The Chinese University of Hong Kong, Shenzhen

PHY 1002

Physics Laboratory

Lab Report for Rotational Inertia

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Contents

1 Experiment 1 Physical Pendulum – Period

1.1 Objective:

This experiment is designed to *explore* the relationship between the *period of a physical pendulum* (a **uniform bar**) and *the distance from the pivot point to the center of mass*. During this process, we will study *how the period changes as pivot is moved from the first hole of the bar to the center.* Then we measure the distance from the *piovt point* to the *center of mass* that produces the *minimum* period of oscillation. Comparing it to the theoretical value, if the error is acceptable, the validity of this experiment is confirmed.

1.2 Theory

Recall that the momentum of inertia of a *long rod* about its center of mass is:

$$I_{\mathrm{c}} = \frac{1}{12} M L^2 \tag{1}$$

where M is its mass, and L is the overall length of rod.
A more precise physical model of the momentum of inertia for the uniform bar as a *rectangle-type* rod is:

$$I_{\mathrm{c}} = \frac{1}{12} M (a^2 + b^2)$$

where a is its length and b is its thickness.
However, in this case $a >> b$, $(\left(\frac{b}{a}\right)^2 < 0.003)$ so $I_{\mathrm{c}} = \frac{1}{12} M L^2$ could be used as a very good approximation.(error is only 0.3%.)
The period of a physical pendulum depends on its momentum of inertia I_{pivot}, its mass M and the distance from the pivot point to the center of mass L_{c}:

$$T = 2\pi \sqrt{\frac{I_{\mathrm{pivot}}}{M g L_{\mathrm{c}}}} \tag{2}$$

And the *parallel axis theorem* enables us to write the momentum of inertia of the bar about a *pivot point* as:

$$I_{\mathrm{pivot}} = I_{\mathrm{c}} + M L_{\mathrm{c}}^2 \tag{3}$$

Combining the Eq.(1), Eq.(2) and Eq.(3), we derive the period of the pendulum:

$$T = 2\pi \sqrt{\frac{\frac{1}{12} L^2 + L_{\mathrm{c}}^2}{g L_{\mathrm{c}}}} \tag{4}$$

Setting the derivative equal to zero and solve for L_c, we obtain the theoratical value which produces the *minimum* period oscillation for the bar:

$$L_c^0 = \frac{1}{\sqrt{12}} L. \tag{5}$$

1.3 Method:

1. We completed the full setup. (Refer to *Capstone Physical Pendulum – Period manual* for detail reading)
2. We gently stated the pendulum bar swing with a *small amplitude* (about 20 degrees total)
3. Then we clicked on "*Record*" on Capstone to begin recording data. After about 25 seconds, click "*STOP*" to stop recording data. Data would appear in the graph of *angular position* versus *time*. From the graph we determined and recorded the *period* of this oscillation.
4. Then we moved the mounting screw to the next hole (4cm from the center hole) and repeated step **2 − 3**. Then we repeated the process for the holes that are 6cm, 8cm, 10cm, 12cm and 14cm from the center hole.

1.4 Raw Data:

The table(1) below shows typical data for the length from the *pivot point* to the *center of mass* of an oscillating pendulum bar and the corresponding *period* of oscillation for each length:

Length (cm) ($\pm$0.05cm)	Period (sec)($\pm 3.16 \times 10^{-4}$s)
2.0	1.195
4.0	0.928
6.0	0.839
8.0	0.823
10.0	0.824
12.0	0.850
14.0	0.878

Table 1: Data for period versus length: Collected from Capstone

- **Error Explanation:** The error for the length δ_{L_c} is due to the *systematic error*, thus $\delta_{L_c} = 5 \times 10^{-4}$m. And the error for the period is due to the *propagation error*:
  - Since $T = 2\pi\sqrt{\frac{\frac{1}{12}L^2+L_c^2}{gL_c}}$, its error could be computed as:

$$\begin{aligned}
\delta T &= \sqrt{\left(\frac{\partial T}{\partial L_c}\right)^2 (\delta_{L_c})^2} \\
&= \left|\frac{\partial T}{\partial L_c}\right| \cdot (\delta_{L_c}) \\
&= \pi \left(\frac{\frac{1}{12}L^2 + L_c^2}{gL_c}\right)^{-1/2} \times \frac{\left|(\frac{1}{12}L^2 + L_c^2) - (2L_c)^2\right|}{(gL_c)^2} \cdot (\delta_{L_c}) \\
&\leq \pi \left(\frac{\frac{1}{12}(0.28\text{m})^2 + (0.14\text{m})^2}{9.8\text{m}/s^2 \cdot (0.14\text{m})}\right)^{-1/2} \times \frac{\left|(\frac{1}{12}(0.28\text{m})^2 + (0.14\text{m})^2) - (0.28\text{m})^2\right|}{(9.8\text{m}/s^2 \cdot (0.14\text{m}))^2} \cdot (\delta_{L_c}) \\
&= 0.6320 \times 5 \times 10^{-4}\text{m} \\
&= 3.16 \times 10^{-4}\text{s}.
\end{aligned}$$

1.5 Data Analysis

We use the data from Table(1) to draw the *period* versus *length* curve. (As Figure(1) shown) As the length from the *pivot point* to the *center of mass* increasing, the period decreased and then increased.

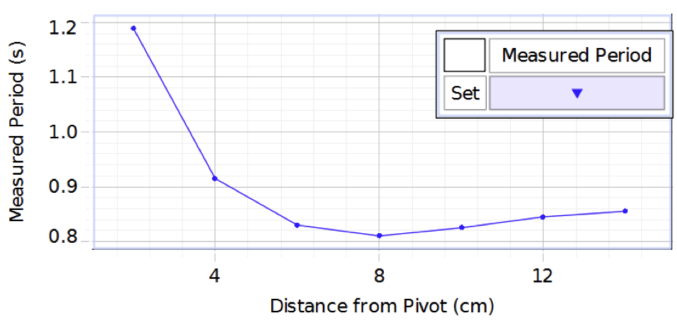

Figure 1: Experimental Period (T) versus Length (L_c) graph: data from Table(1)

The theoretical value for the period is obtained from Eq.(4), where $L = 28$cm, $g = 9.8m/s^2$. Thus we could plot the *theoretical* period versus length L_c graph:

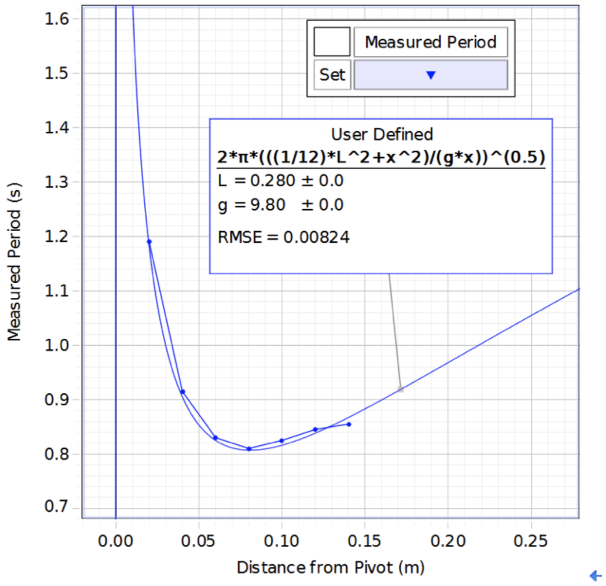

Figure 2: Theoretical Period (T) versus Length (L_c) graph

We observe that there is an error between the experimental and theoretical value for the *period* in Figure(3). In order to reduce the error, we could remove several points from Table(1). Finally, we find the experimental period at length 6cm, 8cm, 10cm fit the theoretical curve better:

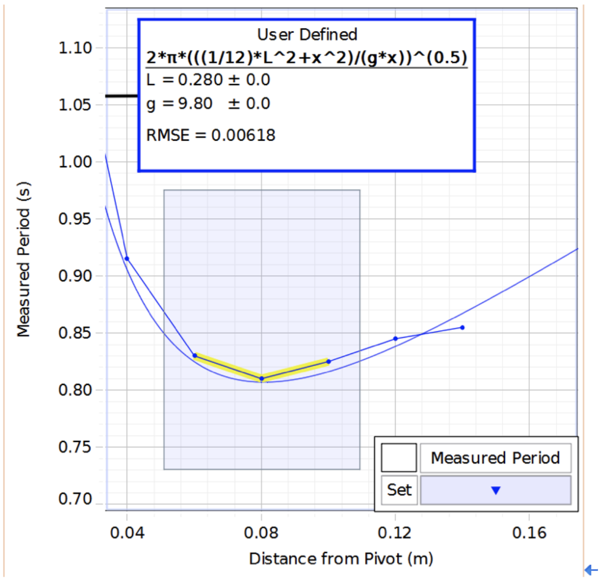

Figure 3: Improved experimental results by ignoring several data

Finally, we unlock the value of length of pendulum and gravity and updated the fitted curve: (Shown in Figure(4))

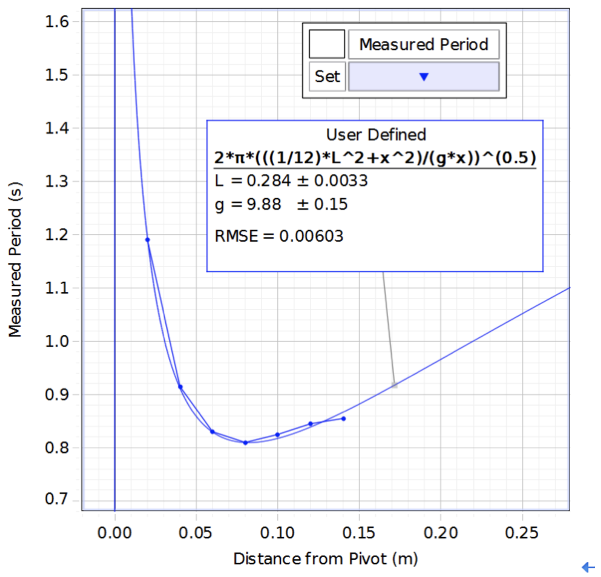

Figure 4: Fitted curves for the data from Table(1)

Thus we find the updated parameter value for the length L and the gravity g:

$$L_{Fitted} = 0.284 \pm 0.0033\text{m}.$$

$$g_{Fitted} = 9.88 \pm 0.15\text{m}/s^2.$$

1.5.1 Difference

In Figure(4), we found the parameters L and g are very close to the actual value. The differnce for L is given by:

$$\begin{aligned}\text{Difference} &= \frac{L_{\text{Fitted}} - L_{\text{Actual Value}}}{L_{\text{Actual Value}}} \times 100\% \\ &= \frac{0.284\text{m} - 0.280\text{m}}{0.280\text{m}} \times 100\% \\ &= 1.43\%\end{aligned}$$

The difference for g is given by:

$$\begin{aligned}\text{Difference} &= \frac{g_{\text{Fitted}} - g_{\text{Actual Value}}}{g_{\text{Actual Value}}} \times 100\% \\ &= \frac{9.88\text{m}/s^2 - 9.80\text{m}/s^2}{9.80\text{m}/s^2} \times 100\% \\ &= 0.82\%\end{aligned}$$

From Table(1), we find the measured value for length that gives minimum period is $L_{\text{measured}} = 8.0 \pm 0.05\text{cm}$. And due to Eq.(5), the theoretical value for length that gives minimum period is:

$$L_{\text{c}}^0 = \frac{1}{\sqrt{12}} \times (28.0 \pm 0.05)\text{cm} = 8.1 \pm 0.01\text{cm}$$

Hence the percent difference between the theoretical and experimental length that produces minimum period is:

$$\begin{aligned}\text{Difference} &= \frac{L_{\text{measured}} - L_{\text{c}}^0}{L_{\text{c}}^0} \times 100\% \\ &= \frac{8.0\text{cm} - 8.1\text{cm}}{8.1\text{cm}} \times 100\% \\ &= 1.23\%\end{aligned}$$

2 Experiment 2 Rotational Inertia Based on Period of Oscillation

2.1 Objective:

During this experiment,

- Firstly, we measured the *mass*, the *dimensions* and the distance from pivot point to its center of mass to derive the *theoretical value of inertia of a physical pendulum.* (By Rotational Inertia Equations)
- Then we measured the *period of oscillation of a physical pendulum* to compute the *momentum of inertia indirectly.*
- And we also calculated the *rotational inertia* of a **irregular shape disk** about the center of mass based on *angular acceleration.*

We compare the results from these different methods, if the error is acceptable, we conclude the validity of this experient.

2.2 Theory:

2.2.1 Method 1:

The theoretical momentum of inertia about its center of mass could be derived from its mass, the dimensions and the distance from pivot point to its center of mass. We have different kinds of disks shown in Figure(5):

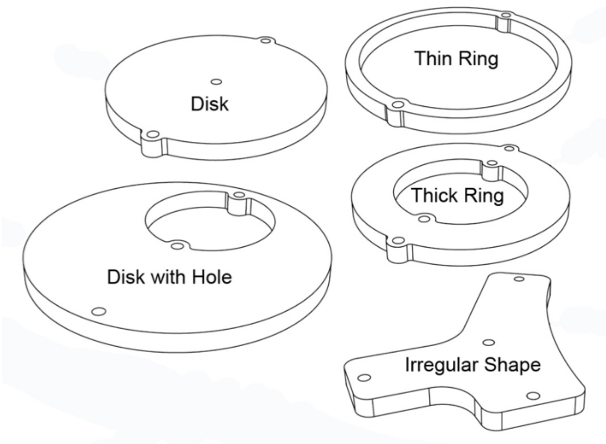

Figure 5: Different kinds of disks

For the first disk, its momentum of inertia about its center of mass is given by:

$$I = \frac{1}{2}MR^2 \tag{6}$$

where M denotes its mass and R denotes its radius.
For the thick ring, its momentum of inertia about its center of mass is given by:

$$I = \frac{1}{2}M(R_1^2 + R_2^2) \tag{7}$$

where M denotes its mass, R_1 denotes its outer radius, R_2 denotes its inner radius.
For the thin ring, its momentum of inertia about its center of mass is given by:

$$I = MR^2 \tag{8}$$

where M denotes its mass and R denotes its radius.
For a disk of radius R with a hole of radius r, we pivoted at the outer edge of the

hole:

$$
\begin{aligned}
I = I_{\text{disk}} - I_{\text{hole}} &= \left(\frac{1}{2}M_{\text{disk}}R^2 + M_{\text{disk}}L_c^2\right) - \left(\frac{1}{2}M_{\text{hole}}r^2 + M_{\text{hole}}r^2\right) \\
&= \frac{1}{2}M_{\text{disk}}R^2 + M_{\text{disk}}L_c^2 - \frac{3}{2}M_{\text{hole}}r^2
\end{aligned}
$$

where R denotes the radius of the disk, r denotes the radius of the hole, M denotes the mass of the disk, L_c denotes the distance from the pivot to the center of mass. Assuming the mass is uniformly distributed on the disk, then we derive:

$$
M_{\text{disk}} = \sigma A_{\text{disk}} = \frac{M}{(\pi R^2 - \pi r^2)}(\pi R^2) = M\left(\frac{R^2}{R^2 - r^2}\right),
$$

$$
M_{\text{hole}} = \sigma A_{\text{hole}} = \frac{M}{(\pi R^2 - \pi r^2)}(\pi r^2) = M\left(\frac{r^2}{R^2 - r^2}\right).
$$

Finally, the momentum of inertia for a disk with a hole is given by:

$$
\begin{aligned}
I &= \frac{1}{2}M\left(\frac{R^2}{R^2 - r^2}\right) + M\left(\frac{R^2}{R^2 - r^2}\right)L_c^2 - \frac{3}{2}M\left(\frac{r^2}{R^2 - r^2}\right)r^2 \qquad (9)\\
&= \frac{1}{2}M\left(\frac{R^4 - 3r^4 + 2R^2L_c^2}{R^2 - r^2}\right) \qquad (10)
\end{aligned}
$$

We **cannot** compute the theoretical momentum of inertia for irregular shape disk in this way.

2.2.2 Method 2:

We could compute the rotational inertia indirectly. By Eq.(2) and Eq.(3), the *period of oscillation* T of a physical pendulum, depends on the *momentum of inertia anout the center of mass* I_{c}, the mass M, the *distance* from the pivot point to the center of mass L_{c}:

$$
T = 2\pi\sqrt{\frac{I_{\text{c}} + ML_c^2}{MgL_c}}
$$

Conversely, the momentum of inertia about its center of mass could be found as follows:

$$
I_{\text{c}} = \frac{T^2MgL_c}{4\pi^2} - ML_c^2. \qquad (11)
$$

2.2.3 Method 3:

We use another way to compute the inertia of **irregular shape of disk**. We apply the formula

$$\tau = I\alpha \implies I = \frac{\tau}{\alpha}$$

where τ is the torque caused by the weight hanging from the string which is wrapped around the large step of the 3-steo pulley of the apparatus. α denotes the angular acceleration. And we also obtain:

$$\tau = rF$$

where r is the radius of the pulley about which the string is wound and F denotes the tension in the string when the apparatus is rotating.
We apply Newton's second law for hanging mass m:

$$\sum F = mg - F = ma \implies F = m(g - a)$$

where a is the linear acceleration of the string and $a = r\alpha$. Finally the momentum of inertia could be expressed as:

$$I = \frac{rm(g - r\alpha)}{\alpha}$$

2.3 Method:

2.3.1 Method 1:

Firstly, I measured the inner and outer radius, mass of each kind of disks shown in Figure(5). Also, I measured the distance from the pivot points (on the edge of the disk) to the center of mass for each disk. Then I use the formula derived from Theory part to compute the theoretical momentum of inertia.

2.3.2 Method 2:

1. We completed the full setup. (Refer to *Capstone Physical Pendulum – Period manual* for detail reading)
2. Then we started the disk to swing with a small amplitude (about 20 degrees)
3. After two seconds, we clicked "*Record*" to begin recording data. Then we clicked "*Stop*" to stop recording data after about 25 seconds. Then we determined the period for this oscillation

4. We repeated the process for three times and recorded the *mean* period for this disk.
5. We replaced the disk with the remaining disks shown in Figure(5) and repeated step **2** to step **4**.

2.3.3 Method 3:

1. We mounted the irregular shape on the Rotary Motion Sensor using the center hole, put about 10g over the pulley and recorded the angular velocity versus time on a graph as the mass falls to the table.
2. Then we selected the part of the graph where the disk was falling to use *lienar regression* to find the straight line that best fits teh data. The slope of the best-fit line was the *angular acceleration* of the apparatus, and we recorded it.
3. Then we use the *angular acceleration* to compute the inertia of the *irregular shape disk*.

2.4 Raw Data:

2.4.1 Method 1:

The data for *inner* and *outer* radius, *mass*, the distance from the *pivot points* (on the edge of each disk) to the center of mass of each disk is given in Table(2):

Pendulum Type	Mass (kg) ($\pm 5 \times 10^{-5}$kg)	Inner Radius (m) ($\pm 5 \times 10^{-4}$m)	Outer Radius (m) ($\pm 5 \times 10^{-4}$m)	Distance (m) ($\pm 5 \times 10^{-4}$m)
Disk	0.08903	0.04000	/	0.04000
Disk with Hole	0.10270	0.04750	0.02250	0.05540
Thin Ring	0.02243	0.04250	0.03750	0.04000
Thick Ring	0.05622	0.04000	0.02500	0.04000
Irregular Shape	0.06340	/	/	0.03600

Table 2: Data collected manually for each type of disk

- **Error Explanation:** The errors for the mass and the length are all due to the *systematic error*.

2.4.2 Method 2:

The average period of oscillation for each disks are given in Table(3)

Pendulum Type	Average Period (s) ($\pm 0.005s$)
Disk	0.495
Disk with Hole	0.549
Thin Ring	0.573
Thick Ring	0.525
Irregular Shape	0.499

Table 3: Average Period for each type of disk: Data collected from Capstone

- **Error Explanation:** The average period is collected from capstone, so its error is due to the *systematic error*, thus $\delta T = 0.005$s.

2.4.3 Method 3:

In this part, we recoreded the *angular velocity* versus *time* graph as the mass falls to the table. Then we used the linear curve to fit on the graph. The stright line that best fits the data is shown in Figure(6):

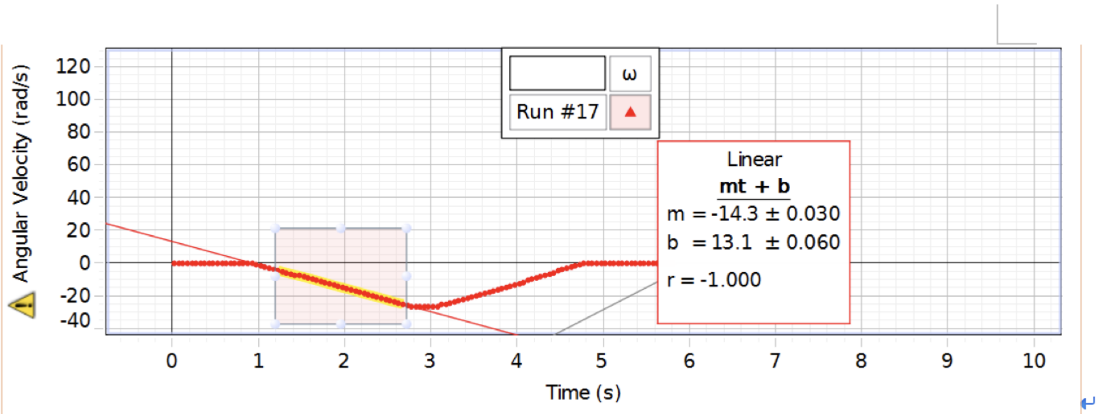

Figure 11

Figure 6: The best-fit line for *angular velocity* versus *time* graph

From the graph we observe that the *angular acceleration* α is obtained:

$$\alpha = 14.3 \pm 0.030\text{rad}/s^2.$$

2.5 Data and Error Analysis

2.5.1 Method 1:

By applying Eq.(6) to Eq.(10) together with the data from Table(2), we could compute the *theoretical* momentum inertia about its center of mass for each type of disk:

Pendulum Type	Theoretical I_{cm} ($\text{kg} \cdot \text{m}^2$)
Disk	$7.1224 \times 10^{-5} \pm 1.8 \times 10^{-6}$
Disk with Hole	$1.0049 \times 10^{-4} \pm 1.2 \times 10^{-6}$
Thin Ring	$4.0514 \times 10^{-5} \pm 1.1 \times 10^{-6}$
Thick Ring	$6.2545 \times 10^{-5} \pm 1.5 \times 10^{-6}$
Irregular Shape	/

Table 4: Theoretical inertia for each type of disk

- **Error Explanation:** For disk, the error is given by:

$$\begin{aligned}
\delta I_{cm} &= \sqrt{\left(\frac{\partial I_{cm}}{\partial M}\right)^2 (\delta M)^2 + \left(\frac{\partial I_{cm}}{\partial R}\right)^2 (\delta R)^2} \\
&= \sqrt{\left(\frac{1}{2}R^2\right)^2 (\delta M)^2 + (MR)^2 (\delta R)^2} \\
&= \sqrt{\left(\frac{1}{2}(0.04m)^2\right)^2 (5 \times 10^{-5}\text{kg})^2 + (0.089 \cdot 0.04)^2 (5 \times 10^{-4}m)^2} \\
&= 1.8 \times 10^{-6}\text{kg} \cdot \text{m}^2.
\end{aligned}$$

Following the similar idea, we could derive the error for the inertia of the remaining kinds of disks.

2.5.2 Method 2:

Using Eq.(11) together with the *average period* data from Table(3) and the data for the *mass* of the disk from Table(2), we could compute the *momentum of inertia* indirectly:

Pendulum Type	Derived I_{cm} ($\text{kg} \cdot \text{m}^2$) ($\pm 3 \times 10^{-6} \text{kg} \cdot \text{m}^2$)
Disk	7.4160×10^{-5}
Disk with Hole	9.9961×10^{-5}
Thin Ring	3.7237×10^{-5}
Thick Ring	6.3912×10^{-5}
Irregular Shape	5.8912×10^{-5}

Table 5: Theoretical inertia for each type of disk

- **Error Explanation:** The error of momentum of inertia is given by:

$$\begin{aligned}
\delta I_{cm} &= \sqrt{\left(\frac{\partial I_{cm}}{\partial T}\right)^2 (\delta T)^2 + \left(\frac{\partial I_{cm}}{\partial M}\right)^2 (\delta M)^2 + \left(\frac{\partial I_{cm}}{\partial L_c}\right)^2 (\delta L_c)^2} \\
&= \sqrt{\left(\frac{TMgL_c}{2\pi^2}\right)^2 (\delta T)^2 + \left(\frac{T^2 g L_c}{4\pi^2} - L_c^2\right)^2 (\delta M)^2 + \left(\frac{T^2 M g}{4\pi^2} - 2ML_c\right)^2 (\delta L_c)^2}
\end{aligned}$$

Following this way we derive the error for the momentum of inertia. The process of plugging in data into this large formula is skipped.

2.5.3 Method 3:

And we could use another way to compute the momentum inertia for irregular shape disk. During this part, the inertia is derived as:

$$\begin{aligned}
I &= \frac{rm(g - r\alpha)}{\alpha} \\
&= \frac{0.014\text{m} \times 0.01\text{kg} \times (9.8\text{m}/s^2 - 0.014\text{m} \times 14.3\text{rad}/s^2)}{14.3\text{rad}/s^2} \\
&= 6.2791 \times 10^{-5} \text{kg} \cdot \text{m}^2.
\end{aligned}$$

The error for the inertia of irregular shape is

$$\begin{aligned}
\delta I &= \sqrt{\left(\frac{\partial I}{\partial \alpha}\right)^2 (\delta\alpha)^2} \\
&= \left|-\frac{rmg}{\alpha^2} - r^2 m\right| \delta\alpha \\
&= \left(\frac{0.014\text{m} \times 0.01\text{kg} \times 9.8\text{m}/s^2}{(14.3\text{rad}/s^2))^2} + (0.014\text{m})^2 \cdot (0.01\text{kg})\right) (0.030\text{rad}/s^2))^2) \\
&= 3.6 \times 10^{-6} \text{kg} \cdot \text{m}^2.
\end{aligned}$$

Hence the inertia of the irregular shape disk is given by:

$$I = 6.2791 \times 10^{-5} \pm 3.6 \times 10^{-6}\text{kg} \cdot \text{m}^2.$$

2.5.4 Difference

We could use **Method 1** and **Method 2** to compute the momentum inertia of the first four types of disk in Figure(5). And we compare these two different results. The difference for two calculated inertia is given by:

$$\text{Difference} = \frac{\text{Theoretical inertia} - \text{Derived inertia}}{\text{Derived inertia}} \times 100\%$$

After plugging in specific data, we obtain the difference for these four difference kinds of disks:

Pendulum Type	Difference
Disk	−3.96%
Disk with Hole	0.53%
Thin Ring	8.80%
Thick Ring	6.17%

Table 6: Theoretical inertia for each type of disk

And we could use **Method 2** and **Method 3** to compute the momentum inertia of the irregular shape disk. The difference for two calculated inertia is given by:

$$\begin{aligned} \text{Difference} &= \frac{5.8912 \times 10^{-5}\text{kg} \cdot \text{m}^2 - 6.2791 \times 10^{-5}\text{kg} \cdot \text{m}^2}{6.2791 \times 10^{-5}\text{kg} \cdot \text{m}^2} \times 100\% \\ &= -6.18\% \end{aligned}$$

3 Conclusion

- In experiment 1, theoretically when the distance between the pivot and the center of mass is 8.1cm, the period of oscillation is minimized. And experimentally we found when the distance is 8.0cm, the period of oscillation is minimized. The difference between the theoretical value and the measured value for the length is 1.23%, which is very small. Hence we conclude the validity of this experiment.
  Moreover, according to Eq.(4), a heavier or lighter bar with the same dimensions should have the same value for the length that gives minimum period of

oscillation. Mass is not a factor in the calculation of the length for minimum period.

- In experiment 2, as we can see in Table(6), the difference between the inertia calculated by period and inertia calculated by dimensions is relatively small. For irregular shape, the difference between the inertia calculated by period and inertia calculated by acceleration is also relatively small. Hence we confirm the validity of our experiment.