Georgia Institute of Technology

ISyE3770 - Midterm 1

Instructor: Jie Wang

2024/02/27 (03:30PM - 04:45PM)

Name: _____

GT Student ID: _____

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1. This midterm **1** contains 9 pages (including this cover page) and 9 questions. Total of points is 100.

2. In taking this examination, you are expected to adhere to the GT academic honor code. At a minimum, this requires that you utilize only the materials supplied to you and that you do not give help to, or accept help from, others.

Distribution of Marks

Question	Points	Score
1	10	
2	10	
3	10	
4	10	
5	10	
6	10	
7	10	
8	10	
9	20	
Total:	100	

Multiple Choice (80 points, 10 points each)
Remark: for each question, one and only one of four given choices (A, B, C, and D) is correct.

1. (10 points) In an urn are 7 blue and 3 red marbles. If you draw 3 marbles without replacement, what is the probability that less than 2 will be red?

     A. $\frac{\binom{7}{3}}{\binom{10}{3}}$

     B. $\frac{\binom{7}{3}+\binom{3}{1}\binom{7}{2}}{\binom{10}{3}}$

     C. $\frac{\binom{7}{3}+\binom{3}{2}\binom{7}{1}}{\binom{10}{3}}$

     D. $\frac{\binom{3}{1}\binom{7}{2}}{\binom{10}{3}}$

                                                      1. —————

2. (10 points) The random variable X has the probability density function

$$f(x) = \begin{cases} cx(1-x), & \text{if } 0 < x < 1, \\ 0, & \text{otherwise.} \end{cases}$$

Find $P(\frac{\sqrt{3}}{3} \leq X \leq \frac{5}{4})$.
A. $1 - \frac{2\sqrt{3}}{9}$ B. $\frac{2\sqrt{3}}{9}$ C. 1   D. 0

                                                      2. —————

3. (10 points) A telephone company employs 5 operators who receive requests independently of one another. The number of the requests received by each operator has a Poisson distribution, and each operator receives on average 1 request every 30 minutes. What is the probability that during a given 2 hour period, exactly 4 of the 5 operators receive no requests?
(Hint: The rate of the Poisson distribution at each operator equals the average number of requests during a given 2 hour period.)

     A. $5(e^{-8} - e^{-10})$

     B. $4(e^{-10} - e^{-16})$

     C. $5(e^{-16} - e^{-20})$

     D. $5(e^{-10} - e^{-16})$

                                                       3. —————

4. (10 points) Find standard deviation of X if X has a Bernoulli distribution such that $2\mathbb{P}(X = 0) = \mathbb{P}(X = 1)$.

    A. $\frac{2}{9}$ B. $\frac{\sqrt{2}}{3}$ C. $\frac{1}{9}$ D. $\frac{1}{3}$

4. _____

5. (10 points) Assume X has moment generating function

$$M_X(t) = e^{16(e^t - 1)}$$

and Y has moment generating function

$$M_Y(t) = \frac{1}{(1 - 2t)^{1/2}}, \quad t < \frac{1}{2}$$

Statement 1: X has standard deviation of 4.
Statement 2: the largest value of d satisfying $P(Y < d) = 0.95$ is 3.84.
Statement 3: $P(X = 0) = e^{-12}$.
How many statements are correct?
    A. 0   B. 1   C. 2   D. 3

5. _____

6. (10 points) Let A and B be two events. Suppose $P(A) = 0.4, P(B) = 0.5$.
Statement 1: $P(A \cap B') = 0.2$ if $P(A \cap B) = 0.2$.
Statement 2: $P(A \mid B) = 0.4$ if A and B are mutually exclusive.
Statement 3: $P(A \cup B) = 0.7$ if A and B are independent.
How many statements are correct?
    A. 0   B. 1   C. 2   D. 3

6. _____

7. (10 points) Suppose that $X \sim N(0, 1)$, find the largest c such that $P(|X| \le c) \le 0.8064$.
    A. 0.8064   B. 1.3000   C. 0.8647   D. 1.4000

7. _____

8. (10 points) Time (in hours) required to repair a machine follows an exponential distribution with parameter $\theta = 2$. What is the probability that a repair needs more than 3 hours?

      A. $e^{-3/2}$

      B. $1 - e^{-3/2}$

      C. $1 - e^{-2/3}$

      D. $e^{-2/3}$

8. _____

Regular Question (20 points)

9. (20 points) Let X and Y be two random variables with the joint pdf

$$f(x,y) = \frac{xy}{8}, 0 \le x \le 4, 1 \le y \le c,$$

where c is a constant to be determined.

   (i) Find the constant c.   (10 points)

   (ii) Find the marginal probability density function of X.   (5 points)

  (iii) Find the marginal probability density function of Y.   (5 points)

Discrete Distributions

Bernoulli $\quad f(x) = p^x(1-p)^{1-x}, \qquad x = 0, 1$

$0 < p < 1$ $\quad M(t) = 1 - p + pe^t, \qquad -\infty < t < \infty$

$\qquad\qquad \mu = p, \qquad \sigma^2 = p(1-p)$

Binomial $\quad f(x) = \dfrac{n!}{x!(n-x)!} p^x(1-p)^{n-x}, \qquad x = 0, 1, 2, \ldots, n$

$b(n, p)$

$0 < p < 1$ $\quad M(t) = (1 - p + pe^t)^n, \qquad -\infty < t < \infty$

$\qquad\qquad \mu = np, \qquad \sigma^2 = np(1-p)$

Geometric $\quad f(x) = (1-p)^{x-1}p, \qquad x = 1, 2, 3, \ldots$

$0 < p < 1$ $\quad M(t) = \dfrac{pe^t}{1 - (1-p)e^t}, \qquad t < -\ln(1-p)$

$\qquad\qquad \mu = \dfrac{1}{p}, \qquad \sigma^2 = \dfrac{1-p}{p^2}$

$$\overline{\qquad\qquad}$$

$$\overline{\quad} \qquad \overline{\quad}\ \overline{\quad}$$

Negative Binomial $\quad f(x) = \dbinom{x-1}{r-1} p^r(1-p)^{x-r}, \qquad x = r, r+1, r+2, \ldots$

$0 < p < 1$

$r = 1, 2, 3, \ldots$ $\quad M(t) = \dfrac{(pe^t)^r}{[1 - (1-p)e^t]^r}, \qquad t < -\ln(1-p)$

$\qquad\qquad \mu = r\left(\dfrac{1}{p}\right), \qquad \sigma^2 = \dfrac{r(1-p)}{p^2}$

Poisson $\quad f(x) = \dfrac{\lambda^x e^{-\lambda}}{x!}, \qquad x = 0, 1, 2, \ldots$

$\lambda > 0$

$\qquad\qquad M(t) = e^{\lambda(e^t - 1)}, \qquad -\infty < t < \infty$

$\qquad\qquad \mu = \lambda, \qquad \sigma^2 = \lambda$

Uniform $\quad f(x) = \dfrac{1}{m}, \qquad x = 1, 2, \ldots, m$

$m > 0$

$\qquad\qquad \mu = \dfrac{m+1}{2}, \qquad \sigma^2 = \dfrac{m^2 - 1}{12}$

Continuous Distributions

Beta
$\alpha > 0$
$\beta > 0$

$$f(x) = \frac{\Gamma(\alpha + \beta)}{\Gamma(\alpha)\Gamma(\beta)} x^{\alpha-1}(1-x)^{\beta-1}, \qquad 0 < x < 1$$

$$\mu = \frac{\alpha}{\alpha + \beta}, \qquad \sigma^2 = \frac{\alpha\beta}{(\alpha + \beta + 1)(\alpha + \beta)^2}$$

Chi-square
$\chi^2(r)$
$r = 1, 2, \ldots$

$$f(x) = \frac{1}{\Gamma(r/2)2^{r/2}} x^{r/2-1}e^{-x/2}, \qquad 0 < x < \infty$$

$$M(t) = \frac{1}{(1 - 2t)^{r/2}}, \qquad t < \frac{1}{2}$$

$$\mu = r, \qquad \sigma^2 = 2r$$

Exponential
$\theta > 0$

$$f(x) = \frac{1}{\theta} e^{-x/\theta}, \qquad 0 \le x < \infty$$

$$M(t) = \frac{1}{1 - \theta t}, \qquad t < \frac{1}{\theta}$$

$$\mu = \theta, \qquad \sigma^2 = \theta^2$$

Gamma
$\alpha > 0$
$\theta > 0$

$$f(x) = \frac{1}{\Gamma(\alpha)\theta^\alpha} x^{\alpha-1}e^{-x/\theta}, \qquad 0 < x < \infty$$

$$M(t) = \frac{1}{(1 - \theta t)^\alpha}, \qquad t < \frac{1}{\theta}$$

$$\mu = \alpha\theta, \qquad \sigma^2 = \alpha\theta^2$$

Normal
$N(\mu, \sigma^2)$
$-\infty < \mu < \infty$
$\sigma > 0$

$$f(x) = \frac{1}{\sigma\sqrt{2\pi}} e^{-(x-\mu)^2/2\sigma^2}, \qquad -\infty < x < \infty$$

$$M(t) = e^{\mu t + \sigma^2 t^2/2}, \qquad -\infty < t < \infty$$

$$E(X) = \mu, \qquad \mathrm{Var}(X) = \sigma^2$$

Uniform
$U(a, b)$
$-\infty < a < b < \infty$

$$f(x) = \frac{1}{b - a}, \qquad a \le x \le b$$

$$M(t) = \frac{e^{tb} - e^{ta}}{t(b - a)}, \qquad t \ne 0; \qquad M(0) = 1$$

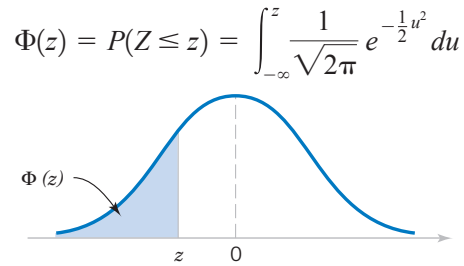$$\mu = \frac{a + b}{2}, \qquad \sigma^2 = \frac{(b - a)^2}{12}$$

$$\Phi(z) = P(Z \le z) = \int_{-\infty}^{z} \frac{1}{\sqrt{2\pi}} e^{-\frac{1}{2}u^2} du$$

Table III   Cumulative Standard Normal Distribution

z	−0.09	−0.08	−0.07	−0.06	−0.05	−0.04	−0.03	−0.02	−0.01	−0.00
−3.9	0.000033	0.000034	0.000036	0.000037	0.000039	0.000041	0.000042	0.000044	0.000046	0.000048
−3.8	0.000050	0.000052	0.000054	0.000057	0.000059	0.000062	0.000064	0.000067	0.000069	0.000072
−3.7	0.000075	0.000078	0.000082	0.000085	0.000088	0.000092	0.000096	0.000100	0.000104	0.000108
−3.6	0.000112	0.000117	0.000121	0.000126	0.000131	0.000136	0.000142	0.000147	0.000153	0.000159
−3.5	0.000165	0.000172	0.000179	0.000185	0.000193	0.000200	0.000208	0.000216	0.000224	0.000233
−3.4	0.000242	0.000251	0.000260	0.000270	0.000280	0.000291	0.000302	0.000313	0.000325	0.000337
−3.3	0.000350	0.000362	0.000376	0.000390	0.000404	0.000419	0.000434	0.000450	0.000467	0.000483
−3.2	0.000501	0.000519	0.000538	0.000557	0.000577	0.000598	0.000619	0.000641	0.000664	0.000687
−3.1	0.000711	0.000736	0.000762	0.000789	0.000816	0.000845	0.000874	0.000904	0.000935	0.000968
−3.0	0.001001	0.001035	0.001070	0.001107	0.001144	0.001183	0.001223	0.001264	0.001306	0.001350
−2.9	0.001395	0.001441	0.001489	0.001538	0.001589	0.001641	0.001695	0.001750	0.001807	0.001866
−2.8	0.001926	0.001988	0.002052	0.002118	0.002186	0.002256	0.002327	0.002401	0.002477	0.002555
−2.7	0.002635	0.002718	0.002803	0.002890	0.002980	0.003072	0.003167	0.003264	0.003364	0.003467
−2.6	0.003573	0.003681	0.003793	0.003907	0.004025	0.004145	0.004269	0.004396	0.004527	0.004661
−2.5	0.004799	0.004940	0.005085	0.005234	0.005386	0.005543	0.005703	0.005868	0.006037	0.006210
−2.4	0.006387	0.006569	0.006756	0.006947	0.007143	0.007344	0.007549	0.007760	0.007976	0.008198
−2.3	0.008424	0.008656	0.008894	0.009137	0.009387	0.009642	0.009903	0.010170	0.010444	0.010724
−2.2	0.011011	0.011304	0.011604	0.011911	0.012224	0.012545	0.012874	0.013209	0.013553	0.013903
−2.1	0.014262	0.014629	0.015003	0.015386	0.015778	0.016177	0.016586	0.017003	0.017429	0.017864
−2.0	0.018309	0.018763	0.019226	0.019699	0.020182	0.020675	0.021178	0.021692	0.022216	0.022750
−1.9	0.023295	0.023852	0.024419	0.024998	0.025588	0.026190	0.026803	0.027429	0.028067	0.028717
−1.8	0.029379	0.030054	0.030742	0.031443	0.032157	0.032884	0.033625	0.034379	0.035148	0.035930
−1.7	0.036727	0.037538	0.038364	0.039204	0.040059	0.040929	0.041815	0.042716	0.043633	0.044565
−1.6	0.045514	0.046479	0.047460	0.048457	0.049471	0.050503	0.051551	0.052616	0.053699	0.054799
−1.5	0.055917	0.057053	0.058208	0.059380	0.060571	0.061780	0.063008	0.064256	0.065522	0.066807
−1.4	0.068112	0.069437	0.070781	0.072145	0.073529	0.074934	0.076359	0.077804	0.079270	0.080757
−1.3	0.082264	0.083793	0.085343	0.086915	0.088508	0.090123	0.091759	0.093418	0.095098	0.096801
−1.2	0.098525	0.100273	0.102042	0.103835	0.105650	0.107488	0.109349	0.111233	0.113140	0.115070
−1.1	0.117023	0.119000	0.121001	0.123024	0.125072	0.127143	0.129238	0.131357	0.133500	0.135666
−1.0	0.137857	0.140071	0.142310	0.144572	0.146859	0.149170	0.151505	0.153864	0.156248	0.158655
−0.9	0.161087	0.163543	0.166023	0.168528	0.171056	0.173609	0.176185	0.178786	0.181411	0.184060
−0.8	0.186733	0.189430	0.192150	0.194894	0.197662	0.200454	0.203269	0.206108	0.208970	0.211855
−0.7	0.214764	0.217695	0.220650	0.223627	0.226627	0.229650	0.232695	0.235762	0.238852	0.241964
−0.6	0.245097	0.248252	0.251429	0.254627	0.257846	0.261086	0.264347	0.267629	0.270931	0.274253
−0.5	0.277595	0.280957	0.284339	0.287740	0.291160	0.294599	0.298056	0.301532	0.305026	0.308538
−0.4	0.312067	0.315614	0.319178	0.322758	0.326355	0.329969	0.333598	0.337243	0.340903	0.344578
−0.3	0.348268	0.351973	0.355691	0.359424	0.363169	0.366928	0.370700	0.374484	0.378281	0.382089
−0.2	0.385908	0.389739	0.393580	0.397432	0.401294	0.405165	0.409046	0.412936	0.416834	0.420740
−0.1	0.424655	0.428576	0.432505	0.436441	0.440382	0.444330	0.448283	0.452242	0.456205	0.460172
0.0	0.464144	0.468119	0.472097	0.476078	0.480061	0.484047	0.488033	0.492022	0.496011	0.500000

$$\Phi(z) = P(Z \le z) = \int_{-\infty}^{z} \frac{1}{\sqrt{2\pi}} e^{-\frac{1}{2}u^2} du$$
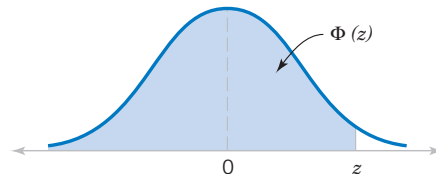
Table III   Cumulative Standard Normal Distribution (*continued*)

z	0.00	0.01	0.02	0.03	0.04	0.05	0.06	0.07	0.08	0.09
0.0	0.500000	0.503989	0.507978	0.511967	0.515953	0.519939	0.532922	0.527903	0.531881	0.535856
0.1	0.539828	0.543795	0.547758	0.551717	0.555760	0.559618	0.563559	0.567495	0.571424	0.575345
0.2	0.579260	0.583166	0.587064	0.590954	0.594835	0.598706	0.602568	0.606420	0.610261	0.614092
0.3	0.617911	0.621719	0.625516	0.629300	0.633072	0.636831	0.640576	0.644309	0.648027	0.651732
0.4	0.655422	0.659097	0.662757	0.666402	0.670031	0.673645	0.677242	0.680822	0.684386	0.687933
0.5	0.691462	0.694974	0.698468	0.701944	0.705401	0.708840	0.712260	0.715661	0.719043	0.722405
0.6	0.725747	0.729069	0.732371	0.735653	0.738914	0.742154	0.745373	0.748571	0.751748	0.754903
0.7	0.758036	0.761148	0.764238	0.767305	0.770350	0.773373	0.776373	0.779350	0.782305	0.785236
0.8	0.788145	0.791030	0.793892	0.796731	0.799546	0.802338	0.805106	0.807850	0.810570	0.813267
0.9	0.815940	0.818589	0.821214	0.823815	0.826391	0.828944	0.831472	0.833977	0.836457	0.838913
1.0	0.841345	0.843752	0.846136	0.848495	0.850830	0.853141	0.855428	0.857690	0.859929	0.862143
1.1	0.864334	0.866500	0.868643	0.870762	0.872857	0.874928	0.876976	0.878999	0.881000	0.882977
1.2	0.884930	0.886860	0.888767	0.890651	0.892512	0.894350	0.896165	0.897958	0.899727	0.901475
1.3	0.903199	0.904902	0.906582	0.908241	0.909877	0.911492	0.913085	0.914657	0.916207	0.917736
1.4	0.919243	0.920730	0.922196	0.923641	0.925066	0.926471	0.927855	0.929219	0.930563	0.931888
1.5	0.933193	0.934478	0.935744	0.936992	0.938220	0.939429	0.940620	0.941792	0.942947	0.944083
1.6	0.945201	0.946301	0.947384	0.948449	0.949497	0.950529	0.951543	0.952540	0.953521	0.954486
1.7	0.955435	0.956367	0.957284	0.958185	0.959071	0.959941	0.960796	0.961636	0.962462	0.963273
1.8	0.964070	0.964852	0.965621	0.966375	0.967116	0.967843	0.968557	0.969258	0.969946	0.970621
1.9	0.971283	0.971933	0.972571	0.973197	0.973810	0.974412	0.975002	0.975581	0.976148	0.976705
2.0	0.977250	0.977784	0.978308	0.978822	0.979325	0.979818	0.980301	0.980774	0.981237	0.981691
2.1	0.982136	0.982571	0.982997	0.983414	0.983823	0.984222	0.984614	0.984997	0.985371	0.985738
2.2	0.986097	0.986447	0.986791	0.987126	0.987455	0.987776	0.988089	0.988396	0.988696	0.988989
2.3	0.989276	0.989556	0.989830	0.990097	0.990358	0.990613	0.990863	0.991106	0.991344	0.991576
2.4	0.991802	0.992024	0.992240	0.992451	0.992656	0.992857	0.993053	0.993244	0.993431	0.993613
2.5	0.993790	0.993963	0.994132	0.994297	0.994457	0.994614	0.994766	0.994915	0.995060	0.995201
2.6	0.995339	0.995473	0.995604	0.995731	0.995855	0.995975	0.996093	0.996207	0.996319	0.996427
2.7	0.996533	0.996636	0.996736	0.996833	0.996928	0.997020	0.997110	0.997197	0.997282	0.997365
2.8	0.997445	0.997523	0.997599	0.997673	0.997744	0.997814	0.997882	0.997948	0.998012	0.998074
2.9	0.998134	0.998193	0.998250	0.998305	0.998359	0.998411	0.998462	0.998511	0.998559	0.998605
3.0	0.998650	0.998694	0.998736	0.998777	0.998817	0.998856	0.998893	0.998930	0.998965	0.998999
3.1	0.999032	0.999065	0.999096	0.999126	0.999155	0.999184	0.999211	0.999238	0.999264	0.999289
3.2	0.999313	0.999336	0.999359	0.999381	0.999402	0.999423	0.999443	0.999462	0.999481	0.999499
3.3	0.999517	0.999533	0.999550	0.999566	0.999581	0.999596	0.999610	0.999624	0.999638	0.999650
3.4	0.999663	0.999675	0.999687	0.999698	0.999709	0.999720	0.999730	0.999740	0.999749	0.999758
3.5	0.999767	0.999776	0.999784	0.999792	0.999800	0.999807	0.999815	0.999821	0.999828	0.999835
3.6	0.999841	0.999847	0.999853	0.999858	0.999864	0.999869	0.999874	0.999879	0.999883	0.999888
3.7	0.999892	0.999896	0.999900	0.999904	0.999908	0.999912	0.999915	0.999918	0.999922	0.999925
3.8	0.999928	0.999931	0.999933	0.999936	0.999938	0.999941	0.999943	0.999946	0.999948	0.999950
3.9	0.999952	0.999954	0.999956	0.999958	0.999959	0.999961	0.999963	0.999964	0.999966	0.999967
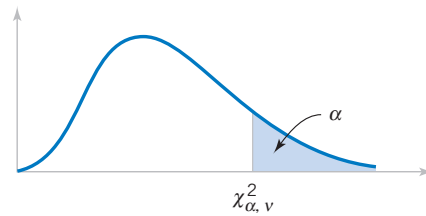
Table IV   Percentage Points $\chi^2_{\alpha,v}$ of the Chi-Squared Distribution

α v	.995	.990	.975	.950	.900	.500	.100	.050	.025	.010	.005
1	.00+	.00+	.00+	.00+	.02	.45	2.71	3.84	5.02	6.63	7.88
2	.01	.02	.05	.10	.21	1.39	4.61	5.99	7.38	9.21	10.60
3	.07	.11	.22	.35	.58	2.37	6.25	7.81	9.35	11.34	12.84
4	.21	.30	.48	.71	1.06	3.36	7.78	9.49	11.14	13.28	14.86
5	.41	.55	.83	1.15	1.61	4.35	9.24	11.07	12.83	15.09	16.75
6	.68	.87	1.24	1.64	2.20	5.35	10.65	12.59	14.45	16.81	18.55
7	.99	1.24	1.69	2.17	2.83	6.35	12.02	14.07	16.01	18.48	20.28
8	1.34	1.65	2.18	2.73	3.49	7.34	13.36	15.51	17.53	20.09	21.96
9	1.73	2.09	2.70	3.33	4.17	8.34	14.68	16.92	19.02	21.67	23.59
10	2.16	2.56	3.25	3.94	4.87	9.34	15.99	18.31	20.48	23.21	25.19
11	2.60	3.05	3.82	4.57	5.58	10.34	17.28	19.68	21.92	24.72	26.76
12	3.07	3.57	4.40	5.23	6.30	11.34	18.55	21.03	23.34	26.22	28.30
13	3.57	4.11	5.01	5.89	7.04	12.34	19.81	22.36	24.74	27.69	29.82
14	4.07	4.66	5.63	6.57	7.79	13.34	21.06	23.68	26.12	29.14	31.32
15	4.60	5.23	6.27	7.26	8.55	14.34	22.31	25.00	27.49	30.58	32.80
16	5.14	5.81	6.91	7.96	9.31	15.34	23.54	26.30	28.85	32.00	34.27
17	5.70	6.41	7.56	8.67	10.09	16.34	24.77	27.59	30.19	33.41	35.72
18	6.26	7.01	8.23	9.39	10.87	17.34	25.99	28.87	31.53	34.81	37.16
19	6.84	7.63	8.91	10.12	11.65	18.34	27.20	30.14	32.85	36.19	38.58
20	7.43	8.26	9.59	10.85	12.44	19.34	28.41	31.41	34.17	37.57	40.00
21	8.03	8.90	10.28	11.59	13.24	20.34	29.62	32.67	35.48	38.93	41.40
22	8.64	9.54	10.98	12.34	14.04	21.34	30.81	33.92	36.78	40.29	42.80
23	9.26	10.20	11.69	13.09	14.85	22.34	32.01	35.17	38.08	41.64	44.18
24	9.89	10.86	12.40	13.85	15.66	23.34	33.20	36.42	39.36	42.98	45.56
25	10.52	11.52	13.12	14.61	16.47	24.34	34.28	37.65	40.65	44.31	46.93
26	11.16	12.20	13.84	15.38	17.29	25.34	35.56	38.89	41.92	45.64	48.29
27	11.81	12.88	14.57	16.15	18.11	26.34	36.74	40.11	43.19	46.96	49.65
28	12.46	13.57	15.31	16.93	18.94	27.34	37.92	41.34	44.46	48.28	50.99
29	13.12	14.26	16.05	17.71	19.77	28.34	39.09	42.56	45.72	49.59	52.34
30	13.79	14.95	16.79	18.49	20.60	29.34	40.26	43.77	46.98	50.89	53.67
40	20.71	22.16	24.43	26.51	29.05	39.34	51.81	55.76	59.34	63.69	66.77
50	27.99	29.71	32.36	34.76	37.69	49.33	63.17	67.50	71.42	76.15	79.49
60	35.53	37.48	40.48	43.19	46.46	59.33	74.40	79.08	83.30	88.38	91.95
70	43.28	45.44	48.76	51.74	55.33	69.33	85.53	90.53	95.02	100.42	104.22
80	51.17	53.54	57.15	60.39	64.28	79.33	96.58	101.88	106.63	112.33	116.32
90	59.20	61.75	65.65	69.13	73.29	89.33	107.57	113.14	118.14	124.12	128.30
100	67.33	70.06	74.22	77.93	82.36	99.33	118.50	124.34	129.56	135.81	140.17

v = degrees of freedom.