

A new first order method

1 Introduction

Professor Yin Zhang talked about a new first order method in his lecture. In particular, see the lecture note “CIE6010”, page 34 in the website

<https://walterbabyrudin.github.io/information/information.html>

In general, this first order method is as follows:

- Generate two initial guess x^1, x^2 .
- For each iteration $T = 1, 2, \dots$
 - Generate x^3 as follows:

$$\begin{aligned} S &\leftarrow x^2 - x^1 \\ y &\leftarrow \nabla f(x^2) - \nabla f(x^1) \\ \alpha &\leftarrow 1 \\ D &\leftarrow \frac{1}{L}I + \frac{(S) \cdot (S)^T}{(y)^T(y)} \succ 0 \\ x^3 &\leftarrow x^2 - \alpha \cdot D \cdot \nabla f(x^2) \end{aligned}$$

- Update $x^1 \leftarrow x^2$ and $x^2 \leftarrow x^3$
- When the stop criteria is satisfied, return x^3 .

This method can be used to efficiently solve the compressive sensing problem (actually the L1 regularization problem)

$$\min_{x \in \mathbb{R}^n} f(x) \equiv \phi_\sigma(Dx) + \frac{\mu}{2} \|Ax - b\|_2^2,$$

where $\phi_\sigma(y) = \sum_i \sqrt{y_i^2 + \sigma}$. Prof. Yin Zhang also writes his MATLAB script to implement this algorithm in file *yzL1reg2d.p*. I also write my own script (see the file *myL1reg2d*.)