying attitudes toward risk. Also, the differences are independent of market fluctuations. Because the ranking measure has these properties, it provides a useful basis for reviewing the performance of fund management.

APPENDIX 10.1

Exhibit 10.8 shows the characteristic line for a typical fund. For each possible value of the market rate of return, the characteristic line predicts the corresponding rate of return for the fund pictures. The slope of the characteristic line is measured by tangent B; the vertical intercept is h. For the particular market rate of return D, the expected fund rate of return is μ. A horizontal line drawn a distance μ^* above the horizontal axis depicts the behavior of a fund consisting solely of fixed income securities. The

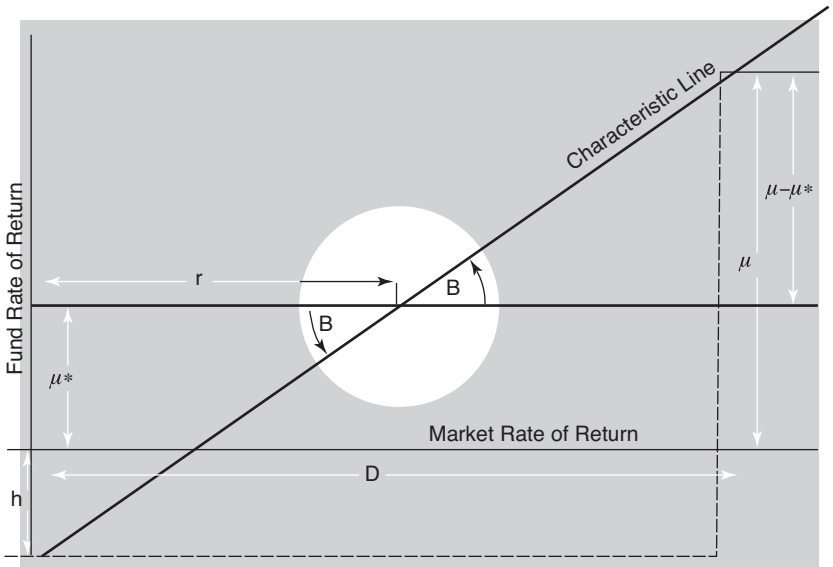

EXHIBIT 10.8 Characteristic Line and Value for a Typical Fund

ranking measure r is determined by the intersection of the characteristic line and the horizontal line at height μ^*.

The question is whether the ranking measure r has the properties specified; that is, whether it will—

- Rank funds in the order of their respective values of tangent α (the slope of the opportunity locus as discussed earlier in the article);
- Have the same value for a given fund, independently of fluctuations in the market rate of return.

A moment's reflection shows that no number can have both properties simultaneously unless the general result alluded to in the main text holds true; that is, unless the *relative* ranking of funds—in terms of the slope of the portfolio-possibility line—is unaffected by fluctuations in the general market. Inasmuch as the proof demonstrates that the number in question does indeed have both properties, the general result follows.

From the geometry of the diagram, we have for the volatility:

$$\sigma = \text{tangent } B = \frac{\mu - \mu^*}{D - r}$$

Solving for r, we obtain:

$$r = D - \left(\frac{\mu - \mu^*}{\sigma}\right)$$

The expression in parentheses is the ranking measure discussed in the section on portfolio-possibility lines (see "Performance Measure," p. 74), with the volatility, σ, serving as the approximate measure of investment risk. We conclude that, for any given level of market rate of return D, r is uniquely related to the ranking fraction.

$$\frac{\mu - \mu^*}{\sigma} \left(\text{which equals tangent } \propto\right)$$

We note that a relatively large value of r signifies a relatively low level of performance for fund management. A second important property of r is obtained when the following relationship, based on the geometry of the diagram, is substituted in the previous expression for r:

$$\mu = D \text{tangent } B + h = D \propto + h$$

Substituting for μ, we find that:

$$r = \frac{\mu^* - h}{\sigma}$$

Now μ^* is the same for all funds and independent of market fluctuations; and h and σ are the intercept and slope, respectively, of the characteristic line. It is clear in

this formulation that r is independent of D, the market rate of return. Hence r tends to have the same value independently of fluctuations in the general market.

You pays your money and you takes your choice.

—*Punch,* Vol. X, p. 16. 1846

Notes

1. In discussing the cumulative performance of investment funds between January 1, 1953, and September 30, 1958, the report says "... the interpretation of the net result is to be made against the background of the movements in security market prices during this period. . .general fund performance and comparisons among funds of different types might be quite different in other time periods . . . ," *A Study of Mutual Funds* (Washington, D.C.: Government Printing Office, 1962), p. 308.
2. See, for example, H. M. Markowitz, *Portfolio Selection: Efficient Diversification of Investments* (New York: John Wiley & Sons, 1959); and D. E. Farrar, *The Investment Decision Under Uncertainty* (Englewood Cliffs: Prentice-Hall, Inc., 1962).
3. For elegant mathematical proof of the validity of indifference curves, see James Tobin, "Liquidity Performance as Behavior Towards Risk," *Review of Economic Studies*, February 1958, p. 65; a subsequently written, unpublished manuscript by the author carries the discussion further.