

# Georgia Institute of Technology

## ISyE3770 - Final Exam Practice

Instructor: Jie Wang

2024/04/30 (02:40PM - 05:30PM)

Name: \_\_\_\_\_

GT Student ID: \_\_\_\_\_

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1. This final exam practice contains 14 pages (including this cover page) and 9 questions. Total of points is 100.
  2. In taking this examination, you are expected to adhere to the GT academic honor code. At a minimum, this requires that you utilize only the materials supplied to you and that you do not give help to, or accept help from, others.

### Distribution of Marks

Question	Points	Score
1	8	
2	4	
3	4	
4	12	
5	6	
6	22	
7	12	
8	20	
9	12	
Total:	100	

**Multiple Choice** (12 points)

**Remark:** for each question, one and only one of four given choices (A, B, C, and D) is correct.

1. A textile fiber manufacturer is investigating a new drapery yarn, which the company claims has a mean thread elongation of 12 kilograms with a standard deviation of 0.5 kilograms. The company wishes to test the hypothesis  $H_0 : \mu = 12$  against  $H_1 : \mu < 12$ , using the random sample of four specimens. Use sample mean  $\bar{x}$  as the test statistic. Assume the critical region is defined as  $\bar{x} < 11.5$  kilogram.

(a) (4 points) Which one below corresponds to the Type-I error of the test:

- A.  $\mathbb{P}\{\bar{X} < 11.5 | \mu = 12\}$
- B.  $\mathbb{P}\{\bar{X} < 11.5 | \mu < 12\}$
- C.  $\mathbb{P}\{\bar{X} > 11.5 | \mu = 12\}$
- D. None above.

(a) \_\_\_\_\_

(b) (4 points) Assuming the true mean is 11.25 kilograms. Which one below corresponds to the power of the test for this assumed parameter value:

- A.  $\mathbb{P}\{\bar{X} < 11.5 | \mu = 11.25\}$
- B.  $\mathbb{P}\{\bar{X} < 11.5 | \mu = 12\}$
- C.  $\mathbb{P}\{\bar{X} > 11.5 | \mu = 11.25\}$
- D. None above.

(b) \_\_\_\_\_

2. (4 points) Color blindness appears in the 1% of the people in a certain population. What is the lower bound on the number of observations from a random sample such that, the probability of containing at least one color-blinded person is 0.95?

A. 299   B. 199   C. 399   D. 499

2. \_\_\_\_\_

3. (4 points) Which of the following **CANNOT** be used to describe the data variability?

A. Sample Range   B. Sample Mean   C. Sample Variance   D. Sample IQR

3. \_\_\_\_\_

**Regular Question** (88 points)

4. Let  $X$  equal the weight in grams of a miniature candy bar. Assume that  $\mu = \mathbb{E}[X] = 24.43$  and  $\sigma^2 = \text{Var}(X) = 2.20$ . Let  $\bar{X}$  be the sample mean of a random sample of  $n = 40$  candy bars. Find
- (a) (4 points)  $\mathbb{E}[\bar{X}]$
  - (b) (4 points)  $\text{Var}(\bar{X})$
  - (c) (4 points)  $\mathbb{P}(24.17 \leq \bar{X} \leq 24.82)$ . approximately.

5. Roll a fair four-sided die twice. Let  $X$  be the outcome on the first roll, and  $Y$  be the sum of the two rolls. Calculate
- (a) (1 point)  $\mu_X = \mathbb{E}[X]$
  - (b) (1 point)  $\mu_Y = \mathbb{E}[Y]$
  - (c) (1 point)  $\sigma_X^2 = \text{Var}(X)$
  - (d) (1 point)  $\sigma_Y^2 = \text{Var}(Y)$
  - (e) (1 point)  $\text{Cov}(X, Y)$
  - (f) (1 point)  $\rho(X, Y)$

6. We investigate the problem of whether people gain weight as they age or not using data. We have  $n = 250$  observations on the age ( $x$ ) and weight ( $y$ ). The linear regression output is attached below:

```
> model <- lm(weight~age)
> summary(model)
```

Call:  
lm(formula = weight ~ age)

Residuals:

Min	1Q	Median	3Q	Max
-70.538	-14.853	0.268	17.469	65.191

Coefficients:

	Estimate	Std. Error	t value	Pr(> t )
(Intercept)	178.46936	3.41128	52.32	<2e-16 ***
age	0.02196	0.08440	0.26	0.795

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Signif. codes:  
0 '\*\*\*' 0.001 '\*\*' 0.01 '\*' 0.05 '.' 0.1 ' ' 1

Residual standard error: 27.03 on 248 degrees of freedom  
Multiple R-squared: 0.000273, Adjusted R-squared: -0.003758  
F-statistic: 0.06771 on 1 and 248 DF, p-value: 0.7949

- (4 points) Write down the regression model found.
- (4 points) Estimate  $\sigma^2$ .
- (4 points) What percentage of the variability in the data does this model account for?
- (3 points) Use the equation of the fitted line to predict the weight that would be observed, on average, for a man who is 25 years old, and for a man who is 65 years old, respectively.
- (3 points) Suppose that the observed weight of a 25-year-old man is 170lb, and the observed weight of a 65-year-old man is 180lb. Find the residual for these two observations.
- (4 points) What is the interpretation of the observation that the value of  $\Pr(> |t|)$  for the age variable is much larger than 0?

**Solution to Problem 6:**

7. (12 points) A random variable has probability density function

$$f(x; \theta) = \frac{1}{\theta^2} x^{\frac{1-\theta^2}{\theta^2}}, \quad 0 < x < 1, 0 < \theta < \infty.$$

Now, given samples  $X_1, \dots, X_n$ , derive the maximum likelihood estimator for the parameter  $\theta$ .

8. Medical researchers have developed a new artificial heart constructed primarily of titanium and plastic. The heart will last and operate almost indefinitely once it is implanted in the patient's body, but the battery pack needs to be recharged about every four hours. A random sample of 10 battery packs is selected and subject to a life test. The results of these 10 tests are as follows:

4.37, 4.19, 3.93, 4.21, 4.11, 3.84, 4.14, 4.01, 4.16, 4.12.

Assume that battery life is normally distributed with standard deviation  $\sigma = 0.2$  hour.

- (a) (10 points) Is there evidence to support the claim that mean battery life exceeds 4 hours? Use significance level  $\alpha = 0.05$ .
- (b) (10 points) Compute the power of the test if the true mean battery life is 4.2 hours.



9. (12 points) To study the pH of rain in Hall County, Georgia, we collect 39 samples:

5.37, 5.47, 5.38, 4.63, 5.37, 3.74, 3.71, 4.96, 4.64, 5.11, 5.65, 5.55, 4.00, 5.62, 4.57, 4.64, 5.48,  
4.60, 4.54, 4.51, 4.86, 4.56, 4.61, 4.32, 3.98, 5.70, 4.15, 3.98, 5.65, 3.11, 5.03, 4.62, 4.50, 4.35,  
4.16, 4.64, 5.12, 3.71, 4.64.

The sample mean of the pH value is 4.698, and the sample variance of the pH value is 0.3963.  
Find the two-sided 95% confidence interval for the variance of pH.

# Discrete Distributions

## Bernoulli

$$0 < p < 1$$

$$f(x) = p^x(1-p)^{1-x}, \quad x = 0, 1$$

$$M(t) = 1 - p + pe^t, \quad -\infty < t < \infty$$

$$\mu = p, \quad \sigma^2 = p(1-p)$$

## Binomial

$$b(n, p)$$

$$0 < p < 1$$

$$f(x) = \frac{n!}{x!(n-x)!} p^x (1-p)^{n-x}, \quad x = 0, 1, 2, \dots, n$$

$$M(t) = (1 - p + pe^t)^n, \quad -\infty < t < \infty$$

$$\mu = np, \quad \sigma^2 = np(1-p)$$

## Geometric

$$0 < p < 1$$

$$f(x) = (1-p)^{x-1}p, \quad x = 1, 2, 3, \dots$$

$$M(t) = \frac{pe^t}{1 - (1-p)e^t}, \quad t < -\ln(1-p)$$

$$\mu = \frac{1}{p}, \quad \sigma^2 = \frac{1-p}{p^2}$$

## Negative Binomial

$$0 < p < 1$$

$$r = 1, 2, 3, \dots$$

$$f(x) = \binom{x-1}{r-1} p^r (1-p)^{x-r}, \quad x = r, r+1, r+2, \dots$$

$$M(t) = \frac{(pe^t)^r}{[1 - (1-p)e^t]^r}, \quad t < -\ln(1-p)$$

$$\mu = r\left(\frac{1}{p}\right), \quad \sigma^2 = \frac{r(1-p)}{p^2}$$

## Poisson

$$\lambda > 0$$

$$f(x) = \frac{\lambda^x e^{-\lambda}}{x!}, \quad x = 0, 1, 2, \dots$$

$$M(t) = e^{\lambda(e^t-1)}, \quad -\infty < t < \infty$$

$$\mu = \lambda, \quad \sigma^2 = \lambda$$

## Uniform

$$m > 0$$

$$f(x) = \frac{1}{m}, \quad x = 1, 2, \dots, m$$

$$\mu = \frac{m+1}{2}, \quad \sigma^2 = \frac{m^2-1}{12}$$

# Continuous Distributions

## Beta

$$\alpha > 0$$

$$\beta > 0$$

$$f(x) = \frac{\Gamma(\alpha + \beta)}{\Gamma(\alpha)\Gamma(\beta)} x^{\alpha-1}(1-x)^{\beta-1}, \quad 0 < x < 1$$

$$\mu = \frac{\alpha}{\alpha + \beta}, \quad \sigma^2 = \frac{\alpha\beta}{(\alpha + \beta + 1)(\alpha + \beta)^2}$$

## Chi-square

$$\chi^2(r)$$

$$r = 1, 2, \dots$$

$$f(x) = \frac{1}{\Gamma(r/2)2^{r/2}} x^{r/2-1} e^{-x/2}, \quad 0 < x < \infty$$

$$M(t) = \frac{1}{(1-2t)^{r/2}}, \quad t < \frac{1}{2}$$

$$\mu = r, \quad \sigma^2 = 2r$$

## Exponential

$$\theta > 0$$

$$f(x) = \frac{1}{\theta} e^{-x/\theta}, \quad 0 \leq x < \infty$$

$$M(t) = \frac{1}{1-\theta t}, \quad t < \frac{1}{\theta}$$

$$\mu = \theta, \quad \sigma^2 = \theta^2$$

## Gamma

$$\alpha > 0$$

$$\theta > 0$$

$$f(x) = \frac{1}{\Gamma(\alpha)\theta^\alpha} x^{\alpha-1} e^{-x/\theta}, \quad 0 < x < \infty$$

$$M(t) = \frac{1}{(1-\theta t)^\alpha}, \quad t < \frac{1}{\theta}$$

$$\mu = \alpha\theta, \quad \sigma^2 = \alpha\theta^2$$

## Normal

$$N(\mu, \sigma^2)$$

$$-\infty < \mu < \infty$$

$$\sigma > 0$$

$$f(x) = \frac{1}{\sigma\sqrt{2\pi}} e^{-(x-\mu)^2/2\sigma^2}, \quad -\infty < x < \infty$$

$$M(t) = e^{\mu t + \sigma^2 t^2/2}, \quad -\infty < t < \infty$$

$$E(X) = \mu, \quad \text{Var}(X) = \sigma^2$$

## Uniform

$$U(a, b)$$

$$-\infty < a < b < \infty$$

$$f(x) = \frac{1}{b-a}, \quad a \leq x \leq b$$

$$M(t) = \frac{e^{tb} - e^{ta}}{t(b-a)}, \quad t \neq 0; \quad M(0) = 1$$

$$\mu = \frac{a+b}{2}, \quad \sigma^2 = \frac{(b-a)^2}{12}$$

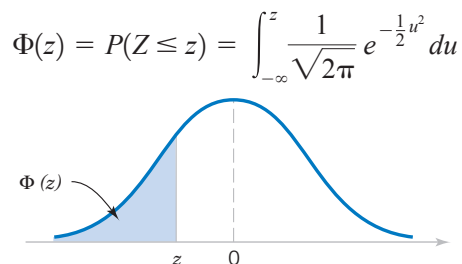
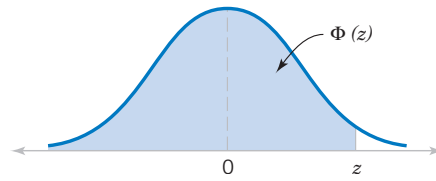


Table III Cumulative Standard Normal Distribution

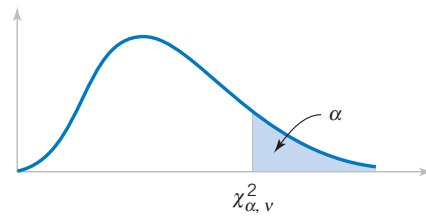
$z$	-0.09	-0.08	-0.07	-0.06	-0.05	-0.04	-0.03	-0.02	-0.01	-0.00
-3.9	0.000033	0.000034	0.000036	0.000037	0.000039	0.000041	0.000042	0.000044	0.000046	0.000048
-3.8	0.000050	0.000052	0.000054	0.000057	0.000059	0.000062	0.000064	0.000067	0.000069	0.000072
-3.7	0.000075	0.000078	0.000082	0.000085	0.000088	0.000092	0.000096	0.000100	0.000104	0.000108
-3.6	0.000112	0.000117	0.000121	0.000126	0.000131	0.000136	0.000142	0.000147	0.000153	0.000159
-3.5	0.000165	0.000172	0.000179	0.000185	0.000193	0.000200	0.000208	0.000216	0.000224	0.000233
-3.4	0.000242	0.000251	0.000260	0.000270	0.000280	0.000291	0.000302	0.000313	0.000325	0.000337
-3.3	0.000350	0.000362	0.000376	0.000390	0.000404	0.000419	0.000434	0.000450	0.000467	0.000483
-3.2	0.000501	0.000519	0.000538	0.000557	0.000577	0.000598	0.000619	0.000641	0.000664	0.000687
-3.1	0.000711	0.000736	0.000762	0.000789	0.000816	0.000845	0.000874	0.000904	0.000935	0.000968
-3.0	0.001001	0.001035	0.001070	0.001107	0.001144	0.001183	0.001223	0.001264	0.001306	0.001350
-2.9	0.001395	0.001441	0.001489	0.001538	0.001589	0.001641	0.001695	0.001750	0.001807	0.001866
-2.8	0.001926	0.001988	0.002052	0.002118	0.002186	0.002256	0.002327	0.002401	0.002477	0.002555
-2.7	0.002635	0.002718	0.002803	0.002890	0.002980	0.003072	0.003167	0.003264	0.003364	0.003467
-2.6	0.003573	0.003681	0.003793	0.003907	0.004025	0.004145	0.004269	0.004396	0.004527	0.004661
-2.5	0.004799	0.004940	0.005085	0.005234	0.005386	0.005543	0.005703	0.005868	0.006037	0.006210
-2.4	0.006387	0.006569	0.006756	0.006947	0.007143	0.007344	0.007549	0.007760	0.007976	0.008198
-2.3	0.008424	0.008656	0.008894	0.009137	0.009387	0.009642	0.009903	0.010170	0.010444	0.010724
-2.2	0.011011	0.011304	0.011604	0.011911	0.012224	0.012545	0.012874	0.013209	0.013553	0.013903
-2.1	0.014262	0.014629	0.015003	0.015386	0.015778	0.016177	0.016586	0.017003	0.017429	0.017864
-2.0	0.018309	0.018763	0.019226	0.019699	0.020182	0.020675	0.021178	0.021692	0.022216	0.022750
-1.9	0.023295	0.023852	0.024419	0.024998	0.025588	0.026190	0.026803	0.027429	0.028067	0.028717
-1.8	0.029379	0.030054	0.030742	0.031443	0.032157	0.032884	0.033625	0.034379	0.035148	0.035930
-1.7	0.036727	0.037538	0.038364	0.039204	0.040059	0.040929	0.041815	0.042716	0.043633	0.044565
-1.6	0.045514	0.046479	0.047460	0.048457	0.049471	0.050503	0.051551	0.052616	0.053699	0.054799
-1.5	0.055917	0.057053	0.058208	0.059380	0.060571	0.061780	0.063008	0.064256	0.065522	0.066807
-1.4	0.068112	0.069437	0.070781	0.072145	0.073529	0.074934	0.076359	0.077804	0.079270	0.080757
-1.3	0.082264	0.083793	0.085343	0.086915	0.088508	0.090123	0.091759	0.093418	0.095098	0.096801
-1.2	0.098525	0.100273	0.102042	0.103835	0.105650	0.107488	0.109349	0.111233	0.113140	0.115070
-1.1	0.117023	0.119000	0.121001	0.123024	0.125072	0.127143	0.129238	0.131357	0.133500	0.135666
-1.0	0.137857	0.140071	0.142310	0.144572	0.146859	0.149170	0.151505	0.153864	0.156248	0.158655
-0.9	0.161087	0.163543	0.166023	0.168528	0.171056	0.173609	0.176185	0.178786	0.181411	0.184060
-0.8	0.186733	0.189430	0.192150	0.194894	0.197662	0.200454	0.203269	0.206108	0.208970	0.211855
-0.7	0.214764	0.217695	0.220650	0.223627	0.226627	0.229650	0.232695	0.235762	0.238852	0.241964
-0.6	0.245097	0.248252	0.251429	0.254627	0.257846	0.261086	0.264347	0.267629	0.270931	0.274253
-0.5	0.277595	0.280957	0.284339	0.287740	0.291160	0.294599	0.298056	0.301532	0.305026	0.308538
-0.4	0.312067	0.315614	0.319178	0.322758	0.326355	0.329969	0.333598	0.337243	0.340903	0.344578
-0.3	0.348268	0.351973	0.355691	0.359424	0.363169	0.366928	0.370700	0.374484	0.378281	0.382089
-0.2	0.385908	0.389739	0.393580	0.397432	0.401294	0.405165	0.409046	0.412936	0.416834	0.420740
-0.1	0.424655	0.428576	0.432505	0.436441	0.440382	0.444330	0.448283	0.452242	0.456205	0.460172
0.0	0.464144	0.468119	0.472097	0.476078	0.480061	0.484047	0.488033	0.492022	0.496011	0.500000

$$\Phi(z) = P(Z \leq z) = \int_{-\infty}^z \frac{1}{\sqrt{2\pi}} e^{-\frac{1}{2}u^2} du$$



**Table III** Cumulative Standard Normal Distribution (*continued*)

$z$	0.00	0.01	0.02	0.03	0.04	0.05	0.06	0.07	0.08	0.09
0.0	0.500000	0.503989	0.507978	0.511967	0.515953	0.519939	0.523922	0.527903	0.531881	0.535856
0.1	0.539828	0.543795	0.547758	0.551717	0.555760	0.559618	0.563559	0.567495	0.571424	0.575345
0.2	0.579260	0.583166	0.587064	0.590954	0.594835	0.598706	0.602568	0.606420	0.610261	0.614092
0.3	0.617911	0.621719	0.625516	0.629300	0.633072	0.636831	0.640576	0.644309	0.648027	0.651732
0.4	0.655422	0.659097	0.662757	0.666402	0.670031	0.673645	0.677242	0.680822	0.684386	0.687933
0.5	0.691462	0.694974	0.698468	0.701944	0.705401	0.708840	0.712260	0.715661	0.719043	0.722405
0.6	0.725747	0.729069	0.732371	0.735653	0.738914	0.742154	0.745373	0.748571	0.751748	0.754903
0.7	0.758036	0.761148	0.764238	0.767305	0.770350	0.773373	0.776373	0.779350	0.782305	0.785236
0.8	0.788145	0.791030	0.793892	0.796731	0.799546	0.802338	0.805106	0.807850	0.810570	0.813267
0.9	0.815940	0.818589	0.821214	0.823815	0.826391	0.828944	0.831472	0.833977	0.836457	0.838913
1.0	0.841345	0.843752	0.846136	0.848495	0.850830	0.853141	0.855428	0.857690	0.859929	0.862143
1.1	0.864334	0.866500	0.868643	0.870762	0.872857	0.874928	0.876976	0.878999	0.881000	0.882977
1.2	0.884930	0.886860	0.888767	0.890651	0.892512	0.894350	0.896165	0.897958	0.899727	0.901475
1.3	0.903199	0.904902	0.906582	0.908241	0.909877	0.911492	0.913085	0.914657	0.916207	0.917736
1.4	0.919243	0.920730	0.922196	0.923641	0.925066	0.926471	0.927855	0.929219	0.930563	0.931888
1.5	0.933193	0.934478	0.935744	0.936992	0.938220	0.939429	0.940620	0.941792	0.942947	0.944083
1.6	0.945201	0.946301	0.947384	0.948449	0.949497	0.950529	0.951543	0.952540	0.953521	0.954486
1.7	0.955435	0.956367	0.957284	0.958185	0.959071	0.959941	0.960796	0.961636	0.962462	0.963273
1.8	0.964070	0.964852	0.965621	0.966375	0.967116	0.967843	0.968557	0.969258	0.969946	0.970621
1.9	0.971283	0.971933	0.972571	0.973197	0.973810	0.974412	0.975002	0.975581	0.976148	0.976705
2.0	0.977250	0.977784	0.978308	0.978822	0.979325	0.979818	0.980301	0.980774	0.981237	0.981691
2.1	0.982136	0.982571	0.982997	0.983414	0.983823	0.984222	0.984614	0.984997	0.985371	0.985738
2.2	0.986097	0.986447	0.986791	0.987126	0.987455	0.987776	0.988089	0.988396	0.988696	0.988989
2.3	0.989276	0.989556	0.989830	0.990097	0.990358	0.990613	0.990863	0.991106	0.991344	0.991576
2.4	0.991802	0.992024	0.992240	0.992451	0.992656	0.992857	0.993053	0.993244	0.993431	0.993613
2.5	0.993790	0.993963	0.994132	0.994297	0.994457	0.994614	0.994766	0.994915	0.995060	0.995201
2.6	0.995339	0.995473	0.995604	0.995731	0.995855	0.995975	0.996093	0.996207	0.996319	0.996427
2.7	0.996533	0.996636	0.996736	0.996833	0.996928	0.997020	0.997110	0.997197	0.997282	0.997365
2.8	0.997445	0.997523	0.997599	0.997673	0.997744	0.997814	0.997882	0.997948	0.998012	0.998074
2.9	0.998134	0.998193	0.998250	0.998305	0.998359	0.998411	0.998462	0.998511	0.998559	0.998605
3.0	0.998650	0.998694	0.998736	0.998777	0.998817	0.998856	0.998893	0.998930	0.998965	0.998999
3.1	0.999032	0.999065	0.999096	0.999126	0.999155	0.999184	0.999211	0.999238	0.999264	0.999289
3.2	0.999313	0.999336	0.999359	0.999381	0.999402	0.999423	0.999443	0.999462	0.999481	0.999499
3.3	0.999517	0.999533	0.999550	0.999566	0.999581	0.999596	0.999610	0.999624	0.999638	0.999650
3.4	0.999663	0.999675	0.999687	0.999698	0.999709	0.999720	0.999730	0.999740	0.999749	0.999758
3.5	0.999767	0.999776	0.999784	0.999792	0.999800	0.999807	0.999815	0.999821	0.999828	0.999835
3.6	0.999841	0.999847	0.999853	0.999858	0.999864	0.999869	0.999874	0.999879	0.999883	0.999888
3.7	0.999892	0.999896	0.999900	0.999904	0.999908	0.999912	0.999915	0.999918	0.999922	0.999925
3.8	0.999928	0.999931	0.999933	0.999936	0.999938	0.999941	0.999943	0.999946	0.999948	0.999950
3.9	0.999952	0.999954	0.999956	0.999958	0.999959	0.999961	0.999963	0.999964	0.999966	0.999967

**Table IV** Percentage Points  $\chi^2_{\alpha, \nu}$  of the Chi-Squared Distribution

$\nu \backslash \alpha$	.995	.990	.975	.950	.900	.500	.100	.050	.025	.010	.005
1	.00+	.00+	.00+	.00+	.02	.45	2.71	3.84	5.02	6.63	7.88
2	.01	.02	.05	.10	.21	1.39	4.61	5.99	7.38	9.21	10.60
3	.07	.11	.22	.35	.58	2.37	6.25	7.81	9.35	11.34	12.84
4	.21	.30	.48	.71	1.06	3.36	7.78	9.49	11.14	13.28	14.86
5	.41	.55	.83	1.15	1.61	4.35	9.24	11.07	12.83	15.09	16.75
6	.68	.87	1.24	1.64	2.20	5.35	10.65	12.59	14.45	16.81	18.55
7	.99	1.24	1.69	2.17	2.83	6.35	12.02	14.07	16.01	18.48	20.28
8	1.34	1.65	2.18	2.73	3.49	7.34	13.36	15.51	17.53	20.09	21.96
9	1.73	2.09	2.70	3.33	4.17	8.34	14.68	16.92	19.02	21.67	23.59
10	2.16	2.56	3.25	3.94	4.87	9.34	15.99	18.31	20.48	23.21	25.19
11	2.60	3.05	3.82	4.57	5.58	10.34	17.28	19.68	21.92	24.72	26.76
12	3.07	3.57	4.40	5.23	6.30	11.34	18.55	21.03	23.34	26.22	28.30
13	3.57	4.11	5.01	5.89	7.04	12.34	19.81	22.36	24.74	27.69	29.82
14	4.07	4.66	5.63	6.57	7.79	13.34	21.06	23.68	26.12	29.14	31.32
15	4.60	5.23	6.27	7.26	8.55	14.34	22.31	25.00	27.49	30.58	32.80
16	5.14	5.81	6.91	7.96	9.31	15.34	23.54	26.30	28.85	32.00	34.27
17	5.70	6.41	7.56	8.67	10.09	16.34	24.77	27.59	30.19	33.41	35.72
18	6.26	7.01	8.23	9.39	10.87	17.34	25.99	28.87	31.53	34.81	37.16
19	6.84	7.63	8.91	10.12	11.65	18.34	27.20	30.14	32.85	36.19	38.58
20	7.43	8.26	9.59	10.85	12.44	19.34	28.41	31.41	34.17	37.57	40.00
21	8.03	8.90	10.28	11.59	13.24	20.34	29.62	32.67	35.48	38.93	41.40
22	8.64	9.54	10.98	12.34	14.04	21.34	30.81	33.92	36.78	40.29	42.80
23	9.26	10.20	11.69	13.09	14.85	22.34	32.01	35.17	38.08	41.64	44.18
24	9.89	10.86	12.40	13.85	15.66	23.34	33.20	36.42	39.36	42.98	45.56
25	10.52	11.52	13.12	14.61	16.47	24.34	34.28	37.65	40.65	44.31	46.93
26	11.16	12.20	13.84	15.38	17.29	25.34	35.56	38.89	41.92	45.64	48.29
27	11.81	12.88	14.57	16.15	18.11	26.34	36.74	40.11	43.19	46.96	49.65
28	12.46	13.57	15.31	16.93	18.94	27.34	37.92	41.34	44.46	48.28	50.99
29	13.12	14.26	16.05	17.71	19.77	28.34	39.09	42.56	45.72	49.59	52.34
30	13.79	14.95	16.79	18.49	20.60	29.34	40.26	43.77	46.98	50.89	53.67
40	20.71	22.16	24.43	26.51	29.05	39.34	51.81	55.76	59.34	63.69	66.77
50	27.99	29.71	32.36	34.76	37.69	49.33	63.17	67.50	71.42	76.15	79.49
60	35.53	37.48	40.48	43.19	46.46	59.33	74.40	79.08	83.30	88.38	91.95
70	43.28	45.44	48.76	51.74	55.33	69.33	85.53	90.53	95.02	100.42	104.22
80	51.17	53.54	57.15	60.39	64.28	79.33	96.58	101.88	106.63	112.33	116.32
90	59.20	61.75	65.65	69.13	73.29	89.33	107.57	113.14	118.14	124.12	128.30
100	67.33	70.06	74.22	77.93	82.36	99.33	118.50	124.34	129.56	135.81	140.17

 $\nu$  = degrees of freedom.