Georgia Institute of Technology

ISyE3770 - Midterm 2 Practice

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Name: _____

GT Student ID: _____

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1. This midterm **2** practice contains 10 pages (including this cover page) and 7 questions. Total of points is 100.

2. In taking this examination, you are expected to adhere to the GT academic honor code. At a minimum, this requires that you utilize only the materials supplied to you and that you do not give help to, or accept help from, others.

Distribution of Marks

Question	Points	Score
1	32	
2	8	
3	8	
4	8	
5	12	
6	20	
7	12	
Total:	100	

Multiple Choice (56 points)
Remark: for each question, one and only one of four given choices (A, B, C, and D) is correct.

1. Suppose you are trying to compare the reliability of two types of batteries: A and B in terms of their battery life. You randomly select 500 batteries of each type, and measured how many hours each of them last. You make the comparison by considering the difference of the sample mean of the two types of batteries, denoted as $\mu_A - \mu_B$.

   (a) (8 points) The 500 selected batteries of each type correspond to
       A. Population   B. Sample   C. Parameter   D. Statistic

       (a) __*B*__ .

   (b) (8 points) The difference of the true mean of the two types of battery life corresponds to
       A. Population   B. Sample   C. Parameter   D. Statistic

       (b) __*C*__

   (c) (8 points) The difference of the sample mean of two types of battery life corresponds to
       A. Population   B. Sample   C. Parameter   D. Statistic

       (c) __*D*__

   (d) (8 points) All batteries of two types and their life times correspond to
       A. Population   B. Sample   C. Parameter   D. Statistic

       (d) __*A*__
       .

2. (8 points) Determine if the following statement is true or not:

   Given two point estimators, if their biases are the same, then the estimator with a smaller variance has a smaller MSE.

   A. True   B. False

   $$MSE = bias^2 + variance$$

   2. __*A*__ .

3. (8 points) Determine if the following statement is true or not:

   Given two point estimators, if their variances are the same, then the estimator with a smaller magnitude of bias has a smaller MSE.

   A. True   B. False

   $$MSE = bias^2 + variance$$

   3. __*A*__ .

4. (8 points) Determine if the following statement is true or not:

   Given two point estimators, if their MSEs are the same, then the estimator with a larger variance has a smaller magnitude of bias.

   A. True   B. False

   $$MSE = bias^2 + variance$$

   4. ___A___ .

Regular Question (44 points)

5. (12 points) A random variable has probability density function

$$f(x; \theta) = \frac{x^3 e^{-x/\theta}}{6\theta^4}, \qquad 0 < x < \infty, 0 < \theta < \infty.$$

Now, given samples $X_1, \ldots, X_n$, derive the maximum likelihood estimator for the parameter θ.

Solution.

$$l(\theta; x) = \sum_{i=1}^{n} \log f(x_i; \theta)$$

$$= \sum_{i=1}^{n} \log \left[\frac{x_i^3 \exp(-\frac{x_i}{\theta})}{6\theta^4} \right]$$

$$= \sum_{i=1}^{n} \left[-\frac{x_i}{\theta} - 4\log\theta \right] + constant$$

$$= -\frac{1}{\theta} \sum_{i=1}^{n} X_i - 4n\log\theta + constant.$$

$$\hat{\theta} = \arg\max_{\theta} l(\theta; x) \Rightarrow l'(\hat{\theta}; x) = 0 \Rightarrow \frac{1}{\theta^2} \sum_{i=1}^{n} X_i - \frac{4n}{\theta} = 0$$

$$\Rightarrow \hat{\theta} = \frac{1}{4n} \sum_{i=1}^{n} X_i = \frac{1}{4}\bar{X}.$$

6. (20 points) A group of wine enthusiasts taste-tested a pinot noir wine from Oregon. The evaluation was to grade the wine on a 0-to-100-point scale. The results from 10 samples are as follows: 94, 90, 92, 91, 91, 86, 89, 91, 91, 80.

   (i) Compute the sample mean, median, range, sample variance, sample standard deviation, Q_1, Q_3, and IQR. (It's allowed to use R or calculator to finish the calculation).
   (15 points)

   (ii) Is there any outlier for the data? Why? (Hint: you can either look at the IQR value or draw the box plot using R to answer this question. If you choose to draw the plot using R, please also try to draw the boxplot in the answer sheet.)
   (5 points)

Solution.

(i) sample mean : 89.5

   median   = 91

   range   = 14

   sample variance = 15.39

   sample standard deviation = 3.92

   Q_1 = 89.25

   Q_3 = 91

   IQR = 1.75

(ii) Since data   80, 86, 94 are outside of

   [$Q_1 - 1.5$ IQR, $Q_3 + 1.5$ IQR] they're outliers

7. (12 points) The weight of a population of workers has a mean of 167 and a standard deviation of 27. Suppose a random sample of 40 workers is chosen, approximate the probability that the sample mean of their weights lies between 163 and 170.

Solution:

By central limit theorem,

$\bar{X}$ approximately follows $N\left(167, \frac{27^2}{40}\right).$

$= N(167, 18.225)$

$\Rightarrow P(163 < \bar{X} < 170) \approx P\left(\frac{163 - 167}{\sqrt{18.225}} < N(0,1) < \frac{170 - 167}{\sqrt{18.225}}\right)$

$= \Phi(0.70) - \Phi(-0.93)$

$= 0.58$

Discrete Distributions

Bernoulli
$0 < p < 1$

$$f(x) = p^x(1-p)^{1-x}, \qquad x = 0, 1$$

$$M(t) = 1 - p + pe^t, \qquad -\infty < t < \infty$$

$$\mu = p, \qquad \sigma^2 = p(1-p)$$

Binomial
$b(n,p)$
$0 < p < 1$

$$f(x) = \frac{n!}{x!(n-x)!} p^x(1-p)^{n-x}, \qquad x = 0, 1, 2, \ldots, n$$

$$M(t) = (1 - p + pe^t)^n, \qquad -\infty < t < \infty$$

$$\mu = np, \qquad \sigma^2 = np(1-p)$$

Geometric
$0 < p < 1$

$$f(x) = (1-p)^{x-1}p, \qquad x = 1, 2, 3, \ldots$$

$$M(t) = \frac{pe^t}{1 - (1-p)e^t}, \qquad t < -\ln(1-p)$$

$$\mu = \frac{1}{p}, \qquad \sigma^2 = \frac{1-p}{p^2}$$

Negative Binomial
$0 < p < 1$
$r = 1, 2, 3, \ldots$

$$f(x) = \binom{x-1}{r-1} p^r(1-p)^{x-r}, \qquad x = r, r+1, r+2, \ldots$$

$$M(t) = \frac{(pe^t)^r}{[1 - (1-p)e^t]^r}, \qquad t < -\ln(1-p)$$

$$\mu = r\left(\frac{1}{p}\right), \qquad \sigma^2 = \frac{r(1-p)}{p^2}$$

Poisson
$\lambda > 0$

$$f(x) = \frac{\lambda^x e^{-\lambda}}{x!}, \qquad x = 0, 1, 2, \ldots$$

$$M(t) = e^{\lambda(e^t - 1)}, \qquad -\infty < t < \infty$$

$$\mu = \lambda, \qquad \sigma^2 = \lambda$$

Uniform
$m > 0$

$$f(x) = \frac{1}{m}, \qquad x = 1, 2, \ldots, m$$

$$\mu = \frac{m+1}{2}, \qquad \sigma^2 = \frac{m^2 - 1}{12}$$

Continuous Distributions

Beta
$\alpha > 0$
$\beta > 0$

$$f(x) = \frac{\Gamma(\alpha + \beta)}{\Gamma(\alpha)\Gamma(\beta)} x^{\alpha-1}(1-x)^{\beta-1}, \qquad 0 < x < 1$$

$$\mu = \frac{\alpha}{\alpha + \beta}, \qquad \sigma^2 = \frac{\alpha\beta}{(\alpha + \beta + 1)(\alpha + \beta)^2}$$

Chi-square
$\chi^2(r)$
$r = 1, 2, \ldots$

$$f(x) = \frac{1}{\Gamma(r/2)2^{r/2}} x^{r/2-1} e^{-x/2}, \qquad 0 < x < \infty$$

$$M(t) = \frac{1}{(1-2t)^{r/2}}, \qquad t < \frac{1}{2}$$

$$\mu = r, \qquad \sigma^2 = 2r$$

Exponential
$\theta > 0$

$$f(x) = \frac{1}{\theta} e^{-x/\theta}, \qquad 0 \le x < \infty$$

$$M(t) = \frac{1}{1-\theta t}, \qquad t < \frac{1}{\theta}$$

$$\mu = \theta, \qquad \sigma^2 = \theta^2$$

Gamma
$\alpha > 0$
$\theta > 0$

$$f(x) = \frac{1}{\Gamma(\alpha)\theta^\alpha} x^{\alpha-1} e^{-x/\theta}, \qquad 0 < x < \infty$$

$$M(t) = \frac{1}{(1-\theta t)^\alpha}, \qquad t < \frac{1}{\theta}$$

$$\mu = \alpha\theta, \qquad \sigma^2 = \alpha\theta^2$$

Normal
$N(\mu, \sigma^2)$
$-\infty < \mu < \infty$
$\sigma > 0$

$$f(x) = \frac{1}{\sigma\sqrt{2\pi}} e^{-(x-\mu)^2/2\sigma^2}, \qquad -\infty < x < \infty$$

$$M(t) = e^{\mu t + \sigma^2 t^2/2}, \qquad -\infty < t < \infty$$

$$E(X) = \mu, \qquad \text{Var}(X) = \sigma^2$$

Uniform
$U(a, b)$
$-\infty < a < b < \infty$

$$f(x) = \frac{1}{b-a}, \qquad a \le x \le b$$

$$M(t) = \frac{e^{tb} - e^{ta}}{t(b-a)}, \qquad t \ne 0; \qquad M(0) = 1$$

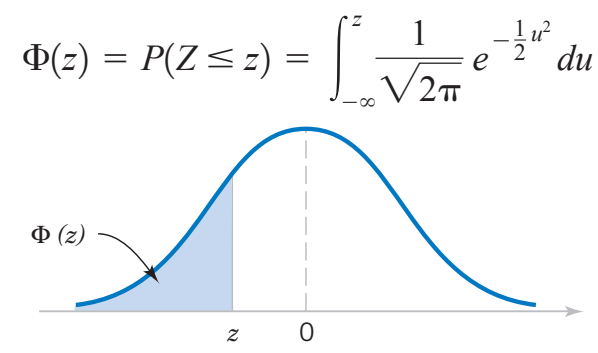$$\mu = \frac{a+b}{2}, \qquad \sigma^2 = \frac{(b-a)^2}{12}$$

$$\Phi(z) = P(Z \le z) = \int_{-\infty}^{z} \frac{1}{\sqrt{2\pi}} e^{-\frac{1}{2}u^2} du$$

Table III Cumulative Standard Normal Distribution

z	−0.09	−0.08	−0.07	−0.06	−0.05	−0.04	−0.03	−0.02	−0.01	−0.00
−3.9	0.000033	0.000034	0.000036	0.000037	0.000039	0.000041	0.000042	0.000044	0.000046	0.000048
−3.8	0.000050	0.000052	0.000054	0.000057	0.000059	0.000062	0.000064	0.000067	0.000069	0.000072
−3.7	0.000075	0.000078	0.000082	0.000085	0.000088	0.000092	0.000096	0.000100	0.000104	0.000108
−3.6	0.000112	0.000117	0.000121	0.000126	0.000131	0.000136	0.000142	0.000147	0.000153	0.000159
−3.5	0.000165	0.000172	0.000179	0.000185	0.000193	0.000200	0.000208	0.000216	0.000224	0.000233
−3.4	0.000242	0.000251	0.000260	0.000270	0.000280	0.000291	0.000302	0.000313	0.000325	0.000337
−3.3	0.000350	0.000362	0.000376	0.000390	0.000404	0.000419	0.000434	0.000450	0.000467	0.000483
−3.2	0.000501	0.000519	0.000538	0.000557	0.000577	0.000598	0.000619	0.000641	0.000664	0.000687
−3.1	0.000711	0.000736	0.000762	0.000789	0.000816	0.000845	0.000874	0.000904	0.000935	0.000968
−3.0	0.001001	0.001035	0.001070	0.001107	0.001144	0.001183	0.001223	0.001264	0.001306	0.001350
−2.9	0.001395	0.001441	0.001489	0.001538	0.001589	0.001641	0.001695	0.001750	0.001807	0.001866
−2.8	0.001926	0.001988	0.002052	0.002118	0.002186	0.002256	0.002327	0.002401	0.002477	0.002555
−2.7	0.002635	0.002718	0.002803	0.002890	0.002980	0.003072	0.003167	0.003264	0.003364	0.003467
−2.6	0.003573	0.003681	0.003793	0.003907	0.004025	0.004145	0.004269	0.004396	0.004527	0.004661
−2.5	0.004799	0.004940	0.005085	0.005234	0.005386	0.005543	0.005703	0.005868	0.006037	0.006210
−2.4	0.006387	0.006569	0.006756	0.006947	0.007143	0.007344	0.007549	0.007760	0.007976	0.008198
−2.3	0.008424	0.008656	0.008894	0.009137	0.009387	0.009642	0.009903	0.010170	0.010444	0.010724
−2.2	0.011011	0.011304	0.011604	0.011911	0.012224	0.012545	0.012874	0.013209	0.013553	0.013903
−2.1	0.014262	0.014629	0.015003	0.015386	0.015778	0.016177	0.016586	0.017003	0.017429	0.017864
−2.0	0.018309	0.018763	0.019226	0.019699	0.020182	0.020675	0.021178	0.021692	0.022216	0.022750
−1.9	0.023295	0.023852	0.024419	0.024998	0.025588	0.026190	0.026803	0.027429	0.028067	0.028717
−1.8	0.029379	0.030054	0.030742	0.031443	0.032157	0.032884	0.033625	0.034379	0.035148	0.035930
−1.7	0.036727	0.037538	0.038364	0.039204	0.040059	0.040929	0.041815	0.042716	0.043633	0.044565
−1.6	0.045514	0.046479	0.047460	0.048457	0.049471	0.050503	0.051551	0.052616	0.053699	0.054799
−1.5	0.055917	0.057053	0.058208	0.059380	0.060571	0.061780	0.063008	0.064256	0.065522	0.066807
−1.4	0.068112	0.069437	0.070781	0.072145	0.073529	0.074934	0.076359	0.077804	0.079270	0.080757
−1.3	0.082264	0.083793	0.085343	0.086915	0.088508	0.090123	0.091759	0.093418	0.095098	0.096801
−1.2	0.098525	0.100273	0.102042	0.103835	0.105650	0.107488	0.109349	0.111233	0.113140	0.115070
−1.1	0.117023	0.119000	0.121001	0.123024	0.125072	0.127143	0.129238	0.131357	0.133500	0.135666
−1.0	0.137857	0.140071	0.142310	0.144572	0.146859	0.149170	0.151505	0.153864	0.156248	0.158655
−0.9	0.161087	0.163543	0.166023	0.168528	0.171056	0.173609	0.176185	0.178786	0.181411	0.184060
−0.8	0.186733	0.189430	0.192150	0.194894	0.197662	0.200454	0.203269	0.206108	0.208970	0.211855
−0.7	0.214764	0.217695	0.220650	0.223627	0.226627	0.229650	0.232695	0.235762	0.238852	0.241964
−0.6	0.245097	0.248252	0.251429	0.254627	0.257846	0.261086	0.264347	0.267629	0.270931	0.274253
−0.5	0.277595	0.280957	0.284339	0.287740	0.291160	0.294599	0.298056	0.301532	0.305026	0.308538
−0.4	0.312067	0.315614	0.319178	0.322758	0.326355	0.329969	0.333598	0.337243	0.340903	0.344578
−0.3	0.348268	0.351973	0.355691	0.359424	0.363169	0.366928	0.370700	0.374484	0.378281	0.382089
−0.2	0.385908	0.389739	0.393580	0.397432	0.401294	0.405165	0.409046	0.412936	0.416834	0.420740
−0.1	0.424655	0.428576	0.432505	0.436441	0.440382	0.444330	0.448283	0.452242	0.456205	0.460172
0.0	0.464144	0.468119	0.472097	0.476078	0.480061	0.484047	0.488033	0.492022	0.496011	0.500000

$$\Phi(z) = P(Z \le z) = \int_{-\infty}^{z} \frac{1}{\sqrt{2\pi}} e^{-\frac{1}{2}u^2} du$$
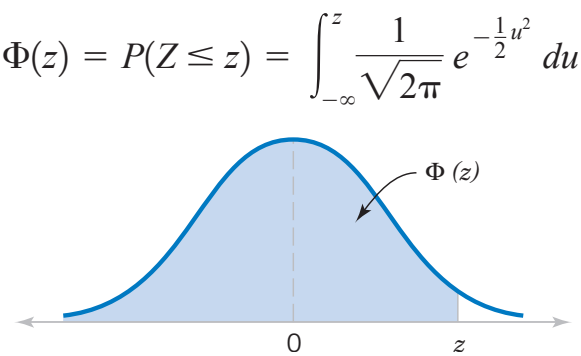
Table III   Cumulative Standard Normal Distribution (*continued*)

| z	0.00	0.01	0.02	0.03	0.04	0.05	0.06	0.07	0.08	0.09
0.0	0.500000	0.503989	0.507978	0.511967	0.515953	0.519939	0.532922	0.527903	0.531881	0.535856
0.1	0.539828	0.543795	0.547758	0.551717	0.555760	0.559618	0.563559	0.567495	0.571424	0.575345
0.2	0.579260	0.583166	0.587064	0.590954	0.594835	0.598706	0.602568	0.606420	0.610261	0.614092
0.3	0.617911	0.621719	0.625516	0.629300	0.633072	0.636831	0.640576	0.644309	0.648027	0.651732
0.4	0.655422	0.659097	0.662757	0.666402	0.670031	0.673645	0.677242	0.680822	0.684386	0.687933
0.5	0.691462	0.694974	0.698468	0.701944	0.705401	0.708840	0.712260	0.715661	0.719043	0.722405
0.6	0.725747	0.729069	0.732371	0.735653	0.738914	0.742154	0.745373	0.748571	0.751748	0.754903
0.7	0.758036	0.761148	0.764238	0.767305	0.770350	0.773373	0.776373	0.779350	0.782305	0.785236
0.8	0.788145	0.791030	0.793892	0.796731	0.799546	0.802338	0.805106	0.807850	0.810570	0.813267
0.9	0.815940	0.818589	0.821214	0.823815	0.826391	0.828944	0.831472	0.833977	0.836457	0.838913
1.0	0.841345	0.843752	0.846136	0.848495	0.850830	0.853141	0.855428	0.857690	0.859929	0.862143
1.1	0.864334	0.866500	0.868643	0.870762	0.872857	0.874928	0.876976	0.878999	0.881000	0.882977
1.2	0.884930	0.886860	0.888767	0.890651	0.892512	0.894350	0.896165	0.897958	0.899727	0.901475
1.3	0.903199	0.904902	0.906582	0.908241	0.909877	0.911492	0.913085	0.914657	0.916207	0.917736
1.4	0.919243	0.920730	0.922196	0.923641	0.925066	0.926471	0.927855	0.929219	0.930563	0.931888
1.5	0.933193	0.934478	0.935744	0.936992	0.938220	0.939429	0.940620	0.941792	0.942947	0.944083
1.6	0.945201	0.946301	0.947384	0.948449	0.949497	0.950529	0.951543	0.952540	0.953521	0.954486
1.7	0.955435	0.956367	0.957284	0.958185	0.959071	0.959941	0.960796	0.961636	0.962462	0.963273
1.8	0.964070	0.964852	0.965621	0.966375	0.967116	0.967843	0.968557	0.969258	0.969946	0.970621
1.9	0.971283	0.971933	0.972571	0.973197	0.973810	0.974412	0.975002	0.975581	0.976148	0.976705
2.0	0.977250	0.977784	0.978308	0.978822	0.979325	0.979818	0.980301	0.980774	0.981237	0.981691
2.1	0.982136	0.982571	0.982997	0.983414	0.983823	0.984222	0.984614	0.984997	0.985371	0.985738
2.2	0.986097	0.986447	0.986791	0.987126	0.987455	0.987776	0.988089	0.988396	0.988696	0.988989
2.3	0.989276	0.989556	0.989830	0.990097	0.990358	0.990613	0.990863	0.991106	0.991344	0.991576
2.4	0.991802	0.992024	0.992240	0.992451	0.992656	0.992857	0.993053	0.993244	0.993431	0.993613
2.5	0.993790	0.993963	0.994132	0.994297	0.994457	0.994614	0.994766	0.994915	0.995060	0.995201
2.6	0.995339	0.995473	0.995604	0.995731	0.995855	0.995975	0.996093	0.996207	0.996319	0.996427
2.7	0.996533	0.996636	0.996736	0.996833	0.996928	0.997020	0.997110	0.997197	0.997282	0.997365
2.8	0.997445	0.997523	0.997599	0.997673	0.997744	0.997814	0.997882	0.997948	0.998012	0.998074
2.9	0.998134	0.998193	0.998250	0.998305	0.998359	0.998411	0.998462	0.998511	0.998559	0.998605
3.0	0.998650	0.998694	0.998736	0.998777	0.998817	0.998856	0.998893	0.998930	0.998965	0.998999
3.1	0.999032	0.999065	0.999096	0.999126	0.999155	0.999184	0.999211	0.999238	0.999264	0.999289
3.2	0.999313	0.999336	0.999359	0.999381	0.999402	0.999423	0.999443	0.999462	0.999481	0.999499
3.3	0.999517	0.999533	0.999550	0.999566	0.999581	0.999596	0.999610	0.999624	0.999638	0.999650
3.4	0.999663	0.999675	0.999687	0.999698	0.999709	0.999720	0.999730	0.999740	0.999749	0.999758
3.5	0.999767	0.999776	0.999784	0.999792	0.999800	0.999807	0.999815	0.999821	0.999828	0.999835
3.6	0.999841	0.999847	0.999853	0.999858	0.999864	0.999869	0.999874	0.999879	0.999883	0.999888
3.7	0.999892	0.999896	0.999900	0.999904	0.999908	0.999912	0.999915	0.999918	0.999922	0.999925
3.8	0.999928	0.999931	0.999933	0.999936	0.999938	0.999941	0.999943	0.999946	0.999948	0.999950
3.9	0.999952	0.999954	0.999956	0.999958	0.999959	0.999961	0.999963	0.999964	0.999966	0.999967
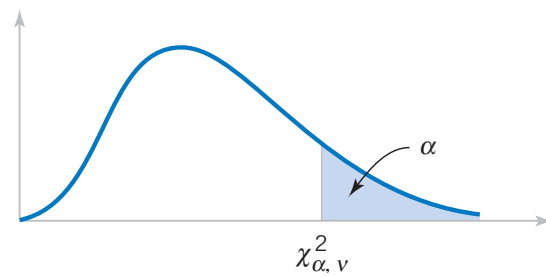
Table IV   Percentage Points $\chi^2_{\alpha,\nu}$ of the Chi-Squared Distribution

ν \ α	.995	.990	.975	.950	.900	.500	.100	.050	.025	.010	.005
1	.00+	.00+	.00+	.00+	.02	.45	2.71	3.84	5.02	6.63	7.88
2	.01	.02	.05	.10	.21	1.39	4.61	5.99	7.38	9.21	10.60
3	.07	.11	.22	.35	.58	2.37	6.25	7.81	9.35	11.34	12.84
4	.21	.30	.48	.71	1.06	3.36	7.78	9.49	11.14	13.28	14.86
5	.41	.55	.83	1.15	1.61	4.35	9.24	11.07	12.83	15.09	16.75
6	.68	.87	1.24	1.64	2.20	5.35	10.65	12.59	14.45	16.81	18.55
7	.99	1.24	1.69	2.17	2.83	6.35	12.02	14.07	16.01	18.48	20.28
8	1.34	1.65	2.18	2.73	3.49	7.34	13.36	15.51	17.53	20.09	21.96
9	1.73	2.09	2.70	3.33	4.17	8.34	14.68	16.92	19.02	21.67	23.59
10	2.16	2.56	3.25	3.94	4.87	9.34	15.99	18.31	20.48	23.21	25.19
11	2.60	3.05	3.82	4.57	5.58	10.34	17.28	19.68	21.92	24.72	26.76
12	3.07	3.57	4.40	5.23	6.30	11.34	18.55	21.03	23.34	26.22	28.30
13	3.57	4.11	5.01	5.89	7.04	12.34	19.81	22.36	24.74	27.69	29.82
14	4.07	4.66	5.63	6.57	7.79	13.34	21.06	23.68	26.12	29.14	31.32
15	4.60	5.23	6.27	7.26	8.55	14.34	22.31	25.00	27.49	30.58	32.80
16	5.14	5.81	6.91	7.96	9.31	15.34	23.54	26.30	28.85	32.00	34.27
17	5.70	6.41	7.56	8.67	10.09	16.34	24.77	27.59	30.19	33.41	35.72
18	6.26	7.01	8.23	9.39	10.87	17.34	25.99	28.87	31.53	34.81	37.16
19	6.84	7.63	8.91	10.12	11.65	18.34	27.20	30.14	32.85	36.19	38.58
20	7.43	8.26	9.59	10.85	12.44	19.34	28.41	31.41	34.17	37.57	40.00
21	8.03	8.90	10.28	11.59	13.24	20.34	29.62	32.67	35.48	38.93	41.40
22	8.64	9.54	10.98	12.34	14.04	21.34	30.81	33.92	36.78	40.29	42.80
23	9.26	10.20	11.69	13.09	14.85	22.34	32.01	35.17	38.08	41.64	44.18
24	9.89	10.86	12.40	13.85	15.66	23.34	33.20	36.42	39.36	42.98	45.56
25	10.52	11.52	13.12	14.61	16.47	24.34	34.28	37.65	40.65	44.31	46.93
26	11.16	12.20	13.84	15.38	17.29	25.34	35.56	38.89	41.92	45.64	48.29
27	11.81	12.88	14.57	16.15	18.11	26.34	36.74	40.11	43.19	46.96	49.65
28	12.46	13.57	15.31	16.93	18.94	27.34	37.92	41.34	44.46	48.28	50.99
29	13.12	14.26	16.05	17.71	19.77	28.34	39.09	42.56	45.72	49.59	52.34
30	13.79	14.95	16.79	18.49	20.60	29.34	40.26	43.77	46.98	50.89	53.67
40	20.71	22.16	24.43	26.51	29.05	39.34	51.81	55.76	59.34	63.69	66.77
50	27.99	29.71	32.36	34.76	37.69	49.33	63.17	67.50	71.42	76.15	79.49
60	35.53	37.48	40.48	43.19	46.46	59.33	74.40	79.08	83.30	88.38	91.95
70	43.28	45.44	48.76	51.74	55.33	69.33	85.53	90.53	95.02	100.42	104.22
80	51.17	53.54	57.15	60.39	64.28	79.33	96.58	101.88	106.63	112.33	116.32
90	59.20	61.75	65.65	69.13	73.29	89.33	107.57	113.14	118.14	124.12	128.30
100	67.33	70.06	74.22	77.93	82.36	99.33	118.50	124.34	129.56	135.81	140.17

ν = degrees of freedom.

Midterm2_Practice

Jie Wang

3/13/2024

Solution to Question 6

```r
x <- c(94, 90, 92, 91, 91, 86, 89, 91, 91, 80)
sample_mean <- mean(x)
Median <- median(x)
range <- max(x) - min(x)
sample_var  <- var(x)
sample_std  <- sqrt(sample_var)
sample_quantile <- quantile(x, probs=c(0.25, 0.75))

print(sample_mean)
```

```
## [1] 89.5
```

```r
print(Median)
```

```
## [1] 91
```

```r
print(range)
```

```
## [1] 14
```

```r
print(sample_var)
```

```
## [1] 15.38889
```

```r
print(sample_std)
```

```
## [1] 3.922867
```

```r
print(sample_quantile[1])
```

```
##    25%
## 89.25
```

```r
print(sample_quantile[2])
```

```
## 75%
##  91
```

```r
print(sample_quantile[2] - sample_quantile[1])
```

```
##  75%
## 1.75
```

```r
x <- c(94, 90, 92, 91, 91, 86, 89, 91, 91, 80)
boxplot(x, main = "Box Plot", ylab = "Values", col = "skyblue", horizontal = TRUE)
```

Box Plot
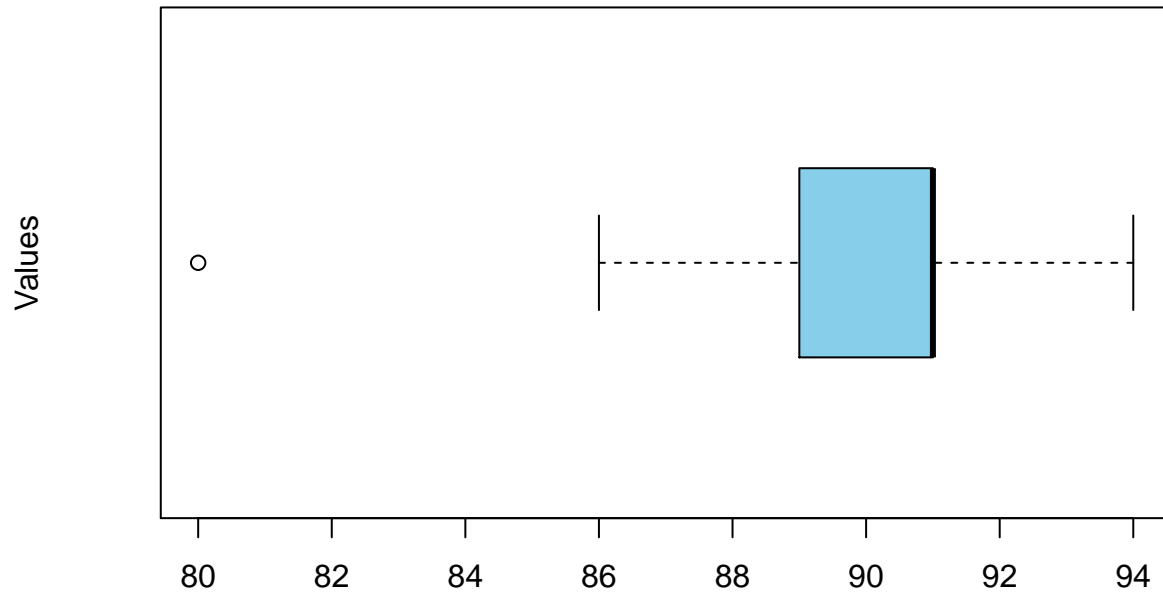
Solution to Question 7

```
sd_sample_mean = sqrt(27^2/40)
pnorm(170, mean=167, sd=sd_sample_mean) - pnorm(163, mean=167, sd=sd_sample_mean)

## [1] 0.5845008
```