ANALYZING PRE-TEST/POST-TEST DESIGNS IN A DIAGNOSTIC CLASSIFICATION MODEL FRAMEWORK

by

MATTHEW JAMES MADISON

(Under the Direction of Laine Bradshaw)

ABSTRACT

Diagnostic classification models (DCMs) are psychometric models that classify examinees according to a specified set of categorical latent traits, or attributes. These classifications can be used to indicate the multifaceted psychological status or multidimensional knowledge state of each examinee. In educational contexts, DCM estimates can identify specific areas for further remediation or instruction where needed. Research on DCMs has been relatively sparse until the past decade or so. As a result, there are fundamental topics related to their application that have not been explored. One such area is the analysis of pre-test/post-test designed experiments. In a pre-test/post-test designed experiment, examinees are administered an assessment before an intervention, then take the same or a parallel form of the assessment after the intervention. In this type of study, the primary objective is to measure growth in examinees, individually and collectively. This dissertation is comprised of two studies, both focusing on developing and examining novel methodology related to the analysis of pre-test/post-test designs in a DCM framework. The first study focuses on the development and application of the Transition Diagnostic Classification Model (TDCM) as a longitudinal and general diagnostic classification model for assessing growth in attribute mastery from pre-test to post-test. The second study extends the TDCM to form the multiple-group TDCM (MG TDCM). In a randomized controlled trial with a control group and an intervention group, the MG TDCM can be used to assess group-differential growth in attribute mastery (i.e., intervention effects). To statistically assess growth and group-differential growth in the TDCM and MG TDCM, respectively, a Wald test was employed. Results of simulation studies show that both models provide highly accurate and reliable classifications, and that the Wald tests have appropriate Type I error and are powerful in the detection of growth, even with small effect sizes and less than ideal sample sizes. To demonstrate their practical utility, both models are applied to analyze pre-test/post-test data from a diagnostic test measuring middle school students' problem solving abilities in mathematics.

INDEX WORDS: Diagnostic classification models, Cognitive diagnosis models, Latent transition analysis, Latent class models, Pre-test, Post-test, Growth, Intervention effects, Wald test, multigroup model, longitudinal analysis, transition diagnostic classification model

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DEDICATION

To my beautiful daughter, Maya.

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CHAPTER 1

INTRODUCTION AND LITERATURE REVIEW

Diagnostic classification models (DCMs; e.g., Rupp, Templin, & Henson, 2010), also referred to as cognitive diagnosis models (CDMs; e.g., Leighton & Gierl, 2007), are psychometric models that classify examinees based on their responses to a set of items measuring a set of specified latent skills, or attributes. Over the past decade, or so, DCMs have received increased attention in educational assessment research and practice due to their potential to provide fine-grained and multidimensional information that educators can use to guide instructional adjustments and improve student learning. Although they have received this attention, DCMs are still in their infancy compared to more traditional and commonly-used item response models. As a result, there are several areas related to their application that have not been fully explored. One such area is in the analysis of longitudinal data sets. This dissertation advances this area by developing novel methodology for the analysis of pre-test/post-test designs in a general DCM framework. This chapter motivates advancement, starting with a description of the pre-test/post-test design, then outlining the conceptual and statistical foundations of DCMs, and concluding with describing the proposed modeling framework.

Pre-test/Post-test Designs in Education

The assessment of intervention effects is an important objective of educational research. One way educational systems can improve is through systematic implementation of products, policies, practices, and programs that quality research has identified as successful. The process of evaluating educational interventions can be a daunting task. Within the Institute of Educational

Sciences (IES; U.S. Department of Education), there are entire organizations dedicated to this effort. For example, the What Works Clearinghouse (WWC) and Regional Education Labs (REL) exist primarily to evaluate educational interventions and report on their effectiveness.

One important issue related to the evaluation of an educational or instructional intervention is quantifying the amount of learning that has taken place. In order to appropriately quantify learning, there must be measures in place to assess student learning before and after an intervention. In an educational context, this is typically completed via some type of pre-test/post-test designed experiment. The most basic type of pre-test/post-test design is the *single-group pre-test/post-test design* (Fraenkel & Wallen, 2003). As the name suggests, there is only one group and the goal is typically to evaluate the post-test scores in relation to the pre-test scores. The difference in performance on the pre-test and post-test can be interpreted as the amount of learning that the examinee has achieved. In assessment, differences in student knowledge over time is referred to as growth and have been studied at length in a variety of contexts. In a classical test theory framework, the difference in pre-test and post-test scores is often interpreted as a quantification of examinee learning (Williams & Zimmerman, 1996). In an item response theory (IRT) framework, longitudinal models have been developed to assess growth in individual and group ability (Anderson, 1985; Embretson, 1991; Fischer, 1976; Fischer, 1989). The weakness of the one-group pre-test/post-test design lies in the fact that whatever growth is exhibited cannot be attributed solely to the intervention because there was no comparison group that did not receive the intervention. Therefore, there is no way to know how much growth the group would have exhibited had they not participated in the intervention.

Another, more rigorous and recommended type of pre-test/post-test design is the *control-group pre-test/post-test design* (Fraenkel & Wallen, 2003). The control group design is stronger than the one-group design because it allows for the evaluation of the counterfactual. That is, because the control group does not receive the intervention, the design allows for the attribution of group differences in outcomes to the intervention. In an IRT framework, there is methodology to analyze this type of pre/post-designed experiment. Namely, variants of longitudinal multi-group (Bock & Zimowski, 1997) and mixture IRT models (Bolt, Cohen, & Wollack, 2001) can be used to assess differential growth in observed or unobserved groups, respectively. In the control-group design, random assignment to treatment and control groups is recommended as a means of minimizing pre-test differences.

Conceptual Foundations of DCMs

The primary and distinguishing feature of DCMs is the distributional specification of the latent traits. More specifically, the latent traits in DCMs are assumed discrete. This is in contrast to traditional and more commonly used psychometric models, where the latent trait is assumed continuous. With standard statistical models, like linear regression, a discrete variable is just a special case that can be accommodated with a simple coding scheme. In the case of psychometric models, this distributional distinction makes the DCM framework slightly different from traditional psychometric models. More specifically, the DCM framework supports a particular belief about the nature of the underlying latent traits, and therefore supports inferences with different interpretations and implications.

Traditional psychometric models, such as the commonly applied unidimensional item response theory models, use item responses to obtain one estimate of examinee ability in a general domain, like mathematics or reading ability. IRT models are designed to locate and order examinees on a continuum of ability. This precise ordering is useful when attempting, for example, to identify the top 10% of examinees for an honors program or the bottom 10% of examinees for remediation. This ordering is not useful, however, in providing detailed information about examinees' specific strengths and weaknesses. In contrast, DCMs are "designed to measure specific knowledge structures and processing skills so as to provide information about their cognitive strengths and weaknesses" (Leighton & Gierl, 2007, p. 3). The design that Leighton and Gierl speak of is initiated by the discrete latent traits modeled in the DCM. The discrete latent traits in DCMs are referred to as *attributes*. For each specified attribute, DCMs probabilistically classify examinees into two groups, termed mastery and non-mastery. When combined with a quality diagnostic test, the feedback from DCMs can have inherent and interpretive meaning; a master is ready to proceed, a non-master likely needs remediation. Therefore, the feedback from DCMs provides fine-grained and actionable information from which educators can make instructional adjustments, thereby improving student learning.

Following the *No Child Left Behind Act* (NCLB; 2001), the *Every Students Succeeds Act* (ESSA; Every Student Succeeds Act, 2015-2016) requires that state assessments provide more detailed and formative information. In particular, it requires that state assessments "produce individual student interpretive, descriptive, and diagnostic reports" (p. 26). Huff and Goldman (2007) found that educators also desire assessments that provide individualized and diagnostic information that informs their instruction. Classroom research has indicated that formative assessment with detailed and diagnostic feedback is effective in improving student learning (Black & Wiliam, 1998). In addition, many states have adopted standards based curricula (e.g., Common Core State Standards, 2010) that clearly outline and describe learning goals and outcomes for what students should know and be able to do at each grade level. With educational standards, comes a need for teachers to assess students' knowledge in a timely manner in relation to the assessed standards. In this context, an IRT model would not be useful because of its inability to support diagnostic inferences and pinpoint students' strengths and weaknesses regarding the targeted standards. DCMs are well-suited to be employed in this context because they are able to provide reliable and accurate diagnostic information with respect to multiple attributes with relatively few items (e.g., 4-6; Templin & Bradshaw, 2013; Madison & Bradshaw, 2014).

Statistical Foundations of DCMs

DCMs are members of a class of statistical models called latent class models (LCMs; e.g., Lazarsfeld & Henry, 1968). Latent class models use item responses to group similar examinees into latent classes. In a typical latent class analysis, the number of latent classes is determined in an exploratory fashion; several models are fit with different numbers of latent classes and the model with the best fit (i.e., most parsimonious) is chosen and subsequently interpreted. The substantive meaning of the detected latent classes is then inferred from the characteristics of its members.

Once a number of latent classes has been chosen as a candidate model, the structural and measurement components of the LCM must be estimated. The structural component of the LCM parametrizes the proportion of examinees belonging to each class, while the measurement component parametrizes how each latent class responds to items. Consider an examinee e responding to I items. In a LCM, the probability of the item response vector $\boldsymbol{x}_e$ is given by:

$$\mathbb{P}(\boldsymbol{X}_e = \boldsymbol{x}_e) = \sum_{c=1}^{C} \nu_c \prod_{i=1}^{I} \pi_{ic}^{x_{ei}} (1 - \pi_{ic})^{1-x_{ei}} \, . \qquad (1.1)$$

In Equation 1.1, ν_c represents the probability of membership in Class c and the collection of ν_c's, called structural parameters, parameterize the proportion of examinees belonging to each latent class. In the product portion, π_{ic} is the probability of a correct response to Item i given membership in Class c, and x_{ei} is Examinee e's dichotomously scored response to Item i. The product across items is a consequence of the local independence assumption, which states that within a latent class, responses to each item are independent. Conditioned on membership in Class c, the probability of an item response vector is the probability of membership in Class c multiplied by the likelihood of the responses under Class c. Equation 1.1 presents the unconditional probability of an item response vector, which is found by summing over the class conditional likelihoods.

DCMs are confirmatory LCMs in two senses: First, the latent classes in a DCM are specified a priori as the patterns of mastery corresponding to the measured attributes. For a test measuring *A* attributes, each examinee is probabilistically classified into one of 2^A patterns of attribute mastery, termed *attribute profiles*. For example, an examinee may be classified into attribute profile [0,1,1,0], with the 1s indicating that she has mastered Attributes 2 and 3, and the 0s indicating that she has not mastered Attributes 1 and 4. In general, attribute profiles are denoted by $\boldsymbol{\alpha_c} = [\alpha_1, \alpha_2, \dots, \alpha_A]$ where $\alpha_a = 0$ indicates non-mastery of Attribute a and $\alpha_a = 1$ indicates mastery of Attribute a. Secondly, DCMs are confirmatory LCMs in that the attribute loading structure is specified a priori in a Q-matrix. The Q-matrix is an item-by-attribute matrix of 0s and 1s indicating which attributes are measured by each item.

DCMs are considered constrained LCMs because they are specified by placing constraints on the parameters in a traditional LCM. Several DCMs have been developed that differ in the way they relate attribute mastery to item response probabilities. The DCM presented in the next section, the log-linear cognitive diagnosis model (LCDM; Henson, Templin, & Willse, 2009), is a general and flexible model that offers a unified framework within which many previously developed DCMs can be specified by placing constraints on the LCDM parameters. We focus on the LCDM in this study because of its modeling flexibility, straightforward interpretation, and estimation capability in commercially available software such as Mplus (Muthén & Muthén, 2012).

Log-linear Cognitive Diagnosis Model

As implied by its name, the LCDM uses a log-linear framework to parametrize the relationship between examinee attribute mastery and item response probabilities. In this log-linear framework, other popular DCMs, like the deterministic-inputs, noisy-and-gate (DINA; e.g., Haertel, 1989; Junker & Sijtsma, 2001) model, are easily obtained by constraining certain LCDM parameters to be zero.

The LCDM item response function is flexible in its treatment of item and attribute effects. To illustrate this flexibility, consider an item measuring two attributes, Attribute 3 (α_3) and Attribute 5 (α_5). The LCDM models the probability of correct response to Item i, conditional on membership in Attribute Profile $\boldsymbol{\alpha_c}$ as:

$$P(X_i = 1|\boldsymbol{\alpha_c}) = \frac{\exp\left(\lambda_{i,0} + \lambda_{i,1,(3)}(\alpha_3) + \lambda_{i,1,(5)}(\alpha_5) + \lambda_{i,2,(3,5)}(\alpha_3 \cdot \alpha_5)\right)}{1 + \exp\left(\lambda_{i,0} + \lambda_{i,1,(3)}(\alpha_3) + \lambda_{i,1,(5)}(\alpha_5) + \lambda_{i,2,(3,5)}(\alpha_3 \cdot \alpha_5)\right)}. \quad (1.2)$$

The item parameters in Equation 1.2 include an intercept ($\lambda_{i,0}$), simple main effects for Attribute 3 ($\lambda_{i,1,(3)}$) and Attribute 5 ($\lambda_{i,1,(5)}$), and an interaction between these two attributes ($\lambda_{i,2,(3,5)}$). These parameters are interpreted similarly to a reference coded analysis of variance model. The intercept represents the log-odds of a correct response for the reference group: examinees who have mastered neither Attribute 3 nor Attribute 5 (i.e., examinees with attribute profiles where $\alpha_3 = 0$ and $\alpha_5 = 0$). The simple main effects for Attribute 3 and Attribute 5 represent the increase in log-odds for examinees who have mastered only Attribute 3 or Attribute 5, respectively. Finally, the interaction term represents the change in log-odds for examinees who have mastered both Attribute 3 and Attribute 5. To obtain the DINA model, both main effects, $\lambda_{i,1,(3)}$ and $\lambda_{i,1,(5)}$, would be constrained to be zero, and only the intercept and interaction term would be estimated.

The LCDM framework currently has a major limitation. Namely, the only way to model longitudinal data in the LCDM is to either *calibrate* the model at each individual testing occasion, or calibrate the model at one testing occasion and obtain classifications by *scoring* responses from the other testing occasions. Calibrating the model involves using the item responses to estimate item parameters and obtain classifications. In a scoring analysis, item parameters are fixed at their respective pre-calibrated estimates, and then used to obtain classifications. In either case, separately modeling the responses at each testing occasion ignores the potential within-person item response dependency, and therefore, valuable information that could lead to more accurate and reliable classifications is lost. For longitudinal item responses, within-person dependency refers to the notion that for any given examinee, item responses at each testing occasion are not independent; rather, they are dependent because they are generated from the same person. In the next section, we present a framework which we will use to specify the LCDM to model longitudinal data.

Latent Transition Analysis

Latent transition analysis (LTA; Collins & Wugalter, 1992) is a longitudinal extension of the general latent class model that simultaneously models latent class prevalence and transitions between different latent classes over time. LTA has been used to study dating and sexual behaviors (Lanza & Collins, 2008), substance use (Lanza, Patrick, & Maggs, 2010; Collins & Lanza, 2010), and the effectiveness of medical interventions (Roberts & Ward, 2011), to name a few examples. To see this extension, Equation 1.3 presents the probability of Examinee e's item response vector in a LTA:

$$\mathbb{P}(\boldsymbol{X}_e = \boldsymbol{x}_e) = \sum_{c_1=1}^{C} \sum_{c_2=1}^{C} \cdots \sum_{c_T=1}^{C} \nu_{c_1} \tau_{c_2|c_1} \tau_{c_3|c_2} \cdots \tau_{c_T|c_{T-1}} \prod_{t=1}^{T} \prod_{i=1}^{I} \pi_{ic_t}^{x_{eit}} \left(1 - \pi_{ic_t}\right)^{1-x_{eit}} \quad (1.3)$$

In Equation 1.3, the same components are present as in the LCM in Equation 1.1. The structural parameters, the ν_{c_1}'s, represent the probability of membership in latent class c at Time Point 1. The π_{ic_t}'s represent the Item i correct response probability at Time Point t for Latent Class c, which are parameterized by the specified measurement model. The distinguishing feature of LTA in comparison to a LCM are the $\tau_{c_t|c_{t-1}}$'s, which are the latent transition probabilities that describe the probability of transitioning from each respective latent class from Time Point $t-1$ to Time Point t. In this study, we focus on a commonly researched longitudinal design: the pre-test/post-test design. In a pre-test/post-test designed study, there are only two time points, and Equation 1.3 is reduced to:

$$\mathbb{P}(\boldsymbol{X}_e = \boldsymbol{x}_e) = \sum_{c_1=1}^{C} \sum_{c_2=1}^{C} \nu_{c_1} \tau_{c_2|c_1} \prod_{t=1}^{2} \prod_{i=1}^{I} \pi_{ic}^{x_{eit}} (1 - \pi_{ic})^{1-x_{eit}}. \quad (1.4)$$

Proposed Modeling Framework

The proposed modeling framework combines LTA with the LCDM specified as the measurement model, a hybrid model that we will refer to as the Transition Diagnostic Classification Model (TDCM). Specifically, we propose this modeling framework as methodology to analyze data from pre-test/post-test designed studies. Specifying the LCDM in a LTA allows us to simultaneously classify examinees as masters/non-masters of attributes, and examine how examinees transition between mastery and non-mastery from pre-test to post-test.

Furthermore, the simultaneous modeling of pre-test and post-test responses allows us to specify a statistical test of growth. This modeling framework should provide several benefits over modeling pre- and post-test responses separately. In particular, because it properly models the within-person dependency, we predict that the TDCM will have increased classification accuracy and reliability, and it will have an appropriately powered statistical test of growth.

Overview of Chapters

The overall goal of this dissertation is to motivate, develop, describe, and assess the efficacy of the proposed modeling framework, the TDCM. The next two chapters are standalone studies focusing on different aspects of the TDCM. Chapter 2 motivates and describes the TDCM, and evaluates its performance in comparison to existing methods of analyzing growth in a DCM framework. Chapter 3 extends the TDCM to the multiple-group TDCM as a means of testing for differential growth between groups, thereby providing methodology for assessing intervention effects in a DCM framework. The concluding chapter summarizes and discusses results from both studies and identifies directions for future research.

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CHAPTER 2

ASSESSING GROWTH IN A DIAGNOSTIC CLASSIFICATION MODEL[1]

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Abstract

A common assessment research design is the single-group pre-test/post-test design in which examinees are administered an assessment before instruction, then take the same, or a parallel form of the assessment after instruction. In this type of study, the primary objective is to measure growth in examinees, individually and collectively. In an item response theory (IRT) framework, longitudinal IRT models can be used to assess growth in examinee ability over time. In a diagnostic classification model (DCM) framework, assessing growth translates to measuring change in attribute mastery over time. This study introduces the Transition Diagnostic Classification Model (TDCM), which combines latent transition analysis with the Log-linear Cognitive Diagnosis Model as novel methodology for analyzing growth in a DCM framework. Simulation study results indicate that the proposed model provides accurate and reliable classifications, and can accurately test for the presence of growth in attribute mastery. The TDCM is used to analyze pre-test/post-test data from a diagnostic mathematics assessment.

Diagnostic classification models (DCMs; Rupp & Templin, 2008b; Rupp, Templin, & Henson, 2010) are becoming an increasingly viable alternative to traditional measurement models. The distinguishing feature of DCMs are the discrete latent traits, or attributes, in place of the continuous latent traits that underlie more commonly applied measurement models, such as item response theory (IRT) models. These classifications can be used to indicate the psychological status or knowledge state of each examinee. Fulfilling the desires of educators for more detailed and actionable feedback from assessments (Huff & Goodman, 2007), DCM estimates can provide specific avenues for further remediation or instruction where needed.

A common assessment research design is the pre-test/post-test design in which examinees are administered an assessment before instruction, then take the same, or a parallel form of the assessment after instruction. In this type of study, the primary objective is to measure growth in examinees, individually and collectively. In an item response theory (IRT) framework, longitudinal IRT models can be used to assess growth in examinee ability over time (Anderson, 1985; Embretson, 1991; Fischer, 1976, 1989). In a DCM framework, assessing growth translates to measuring change in attribute mastery over time. The focus of this study is to develop and examine new methodology to accommodate this type of longitudinal and dependent data in a DCM framework.

For latent class models, of which DCMs are restricted versions, there exists methodology to assess change in latent class prevalence over time. Namely, latent transition analysis (LTA; Collins & Wugalter, 1992) can be used to model change over time in a latent class model framework. In addition to the parameters estimated in a traditional latent class analysis, which includes the detection of latent classes and item parameters, LTA models how members of each latent class transition from one latent class to another between each measurement occasions. For example, it could be the case that a respondent is classified into Latent Class 1 at Time Point 1, then transitions into Latent Class 3 at Time Point 2, and then back to Latent Class 1 at Time Point 3. LTA has been used to study dating and sexual behaviors (Lanza & Collins, 2008), substance use (Lanza, Patrick, & Maggs, 2010; Collins & Lanza, 2010), and the effectiveness of medical interventions (Roberts & Ward, 2011), to name a few examples.

In addition to LTA, *Bayesian Knowledge Tracing* (BKT; Corbett & Anderson, 1995) is another method for assessing change over time in discrete latent traits. BKT is an algorithm used in intelligent tutoring systems to update estimates of user knowledge components as they progress through learning modules. ‘Items’ in an intelligent tutoring system are moves or actions that a user makes. User moves may include selecting an answer option, inputting a free response, or requesting a hint. Typically, BKT assumes that each item isolates one skill that is being measured (Baker, Corbett, & Aleven, 2008; Xu & Mostow, 2011). For each specified latent skill, BKT consists of four parameters: (1) the initial probability of skill mastery for the user, (2) the probability of transitioning into skill mastery, (3) the probability of responding correctly while not having skill mastery (guess), and (4) the probability of responding incorrectly while having skill mastery (slip). BKT is useful in intelligent tutoring systems because the updated estimate can be used to guide the system’s next move (e.g., next item, hint, feedback) in an effort to individualize the learning experience for each user. In a BKT modeling framework, the goal is to assess student learning from item to item within an assessment. This is accomplished by updating the user’s skill probability estimate after each move within the system. For example, before responding to an item, a user may have an estimated probability of skill mastery of .4. After a correct response, this estimated probability may increase to .58. While the BKT modeling framework is similar in form to using LTA, the difference lies in the overall goal of the model. In

BKT, the goal is to assess student learning from item to item *within* an assessment. This study proposes a model combining LTA and a DCM, where the goal is to assess student learning *between* assessments.

There have been relatively few studies applying DCMs in pre-test/post-test designed studies. Jang (2005, 2006) used a non-compensatory DCM to diagnose students' English reading skills before and after instruction. Because the items on the pre-test and post-test were different, Jang calibrated the DCMs separately at the pre-test and post-test, then descriptively compared and analyzed change in attribute mastery. This modeling approach assumes that the meaning of attribute mastery is constant for both tests and assumes that pre-test and post-test responses are independent. More recently, Jurich and Bradshaw (2014) employed a general DCM to examine four psychoeducational learning outcomes in undergraduate students. They first calibrated item parameters and obtained examinee classifications for the pre-test, then obtained post-test classifications by scoring the post-test item responses with item parameters fixed at their respective pre-test estimates. This modeling approach assumes parameter equality across the pre-test and post-test, and similar to Jang, also assumes that pre-test and post-test item responses are independent. Li, Cohen, Bottge, and Templin (2015) combined LTA with a constrained DCM to assess change in four attributes over four time points on a mathematics assessment. The DCM applied by Li et al. assumes a particular structure to the item responses and imposes extreme parameter constraints a priori across all items and attributes. Under these assumptions, for each item, examinees are classified into exactly two classes: examinees who have mastered of *all* attributes measured by the item and examinees who have not. Only examinees who have mastered all the attributes measured by the item have an increased correct response probability. In the model used by Li et al., partial masters are those who have mastered some, but not all, the measured attributes by an item. These are modeled as having the same correct response probability as examinees who have mastered none of the measured attributes. When these strict assumptions are not met, model classification accuracy decreases significantly and item parameters are inaccurate and misleading (Bradshaw & Templin, 2014). Li et al.'s (2015) use of LTA in conjunction with a DCM is a significant advancement for DCM methodology; however, the use of a constrained DCM limits its application and utility to the special—and likely rare—cases where its strict assumptions are met.

This study develops new methodology for assessing growth in a diagnostic classification model framework by

(1) proposing a model that combines LTA with a general DCM,
(2) modeling attributes and attribute transition dependencies,
(3) examining model performance in a variety of assessment conditions,
(4) employing null hypothesis significance testing (NHST) to assess statistical significance of growth in attribute mastery,
(5) evaluating methods for controlling Type I error for the NHST,
(6) comparing the proposed model to alternative methods of assessing change in attribute mastery over time, and
(7) applying the proposed model to empirical data.

In the next section is a description of (1) the general DCM used in this study, the log-linear cognitive diagnosis model (LCDM; Henson, Templin, & Willse, 2009); (2) latent transition analysis; and (3) the proposed model, the Transition Diagnostic Classification Model (TDCM). Following that, we provide the simulation study design and empirical analysis description.

Method

Log-linear Cognitive Diagnosis Model

The LCDM is a general DCM that parameterizes the probability of a correct response as a function of examinee attribute mastery, the attributes measured by the item, and the item parameters. It is similar in form to a multi-dimensional IRT model, with the main difference being that the latent traits are not continuous; rather, they are categorical and typically dichotomous. In the LCDM, the latent traits are collectively referred to as an *attribute profile*. On a test measuring A attributes, each examinee possesses an A length vector, denoted $\boldsymbol{\alpha_c} = [\alpha_{c1}, \alpha_{c2}, \dots, \alpha_{cA}]$, where c refers to the index of the specific attribute profile, $\alpha_{ca} = 0$ indicates attribute non-mastery of Attribute a, and $\alpha_{ca} = 1$ indicates attribute mastery of Attribute a. For example, on a test measuring five attributes, an attribute profile of $[1,1,0,1,0]$ indicates that the examinee has mastered Attributes 1, 2, and 4, and has not mastered Attributes 3 and 5. The LCDM uses item responses to probabilistically classify each examinee into one of the 2^A attribute profiles.

The item parameters in the LCDM have similar interpretation to a reference-coded ANOVA model. That is, for each item measured on a test, the LCDM item response function is comprised of an intercept, a main effect for each attribute measured by the item, and interaction term(s) that correspond to each pair, triplet, etc. of attributes measured by the item. To demonstrate the item response function, consider an item measuring two attributes, Attribute 2 and Attribute 3. The LCDM item response function models the conditional probability of a correct response as

$$P(X_{ic} = 1|\boldsymbol{\alpha_c}) = \frac{\exp\left(\lambda_{i,0} + \lambda_{i,1,(2)}(\alpha_{c2}) + \lambda_{i,1,(3)}(\alpha_{c3}) + \lambda_{i,2,(2,3)}(\alpha_{c2} \cdot \alpha_{c3})\right)}{1 + \exp\left(\lambda_{i,0} + \lambda_{i,1,(2)}(\alpha_{c2}) + \lambda_{i,1,(3)}(\alpha_{c3}) + \lambda_{i,2,(2,3)}(\alpha_{c2} \cdot \alpha_{c3})\right)}. \quad (2.1)$$

In Equation 2.1, X_{ic} represents the random variable for the response to item i by an examinee with attribute profile $\boldsymbol{\alpha_c}$. The first parameter in Equation 2.1 is the intercept, $\lambda_{i,0}$. The intercept represents the log-odds of a correct response for the reference group, which is composed of examinees with attribute profile $[\alpha_{c1} =*, \alpha_{c2} = 0, \alpha_{c3} = 0, \alpha_{c4} =*]$, that is, examinees who possess neither Attribute 3 nor Attribute 4. The asterisks in the profile indicate either a 0 or a 1. Since this item only measures Attributes 2 and 3, the Attribute 1 and 4 mastery statuses do not affect the item response probabilities. The next parameters, $\lambda_{i,1,(2)}$ and $\lambda_{i,1,(3)}$, are the main effects for Attribute 2 and 3, respectively. They represent the increase in log-odds of a correct response for examinees who have mastered either Attribute 2 or 3. The last parameter, $\lambda_{i,2,(2,3)}$, is the interaction term and represents the additional change in log-odds of a correct response for examinees who have mastered both Attribute 2 and Attribute 3. The subscript i on the parameters represents the corresponding item. The first numerical subscript on the parameters represents the level of the parameter. Intercepts have a second subscript of 0, main effects have subscript of 1, two-way interactions have a subscript of 2, three-way interactions have a subscript of 3, etc. The subscripts in parentheses on the main effects and interaction parameters represent the attributes for the respective parameters. For example, $\lambda_{i,1,(3)}$ is the Attribute 3 main effect and $\lambda_{i,2,(2,3)}$ is the interaction for Attributes 2 and 3. The magnitude of these parameters reflect the degree to which attribute mastery statuses affect correct response probabilities.

The example above presents an item measuring two attributes. The LCDM, however, can accommodate up to *A* attributes on any given item. Computational demands and model estimation time are the only theoretical limitations to the number of attributes specified on an item or test in the LCDM. In its general form, the LCDM item response function can be expressed as:

$$P(X_{ic} = 1|\boldsymbol{\alpha_c}) = \frac{\exp\left(\lambda_{i,0} + \boldsymbol{\lambda}_i^T \boldsymbol{h}(\boldsymbol{\alpha}_c, \boldsymbol{q}_i)\right)}{1 + \exp\left(\lambda_{i,0} + \boldsymbol{\lambda}_i^T \boldsymbol{h}(\boldsymbol{\alpha}_c, \boldsymbol{q}_i)\right)}, \tag{2.2}$$

where $\lambda_{i,0}$ is the intercept parameter described above, $\boldsymbol{\lambda}_i^T$ is a column vector containing all possible $2^A - 1$ main effect and interactions terms for Item i, and $\boldsymbol{q}_i$ represents the i^{th} row of the Q-matrix. The Q-matrix is an item-by-attribute matrix of 0s and 1s indicating which attributes are measured by each item. If item i requires Attribute a, then cell ia in the Q-matrix will be 1, and 0 otherwise. The function $\boldsymbol{h}(\boldsymbol{\alpha}_c, \boldsymbol{q}_i)$ results in a column vector of length $2^A - 1$ whose elements are linear combinations of $\boldsymbol{\alpha_c}$ and $\boldsymbol{q_i}$. This combination equals 1 only when the examinee possesses the attributes corresponding to a given parameter λ and these attributes are measured by Item i. If the examinee has not mastered the attributes corresponding to a particular λ, or the attributes are not measured by the item, then the linear combination will result in a value of 0. When multiplied by the vector of main effect and interaction parameters, $\boldsymbol{\lambda}_i^T$, this general expansion becomes:

$$\boldsymbol{\lambda}_i^T \boldsymbol{h}(\boldsymbol{\alpha}_c, \boldsymbol{q}_i) = \sum_{a=1}^{A} \lambda_{i,1,(a)} \alpha_{ca} q_{ia} + \sum_{a=1}^{A-1} \sum_{a'=a+1}^{A} \lambda_{i,2,(a,a')} \alpha_{ca} \alpha_{ca'} q_{ia} q_{ia'} + \cdots. \tag{2.3}$$

The first summation includes the $\binom{A}{1} = A$ main effect parameters, the double summation includes the $\binom{A}{2}$ interaction parameters, and the ellipses represent all three-way and higher interaction terms. In the estimation of the LCDM, main effect parameters are constrained to be greater than zero so that examinees who have mastered more of the measured attributes on an item have increasing predicted correct response probabilities, and to prevent latent class switching (Lao & Templin, 2016). Additionally, interaction terms are constrained to be greater than the negative of the main effects so that the mastery of additional attributes cannot result in decreased item response probabilities.

Constrained Versions of the LCDM

The LCDM is a general DCM in that it subsumes many other common DCMs, including the deterministic-inputs, noisy-and-gate (DINA; e.g., Haertel, 1989; Junker & Sijtsma, 2001) model. In the DINA model, the main effects in Equation 2.1, $\lambda_{i,1,(2)}$ and $\lambda_{i,1,(3)}$, are constrained a priori to be zero. This constraint forces attributes to behave in a non-compensatory fashion, meaning that non-mastery of one measured attribute cannot be compensated for by mastery of other measured attributes. For each item, the DINA model separates all examinees into exactly two groups: those who have mastered all the measured attributes, and those who have not. For example, on the item illustrated in Equation 2.1, the DINA model would have two item response probabilities: one for masters of both Attributes 2 and 3, i.e., profile $[*,1,1,*]$, and another for masters of either or none, i.e., profiles $[*,1,0,*]$, $[*,0,1,*]$, and $[*,0,0,*]$. In contrast, in the LCDM, each of the four attribute profiles, $[*,1,1,*], [*,1,0,*], [*,0,1,*], \text{and } [*,0,0,*]$, have distinct item response probabilities, with the size of these distinctions being dictated by the magnitude of the main effect and interaction parameters. Because general DCMs such as the LCDM, the General Diagnostic Model (GDM; von Davier, 2005), or the Generalized DINA model (G-DINA; de la Torre, 2011) subsume many core DCMs, including the DINA model, they are better able to accurately represent the attribute mastery processes underlying the item responses. The benefit of using a general model is that attribute and item effects are freed and item parameter estimates can indicate whether these constraints are warranted. As noted earlier, when these constraints are imposed and do not fit the data, the model can be severely misspecified and classification accuracy decreases (Bradshaw & Templin, 2014).

Latent Transition Analysis

LTA is an expansion of the latent or hidden Markov model (HMM; Baum & Petrie, 1966). HMMs are a class of statistical models for analyzing sequences of observations over time. The observed data in HMMs are assumed to be generated by a latent, or hidden, process. In LTA, class membership at each time point is latent, but measured with a set of observed item responses. The measurement model that parameterizes item response probabilities in LTA at each time point is a latent class model, and class-sequential progressions are provided via latent class transition probabilities from each time point to the next. In a typical latent class analysis, the number of latent classes is determined in an exploratory fashion; several models are fit with different numbers of latent classes and the model with the best fit (i.e., greatest parsimony) is chosen and subsequently interpreted (e.g., see Lanza, et al., 2010; Collins & Lanza, 2010). In contrast, the latent classes in a DCM framework are specified a priori as the attribute profiles. For example, on a test measuring $A = 2$ dichotomous attributes, there are $2^A = 2^2 = 4$ attribute profiles that serve as the latent classes in the LTA. Namely, profiles [0,0], [0,1], [1,0], and [1,1] serve as the four latent classes in the LTA. The transition probabilities model examinee transition between the attribute profiles from each time point to the next. In this example, there are $4 \times 4 = 16$ transition probabilities. These transition probabilities would describe examinee transition from [0,0] to [0,1], [0,0] to [1,0], [0,0] to [1,1], and so on.

The latent transition probabilities are typically displayed in $2^A \times 2^A$ matrix with the jk^{th} cell representing the probability of transitioning from latent class j to latent class k. To illustrate, Table 2.1 shows an example transition matrix for the four attribute profiles from the previous paragraph. Here, the probability of transition from attribute profile [0,0] at pre-test into attribute profile [1,1] at post-test is .27, while the probability of transitioning from [0,1] to [1,1] is .60.

Although this transition matrix is hypothetical, note that transitions from attribute profiles with more mastered attributes to a profile with fewer mastered attributes also are possible, but far generally less likely. For example, the probability of transitioning from the attribute profile [0,0] to [1,0] is .35, while the probability of transitioning from [1,0] to [0,0] is much smaller at .05. In educational contexts, we would expect this to be the case because students are more likely to obtain or retain knowledge over short periods of targeted instruction than they are to forget or unlearn knowledge (e.g., see Li et al., 2015). In other contexts, such as adolescent depression or substance abuse patterns, one might expect, and studies have observed, more uniform transition probabilities (e.g., see Collins & Lanza, 2010).

The transition probability matrix can be used to evaluate learning across the examinee sample. For example, in Table 2.1, 80% (18% + 35% + 27%) of examinees starting as non-masters of both attributes transitioned into mastery of one or both attributes. This is taken as an indicator of successful learning. In this two-attribute example, this table is still easily interpretable. As the number of specified attributes becomes large, however, this matrix can quickly become unruly and difficult to interpret. In an educational assessment context, the individual attribute mastery transitions are often of interest because they can be used to evaluate learning for each measured attribute. This information can then be used to guide instructional adjustments. The transition probability matrix, combined with the initial time point attribute profile membership proportions, can be used to compute individual attribute mastery transitions. For example, suppose in the Table 2.1 example that the pre-test attribute profile membership proportions are .40, .25, .20, and .15 for [0,0], [0,1], [1,0], and [1,1], respectively. Table 2.2 shows the resulting Attribute 1 transition probability matrix. The calculation of the conditional probabilities in Table 2.2 is shown in Appendix A. Focusing on individual attribute mastery transitions does not ease the computational burden, but does provide a more interpretable result. The most likely transition is remaining a master of Attribute 1 at .87. The least likely transition is losing or forgetting Attribute 1 at .13. Note that each row sums to one, as the row probabilities are conditional on the attribute mastery state at pre-test.

The LTA model consists of three groups of parameters. The first are the probabilities of belonging to each latent class at the first time point. The second group consists of the latent transition probabilities, or the probabilities of transitioning from one latent class to another across each time point. Lastly, the LTA model employs a measurement model to estimate the item response probabilities at each time point. Consider an examinee e responding to I items over T time points. The probability of the item response vector $\boldsymbol{x}_e$ is given by:

$$\mathbb{P}(\boldsymbol{X}_e = \boldsymbol{x}_e) = \sum_{c_1=1}^{C} \sum_{c_2=1}^{C} \cdots \sum_{c_T=1}^{C} \nu_{c_1} \tau_{c_2|c_1} \tau_{c_3|c_2} \cdots \tau_{c_T|c_{T-1}} \prod_{t=1}^{T} \prod_{i=1}^{I} \pi_{ic_t}^{x_{eit}} \left(1 - \pi_{ic_t}\right)^{1-x_{eit}}. \quad (2.4)$$

In Equation 2.4, ν_{c_1} represents the probability of membership in Class c at Time Point 1. Each sum ranges over each of the C latent classes. Each $\tau_{c_t|c_{t-1}}$ represents the transition probability from Time Point $t-1$ to Time Point t, π_{ic_t} is the probability of a correct response to Item i given membership in Latent Class c at Time t, and x_{eit} is Examinee e's response to Item i at Time Point t. The π_{ict}'s are estimated with a measurement model, which in this study is the LCDM. For the special case of $T = 2$ (two time points), the focus of this paper, Equation 2.4 reduces to:

$$\mathbb{P}(\boldsymbol{X}_e = \boldsymbol{x}_e) = \sum_{c_1=1}^{C} \sum_{c_2=1}^{C} \nu_{c_1} \tau_{c_2|c_1} \prod_{t=1}^{2} \prod_{i=1}^{I} \pi_{ic_t}^{x_{eit}} \left(1 - \pi_{ic_t}\right)^{1-x_{eit}}. \quad (2.5)$$

Within the context of an LTA, LCDM item parameters can be estimated in three ways. The item parameters from each testing occasion could be (1) estimated concurrently with no constraints (e.g., Collins & Lanza, 2010), (2) estimated concurrently with equality constraints (e.g., Collins & Lanza, 2010; Li, et al., 2015), or (3) fixed to pre-calibrated estimates from a specific time point. In a DCM framework, all three estimation options are viable, assuming that the model fits the data at both time points. This is because DCM classifications are item-invariant (Madison & Bradshaw, 2015). That is, when the model fits the data, LCDM classifications are invariant with respect to the set of items used to obtain them.

Each estimation option, however, has advantages and disadvantages. Estimating the item parameters at each time point concurrently with no constraints allows for the greatest model likelihood because the model is free to estimate unique item parameters for each testing occasion. However, this method may result in item parameter estimates that are not invariant across time points. When item parameter invariance does not hold across time points in a latent transition analysis, the interpretation of latent class memberships and transitions can become complicated, because the meaning of each latent class can change between testing occasions. In a DCM context, item parameter invariance over time points in the LTA implies that the meaning of attribute mastery does not change across time points, and therefore, the attribute profiles retain the same meaning over time points.

Estimating the item parameters concurrently with equality constraints may consist of constraining item parameters to be equal at pre- and post-test. This method maintains (i.e., forces) parameter invariance, but if the item parameters vary greatly between time points, constraining the item parameters to be equal could result in a misspecified model and, thus, invalid conclusions. Fixing item parameters to the pre-calibrated estimates from one particular time point may include estimating pre-test parameters and fixing post-test parameters, or vice versa. This method significantly reduces the number of estimated parameters compared to concurrently estimating the item parameters without equality constraints. This approach can eliminate estimation bias if items parameters are biased at a particular time point, however, this method bears the same risk of model misspecification due to forced item parameter invariance.

In a typical pre-test/post-test designed study, pre-test item responses may contain random noise because examinees have not had an opportunity to learn the concepts or skills being assessed and, as a result, are more likely to guess or respond haphazardly. Because the LCDM does not directly account for guessing, this can result in item parameter bias and reduced reliability and validity (Rogers, 1999; Santelices & Wilson, 2012).

In this study, we focus on the case that examinees are administered the same collection of items at pre- and post-test. In addition, we fix pre-test item parameters at their respective pre-calibrated post-test item parameter estimates. Since pre-test items are identical to post-test items in this study, it is reasonable to assume that, if students had an opportunity to learn, then items would behave similarly at pre- and post-test. If examinees were given a different set of items at pre- and post-test, we would have to concurrently estimate the item parameters at each time point with no constraints, and rely on the item-invariance of LCDM classifications (Bradshaw & Madison, 2015). Fixing the pre-test parameters at their respective pre-calibrated post-test estimates reduces the number of estimated parameters, forces measurement invariance from pre-test to post-test, and does not contaminate the item parameter estimates with the effects of guessing from the pre-test responses. When item parameters are fixed at pre-calibrated estimates, the probability of Examinee e belonging to Class j at pre-test and k at post-test, conditional on their pre- and post-test responses is given by:

$$\mathbb{P}(\boldsymbol{\alpha}_j, \boldsymbol{\alpha}_k | \boldsymbol{x}_e) = \frac{\nu_{c_j} \tau_{c_k|c_j} \prod_{t=1}^{2} \prod_{i=1}^{I} \pi_{ic}^{x_{eit}} (1-\pi_{ic})^{1-x_{eit}}}{\sum_{c_1=1}^{C} \sum_{c_2=1}^{C} \nu_{c_1} \tau_{c_2|c_1} \prod_{t=1}^{2} \prod_{i=1}^{I} \pi_{ic}^{x_{eit}} (1-\pi_{ic})^{1-x_{eit}}}. \tag{2.6}$$

Transition Diagnostic Classification Model

The proposed model combines LTA with the LCDM as the measurement model. We will refer to this model as the Transition Diagnostic Classification Model (TDCM). In the Li et al. study, the DINA model is used in conjunction with LTA (2015), referred to as the LTA-DINA model. As noted above, the DINA model is a submodel or special case of the LCDM that is obtained by constraining parameters in the LCDM. Therefore, the TDCM is a generalization of the LTA-DINA model. In developing the TDCM, we are also developing LTA hybrid versions of several other DCMs subsumed by the LCDM. Namely, parameters in the TDCM can be constrained to produce LTA hybrids of the commonly used DCMs such as the Noisy Inputs Deterministic And Gate model (NIDA; Maris 1999), the Reduced Reparameterized Unified Model (or rRUM; Roussos, DiBello, Stout, Hartz, Henson, & Templin 2007), the Deterministic Inputs Noisy Or Gate model (DINO; Templin & Henson 2006), the Noisy Inputs Deterministic Or Gate model (NIDO; Templin 2006), and the Compensatory Reparameterized Unified Model (C-RUM; Hartz 2002).

Assessing Statistical Significance of Growth in the TDCM Framework

The TDCM is designed to assess growth in attribute mastery across time points. As such, the output from the TDCM provides attribute mastery proportions for each time point. These mastery proportions can be used to compute effect sizes to assess the practical significance of the observed growth. The TDCM, however, does not directly assess the statistical significance of the observed growth. An assessment of statistical significance is needed to facilitate a more complete evaluation of the observed growth in attribute mastery. In particular, a null hypothesis significance test (NSHT) would be useful to test whether the pre-test attribute mastery proportion is significantly different from the post-test attribute mastery proportion.

In traditional statistical models like linear regression, Wald tests are often used to assess the significance of model parameters. In the TDCM, Wald tests can be used to assess growth in attribute mastery. Let the pre-test mastery proportion (p_{pre}) be compared to the post-test mastery proportion (p_{post}). The Wald test statistic for testing $H_0\colon p_{pre} = p_{post}$ vs. $H_a\colon p_{pre} \neq p_{post}$, is given by:

$$z = \frac{\hat{p}_{pre} - \hat{p}_{post}}{se(\hat{p}_{pre} - \hat{p}_{post})} \;, \tag{2.7}$$

where $\hat{p}_{pre} - \hat{p}_{post}$ is the maximum likelihood estimate of $p_{pre} - p_{post}$ and $se(\hat{p}_{pre} - \hat{p}_{post})$ is its standard error. For maximum likelihood estimates, the Wald test statistic follows a standard normal distribution. Therefore, the statistical significance of the Wald test statistic can be assessed by computing its two-tailed standard normal probability and comparing it to the specified significance level. In the TDCM, the attribute mastery proportions for the pre- and post-test are functions of the latent class means and transition probabilities. In Mplus, the standard errors of the attribute mastery proportions are computed using the multivariate delta method. In Appendix B, we provide an illustration of the multivariate delta method for the computation of the estimate and standard error of an attribute mastery proportion in the two-attribute case.

Alternative Methods of Assessing Growth in a DCM Framework

This study focuses on the proposed model, the TDCM, as a generalization of the LTA-DINA model (Li et al., 2015). Above, we described how the Wald test can be used with the TDCM to assess the statistical significance of observed growth in attribute mastery. The Wald test uses distributional theory to obtain standard errors and the resulting Wald test statistic. There are, however, other methods by which one could statistically assess growth in a DCM framework. This study examines two such methods for comparison to the Wald test, which we refer to as the Naïve Method (NM) and the Resampling Method (RM).

In the NM, pre-test ($\hat{\alpha}_{e,pre}$) and post-test ($\hat{\alpha}_{e,post}$) classifications are obtained by first calibrating the DCM with pre-test (or post-test) item responses, then scoring the post-test (or pre-test) responses with item parameters fixed at their pre-calibrated estimates. Then for each attribute, a dependent samples proportions test is used to assess the significance of growth from pre-test to post-test. We term this method the Naïve Method because it ignores the error associated with the model-based classifications, and does not account for the dependence in pre-test and post-test responses within a given examinee. As a result, we expect the Type 1 error rates of the statistical tests of attribute mastery growth for the NM to be higher than the Wald test in the TDCM.

The Resampling Method, similar to pseudo-class draws (Wang, Brown, & Bandeen-Roche, 2005), uses a more sophisticated approach to assessing growth than the Naïve Method. The RM resamples, with replacement, from the model-based pre-test and post-test posterior attribute mastery probabilities. This resampling process is repeated many times. For each new sample, the difference in pre-test and post-test mastery proportions is recorded. Once completed, the mean and standard deviation of these differences across the replications is computed and are input as the numerator and denominator in a resampling analogue of the Wald test described above. Because the RM accounts for measurement error in classifications and sampling error in the resampling procedure, the Wald-like test in the RM is expected to perform better than the dependent samples test within the NM with respect to Type I error, but not as well as the Wald test in the TDCM. One reason is that it is heavily dependent on the uniqueness of the given sample, which may contain idiosyncrasies, especially with smaller samples.

Simulation Study

To examine the performance of the TDCM in a variety of assessment contexts, we conducted a simulation study. Li et al. did not conduct a simulation study to assess the performance of the LTA-DINA under varying conditions, so information about the theoretical performance of these types of models is not yet available. Results of this study will provide such information, as well as evidence as to why this generalization is worthy of development. Lastly, the simulation study results will compare the TDCM's Wald test to the NM and RM for detecting statistical significance of growth in attribute mastery over time.

We manipulated key factors including the number of attributes, sample size, Q-matrix, attribute pre-test and post-test mastery proportions or *base-rates*, and marginal mastery transition probabilities. Factors including item parameter magnitude and pre-test attribute correlations were fixed. These simulation conditions are described in detail below.

Number of Attributes

The simulation study used two attribute-number conditions: a two-attribute condition and a four-attribute condition. Two- and four-attribute tests fall within the range of recent LCDM applications and simulation studies (Bradshaw & Templin, 2014; Choi, 2009; de la Torre, 2011; Henson, et al., 2009; Kunina-Habenicht, Rupp, & Wilhem, 2012; Madison & Bradshaw, 2014; Bradshaw & Madison, 2015; Templin & Hoffman, 2013). Table 2.3 outlines the simulation conditions for the two-attribute and four-attribute conditions.

Q-matrix Designs

The Q-matrix is an item by attribute matrix that specifies which items measure which attributes. In the Q-matrix, if Item i measures Attribute a, then cell ia will be a 1, and 0 otherwise. The two-attribute condition had three 10-item Q-matrices and the four-attribute condition had three 20-item Q-matrices (See Appendix C). The first Q-matrix condition has simple structure, meaning that each item measures exactly one attribute. The second Q-matrix condition is moderately complex, with 40% of the items measuring two attributes. The third Q-matrix condition is highly complex, with 80% of the items measuring two attributes. None of the Q-matrices include items measuring more than two attributes.

The three different Q-matrices were selected to examine the model performance under a variety of plausible test complexities, as well as to illustrate testing scenarios where the TDCM provides utility beyond the LTA-DINA model. For the simple structure Q-matrix, the LTA-DINA model is equivalent to the TDCM. As the percentage of complex items in the Q-matrix increases, we expect the differences in results for the LTA-DINA model and TDCM to also increase. Since data in this simulation study are generated from the LCDM, as the complexity of the Q-matrix increases, the flexibility of the TDCM will provide benefits with respect to classification quality and detection of item parameter effects that the LTA-DINA cannot provide.

Attribute Pre-test Base-rates and Correlations

An attribute base-rate is the proportion of examinees who are masters of the attribute. For example, an easier attribute may have a base-rate of .65, while a more difficult attribute may have a base-rate of .35. Regarding attribute base-rates, we assumed that the majority of examinees are non-masters at pre-test. Therefore, attribute base-rates were all less than .50 at the pre-test occasion. For the two-attribute conditions, base-rates were .20 and .40 for Attributes 1 and 2, respectively. In the four-attribute conditions, the base-rates were .20, .40, .20, and .40 for Attributes 1, 2, 3, and 4, respectively. Examinee pre-test attribute profiles were generated with an attribute correlation of .50.

Marginal Transition Probabilities and Attribute Mastery Growth

To assess the ability of the TDCM to detect growth in attribute mastery from pre-test to post-test, we manipulated transition probabilities for each attribute. Combined with the pre-test attribute base-rates, the marginal attribute transition probabilities were chosen to produce different amounts of growth in attribute mastery. With respect to marginal attribute transitions, we assumed that over the course of a relatively short instructional period, examinees are unlikely to unlearn or forget attributes from pre-test to post-test. Therefore, in the two-attribute conditions, the conditional transition probabilities for attribute loss were fixed at .10 and .20, for Attribute 1 and 2, respectively. In the four-attribute condition, the transition probabilities for attribute loss were fixed at .10, .20, .10, and .20 for Attributes 1, 2, 3, and 4, respectively. Tables 2.4 and 2.5 show the marginal attribute transitions and attribute mastery growth for the two-attribute and four-attribute conditions. The no growth transition condition was designed to assess the Type I error of the statistical tests used to assess growth in attribute mastery from pre-test to post-test.

The variance in growth for the other attributes was manipulated to assess the power of the statistical tests under different amounts of growth. The different amounts of growth were selected in accordance with different levels of odds ratios. In this context, the odds ratio (OR) is used as an effect size measuring the difference in likelihood of attribute mastery at post-test relative to pre-test. For example, an odds ratio of two would indicate that the odds of an examinee being a master at post-test is twice the odds of being a master at pre-test. Let p_{pre} and p_{post} be the respective pre-test and post-test base-rates of attribute mastery. The OR is calculated as:

$$OR = \frac{\text{odds(mastery at post-test)}}{\text{odds(mastery at pre-test)}} = \frac{\left(\frac{p_{post}}{1 - p_{post}}\right)}{\left(\frac{p_{pre}}{1 - p_{pre}}\right)} \tag{2.8}$$

According to Cohen (1988), odds ratios of 1.5, 2.5, and 4.3 are small, medium, and large effect sizes, respectively. Marginal transition probabilities were manipulated to produce different levels of growth in attribute mastery corresponding to Cohen's three odds ratio effect size levels.

Attribute Post-test Base-rates and Correlations

Post-test attribute profiles were generated in conjunction with the pre-test attribute profiles and according to the marginal transition probabilities in Tables 2.4 and 2.5. In some conditions, due to examinee mastery transitions, the attribute correlation increased slightly at post-test. In the two-attribute no growth condition, the attribute correlation remained at .50. In the two-attribute varied growth condition, the correlation increased from .50 to .53. In the four-attribute no growth condition, the correlation increased from .50 to .56. In the four attribute varied growth condition, the correlation increased from .50 to .57.

Item Parameters

Item responses for the pre-test and post-test were generated from the full LCDM, which includes main effects and up to two-way interactions for items measuring two attributes. Item parameters were randomly generated so that complete non-masters, examinees who have mastered none of the required attributes on an item, have between .15 and .30 probability of a correct response and complete masters, examinees who have mastered all the required attributes on an item, have between .60 and .90 probability of a correct response. Partial masters, examinees who have mastered one of the two required attributes on two-attribute items, have between .30 and .60 probability of a correct response.

Sample Sizes

The sample sizes for the two-attribute conditions were 250, 500, and 1000, while the samples sizes for the four-attribute conditions were 750, 1500, and 3000. These sample size ranges were chosen to be consistent with and satisfy data requirements suggested by previous studies (Rupp & Templin, 2008; Bradshaw & Templin, 2012; Cui, Gierl, & Chang, 2012; Kunina-Habenicht, Rupp, & Wilhem, 2012; Madison & Bradshaw, 2014; Bradshaw & Madison, 2015), as well as to reflect a variety of assessment contexts from research applications to operational conditions.

Calibration and Estimation Models

The DINA model and LCDM were calibrated to obtain post-test item parameter estimates. Then pre-test and post-test classifications were obtained with the LTA-DINA model and TDCM with parameters fixed at their pre-calibrated estimates. To demonstrate the need for the TDCM over the LCDM, we also modeled the pre-test and post-test responses separately with the LCDM by first calibrating the LCDM with post-test responses, and then scoring the pre-test responses with the parameters fixed (see Jurich & Bradshaw, 2014). For convenience, we will refer to this separate modeling of pre- and post-test responses with the LCDM as $LCDM_2$. For comparison of the statistical tests of attribute mastery growth, the Naïve and Resampling Methods were used to statistically assess growth in attribute mastery. The Naïve and Resampling Methods both use pre-test and post-test classifications obtained from LCDM_2. The Naïve Method then uses a dependent samples proportions test to test for attribute mastery growth from pre-test to post-test. The Resampling Method uses a resampling analogue of a Wald-test, which we refer to as a “Wald-like” test.

Data were generated in R, Version 3.1.1 and all models were estimated in Mplus, Version 7 (Muthén & Muthén, 2012) using marginal maximum likelihood. Each condition was replicated 250 times. This is more replications than typical LCDM simulation studies, but necessary to obtain stable estimates of Type I error and power. Sample Mplus syntax for the two-attribute TDCM is attached in Appendix D. In sum, the simulation study had $2 \times 3 \times 3 \times 2 = 36$ conditions, 250 replications, and three different modeling approaches, totaling to more than 50,000 analyses.

Evaluation of TDCM Performance

We used several measures to assess the performance of the TDCM. Given that this is a novel and complex model extension, we first assessed the convergence of the proposed TDCM compared to the LTA-DINA model and $LCDM_2$. Next, we assessed classification quality by computing classification accuracy and reliability. Lastly, we investigated the power and Type I error the three methods of statistically assessing growth in attribute mastery.

Convergence Rates

To estimate the LTA-DINA and TDCM, Mplus obtains maximum likelihood estimates via the Expectation-Maximization algorithm (EM; Dempster, Laird, & Rubin, 1977). With maximum likelihood estimation in Mplus (assuming default specifications), the convergence of each replication is a binary outcome (converged or did not converge). The convergence rate is the proportion of replications for which the estimation algorithm converged on a unique solution. Factors expected to impact convergence rates are number of attributes (2 vs. 4), modeling approach (LTA-DINA, TDCM, $LCDM_2$), sample size, and Q-matrix complexity. We evaluated convergence rates across each level of these four simulation design factors.

Classification accuracy and reliability

We evaluated the quality of classifications using marginal classification accuracy and reliability. To calculate classification accuracy, we compared the generated attribute mastery statuses to the estimated attribute mastery statuses. For each individual attribute, the proportion of examinees for which the model-estimated attribute mastery state agrees with the generated mastery state is the correct classification rate. Reliability for classifications refers to the stability of the model-based classifications. For example, posterior probabilities of mastery that are close to 0 or 1 indicate stable classifications that are unlikely to change upon a hypothetical re-examination. Posterior probabilities close to .50 represent model uncertainty and unreliability. Reliabilities of the classifications were calculated according to the tetrachoric correlation-based metric defined by Templin and Bradshaw (2013). Though no differences are expected because pre- and post-test responses were generated with identical parameters, we evaluated classification accuracy and reliability at pre- and post-test. We compared the TDCM's performance to the LTA-DINA model and $LCDM_2$. Because data were generated using the LCDM, the TDCM will have higher classification accuracy than the LTA-DINA model. Also, because the TDCM obtains pre-test and post-test classifications simultaneously, we expected the classification accuracy to be higher than LCDM_2.

Statistical Testing of Growth

For each modeling approach, we examined the Type I error and power of the statistical tests of attribute mastery growth at the .01, .05, and .10 significance levels. Within the TDCM, we expect that the Type I error for each individual Wald test of attribute mastery growth will be close to the specified significance level. It is expected that the dependent samples proportions test in the Naïve Method will suffer from severely inflated Type I error because it does not account for measurement error in the classifications. The Wald-like test in the RM does account for measurement and sampling error, albeit empirically though resampling. So the RM may or may not perform similarly to the TDCM depending on the efficacy of the resampling procedure in this context. Next, we use the varied growth conditions to evaluate the power of statistical tests in the three different modeling approaches. As pointed out by Wollack, Cohen, and Serlin (2001), only when error rates are controlled can power be evaluated. In this study, Type I and family-wise error rates for a given significance level (α) were considered controlled if they fell within Bradley's (1978) liberal criteria: $\alpha \pm 0.50 \cdot \alpha$. We expect that in the conditions where error rates are controlled, the attribute growth effect size and sample size will have a positive relationship with the respective tests' power.

In the two-attribute conditions, we are testing for growth from pre-test to post-test for each of the two individual attributes, and for each of the four attributes in the four-attribute conditions. Because there are multiple attributes, and hence multiple tests, the family-wise error rate could rise above the nominal rate. The family-wise error is the probability of making at least one Type I error across the family of hypothesis tests. In this context, we consider the two tests of mastery growth in the two-attribute conditions, and the four tests of mastery growth in the four-attribute conditions, families of hypothesis tests. If each individual test has Type I error at the specified nominal level, then the family-wise error could possibly rise above the specified nominal level. Therefore, we examined and evaluated methods for controlling family-wise error rate across the multiple tests of attribute mastery growth.

More specifically, we will evaluate the Bonferroni method, Šidák method, and Hochberg's stepwise method for controlling family-wise error rates. Each of these methods adjusts the significance level in attempt to maintain a desired family-wise error rate. Suppose the specified significance level is α and there are m different hypothesis tests. In the traditional Bonferroni method (Dunn, 1961), each test is evaluated at $\tilde{\alpha} = \frac{\alpha}{m}$. In the Šidák method (1967), the family-wise error rate is computed as $\tilde{\alpha} = 1 - (1 - \alpha)^{\frac{1}{m}}$. In Hochberg's stepwise method (Hochberg, 1988), the largest obtained *p*-value is compared to α. If it is less than α, then the mull hypothesis corresponding to the largest *p*-value, and all other null hypothesis tests are rejected. If the null hypothesis corresponding to the largest *p*-value is not rejected, then Hochberg's method proceeds to the second largest *p*-value. If the second largest *p*-value is less than $\frac{\alpha}{2}$, then the corresponding null hypotheses, and null hypotheses corresponding to smaller *p*-values are rejected. Then we proceed to comparing the third largest *p*-value to $\frac{\alpha}{3}$, fourth largest to $\frac{\alpha}{4}$, and so on. We expect that these methods will exhibit more differences in four-attribute conditions than the two-attribute conditions due to the greater number of tests in the family (4 vs. 2). In evaluating these different family-wise error control methods, we considered rates controlled according to Bradley's (1978) liberal criteria.

Simulation Study Results

The results of the simulation study are reported according to the measures described in the previous section. The results will be discussed with a focus on comparing the proposed model, the TDCM, to existing methods of modeling growth in a DCM framework. We first provide results on convergence rates, and subsequent results are based on replications that successfully converged. We then move to classification accuracy and reliability, where results are reported separately for each number of attribute, Q-matrix complexity, and sample size condition.

Next, we evaluate the Type I error, family-wise error, and power of the different statistical tests of attribute mastery growth. Type I and family-wise error rates are evaluated in the no growth conditions. Type I error rates are aggregated over individual attributes. Bonferroni, Šidák, and Hochberg's procedures are used to control family-wise error. Power is evaluated in the varied growth conditions. In varied growth conditions, each attribute has a different growth effect size, so power is evaluated for each individual attribute.

Convergence Rates

Table 2.6 shows the convergence rates for the three different modeling approaches. With respect to Q-matrix complexity, we see that for all three modeling approaches, as the Q-matrix becomes more complex, the convergence rates decrease. In the low and medium Q-matrix complexity conditions, convergence rates are all greater than .90 and mostly approach 1 for each modeling approach and across both the two-attribute and four-attribute conditions. But in the high Q-matrix conditions, convergence rates drop significantly, especially in the lower sample size conditions. This result is consistent with Madison and Bradshaw (2014), who found that Q-matrix complexity had a negative relationship with overall model performance, including convergence rates. In the two-attribute conditions, the LTA-DINA model converged more often than $LCDM_2$ and the TDCM. In the four-attribute conditions, the LTA-DINA model converged less often than $LCDM_2$ and the TDCM. Only in the high complexity Q-matrix conditions, did sample size have an effect on convergence rates, with larger sample sizes converging more often.

Classification Accuracy and Reliability

Table 2.7 shows the marginal attribute correct classification rates (CCR) for the different modeling approaches. For each modeling condition, CCRs are almost identical across the pre-test and post-test. Overall, CCRs are strong, with averages of .90 and .93 in the two-Attribute and four-Attribute conditions. Similar to the convergence rates, we see that Q-matrix complexity plays a large role with classification accuracy rates decreasing as the Q-matrix complexity increases. Across number of attribute, modeling approach, and sample size conditions, CCRs drop by an average of 7.5% from the low Q-matrix conditions to the high Q-matrix conditions. Within each modeling approach and Q-matrix complexity condition, sample size had a slight positive effect on CCRs. With respect to the different modeling approaches, the TDCM had higher CCRs across all conditions. In the low Q-matrix conditions, the LTA-DINA model and the TDCM have identical CCRs. This is because in this simple-structure Q-matrix condition, these two models are equivalent. In the medium and high Q-matrix conditions, however, the TDCM CCRs are greater than the LTA-DINA model by an average of 1% and 4%, respectively. The TDCM was expected to outperform the LTA-DINA model because the data were generated from the saturated LCDM. But results show that the TDCM also has higher CCRs when compared to $LCDM_2$. This result shows that modeling the pre-test and post-test item responses simultaneously, thereby capturing the within-person dependencies, provides additional information that the TDCM uses to more accurately classify examinees.

Tables 2.8 displays the marginal attribute classification reliability for the different modeling approaches. As expected, the pre-test and post-test classification reliabilities are extremely similar, within 1-2% of each other. Overall, classification reliability was high, with an average of .916 across all conditions, and only dropping below .80 in the LTA-DINA model and high complexity Q-matrix conditions. Following the trend seen in convergence rates and classification accuracy, we see that as Q-matrix complexity increases, reliability decreases. Across number of attribute, modeling approach, and sample size conditions, classification reliability drops by an average of 9.4% from the low Q-matrix conditions to the high Q-matrix conditions. Again, we see that the TDCM (.94) outperforms the LTA-DINA (.90) model and $LCDM_2$ (.90).

Type I and Family-wise Error Rates

Tables 2.9 and 2.10 display the Type I error rates for the two-attribute and four-attribute, no growth conditions, respectively. These Type I error rates are without multiple test corrections or adjustments. Values in bold are Type I error rates that are outside the bounds of Bradley's (1978) liberal criteria for acceptable or robust Type I error rates for each respective significance level. It is also clear that Q-matrix complexity plays a large role in Type I error rates; across all considered factors and modeling approaches, Type I error rates increase as the Q-matrix becomes more complex. This is perhaps due to decreasing classification accuracy and reliability in the increasing Q-matrix complexity conditions. Sample size does not appear to have a clear and consistent trend in affecting Type I error rates.

With respect to the three different growth testing approaches, what is immediately evident is that the Naïve Method's Type I error rates are severely inflated across all significance levels, Q-matrix, and sample size conditions. This was expected as the Naïve Method does not account for the measurement error in the model-based classifications. The Resampling Method's results are mixed, with Type I error rates be mostly deflated in the low and medium Q-matrix complexity conditions, and inflated in the high complexity Q-matrix conditions. For the Wald test in the TDCM, Type I error rates were controlled in all but 2/54 conditions; only in the High complexity Q-matrix, two-attribute conditions, with sample sizes of 250 and 500, were the Wald test's Type I error rates are inflated above the range for $\alpha = .01$. Overall, these results suggest that the Wald test within the TDCM is an appropriate statistic for testing attribute mastery growth. In addition, these results suggest that testing for attribute mastery growth while modeling pre- and post-test item responses separately, as in the Naïve and Resampling Methods, is not appropriate as Type I error rates are not controlled, and are mostly inflated above the specified significance level.

Tables 2.11 and 2.12 display the family-wise error rates for the two-attribute and four-attribute, no growth conditions, respectively. These family-wise error rates are also without multiplicity corrections or adjustments. Values in bold are family-wise error rates that were outside the bounds of Bradley's (1978) liberal criteria for acceptable or robust error rates for the respective significance level. In the two-attribute conditions, there are two tests of attribute mastery growth, and in the four-attribute conditions, there are four tests of attribute mastery growth. As there are more tests of growth in the four-attribute conditions, the family-wise error rates were higher in the four-attribute conditions than the two-attribute conditions. In the two-attribute conditions, only the Resampling Method had more than two instances where family-wise error was controlled. In the main, this occurred because the Type I error rates of the Resampling Method were conservative in many conditions. In the four-attribute conditions, every condition resulted in an uncontrolled family-wise error rate. Given that family-wise error rates are a function of Type I error rates, we expected and observed similar trends. As suspected, across most conditions, the family-wise error rate across the multiple tests of growth rise above the specified significance level. We observed the same effect of Q-matrix complexity for family-wise error rates with family-wise error rates increasing as Q-matrix complexity increases.

Regarding the three different testing approaches, the Naïve Method's family-wise error rates were not controlled in any condition and much higher than the specified significance levels, and the highest among the three methods. Similar to the Type I error rates, the family-wise error rates for the Resampling method were sometimes controlled, but mostly inflated. Lastly, the TDCM Wald tests' family-wise error rates are inflated in all but 3/54 conditions. These results motivate the need for a method that will control the familywise error when tests of attribute mastery growth are employed in practice.

Methods for Controlling Family-wise Error Rates

Results for the three different significance levels were generally consistent. Therefore, for space considerations, we only report results for the most commonly used significance level, $\alpha = .05$. Results for $\alpha = .01$ and $\alpha = .10$ are available in the supplementary materials (see Appendix E). Tables 2.13, 2.14, and 2.15 display the observed family-wise error rates for the two-attribute and four-attribute no growth conditions after correction by the Bonferroni, Šidák, and Hochberg methods, respectively. Values in bold are family-wise error rates that are outside the bounds of Bradley's (1978) liberal range for acceptable or robust error rates for $\alpha = .05$, which is (.025, .075). First, we note than none of family-wise error control methods controlled the error rates of the Naïve Method. Across all sample size and Q-matrix complexity conditions, the Naïve Method's family-wise error rates are severely inflated to the point that none of the three family-wise error rate methods could reduce them enough to be considered controlled according to Bradley's liberal criteria. For the Resampling Method, family-wise error rates were mostly either conservative or inflated. Across the three family-wise error control methods, in the low Q-matrix complexity conditions, the RM error rates were conservative, but in the medium and high Q-matrix complexity conditions, error rates were mostly inflated. The Resampling Method's error rates were controlled in only 4 of the 18 conditions (22%) for each of the three multiple test correction methods. These results suggest that even after multiplicity corrections, modeling pre-test and post-test item responses separately, as in the Naïve and Resampling Method, is not an appropriate modeling approach for testing for attribute mastery growth.

The Wald test employed in the TDCM performed quite well. In each of the family-wise error control procedures, the family-wise error rates for the TDCM were controlled in 15 of 18 conditions (83%). In the two-attribute, high complexity Q-matrix, smaller sample size conditions, the family-wise error rates for the Wald test in the TDCM were inflated after multiplicity correction. This is likely due to the decreased accuracy resulting from the increased Q-matrix complexity. Only in the two-attribute, low complexity Q-matrix condition, with sample size of 500, were the Wald test's family-wise error rates deflated. Consistent with the Type I error results, the Wald test employed in the TDCM seems to be an appropriate method for statistically testing for attribute mastery growth from pre-test to post-test.

Power Analysis

Because it was the only statistical test of attribute mastery growth with controlled Type I and family-wise error rates, we only evaluated power for the Wald test in the TDCM. The three family-wise error control methods performed equally with respect to family-wise error control; therefore, we evaluated the power of the Wald test in the TDCM for the Bonferroni, Šidák, and Hochberg methods. Tables 2.16 and 2.17 present the power of the Wald test in the TDCM in the varied growth conditions in the two-attribute and four-attribute conditions, respectively. For space considerations, and because results were generally consistent, we only present results for $\alpha = .05$. Results for $\alpha = .01$ and $\alpha = .10$ are available in the supplementary materials (Appendix E). Power was evaluated in the varied growth conditions, were each attribute had a different growth effect. Growth effect sizes were chosen according to Cohen's (1988) odds ratio effect sizes (small = 1.5, medium = 2.5, large = 4.3). In the two-attribute conditions, Attribute 1 had a small growth effect, and Attribute 2 had a medium growth effect. In the four-attribute conditions, Attributes 1, 2, 3, and 4 had no, small, medium, and large growth effects.

In the two- and four-attribute conditions, we expected and observed a positive effect of sample size with power increasing with sample size. In the two-attribute, small growth conditions, power increased steadily with averages of .60, .89, and .98 for N=250, 500, and 1000, respectively. In the four-attribute conditions, sample size had a smaller, but still positive, effect as power was already in the upper .90s for the small sample size condition (N=750). We also expected and observed that attributes with larger effect sizes had increased power. In the two-attribute conditions, aggregated over sample size and correction method, average power for the medium growth effect was .94, which was greater than power for the small growth effect of .82. In the four-attribute conditions, power approached one for small, medium, and large growth effect conditions. Overall, power for the Wald test in the TDCM was very strong. Only in the two-attribute, small growth effect condition with smaller sample sizes, was power consistently less than .90.

While there is not much distinction between the power of the three family-wise error control procedures, Hochberg's stepwise method had slightly more power, and controlled Type I error rates for Attribute 1 (no growth) in the four-attribute conditions better than the Šidák, and Hochberg methods. The results from this power analysis suggest that the Wald test employed in the TDCM, together with Hochberg's family-wise error control procedure, is an appropriate and adequately powered test for testing growth in attribute mastery.

Empirical Data Analysis

To demonstrate the utility of the TDCM, we analyzed pre-test/post-test data collected over two years in two large-scale mathematics education research studies (Bottge, Ma, Gassaway, Toland, Butler, & Cho, 2014; Bottge, Toland, Gassaway, Butler, Choo, Griffen, & Ma, 2015). The first study included 407 students with disabilities, and the second study had 472 students, 135 of which had disabilities. The overall goal of these studies was to examine the difficulties associated with and improve mathematics problem solving skills for students with disabilities (SWD). In particular, these studies examine an innovative instructional program called *Enhanced Anchored Instruction* (EAI; Bottge, Heinrichs, Chan, Mehta, & Watson, 2003). In EAI, students engage in authentic problem solving sessions where students watch a 10 to 15-minute video presenting a group of adolescents encountering and attempting to solve a problem. Students then search the video for relevant information and use their mathematics knowledge to help the characters in the video develop a solution. Research has shown that embedding mathematics problems in videos can help SWDs unlock the meaning of text-based problems (Bottge, Heinrichs, Chan, & Serlin, 2001). It is not known, however, whether engaging in these types of problem solving activities over a period of instructional time actually improves the problems solving skills of SWDs.

This study provides a secondary analysis of pre-test/post-test data from a researcher-developed mathematics problem-solving test administered to 879 middle school students (6-8 grades) at the beginning and end of a 16-week instructional period. Of the 879 students, 572 (62%) had a documented disability. This problem-solving test had 21 simple structure items measuring four attributes: *ratios and proportional relationships, measurement and data*, *number system (fractions)*, and *geometry (graphing)*. The four attributes were measured by four, six, five, and six item, respectively. As each item was simple structured, the TDCM employs the full LCDM, which included intercepts and main effects. Each item was open-ended and dichotomously scored (correct/incorrect). The goal of this analysis is to analyze the growth of the examinees with respect to attribute mastery. We will provide item parameter estimates, pre- and post-test attribute reliabilities and correlations, pre- and post-test mastery proportions and tests of growth, and individual attribute transition probability matrices.

Empirical Data Analysis Results

Item Parameters

Table 2.18 presents the LCDM item parameters, which were calibrated at post-test and fixed in the TDCM. The median intercept and main effect estimates were -2.05 and 3.03, respectively. These median parameter estimates correspond to correct response probabilities of .11 and .73 for non-masters and masters, respectively. Item quality can be measured by how well they discriminate between non-masters and masters of the required attribute. For example, on Item 1, non-masters of ratios and proportional relationships had a 9% chance of answering correctly, while masters had a 62% chance, resulting in an item discrimination of .53. Overall, the items discriminated well, with item discriminations ranging from .30 to .86, and a median discrimination of .60.

Though it discriminated well (.61), Item 19 was the only item that appeared to have an non-significant main effect according to a Wald test of significance ($\lambda = 15.39, SE = 131.45, p = .907$). Because main effects in the LCDM are constrained to be greater than 0, the null hypothesis of the Wald test ($H_0: \lambda = 0$ vs. $H_a: \lambda \neq 0$) is on the boundary of the parameter space, and therefore, not appropriate. To more appropriately assess the significance of Item 19's main effect, we conducted a likelihood ratio test comparing the model with the Item 19 main effect, to a nested model without the main effect for Item 19. Results of the likelihood ratio test indicated that Item 19's main effect was indeed significant ($\chi_1^2 = 227.19, p < .001$). Therefore, Item 19 was not removed from the test. All 21 items were included in the TDCM analysis.

Attribute Reliability

Reliability is an important characteristic for supporting the interpretation and usage of test scores (AERA, APA, & NCME 2014). For DCMs, the examinee estimates are attribute classifications, which must be reliable for valid interpretation and usage. We quantified DCM reliability using the tetrachoric correlation-based metric developed by Templin and Bradshaw (2013). The pre-test classifications were highly reliable, with reliabilities of .955, .990, .969, .968 for ratios and proportional relationships, measurement and data, number system (fractions), and geometry (graphing), respectively. Classifications were also highly reliable for the post-test, with reliabilities of .968, .996, .973, and .991 for the four attributes, respectively. With each respective attribute being measured by between 4-6 items, these high reliability estimates highlight the utility and efficiency of using DCMs when classification is the goal of the assessment.

Attribute Correlations

In educational settings, when tests measure attributes belonging to a common discipline, moderate to high attribute correlations are commonplace. When a student is a master of an attribute, it is likely that she is a master of other attributes as well. With this mathematics problem-solving test, we observed moderate to high attribute correlations. Table 2.19 displays the pre-test and post-test attribute correlations. Pre-test correlations were lower than the post-test, ranging from .554 to .810. At pre-test, the most correlated attributes were ratios and proportional relationships and number systems (fractions). The least correlated attributes at pre-test were measurement and data and geometry (graphing). The post-test correlations ranged from .720 to .897. At post-test, the most correlated attributes were ratios and proportions and geometry (graphing). The least correlated attributes at post-test were number systems (fractions) and geometry (graphing). While the correlations were higher and approaching .900 at post-test, the overall average correlation was .763, which indicates that there was distinction among attributes and evidence of multidimensionality in the test.

Attribute Mastery Classifications and Growth

Figure 2.1 shows the pre-test and post-test mastery proportions for each attribute. Pre-test attribute mastery proportions range from .38 to .64, with an average of .49. The most mastered attribute at pre-test was measurement and data, while the least mastered was number systems (fractions). Post-test attribute mastery proportions range from .51 to .74, with an average of .63. The most mastered attributed at post-test was geometry (graphing), while the least mastered at post-test was number systems (fractions).

Attribute mastery growth ranged from 5.9% to 20.8%. Students exhibited the most growth for ratios and proportional relationships (20.8%, $\mathrm{OR} = 2.33$) and the least with measurement and data (5.9%, $\mathrm{OR} = 1.31$). Measurement data was in one sense the easiest attribute with the highest pre-test mastery proportion at .64, but had the least growth from pre-test to post-test (5.9%). To assess the statistical significance of mastery growth for each attribute, we employed the Wald test in the TDCM with Hochberg's multiple test correction, as demonstrated in the simulation study. Table 2.20 shows the mastery growth and Wald test results for each individual attribute. As demonstrated in the simulation study, even with a small growth effect size (5.9%, OR = 1.31) for measurement and data, the Wald test still results in a significant *p*-value of .001. Examinees exhibited significant growth in mastery from pre-test to post-test for all attributes as indicated by all attribute growth tests resulting in *p*-values less than or equal to .001.

Attribute Mastery Transitions

While the previous section gives information on overall attribute mastery growth, it does not give details on the individual attribute mastery transitions. One benefit of using the TDCM is that the transition probabilities are estimated and accounted for in the estimation of posterior probabilities of mastery. In this case, there were $4^2 \times 4^2 = 256$ transition probabilities. Using the steps outlined in Appendix A, mastery transition probability matrices were calculated for each individual attribute. These matrices are shown in Table 2.21. The second cell (top right) in each 2×2 matrix represents the conditional probability of transitioning from non-master at pre-test to mastery at post-test. For the first three attributes (ratios and proportional relationships, measurement and data, number systems (fractions)), this cell ranges from .41 to .47, indicating that pre-test non-masters of these attributes are more likely to remain non-masters at post-test than to transition into mastery. This is not the case for geometry (graphing), where the probability of transitioning to mastery at post-test, given non-mastery at pre-test, is .61.

As pointed out earlier, measurement and data was the most mastered attribute at post-test, but here we see it also one of the hardest attributes to master for examinees who were not masters at pre-test. Number systems (fractions) was consistently difficult to master; it had the fewest masters at pre-test (.36) and was the hardest attribute to master given pre-test non-mastery (.41). Also of note is that pre-test masters of number systems (fractions) were more likely to regress into non-mastery (.32) than the other three attributes (.22, .16, .17).

Discussion

This study presents novel methodology for analyzing longitudinal data in a DCM framework. In this study, we focused on the pre-test/post-test design where students are administered the same set of items before and after instruction. In the pre-test/post-test designed experiment in this study, the goal was to evaluate student learning, commonly referred to as growth. In a DCM framework, this means analyzing growth in attribute mastery. Until now, the only way to model growth in the LCDM was to separately model pre-test and post-test item responses. Moreover, statistical tests of growth had never been employed in this context. This study extended the LCDM to the TDCM that is able to simultaneously model data from pre-test/post-test designed experiments, and statistically test for attribute mastery growth.

Results from the simulation study clearly indicate that the TDCM has clear advantages over other methods of analyzing pre-test/post-test designs in a DCM framework. Showing its flexibility when the data is not DINA generated, the TDCM had greater classification accuracy and reliability compared to the LTA-DINA model. Compared to modeling pre- and post-test item responses separately, the TDCM also had increased accuracy and reliability. Modeling the dependency in item responses added information, which enhanced the model's ability to classify examinees accurately and reliably compared to modeling separately.

Results from the simulation study also demonstrated that the Wald test employed in the TDCM is an appropriate and powerful method for statistically testing for growth in attribute mastery. Modeling the pre-test and post-test item responses separately resulted in tests with uncontrolled and inflated Type I and family-wise error rates. When combined with Hochberg's multiple tests correction, the Wald test employed with the TDCM controlled family-wise error rates and still maintained consistently high power, even in conditions with small growth effects and less than ideal sample sizes.

In the empirical data analysis, the utility of the TDCM was demonstrated for investigating effects from a mathematics problem-solving test. The TDCM provided highly reliable classifications, even though attributes were only measured by 4-6 items. Statistical tests of attribute mastery growth indicated that students improved from pre-test to post-test with respect to all four attributes. Mastery transition matrices for each attribute gave information detailing how students transitioned between mastery and non-mastery, and gave information regarding which attributes were the most difficult to learn.

The TDCM modeling framework is not without limitations. The first is that it in cases where the pre- and post-test have different items, the TDCM must rely on an assumption of item invariance for making LCDM classifications. DCM item invariance states that when the model fits, LCDM classifications are invariant to the particular set of items administered (Bradshaw & Madison, 2015). If the DCM does not fit the data, and the TDCM is used under this assumption, this could result in the meaning of attribute mastery changing from pre-test to post-test. Another limitation is model complexity and estimation time. In this study, we focused on the case of pre-test/post-test design with a maximum of four attributes. In the simulation study, some individual replication in the four-attribute conditions took up to 11 hours. If there are more attributes, and/or more time points, the number of estimated parameters increases exponentially. Data requirements and estimation time for additional attributes and times points are unknown, but are likely to be more demanding. In the future, we hope to examine these extensions of the TDCM. First, examining how the model extends to multiple times points and the statistical issues that arise would be worthwhile. In addition, a multiple-group extension of the TDCM would provide an inferential way of assessing intervention effects in a DCM framework. A multiple-group TDCM would allow for the evaluation of differential growth between a treatment and control group, for example. If the treatment group exhibited more growth with respect to attribute mastery, this could be an indication of a successful intervention or treatment effect.

In sum, the TDCM is a novel and general model that can be used to model and test for attribute mastery growth in pre-test/post-test designed assessment studies. The TDCM expands the psychometric toolbox available to researchers and psychometricians by adding a modeling option that addresses and is appropriate for modeling data from a commonly-used assessment design. Now that this methodology is available and estimable in commercially available software (Mplus, see Appendix D), it is hoped that researchers will design diagnostic tests in the context of pre-test/post-test designs, and use the fine-grained feedback to improve instruction, thereby improving student learning.

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CHAPTER 3

ASSESSING INTERVENTION EFFECTS IN A DIAGNOSTIC CLASSIFICATION MODEL[2]

[2] Madison, M.J. and L.P. Bradshaw. To be submitted to *Journal of Educational Measurement*.

Abstract

The assessment of intervention effects is an important objective of educational research. One way to evaluate the effectiveness of an intervention is to conduct an experiment that assigns individuals to a control group and treatment group. In the context of pre-test/post-test designed studies, this is referred to as a control-group pre-test/post-test design. The Transition Diagnostic Classification Model (TDCM) was recently developed to assess growth in a diagnostic classification model framework. The TDCM, however, does not model multiple groups, and therefore is not able to analyze data from a control-group pre-test/post-test designed experiment. In this study, we extend the TDCM to model multiple groups, thereby enabling the examination of group-differential growth and assessment of intervention effects. Results from a simulation study show that the multiple-group TDCM provides accurate classifications, and adequately powered tests of differential growth. The utility of the multiple-group TDCM is demonstrated in the evaluation of an innovative instructional method in mathematics.

The assessment of intervention effects is an important objective of educational research. One way educational systems can improve is through systematic implementation of products, policies, practices, and programs that quality research has identified as successful. The process of evaluating educational interventions can be a daunting task. Within the Institute of Educational Sciences (U.S. Department of Education), there are entire organizations dedicated to this effort. For example, the What Works Clearinghouse (WWC) and Regional Education Labs (REL) exist primarily to evaluate educational interventions and report on their effectiveness.

One important issue related to the evaluation of an educational or instructional intervention is quantifying the amount of learning that has taken place. In order to appropriately quantify learning, there must be measures in place to assess student learning before and after an intervention. Typically, this is completed via some type of pre-test/post-test designed experiment. For each examinee, the difference in performance on the pre-test and post-test can be interpreted as the amount of learning that the examinee has achieved. In assessment, changes in student knowledge over time are referred to as growth and have been studied at length in a variety of contexts. In this study, we will refer to change in student knowledge as growth, even though changes can be negative or decreasing. In a classical test theory framework, the difference in pre-test and post-test scores is often interpreted as a quantification of examinee learning (Williams & Zimmerman, 1996). In an item response theory (IRT) framework, longitudinal models have been developed to assess growth in individual and group ability (Anderson, 1985; Embretson, 1991; Fischer, 1976; Fischer, 1989). In a diagnostic classification model (DCM; Rupp, Templin, & Henson, 2010) framework, recent developments have combined latent transition analysis (LTA; Collins & Wugalter, 1992) in conjunction with a DCM as the measurement model to assess change in attribute mastery over time (Li, Cohen, Bottge, &

Templin, 2015). Li et al. (2015) used LTA in conjunction with the deterministic-inputs, noisy-and-gate (DINA; e.g., Haertel, 1989; Junker & Sijtsma, 2001) model to form what they refer to as the LTA-DINA model. In Chapter 2, we generalized the LTA-DINA by using a general DCM, the log-linear cognitive diagnosis model (LCDM; Henson, Templin, & Willse, 2009), to form the Transition Diagnostic Classification Model (TDCM).

According to the WWC, the most valid way to evaluate the effectiveness of an intervention is to conduct an experiment by assigning individuals to a control group and to an intervention, or treatment, group (What Works Clearinghouse, 2014). In the context of pre-test/post-test designs, this is referred to as a *control-group pre-test/post-test* design (Fraenkel & Wallen, 2003). When the assignment of individuals to groups is done randomly, this type of experiment is referred to as a Randomized Controlled Trial (RCT). Figure 3.1 illustrates an RCT in the context of a control-group pre-test/post-test research design. In Figure 3.1, there are two inherent outcomes: growth for the control group and growth for the treatment group. To analyze data from this type of experiment, in addition to assessing the change exhibited by both groups, the difference in growth between groups needs to be evaluated. Moreover, there needs to be methodology for evaluating the statistical significance of the intervention effect, in addition to its practical significance. If the growth exhibited by the treatment group is determined to be practically or statistically greater than that of the control group, this can be interpreted as evidence of an effective intervention.

In an IRT framework, there is methodology to analyze this type of pre/post-designed experiment. Namely, variants of longitudinal multi-group (Bock & Zimowski, 1997) and mixture IRT models (Bolt, Cohen, & Wollack, 2001) can be used to assess differential growth in observed or unobserved groups, respectively. von Davier, Xu, and Carstensen (2011) used the general diagnostic model (GDM; von Davier, 2005) to compare observed and unobserved group-specific growth of continuous ability estimates in a large-scale assessment of educational outcomes. More recently, Cho, Cohen, and Bottge (2013) combined a multilevel LTA with a mixture IRT model to assess differential growth in continuous ability estimates in observed and unobserved groups.

In a DCM framework, the GDM has multiple-group capabilities that can also analyze growth in multiple-group longitudinal data measuring discrete latent traits. The GDM, however, does not directly model latent class transitions, which give more detailed information on how growth is occurring using transition probabilities. The TDCM combines LTA with a general DCM, and therefore is designed to model attribute mastery transitions and growth over time. The goal of this study is to extend the TDCM to provide comparisons of growth in attribute mastery over time for one or more groups and, thus, accommodate the control-group pre-test/post-test designed experiment described above. We accomplish this by combining a multiple-group LTA with the LCDM.

Method

Diagnostic Classification Models

Diagnostic classification models (DCMs) are multivariate, confirmatory latent class models designed to classify examinees into latent groups according to their performance on a set of items measuring specified categorical latent traits. DCMs are similar to IRT models, with the primary difference being that in DCMs, the examinee latent traits are categorical, as opposed to continuous in IRT models. While other psychometric models can be and are used to classify examinees, DCMs are designed to directly classify examinees, and therefore have greater efficiency and reliability for classification (e.g., Templin & Bradshaw, 2013). DCMs have been applied in educational contexts to diagnose teachers' multiplicative reasoning (Bradshaw, Izsák, Templin, & Jacobson, 2014), to analyze data from a large-scale language assessment (von Davier, 2005), and in psychological contexts to identify psychological disorders (Templin & Henson, 2006).

In this study, we focus on dichotomized latent traits, although DCMs can be extended to accommodate polytomous modeling of latent traits. These dichotomous latent traits are often referred to as *attributes*, and can represent examinee skills, knowledge components, or mental states. For a test measuring A attributes, each examinee is probabilistically classified into one of 2^A attribute profiles. Attribute profiles are A-length vectors that indicate the mastery pattern of each examinee with respect to each measured attribute. For example, an examinee may be classified into attribute profile $[0,1,1,0]$, with the 1s indicating that she has mastered Attributes 2 and 3, and the 0s indicating that she has not mastered Attributes 1 and 4. In general, attribute profiles are denoted by $\boldsymbol{\alpha_c} = [\alpha_1, \alpha_2, \dots, \alpha_A]$ where $\alpha_a = 0$ indicates non-mastery of Attribute a and $\alpha_a = 1$ indicates mastery of Attribute a. DCMs are confirmatory models in the sense that latent classes are specified a priori as the attribute profiles. This is in contrast to traditional latent class analyses where the number and interpretation of latent classes is usually determined through likelihood-based model comparisons.

There exist several DCMs that differ in the manner in which they characterize the relationships between examinee attribute mastery and item responses. In this study, we utilize a general diagnostic model, the log-linear cognitive diagnosis model (LCDM). As implied by its name, the LCDM uses a log-linear framework to parametrize the relationship between examinee attribute mastery and item response probabilities. In this log-linear framework, other popular DCMs, like the DINA model, are easily obtained by constraining certain LCDM parameters.

The LCDM item response function is flexible in its treatment of item and attribute effects. To illustrate this flexibility, consider an item measuring two attributes Attribute 3 (α_3) and Attribute 5 (α_5). The LCDM models the probability of correct response to item i, conditional on membership in Attribute Profile $\boldsymbol{\alpha_c}$ as:

$$P(X_i = 1|\boldsymbol{\alpha_c}) = \frac{\exp\left(\lambda_{i,0} + \lambda_{i,1,(3)}(\alpha_3) + \lambda_{i,1,(5)}(\alpha_5) + \lambda_{i,2,(3,5)}(\alpha_3 \cdot \alpha_5)\right)}{1 + \exp\left(\lambda_{i,0} + \lambda_{i,1,(3)}(\alpha_3) + \lambda_{i,1,(5)}(\alpha_5) + \lambda_{i,2,(3,5)}(\alpha_3 \cdot \alpha_5)\right)}. \tag{3.1}$$

The item parameters in Equation 3.1 include an intercept ($\lambda_{i,0}$), main effects for Attribute 3 ($\lambda_{i,1,(3)}$) and Attribute 5 ($\lambda_{i,1,(5)}$), and an interaction between these two attributes ($\lambda_{i,2,(3,5)}$). These parameters are interpreted similarly to a reference coded analysis of variance model. The intercept is the log-odds of a correct response for the reference group: examinees who have mastered neither Attribute 3 nor Attribute 5 (i.e., examinees with attribute profiles where $\alpha_3 = 0$ and $\alpha_5 = 0$). The main effects for Attribute 3 and Attribute 5 represent the increase in the log-odds of a correct response for examinees who have mastered either Attribute 3 or Attribute 5, respectively. Lastly, the interaction term is the change in log-odds for examinees who have mastered both Attribute 3 and Attribute 5. In the estimation of the LCDM, attribute main effects are typically constrained to be greater than zero. In this example, this constraint forces examinees with different patterns of mastery with respect to Attributes 3 and 5 to have different item response probabilities. This parametrization is more flexible than other DCMs because it allows for the statistical evaluation of whether these attribute profiles have significantly different item responses probabilities. Other DCMs constrain attribute profiles to have equal item response probabilities a priori, and therefore, do not permit such an evaluation.

Latent Transition Analysis

In a traditional latent class model (LCM), there are two components: the structural model and the measurement model. The structural model parameterizes the probability of belonging to each latent class while the measurement model parameterizes how each latent class responds to test items. More formally, in a LCM, the probability of Examinee e's item response vector is given by:

$$\mathbb{P}(\boldsymbol{X}_e = \boldsymbol{x}_e) = \sum_{c=1}^{C} \nu_c \prod_{i=1}^{I} \pi_{ic}^{x_{ei}} (1 - \pi_{ic})^{1-x_{ei}}. \tag{3.2}$$

In Equation 3.2, the ν_c's are the structural model parameters that represent the probability of belonging to latent class c. The π_{ic}'s are the item response probabilities that are parameterized via the specified measurement model.

Latent transition analysis (LTA; Collins & Wugalter, 1992) is a longitudinal extension of latent class models that is designed to simultaneously classify examinees into latent classes and model latent class transition over time. To see this extension, Equation 3.3 presents the probability of Examinee e's item response vector in a LTA:

$$\mathbb{P}(\boldsymbol{X}_e = \boldsymbol{x}_e) = \sum_{c_1=1}^{C} \sum_{c_2=1}^{C} \cdots \sum_{c_T=1}^{C} \nu_{c_1} \tau_{c_2|c_1} \tau_{c_3|c_2} \cdots \tau_{c_T|c_{T-1}} \prod_{t=1}^{T} \prod_{i=1}^{I} \pi_{ic_t}^{x_{eit}} \left(1 - \pi_{ic_t}\right)^{1-x_{eit}} \tag{3.3}$$

In Equation 3.3, present are the same components as the LCM in Equation 3.2. The structural parameters, the ν_{c_1}'s, represent the probability of membership in latent class c at Time Point 1. The π_{ic_t}'s represent the Item i correct response probability at Time Point t. The distinguishing feature of LTA in comparison to a LCM are the $\tau_{c_t|c_{t-1}}$'s, which are the latent class transition probabilities that describe the probability of transitioning from each respective latent class from Time Point $t-1$ to Time Point t. Combined with the Time Point 1 latent class proportions, these latent transition probabilities can be used to calculate the latent class proportions at each time point.

Multiple-group Latent Transition Analysis

LTA can be straightforwardly extended to accommodate grouping variables as a means of observing differences in latent class prevalences and transitions across observed groups (Humphreys & Janson, 2000; Reboussin, Liang, & Reboussin, 1999; Vermunt, Langeheine, & Bockenholt, 1999). Multiple-group LTA has been used to examine differential transitions and latent class prevalences among adolescent depression in 10th and 11th grader cohorts (Collins & Wugalter, 1992) and to model transition in health-risk behavior in eight and ten-year-old males (Reboussin, Reboussin, Liang, & Anthony, 1998). The formulation of the model is similar to Equation 3.3, but the parameters are conditional on observed group membership, which we denote G:

$$\mathbb{P}(\boldsymbol{X}_e = \boldsymbol{x}_e | G = g) = \sum_{c_1=1}^{C}\sum_{c_2=1}^{C}\cdots\sum_{c_T=1}^{C} \nu_{c_1|g}\tau_{c_2|c_1,g}\tau_{c_3|c_2,g}\cdots\tau_{c_T|c_{T-1},g}\prod_{t=1}^{T}\prod_{i=1}^{I}\pi_{ic_t,g}^{x_{eit}}\left(1-\pi_{ic_t,g}\right)^{1-x_{eit}} \tag{3.4}$$

In Equation 3.4, $\nu_{c_1,g}$ represents the probability of membership in Latent Class c at Time Point 1, conditional on membership in observed Group g; and $\pi_{ic_t,g}$ represents the Item i correct response probability at Time Point t, conditional on membership in Latent Class c and membership in observed Group g.

Transition Diagnostic Classification Model

In Chapter 2, we specified the LCDM as the measurement model in LTA to form the Transition Diagnostic Classification Model (TDCM). In the TDCM, the latent classes at each time are specified as the attribute profiles. Therefore, the TDCM serves as a longitudinal and general diagnostic classification model by simultaneously classifying examinees into attribute profiles and modeling examinee transitions to and from different attribute profiles across time points.

Multiple-group TDCM

This study proposes and introduces the multiple-group LDTM as a multiple-group extension of the TDCM developed in Chapter 2. This multiple-group extension enables examination of differential growth across groups as a means of evaluating the effectiveness of an intervention. In this study, we focus on the pre-test/post-test randomized controlled trial depicted in Figure 3.1. The overall goal of the multiple-group TDCM will be to assess differential growth between the treatment group and control group in the pre-test/post-test design. The pre-test/post-test context enables a reduction of Equation 3.4. In a pre-test/post-test designed study, there are only two time points for which transitions are modeled. Thus, Equation 3.4 can be reduced to:

$$\mathbb{P}(\boldsymbol{X}_e = \boldsymbol{x}_e | G = g) = \sum_{c_1=1}^{C} \sum_{c_2=1}^{C} \nu_{c_1|g} \tau_{c_2|c_1,g} \prod_{t=1}^{2} \prod_{i=1}^{I} \pi_{ic_t,g}^{x_{eit}} \left(1 - \pi_{ic_t,g}\right)^{1-x_{eit}}. \quad (3.5)$$

In a pre-test/post-test designed study, pre-test item parameters may be biased because students may not have had an opportunity to learn the measured attributes, which can lead to guessing or other types of indiscriminate responses (Rogers, 1999; Santelices & Wilson, 2012). In this study, we focus on the case that pre-test and post-test items are identical. Therefore, pre-test item parameters will be fixed at their respective, pre-calibrated post-test item parameter estimates (see Chapter 2). In addition, to ensure that attribute mastery maintains the same meaning across groups, item parameters are calibrated with both groups and constrained to be equal across groups. The only differences across groups allowed in this formulation are latent class proportions and latent class transitions. The appropriateness of this assumption can be tested in this framework using a likelihood ratio test comparing the group-constrained model to the unconstrained model. Because DCMs are group-invariant (Bradshaw & Madison, 2015), any differential item functioning would be an issue of model-data fit. Making time-invariance and group-invariance assumptions allows us compare attribute mastery transitions and proportions across groups and across time points and allows us to further reduce Equation 3.5:

$$\mathbb{P}(\boldsymbol{X}_e = \boldsymbol{x}_e | G = g) = \sum_{c_1=1}^{C} \sum_{c_2=1}^{C} \nu_{c_1|g} \tau_{c_2|c_1,g} \prod_{t=1}^{2} \prod_{i=1}^{I} \pi_{ic}^{x_{eit}} (1 - \pi_{ic})^{1-x_{eit}}. \qquad (3.6)$$

In Equation 3.6, the grouping variable is only present in the structural portion of the model and in the latent transition portion of the model.

Assessing Differential Growth in the Multiple-group TDCM

In the randomized control trial assessment design presented in Figure 3.1, there are three outcomes to assess. The first is the attribute mastery growth of the control group; the second is the mastery growth of the treatment group. In Chapter 2, this was accomplished in the single-group TDCM by employing Wald tests of growth in attribute mastery. The third outcome to assess is the difference in growth between the control group and the treatment group. The multiple-group TDCM's primary objective is to assess this differential growth between groups. The model directly provides mastery proportions for each time point and each group. This information can be used to provide a point estimate and an effect size of differential growth. In order facilitate a more comprehensive assessment of differential growth, however, a statistical assessment of significance is needed. We accomplish this via a Wald test. Suppose we are interested in group differential growth of Attribute a. The control group will exhibit some growth in Attribute a mastery ($\delta_{g_1 a}$) and the treatment group will exhibit some as well, ($\delta_{g_2 a}$).

The respective attribute pre-test and post-test mastery proportions are used to calculate growth for each group. Let $p_{(g_1,a)pre}$ and $p_{(g_1,a)post}$ be the control group's pre-test and post-test mastery proportions for Attribute a, respectively. And let $p_{(g_2,a)pre}$ and $p_{(g_2,a)post}$ be the treatments group's pre-test and post-test mastery proportions for Attribute a, respectively. Then $\delta_{g_1a} = p_{(g_1,a)post} - p_{(g_1,a)pre}$ and $\delta_{g_2a} = p_{(g_2,a)post} - p_{(g_2,a)pre}$; the growth for each group is the respective difference in pre-test and post-test attribute mastery. The Wald test tests the hypothesis $H_0\colon \delta_{g_1a} = \delta_{g_2a}$ vs. $H_a\colon \delta_{g_1a} \neq \delta_{g_2a}$. The Wald test statistic in this case is given by:

$$z = \frac{|\hat{\delta}_{g_1a} - \hat{\delta}_{g_2a}|}{se(\hat{\delta}_{g_1a} - \hat{\delta}_{g_2a})}, \tag{3.7}$$

where $\hat{\delta}_{g_1a} - \hat{\delta}_{g_2a}$ is the maximum likelihood estimate of $\delta_{g_1a} - \delta_{g_2a}$ and $se(\hat{\delta}_{g_1a} - \hat{\delta}_{g_2a})$ is its standard error. The Wald statistic z follows a standard normal distribution.

Simulation Study

The goal of this study is the development, examination, and application of the TDCM, which combines the LCDM with multiple-group LTA. The primary objective of this model is to assess differential growth in attribute mastery between the treatment and control group. Since the LCDM has been previously examined under many different conditions, we manipulated simulation design factors that we expected to affect the performance of the multiple-group LTA portion of the model and the Wald test of group differential growth in attribute mastery. The first manipulated factor was the group sample size. Three sample sizes were simulated for mastery and non-mastery groups, respectively: 500, 1000, and 1500. Sample sizes in each condition for both groups (i.e., mastery and non-mastery groups) were kept equal, as keeping group sizes equal was not expected to affect results. The corresponding total sample sizes were 1000, 2000, and 3000.

We also manipulated group-specific pre-test base-rates and levels of growth. We assumed that at the pre-test occasion, most examinees are not masters of the attributes, with base rates ranging from .10 (10% masters) to .40 (40% masters). As shown in Table 3.1, the control group's pre-test base-rates were $\{.10, .20, .30, .40\}$ in all simulation conditions. In one half of the conditions, the treatment group had the same set of attribute pre-test base-rates (i.e., $\{.10, .20, .30, .40\}$); in the other half, the treatment group had different pre-test base-rates of $\{.40, .30, .20, .10\}$; Regarding attribute mastery growth, each group either had no growth, small (10% more examinee masters at post-test compared to pre-test), medium (20% more examinee masters at post-test), or large (30% more examinee masters at post-test) growth for each attribute. The manipulated group-specific base-rates and mastery growth are summarized in Table 3.1.

Conditions 1-4 were designed to assess the Type I error of the Wald test. In these conditions, both groups were simulated to exhibit equal growth in attribute mastery. Therefore, the Wald test should not frequently detect differential growth between groups. In Conditions 5-16, the control group was simulated to exhibit less growth than the treatment group. The amount of differential growth was simulated to differ across the conditions. For example, in Condition 5, the control group had no growth, while the treatment group had small growth. In Condition 6, the control group had no growth, while the treatment group had medium growth. These conditions were designed to assess the Wald test's power to detect differential growth in attribute mastery. As for any statistical test, it was expected that the larger the difference in growth between the two groups, the more likely the test will be to detect it.

There have been several simulation studies examining properties and the performance of general DCMs under different conditions (Bradshaw & Madison, 2015; Bradshaw & Templin, 2012; Choi, 2009; Kunina-Habenicht, Rupp, & Wilhem, 2012; Madison & Bradshaw, 2014; Rupp & Templin, 2008). The following factors were fixed in the present simulation study: the number of groups, Q-matrix, number of attributes, attribute correlations, and item parameter quality. The fixed design factors are described in detail below.

The number of groups was fixed at two to represent a control group versus a treatment group experimental design. Though the model can handle more than two groups (as shown in Equation 3.6), this study was limited to examination of the control group versus treatment group context. The number of measured attributes was fixed at four. A test measuring four attributes was chosen to emulate tests used in educational research applications (Bradshaw, Izsák, Templin, & Jacobson, 2014; Kunina-Habenicht, Rupp, & Wilhelm, 2009; Li et al., 2015). Four dichotomous attributes generate at total of 16 attribute profiles, which were fully crossed at both time points to make 256 latent classes for each group.

The Q-matrix was fixed to have 20 items; 12 items measure only one attribute and eight measure two attributes. The Q-matrix is displayed in Table 3.2. Though the structure of the Q-matrix can have a drastic effect on model performance (DeCarlo, 2011; Madison & Bradshaw, 2014), we chose to fix it at a moderate complexity (i.e., 1.4 attributes per item) and manipulate factors more pertinent to the proposed model.

The next fixed factor was the item quality. Item quality in a DCM framework can be measured via item discrimination. Item discrimination is the difference in item response probabilities for different groups of examinees (e.g., non-masters of an attribute compared to masters of the attribute). Item parameters were generated so that complete non-masters, that is, examinees who have mastered none of the required attributes on an item, have probabilities of a correct response between .15 and .30. Probabilities of a correct response for complete masters, that is, examinees who have mastered all the required attributes on an item, were simulated to be between .60 and .90. Partial masters, that is, examinees mastering one but not both attributes on item requiring two attributes, had between .30 and .60 probability of a correct response. The average discrimination between complete masters and non-masters over all items was simulated to be .50.

Lastly, we fixed the pre-test and post-test pairwise attribute correlations at .50. In this context, a correlation of .50 is considered a moderate sized correlation. Similar attribute/dimension correlations have been observed in educational contexts (see Sinharay, Puhan, & Haberman, 2011; Bradshaw, Izsák, Templin, & Jacobson, 2014) and used in many of the aforementioned DCM simulation studies. These fixed factors are summarized in Table 3.3.

As seen in Table 3.1, there were 16 combinations of base-rate and attribute mastery growth, each representing a different condition. Fully crossed with the three sample size conditions, this made for $16 \times 3 = 48$ conditions. Examinees and item responses were generated in R, Version 3.1.1. One hundred replications of each condition were estimated in Mplus, Version 7 (Muthén & Muthén, 2012). TYPE = MIXTURE instructs Mplus to estimate different latent classes, which correspond to the different attribute profiles. When TYPE = MIXTURE is specified, multiple groups are indicated using the KNOWNCLASS option. In Mplus, the default estimator for TYPE = MIXTURE is maximum likelihood with robust standard errors. Mplus defaults were used for all estimation options. Sample Mplus syntax for the multiple-group TDCM is displayed in Appendix F.

Evaluation of Multiple-group TDCM Performance

Since item parameters were pre-calibrated using the LCDM, and fixed in the multiple-group TDCM, we did not evaluate item parameter recovery, expecting it to be on par with previous LCDM and general DCM simulation studies (Bradshaw & Madison, 2015; Bradshaw & Templin, 2012; Kunina-Habenicht, Rupp, & Wilhem, 2012). The first component in evaluation the multiple-group TDCM was the convergence of the model. Compared to the single group TDCM developed in Chapter 2, the multiple-group extension to the TDCM requires the estimation of more parameters. For example, in the four-attribute case, there are a total of $4^2 = 16$ attribute profiles. The multiple-group TDCM requires the estimation of $16^2 = 256$ additional transition probabilities, and additional group-specific structural parameters. These additional parameters make the model more complex and difficult to estimate, and therefore, we evaluated how often the estimation algorithm was able to successfully converge on a solution.

Next in the evaluation of the multiple-group TDCM are the accuracy and reliability of classifications. If the model is performing properly, then classifications should be at least as accurate and reliable as previous studies have found the LCDM to be under similar conditions. Under the simulation conditions specified, we expect classification accuracy and reliability results to be slightly better than results found for similar LCDM studies because the multiple-group TDCM has additional information (data from two time points instead of one) from which to help classify examinees. Analogous to a linear regression model, where the inclusion of additional meaningful predictors helps the regression model make better predictions, data from additional time points helps the TDCM make more accurate and reliable classifications. Although there is no reason to suspect any differences, due to the lack of model misspecification for the simulated data, accuracy and reliability will be computed separately for the treatment and control group, and for the pre- and post-test. Accurate classification is a pre-requisite for the detection of differential growth, which is discussed below.

The last component in the evaluation of the proposed model is perhaps the most important. In Chapter 2, we found that for the TDCM, a Wald test employed to examine growth in attribute mastery performed well with respect to Type I error and power after correction by a multiple testing procedure. Therefore, in this study, we focused on the Type I error and power of the Wald test for assessing detection of differential growth between groups. Because there is a family of hypothesis tests, namely, a test for each of the four attributes, the family-wise error rate could rise above the specified significance level. Hence, we used the Bonferroni (Dunn, 1961), Šidák (Šidák, 1967), and Hochberg (Hochberg, 1988) stepwise methods for controlling the family-wise error rate. Type I error rates were considered controlled if they fall within Bradley's liberal criteria for a given significance level (α): $\alpha \pm 0.50 \cdot \alpha$. For conditions with group-differential growth in attribute mastery, we will evaluate the power of the Wald test.

Simulation Study Results

We first report on the convergence rates of the model for each sample size condition, then results are reported for the replications that successfully converged. Next, we summarize results for classification accuracy and reliability. These first two evaluations were essentially to ensure that the model was estimable and performing properly. Lastly, we report on results from the Type I error, family-wise error, and power analyses.

Convergence rates

In Mplus, the multiple-group TDCM is estimated using marginal maximum likelihood. In Mplus' implementation of maximum likelihood estimation, either the model converges according to the preset convergence criteria, or it does not. This is in contrast to other methods of estimation where convergence is not a binary outcome. For all conditions and replications, convergence criteria were set at Mplus' defaults. The convergence rates presented in Table 3.4 indicate the proportions of the 100 replications for each condition for which the estimation algorithm converged. Table 3.4 shows the convergence rates by sample size condition. Convergence rates were high for each sample size condition, ranging from .954 in the smallest sample size condition to .976 in the largest sample size condition. These results suggest that when the model fits the data, the multiple-group TDCM is indeed estimable with adequate sample sizes.

Classification Accuracy and Reliability

Classification accuracy is calculated by comparing the simulated examinee attribute mastery statuses to the model estimated attribute mastery statuses. The correct classification rate (CCR) is the proportion of examinees for which the model-based attribute classifications agreed with the simulated attribute mastery status. Table 3.5 shows the CCRs for each sample size, separately for pre- and post-test, as well as for both groups. Overall, classification accuracy was above .90 for all conditions, with an aggregate average of .917. CCRs increased slightly with each increasing sample size condition. As expected, there was minimal effect due to pre- and post-test; CCRs were very similar, with an average absolute deviation of .004. There was also a negligible effect of group, with an average CCR deviation of .006.

Classification reliability refers to the precision and stability of model-based classifications, and is a measure of model certainty in classifications. Reliability was calculated as described by Templin and Bradshaw (2013). In this reliability calculation, examinee test-retest attribute mastery probabilities are aggregated into one $2x2$ contingency table with 4 cells for each test-retest mastery pattern ([0,0], [0,1], [1,0], [1,1]). Then the tetrachoric correlation between the [0,0] and [1,1] cells is computed as the reliability. Table 3.6 displays classification reliability for each sample size, separately for pre- and post-test, as well as for both groups. Overall, classification reliability was high, with an aggregated average of .919, across all conditions. As expected, there were minimal differences between groups and between pre- and post-test, with average reliability deviations of .009 and .004, respectively. Unexpectedly, reliability decreased slightly with each increasing sample size condition. In the smallest sample size condition, the average reliability was .928, and in the largest sample size condition, average reliability was .912.

Type I and Family-wise Error Rates

For testing differential growth in the multiple-group TDCM, a Type I error rate is committed when the Wald test incorrectly determines that the groups exhibited differential growth in attribute mastery. In the simulation study, the first four conditions for each sample size condition were generated as having no differential growth between treatment and control groups. Table 3.7 shows the Type I error rates for these conditions at the .01, .05, and .10 significance levels. Overall, the Wald test had mostly controlled Type I error rates, with Type I error being controlled in 33 of 36 (91.6%) conditions. At $\alpha = .01$, there were three conditions that resulted in Type I error rates that were slightly outside of Bradley's bounds. At $\alpha = .05$ and $.10$, all conditions had controlled Type I error. Control and treatment group base-rates had no effect on Type I error rates. Also, whether the control and treatment groups both had no growth, or had equal and positive growth in attribute mastery, had no effect on Type I error rates. Sample size did not appear to have any effect on Type I error rates.

In this four-attribute simulation study, the family-wise error rate is the probability that, for at least one of the four attributes, Wald tests of differential growth incorrectly determined that the groups exhibited differential growth in attribute mastery. Table 3.8 shows the family-wise error rates for the no group-differential growth conditions. Immediately noticeable is that the family-wise error rates for all conditions were not controlled, and severely inflated. In this context, the four tests of group-differential growth resulted in family-wise error rates that were severely inflated above Bradley's acceptable criteria. These results prompted an examination of family-wise error control methods.

Methods for Controlling Family-wise Error Rates

In an attempt to control family-wise error rates, we employed Bonferroni, Šidák, and Hochberg's multiple test corrections to the Wald tests of group-differential growth in attribute mastery. Table 3.9 shows the family-wise error rates, after correction by these three methods. Only one table is needed because the rates for each method were identical. Other than at the .01 significance level, the family-wise error control methods reduced the error rates, sometimes reducing them to below the lower bound on Bradley's criterion. Overall, in only six of the 36 conditions was the error rate inflated above Bradley's criteria. For the .01 significance level, corrected family-wise error rates were overly conservative in five of the 12 conditions, and inflated in the four conditions. Error rates for the .05 significance level indicated more control than the .01 significance level, with rates being controlled in nine of 12 conditions. For $\alpha = .05$, error rates were inflated in only two of the 12 conditions. Results for the .10 significance level were also improved with the familywise analysis, with rates being controlled in all of the 12 conditions. Overall, the Type I error rates were mostly controlled, and after the correction for multiple tests, family-wise error rates were mostly controlled for $\alpha = .05$ and .10. Thus, the Wald test appears to be an appropriate test for differential attribute mastery growth in the multiple-group TDCM.

Power Analysis

In the simulation study, conditions 5-16 were generated with differing amounts of group-differential growth. Since each multiple tests correction method performed equally with respect to family-wise error rates, we evaluate power for each of the three methods to determine which one should be employed in practice. Tables 3.10, 3.11, and 3.12 display the power of the Wald test for each multiple tests correction method at $\alpha = .05$ for the three sample size conditions, respectively. For space considerations, and because results were consistent, power for $\alpha = .10$ are displayed in the supplementary materials (Appendix G). Since family-wise error rates for $\alpha = .01$ were not controlled, power is not evaluated for the .01 significance level. As expected, sample size and size of the differential growth effect had an impact on power. As sample size increases, power increases. Similarly, as the size of the differential growth effect increases, the power to detect it increases as well. For a sample size of 500, the group differential growth effect was .10, and power was an average of .374. This is in comparison to sample sizes of 1000 and 1500, for which power was an average of .682 and .956, respectively. In all three sample size conditions, when the differential growth effect was .20, power was high, with average of .952, 1, and 1 in the three respective sample size conditions. When the group-differential growth effect was .30, the power for each sample size condition was 1. Pre-test base rates did not have an effect on power. Power was approximately equal for the same and different pre-test base-rate conditions.

With respect to multiple test correction methods, Hochberg's stepwise method consistently had the highest power across all sample size and differential growth effect conditions. This result is consistent with the Chapter 2 application of Hochberg's method to a Wald test of overall growth in the TDCM. Aggregating over all conditions, Bonferroni and

Šidák’s methods had identical average power of .813, while Hochberg’s method had average power of .864. The biggest difference between the methods occurred when group sizes were the smallest at 500 and differential growth was small at .10. In these conditions, Hochberg’s method overpowered Bonferroni and Šidák’s methods by an average of .108 (.447 vs. .339). The results from this power analysis indicate that the Wald test of group-differential growth in attribute mastery, coupled with Hochberg’s family-wise error control method, is an appropriately powered test for evaluation group-differential growth in the multiple-group TDCM.

Empirical Data Analysis

In this section, we present an application of the multiple group TDCM to assess an intervention effect in a randomized controlled trial. We apply it to analyze data collected in two mathematics education research studies (Bottge, Ma, Gassaway, Toland, Butler, & Cho, 2014; Bottge, Toland, Gassaway, Butler, Choo, Griffen, & Ma, 2015). We refer to these as Study 1 and Study 2. In both studies, the overall goal was to evaluate an instructional program called *Enhanced Anchored Instruction* (EAI; Bottge, Heinrichs, Chan, Mehta, & Watson, 2003) and assess its effectiveness in improving the mathematics achievement of students with disabilities (SWD). As mentioned earlier, the only way to truly evaluate an intervention is to conduct an experiment by randomly assigning individuals to a control group and to a treatment group. As such, both studies employed cluster-randomized controlled trials, where clusters of students (nested within teachers) were assigned to a treatment group (EAI) or a control group that received what is referred to as *business as usual* (BAU) instruction. We first describe the treatment and control conditions, and then describe the instrument and examinee sample.

Enhanced Anchored Instruction

EAI is an extension of anchored instruction (AI; Cognition and Technology Group at Vanderbilt, 1997). In anchored instruction, problems are presented, or anchored, in realistic contexts that students can investigate to develop solutions. EAI enhances AI by utilizing sophisticated technology, and by providing students with more opportunities to practice solving challenging problems in engaging and interactive contexts. In the EAI conditions, students watched short video clips of same-age adolescents attempting to solve a problem requiring mathematics. Students then used the relevant information in the video to attempt to solve the problem. Students in EAI also engaged in computer-based activities and hands-on applied projects.

Business as Usual

Classrooms in the BAU group followed the traditional curriculum while covering the same mathematical content as classrooms in the EAI group. In most BAU classrooms, in contrast to EAI classrooms, teachers primarily used traditional teaching methods, first providing procedural instruction, then giving formative assessments to check student learning. Students in BAU instruction also completed project-based activities to supplement their learning (Bottge, et al., 2014, 2015). Measures were taken to ensure that the only difference between the instructional conditions was the use of problem-based instruction in the EAI classrooms verses procedure-based instruction in the BAU classrooms.

Instrument and Sample

In this study, we used a multiple group TDCM to analyze pre-test and post-test data from a researcher-developed problem solving test. Each item on the problem solving test was aligned with a specific Common Core State Standard, and therefore, was well suited to support the inferences obtained from a DCM analysis. The problem solving test had 21 items measuring four attributes: *ratios and proportional relationships*, *measurement and data*, *number system (fractions)*, and *geometry (graphing)*. Each item was designed to measure a single attribute. The four attributes were measured by four, six, five, and six items, respectively. Each item was open-ended and dichotomously scored. Combining data from both Studies 1 and 2, the test was administered to 879 middle school students (i.e., grades 6-8) at the beginning and end of 18-week (Study 1) and 13-week (Study 2) instructional periods. There were 423 students in the EAI classrooms and 456 in the BAU classrooms.

Comparison with Prior Analyses

Previous results from both Study 1 and Study 2 found students in the EAI condition learned more than students in the BAU condition (Bottge, et al., 2014, 2015). These results were reported using subscale gain scores for each attribute in a classical test theory framework to assess. In Chapter 2, the TDCM was used to find that the student sample as a whole exhibited significant growth in mastery for each attribute. Therefore, in Chapter 2, there was no distinction made between students in EAI versus BAU instruction. In this study, we used the multiple group extension to the TDCM to assess the effect of EAI on students' problem solving skills as measured by the problem solving test described above. First, we provide descriptive statistics summarizing item parameter estimates, group-specific pre-test and post-test attribute mastery proportions, and group-specific attribute mastery transition matrices. Then we report on statistical tests assessing the overall growth in attribute mastery exhibited by each group, and group-differential growth in attribute mastery.

Empirical Data Analysis Results

Item Parameters

Table 3.13 displays the LCDM item parameters. Item parameters were calibrated using the post-test item responses, and fixed in the multigroup TDCM. Item parameters were held constant in across BAU (control) and EAI (treatment) groups. They are reported in detail in Chapter 2.

Attribute Mastery Classifications and Growth

Table 3.14 shows the pre-test and post-test attribute mastery proportions, as well as attribute mastery growth by group. At pre-test, the most frequently mastered attribute by both groups was measurement and data, with mastery proportions of .608 and .677 for the BAU and EAI groups, respectively. For both groups, the least mastered attribute at pre-test was number systems (fractions) with mastery proportions of .346 and .375 for the BAU and EAI groups respectively. With respect to growth, each group exhibited significant growth for each individual attribute as evinced by all Hochberg corrected Wald test *p*-values being less than .05. The BAU group exhibited the most growth for number system (fractions) (.134), while the EAI group exhibited the most growth for ratios and proportional relationships (.299).

To facilitate a practical evaluation of the growth in attribute mastery, we computed odds ratio effect sizes for growth. The odds ratio was calculated as the odds of mastery at post-test divided by the odds ratio at pre-test. An odds ratio of two would indicate that mastery at post-test is twice as likely as the mastery at pre-test. The three largest gains in attribute mastery, as measured by the odds ratio effect size, were exhibited by the EAI group for ratios and proportional relationships, number systems (fractions), and geometry (graphing), with odds ratios of 3.47, 2.07, and 2.57, respectively. These would be medium to large size growth effects, according to Cohen's recommendations for effect size (Cohen, 1988). The effects sizes for the BAU group were all less than 1.74, which would be considered small.

Group-differential Growth

The primary objective of the multiple-group TDCM is to model and test for group-differential growth in attribute mastery. Table 3.15 shows the results of the Wald tests of group-differential growth in attribute mastery for each attribute. The EAI group exhibited more growth than the BAU group on each individual attribute. However, the only attribute for which the EAI group showed significantly more growth than the BAU group was ratios and proportional relationships ($\Delta = .176$, SE = .046, $p<.001$).

Attribute Mastery Transitions

The previous sections gave descriptions and tests of overall growth in attribute mastery, but did not detail how examinees are transitioning between mastery and non-mastery for each attribute. One benefit of using the multiple-group TDCM is that transition probabilities are modeled and estimated. For each group, the model estimates 256 transition probabilities that represent transitions from 16 possible pre-test profiles to 16 possible post-test profiles. These profile transition probabilities can be combined with the pre-test attribute profile proportions to produce mastery transition matrices for each individual attribute. Table 3.16 shows the attribute mastery transitions matrices for each group. To interpret the matrices, the cells represent conditional probabilities of transition. For example, for ratios and proportional relationships for the BAU group, the .37 in the upper right corner indicates that 37% of BAU non-masters at pre-test transitioned to mastery at post-test. The .29 in the lower left cell indicates that 29% of BAU masters at pre-test regressed to non-mastery at post-test. For each of the four attributes, pre-test non-masters in the EAI group were more likely to transition into mastery at post-test than pre-test non-masters in the BAU group (upper right cell). For example, pre-test non-masters in the EAI group had a .73 probability of transitioning into mastery at post-test, compared to .50 for pre-test non-masters in the BAU group. Also, for each attribute, pre-test masters in the EAI group were less likely to regress into non-mastery at post-test than pre-test masters in the BAU group (lower left cell). For example, for ratios and proportional relationships, pre-test masters in the BAU group were almost twice as likely (.29) to regress into non-mastery as pre-test masters in the EAI group (.15). Table 3.17 displays the unconditional pre- and post-test attribute mastery proportions by group. The right column of each matrix in Table 3.17 indicate proportions of masters at post-test. For each attribute, the EAI group has higher proportions of masters.

Discussion

This study extended the recently developed TDCM (Chapter 2) to the multiple-group TDCM. In Chapter 2, we developed and applied the TDCM to statistically model and test growth in attribute mastery. The multiple-group extension presented in this study enables the examination of group-differential growth in attribute mastery, and therefore enables the evaluation of intervention effects. More specifically, in a randomized controlled trial, the multiple-group TDCM can be used to statistically assess whether a treatment group exhibits more growth in attribute mastery than a control group, which is the standard that the What Works Clearing House sets for educational researchers in evaluating intervention effects (What Works Clearinghouse, 2014).

Results from the simulation study suggest that the multiple-group TDCM functions properly, as shown by its frequent convergence and high classification accuracy and reliability. Even with group sample sizes of 500, the multiple-group TDCM was able to accurately and reliably classify examinees in both groups, and for both pre-test and post-test. Results from the simulation study also suggest that the Wald test of group-differential growth, combined with Hochberg's (1988) multiple tests correction, had mostly controlled Type I error rates and was powerful in the detection of group-differential growth.

In the empirical data analysis, we presented an application of the multiple-group TDCM to analyze data from a pre-test/post-test, cluster-randomized control trial. The multiple-group TDCM is simultaneously able to assess growth for each group (as in the single group TDCM), and assess group-differential growth. Statistical tests of growth indicated that both the control and treatment group exhibited significant growth in attribute mastery for each attribute. Tests of group-differential growth in attribute mastery indicated that while the EAI (treatment) group exhibited more growth than the BAU (control) group for each attribute, only for ratios and proportional relationships was the difference significant. Additional tests showed that non-masters of ratios and proportional relationships, and non-masters of geometry (graphing) in the EAI group were more likely to transition to mastery over the course of instruction than non-masters in the BAU group. These results from the multiple-group TDCM suggest that EAI did have a significant positive effect on students' problem solving skills. The investigation of the diagnostic problem solving test does not evaluate all aspects of the EAI system, but results from the multiple group TDCM provides one useful indicator of a successful intervention.

In Chapter 2, we mentioned a few limitations of the TDCM. We discussed the reliance on item parameter invariance for pre- and post-test items, and to the estimation time and complexity of the models. The multiple-group model also suffers from these limitations to a higher degree. The multiple-group extension relies on group invariance in addition to the item parameter invariance. In the LTA framework, these constraints can be tested empirically, but if item parameters are found to be different across testing occasions, or across groups, the comparisons and interpretations of attribute mastery and attribute mastery transitions between groups and time points may not be as straightforward as they are in the studies discussed in this paper.

The multiple-group extension is a more complex model than the TDCM and estimation time is an issue given current software and hardware capabilities. In the simulation study, some replications took up to 72 hours to converge. More research needs to be completed to explore data requirements and estimation time for instances with more than two groups, and/or more than two time points.

There is one additional limitation unique to the multiple-group TDCM. The length of the intervention across groups must be held constant. Otherwise, the intervention effect would be confounded with a time effect. For example, suppose the treatment group has an 8-week intervention, while the control group has a 4-week intervention. If the treatment group exhibits more growth than the control group, the potential effect of the intervention cannot be distinguished from the effect of the additional time.

To summarize, this study presented a multiple-group extension to the recently developed TDCM. This extension creates methodology for analyzing data from one of the most popular educational research designs: the pre-test/post-test control group design. Educators and curriculum developers are constantly developing and implementing innovative instructional methods and materials. Now that this methodology exists, we hope it will be useful for educators who seek to evaluate the effectiveness of educational interventions in a DCM framework

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CHAPTER 4

DISCUSSION

There have been many recent advances in the area of diagnostic classification models (DCMs; Rupp, Templin, & Henson, 2010). Arguably, the most significant was the development of the log linear cognitive diagnosis model (LCDM; Henson, Templin, & Willse, 2009). This development was significant because it provided a unified and general framework upon which many previously developed DCMs could be specified by constraining different parameters. One limitation of the LCDM is that it is not directly suited to accommodate longitudinal data. The focus of this dissertation was to address this limitation by developing methodology by which longitudinal data could be modeled in the LCDM framework. In particular, this dissertation is comprised of two separate, but related, studies, both focuses on a longitudinal research design commonly used in educational research: the pre-test/post-test design. In this chapter, we summarize findings from both studies, discuss the implications and significance of this work, and offer future directions for further research.

Study 1: Assessing Growth in a DCM

Traditionally, the term 'growth' has been reserved for continuous measures. For example, in a classical test theory framework, growth refers to an increase in number of items correct, a continuous measure, across different testing occasions. In an item response theory (IRT) framework, growth refers to an increase in estimated ability, another continuous measure, across different testing occasions. In this study, we used the term growth in a DCM framework to represent change in attribute mastery over time. The purpose of this study was to develop methodology to assess growth in attribute mastery in a pre-test/post-test designed study.

As mentioned earlier, the LCDM does not accommodate any type of longitudinal data. Only one study conducted by Jurich and Bradshaw (2014) analyzed pre-test/post-test data in the LCDM. They did so by calibrating the LCDM with pre-test responses, and then separately scoring the post-test responses. This approach ignores the within-person dependency in pre-test and post-test item responses, thereby losing valuable information that could be used to obtain more accuracy and reliable classifications. Further, the approach does not provide a method for assessing the statistical significance of growth.

To address this limitation of the LCDM, we utilized latent transition analysis (LTA; Collins & Wugalter, 1992): a longitudinal extension of the general latent class model that simultaneously models latent class prevalence and transitions between different latent classes over time. In LTA, the measurement portion of standard latent class model is used to parameterize the item response probabilities. The proposed model combines LTA with the LCDM specified as the measurement model, thereby creating a method by which the LCDM can accommodate longitudinal data. This hybrid model is termed the Transition Diagnostic Classification Model (TDCM). The idea to combine LTA with a DCM was initiated by Li, Cohen, Bottge, and Templin (2015), who combined LTA with the deterministic-inputs, noisy-and-gate (DINA; e.g., Haertel, 1989; Junker & Sijtsma, 2001) model, referred to as the LTA-DINA model. The DINA model is a submodel and constrained version of the LCDM. Therefore, the TDCM serves as a generalized and longitudinal version of the DINA model, and many other previously developed non-general DCMs. In addition to not being able to model longitudinal data, there was no way to statistically test the significance of growth in attribute mastery over time in a DCM framework. To address this limitation, we used a Wald test to test the statistical significance of parameters in the TDCM as a means to statistically evaluated the growth in attribute mastery from pre-test to post-test.

Results from the TDCM simulation study showed that the TDCM has clear advantages over other methods of analyzing pre-test/post-test designs in a DCM framework. When data were generated from the LCDM, the TDCM had greater classification accuracy and reliability compared to the LTA-DINA model. Compared to modeling pre- and post-test item responses separately, the TDCM had increased accuracy and reliability. Modeling the dependency in item responses added information, which enhanced the model's ability to classify examinees accurately and reliably. The Wald test employed in the TDCM was an appropriately powered method for statistically testing for growth in attribute mastery. Modeling the pre-test and post-test item responses separately resulted in tests with uncontrollable and inflated error rates. When combined with Hochberg's multiple tests correction (Hochberg, 1988), the Wald test employed with the TDCM controlled error rates and still maintained consistently high power, even in conditions with small growth effects and less than ideal sample sizes.

To demonstrate the utility of the TDCM, we analyzed pre-test/post-test data from a researcher-developed diagnostic mathematics test. The TDCM provided highly reliable classifications, even though attributes were only measured by 4-6 items. Statistical tests of attribute mastery growth indicated that students improved from pre-test to post-test with respect to all four attributes. Mastery transition matrices for each attribute gave information detailing how students transitioned between mastery and non-mastery, and gave information regarding which attributes were the most difficult to master.

Study 2: Assessing Intervention Effects in a DCM

In Study 1, the TDCM provided methodology for assessing growth in the LCDM. For example, if students are administered a diagnostic test at the beginning and end of an instructional period, the TDCM can be used to models students' transitions between non-mastery and mastery of each specified attribute. But suppose that students were randomly assigned to different instructional methods, and of interest was the potential advantages of one instructional method relative to another. This type of experiment is the commonly-used pre-test/post-test randomized control trial, and the TDCM is not designed to assess differential growth exhibited by different groups of students.

To address this limitation of the TDCM, we extended the TDCM to a multiple-group TDCM. This multiple-group extension enables the specification and modeling of two or more groups. By accounting for group differences in attribute mastery and growth, the multiple-group TDCM can assess whether one group of examinees, say a treatment group, exhibits more growth in attribute mastery than another group, say a control group. This procedure is how the What Works Clearing House suggests that educational researchers evaluate intervention effects. We examined the multiple-group TDCM as methodology for analyzing data from a pre-test/post-test randomized controlled trial with a treatment group and a control group. To statistically assess differential growth in attribute mastery, we used a Wald test with Hochberg's (1988) stepwise multiple tests correction.

Results from a simulation study suggested that the multiple-group TDCM functions properly, as evinced by its frequent convergence and high classification accuracy and reliability. Even with group sample sizes of 500, the multiple-group TDCM was able to accurately and reliably classify examinees in both groups, and for both pre-test and post-test. Results from the simulation study also indicated that the Wald test of group-differential growth, combined with Hochberg's (1988) multiple tests correction, had mostly controlled Type I error rates and was powerful in the detection of group-differential growth.

In the empirical data analysis, we presented an application of the multiple-group TDCM to analyze data from a pre-test/post-test, cluster-randomized control trial. The purpose of this experiment was to evaluate the effectiveness of an instructional method called Enhanced Anchored Instruction (EAI; Bottge, Heinrichs, Chan, Mehta, & Watson, 2003) relative the Business as Usual (BAU) instruction. The multiple-group TDCM was able to simultaneously assess growth for each group (as in the single group TDCM) and to assess group-differential growth. Statistical tests of growth indicated that both the EAI and BAU groups exhibited significant growth in attribute mastery for each attribute. Tests of group-differential growth in attribute mastery indicated that while the EAI group exhibited more growth than the BAU group for each attribute, only for one attribute was the difference significant. Additional tests showed that non-masters of two attributes in the EAI group were statistically significantly more likely to transition to mastery over the course of instruction than non-masters in the BAU group. Using results from the multiple-group TDCM, we concluded that EAI had a significant positive effect on students' problem solving skills.

Educational Significance

We believe the TDCM expands the methodological toolbox available to researchers, practitioners, and psychometricians by providing a framework for analyzing longitudinal data in a DCM framework. Previous methods ignored the within-person dependency in item responses, which can reduce classification accuracy and reliability. Furthermore, the multiple-group extension enables the assessment of intervention effects in a DCM framework. For other psychometric models like IRT, methodologies for accommodating longitudinal and multiple group data have existed for decades (Anderson, 1985; Bock & Zimowski, 1997; Bolt, Cohen, & Wollack, 2001; Embretson, 1991; Fischer, 1976, 1989). Until now, there had been no such methodology for general DCMs. DCMs have emerged as a viable psychometric alternative to IRT due to their ability to provide fine-grained, multidimensional, and actionable feedback for teachers and students. DCMs provide a parametric approach to obtain classifications that have improved efficiency, accuracy, and reliability compared to various other methods (e.g., Templin & Bradshaw, 2013). Now these benefits of DCMs are available when analyzing longitudinal data in pre-test/post-test designed study. We note that all the analyses in this dissertation were completed in commercially available software (Mplus; Muthén & Muthén, 2012).

Future Directions

Study 1 of this dissertation focused on the pre-test/post-test design (two time points). The extension in Study 2 focused on the pre-test/post-test randomized controlled trial (two time points with two groups). Therefore, the most immediate direction for future research would be an investigation into the extension of the TDCM and multiple-group TDCM to accommodate more than two time points and more than two groups, respectively. Both models are very complex; in an empirical analysis with four attributes, 21 items, and two time points, the multiple-group TDCM had 543 estimated parameters, and took 59 hours for estimation to complete. The number of estimated parameters grows exponentially with the number of time points and attributes. In order to apply the TDCM and multiple group extension in practice, further research needs to determine data requirements for acceptable performance. Given the increased estimation time that could ensue with this generalization, estimation that is more efficient is another direction for future research.

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Table 2.1

Example Transition Probability Matrix for Four Attribute Profiles

		Post-test			
	Attribute Profile	[0,0]	[0,1]	[1,0]	[1,1]
Pre-test	[0,0]	.20	.18	.35	.27
	[0,1]	.10	.25	.05	.60
	[1,0]	.05	.15	.10	.70
	[1,1]	.02	.01	.01	.96

Table 2.2

Example Attribute 1 Conditional Transition Matrix

		Post-test	
	Mastery State	0	1
Pre-test	0	.37	.63
	1	.13	.87

Table 2.3

Simulation Study Design

	Number of Attributes	
Design Factor	Two	Four
Number items	10	20
Sample sizes	250, 500, 1000	750, 1500, 3000
Q-matrix % complex items	0, 40%, 80%	0, 40%, 80%
Attribute correlation (pre-test)	.50	.50
Attribute base-rates (pre-test)	Attribute 1: .20 Attribute 2: .40	Attribute 1: .20 Attribute 2: .40 Attribute 3: .20 Attribute 4: .40
Generating model	Saturated LCDM	Saturated LCDM
Correct response probability interval for complete non-masters	(.15, .30)	(.15, .30)
Correct response probability interval for complete masters	(.60, .90)	(.60, .90)
Estimating Model	DINA, LCDM, LTA-DINA, TDCM	DINA, LCDM, LTA-DINA, TDCM

Table 2.4

Two-Attribute Conditional Transition Probabilities

<table>
<tr><th>Condition</th><th>Attribute 1</th><th>Attribute 2</th></tr>
<tr><td>No growth</td><td>Attribute 1: base-rate = .20
Mastery increase = 0
<table><tr><td></td><td></td><td colspan="2">Post-test</td></tr><tr><td></td><td></td><td>0</td><td>1</td></tr><tr><td rowspan="2">Pre-test</td><td>0</td><td>.98</td><td>.02</td></tr><tr><td>1</td><td>.10</td><td>.90</td></tr></table></td><td>Attribute 2: base-rate = .40
Mastery increase = 0
<table><tr><td></td><td></td><td colspan="2">Post-test</td></tr><tr><td></td><td></td><td>0</td><td>1</td></tr><tr><td rowspan="2">Pre-test</td><td>0</td><td>.87</td><td>.13</td></tr><tr><td>1</td><td>.20</td><td>.80</td></tr></table></td></tr>
<tr><td>Varied Growth</td><td>Attribute 1: base-rate = .20
Mastery increase = .07
$OR = 1.5$ (small)
<table><tr><td></td><td></td><td colspan="2">Post-test</td></tr><tr><td></td><td></td><td>0</td><td>1</td></tr><tr><td rowspan="2">Pre-test</td><td>0</td><td>.88</td><td>.12</td></tr><tr><td>1</td><td>.10</td><td>.90</td></tr></table></td><td>Attribute 2: base-rate = .40
Mastery increase = .23
$OR = 2.5$ (medium)
<table><tr><td></td><td></td><td colspan="2">Post-test</td></tr><tr><td></td><td></td><td>0</td><td>1</td></tr><tr><td rowspan="2">Pre-test</td><td>0</td><td>.49</td><td>.51</td></tr><tr><td>1</td><td>.20</td><td>.80</td></tr></table></td></tr>
</table>

Note. OR = odds ratio

Table 2.5

Four-Attribute Marginal Transition Probabilities

Condition	Attribute	Parameters	Pre-test	Post-test 0	Post-test 1
No growth	Attribute 1	Attribute 1: base-rate = .20 Mastery increase = 0	0	.98	.02
			1	.10	.90
	Attribute 2	Attribute 2: base-rate = .40 Mastery increase = 0	0	.87	.13
			1	.20	.80
	Attribute 3	Attribute 3: base-rate = .20 Mastery increase = 0	0	.98	.02
			1	.10	.90
	Attribute 4	Attribute 4: base-rate = .40 Mastery increase = 0	0	.87	.13
			1	.20	.80
Varied Growth	Attribute 1	Attribute 1: base-rate = .20 Mastery increase = 0 $OR = 1$ (none)	0	.98	.02
			1	.10	.90
	Attribute 2	Attribute 2: base-rate = .40 Mastery increase = .10 $OR = 1.5$ (small)	0	.70	.30
			1	.20	.80
	Attribute 3	Attribute 3: base-rate = .20 Mastery increase = .18 $OR = 2.5$ (medium)	0	.74	.26
			1	.10	.90
	Attribute 4	Attribute 4: base-rate = .40 Mastery increase = .34 $OR = 4.3$ (large)	0	.30	.70
			1	.20	.10

Table 2.6

Convergence Rates for Different Modeling Approaches

Number of Attributes	Q-matrix Complexity	Sample Size	Model		
			$LCDM_2$	LTA-DINA	TDCM
Two	Low	250	1	.992	.992
		500	.998	.994	.994
		1000	1	.988	.988
	Medium	250	.986	.982	.974
		500	.996	.994	.986
		1000	.998	.988	.984
	High	250	.684	.934	.638
		500	.758	.968	.728
		1000	.860	.982	.848
Four	Low	750	1	.976	.976
		1500	.998	.960	.960
		3000	1	.958	.958
	Medium	750	.992	.978	.924
		1500	1	.984	.924
		3000	1	.978	.970
	High	750	.740	.718	.724
		1500	.916	.726	.768
		3000	.968	.660	.856

Note. $LCDM_2$ is calibration of LCDM with post-test responses, followed by scoring of pre-test responses.

Table 2.7

Marginal Attribute Classification Accuracy for Different Modeling Approaches

Number of Attributes	Q-matrix Complexity	Sample Size	Pre-test			Post-test		
			$LCDM_2$	LTA-DINA	TDCM	$LCDM_2$	LTA-DINA	TDCM
Two	Low	250	.915	.933	.933	.915	.933	.933
		500	.919	.939	.939	.919	.938	.938
		1000	.922	.942	.942	.921	.940	.940
	Medium	250	.897	.909	.915	.900	.911	.917
		500	.906	.918	.926	.906	.918	.926
		1000	.909	.922	.93	.910	.922	.931
	High	250	.834	.823	.842	.833	.826	.842
		500	.853	.831	.866	.854	.834	.866
		1000	.864	.834	.88	.865	.839	.881
Four	Low	750	.942	.958	.958	.946	.965	.965
		1500	.943	.960	.960	.947	.968	.968
		3000	.943	.961	.961	.948	.969	.969
	Medium	750	.913	.913	.931	.924	.925	.943
		1500	.916	.919	.936	.926	.931	.948
		3000	.917	.922	.938	.926	.934	.950
	High	750	.878	.854	.895	.894	.868	.912
		1500	.884	.862	.904	.900	.878	.922
		3000	.888	.869	.910	.903	.887	.927

Note. Classification accuracy rates are averaged over individual attributes. $LCDM_2$ is calibration of LCDM with post-test responses, followed by scoring of pre-test responses.

Table 2.8

Marginal Attribute Classification Reliability for Different Modeling Approaches

Number of Attributes	Q-matrix Complexity	Sample Size	Pre-test			Post-test		
			$LCDM_2$	LTA-DINA	TDCM	$LCDM_2$	LTA-DINA	TDCM
Two	Low	250	.914	.952	.952	.925	.955	.955
		500	.912	.951	.951	.918	.953	.953
		1000	.911	.952	.952	.915	.952	.952
	Medium	250	.896	.913	.935	.916	.926	.944
		500	.892	.913	.935	.905	.922	.940
		1000	.889	.913	.934	.899	.921	.939
	High	250	.871	.803	.900	.898	.809	.916
		500	.851	.790	.886	.865	.791	.892
		1000	.847	.786	.883	.851	.779	.884
Four	Low	750	.946	.979	.979	.953	.985	.985
		1500	.945	.978	.978	.952	.985	.985
		3000	.945	.977	.977	.951	.985	.985
	Medium	750	.899	.919	.948	.921	.941	.964
		1500	.897	.917	.947	.918	.939	.963
		3000	.895	.916	.945	.915	.939	.963
	High	750	.846	.845	.902	.880	.871	.927
		1500	.840	.835	.897	.869	.863	.923
		3000	.835	.827	.894	.865	.857	.922

Note. Classification reliabilities are averaged over individual attributes. $LCDM_2$ is calibration of LCDM with post-test responses, followed by scoring of pre-test responses.

Table 2.9

Attribute Mastery Growth Test Type I Error Rates for Two-Attribute No Growth Conditions

Q-matrix Complexity	Sample Size	$\alpha = .01$ (.005, .015)			$\alpha = .05$ (.025, .075)			$\alpha = .10$ (.05, .15)		
		NM	RM	TDCM	NM	RM	TDCM	NM	RM	TDCM
Low	250	**.032**	**.000**	.014	**.126**	**.014**	.041	**.194**	.052	.080
	500	**.046**	**.000**	.006	**.098**	**.014**	.026	**.156**	**.048**	.067
	1000	**.074**	.006	.014	**.140**	**.008**	.038	**.228**	.052	.099
Medium	250	**.059**	**.004**	.012	**.176**	**.022**	.051	**.245**	.087	.086
	500	**.068**	**.000**	.014	**.177**	.028	.043	**.247**	.072	.084
	1000	**.072**	**.000**	.010	**.152**	.026	.039	**.224**	.068	.081
High	250	**.134**	**.017**	**.031**	**.236**	**.082**	.064	**.355**	**.205**	.138
	500	**.142**	**.019**	**.029**	**.240**	.063	.060	**.317**	.147	.110
	1000	**.128**	.011	.009	**.258**	.066	.057	**.319**	**.185**	.120

Note. NM = Naïve Method, RM = Resampling Method; α = significance level. The Naïve Method employs a dependent samples proportions test. The Resampling Method employs a resampling analogue of a Wald test. The TDCM employs a traditional Wald test. The intervals in parentheses represent the range of acceptable values of the observed Type I error rate for a given α level according to Bradley's (1978) liberal criteria. Bold values are outside the range of Bradley's acceptable Type I error rates.

Table 2.10

Attribute Mastery Growth Test Type I Error Rates for Four-Attribute No Growth Conditions

Q-matrix Complexity	Sample Size	$\alpha = .01$ (.005, .015)			$\alpha = .05$ (.025, .075)			$\alpha = .10$ (.05, .15)		
		NM	RM	TDCM	NM	RM	TDCM	NM	RM	TDCM
Low	250	**.035**	**.004**	.007	**.108**	.030	.045	**.170**	.091	.086
	500	**.032**	**.003**	.007	**.105**	.025	.055	**.191**	.091	.091
	1000	**.045**	**.004**	.008	**.123**	.037	.053	**.189**	.107	.105
Medium	250	**.063**	.008	.005	**.140**	.064	.041	**.210**	**.152**	.089
	500	**.065**	.010	.008	**.161**	.071	.036	**.239**	**.174**	.091
	1000	**.057**	**.018**	.008	**.143**	.063	.054	**.224**	**.157**	.095
High	250	**.119**	**.048**	.011	**.228**	**.152**	.046	**.310**	**.263**	.085
	500	**.106**	**.047**	.009	**.222**	**.126**	.047	**.297**	**.254**	.095
	1000	**.105**	**.040**	.007	**.215**	**.132**	.037	**.301**	**.247**	.089

Note. NM = Naïve Method, RM = Resampling Method; α = significance level. The Naïve Method employs a dependent samples proportions test. The Resampling Method employs a resampling analogue of a Wald test. The TDCM employs a traditional Wald test. The intervals in parentheses represent the range of acceptable values of the observed Type I error rate for a given α level according to Bradley's (1978) liberal criteria. Bold values are outside the range of Bradley's acceptable Type I error rates.

Table 2.11

Attribute Mastery Growth Test Family-wise Error Rates for Two-Attribute No Growth Conditions

Q-matrix Complexity	Sample Size	$\alpha = .01$ (.005, .015)			$\alpha = .05$ (.025, .075)			$\alpha = .10$ (.05, .15)		
		NM	RM	TDCM	NM	RM	TDCM	NM	RM	TDCM
Low	250	**.064**	**.000**	**.029**	**.236**	.028	**.082**	**.356**	.104	**.155**
	500	**.092**	**.000**	.012	**.192**	.028	.053	**.292**	.096	.134
	1000	**.132**	.012	**.028**	**.244**	**.016**	**.077**	**.392**	.096	**.190**
Medium	250	**.117**	.008	**.025**	**.328**	.045	**.099**	**.425**	**.166**	**.169**
	500	**.133**	**.000**	**.029**	**.341**	.056	**.086**	**.434**	.141	**.165**
	1000	**.136**	**.000**	**.020**	**.284**	.048	**.077**	**.404**	.132	**.154**
High	250	**.244**	**.034**	**.061**	**.403**	**.159**	**.129**	**.563**	**.364**	**.233**
	500	**.264**	**.038**	**.058**	**.438**	**.125**	**.115**	**.543**	**.269**	**.204**
	1000	**.238**	**.022**	**.018**	**.449**	**.128**	**.109**	**.546**	**.330**	**.222**

Note. NM = Naïve Method, RM = Resampling Method; α = significance level. The Naïve Method employs a dependent samples proportions test. The Resampling Method employs a resampling analogue of a Wald test. The TDCM employs a traditional Wald test. The intervals in parentheses represent the range of acceptable values of the observed family-wise error rate for a given α level according to Bradley's (1978) liberal criteria. Bold values are outside the range of Bradley's acceptable family-wise error rates.

Table 2.12

Attribute Mastery Growth Test Family-wise Error Rates for Four-Attribute No Growth Conditions

Q-matrix Complexity	Sample Size	$\alpha = .01$ (.005, .015)			$\alpha = .05$ (.025, .075)			$\alpha = .10$ (.05, .15)		
		NM	RM	TDCM	NM	RM	TDCM	NM	RM	TDCM
Low	250	**.132**	**.016**	**.020**	**.360**	**.116**	**.166**	**.532**	**.312**	**.304**
	500	**.112**	.012	**.030**	**.360**	**.092**	**.186**	**.580**	**.312**	**.297**
	1000	**.164**	**.016**	**.033**	**.416**	**.132**	**.188**	**.572**	**.360**	**.342**
Medium	250	**.218**	**.032**	**.022**	**.468**	**.222**	**.154**	**.605**	**.492**	**.285**
	500	**.232**	**.040**	**.030**	**.476**	**.256**	**.139**	**.636**	**.504**	**.322**
	1000	**.216**	**.064**	**.034**	**.448**	**.236**	**.191**	**.640**	**.488**	**.335**
High	250	**.368**	**.182**	**.042**	**.668**	**.477**	**.173**	**.800**	**.736**	**.308**
	500	**.358**	**.183**	**.034**	**.630**	**.402**	**.172**	**.744**	**.679**	**.325**
	1000	**.352**	**.160**	**.029**	**.632**	**.440**	**.139**	**.772**	**.680**	**.332**

Note. NM = Naïve Method, RM = Resampling Method; α = significance level. The Naïve Method employs a dependent samples proportions test. The Resampling Method employs a resampling analogue of a Wald test. The TDCM employs a traditional Wald test. The intervals in parentheses represent the range of acceptable values of the observed family-wise error rate for a given α level according to Bradley's (1978) liberal criteria. Bold values are outside the range of Bradley's acceptable family-wise error rates.

Table 2.13

Attribute Mastery Growth Test Family-wise Error Rates after Bonferroni Correction at $\alpha = .05$

Q-matrix Complexity	Sample Size	Two-Attribute Conditions			Four-Attribute Conditions		
		NM	RM	TDCM	NM	RM	TDCM
Low	N_1	**.132**	**.008**	.053	**.140**	**.016**	.028
	N_2	**.116**	**.004**	**.020**	**.148**	**.020**	.042
	N_3	**.172**	**.012**	.049	**.188**	**.024**	.046
Medium	N_1	**.186**	**.024**	.066	**.250**	.048	.026
	N_2	**.257**	**.020**	.041	**.264**	.056	.043
	N_3	**.188**	**.016**	.028	**.244**	**.080**	.055
High	N_1	**.324**	**.091**	**.092**	**.395**	**.214**	.051
	N_2	**.337**	.067	**.094**	**.398**	**.211**	.044
	N_3	**.339**	.044	.054	**.376**	**.188**	.043

Note. NM = Naïve Method, RM = Resampling Method; α = significance level. For the two-attribute conditions, N_1, N_2, and N_3 are 250, 500, 1000. For the four-attribute conditions, N_1, N_2, and N_3 are 750, 1500, 3000. The Naïve Method employs a dependent samples proportions test. The Resampling Method employs a resampling analogue of a Wald test. The TDCM employs a traditional Wald test. Bold values are outside the range of Bradley's (1978) acceptable family-wise error rates for $\alpha = .05$: (.025, .075).

Table 2.14

Attribute Mastery Growth Test Family-wise Error Rates after Šidák *Correction at* $\alpha = .05$

Q-matrix Complexity	Sample Size	Two-Attribute Conditions			Four-Attribute Conditions		
		NM	RM	TDCM	NM	RM	TDCM
Low	N_1	**.132**	**.008**	.057	**.152**	**.016**	.028
	N_2	**.116**	**.004**	**.024**	**.152**	**.020**	.042
	N_3	**.172**	**.012**	.053	**.192**	**.024**	.046
Medium	N_1	**.186**	**.024**	.066	**.258**	.048	.026
	N_2	**.257**	**.020**	.041	**.264**	.056	.043
	N_3	**.188**	**.016**	.033	**.244**	**.080**	.055
High	N_1	**.324**	**.091**	**.092**	**.395**	**.214**	.051
	N_2	**.337**	.067	**.094**	**.398**	**.211**	.044
	N_3	**.344**	.044	.068	**.376**	**.192**	.043

Note. NM = Naïve Method, RM = Resampling Method; α = significance level. For the two-attribute conditions, N_1, N_2, and N_3 are 250, 500, 1000. For the four-attribute conditions, N_1, N_2, and N_3 are 750, 1500, 3000. The Naïve Method employs a dependent samples proportions test. The Resampling Method employs a resampling analogue of a Wald test. The TDCM employs a traditional Wald test. Bold values are outside the range of Bradley's (1978) acceptable family-wise error rates for $\alpha = .05$: (.025, .075).

Table 2.15

Attribute Mastery Growth Test Family-wise Error Rates after Hochberg *Correction at* $\alpha = .05$

Q-matrix Complexity	Sample Size	Two-Attribute Conditions			Four-Attribute Conditions		
		NM	RM	TDCM	NM	RM	TDCM
Low	N_1	**.132**	**.008**	.053	**.140**	**.016**	.028
	N_2	**.120**	**.004**	**.020**	**.148**	**.020**	.042
	N_3	**.172**	**.012**	.049	**.188**	**.024**	.046
Medium	N_1	**.186**	**.024**	.066	**.254**	.048	.026
	N_2	**.257**	**.020**	.041	**.264**	.056	.043
	N_3	**.188**	**.016**	.028	**.244**	**.080**	.055
High	N_1	**.324**	**.091**	**.092**	**.395**	**.214**	.051
	N_2	**.337**	.067	**.094**	**.398**	**.211**	.044
	N_3	**.348**	.044	.059	**.376**	**.192**	.043

Note. NM = Naïve Method, RM = Resampling Method; α = significance level. For the two-attribute conditions, N_1, N_2, and N_3 are 250, 500, 1000. For the four-attribute conditions, N_1, N_2, and N_3 are 750, 1500, 3000. The Naïve Method employs a dependent samples proportions test. The Resampling Method employs a resampling analogue of a Wald test. The TDCM employs a traditional Wald test. Bold values are outside the range of Bradley's (1978) acceptable family-wise error rates for $\alpha = .05$: (.025, .075).

Table 2.16

Power of Wald Test in TDCM in Two-Attribute Varied Growth Conditions at $\alpha = .05$

Q-matrix Complexity	Sample Size	Bonferroni		Šidák		Hochberg	
		Att 1 (small)	Att 2 (medium)	Att 1 (small)	Att 2 (medium)	Att 1 (small)	Att 2 (medium)
Low	250	.517	.945	.525	.945	.643	.950
	500	.894	.966	.898	.966	.945	.966
	1000	.979	.945	.979	.945	.983	.945
Medium	250	.556	.931	.560	.931	.621	.935
	500	.842	.946	.846	.946	.883	.950
	1000	.983	.979	.983	.979	.987	.979
High	250	.632	.829	.632	.829	.691	.849
	500	.906	.981	.913	.981	.925	.981
	1000	.979	.947	.979	.947	.979	.947

Note. Att = attribute, α = significance level. Description in parentheses represent odds ratio effect sizes for attribute mastery growth according to Cohen (1988). Small = 1.5, medium = 2.5.

Table 2.17

Power of Wald Test in TDCM in Four-Attribute Varied Growth Conditions at $\alpha = .05$

Q-matrix Complexity	Sample Size	Bonferroni				Šidák				Hochberg			
		Att 1	Att 2	Att 3	Att 4	Att 1	Att 2	Att 3	Att 4	Att 1	Att 2	Att 3	Att 4
Low	750	**.008**	.975	.996	1	**.008**	.975	.996	1	.033	.983	.996	1
	1500	**.000**	1	1	1	**.000**	1	1	1	**.016**	1	1	1
	3000	**.004**	1	1	1	**.004**	1	1	1	.034	1	1	1
Medium	750	**.004**	.953	1	1	**.004**	.953	1	1	**.013**	.970	1	1
	1500	**.000**	.996	1	1	**.000**	.996	1	1	.039	.996	1	1
	3000	**.004**	1	1	1	**.004**	1	1	1	.040	1	1	1
High	750	**.020**	.919	1	.993	**.020**	.919	1	.993	.054	.939	1	.993
	1500	**.006**	.994	1	1	**.006**	.994	1	1	.050	1	1	1
	3000	**.014**	1	1	1	**.014**	1	1	1	.055	1	1	1

Note. Att = attribute, α = significance level. Attribute 1 had no growth, Attribute 2 had a small growth effect, Attribute 3 had a medium growth effect, and Attribute 4 had a large growth effect. Growth effect sizes were chosen according to Cohen's odds ratio effect sizes (1988): Small = 1.5, medium = 2.5, large = 4.3. Bold values for Attribute 1 are outside the range of Bradley's (1978) acceptable Type I error rates for $\alpha = .05$: (.025, .075).

Table 2.18

Problem Solving Test LCDM Item Parameters

Item	Intercept	Main Effects			
		RPR	MD	NSF	GG
1	-2.33 (0.25)	2.82 (0.25)			
2	-0.56 (0.15)		2.65 (0.20)		
3	-3.4 (0.39)		2.92 (0.40)		
4	-1.93 (0.21)			2.34 (0.23)	
5	-1.35 (0.22)			3.97 (0.29)	
6	-1.08 (0.14)			3.20 (0.29)	
7	-4.98 (0.97)			5.34 (0.95)	
8	-1.28 (0.14)			1.89 (0.17)	
9	-1.34 (0.18)		2.90 (0.21)		
10	-2.55 (0.39)		5.07 (0.40)		
11	-2.05 (0.31)		5.57 (0.40)		
12	-2.52 (0.28)		4.07 (0.29)		
13	-0.84 (0.21)	3.03 (0.25)			
14	-4.18 (0.69)	4.98 (0.67)			
15	-1.75 (0.3)				2.75 (0.30)
16	-1.52 (0.27)				2.70 (0.28)
17	-3.67 (0.42)	2.94 (0.43)			
18	-1.66 (0.27)				4.65 (0.33)
19	-14.94 (131.47)				15.39 (131.45)
20	-2.78 (0.45)				3.48 (0.44)
21	-2.71 (0.32)				2.82 (0.33)
Median	-2.05	2.98	3.49	3.20	3.15

Note. RPR = ratios and proportional relationships, MD = measurement and data, NSF = number systems (fractions) GG = geometry (graphing). Standard errors for parameters are given in parenthesis.

Table 2.19

Pre-test and Post-test Attribute Correlations

Attribute	1	2	3	4
1. Ratios and Proportional Relationships	--	.869	.856	.897
2. Measurement and Data	.754	--	.891	.806
3. Number Systems (Fractions)	.810	.734	--	.720
4. Geometry (Graphing)	.674	.554	.588	--

Note. Pre-test correlations in lower triangle, post-test correlations in upper triangle.

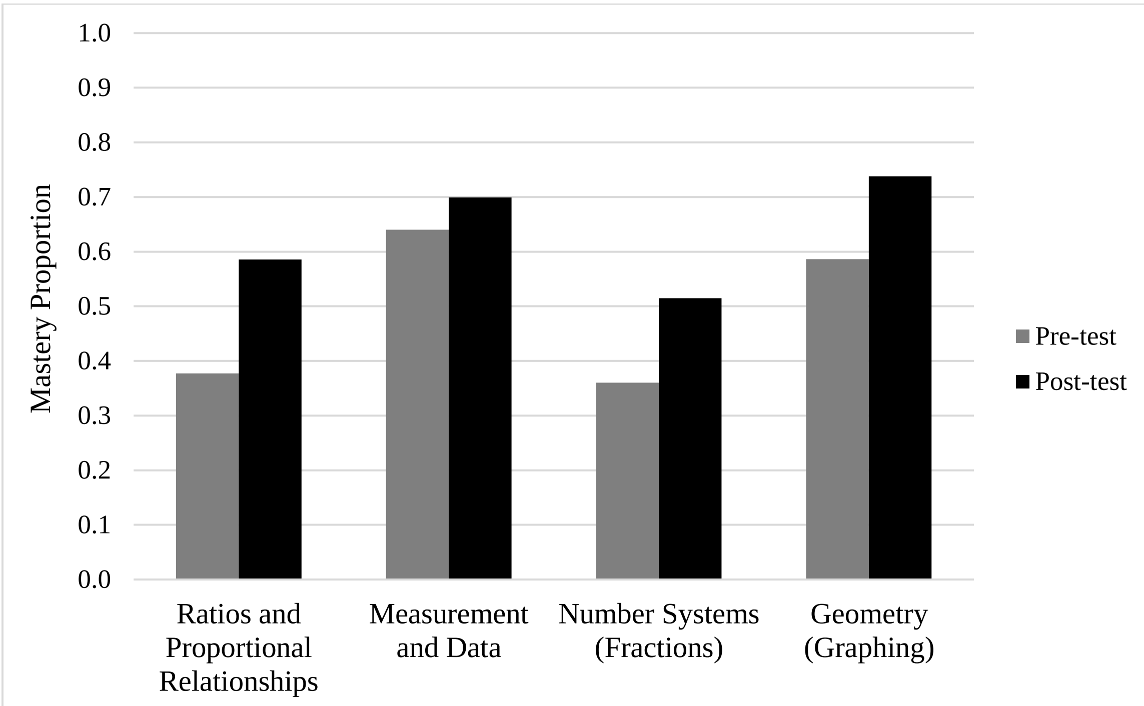

Figure 2.1. Pre-test (grey) and post-test (black) marginal attribute mastery proportions.

Table 2.20

Attribute Mastery Growth from Pre-test to Post-test

Attribute	Pre-test Mastery	Post-test Mastery	Growth	Odds Ratio	*p*-value
Ratios and Proportional Relationships	.377	.586	.208 (0.024)	2.33	<.001
Measurement and Data	.640	.699	.059 (0.018)	1.31	.001
Number Systems - Fractions	.360	.515	.155 (0.023)	1.89	<.001
Geometry - Graphing	.586	.738	.152 (0.022)	1.99	<.001

Note. Odds ratio = odds(mastery at post-test)/odds(mastery at pre-test). Parameter standard errors are in parentheses. Wald test *p*-values are adjusted by Hochberg's (1988) stepwise multiple tests procedure.

Table 2.21

Attribute Mastery Transition Probability Matrices

Attribute	Mastery Transition Matrix			
			Post-test	
			0	1
Ratios and Proportional Relationships	Pre-test	0	.53	.47
		1	.22	.78
			Post-test	
			0	1
Measurement and Data	Pre-test	0	.56	.44
		1	.16	.84
			Post-test	
			0	1
Number Systems (Fractions)	Pre-test	0	.59	.41
		1	.31	.69
			Post-test	
			0	1
Geometry (Graphing)	Pre-test	0	.39	.61
		1	.17	.83

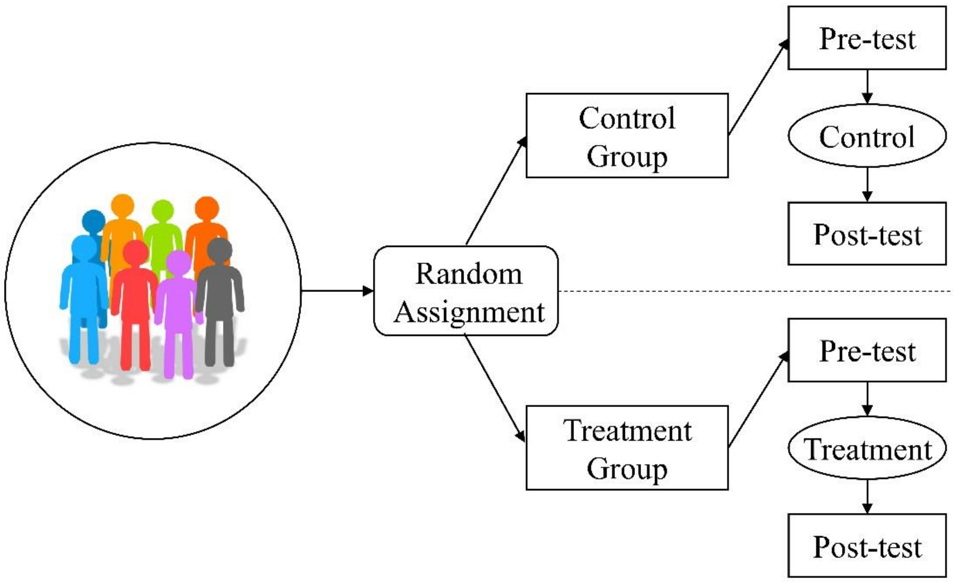

Figure 3.1. A pre-test/post-test randomized controlled trial.

Table 3.1

Group-specific attribute base-rates and mastery growth conditions

Condition Number	Pre-test Base-rates		Mastery Growth	
	Control Group	Treatment Group	Control Group	Treatment Group
1	{.10, .20, .30, .40}	{.10, .20, .30, .40}	none	none
2	{.10, .20, .30, .40}	{.40, .30, .20, .10}	none	none
3	{.10, .20, .30, .40}	{.10, .20, .30, .40}	{0, .10, .20, .30}	{0, .10, .20, .30}
4	{.10, .20, .30, .40}	{.40, .30, .20, .10}	{0, .10, .20, .30}	{0, .10, .20, .30}
5	{.10, .20, .30, .40}	{.10, .20, .30, .40}	0	.10
6	{.10, .20, .30, .40}	{.10, .20, .30, .40}	0	.20
7	{.10, .20, .30, .40}	{.10, .20, .30, .40}	0	.30
8	{.10, .20, .30, .40}	{.10, .20, .30, .40}	.10	.20
9	{.10, .20, .30, .40}	{.10, .20, .30, .40}	.10	.30
10	{.10, .20, .30, .40}	{.10, .20, .30, .40}	.20	.30
11	{.10, .20, .30, .40}	{.40, .30, .20, .10}	0	.10
12	{.10, .20, .30, .40}	{.40, .30, .20, .10}	0	.20
13	{.10, .20, .30, .40}	{.40, .30, .20, .10}	0	.30
14	{.10, .20, .30, .40}	{.40, .30, .20, .10}	.10	.20
15	{.10, .20, .30, .40}	{.40, .30, .20, .10}	.10	.30
16	{.10, .20, .30, .40}	{.40, .30, .20, .10}	.20	.30

Note. The four decimals in braces represent pre-test base-rates for Attributes 1, 2, 3, and 4, respectively. In Conditions 1-2, and 5-16, the Mastery Growth description applies to all four attributes. In Conditions 3-4, the mastery growth descriptions in braces refer to Attribute 1, 2, 3, and 4, respectively. Conditions 1-16 are repeated for all three each sample size conditions.

Table 3.2

Q-matrix for All Simulation Conditions

Item	Attribute 1	Attribute 2	Attribute 3	Attribute 4
1	1	0	0	0
2	1	0	0	0
3	1	0	0	0
4	1	1	0	0
5	1	0	1	0
6	0	1	0	0
7	0	1	0	0
8	0	1	0	0
9	0	1	1	0
10	0	1	0	1
11	0	0	1	0
12	0	0	1	0
13	0	0	1	0
14	0	0	1	1
15	1	0	1	0
16	0	0	0	1
17	0	0	0	1
18	0	0	0	1
19	1	0	0	1
20	0	1	0	1

Table 3.2

Fixed Simulation Design Factors

Design Factor	Value/Interval
Number of groups	2
Number of attributes	4
Q-matrix	20 items 1.4 attributes/item
Attribute correlation	.50
Correct response probability interval for complete non-masters	(.15, .30)
Correct response probability interval for complete masters	(.60, .90)
Correct response probability interval for partial masters	(.30, .60)

Table 3.4

Convergence Rates by Sample Size

Group Sample Size	Total Sample Size	Convergence Rate
500	1000	.954
1000	2000	.975
1500	3000	.976

Table 3.5

Marginal Attribute Classification Accuracy

Group Size	Total Sample Size	Pre-test		Post-test	
		Control	Treatment	Control	Treatment
500	1000	.918	.915	.916	.908
1000	2000	.922	.918	.919	.911
1500	3000	.924	.920	.922	.914

Note. Classification accuracy rates are averaged over individual attributes.

Table 3.6

Marginal Attribute Classification Reliability

Group Size	Total Sample Size	Pre-test		Post-test	
		Control	Treatment	Control	Treatment
500	1000	.925	.929	.923	.935
1000	2000	.915	.920	.912	.925
1500	3000	.909	.913	.905	.920

Note. Classification reliability is averaged over individual attributes.

Table 3.7

Type I Error Rates for Wald Test of Group-differential Growth in Attribute Mastery

Condition Number	Group Size	Pre-test Base-rates	Attribute Mastery Growth		$\alpha = .01$ (.005, .015)	$\alpha = .05$ (.025, .075)	$\alpha = .10$ (.05, .15)
			Control	Treatment			
1	500	Same	none	none	.008	.052	.081
2		Different	varied	varied	.013	.043	.082
3		Same	none	none	.012	.066	.108
4		Different	varied	varied	.013	.049	.107
1	1000	Same	none	none	.008	.055	.118
2		Different	varied	varied	.005	.055	.094
3		Same	none	none	.008	.030	.100
4		Different	varied	varied	**.018**	.057	.106
1	1500	Same	none	none	.013	.059	.108
2		Different	varied	varied	.013	.054	.116
3		Same	none	none	**.003**	.049	.099
4		Different	varied	varied	**.018**	.059	.124

Note. α = significance level. The intervals in parentheses represent the range of acceptable values of the observed family-wise error rate for a given α level according to Bradley's (1978) liberal criteria. Bold values are outside the range of Bradley's acceptable family-wise error rates.

Table 3.8

Family-wise Error Rates for Wald Test of Group-differential Growth in Attribute Mastery

Condition Number	Group Size	Pre-test Base-rates	Attribute Mastery Growth		$\alpha = .01$ (.005, .015)	$\alpha = .05$ (.025, .075)	$\alpha = .10$ (.05, .15)
			Control	Treatment			
1	500	Same	none	none	**.031**	**.167**	**.260**
2		Different	varied	varied	**.053**	**.158**	**.289**
3		Same	none	none	**.048**	**.253**	**.361**
4		Different	varied	varied	**.052**	**.177**	**.333**
1	1000	Same	none	none	**.032**	**.211**	**.368**
2		Different	varied	varied	**.021**	**.219**	**.323**
3		Same	none	none	**.030**	**.120**	**.350**
4		Different	varied	varied	**.072**	**.196**	**.361**
1	1500	Same	none	none	**.052**	**.216**	**.371**
2		Different	varied	varied	**.052**	**.206**	**.412**
3		Same	none	none	**.016**	**.188**	**.323**
4		Different	varied	varied	**.072**	**.227**	**.423**

Note. α = significance level. The intervals in parentheses represent the range of acceptable values of the observed family-wise error rate for a given α level according to Bradley's (1978) liberal criteria. Bold values are outside the range of Bradley's acceptable family-wise error rates.

Table 3.9

Family-wise Error Rates for Wald Test of Group-differential Growth in Attribute Mastery after Multiple Tests Correction

Condition Number	Group Size	Pre-test Base-rates	Attribute Mastery Growth		$\alpha = .01$ (.005, .015)	$\alpha = .05$ (.025, .075)	$\alpha = .10$ (.05, .15)
			Control	Treatment			
1	500	Same	none	none	**.000**	.042	.094
2		Different	varied	varied	.013	.053	.092
3		Same	none	none	**.000**	.072	.108
4		Different	varied	varied	**.021**	.073	.083
1	1000	Same	none	none	**.021**	.042	.084
2		Different	varied	varied	.010	.042	.104
3		Same	none	none	**.000**	.030	.050
4		Different	varied	varied	.010	**.082**	.124
1	1500	Same	none	none	**.021**	.062	.134
2		Different	varied	varied	**.000**	.052	.093
3		Same	none	none	**.000**	**.010**	.115
4		Different	varied	varied	**.031**	**.082**	.144

Note. Error rates were identical for Bonferroni, Šidák, and Hochberg's multiple tests correction. α = significance level. The intervals in parentheses represent the range of acceptable values of the observed family-wise error rate for a given α level according to Bradley's (1978) liberal criteria. Bold values are outside the range of Bradley's acceptable family-wise error rates.

Table 3.10

Power of Wald Test of Group-differential Growth after Multiple Tests Correction in Small Sample Size Condition at α = .05

Condition Number	Pre-test Base-rates	Attribute Mastery Growth		Multiple Tests Correction Method											
		Control	Treatment	Bonferroni				Šidák				Hochberg			
				Att1	Att2	Att3	Att4	Att1	Att2	Att3	Att4	Att1	Att2	Att3	Att4
5	Same	.00	.10	.375	.375	.271	.313	.375	.375	.271	.313	.500	.458	.396	.479
6	Same	.00	.20	1	.940	.920	.820	1	.940	.920	.820	1	.980	.960	.940
7	Same	.00	.30	1	1	1	1	1	1	1	1	1	1	1	1
8	Same	.10	.20	.400	.280	.160	.240	.400	.280	.160	.240	.440	.420	.260	.340
9	Same	.10	.30	1	.980	.940	.900	1	.980	.940	.900	1	1	.960	.960
10	Same	.20	.30	.542	.313	.354	.292	.542	.313	.354	.292	.667	.438	.458	.438
11	Different	.00	.10	.327	.429	.347	.265	.327	.429	.347	.265	.429	.469	.490	.388
12	Different	.00	.20	.918	.939	.939	.939	.918	.939	.939	.939	.980	.980	1	1
13	Different	.00	.30	1	1	1	1	1	1	1	1	1	1	1	1
14	Different	.10	.20	.347	.469	.306	.327	.347	.469	.306	.327	.429	.551	.449	.449
15	Different	.10	.30	.957	.957	.894	.936	.957	.957	.894	.936	1	1	.957	1
16	Different	.20	.30	.340	.440	.320	.280	.340	.440	.320	.280	.380	.520	.460	.420

Note. Group sample sizes were 500. Att = attribute. In same base-rate conditions, both group had base-rates of {.10,.20,.30,.40}. In different base-rate conditions, Control and Treatment groups had base-rates of {.10, .20, .30, .40} and {.40, .30, .20, .10}, respectively.

Table 3.11

Power of Wald Test of Group-differential Growth after Multiple Tests Correction in Medium Sample Size Condition at α = .05

Condition Number	Pre-test Base-rates	Attribute Mastery Growth		Multiple Tests Correction Method											
		Control	Treatment	Bonferroni				Šidák				Hochberg			
				Att1	Att2	Att3	Att4	Att1	Att2	Att3	Att4	Att1	Att2	Att3	Att4
5	Same	.00	.10	.826	.826	.543	.457	.826	.826	.543	.457	.891	.935	.630	.696
6	Same	.00	.20	1	1	1	1	1	1	1	1	1	1	1	1
7	Same	.00	.30	1	1	1	1	1	1	1	1	1	1	1	1
8	Same	.10	.20	.792	.646	.583	.646	.792	.646	.583	.646	.896	.792	.708	.708
9	Same	.10	.30	1	1	1	1	1	1	1	1	1	1	1	1
10	Same	.20	.30	.776	.633	.612	.551	.776	.633	.612	.551	.898	.735	.796	.755
11	Different	.00	.10	.653	.653	.571	.735	.653	.653	.571	.735	.735	.714	.755	.816
12	Different	.00	.20	1	1	1	1	1	1	1	1	1	1	1	1
13	Different	.00	.30	1	1	1	1	1	1	1	1	1	1	1	1
14	Different	.10	.20	.750	.521	.604	.500	.750	.521	.604	.500	.792	.708	.750	.708
15	Different	.10	.30	1	1	1	1	1	1	1	1	1	1	1	1
16	Different	.20	.30	.680	.560	.580	.620	.680	.560	.580	.620	.800	.760	.760	.720

Note. Group sample sizes were 100. Att = attribute. In same base-rate conditions, both group had base-rates of {.10,.20,.30,.40}. In different base-rate conditions, Control and Treatment groups had base-rates of {.10, .20, .30, .40} and {.40, .30, .20, .10}, respectively.

Table 3.12

Power of Wald Test of Group-differential Growth after Multiple Tests Correction in Large Sample Size Condition at $\alpha = .05$

Condition Number	Pre-test Base-rates	Attribute Mastery Growth		Multiple Tests Correction Method											
		Control	Treatment	Bonferroni				Šidák				Hochberg			
				Att1	Att2	Att3	Att4	Att1	Att2	Att3	Att4	Att1	Att2	Att3	Att4
5	Same	.00	.10	.980	.959	.980	.939	.980	.959	.980	.939	1	.980	1	1
6	Same	.00	.20	1	1	1	1	1	1	1	1	1	1	1	1
7	Same	.00	.30	1	1	1	1	1	1	1	1	1	1	1	1
8	Same	.10	.20	1	.958	.958	.833	1	.958	.958	.833	1	.958	.979	.938
9	Same	.10	.30	1	1	1	1	1	1	1	1	1	1	1	1
10	Same	.20	.30	.960	.960	.940	.920	.960	.960	.940	.920	.980	1	.980	.980
11	Different	.00	.10	.980	.880	.980	.980	.980	.880	.980	.980	1	.980	1	.980
12	Different	.00	.20	1	1	1	1	1	1	1	1	1	1	1	1
13	Different	.00	.30	1	1	1	1	1	1	1	1	1	1	1	1
14	Different	.10	.20	.959	.939	.939	.939	.959	.939	.939	.939	1	1	.959	.939
15	Different	.10	.30	1	1	1	1	1	1	1	1	1	1	1	1
16	Different	.20	.30	.960	.880	.880	.940	.960	.880	.880	.940	.980	.920	.980	1

Note. Group sample sizes were 150. Att = attribute. In same base-rate conditions, both group had base-rates of {.10,.20,.30,.40}. In different base-rate conditions, Control and Treatment groups had base-rates of {.10, .20, .30, .40} and {.40, .30, .20, .10}, respectively.

Table 3.13

Problem Solving Test LCDM Item Parameters

Item	Intercept	Main Effects			
		RPR	MD	NSF	GG
1	-2.33 (0.25)	2.82 (0.25)			
2	-0.56 (0.15)		2.65 (0.20)		
3	-3.4 (0.39)		2.92 (0.40)		
4	-1.93 (0.21)			2.34 (0.23)	
5	-1.35 (0.22)			3.97 (0.29)	
6	-1.08 (0.14)			3.20 (0.29)	
7	-4.98 (0.97)			5.34 (0.95)	
8	-1.28 (0.14)			1.89 (0.17)	
9	-1.34 (0.18)		2.90 (0.21)		
10	-2.55 (0.39)		5.07 (0.40)		
11	-2.05 (0.31)		5.57 (0.40)		
12	-2.52 (0.28)		4.07 (0.29)		
13	-0.84 (0.21)	3.03 (0.25)			
14	-4.18 (0.69)	4.98 (0.67)			
15	-1.75 (0.3)				2.75 (0.30)
16	-1.52 (0.27)				2.70 (0.28)
17	-3.67 (0.42)	2.94 (0.43)			
18	-1.66 (0.27)				4.65 (0.33)
19	-14.94 (131.47)				15.39 (131.45)
20	-2.78 (0.45)				3.48 (0.44)
21	-2.71 (0.32)				2.82 (0.33)
Median	-2.05	2.98	3.49	3.20	3.15

Note. RPR = ratios and proportional relationships, MD = measurement and data, NSF = number systems (fractions) GG = geometry (graphing). Standard errors for parameters are given in parenthesis.

Table 3.14

Attribute Mastery Proportions and Growth by Group

Group	Attribute	Pre-test Mastery	Post-test Mastery	Growth	Odds Ratio	*p*-value
BAU (Control)	Ratios and Proportional Relationships	.372	.495	.123 (0.033)	1.65	<.001
	Measurement and Data	.608	.663	.055 (0.026)	1.27	.031
	Number Systems (Fractions)	.346	.479	.134 (0.032)	1.74	<.001
	Geometry (Graphing)	.569	.671	.102 (0.030)	1.54	<.001
EAI (Treatment)	Ratios and Proportional Relationships	.382	.682	.299 (0.033)	3.47	<.001
	Measurement and Data	.677	.737	.059 (0.026)	1.34	.024
	Number Systems (Fractions)	.375	.554	.179 (0.033)	2.07	<.001
	Geometry (Graphing)	.605	.796	.191 (0.032)	2.57	<.001

Note. BAU = Business as Usual; EAI = Enhanced Anchored Instruction. Odds ratio = odds(mastery at post-test)/odds(mastery at pre-test). Parameter standard errors are in parentheses. Wald test *p*-values are adjusted by Hochberg’s (1988) stepwise multiple tests procedure.

Table 3.15

Group-differential Growth in Attribute Mastery

Attribute	Growth		Difference (EAI - BAU)	*p*-value
	BAU	EAI		
Ratios and Proportional Relationships	.123	.299	.176 (.046)	<.001
Measurement and Data	.055	.059	.004 (.037)	.911
Number Systems (Fractions)	.134	.179	.045 (.046)	.651
Geometry (Graphing)	.102	.191	.089 (.044)	.127

Note. BAU = Business as Usual; EAI = Enhanced Anchored Instruction. Parameter standard errors are in parentheses. Wald test *p*-values are adjusted by Hochberg's (1988) stepwise multiple tests procedure.

Table 3.16

Attribute Mastery Transition Probability Matrices by Group

Attribute	BAU (Control)				EAI (Treatment)			
			Post-test				Post-test	
			0	1			0	1
Ratios and Proportional Relationships	Pre-test	0	.63	.37	Pre-test	0	.42	.58
		1	.29	.71		1	.15	.86
Measurement and Data	Pre-test	0	.60	.40	Pre-test	0	.50	.50
		1	.17	.83		1	.15	.85
Number Systems (Fractions)	Pre-test	0	.62	.38	Pre-test	0	.55	.46
		1	.33	.67		1	.28	.72
Geometry (Graphing)	Pre-test	0	.50	.50	Pre-test	0	.28	.73
		1	.20	.80		1	.16	.84

Note. BAU = Business as Usual; EAI = Enhanced Anchored Instruction.

Table 3.17

Attribute Mastery Proportions at Pre- and Post-test by Group

Attribute		BAU (Control)				EAI (Treatment)		
			Post-test				Post-test	
			0	1			0	1
Ratios and Proportional Relationships	Pre-test	0	.40	.23	Pre-test	0	.27	.35
		1	.10	.27		1	.05	.32
Measurement and Data	Pre-test	0	.24	.15	Pre-test	0	.16	.16
		1	.10	.51		1	.10	.57
Number Systems (Fractions)	Pre-test	0	.40	.25	Pre-test	0	.33	.29
		1	.11	.23		1	.10	.28
Geometry (Graphing)	Pre-test	0	.22	.23	Pre-test	0	.11	.30
		1	.11	.44		1	.09	.49

Note. BAU = Business as Usual; EAI = Enhanced Anchored Instruction. For the BAU group, there were a total of 456 students, and a total of 423 in the EAI group.

APPENDIX A

CALCULATING MARGINAL ATTRIBUTE MASTERY TRANSTION PROBABILITIES

To obtain individual attribute transition probabilities, one needs the pre-test attribute profile proportions and the complete transition probability matrix. In the running example, there were two attributes. Table A1 shows the pre-test attribute profile proportions and the transition probabilities.

Table A1

Two-attribute Transition Matrix

			Post-test			
	Pre-test Proportion	Attribute Profile	[0,0]	[0,1]	[1,0]	[1,1]
Pre-test	.40	[0,0]	0.20	0.18	0.35	0.27
	.25	[0,1]	0.10	0.25	0.05	0.60
	.20	[1,0]	0.05	0.15	0.10	0.70
	.15	[1,1]	0.02	0.01	0.01	0.96

The probabilities in the table above are all conditional on the pre-test attribute profile membership. The attribute profiles [0,0] and [0,1] are non-masters of Attribute 1, which have pre-test proportions of .40 and .25, respectively. To calculate the conditional probability that a non-master of Attribute 1 at pre-test remains a non-master at post-test, we calculate the unconditional probability of an examinee transitioning from [0,0] to [0,0] or [0,1] and the probability of transition from [0,1] to [0,0] or [0,1].

$$\mathbb{P}([0,0] \to [0,0]) = .40 \cdot .20 = .08$$

$$\mathbb{P}([0,0] \to [0,1]) = .40 \cdot .18 = .072$$

$$\mathbb{P}([0,1] \to [0,0]) = .25 \cdot .10 = .025$$

$$\mathbb{P}([0,1] \rightarrow [0,1]) = .25 \cdot .25 = .0625$$

If we sum these probabilities, we get an unconditional probability of .2395. Finally, to obtain the probability of being a non-master of Attribute 1 at post-test, conditioned on being a non-master at pre-test, we divide by the overall probability of being a non-master of Attribute 1, which is $.40 + .25 = .65$.

$$\mathbb{P}([0,*] \rightarrow [0,*] \mid [0,*]) = \frac{.2395}{.65} = .368 \approx .37$$

If we repeat this procedure, we will obtain all the probabilities seen in Table 2.

In general, for any number of attributes, the procedure is to first compute the unconditional probabilities of transitioning from one attribute profile to another. Then, sum the probabilities of the attribute profiles corresponding to mastery state of interest. Lastly, obtain the conditional probability by dividing by the pre-test probability of the corresponding mastery state of interest.

APPENDIX B

MULTIVARIATE DELTA METHOD

The delta method is a method by which one can find the distribution of a function of a normally distributed parameter. Since Mplus uses maximum likelihood estimation, and maximum likelihood estimates (MLEs) are asymptotically normal, it can be applied to diagnostic classification model parameters estimated by Mplus. The multivariate delta method is an extension of the delta method to accommodate parameters that are functions of multiple normally distributed parameters. Below, we state the formal theorem of the multivariate delta method, and then give an illustration using DCM estimates.

Theorem 1 (Wasserman, 2004). Suppose that $Y_n = (Y_{n1}, \ldots, Y_{nk})$ is a sequence of random vectors such that

$$\sqrt{n}(Y_n - \mu) \rightsquigarrow N(0, \Sigma).$$

Let $g: \mathbb{R}^k \to \mathbb{R}$ and let

$$\nabla g(y) = \begin{pmatrix} \frac{\partial g}{\partial y_1} \\ \vdots \\ \frac{\partial g}{\partial y_k} \end{pmatrix}.$$

Let ∇_μ denote $\nabla g(y)$ evaluated at $y = \mu$ and assume that the elements of ∇_μ are nonzero. Then

$$\sqrt{n}(g(Y_n) - g(\mu)) \rightsquigarrow N\left(0, \nabla_\mu^{\mathrm{T}} \Sigma \nabla_\mu\right).$$

Put simply, the multivariate delta method allows us find the distribution of functions of MLEs. All we need is the parameter estimates, their variance-covariance matrix, and the gradient vector of the function.

To illustrate the multivariate delta method, we consider a simple example. On a two-attribute diagnostic assessment, there are four latent classes, or attribute profiles ($[0,0], [0,1], [1,0], [1,1]$). The proportion of examinees in each attribute profile is a function of the latent class means. Let μ_1, μ_2, μ_3, and μ_4 be the latent class means for $[0,0], [0,1], [1,0]$, and $[1,1]$, respectively. Then the attribute profile proportions can be computed as:

$$\mathbb{P}([0,0]) = \frac{\exp(\mu_1)}{\sum \exp(\mu_i)} = \frac{\exp(\mu_1)}{\exp(\mu_1) + \exp(\mu_2) + \exp(\mu_3) + \exp(\mu_4)}$$

$$\mathbb{P}([0,1]) = \frac{\exp(\mu_2)}{\sum \exp(\mu_i)} = \frac{\exp(\mu_2)}{\exp(\mu_1) + \exp(\mu_2) + \exp(\mu_3) + \exp(\mu_4)}$$

$$\mathbb{P}([1,0]) = \frac{\exp(\mu_3)}{\sum \exp(\mu_i)} = \frac{\exp(\mu_3)}{\exp(\mu_1) + \exp(\mu_2) + \exp(\mu_3) + \exp(\mu_4)}$$

$$\mathbb{P}([1,1]) = \frac{\exp(\mu_4)}{\sum \exp(\mu_i)} = \frac{\exp(\mu_4)}{\exp(\mu_1) + \exp(\mu_2) + \exp(\mu_3) + \exp(\mu_4)}$$

In this example, suppose Mplus estimates latent class means of $\widehat{\mu_1} = 0.872, \widehat{\mu_2} = -0.342$, $\widehat{\mu_3} = 0.070$, and the last latent class mean is fixed at $\mu_4 = 0$ to identify the model. Then it is simple to find the MLEs of the attribute profile proportions:

$$\mathbb{P}(\widehat{[0,0]}) = \frac{\exp(\widehat{\mu_1})}{\sum \exp(\widehat{\mu_i})} = \frac{2.392}{5.175} = .462$$

$$\mathbb{P}(\widehat{[0,1]}) = \frac{\exp(\widehat{\mu_2})}{\sum \exp(\widehat{\mu_i})} = \frac{0.710}{5.175} = .137$$

$$\mathbb{P}(\widehat{[1,0]}) = \frac{\exp(\widehat{\mu_3})}{\sum \exp(\widehat{\mu_i})} = \frac{1.073}{5.175} = .207$$

$$\mathbb{P}(\widehat{[1,1]}) = \frac{\exp(0)}{\sum \exp(\widehat{\mu_i})} = \frac{1}{5.175} = .193$$

Suppose we are interested in the standard error of $\widehat{\mathbb{P}([0,0])}$. First, we need the function. As noted above, with $\mu_4 = 0$,

$$\mathbb{P}([0,0]) = g(\mu_1, \mu_2, \mu_3) = \frac{\exp(\mu_1)}{\exp(\mu_1) + \exp(\mu_2) + \exp(\mu_3) + 1}.$$

Therefore, the gradient vector is given by

$$\nabla g(\mu_1, \mu_2, \mu_3) = \begin{pmatrix} \frac{\partial g}{\partial \mu_1} \\ \frac{\partial g}{\partial \mu_2} \\ \frac{\partial g}{\partial \mu_3} \end{pmatrix} = \begin{pmatrix} \frac{(1 + e^{\mu_1} + e^{\mu_2} + e^{\mu_3})(e^{\mu_1}) - (e^{\mu_1})^2}{(1 + e^{\mu_1} + e^{\mu_2} + e^{\mu_3})^2} \\ \frac{(1 + e^{\mu_1} + e^{\mu_2} + e^{\mu_3})(e^{\mu_2}) - (e^{\mu_2})^2}{(1 + e^{\mu_1} + e^{\mu_2} + e^{\mu_3})^2} \\ \frac{(1 + e^{\mu_1} + e^{\mu_2} + e^{\mu_3})(e^{\mu_3}) - (e^{\mu_3})^2}{(1 + e^{\mu_1} + e^{\mu_2} + e^{\mu_3})^2} \end{pmatrix}.$$

If we plug in the Mplus estimates of $\widehat{\mu_1} = 0.872, \widehat{\mu_2} = -0.342$, $\widehat{\mu_3} = 0.070$, we get a gradient vector of

$$\nabla g(\widehat{\mu_1}, \widehat{\mu_2}, \widehat{\mu_3}) = \begin{pmatrix} \frac{\partial g}{\partial \widehat{\mu_1}} \\ \frac{\partial g}{\partial \widehat{\mu_2}} \\ \frac{\partial g}{\partial \widehat{\mu_3}} \end{pmatrix} = \begin{pmatrix} \frac{(1 + e^{\mu_1} + e^{\mu_2} + e^{\mu_3})(e^{\mu_1}) - (e^{\mu_1})^2}{(1 + e^{\mu_1} + e^{\mu_2} + e^{\mu_3})^2} \\ \frac{(1 + e^{\mu_1} + e^{\mu_2} + e^{\mu_3})(e^{\mu_2}) - (e^{\mu_2})^2}{(1 + e^{\mu_1} + e^{\mu_2} + e^{\mu_3})^2} \\ \frac{(1 + e^{\mu_1} + e^{\mu_2} + e^{\mu_3})(e^{\mu_3}) - (e^{\mu_3})^2}{(1 + e^{\mu_1} + e^{\mu_2} + e^{\mu_3})^2} \end{pmatrix} = \begin{pmatrix} 0.248 \\ -0.063 \\ -0.095 \end{pmatrix}.$$

Next, we take the variance-covariance matrix of the parameter estimates that Mplus outputs with the TECH3 command:

$$\Sigma = \begin{pmatrix} \mathrm{var}(\widehat{\mu_1}) & \mathrm{cov}(\widehat{\mu_1}, \widehat{\mu_2}) & \mathrm{cov}(\widehat{\mu_1}, \widehat{\mu_3}) \\ \mathrm{cov}(\widehat{\mu_1}, \widehat{\mu_2}) & \mathrm{var}(\widehat{\mu_2}) & \mathrm{cov}(\widehat{\mu_2}, \widehat{\mu_3}) \\ \mathrm{cov}(\widehat{\mu_1}, \widehat{\mu_3}) & \mathrm{cov}(\widehat{\mu_2}, \widehat{\mu_3}) & \mathrm{var}(\widehat{\mu_3}) \end{pmatrix} = \begin{pmatrix} 0.068 & 0.053 & 0.052 \\ 0.053 & 0.112 & 0.039 \\ 0.052 & 0.039 & 0.091 \end{pmatrix}.$$

Lastly, to compute the standard error of $\widehat{\mathbb{P}([0,0])}$, we multiply:

$$\sqrt{\nabla_\mu^{\mathrm{T}} \Sigma \nabla_\mu} = \sqrt{\begin{pmatrix} 0.248 \\ -0.063 \\ -0.095 \end{pmatrix}^T \begin{pmatrix} 0.068 & 0.053 & 0.052 \\ 0.053 & 0.112 & 0.039 \\ 0.052 & 0.039 & 0.091 \end{pmatrix} \begin{pmatrix} 0.248 \\ -0.063 \\ -0.095 \end{pmatrix}} = .0426.$$

To summarize, we used the Mplus MLEs, their variance-covariance matrix, and the gradient vector in the multivariate delta method to find the standard error of $\mathbb{P}(\widehat{[0,0]})$. Using the multivariate delta method, we found $\mathbb{P}(\widehat{[0,0]})$ to be .462, with a standard error of .0426. This computation was done in R, and matches the estimate and standard error from Mplus below:

New/Additional Parameters		
D1	5.175	1.027
LOG11	0.872	0.260
LOG12	-0.342	0.334
LOG13	0.070	0.302
P11	0.462	0.042
P12	0.137	0.032
P13	0.207	0.038
P14	0.193	0.038

In the two- and four-attribute conditions, the pre-test and post-test attribute profile proportions were computed in the Model Constraint portion of the Mplus syntax (see Appendix D). These proportions were then aggregated to compute the pre-test and post-test attribute mastery proportions. Finally, these proportions were tested for significant differences using a Wald test.

Wasserman, L. A. (2004). *All of statistics: A concise course in statistical inference*. New York, NY: Springer.

APPENDIX C:

STUDY 1 SIMULATION STUDY Q-MATRICES

Two- Attribute Q-matrices

Q-matrix 2.1

Item	α_1	α_2
1	1	0
2	1	0
3	1	0
4	1	0
5	1	0
6	0	1
7	0	1
8	0	1
9	0	1
10	0	1

Q-matrix 2.2

Item	α_1	α_2
1	1	0
2	1	0
3	1	0
4	1	1
5	1	1
6	1	1
7	1	1
8	0	1
9	0	1
10	0	1

Q-matrix 2.3

Item	α_1	α_2
1	1	0
2	1	1
3	1	1
4	1	1
5	1	1
6	1	1
7	1	1
8	1	1
9	1	1
10	0	1

Four- Attribute Q-matrices

Q-matrix 4.1

Item	α_1	α_2	α_3	α_4
1	1	0	0	0
2	1	0	0	0
3	1	0	0	0
4	1	0	0	0
5	1	0	0	0
6	0	1	0	0
7	0	1	0	0
8	0	1	0	0
9	0	1	0	0
10	0	1	0	0
11	0	0	1	0
12	0	0	1	0
13	0	0	1	0
14	0	0	1	0
15	0	0	1	0
16	0	0	0	1
17	0	0	0	1
18	0	0	0	1
19	0	0	0	1
20	0	0	0	1

Q-matrix 4.2

Item	α_1	α_2	α_3	α_4
1	1	0	0	0
2	1	0	0	0
3	1	0	0	0
4	1	1	0	0
5	1	0	1	0
6	0	1	0	0
7	0	1	0	0
8	0	1	0	0
9	0	1	1	0
10	0	1	0	1
11	0	0	1	0
12	0	0	1	0
13	0	0	1	0
14	0	0	1	1
15	1	0	1	0
16	0	0	0	1
17	0	0	0	1
18	0	0	0	1
19	1	0	0	1
20	0	1	0	1

Q-matrix 4.3

Item	α_1	α_2	α_3	α_4
1	1	1	0	0
2	1	0	1	0
3	1	0	0	1
4	1	1	0	0
5	1	0	0	0
6	0	1	1	0
7	0	1	0	1
8	1	1	0	0
9	0	1	1	0
10	0	1	0	0
11	0	0	1	1
12	1	0	1	0
13	0	1	1	0
14	0	0	1	1
15	0	0	1	0
16	1	0	0	1
17	0	1	0	1
18	0	0	1	1
19	1	0	0	1
20	0	0	0	1

APPENDIX D

SAMPLE MPLUS SYNTAX FOR TWO-ATTRIBUTE TDCM

Comments in Mplus are preceded by an exclamation point (!).

Two-Attribute TDCM

```
! Author: Matthew Madison

TITLE: Pre/Post TDCM
      Two attributes and 10 items
      Pre-calibrated item parameters

DATA:  ! Location of free format data file
   FILE = prepost1.txt;

VARIABLE:
   NAMES = ID mitem1-mitem10 mitem11-mitem20;   !data contains ID variable
         USEVARIABLE = mitem1-mitem10 mitem11-mitem20;  !10 pre- and post-test items
         CATEGORICAL = mitem1-mitem10 mitem11-mitem20;  !each item is dichotomous
         CLASSES = c1(4) c2(4);        !Four latent classes at pre- and post-test
         IDvariable = ID;

ANALYSIS:
   TYPE = MIXTURE;                ! Estimates latent classes
         STARTS = 0;              ! Number of random starts
         PROCESSORS = 8;              ! Number of processors available

MODEL:

         %OVERALL%
```

```
[c1#1] (m11);  ! Latent variable mean for class 1, pre-test
[c1#2] (m12);  ! Latent variable mean for class 2, pre-test
[c1#3] (m13);  ! Latent variable mean for class 3, pre-test

[c2#1] (a1);  ! Latent variable mean for class 1, post-test
[c2#2] (a2);  ! Latent variable mean for class 2, post-test
[c2#3] (a3);  ! Latent variable mean for class 3, post-test

c2 on c1 (b1-b9);    !b1-b9 are logistic regression coefficientss
!9 logistic regressions predicting pre-test attribute profile
!membership form post-test attribute proile membership

Model c1: !item parameters for pre-test (same as post-test)
%C1#1%

!item thresholds fixed at pre-calibrated post-test estimates
[mitem1$1@  1.189740        ] (T1_1);
[mitem2$1@  1.052060        ] (T2_1);
[mitem3$1@  1.064270        ] (T3_1);
[mitem4$1@  1.095270        ] (T4_1);
[mitem5$1@  1.484930        ] (T5_1);
[mitem6$1@  1.288930        ] (T6_1);
[mitem7$1@  1.073860        ] (T7_1);
[mitem8$1@  1.311970        ] (T8_1);
[mitem9$1@  1.203830        ] (T9_1);
[mitem10$1@  1.198540       ] (T10_1);

   %C1#2%

[mitem1$1@  1.189740        ] (T1_1);
[mitem2$1@  1.052060        ] (T2_1);
[mitem3$1@  1.064270        ] (T3_1);
```

```
[mitem4$1@  1.095270         ] (T4_1);
[mitem5$1@  1.484930         ] (T5_1);
[mitem6$1@ -0.465080         ] (T6_2);
[mitem7$1@ -0.600460         ] (T7_2);
[mitem8$1@ -1.098310         ] (T8_2);
[mitem9$1@ -1.991390         ] (T9_2);
[mitem10$1@ -2.320900        ] (T10_2);

    %C1#3%

[mitem1$1@ -0.703990         ] (T1_2);
[mitem2$1@ -1.017630         ] (T2_2);
[mitem3$1@ -1.697440         ] (T3_2);
[mitem4$1@ -0.982250         ] (T4_2);
[mitem5$1@ -1.366870         ] (T5_2);
[mitem6$1@  1.288930         ] (T6_1);
[mitem7$1@  1.073860         ] (T7_1);
[mitem8$1@  1.311970         ] (T8_1);
[mitem9$1@  1.203830         ] (T9_1);
[mitem10$1@  1.198540        ] (T10_1);

    %C1#4%

[mitem1$1@ -0.703990         ] (T1_2);
[mitem2$1@ -1.017630         ] (T2_2);
[mitem3$1@ -1.697440         ] (T3_2);
[mitem4$1@ -0.982250         ] (T4_2);
[mitem5$1@ -1.366870         ] (T5_2);
[mitem6$1@ -0.465080         ] (T6_2);
[mitem7$1@ -0.600460         ] (T7_2);
[mitem8$1@ -1.098310         ] (T8_2);
[mitem9$1@ -1.991390         ] (T9_2);
```

```
[mitem10$1@ -2.320900      ] (T10_2);

Model c2:       !item parameters for pre-test (same as post-test)
%C2#1%

[mitem11$1@  1.189740      ] (T1_1);
[mitem12$1@  1.052060      ] (T2_1);
[mitem13$1@  1.064270      ] (T3_1);
[mitem14$1@  1.095270      ] (T4_1);
[mitem15$1@  1.484930      ] (T5_1);
[mitem16$1@  1.288930      ] (T6_1);
[mitem17$1@  1.073860      ] (T7_1);
[mitem18$1@  1.311970      ] (T8_1);
[mitem19$1@  1.203830      ] (T9_1);
[mitem20$1@  1.198540      ] (T10_1);

    %C2#2%

[mitem11$1@  1.189740      ] (T1_1);
[mitem12$1@  1.052060      ] (T2_1);
[mitem13$1@  1.064270      ] (T3_1);
[mitem14$1@  1.095270      ] (T4_1);
[mitem15$1@  1.484930      ] (T5_1);
[mitem16$1@ -0.465080      ] (T6_2);
[mitem17$1@ -0.600460      ] (T7_2);
[mitem18$1@ -1.098310      ] (T8_2);
[mitem19$1@ -1.991390      ] (T9_2);
[mitem20$1@ -2.320900      ] (T10_2);

    %C2#3%

[mitem11$1@ -0.703990      ] (T1_2);
```

```
[mitem12$1@ -1.017630      ] (T2_2);
[mitem13$1@ -1.697440      ] (T3_2);
[mitem14$1@ -0.982250      ] (T4_2);
[mitem15$1@ -1.366870      ] (T5_2);
[mitem16$1@  1.288930      ] (T6_1);
[mitem17$1@  1.073860      ] (T7_1);
[mitem18$1@  1.311970      ] (T8_1);
[mitem19$1@  1.203830      ] (T9_1);
[mitem20$1@  1.198540      ] (T10_1);

    %C2#4%

[mitem11$1@ -0.703990      ] (T1_2);
[mitem12$1@ -1.017630      ] (T2_2);
[mitem13$1@ -1.697440      ] (T3_2);
[mitem14$1@ -0.982250      ] (T4_2);
[mitem15$1@ -1.366870      ] (T5_2);
[mitem16$1@ -0.465080      ] (T6_2);
[mitem17$1@ -0.600460      ] (T7_2);
[mitem18$1@ -1.098310      ] (T8_2);
[mitem19$1@ -1.991390      ] (T9_2);
[mitem20$1@ -2.320900      ] (T10_2);


MODEL CONSTRAINT:  ! Used to define LCDM parameters
! Mplus uses P(X=0) rather than P(X=1) so multiply by -1

!Time 1 latent class probs
NEW(d1 log11 log12 log13 p11 p12 p13 p14 t1a1 t1a2);
log11 = m11;  !pre-test latent class mean for attribute profile 1
log12 = m12;  !pre-test latent class mean for attribute profile 2
log13 = m13;  !pre-test latent class mean for attribute profile 3
```

```
d1 = 1 + exp(log11) + exp(log12) + exp(log13);       ! denominator is sum of exp(class means)
p11 = exp(log11)/d1;   !pre-test, probability att profile [0,0]
p12 = exp(log12)/d1;   !pre-test, probability att profile [0,1]
p13 = exp(log13)/d1;   !pre-test, probability att profile [1,0]
p14 = 1/d1;            !pre-test, probability att profile [1,1]
t1a1 = p13 + p14;          !pre-test att 1 mastery proportion
t1a2 = p12 + p14;          !pre-test att 2 mastery proportion

!Time 2 latent class probs
NEW(l11 l12 l13 l21 l22 l23 l31 l32 l33 p21 p22 p23 p24 t2a1 t2a2 diff1 diff2);
l11 = a1 + b1;        !transition prob from [0,0] to [0,0]
l12 = a2 + b4;  !transition prob from [0,0] to [0,1]
l13 = a3 + b7;  !transition prob from [0,0] to [1,0]
l21 = a1 + b2;  !transition prob from [0,1] to [0,0]
l22 = a2 + b5;  !transition prob from [0,1] to [0,1]
l23 = a3 + b8;  !transition prob from [0,1] to [1,0]
l31 = a1 + b3;  !transition prob from [1,0] to [0,0]
l32 = a2 + b6;  !transition prob from [1,0] to [0,1]
l33 = a3 + b9;  !transition prob from [1,0] to [1,0]

!post-test att profile probs are pre-test*transition
!post-test p([0,0])
p21 = p11*exp(l11)/(1 + exp(l11) + exp(l12) + exp(l13)) +
    p12*exp(l21)/(1 + exp(l21) + exp(l22) + exp(l23)) +
    p13*exp(l31)/(1 + exp(l31) + exp(l32) + exp(l33)) +
    p14*exp(a1)/(1 + exp(a1) + exp(a2) + exp(a3));
!post-test p([0,1])
p22 = p11*exp(l12)/(1 + exp(l11) + exp(l12) + exp(l13)) +
    p12*exp(l22)/(1 + exp(l21) + exp(l22) + exp(l23)) +
    p13*exp(l32)/(1 + exp(l31) + exp(l32) + exp(l33)) +
    p14*exp(a2)/(1 + exp(a1) + exp(a2) + exp(a3));
!post-test p([1,0])
```

```
    p23 = p11*exp(l13)/(1 + exp(l11) + exp(l12) + exp(l13)) +
        p12*exp(l23)/(1 + exp(l21) + exp(l22) + exp(l23)) +
        p13*exp(l33)/(1 + exp(l31) + exp(l32) + exp(l33)) +
        p14*exp(a3)/(1 + exp(a1) + exp(a2) + exp(a3));
!post-test p([1,1])
    p24 = 1 - p21 - p22 - p23;

    t2a1 = p23 + p24;   !post-test att 1 mastery proportion
    t2a2 = p22 + p24;       !post-test att 2 mastery proportion

!Wald test for difference in pre- and post-test att 1 mastery
    diff1 = t2a1 - t1a1;

!Wald test for difference in pre- and post-test att 2 mastery
    diff2 = t2a2 - t1a2;

    SAVEDATA: ! Format, name of posterior probabilities of class membership file
      FORMAT = F10.5;
      FILE = prepost1_out.txt;
      SAVE = CPROBABILITIES;
```

APPENDIX E

CHAPTER 2 SUPPLEMENTARY MATERIALS

Attribute Mastery Growth Test Family-wise Error Rates after Bonferroni Correction at $\alpha = .01$

Q-matrix Complexity	Sample Size	Two-Attribute Conditions			Four-Attribute Conditions		
		NM	RM	TDCM	NM	RM	TDCM
Low	N_1	**.036**	**.000**	.012	**.060**	**.000**	**.016**
	N_2	**.052**	**.000**	**.004**	**.044**	**.004**	**.000**
	N_3	**.116**	.008	**.028**	**.064**	**.000**	**.000**
Medium	N_1	**.089**	**.000**	**.004**	**.077**	.008	**.000**
	N_2	**.080**	**.000**	**.021**	**.116**	.012	.009
	N_3	**.108**	**.000**	.008	**.124**	**.016**	.008
High	N_1	**.182**	**.017**	**.018**	**.236**	**.077**	**.019**
	N_2	**.168**	**.019**	**.026**	**.240**	**.089**	**.020**
	N_3	**.194**	**.018**	.014	**.248**	**.068**	.005

Note. NM = Naïve Method, RM = Resampling Method; α = significance level. For the two-attribute conditions, N_1, N_2, and N_3 are 250, 500, 1000. For the four-attribute conditions, N_1, N_2, and N_3 are 750, 1500, 3000. The Naïve Method employs a dependent samples proportions test. The Resampling Method employs a resampling analogue of a Wald test. The TDCM employs a traditional Wald test. Bold values are outside the range of Bradley's (1978) acceptable family-wise error rates for $\alpha = .01$: (.005, .015).

Attribute Mastery Growth Test Family-wise Error Rates after Šidák *Correction at* $\alpha = .01$

Q-matrix Complexity	Sample Size	Two-Attribute Conditions			Four-Attribute Conditions		
		NM	RM	TDCM	NM	RM	TDCM
Low	N_1	**.036**	**.000**	**.016**	**.080**	**.000**	**.016**
	N_2	**.052**	**.000**	**.004**	**.076**	**.004**	**.000**
	N_3	**.116**	.008	**.028**	**.112**	.008	**.000**
Medium	N_1	**.089**	**.000**	.008	**.137**	**.016**	**.000**
	N_2	**.080**	**.000**	**.021**	**.148**	**.024**	.009
	N_3	**.108**	**.000**	.008	**.168**	**.040**	.008
High	N_1	**.182**	**.017**	**.025**	**.314**	**.136**	**.019**
	N_2	**.168**	**.019**	**.042**	**.301**	**.126**	**.020**
	N_3	**.194**	**.018**	.014	**.300**	**.100**	.005

Note. NM = Naïve Method, RM = Resampling Method; α = significance level. For the two-attribute conditions, N_1, N_2, and N_3 are 250, 500, 1000. For the four-attribute conditions, N_1, N_2, and N_3 are 750, 1500, 3000. The Naïve Method employs a dependent samples proportions test. The Resampling Method employs a resampling analogue of a Wald test. The TDCM employs a traditional Wald test. Bold values are outside the range of Bradley's (1978) acceptable family-wise error rates for $\alpha = .01$: (.005, .015).

Attribute Mastery Growth Test Family-wise Error Rates after Hochberg *Correction at* $\alpha = .01$

Q-matrix Complexity	Sample Size	Two-Attribute Conditions			Four-Attribute Conditions		
		NM	RM	TDCM	NM	RM	TDCM
Low	N_1	**.036**	**.000**	.012	**.060**	**.000**	**.016**
	N_2	**.052**	**.000**	**.004**	**.048**	**.004**	**.000**
	N_3	**.116**	.008	**.028**	**.064**	**.000**	**.000**
Medium	N_1	**.089**	**.000**	**.004**	**.081**	.008	**.000**
	N_2	**.080**	**.000**	**.021**	**.116**	.012	.009
	N_3	**.108**	**.000**	.008	**.124**	**.016**	.008
High	N_1	**.182**	**.017**	**.018**	**.236**	**.077**	**.019**
	N_2	**.173**	**.019**	**.026**	**.240**	**.089**	**.020**
	N_3	**.198**	**.018**	.014	**.248**	**.068**	.005

Note. NM = Naïve Method, RM = Resampling Method; α = significance level. For the two-attribute conditions, N_1, N_2, and N_3 are 250, 500, 1000. For the four-attribute conditions, N_1, N_2, and N_3 are 750, 1500, 3000. The Naïve Method employs a dependent samples proportions test. The Resampling Method employs a resampling analogue of a Wald test. The TDCM employs a traditional Wald test. Bold values are outside the range of Bradley's (1978) acceptable family-wise error rates for $\alpha = .01$: (.005, .015).

Attribute Mastery Growth Test Family-wise Error Rates after Bonferroni Correction at $\alpha = .10$

Q-matrix Complexity	Sample Size	Two-Attribute Conditions			Four-Attribute Conditions		
		NM	RM	TDCM	NM	RM	TDCM
Low	N_1	**.236**	**.028**	.082	**.256**	**.044**	.073
	N_2	**.192**	**.028**	.053	**.224**	**.036**	.072
	N_3	**.244**	**.016**	.077	**.264**	**.044**	.100
Medium	N_1	**.328**	**.045**	.099	**.335**	.121	.061
	N_2	**.341**	.056	.086	**.360**	.120	.087
	N_3	**.284**	.048	.077	**.332**	.128	.110
High	N_1	**.403**	**.159**	.129	**.541**	**.300**	.098
	N_2	**.438**	**.125**	.115	**.484**	**.272**	.069
	N_3	**.449**	.128	.109	**.516**	**.280**	.072

Note. NM = Naïve Method, RM = Resampling Method; α = significance level. For the two-attribute conditions, N_1, N_2, and N_3 are 250, 500, 1000. For the four-attribute conditions, N_1, N_2, and N_3 are 750, 1500, 3000. The Naïve Method employs a dependent samples proportions test. The Resampling Method employs a resampling analogue of a Wald test. The TDCM employs a traditional Wald test. Bold values are outside the range of Bradley's (1978) acceptable family-wise error rates for $\alpha = .01$: (.05, .15).

Attribute Mastery Growth Test Family-wise Error Rates after Šidák *Correction at* $\alpha = .10$

Q-matrix Complexity	Sample Size	Two-Attribute Conditions			Four-Attribute Conditions		
		NM	RM	TDCM	NM	RM	TDCM
Low	N_1	**.236**	**.028**	.082	**.364**	.120	.073
	N_2	**.196**	**.028**	.057	**.372**	.100	.085
	N_3	**.256**	**.016**	.077	**.424**	.140	.104
Medium	N_1	**.328**	**.049**	.103	**.480**	**.226**	.061
	N_2	**.345**	.056	.099	**.484**	**.280**	.087
	N_3	**.292**	**.048**	.077	**.448**	**.240**	.110
High	N_1	**.403**	**.170**	.135	**.673**	**.482**	.103
	N_2	**.447**	.130	.115	**.630**	**.411**	.074
	N_3	**.458**	.132	.113	**.636**	**.444**	.077

Note. NM = Naïve Method, RM = Resampling Method; α = significance level. For the two-attribute conditions, N_1, N_2, and N_3 are 250, 500, 1000. For the four-attribute conditions, N_1, N_2, and N_3 are 750, 1500, 3000. The Naïve Method employs a dependent samples proportions test. The Resampling Method employs a resampling analogue of a Wald test. The TDCM employs a traditional Wald test. Bold values are outside the range of Bradley's (1978) acceptable family-wise error rates for $\alpha = .01$: (.05, .15).

Attribute Mastery Growth Test Family-wise Error Rates after Hochberg *Correction at* $\alpha = .10$

Q-matrix Complexity	Sample Size	Two-Attribute Conditions			Four-Attribute Conditions		
		NM	RM	TDCM	NM	RM	TDCM
Low	N_1	**.236**	**.028**	.082	**.264**	**.044**	.073
	N_2	**.196**	**.028**	.053	**.224**	**.040**	.072
	N_3	**.248**	**.024**	.077	**.264**	**.048**	.100
Medium	N_1	**.332**	**.045**	.099	**.335**	.121	.066
	N_2	**.345**	.060	.091	**.360**	.128	.087
	N_3	**.284**	**.048**	.085	**.332**	.128	.114
High	N_1	**.420**	**.165**	.135	**.545**	**.305**	.098
	N_2	**.442**	.130	.120	**.488**	**.280**	.069
	N_3	**.449**	.137	.113	**.520**	**.280**	.072

Note. NM = Naïve Method, RM = Resampling Method; α = significance level. For the two-attribute conditions, N_1, N_2, and N_3 are 250, 500, 1000. For the four-attribute conditions, N_1, N_2, and N_3 are 750, 1500, 3000. The Naïve Method employs a dependent samples proportions test. The Resampling Method employs a resampling analogue of a Wald test. The TDCM employs a traditional Wald test. Bold values are outside the range of Bradley's (1978) acceptable family-wise error rates for $\alpha = .01$: (.05, .15).

Power of Wald Test in TDCM in Four-Attribute Varied Growth Conditions at $\alpha = .01$

Q-matrix Complexity	Sample Size	Bonferroni				Šidák				Hochberg			
		Att 1	Att 2	Att 3	Att 4	Att 1	Att 2	Att 3	Att 4	Att 1	Att 2	Att 3	Att 4
Low	750	**.004**	.913	.996	1	**.004**	.913	.996	1	**.004**	.921	.996	1
	1500	**.000**	1	1	1	**.000**	1	1	1	**.000**	1	1	1
	3000	**.000**	1	1	1	**.000**	1	1	1	**.000**	1	1	1
Medium	750	**.000**	.884	1	1	**.000**	.884	1	1	**.000**	.918	1	1
	1500	**.000**	.996	1	1	**.000**	.996	1	1	**.000**	.996	1	1
	3000	**.004**	1	1	1	**.004**	1	1	1	**.004**	1	1	1
High	750	.007	.851	1	.993	.007	.851	1	.993	.014	.885	1	.993
	1500	**.000**	.989	1	1	**.000**	.989	1	1	**.000**	.989	1	1
	3000	.005	1	1	1	.005	1	1	1	.009	1	1	1

Note. Att = attribute, α = significance level. Attribute 1 had no growth, Attribute 2 had a small growth effect, Attribute 3 had a medium growth effect, and Attribute 4 had a large growth effect. Growth effect sizes were chosen according to Cohen's odds ratio effect sizes (1988): Small = 1.5, medium = 2.5, large = 4.3. Bold values are outside the range of Bradley's (1978) acceptable Type I error rates for $\alpha = .05$: (.005, .015).

Power of Wald Test in TDCM in Four-Attribute Varied Growth Conditions at $\alpha = .10$

Q-matrix Complexity	Sample Size	Bonferroni				Šidák				Hochberg			
		Att 1	Att 2	Att 3	Att 4	Att 1	Att 2	Att 3	Att 4	Att 1	Att 2	Att 3	Att 4
Low	750	**.008**	.983	.996	1	**.008**	.983	.996	1	.079	.988	.996	1
	1500	**.000**	1	1	1	**.000**	1	1	1	.078	1	1	1
	3000	**.013**	1	1	1	**.013**	1	1	1	**.046**	1	1	1
Medium	750	**.013**	.970	1	1	**.013**	.974	1	1	.069	.987	1	1
	1500	**.022**	.996	1	1	**.022**	.996	1	1	.073	1	1	1
	3000	**.024**	1	1	1	**.024**	1	1	1	.105	1	1	1
High	750	**.034**	.932	1	.993	**.034**	.932	1	.993	.128	.953	1	.993
	1500	**.022**	1	1	1	**.028**	1	1	1	.099	1	1	1
	3000	**.023**	1	1	1	**.023**	1	1	1	.086	1	1	1

Note. Att = attribute, α = significance level. Attribute 1 had no growth, Attribute 2 had a small growth effect, Attribute 3 had a medium growth effect, and Attribute 4 had a large growth effect. Growth effect sizes were chosen according to Cohen's odds ratio effect sizes (1988): Small = 1.5, medium = 2.5, large = 4.3. Bold values are outside the range of Bradley's (1978) acceptable Type I error rates for $\alpha = .05$: (.05, .15).

APPENDIX F:

SAMPLE MPLUS SYNTAX FOR MULTIPLE-GROUP TDCM

Comments in Mplus are preceded by an exclamation point (!).

```
! Author: Matthew Madison

TITLE: Pre/Post Multigroup TDCM

DATA:  ! Location of free format data file
    FILE = prepost1.txt;

VARIABLE:
            NAMES = ID mitem1-mitem20 xitem1-xitem20 g;
            USEVARIABLE = mitem1-mitem20 xitem1-xitem20;
            CATEGORICAL = mitem1-mitem20 xitem1-xitem20;
            CLASSES =  group(2) c1(16) c2(16) ;
            IDvariable = ID;
            KNOWNCLASS = group (g=1 g=2);

ANALYSIS:
            TYPE = MIXTURE;                 ! Estimates latent classes
            STARTS = 0;                 ! Turn off multiple random start feature
            PROCESSORS = 8;                 ! Number of processors available

MODEL:
%OVERALL%
[c1#1] (m11);  ! Latent variable mean for class 1 (time 1)
[c1#2] (m12);  ! Latent variable mean for class 2 (time 1)
[c1#3] (m13);  ! Latent variable mean for class 3 (time 1)
[c1#4] (m14);  ! Latent variable mean for class 4 (time 1)
```

```
[c1#5] (m15);  ! Latent variable mean for class 5 (time 1)
[c1#6] (m16);  ! Latent variable mean for class 6 (time 1)
[c1#7] (m17);  ! Latent variable mean for class 7 (time 1)
[c1#8] (m18);  ! Latent variable mean for class 8 (time 1)
[c1#9] (m19);  ! Latent variable mean for class 9 (time 1)
[c1#10] (m110);  ! Latent variable mean for class 10 (time 1)
[c1#11] (m111);  ! Latent variable mean for class 11 (time 1)
[c1#12] (m112);  ! Latent variable mean for class 12 (time 1)
[c1#13] (m113);  ! Latent variable mean for class 13 (time 1)
[c1#14] (m114);  ! Latent variable mean for class 14 (time 1)
[c1#15] (m115);  ! Latent variable mean for class 15 (time 1)

[c2#1] (m21);  ! Latent variable mean for class 1 (time 2)
[c2#2] (m22);  ! Latent variable mean for class 2 (time 2)
[c2#3] (m23);  ! Latent variable mean for class 3 (time 2)
[c2#4] (m24);  ! Latent variable mean for class 4 (time 2)
[c2#5] (m25);  ! Latent variable mean for class 5 (time 2)
[c2#6] (m26);  ! Latent variable mean for class 6 (time 2)
[c2#7] (m27);  ! Latent variable mean for class 7 (time 2)
[c2#8] (m28);  ! Latent variable mean for class 8 (time 2)
[c2#9] (m29);  ! Latent variable mean for class 9 (time 2)
[c2#10] (m210);  ! Latent variable mean for class 10 (time 2)
[c2#11] (m211);  ! Latent variable mean for class 11 (time 2)
[c2#12] (m212);  ! Latent variable mean for class 12 (time 2)
[c2#13] (m213);  ! Latent variable mean for class 13 (time 2)
[c2#14] (m214);  ! Latent variable mean for class 14 (time 2)
[c2#15] (m215);  ! Latent variable mean for class 15 (time 2)

c1 c2 ON group (aa1-aa30);

Model group:
%group#1%
```

```
c2 on c1 (bb1-bb225);

%group#2%
c2 on c1 (r1-r225);

Model c1:
%C1#1%

        [mitem1$1@  1.719170        ](t1_1);
        [mitem2$1@  1.634080        ](t2_1);
        [mitem3$1@  1.588670        ](t3_1);
        [mitem4$1@  1.299340        ](t4_1);
        [mitem5$1@  1.500100        ](t5_1);
        [mitem6$1@  1.517960        ](t6_1);
        [mitem7$1@  1.349330        ](t7_1);
        [mitem8$1@  1.743740        ](t8_1);
        [mitem9$1@  1.345310        ](t9_1);
        [mitem10$1@  1.509530       ](t10_1);
        [mitem11$1@  1.040810       ](t11_1);
        [mitem12$1@  1.077540       ](t12_1);
        [mitem13$1@  1.245470       ](t13_1);
        [mitem14$1@  1.371850       ](t14_1);
        [mitem15$1@  1.218120       ](t15_1);
        [mitem16$1@  1.626590       ](t16_1);
        [mitem17$1@  1.058700       ](t17_1);
        [mitem18$1@  0.936850       ](t18_1);
        [mitem19$1@  1.053960       ](t19_1);
        [mitem20$1@  1.713360       ](t20_1);

           %C1#2%

        [mitem1$1@  1.719170        ](t1_1);
```

```
[mitem2$1@  1.634080         ](t2_1);
[mitem3$1@  1.588670         ](t3_1);
[mitem4$1@  1.299340         ](t4_1);
[mitem5$1@  1.500100         ](t5_1);
[mitem6$1@  1.517960         ](t6_1);
[mitem7$1@  1.349330         ](t7_1);
[mitem8$1@  1.743740         ](t8_1);
[mitem9$1@  1.345310         ](t9_1);
[mitem10$1@  0.322110        ](t10_2);
[mitem11$1@  1.040810        ](t11_1);
[mitem12$1@  1.077540        ](t12_1);
[mitem13$1@  1.245470        ](t13_1);
[mitem14$1@  0.440650        ](t14_2);
[mitem15$1@  1.218120        ](t15_1);
[mitem16$1@ -0.372900        ](t16_2);
[mitem17$1@ -2.321100        ](t17_2);
[mitem18$1@ -2.147900        ](t18_2);
[mitem19$1@  0.137010        ](t19_2);
[mitem20$1@  0.596180        ](t20_2);

    %C1#3%

[mitem1$1@  1.719170         ](t1_1);
[mitem2$1@  1.634080         ](t2_1);
[mitem3$1@  1.588670         ](t3_1);
[mitem4$1@  1.299340         ](t4_1);
[mitem5$1@  0.357610         ](t5_2);
[mitem6$1@  1.517960         ](t6_1);
[mitem7$1@  1.349330         ](t7_1);
[mitem8$1@  1.743740         ](t8_1);
[mitem9$1@  0.107270         ](t9_2);
[mitem10$1@  1.509530        ](t10_1);
```

```
[mitem11$1@ -1.225800       ](t11_2);
[mitem12$1@ -1.207300       ](t12_2);
[mitem13$1@ -0.921500       ](t13_2);
[mitem14$1@  0.396180       ](t14_3);
[mitem15$1@  0.279290       ](t15_2);
[mitem16$1@  1.626590       ](t16_1);
[mitem17$1@  1.058700       ](t17_1);
[mitem18$1@  0.936850       ](t18_1);
[mitem19$1@  1.053960       ](t19_1);
[mitem20$1@  1.713360       ](t20_1);

   %C1#4%

[mitem1$1@  1.719170        ](t1_1);
[mitem2$1@  1.634080        ](t2_1);
[mitem3$1@  1.588670        ](t3_1);
[mitem4$1@  1.299340        ](t4_1);
[mitem5$1@  0.357610        ](t5_2);
[mitem6$1@  1.517960        ](t6_1);
[mitem7$1@  1.349330        ](t7_1);
[mitem8$1@  1.743740        ](t8_1);
[mitem9$1@  0.107270        ](t9_2);
[mitem10$1@  0.322110       ](t10_2);
[mitem11$1@ -1.225800       ](t11_2);
[mitem12$1@ -1.207300       ](t12_2);
[mitem13$1@ -0.921500       ](t13_2);
[mitem14$1@ -1.294900       ](t14_4);
[mitem15$1@  0.279290       ](t15_2);
[mitem16$1@ -0.372900       ](t16_2);
[mitem17$1@ -2.321100       ](t17_2);
[mitem18$1@ -2.147900       ](t18_2);
[mitem19$1@  0.137010       ](t19_2);
```

```
[mitem20$1@  0.596180       ](t20_2);

   %C1#5%

[mitem1$1@  1.719170        ](t1_1);
[mitem2$1@  1.634080        ](t2_1);
[mitem3$1@  1.588670        ](t3_1);
[mitem4$1@  0.523780        ](t4_2);
[mitem5$1@  1.500100        ](t5_1);
[mitem6$1@ -0.589180        ](t6_2);
[mitem7$1@ -1.249380        ](t7_2);
[mitem8$1@ -1.488680        ](t8_2);
[mitem9$1@  0.307240        ](t9_3);
[mitem10$1@  0.743250       ](t10_3);
[mitem11$1@  1.040810       ](t11_1);
[mitem12$1@  1.077540       ](t12_1);
[mitem13$1@  1.245470       ](t13_1);
[mitem14$1@  1.371850       ](t14_1);
[mitem15$1@  1.218120       ](t15_1);
[mitem16$1@  1.626590       ](t16_1);
[mitem17$1@  1.058700       ](t17_1);
[mitem18$1@  0.936850       ](t18_1);
[mitem19$1@  1.053960       ](t19_1);
[mitem20$1@  0.385480       ](t20_3);

   %C1#6%

[mitem1$1@  1.719170        ](t1_1);
[mitem2$1@  1.634080        ](t2_1);
[mitem3$1@  1.588670        ](t3_1);
[mitem4$1@  0.523780        ](t4_2);
[mitem5$1@  1.500100        ](t5_1);
```

```
[mitem6$1@ -0.589180        ](t6_2);
[mitem7$1@ -1.249380        ](t7_2);
[mitem8$1@ -1.488680        ](t8_2);
[mitem9$1@  0.307240        ](t9_3);
[mitem10$1@ -1.494000       ](t10_4);
[mitem11$1@  1.040810       ](t11_1);
[mitem12$1@  1.077540       ](t12_1);
[mitem13$1@  1.245470       ](t13_1);
[mitem14$1@  0.440650       ](t14_2);
[mitem15$1@  1.218120       ](t15_1);
[mitem16$1@ -0.372900       ](t16_2);
[mitem17$1@ -2.321100       ](t17_2);
[mitem18$1@ -2.147900       ](t18_2);
[mitem19$1@  0.137010       ](t19_2);
[mitem20$1@ -1.254200       ](t20_4);

    %C1#7%

[mitem1$1@  1.719170        ](t1_1);
[mitem2$1@  1.634080        ](t2_1);
[mitem3$1@  1.588670        ](t3_1);
[mitem4$1@  0.523780        ](t4_2);
[mitem5$1@  0.357610        ](t5_2);
[mitem6$1@ -0.589180        ](t6_2);
[mitem7$1@ -1.249380        ](t7_2);
[mitem8$1@ -1.488680        ](t8_2);
[mitem9$1@ -1.932170        ](t9_4);
[mitem10$1@  0.743250       ](t10_3);
[mitem11$1@ -1.225800       ](t11_2);
[mitem12$1@ -1.207300       ](t12_2);
[mitem13$1@ -0.921500       ](t13_2);
[mitem14$1@  0.396180       ](t14_3);
```

```
[mitem15$1@  0.279290       ](t15_2);
[mitem16$1@  1.626590       ](t16_1);
[mitem17$1@  1.058700       ](t17_1);
[mitem18$1@  0.936850       ](t18_1);
[mitem19$1@  1.053960       ](t19_1);
[mitem20$1@  0.385480       ](t20_3);

    %C1#8%

[mitem1$1@  1.719170        ](t1_1);
[mitem2$1@  1.634080        ](t2_1);
[mitem3$1@  1.588670        ](t3_1);
[mitem4$1@  0.523780        ](t4_2);
[mitem5$1@  0.357610        ](t5_2);
[mitem6$1@ -0.589180        ](t6_2);
[mitem7$1@ -1.249380        ](t7_2);
[mitem8$1@ -1.488680        ](t8_2);
[mitem9$1@ -1.932170        ](t9_4);
[mitem10$1@ -1.494000       ](t10_4);
[mitem11$1@ -1.225800       ](t11_2);
[mitem12$1@ -1.207300       ](t12_2);
[mitem13$1@ -0.921500       ](t13_2);
[mitem14$1@ -1.294900       ](t14_4);
[mitem15$1@  0.279290       ](t15_2);
[mitem16$1@ -0.372900       ](t16_2);
[mitem17$1@ -2.321100       ](t17_2);
[mitem18$1@ -2.147900       ](t18_2);
[mitem19$1@  0.137010       ](t19_2);
[mitem20$1@ -1.254200       ](t20_4);

    %C1#9%
```

```
[mitem1$1@ -2.181330        ](t1_2);
[mitem2$1@ -0.343250        ](t2_2);
[mitem3$1@ -0.752410        ](t3_2);
[mitem4$1@  0.919880        ](t4_3);
[mitem5$1@  1.500010        ](t5_3);
[mitem6$1@  1.517960        ](t6_1);
[mitem7$1@  1.349330        ](t7_1);
[mitem8$1@  1.743740        ](t8_1);
[mitem9$1@  1.345310        ](t9_1);
[mitem10$1@  1.509530       ](t10_1);
[mitem11$1@  1.040810       ](t11_1);
[mitem12$1@  1.077540       ](t12_1);
[mitem13$1@  1.245470       ](t13_1);
[mitem14$1@  1.371850       ](t14_1);
[mitem15$1@  0.919310       ](t15_3);
[mitem16$1@  1.626590       ](t16_1);
[mitem17$1@  1.058700       ](t17_1);
[mitem18$1@  0.936850       ](t18_1);
[mitem19$1@  0.421690       ](t19_3);
[mitem20$1@  1.713360       ](t20_1);

  %C1#10%

[mitem1$1@ -2.181330        ](t1_2);
[mitem2$1@ -0.343250        ](t2_2);
[mitem3$1@ -0.752410        ](t3_2);
[mitem4$1@  0.919880        ](t4_3);
[mitem5$1@  1.500010        ](t5_3);
[mitem6$1@  1.517960        ](t6_1);
[mitem7$1@  1.349330        ](t7_1);
[mitem8$1@  1.743740        ](t8_1);
[mitem9$1@  1.345310        ](t9_1);
```

```
[mitem10$1@  0.322110       ](t10_2);
[mitem11$1@  1.040810       ](t11_1);
[mitem12$1@  1.077540       ](t12_1);
[mitem13$1@  1.245470       ](t13_1);
[mitem14$1@  0.440650       ](t14_2);
[mitem15$1@  0.919310       ](t15_3);
[mitem16$1@ -0.372900       ](t16_2);
[mitem17$1@ -2.321100       ](t17_2);
[mitem18$1@ -2.147900       ](t18_2);
[mitem19$1@ -0.819400       ](t19_4);
[mitem20$1@  0.596180       ](t20_2);

    %C1#11%

[mitem1$1@ -2.181330        ](t1_2);
[mitem2$1@ -0.343250        ](t2_2);
[mitem3$1@ -0.752410        ](t3_2);
[mitem4$1@  0.919880        ](t4_3);
[mitem5$1@ -1.831390        ](t5_4);
[mitem6$1@  1.517960        ](t6_1);
[mitem7$1@  1.349330        ](t7_1);
[mitem8$1@  1.743740        ](t8_1);
[mitem9$1@  0.107270        ](t9_2);
[mitem10$1@  1.509530       ](t10_1);
[mitem11$1@ -1.225800       ](t11_2);
[mitem12$1@ -1.207300       ](t12_2);
[mitem13$1@ -0.921500       ](t13_2);
[mitem14$1@  0.396180       ](t14_3);
[mitem15$1@ -2.276100       ](t15_4);
[mitem16$1@  1.626590       ](t16_1);
[mitem17$1@  1.058700       ](t17_1);
[mitem18$1@  0.936850       ](t18_1);
```

```
[mitem19$1@  0.421690       ](t19_3);
[mitem20$1@  1.713360       ](t20_1);

    %C1#12%

[mitem1$1@ -2.181330        ](t1_2);
[mitem2$1@ -0.343250        ](t2_2);
[mitem3$1@ -0.752410        ](t3_2);
[mitem4$1@  0.919880        ](t4_3);
[mitem5$1@ -1.831390        ](t5_4);
[mitem6$1@  1.517960        ](t6_1);
[mitem7$1@  1.349330        ](t7_1);
[mitem8$1@  1.743740        ](t8_1);
[mitem9$1@  0.107270        ](t9_2);
[mitem10$1@  0.322110       ](t10_2);
[mitem11$1@ -1.225800       ](t11_2);
[mitem12$1@ -1.207300       ](t12_2);
[mitem13$1@ -0.921500       ](t13_2);
[mitem14$1@ -1.294900       ](t14_4);
[mitem15$1@ -2.276100       ](t15_4);
[mitem16$1@ -0.372900       ](t16_2);
[mitem17$1@ -2.321100       ](t17_2);
[mitem18$1@ -2.147900       ](t18_2);
[mitem19$1@ -0.819400       ](t19_4);
[mitem20$1@  0.596180       ](t20_2);

    %C1#13%

[mitem1$1@ -2.181330        ](t1_2);
[mitem2$1@ -0.343250        ](t2_2);
[mitem3$1@ -0.752410        ](t3_2);
[mitem4$1@ -1.143880        ](t4_4);
```

```
[mitem5$1@  1.500010         ](t5_3);
[mitem6$1@ -0.589180         ](t6_2);
[mitem7$1@ -1.249380         ](t7_2);
[mitem8$1@ -1.488680         ](t8_2);
[mitem9$1@  0.307240         ](t9_3);
[mitem10$1@  0.743250        ](t10_3);
[mitem11$1@  1.040810        ](t11_1);
[mitem12$1@  1.077540        ](t12_1);
[mitem13$1@  1.245470        ](t13_1);
[mitem14$1@  1.371850        ](t14_1);
[mitem15$1@  0.919310        ](t15_3);
[mitem16$1@  1.626590        ](t16_1);
[mitem17$1@  1.058700        ](t17_1);
[mitem18$1@  0.936850        ](t18_1);
[mitem19$1@  0.421690        ](t19_3);
[mitem20$1@  0.385480        ](t20_3);

   %C1#14%

[mitem1$1@ -2.181330         ](t1_2);
[mitem2$1@ -0.343250         ](t2_2);
[mitem3$1@ -0.752410         ](t3_2);
[mitem4$1@ -1.143880         ](t4_4);
[mitem5$1@  1.500010         ](t5_3);
[mitem6$1@ -0.589180         ](t6_2);
[mitem7$1@ -1.249380         ](t7_2);
[mitem8$1@ -1.488680         ](t8_2);
[mitem9$1@  0.307240         ](t9_3);
[mitem10$1@ -1.494000        ](t10_4);
[mitem11$1@  1.040810        ](t11_1);
[mitem12$1@  1.077540        ](t12_1);
[mitem13$1@  1.245470        ](t13_1);
```

```
[mitem14$1@  0.440650       ](t14_2);
[mitem15$1@  0.919310       ](t15_3);
[mitem16$1@ -0.372900       ](t16_2);
[mitem17$1@ -2.321100       ](t17_2);
[mitem18$1@ -2.147900       ](t18_2);
[mitem19$1@ -0.819400       ](t19_4);
[mitem20$1@ -1.254200       ](t20_4);

   %C1#15%

[mitem1$1@ -2.181330        ](t1_2);
[mitem2$1@ -0.343250        ](t2_2);
[mitem3$1@ -0.752410        ](t3_2);
[mitem4$1@ -1.143880        ](t4_4);
[mitem5$1@ -1.831390        ](t5_4);
[mitem6$1@ -0.589180        ](t6_2);
[mitem7$1@ -1.249380        ](t7_2);
[mitem8$1@ -1.488680        ](t8_2);
[mitem9$1@ -1.932170        ](t9_4);
[mitem10$1@  0.743250       ](t10_3);
[mitem11$1@ -1.225800       ](t11_2);
[mitem12$1@ -1.207300       ](t12_2);
[mitem13$1@ -0.921500       ](t13_2);
[mitem14$1@  0.396180       ](t14_3);
[mitem15$1@ -2.276100       ](t15_4);
[mitem16$1@  1.626590       ](t16_1);
[mitem17$1@  1.058700       ](t17_1);
[mitem18$1@  0.936850       ](t18_1);
[mitem19$1@  0.421690       ](t19_3);
[mitem20$1@  0.385480       ](t20_3);

   %C1#16%
```

```
[mitem1$1@ -2.181330        ](t1_2);
[mitem2$1@ -0.343250        ](t2_2);
[mitem3$1@ -0.752410        ](t3_2);
[mitem4$1@ -1.143880        ](t4_4);
[mitem5$1@ -1.831390        ](t5_4);
[mitem6$1@ -0.589180        ](t6_2);
[mitem7$1@ -1.249380        ](t7_2);
[mitem8$1@ -1.488680        ](t8_2);
[mitem9$1@ -1.932170        ](t9_4);
[mitem10$1@ -1.494000       ](t10_4);
[mitem11$1@ -1.225800       ](t11_2);
[mitem12$1@ -1.207300       ](t12_2);
[mitem13$1@ -0.921500       ](t13_2);
[mitem14$1@ -1.294900       ](t14_4);
[mitem15$1@ -2.276100       ](t15_4);
[mitem16$1@ -0.372900       ](t16_2);
[mitem17$1@ -2.321100       ](t17_2);
[mitem18$1@ -2.147900       ](t18_2);
[mitem19$1@ -0.819400       ](t19_4);
[mitem20$1@ -1.254200       ](t20_4);

Model c2:
%C2#1%

[xitem1$1@  1.719170        ](t1_1);
[xitem2$1@  1.634080        ](t2_1);
[xitem3$1@  1.588670        ](t3_1);
[xitem4$1@  1.299340        ](t4_1);
[xitem5$1@  1.500100        ](t5_1);
[xitem6$1@  1.517960        ](t6_1);
[xitem7$1@  1.349330        ](t7_1);
```

```
[xitem8$1@  1.743740       ](t8_1);
[xitem9$1@  1.345310       ](t9_1);
[xitem10$1@  1.509530      ](t10_1);
[xitem11$1@  1.040810      ](t11_1);
[xitem12$1@  1.077540      ](t12_1);
[xitem13$1@  1.245470      ](t13_1);
[xitem14$1@  1.371850      ](t14_1);
[xitem15$1@  1.218120      ](t15_1);
[xitem16$1@  1.626590      ](t16_1);
[xitem17$1@  1.058700      ](t17_1);
[xitem18$1@  0.936850      ](t18_1);
[xitem19$1@  1.053960      ](t19_1);
[xitem20$1@  1.713360      ](t20_1);

   %C2#2%

[xitem1$1@  1.719170       ](t1_1);
[xitem2$1@  1.634080       ](t2_1);
[xitem3$1@  1.588670       ](t3_1);
[xitem4$1@  1.299340       ](t4_1);
[xitem5$1@  1.500100       ](t5_1);
[xitem6$1@  1.517960       ](t6_1);
[xitem7$1@  1.349330       ](t7_1);
[xitem8$1@  1.743740       ](t8_1);
[xitem9$1@  1.345310       ](t9_1);
[xitem10$1@  0.322110      ](t10_2);
[xitem11$1@  1.040810      ](t11_1);
[xitem12$1@  1.077540      ](t12_1);
[xitem13$1@  1.245470      ](t13_1);
[xitem14$1@  0.440650      ](t14_2);
[xitem15$1@  1.218120      ](t15_1);
[xitem16$1@ -0.372900      ](t16_2);
```

```
[xitem17$1@ -2.321100        ](t17_2);
[xitem18$1@ -2.147900        ](t18_2);
[xitem19$1@  0.137010        ](t19_2);
[xitem20$1@  0.596180        ](t20_2);

    %C2#3%

[xitem1$1@  1.719170         ](t1_1);
[xitem2$1@  1.634080         ](t2_1);
[xitem3$1@  1.588670         ](t3_1);
[xitem4$1@  1.299340         ](t4_1);
[xitem5$1@  0.357610         ](t5_2);
[xitem6$1@  1.517960         ](t6_1);
[xitem7$1@  1.349330         ](t7_1);
[xitem8$1@  1.743740         ](t8_1);
[xitem9$1@  0.107270         ](t9_2);
[xitem10$1@  1.509530        ](t10_1);
[xitem11$1@ -1.225800        ](t11_2);
[xitem12$1@ -1.207300        ](t12_2);
[xitem13$1@ -0.921500        ](t13_2);
[xitem14$1@  0.396180        ](t14_3);
[xitem15$1@  0.279290        ](t15_2);
[xitem16$1@  1.626590        ](t16_1);
[xitem17$1@  1.058700        ](t17_1);
[xitem18$1@  0.936850        ](t18_1);
[xitem19$1@  1.053960        ](t19_1);
[xitem20$1@  1.713360        ](t20_1);

    %C2#4%

[xitem1$1@  1.719170         ](t1_1);
[xitem2$1@  1.634080         ](t2_1);
```

```
[xitem3$1@  1.588670         ](t3_1);
[xitem4$1@  1.299340         ](t4_1);
[xitem5$1@  0.357610         ](t5_2);
[xitem6$1@  1.517960         ](t6_1);
[xitem7$1@  1.349330         ](t7_1);
[xitem8$1@  1.743740         ](t8_1);
[xitem9$1@  0.107270         ](t9_2);
[xitem10$1@  0.322110        ](t10_2);
[xitem11$1@ -1.225800        ](t11_2);
[xitem12$1@ -1.207300        ](t12_2);
[xitem13$1@ -0.921500        ](t13_2);
[xitem14$1@ -1.294900        ](t14_4);
[xitem15$1@  0.279290        ](t15_2);
[xitem16$1@ -0.372900        ](t16_2);
[xitem17$1@ -2.321100        ](t17_2);
[xitem18$1@ -2.147900        ](t18_2);
[xitem19$1@  0.137010        ](t19_2);
[xitem20$1@  0.596180        ](t20_2);

   %C2#5%

[xitem1$1@  1.719170         ](t1_1);
[xitem2$1@  1.634080         ](t2_1);
[xitem3$1@  1.588670         ](t3_1);
[xitem4$1@  0.523780         ](t4_2);
[xitem5$1@  1.500100         ](t5_1);
[xitem6$1@ -0.589180         ](t6_2);
[xitem7$1@ -1.249380         ](t7_2);
[xitem8$1@ -1.488680         ](t8_2);
[xitem9$1@  0.307240         ](t9_3);
[xitem10$1@  0.743250        ](t10_3);
[xitem11$1@  1.040810        ](t11_1);
```

```
[xitem12$1@  1.077540        ](t12_1);
[xitem13$1@  1.245470        ](t13_1);
[xitem14$1@  1.371850        ](t14_1);
[xitem15$1@  1.218120        ](t15_1);
[xitem16$1@  1.626590        ](t16_1);
[xitem17$1@  1.058700        ](t17_1);
[xitem18$1@  0.936850        ](t18_1);
[xitem19$1@  1.053960        ](t19_1);
[xitem20$1@  0.385480        ](t20_3);

   %C2#6%

[xitem1$1@  1.719170         ](t1_1);
[xitem2$1@  1.634080         ](t2_1);
[xitem3$1@  1.588670         ](t3_1);
[xitem4$1@  0.523780         ](t4_2);
[xitem5$1@  1.500100         ](t5_1);
[xitem6$1@ -0.589180         ](t6_2);
[xitem7$1@ -1.249380         ](t7_2);
[xitem8$1@ -1.488680         ](t8_2);
[xitem9$1@  0.307240         ](t9_3);
[xitem10$1@ -1.494000        ](t10_4);
[xitem11$1@  1.040810        ](t11_1);
[xitem12$1@  1.077540        ](t12_1);
[xitem13$1@  1.245470        ](t13_1);
[xitem14$1@  0.440650        ](t14_2);
[xitem15$1@  1.218120        ](t15_1);
[xitem16$1@ -0.372900        ](t16_2);
[xitem17$1@ -2.321100        ](t17_2);
[xitem18$1@ -2.147900        ](t18_2);
[xitem19$1@  0.137010        ](t19_2);
[xitem20$1@ -1.254200        ](t20_4);
```

```
%C2#7%

[xitem1$1@  1.719170          ](t1_1);
[xitem2$1@  1.634080          ](t2_1);
[xitem3$1@  1.588670          ](t3_1);
[xitem4$1@  0.523780          ](t4_2);
[xitem5$1@  0.357610          ](t5_2);
[xitem6$1@ -0.589180          ](t6_2);
[xitem7$1@ -1.249380          ](t7_2);
[xitem8$1@ -1.488680          ](t8_2);
[xitem9$1@ -1.932170          ](t9_4);
[xitem10$1@  0.743250         ](t10_3);
[xitem11$1@ -1.225800         ](t11_2);
[xitem12$1@ -1.207300         ](t12_2);
[xitem13$1@ -0.921500         ](t13_2);
[xitem14$1@  0.396180         ](t14_3);
[xitem15$1@  0.279290         ](t15_2);
[xitem16$1@  1.626590         ](t16_1);
[xitem17$1@  1.058700         ](t17_1);
[xitem18$1@  0.936850         ](t18_1);
[xitem19$1@  1.053960         ](t19_1);
[xitem20$1@  0.385480         ](t20_3);

%C2#8%

[xitem1$1@  1.719170          ](t1_1);
[xitem2$1@  1.634080          ](t2_1);
[xitem3$1@  1.588670          ](t3_1);
[xitem4$1@  0.523780          ](t4_2);
[xitem5$1@  0.357610          ](t5_2);
[xitem6$1@ -0.589180          ](t6_2);
```

```
[xitem7$1@ -1.249380         ](t7_2);
[xitem8$1@ -1.488680         ](t8_2);
[xitem9$1@ -1.932170         ](t9_4);
[xitem10$1@ -1.494000        ](t10_4);
[xitem11$1@ -1.225800        ](t11_2);
[xitem12$1@ -1.207300        ](t12_2);
[xitem13$1@ -0.921500        ](t13_2);
[xitem14$1@ -1.294900        ](t14_4);
[xitem15$1@  0.279290        ](t15_2);
[xitem16$1@ -0.372900        ](t16_2);
[xitem17$1@ -2.321100        ](t17_2);
[xitem18$1@ -2.147900        ](t18_2);
[xitem19$1@  0.137010        ](t19_2);
[xitem20$1@ -1.254200        ](t20_4);

   %C2#9%

[xitem1$1@ -2.181330         ](t1_2);
[xitem2$1@ -0.343250         ](t2_2);
[xitem3$1@ -0.752410         ](t3_2);
[xitem4$1@  0.919880         ](t4_3);
[xitem5$1@  1.500010         ](t5_3);
[xitem6$1@  1.517960         ](t6_1);
[xitem7$1@  1.349330         ](t7_1);
[xitem8$1@  1.743740         ](t8_1);
[xitem9$1@  1.345310         ](t9_1);
[xitem10$1@  1.509530        ](t10_1);
[xitem11$1@  1.040810        ](t11_1);
[xitem12$1@  1.077540        ](t12_1);
[xitem13$1@  1.245470        ](t13_1);
[xitem14$1@  1.371850        ](t14_1);
[xitem15$1@  0.919310        ](t15_3);
```

```
[xitem16$1@  1.626590        ](t16_1);
[xitem17$1@  1.058700        ](t17_1);
[xitem18$1@  0.936850        ](t18_1);
[xitem19$1@  0.421690        ](t19_3);
[xitem20$1@  1.713360        ](t20_1);

   %C2#10%

[xitem1$1@ -2.181330         ](t1_2);
[xitem2$1@ -0.343250         ](t2_2);
[xitem3$1@ -0.752410         ](t3_2);
[xitem4$1@  0.919880         ](t4_3);
[xitem5$1@  1.500010         ](t5_3);
[xitem6$1@  1.517960         ](t6_1);
[xitem7$1@  1.349330         ](t7_1);
[xitem8$1@  1.743740         ](t8_1);
[xitem9$1@  1.345310         ](t9_1);
[xitem10$1@  0.322110        ](t10_2);
[xitem11$1@  1.040810        ](t11_1);
[xitem12$1@  1.077540        ](t12_1);
[xitem13$1@  1.245470        ](t13_1);
[xitem14$1@  0.440650        ](t14_2);
[xitem15$1@  0.919310        ](t15_3);
[xitem16$1@ -0.372900        ](t16_2);
[xitem17$1@ -2.321100        ](t17_2);
[xitem18$1@ -2.147900        ](t18_2);
[xitem19$1@ -0.819400        ](t19_4);
[xitem20$1@  0.596180        ](t20_2);

   %C2#11%

[xitem1$1@ -2.181330         ](t1_2);
```

```
[xitem2$1@ -0.343250         ](t2_2);
[xitem3$1@ -0.752410         ](t3_2);
[xitem4$1@  0.919880         ](t4_3);
[xitem5$1@ -1.831390         ](t5_4);
[xitem6$1@  1.517960         ](t6_1);
[xitem7$1@  1.349330         ](t7_1);
[xitem8$1@  1.743740         ](t8_1);
[xitem9$1@  0.107270         ](t9_2);
[xitem10$1@  1.509530        ](t10_1);
[xitem11$1@ -1.225800        ](t11_2);
[xitem12$1@ -1.207300        ](t12_2);
[xitem13$1@ -0.921500        ](t13_2);
[xitem14$1@  0.396180        ](t14_3);
[xitem15$1@ -2.276100        ](t15_4);
[xitem16$1@  1.626590        ](t16_1);
[xitem17$1@  1.058700        ](t17_1);
[xitem18$1@  0.936850        ](t18_1);
[xitem19$1@  0.421690        ](t19_3);
[xitem20$1@  1.713360        ](t20_1);

   %C2#12%

[xitem1$1@ -2.181330         ](t1_2);
[xitem2$1@ -0.343250         ](t2_2);
[xitem3$1@ -0.752410         ](t3_2);
[xitem4$1@  0.919880         ](t4_3);
[xitem5$1@ -1.831390         ](t5_4);
[xitem6$1@  1.517960         ](t6_1);
[xitem7$1@  1.349330         ](t7_1);
[xitem8$1@  1.743740         ](t8_1);
[xitem9$1@  0.107270         ](t9_2);
[xitem10$1@  0.322110        ](t10_2);
```

```
[xitem11$1@ -1.225800       ](t11_2);
[xitem12$1@ -1.207300       ](t12_2);
[xitem13$1@ -0.921500       ](t13_2);
[xitem14$1@ -1.294900       ](t14_4);
[xitem15$1@ -2.276100       ](t15_4);
[xitem16$1@ -0.372900       ](t16_2);
[xitem17$1@ -2.321100       ](t17_2);
[xitem18$1@ -2.147900       ](t18_2);
[xitem19$1@ -0.819400       ](t19_4);
[xitem20$1@  0.596180       ](t20_2);

   %C2#13%

[xitem1$1@ -2.181330        ](t1_2);
[xitem2$1@ -0.343250        ](t2_2);
[xitem3$1@ -0.752410        ](t3_2);
[xitem4$1@ -1.143880        ](t4_4);
[xitem5$1@  1.500010        ](t5_3);
[xitem6$1@ -0.589180        ](t6_2);
[xitem7$1@ -1.249380        ](t7_2);
[xitem8$1@ -1.488680        ](t8_2);
[xitem9$1@  0.307240        ](t9_3);
[xitem10$1@  0.743250       ](t10_3);
[xitem11$1@  1.040810       ](t11_1);
[xitem12$1@  1.077540       ](t12_1);
[xitem13$1@  1.245470       ](t13_1);
[xitem14$1@  1.371850       ](t14_1);
[xitem15$1@  0.919310       ](t15_3);
[xitem16$1@  1.626590       ](t16_1);
[xitem17$1@  1.058700       ](t17_1);
[xitem18$1@  0.936850       ](t18_1);
[xitem19$1@  0.421690       ](t19_3);
```

```
[xitem20$1@  0.385480        ](t20_3);

 %C2#14%

[xitem1$1@ -2.181330         ](t1_2);
[xitem2$1@ -0.343250         ](t2_2);
[xitem3$1@ -0.752410         ](t3_2);
[xitem4$1@ -1.143880         ](t4_4);
[xitem5$1@  1.500010         ](t5_3);
[xitem6$1@ -0.589180         ](t6_2);
[xitem7$1@ -1.249380         ](t7_2);
[xitem8$1@ -1.488680         ](t8_2);
[xitem9$1@  0.307240         ](t9_3);
[xitem10$1@ -1.494000        ](t10_4);
[xitem11$1@  1.040810        ](t11_1);
[xitem12$1@  1.077540        ](t12_1);
[xitem13$1@  1.245470        ](t13_1);
[xitem14$1@  0.440650        ](t14_2);
[xitem15$1@  0.919310        ](t15_3);
[xitem16$1@ -0.372900        ](t16_2);
[xitem17$1@ -2.321100        ](t17_2);
[xitem18$1@ -2.147900        ](t18_2);
[xitem19$1@ -0.819400        ](t19_4);
[xitem20$1@ -1.254200        ](t20_4);

 %C2#15%

[xitem1$1@ -2.181330         ](t1_2);
[xitem2$1@ -0.343250         ](t2_2);
[xitem3$1@ -0.752410         ](t3_2);
[xitem4$1@ -1.143880         ](t4_4);
[xitem5$1@ -1.831390         ](t5_4);
```

```
[xitem6$1@ -0.589180          ](t6_2);
[xitem7$1@ -1.249380          ](t7_2);
[xitem8$1@ -1.488680          ](t8_2);
[xitem9$1@ -1.932170          ](t9_4);
[xitem10$1@  0.743250         ](t10_3);
[xitem11$1@ -1.225800         ](t11_2);
[xitem12$1@ -1.207300         ](t12_2);
[xitem13$1@ -0.921500         ](t13_2);
[xitem14$1@  0.396180         ](t14_3);
[xitem15$1@ -2.276100         ](t15_4);
[xitem16$1@  1.626590         ](t16_1);
[xitem17$1@  1.058700         ](t17_1);
[xitem18$1@  0.936850         ](t18_1);
[xitem19$1@  0.421690         ](t19_3);
[xitem20$1@  0.385480         ](t20_3);

   %C2#16%

[xitem1$1@ -2.181330          ](t1_2);
[xitem2$1@ -0.343250          ](t2_2);
[xitem3$1@ -0.752410          ](t3_2);
[xitem4$1@ -1.143880          ](t4_4);
[xitem5$1@ -1.831390          ](t5_4);
[xitem6$1@ -0.589180          ](t6_2);
[xitem7$1@ -1.249380          ](t7_2);
[xitem8$1@ -1.488680          ](t8_2);
[xitem9$1@ -1.932170          ](t9_4);
[xitem10$1@ -1.494000         ](t10_4);
[xitem11$1@ -1.225800         ](t11_2);
[xitem12$1@ -1.207300         ](t12_2);
[xitem13$1@ -0.921500         ](t13_2);
[xitem14$1@ -1.294900         ](t14_4);
```

```
[xitem15$1@ -2.276100        ](t15_4);
[xitem16$1@ -0.372900        ](t16_2);
[xitem17$1@ -2.321100        ](t17_2);
[xitem18$1@ -2.147900        ](t18_2);
[xitem19$1@ -0.819400        ](t19_4);
[xitem20$1@ -1.254200        ](t20_4);

Model Constraint:
!!!!!!!!!!!!!!!!!!!!!!!!!!!!!!!!!!!!!!!!!!!!!!!!!!!!!!!!!!!!!!!!!!
!!!!! Treatment group pre-post analysis !!!!!

!16 tpre-test Attribute Profile Proportions
NEW(s1 tpre1 tpre2 tpre3 tpre4 tpre5 tpre6 tpre7 tpre8 tpre9 tpre10 tpre11 tpre12 tpre13
tpre14 tpre15 tpre16);

s1 = 1 + exp(m11) + exp(m12) + exp(m13) + exp(m14) + exp(m15) + exp(m16) +
exp(m17) + exp(m18) + exp(m19) + exp(m110) + exp(m111) + exp(m112) +
exp(m113) + exp(m114) + exp(m115);

tpre1 = exp(m11)/s1;
tpre2 = exp(m12)/s1;
tpre3 = exp(m13)/s1;
tpre4 = exp(m14)/s1;
tpre5 = exp(m15)/s1;
tpre6 = exp(m16)/s1;
tpre7 = exp(m17)/s1;
tpre8 = exp(m18)/s1;
tpre9 = exp(m19)/s1;
tpre10 = exp(m110)/s1;
tpre11 = exp(m111)/s1;
tpre12 = exp(m112)/s1;
tpre13 = exp(m113)/s1;
```

```
tpre14 = exp(m114)/s1;
tpre15 = exp(m115)/s1;
tpre16 = 1/s1;

!attribute profile transition probabilities
New(t11a t12a t13a t14a t15a t16a t17a t18a
t19a t110a t111a t112a t113a t114a t115a t116a
t21b t22b t23b t24b t25b t26b t27b t28b t29b t210b t211b t212b t213b t214b t215b t216b
t31e t32e t33e t34e t35e t36e t37e t38e t39e t310e t311e t312e t313e t314e t315e t316e
t41f t42f t43f t44f t45f t46f t47f t48f t49f t410f t411f t412f t413f t414f t415f t416f
t51h t52h t53h t54h t55h t56h t57h t58h t59h t510h t511h t512h t513h t514h t515h t516h
t61i t62i t63i t64i t65i t66i t67i t68i t69i t610i t611i t612i t613i t614i t615i t616i
t71j t72j t73j t74j t75j t76j t77j t78j t79j t710j t711j t712j t713j t714j t715j t716j
t81k t82k t83k t84k t85k t86k t87k t88k t89k t810k t811k t812k t813k t814k t815k t816k
t91n t92n t93n t94n t95n t96n t97n t98n t99n t910n t911n t912n t913n t914n t915n t916n
t101o t102o t103o t104o t105o t106o t107o t108o t109o
t1010o t1011o t1012o t1013o t1014o t1015o t1016o
t111q t112q t113q t114q t115q t116q t117q t118q t119q
t1110q t1111q t1112q t1113q t1114q t1115q t1116q
t121u t122u t123u t124u t125u t126u t127u t128u t129u
t1210u t1211u t1212u t1213u t1214u t1215u t1216u
t131v t132v t133v t134v t135v t136v t137v t138v t139v
t1310v t1311v t1312v t1313v t1314v t1315v t1316v
t141w t142w t143w t144w t145w t146w t147w t148w t149w
t1410w t1411w t1412w t1413w t1414w t1415w t1416w
t151y t152y t153y t154y t155y t156y t157y t158y t159y
t1510y t1511y t1512y t1513y t1514y t1515y t1516y
t161z t162z t163z t164z t165z t166z t167z t168z t169z
t1610z t1611z t1612z t1613z t1614z t1615z t1616z);

!row 1
t11a=exp(r1 + m21)/(1+exp(r1 + m21)+exp(r16 + m22)+exp(r31 + m23)+exp(r46 + m24)
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+exp(r61 + m25)+exp(r76 + m26)+exp(r91 + m27)+exp(r106 + m28)+exp(r121 + m29)+
exp(r136 + m210)+exp(r151 + m211)+exp(r166 + m212)+exp(r181 + m213)+exp(r196 + m214)+
exp(r211 + m215));
t12a=exp(r16 + m22)/(1+exp(r1 + m21)+exp(r16 + m22)+exp(r31 + m23)+exp(r46 + m24)+
exp(r61 + m25)+exp(r76 + m26)+exp(r91 + m27)+exp(r106 + m28)+exp(r121 + m29)+
exp(r136 + m210)+exp(r151 + m211)+exp(r166 + m212)+exp(r181 + m213)+exp(r196 + m214)+
exp(r211 + m215));
t13a=exp(r31 + m23)/(1+exp(r1 + m21)+exp(r16 + m22)+exp(r31 + m23)+exp(r46 + m24)+
exp(r61 + m25)+exp(r76 + m26)+exp(r91 + m27)+exp(r106 + m28)+exp(r121 + m29)+
exp(r136 + m210)+exp(r151 + m211)+exp(r166 + m212)+exp(r181 + m213)+exp(r196 + m214)
+exp(r211 + m215));
t14a=exp(r46 + m24)/(1+exp(r1 + m21)+exp(r16 + m22)+exp(r31 + m23)+exp(r46 + m24)
+exp(r61 + m25)+exp(r76 + m26)+exp(r91 + m27)+exp(r106 + m28)+exp(r121 + m29)+
exp(r136 + m210)+exp(r151 + m211)+exp(r166 + m212)+exp(r181 + m213)+exp(r196 + m214)
+exp(r211 + m215));
t15a=exp(r61 + m25)/(1+exp(r1 + m21)+exp(r16 + m22)+exp(r31 + m23)+exp(r46 + m24)+
exp(r61 + m25)+exp(r76 + m26)+exp(r91 + m27)+exp(r106 + m28)+exp(r121 + m29)+
exp(r136 + m210)+exp(r151 + m211)+exp(r166 + m212)+exp(r181 + m213)+exp(r196 + m214)
+exp(r211 + m215));
t16a=exp(r76 + m26)/(1+exp(r1 + m21)+exp(r16 + m22)+exp(r31 + m23)+exp(r46 + m24)
+exp(r61 + m25)+exp(r76 + m26)+exp(r91 + m27)+exp(r106 + m28)+exp(r121 + m29)+
exp(r136 + m210)+exp(r151 + m211)+exp(r166 + m212)+exp(r181 + m213)+exp(r196 + m214)+
exp(r211 + m215));
t17a=exp(r91 + m27)/(1+exp(r1 + m21)+exp(r16 + m22)+exp(r31 + m23)+exp(r46 + m24)+
exp(r61 + m25)+exp(r76 + m26)+exp(r91 + m27)+exp(r106 + m28)+exp(r121 + m29)+
exp(r136 + m210)+exp(r151 + m211)+exp(r166 + m212)+exp(r181 + m213)+exp(r196 + m214)
+exp(r211 + m215));
t18a=exp(r106 + m28)/(1+exp(r1 + m21)+exp(r16 + m22)+exp(r31 + m23)+exp(r46 + m24)
+exp(r61 + m25)+exp(r76 + m26)+exp(r91 + m27)+exp(r106 + m28)+exp(r121 + m29)+
exp(r136 + m210)+exp(r151 + m211)+exp(r166 + m212)+exp(r181 + m213)+exp(r196 + m214)
+exp(r211 + m215));
t19a=exp(r121 + m29)/(1+exp(r1 + m21)+exp(r16 + m22)+exp(r31 + m23)+exp(r46 + m24)
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+exp(r61 + m25)+exp(r76 + m26)+exp(r91 + m27)+exp(r106 + m28)+exp(r121 + m29)+
exp(r136 + m210)+exp(r151 + m211)+exp(r166 + m212)+exp(r181 + m213)+exp(r196 + m214)
+exp(r211 + m215));
t110a=exp(r136 + m210)/(1+exp(r1 + m21)+exp(r16 + m22)+exp(r31 + m23)+exp(r46 + m24)
+exp(r61 + m25)+exp(r76 + m26)+exp(r91 + m27)+exp(r106 + m28)+exp(r121 + m29)+
exp(r136 + m210)+exp(r151 + m211)+exp(r166 + m212)+exp(r181 + m213)+exp(r196 + m214)
+exp(r211 + m215));
t111a=exp(r151 + m211)/(1+exp(r1 + m21)+exp(r16 + m22)+exp(r31 + m23)+exp(r46 + m24)
+exp(r61 + m25)+exp(r76 + m26)+exp(r91 + m27)+exp(r106 + m28)+exp(r121 + m29)+
exp(r136 + m210)+exp(r151 + m211)+exp(r166 + m212)+exp(r181 + m213)+exp(r196 + m214)
+exp(r211 + m215));
t112a=exp(r166 + m212)/(1+exp(r1 + m21)+exp(r16 + m22)+exp(r31 + m23)+exp(r46 + m24)
+exp(r61 + m25)+exp(r76 + m26)+exp(r91 + m27)+exp(r106 + m28)+exp(r121 + m29)+
exp(r136 + m210)+exp(r151 + m211)+exp(r166 + m212)+exp(r181 + m213)+exp(r196 + m214)
+exp(r211 + m215));
t113a=exp(r181 + m213)/(1+exp(r1 + m21)+exp(r16 + m22)+exp(r31 + m23)+exp(r46 + m24)+
exp(r61 + m25)+exp(r76 + m26)+exp(r91 + m27)+exp(r106 + m28)+exp(r121 + m29)+
exp(r136 + m210)+exp(r151 + m211)+exp(r166 + m212)+exp(r181 + m213)+exp(r196 + m214)
+exp(r211 + m215));
t114a=exp(r196 + m214)/(1+exp(r1 + m21)+exp(r16 + m22)+exp(r31 + m23)+exp(r46 + m24)
+exp(r61 + m25)+exp(r76 + m26)+exp(r91 + m27)+exp(r106 + m28)+exp(r121 + m29)+
exp(r136 + m210)+exp(r151 + m211)+exp(r166 + m212)+exp(r181 + m213)+exp(r196 + m214)
+exp(r211 + m215));
t115a=exp(r211 + m215)/(1+exp(r1 + m21)+exp(r16 + m22)+exp(r31 + m23)+exp(r46 + m24)
+exp(r61 + m25)+exp(r76 + m26)+exp(r91 + m27)+exp(r106 + m28)+exp(r121 + m29)+
exp(r136 + m210)+exp(r151 + m211)+exp(r166 + m212)+exp(r181 + m213)+exp(r196 + m214)
+exp(r211 + m215));
t116a=1/(1+exp(r1 + m21)+exp(r16 + m22)+exp(r31 + m23)+exp(r46 + m24)+exp(r61 + m25)
+exp(r76 + m26)+exp(r91 + m27)+exp(r106 + m28)+exp(r121 + m29)+exp(r136 + m210)+
exp(r151 + m211)+exp(r166 + m212)+exp(r181 + m213)+exp(r196 + m214)+exp(r211 + m215));
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!row 2
t21b=exp(r2+m21)/(1+exp(r2+m21)+exp(r17+m22)+exp(r32+m23)+exp(r47+m24)+exp(r62+m25)
+exp(r77+m26)+exp(r92+m27)+exp(r107+m28)+exp(r122+m29)+exp(r137+m210)+exp(r152+m211)
+exp(r167+m212)+exp(r182+m213)+exp(r197+m214)+exp(r212+m215));
t22b=exp(r17+m22)/(1+exp(r2+m21)+exp(r17+m22)+exp(r32+m23)+exp(r47+m24)+exp(r62+m25)
+exp(r77+m26)+exp(r92+m27)+exp(r107+m28)+exp(r122+m29)+exp(r137+m210)+exp(r152+m211)
+exp(r167+m212)+exp(r182+m213)+exp(r197+m214)+exp(r212+m215));
t23b=exp(r32+m23)/(1+exp(r2+m21)+exp(r17+m22)+exp(r32+m23)+exp(r47+m24)+exp(r62+m25)
+exp(r77+m26)+exp(r92+m27)+exp(r107+m28)+exp(r122+m29)+exp(r137+m210)+exp(r152+m211)
+exp(r167+m212)+exp(r182+m213)+exp(r197+m214)+exp(r212+m215));
t24b=exp(r47+m24)/(1+exp(r2+m21)+exp(r17+m22)+exp(r32+m23)+exp(r47+m24)+exp(r62+m25)
+exp(r77+m26)+exp(r92+m27)+exp(r107+m28)+exp(r122+m29)+exp(r137+m210)+exp(r152+m211)
+exp(r167+m212)+exp(r182+m213)+exp(r197+m214)+exp(r212+m215));
t25b=exp(r62+m25)/(1+exp(r2+m21)+exp(r17+m22)+exp(r32+m23)+exp(r47+m24)+exp(r62+m25)
+exp(r77+m26)+exp(r92+m27)+exp(r107+m28)+exp(r122+m29)+exp(r137+m210)+exp(r152+m211)
+exp(r167+m212)+exp(r182+m213)+exp(r197+m214)+exp(r212+m215));
t26b=exp(r77+m26)/(1+exp(r2+m21)+exp(r17+m22)+exp(r32+m23)+exp(r47+m24)+exp(r62+m25)
+exp(r77+m26)+exp(r92+m27)+exp(r107+m28)+exp(r122+m29)+exp(r137+m210)+exp(r152+m211)
+exp(r167+m212)+exp(r182+m213)+exp(r197+m214)+exp(r212+m215));
t27b=exp(r92+m27)/(1+exp(r2+m21)+exp(r17+m22)+exp(r32+m23)+exp(r47+m24)+exp(r62+m25)
+exp(r77+m26)+exp(r92+m27)+exp(r107+m28)+exp(r122+m29)+exp(r137+m210)+exp(r152+m211)
+exp(r167+m212)+exp(r182+m213)+exp(r197+m214)+exp(r212+m215));
t28b=exp(r107+m28)/(1+exp(r2+m21)+exp(r17+m22)+exp(r32+m23)+exp(r47+m24)+exp(r62+m25)
+exp(r77+m26)+exp(r92+m27)+exp(r107+m28)+exp(r122+m29)+exp(r137+m210)+exp(r152+m211)+
exp(r167+m212)+exp(r182+m213)+exp(r197+m214)+exp(r212+m215));
t29b=exp(r122+m29)/(1+exp(r2+m21)+exp(r17+m22)+exp(r32+m23)+exp(r47+m24)+exp(r62+m25)+
exp(r77+m26)+exp(r92+m27)+exp(r107+m28)+exp(r122+m29)+exp(r137+m210)+exp(r152+m211)+
exp(r167+m212)+exp(r182+m213)+exp(r197+m214)+exp(r212+m215));
t210b=exp(r137+m210)/(1+exp(r2+m21)+exp(r17+m22)+exp(r32+m23)+exp(r47+m24)+exp(r62+m25)
+exp(r77+m26)+exp(r92+m27)+exp(r107+m28)+exp(r122+m29)+exp(r137+m210)+exp(r152+m211)+
exp(r167+m212)+exp(r182+m213)+exp(r197+m214)+exp(r212+m215));
t211b=exp(r152+m211)/(1+exp(r2+m21)+exp(r17+m22)+exp(r32+m23)+exp(r47+m24)+exp(r62+m25)
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+exp(r77+m26)+exp(r92+m27)+exp(r107+m28)+exp(r122+m29)+exp(r137+m210)+exp(r152+m211)+
exp(r167+m212)+exp(r182+m213)+exp(r197+m214)+exp(r212+m215));
t212b=exp(r167+m212)/(1+exp(r2+m21)+exp(r17+m22)+exp(r32+m23)+exp(r47+m24)+exp(r62+m25)
+exp(r77+m26)+exp(r92+m27)+exp(r107+m28)+exp(r122+m29)+exp(r137+m210)+exp(r152+m211)+
exp(r167+m212)+exp(r182+m213)+exp(r197+m214)+exp(r212+m215));
t213b=exp(r182+m213)/(1+exp(r2+m21)+exp(r17+m22)+exp(r32+m23)+exp(r47+m24)+exp(r62+m25)
+exp(r77+m26)+exp(r92+m27)+exp(r107+m28)+exp(r122+m29)+exp(r137+m210)+exp(r152+m211)+
exp(r167+m212)+exp(r182+m213)+exp(r197+m214)+exp(r212+m215));
t214b=exp(r197+m214)/(1+exp(r2+m21)+exp(r17+m22)+exp(r32+m23)+exp(r47+m24)+exp(r62+m25)
+exp(r77+m26)+exp(r92+m27)+exp(r107+m28)+exp(r122+m29)+exp(r137+m210)+exp(r152+m211)+
exp(r167+m212)+exp(r182+m213)+exp(r197+m214)+exp(r212+m215));
t215b=exp(r212+m215)/(1+exp(r2+m21)+exp(r17+m22)+exp(r32+m23)+exp(r47+m24)+exp(r62+m25)
+exp(r77+m26)+exp(r92+m27)+exp(r107+m28)+exp(r122+m29)+exp(r137+m210)+exp(r152+m211)+
exp(r167+m212)+exp(r182+m213)+exp(r197+m214)+exp(r212+m215));
t216b = 1/(1+exp(r2+m21)+exp(r17+m22)+exp(r32+m23)+exp(r47+m24)+exp(r62+m25)
+exp(r77+m26)+exp(r92+m27)+exp(r107+m28)+exp(r122+m29)+exp(r137+m210)+exp(r152+m211)+
exp(r167+m212)+exp(r182+m213)+exp(r197+m214)+exp(r212+m215));


!row 3
t31e=exp(r3+m21)/(1+exp(r3+m21)+exp(r18+m22)+exp(r33+m23)+exp(r48+m24)+exp(r63+m25)
+exp(r78+m26)+exp(r93+m27)+exp(r108+m28)+exp(r123+m29)+exp(r138+m210)+exp(r153+m211)
+exp(r168+m212)+exp(r183+m213)+exp(r198+m214)+exp(r213+m215));
t32e=exp(r18+m22)/(1+exp(r3+m21)+exp(r18+m22)+exp(r33+m23)+exp(r48+m24)+exp(r63+m25)
+exp(r78+m26)+exp(r93+m27)+exp(r108+m28)+exp(r123+m29)+exp(r138+m210)+exp(r153+m211)
+exp(r168+m212)+exp(r183+m213)+exp(r198+m214)+exp(r213+m215));
t33e=exp(r33+m23)/(1+exp(r3+m21)+exp(r18+m22)+exp(r33+m23)+exp(r48+m24)+exp(r63+m25)
+exp(r78+m26)+exp(r93+m27)+exp(r108+m28)+exp(r123+m29)+exp(r138+m210)+exp(r153+m211)
+exp(r168+m212)+exp(r183+m213)+exp(r198+m214)+exp(r213+m215));
t34e=exp(r48+m24)/(1+exp(r3+m21)+exp(r18+m22)+exp(r33+m23)+exp(r48+m24)+exp(r63+m25)
+exp(r78+m26)+exp(r93+m27)+exp(r108+m28)+exp(r123+m29)+exp(r138+m210)+exp(r153+m211)
+exp(r168+m212)+exp(r183+m213)+exp(r198+m214)+exp(r213+m215));
```

t35e=exp(r63+m25)/(1+exp(r3+m21)+exp(r18+m22)+exp(r33+m23)+exp(r48+m24)+exp(r63+m25)
+exp(r78+m26)+exp(r93+m27)+exp(r108+m28)+exp(r123+m29)+exp(r138+m210)+exp(r153+m211)
+exp(r168+m212)+exp(r183+m213)+exp(r198+m214)+exp(r213+m215));
t36e=exp(r78+m26)/(1+exp(r3+m21)+exp(r18+m22)+exp(r33+m23)+exp(r48+m24)+exp(r63+m25)
+exp(r78+m26)+exp(r93+m27)+exp(r108+m28)+exp(r123+m29)+exp(r138+m210)+exp(r153+m211)
+exp(r168+m212)+exp(r183+m213)+exp(r198+m214)+exp(r213+m215));
t37e=exp(r93+m27)/(1+exp(r3+m21)+exp(r18+m22)+exp(r33+m23)+exp(r48+m24)+exp(r63+m25)
+exp(r78+m26)+exp(r93+m27)+exp(r108+m28)+exp(r123+m29)+exp(r138+m210)+exp(r153+m211)
+exp(r168+m212)+exp(r183+m213)+exp(r198+m214)+exp(r213+m215));
t38e=exp(r108+m28)/(1+exp(r3+m21)+exp(r18+m22)+exp(r33+m23)+exp(r48+m24)+exp(r63+m25)
+exp(r78+m26)+exp(r93+m27)+exp(r108+m28)+exp(r123+m29)+exp(r138+m210)+exp(r153+m211)+
exp(r168+m212)+exp(r183+m213)+exp(r198+m214)+exp(r213+m215));
t39e=exp(r123+m29)/(1+exp(r3+m21)+exp(r18+m22)+exp(r33+m23)+exp(r48+m24)+exp(r63+m25)+
exp(r78+m26)+exp(r93+m27)+exp(r108+m28)+exp(r123+m29)+exp(r138+m210)+exp(r153+m211)+
exp(r168+m212)+exp(r183+m213)+exp(r198+m214)+exp(r213+m215));
t310e=exp(r138+m210)/(1+exp(r3+m21)+exp(r18+m22)+exp(r33+m23)+exp(r48+m24)+exp(r63+m25)+
exp(r78+m26)+exp(r93+m27)+exp(r108+m28)+exp(r123+m29)+exp(r138+m210)+exp(r153+m211)+
exp(r168+m212)+exp(r183+m213)+exp(r198+m214)+exp(r213+m215));
t311e=exp(r153+m211)/(1+exp(r3+m21)+exp(r18+m22)+exp(r33+m23)+exp(r48+m24)+exp(r63+m25)
+exp(r78+m26)+exp(r93+m27)+exp(r108+m28)+exp(r123+m29)+exp(r138+m210)+exp(r153+m211)+
exp(r168+m212)+exp(r183+m213)+exp(r198+m214)+exp(r213+m215));
t312e=exp(r168+m212)/(1+exp(r3+m21)+exp(r18+m22)+exp(r33+m23)+exp(r48+m24)+exp(r63+m25)+
exp(r78+m26)+exp(r93+m27)+exp(r108+m28)+exp(r123+m29)+exp(r138+m210)+exp(r153+m211)+
exp(r168+m212)+exp(r183+m213)+exp(r198+m214)+exp(r213+m215));
t313e=exp(r183+m213)/(1+exp(r3+m21)+exp(r18+m22)+exp(r33+m23)+exp(r48+m24)+exp(r63+m25)+
exp(r78+m26)+exp(r93+m27)+exp(r108+m28)+exp(r123+m29)+exp(r138+m210)+exp(r153+m211)+
exp(r168+m212)+exp(r183+m213)+exp(r198+m214)+exp(r213+m215));
t314e=exp(r198+m214)/(1+exp(r3+m21)+exp(r18+m22)+exp(r33+m23)+exp(r48+m24)+exp(r63+m25)
+exp(r78+m26)+exp(r93+m27)+exp(r108+m28)+exp(r123+m29)+exp(r138+m210)+exp(r153+m211)+
exp(r168+m212)+exp(r183+m213)+exp(r198+m214)+exp(r213+m215));
t315e=exp(r213+m215)/(1+exp(r3+m21)+exp(r18+m22)+exp(r33+m23)+exp(r48+m24)+exp(r63+m25)
+exp(r78+m26)+exp(r93+m27)+exp(r108+m28)+exp(r123+m29)+exp(r138+m210)+exp(r153+m211)+

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exp(r168+m212)+exp(r183+m213)+exp(r198+m214)+exp(r213+m215));
t316e = 1/(1+exp(r3+m21)+exp(r18+m22)+exp(r33+m23)+exp(r48+m24)+exp(r63+m25)
+exp(r78+m26)+exp(r93+m27)+exp(r108+m28)+exp(r123+m29)+exp(r138+m210)+exp(r153+m211)+
exp(r168+m212)+exp(r183+m213)+exp(r198+m214)+exp(r213+m215));

!row 4
t41f=exp(r4+m21)/(1+exp(r4+m21)+exp(r19+m22)+exp(r34+m23)+exp(r49+m24)+exp(r64+m25)
+exp(r79+m26)+exp(r94+m27)+exp(r109+m28)+exp(r124+m29)+exp(r139+m210)+exp(r154+m211)
+exp(r169+m212)+exp(r184+m213)+exp(r199+m214)+exp(r214+m215));
t42f=exp(r19+m22)/(1+exp(r4+m21)+exp(r19+m22)+exp(r34+m23)+exp(r49+m24)+exp(r64+m25)
+exp(r79+m26)+exp(r94+m27)+exp(r109+m28)+exp(r124+m29)+exp(r139+m210)+exp(r154+m211)
+exp(r169+m212)+exp(r184+m213)+exp(r199+m214)+exp(r214+m215));
t43f=exp(r34+m23)/(1+exp(r4+m21)+exp(r19+m22)+exp(r34+m23)+exp(r49+m24)+exp(r64+m25)
+exp(r79+m26)+exp(r94+m27)+exp(r109+m28)+exp(r124+m29)+exp(r139+m210)+exp(r154+m211)
+exp(r169+m212)+exp(r184+m213)+exp(r199+m214)+exp(r214+m215));
t44f=exp(r49+m24)/(1+exp(r4+m21)+exp(r19+m22)+exp(r34+m23)+exp(r49+m24)+exp(r64+m25)
+exp(r79+m26)+exp(r94+m27)+exp(r109+m28)+exp(r124+m29)+exp(r139+m210)+exp(r154+m211)
+exp(r169+m212)+exp(r184+m213)+exp(r199+m214)+exp(r214+m215));
t45f=exp(r64+m25)/(1+exp(r4+m21)+exp(r19+m22)+exp(r34+m23)+exp(r49+m24)+exp(r64+m25)
+exp(r79+m26)+exp(r94+m27)+exp(r109+m28)+exp(r124+m29)+exp(r139+m210)+exp(r154+m211)
+exp(r169+m212)+exp(r184+m213)+exp(r199+m214)+exp(r214+m215));
t46f=exp(r79+m26)/(1+exp(r4+m21)+exp(r19+m22)+exp(r34+m23)+exp(r49+m24)+exp(r64+m25)
+exp(r79+m26)+exp(r94+m27)+exp(r109+m28)+exp(r124+m29)+exp(r139+m210)+exp(r154+m211)
+exp(r169+m212)+exp(r184+m213)+exp(r199+m214)+exp(r214+m215));
t47f=exp(r94+m27)/(1+exp(r4+m21)+exp(r19+m22)+exp(r34+m23)+exp(r49+m24)+exp(r64+m25)
+exp(r79+m26)+exp(r94+m27)+exp(r109+m28)+exp(r124+m29)+exp(r139+m210)+exp(r154+m211)
+exp(r169+m212)+exp(r184+m213)+exp(r199+m214)+exp(r214+m215));
t48f=exp(r109+m28)/(1+exp(r4+m21)+exp(r19+m22)+exp(r34+m23)+exp(r49+m24)+exp(r64+m25)
+exp(r79+m26)+exp(r94+m27)+exp(r109+m28)+exp(r124+m29)+exp(r139+m210)+exp(r154+m211)
+exp(r169+m212)+exp(r184+m213)+exp(r199+m214)+exp(r214+m215));
t49f=exp(r124+m29)/(1+exp(r4+m21)+exp(r19+m22)+exp(r34+m23)+exp(r49+m24)+exp(r64+m25)
+exp(r79+m26)+exp(r94+m27)+exp(r109+m28)+exp(r124+m29)+exp(r139+m210)+exp(r154+m211)
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+exp(r169+m212)+exp(r184+m213)+exp(r199+m214)+exp(r214+m215));
t410f=exp(r139+m210)/(1+exp(r4+m21)+exp(r19+m22)+exp(r34+m23)+exp(r49+m24)+exp(r64+m25)
+exp(r79+m26)+exp(r94+m27)+exp(r109+m28)+exp(r124+m29)+exp(r139+m210)+exp(r154+m211)+
exp(r169+m212)+exp(r184+m213)+exp(r199+m214)+exp(r214+m215));
t411f=exp(r154+m211)/(1+exp(r4+m21)+exp(r19+m22)+exp(r34+m23)+exp(r49+m24)+exp(r64+m25)
+exp(r79+m26)+exp(r94+m27)+exp(r109+m28)+exp(r124+m29)+exp(r139+m210)+exp(r154+m211)+
exp(r169+m212)+exp(r184+m213)+exp(r199+m214)+exp(r214+m215));
t412f=exp(r169+m212)/(1+exp(r4+m21)+exp(r19+m22)+exp(r34+m23)+exp(r49+m24)+exp(r64+m25)
+exp(r79+m26)+exp(r94+m27)+exp(r109+m28)+exp(r124+m29)+exp(r139+m210)+exp(r154+m211)+
exp(r169+m212)+exp(r184+m213)+exp(r199+m214)+exp(r214+m215));
t413f=exp(r184+m213)/(1+exp(r4+m21)+exp(r19+m22)+exp(r34+m23)+exp(r49+m24)+exp(r64+m25)
+exp(r79+m26)+exp(r94+m27)+exp(r109+m28)+exp(r124+m29)+exp(r139+m210)+exp(r154+m211)+
exp(r169+m212)+exp(r184+m213)+exp(r199+m214)+exp(r214+m215));
t414f=exp(r199+m214)/(1+exp(r4+m21)+exp(r19+m22)+exp(r34+m23)+exp(r49+m24)+exp(r64+m25)+
exp(r79+m26)+exp(r94+m27)+exp(r109+m28)+exp(r124+m29)+exp(r139+m210)+exp(r154+m211)+
exp(r169+m212)+exp(r184+m213)+exp(r199+m214)+exp(r214+m215));
t415f=exp(r214+m215)/(1+exp(r4+m21)+exp(r19+m22)+exp(r34+m23)+exp(r49+m24)+exp(r64+m25)
+exp(r79+m26)+exp(r94+m27)+exp(r109+m28)+exp(r124+m29)+exp(r139+m210)+exp(r154+m211)+
exp(r169+m212)+exp(r184+m213)+exp(r199+m214)+exp(r214+m215));
t416f = 1/(1+exp(r4+m21)+exp(r19+m22)+exp(r34+m23)+exp(r49+m24)+exp(r64+m25)
+exp(r79+m26)+exp(r94+m27)+exp(r109+m28)+exp(r124+m29)+exp(r139+m210)+exp(r154+m211)+
exp(r169+m212)+exp(r184+m213)+exp(r199+m214)+exp(r214+m215));

!row 5
t51h=exp(r5+m21)/(1+exp(r5+m21)+exp(r20+m22)+exp(r35+m23)+exp(r50+m24)+exp(r65+m25)
+exp(r80+m26)+exp(r95+m27)+exp(r110+m28)+exp(r125+m29)+exp(r140+m210)+exp(r155+m211)
+exp(r170+m212)+exp(r185+m213)+exp(r200+m214)+exp(r215+m215));
t52h=exp(r20+m22)/(1+exp(r5+m21)+exp(r20+m22)+exp(r35+m23)+exp(r50+m24)+exp(r65+m25)
+exp(r80+m26)+exp(r95+m27)+exp(r110+m28)+exp(r125+m29)+exp(r140+m210)+exp(r155+m211)
+exp(r170+m212)+exp(r185+m213)+exp(r200+m214)+exp(r215+m215));
t53h=exp(r35+m23)/(1+exp(r5+m21)+exp(r20+m22)+exp(r35+m23)+exp(r50+m24)+exp(r65+m25)
+exp(r80+m26)+exp(r95+m27)+exp(r110+m28)+exp(r125+m29)+exp(r140+m210)+exp(r155+m211)
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+exp(r170+m212)+exp(r185+m213)+exp(r200+m214)+exp(r215+m215));
t54h=exp(r50+m24)/(1+exp(r5+m21)+exp(r20+m22)+exp(r35+m23)+exp(r50+m24)+exp(r65+m25)
+exp(r80+m26)+exp(r95+m27)+exp(r110+m28)+exp(r125+m29)+exp(r140+m210)+exp(r155+m211)
+exp(r170+m212)+exp(r185+m213)+exp(r200+m214)+exp(r215+m215));
t55h=exp(r65+m25)/(1+exp(r5+m21)+exp(r20+m22)+exp(r35+m23)+exp(r50+m24)+exp(r65+m25)
+exp(r80+m26)+exp(r95+m27)+exp(r110+m28)+exp(r125+m29)+exp(r140+m210)+exp(r155+m211)
+exp(r170+m212)+exp(r185+m213)+exp(r200+m214)+exp(r215+m215));
t56h=exp(r80+m26)/(1+exp(r5+m21)+exp(r20+m22)+exp(r35+m23)+exp(r50+m24)+exp(r65+m25)
+exp(r80+m26)+exp(r95+m27)+exp(r110+m28)+exp(r125+m29)+exp(r140+m210)+exp(r155+m211)
+exp(r170+m212)+exp(r185+m213)+exp(r200+m214)+exp(r215+m215));
t57h=exp(r95+m27)/(1+exp(r5+m21)+exp(r20+m22)+exp(r35+m23)+exp(r50+m24)+exp(r65+m25)
+exp(r80+m26)+exp(r95+m27)+exp(r110+m28)+exp(r125+m29)+exp(r140+m210)+exp(r155+m211)
+exp(r170+m212)+exp(r185+m213)+exp(r200+m214)+exp(r215+m215));
t58h=exp(r110+m28)/(1+exp(r5+m21)+exp(r20+m22)+exp(r35+m23)+exp(r50+m24)+exp(r65+m25)
+exp(r80+m26)+exp(r95+m27)+exp(r110+m28)+exp(r125+m29)+exp(r140+m210)+exp(r155+m211)+
exp(r170+m212)+exp(r185+m213)+exp(r200+m214)+exp(r215+m215));
t59h=exp(r125+m29)/(1+exp(r5+m21)+exp(r20+m22)+exp(r35+m23)+exp(r50+m24)+exp(r65+m25)
+exp(r80+m26)+exp(r95+m27)+exp(r110+m28)+exp(r125+m29)+exp(r140+m210)+exp(r155+m211)+
exp(r170+m212)+exp(r185+m213)+exp(r200+m214)+exp(r215+m215));
t510h=exp(r140+m210)/(1+exp(r5+m21)+exp(r20+m22)+exp(r35+m23)+exp(r50+m24)+exp(r65+m25)
+exp(r80+m26)+exp(r95+m27)+exp(r110+m28)+exp(r125+m29)+exp(r140+m210)+exp(r155+m211)+
exp(r170+m212)+exp(r185+m213)+exp(r200+m214)+exp(r215+m215));
t511h=exp(r155+m211)/(1+exp(r5+m21)+exp(r20+m22)+exp(r35+m23)+exp(r50+m24)+exp(r65+m25)
+exp(r80+m26)+exp(r95+m27)+exp(r110+m28)+exp(r125+m29)+exp(r140+m210)+exp(r155+m211)+
exp(r170+m212)+exp(r185+m213)+exp(r200+m214)+exp(r215+m215));
t512h=exp(r170+m212)/(1+exp(r5+m21)+exp(r20+m22)+exp(r35+m23)+exp(r50+m24)+exp(r65+m25)+
exp(r80+m26)+exp(r95+m27)+exp(r110+m28)+exp(r125+m29)+exp(r140+m210)+exp(r155+m211)+
exp(r170+m212)+exp(r185+m213)+exp(r200+m214)+exp(r215+m215));
t513h=exp(r185+m213)/(1+exp(r5+m21)+exp(r20+m22)+exp(r35+m23)+exp(r50+m24)+exp(r65+m25)
+exp(r80+m26)+exp(r95+m27)+exp(r110+m28)+exp(r125+m29)+exp(r140+m210)+exp(r155+m211)+
exp(r170+m212)+exp(r185+m213)+exp(r200+m214)+exp(r215+m215));
t514h=exp(r200+m214)/(1+exp(r5+m21)+exp(r20+m22)+exp(r35+m23)+exp(r50+m24)+exp(r65+m25)

+exp(r80+m26)+exp(r95+m27)+exp(r110+m28)+exp(r125+m29)+exp(r140+m210)+exp(r155+m211)+
exp(r170+m212)+exp(r185+m213)+exp(r200+m214)+exp(r215+m215));
t515h=exp(r215+m215)/(1+exp(r5+m21)+exp(r20+m22)+exp(r35+m23)+exp(r50+m24)+exp(r65+m25)
+exp(r80+m26)+exp(r95+m27)+exp(r110+m28)+exp(r125+m29)+exp(r140+m210)+exp(r155+m211)+
exp(r170+m212)+exp(r185+m213)+exp(r200+m214)+exp(r215+m215));
t516h = 1/(1+exp(r5+m21)+exp(r20+m22)+exp(r35+m23)+exp(r50+m24)+exp(r65+m25)
+exp(r80+m26)+exp(r95+m27)+exp(r110+m28)+exp(r125+m29)+exp(r140+m210)+exp(r155+m211)+
exp(r170+m212)+exp(r185+m213)+exp(r200+m214)+exp(r215+m215));

!row 6
t61i=exp(r6+m21)/(1+exp(r6+m21)+exp(r21+m22)+exp(r36+m23)+exp(r51+m24)+exp(r66+m25)
+exp(r81+m26)+exp(r96+m27)+exp(r111+m28)+exp(r126+m29)+exp(r141+m210)+exp(r156+m211)
+exp(r171+m212)+exp(r186+m213)+exp(r201+m214)+exp(r216+m215));
t62i=exp(r21+m22)/(1+exp(r6+m21)+exp(r21+m22)+exp(r36+m23)+exp(r51+m24)+exp(r66+m25)
+exp(r81+m26)+exp(r96+m27)+exp(r111+m28)+exp(r126+m29)+exp(r141+m210)+exp(r156+m211)
+exp(r171+m212)+exp(r186+m213)+exp(r201+m214)+exp(r216+m215));
t63i=exp(r36+m23)/(1+exp(r6+m21)+exp(r21+m22)+exp(r36+m23)+exp(r51+m24)+exp(r66+m25)
+exp(r81+m26)+exp(r96+m27)+exp(r111+m28)+exp(r126+m29)+exp(r141+m210)+exp(r156+m211)
+exp(r171+m212)+exp(r186+m213)+exp(r201+m214)+exp(r216+m215));
t64i=exp(r51+m24)/(1+exp(r6+m21)+exp(r21+m22)+exp(r36+m23)+exp(r51+m24)+exp(r66+m25)
+exp(r81+m26)+exp(r96+m27)+exp(r111+m28)+exp(r126+m29)+exp(r141+m210)+exp(r156+m211)
+exp(r171+m212)+exp(r186+m213)+exp(r201+m214)+exp(r216+m215));
t65i=exp(r66+m25)/(1+exp(r6+m21)+exp(r21+m22)+exp(r36+m23)+exp(r51+m24)+exp(r66+m25)
+exp(r81+m26)+exp(r96+m27)+exp(r111+m28)+exp(r126+m29)+exp(r141+m210)+exp(r156+m211)
+exp(r171+m212)+exp(r186+m213)+exp(r201+m214)+exp(r216+m215));
t66i=exp(r81+m26)/(1+exp(r6+m21)+exp(r21+m22)+exp(r36+m23)+exp(r51+m24)+exp(r66+m25)
+exp(r81+m26)+exp(r96+m27)+exp(r111+m28)+exp(r126+m29)+exp(r141+m210)+exp(r156+m211)
+exp(r171+m212)+exp(r186+m213)+exp(r201+m214)+exp(r216+m215));
t67i=exp(r96+m27)/(1+exp(r6+m21)+exp(r21+m22)+exp(r36+m23)+exp(r51+m24)+exp(r66+m25)
+exp(r81+m26)+exp(r96+m27)+exp(r111+m28)+exp(r126+m29)+exp(r141+m210)+exp(r156+m211)
+exp(r171+m212)+exp(r186+m213)+exp(r201+m214)+exp(r216+m215));
t68i=exp(r111+m28)/(1+exp(r6+m21)+exp(r21+m22)+exp(r36+m23)+exp(r51+m24)+exp(r66+m25)

+exp(r81+m26)+exp(r96+m27)+exp(r111+m28)+exp(r126+m29)+exp(r141+m210)+exp(r156+m211)
+exp(r171+m212)+exp(r186+m213)+exp(r201+m214)+exp(r216+m215));
t69i=exp(r126+m29)/(1+exp(r6+m21)+exp(r21+m22)+exp(r36+m23)+exp(r51+m24)+exp(r66+m25)
+exp(r81+m26)+exp(r96+m27)+exp(r111+m28)+exp(r126+m29)+exp(r141+m210)+exp(r156+m211)
+exp(r171+m212)+exp(r186+m213)+exp(r201+m214)+exp(r216+m215));
t610i=exp(r141+m210)/(1+exp(r6+m21)+exp(r21+m22)+exp(r36+m23)+exp(r51+m24)+exp(r66+m25)
+exp(r81+m26)+exp(r96+m27)+exp(r111+m28)+exp(r126+m29)+exp(r141+m210)+exp(r156+m211)+
exp(r171+m212)+exp(r186+m213)+exp(r201+m214)+exp(r216+m215));
t611i=exp(r156+m211)/(1+exp(r6+m21)+exp(r21+m22)+exp(r36+m23)+exp(r51+m24)+exp(r66+m25)
+exp(r81+m26)+exp(r96+m27)+exp(r111+m28)+exp(r126+m29)+exp(r141+m210)+exp(r156+m211)+
exp(r171+m212)+exp(r186+m213)+exp(r201+m214)+exp(r216+m215));
t612i=exp(r171+m212)/(1+exp(r6+m21)+exp(r21+m22)+exp(r36+m23)+exp(r51+m24)+exp(r66+m25)
+exp(r81+m26)+exp(r96+m27)+exp(r111+m28)+exp(r126+m29)+exp(r141+m210)+exp(r156+m211)+
exp(r171+m212)+exp(r186+m213)+exp(r201+m214)+exp(r216+m215));
t613i=exp(r186+m213)/(1+exp(r6+m21)+exp(r21+m22)+exp(r36+m23)+exp(r51+m24)+exp(r66+m25)
+exp(r81+m26)+exp(r96+m27)+exp(r111+m28)+exp(r126+m29)+exp(r141+m210)+exp(r156+m211)+
exp(r171+m212)+exp(r186+m213)+exp(r201+m214)+exp(r216+m215));
t614i=exp(r201+m214)/(1+exp(r6+m21)+exp(r21+m22)+exp(r36+m23)+exp(r51+m24)+exp(r66+m25)
+exp(r81+m26)+exp(r96+m27)+exp(r111+m28)+exp(r126+m29)+exp(r141+m210)+exp(r156+m211)+
exp(r171+m212)+exp(r186+m213)+exp(r201+m214)+exp(r216+m215));
t615i=exp(r216+m215)/(1+exp(r6+m21)+exp(r21+m22)+exp(r36+m23)+exp(r51+m24)+exp(r66+m25)
+exp(r81+m26)+exp(r96+m27)+exp(r111+m28)+exp(r126+m29)+exp(r141+m210)+exp(r156+m211)+
exp(r171+m212)+exp(r186+m213)+exp(r201+m214)+exp(r216+m215));
t616i = 1/(1+exp(r6+m21)+exp(r21+m22)+exp(r36+m23)+exp(r51+m24)+exp(r66+m25)
+exp(r81+m26)+exp(r96+m27)+exp(r111+m28)+exp(r126+m29)+exp(r141+m210)+exp(r156+m211)+
exp(r171+m212)+exp(r186+m213)+exp(r201+m214)+exp(r216+m215));

!row 7
t71j=exp(r7+m21)/(1+exp(r7+m21)+exp(r22+m22)+exp(r37+m23)+exp(r52+m24)+exp(r67+m25)+
exp(r82+m26)+exp(r97+m27)+exp(r112+m28)+exp(r127+m29)+exp(r142+m210)+exp(r157+m211)+
exp(r172+m212)+exp(r187+m213)+exp(r202+m214)+exp(r217+m215));
t72j=exp(r22+m22)/(1+exp(r7+m21)+exp(r22+m22)+exp(r37+m23)+exp(r52+m24)+exp(r67+m25)+

exp(r82+m26)+exp(r97+m27)+exp(r112+m28)+exp(r127+m29)+exp(r142+m210)+exp(r157+m211)+
exp(r172+m212)+exp(r187+m213)+exp(r202+m214)+exp(r217+m215));
t73j=exp(r37+m23)/(1+exp(r7+m21)+exp(r22+m22)+exp(r37+m23)+exp(r52+m24)+exp(r67+m25)+
exp(r82+m26)+exp(r97+m27)+exp(r112+m28)+exp(r127+m29)+exp(r142+m210)+exp(r157+m211)+
exp(r172+m212)+exp(r187+m213)+exp(r202+m214)+exp(r217+m215));
t74j=exp(r52+m24)/(1+exp(r7+m21)+exp(r22+m22)+exp(r37+m23)+exp(r52+m24)+exp(r67+m25)+
exp(r82+m26)+exp(r97+m27)+exp(r112+m28)+exp(r127+m29)+exp(r142+m210)+exp(r157+m211)+
exp(r172+m212)+exp(r187+m213)+exp(r202+m214)+exp(r217+m215));
t75j=exp(r67+m25)/(1+exp(r7+m21)+exp(r22+m22)+exp(r37+m23)+exp(r52+m24)+exp(r67+m25)+
exp(r82+m26)+exp(r97+m27)+exp(r112+m28)+exp(r127+m29)+exp(r142+m210)+exp(r157+m211)+
exp(r172+m212)+exp(r187+m213)+exp(r202+m214)+exp(r217+m215));
t76j=exp(r82+m26)/(1+exp(r7+m21)+exp(r22+m22)+exp(r37+m23)+exp(r52+m24)+exp(r67+m25)+
exp(r82+m26)+exp(r97+m27)+exp(r112+m28)+exp(r127+m29)+exp(r142+m210)+exp(r157+m211)+
exp(r172+m212)+exp(r187+m213)+exp(r202+m214)+exp(r217+m215));
t77j=exp(r97+m27)/(1+exp(r7+m21)+exp(r22+m22)+exp(r37+m23)+exp(r52+m24)+exp(r67+m25)+
exp(r82+m26)+exp(r97+m27)+exp(r112+m28)+exp(r127+m29)+exp(r142+m210)+exp(r157+m211)+
exp(r172+m212)+exp(r187+m213)+exp(r202+m214)+exp(r217+m215));
t78j=exp(r112+m28)/(1+exp(r7+m21)+exp(r22+m22)+exp(r37+m23)+exp(r52+m24)+exp(r67+m25)+
exp(r82+m26)+exp(r97+m27)+exp(r112+m28)+exp(r127+m29)+exp(r142+m210)+exp(r157+m211)+
exp(r172+m212)+exp(r187+m213)+exp(r202+m214)+exp(r217+m215));
t79j=exp(r127+m29)/(1+exp(r7+m21)+exp(r22+m22)+exp(r37+m23)+exp(r52+m24)+exp(r67+m25)+
exp(r82+m26)+exp(r97+m27)+exp(r112+m28)+exp(r127+m29)+exp(r142+m210)+exp(r157+m211)+
exp(r172+m212)+exp(r187+m213)+exp(r202+m214)+exp(r217+m215));
t710j=exp(r142+m210)/(1+exp(r7+m21)+exp(r22+m22)+exp(r37+m23)+exp(r52+m24)+exp(r67+m25)
+exp(r82+m26)+exp(r97+m27)+exp(r112+m28)+exp(r127+m29)+exp(r142+m210)+exp(r157+m211)+
exp(r172+m212)+exp(r187+m213)+exp(r202+m214)+exp(r217+m215));
t711j=exp(r157+m211)/(1+exp(r7+m21)+exp(r22+m22)+exp(r37+m23)+exp(r52+m24)+exp(r67+m25)
+exp(r82+m26)+exp(r97+m27)+exp(r112+m28)+exp(r127+m29)+exp(r142+m210)+exp(r157+m211)+
exp(r172+m212)+exp(r187+m213)+exp(r202+m214)+exp(r217+m215));
t712j=exp(r172+m212)/(1+exp(r7+m21)+exp(r22+m22)+exp(r37+m23)+exp(r52+m24)+exp(r67+m25)
+exp(r82+m26)+exp(r97+m27)+exp(r112+m28)+exp(r127+m29)+exp(r142+m210)+exp(r157+m211)
+exp(r172+m212)+exp(r187+m213)+exp(r202+m214)+exp(r217+m215));

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t713j=exp(r187+m213)/(1+exp(r7+m21)+exp(r22+m22)+exp(r37+m23)+exp(r52+m24)+exp(r67+m25)
+exp(r82+m26)+exp(r97+m27)+exp(r112+m28)+exp(r127+m29)+exp(r142+m210)+exp(r157+m211)
+exp(r172+m212)+exp(r187+m213)+exp(r202+m214)+exp(r217+m215));
t714j=exp(r202+m214)/(1+exp(r7+m21)+exp(r22+m22)+exp(r37+m23)+exp(r52+m24)+exp(r67+m25)
+exp(r82+m26)+exp(r97+m27)+exp(r112+m28)+exp(r127+m29)+exp(r142+m210)+exp(r157+m211)+
exp(r172+m212)+exp(r187+m213)+exp(r202+m214)+exp(r217+m215));
t715j=exp(r217+m215)/(1+exp(r7+m21)+exp(r22+m22)+exp(r37+m23)+exp(r52+m24)+exp(r67+m25)
+exp(r82+m26)+exp(r97+m27)+exp(r112+m28)+exp(r127+m29)+exp(r142+m210)+exp(r157+m211)+
exp(r172+m212)+exp(r187+m213)+exp(r202+m214)+exp(r217+m215));
t716j=1/(1+exp(r7+m21)+exp(r22+m22)+exp(r37+m23)+exp(r52+m24)+exp(r67+m25)
+exp(r82+m26)+exp(r97+m27)+exp(r112+m28)+exp(r127+m29)+exp(r142+m210)+exp(r157+m211)+
exp(r172+m212)+exp(r187+m213)+exp(r202+m214)+exp(r217+m215));

!row 8
t81k=exp(r8+m21)/(1+exp(r8+m21)+exp(r23+m22)+exp(r38+m23)+exp(r53+m24)+exp(r68+m25)
+exp(r83+m26)+exp(r98+m27)+exp(r113+m28)+exp(r128+m29)+exp(r143+m210)+exp(r158+m211)
+exp(r173+m212)+exp(r188+m213)+exp(r203+m214)+exp(r218+m215));
t82k=exp(r23+m22)/(1+exp(r8+m21)+exp(r23+m22)+exp(r38+m23)+exp(r53+m24)+exp(r68+m25)
+exp(r83+m26)+exp(r98+m27)+exp(r113+m28)+exp(r128+m29)+exp(r143+m210)+exp(r158+m211)
+exp(r173+m212)+exp(r188+m213)+exp(r203+m214)+exp(r218+m215));
t83k=exp(r38+m23)/(1+exp(r8+m21)+exp(r23+m22)+exp(r38+m23)+exp(r53+m24)+exp(r68+m25)
+exp(r83+m26)+exp(r98+m27)+exp(r113+m28)+exp(r128+m29)+exp(r143+m210)+exp(r158+m211)
+exp(r173+m212)+exp(r188+m213)+exp(r203+m214)+exp(r218+m215));
t84k=exp(r53+m24)/(1+exp(r8+m21)+exp(r23+m22)+exp(r38+m23)+exp(r53+m24)+exp(r68+m25)
+exp(r83+m26)+exp(r98+m27)+exp(r113+m28)+exp(r128+m29)+exp(r143+m210)+exp(r158+m211)
+exp(r173+m212)+exp(r188+m213)+exp(r203+m214)+exp(r218+m215));
t85k=exp(r68+m25)/(1+exp(r8+m21)+exp(r23+m22)+exp(r38+m23)+exp(r53+m24)+exp(r68+m25)
+exp(r83+m26)+exp(r98+m27)+exp(r113+m28)+exp(r128+m29)+exp(r143+m210)+exp(r158+m211)
+exp(r173+m212)+exp(r188+m213)+exp(r203+m214)+exp(r218+m215));
t86k=exp(r83+m26)/(1+exp(r8+m21)+exp(r23+m22)+exp(r38+m23)+exp(r53+m24)+exp(r68+m25)
+exp(r83+m26)+exp(r98+m27)+exp(r113+m28)+exp(r128+m29)+exp(r143+m210)+exp(r158+m211)
+exp(r173+m212)+exp(r188+m213)+exp(r203+m214)+exp(r218+m215));
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t87k=exp(r98+m27)/(1+exp(r8+m21)+exp(r23+m22)+exp(r38+m23)+exp(r53+m24)+exp(r68+m25)
+exp(r83+m26)+exp(r98+m27)+exp(r113+m28)+exp(r128+m29)+exp(r143+m210)+exp(r158+m211)
+exp(r173+m212)+exp(r188+m213)+exp(r203+m214)+exp(r218+m215));
t88k=exp(r113+m28)/(1+exp(r8+m21)+exp(r23+m22)+exp(r38+m23)+exp(r53+m24)+exp(r68+m25)
+exp(r83+m26)+exp(r98+m27)+exp(r113+m28)+exp(r128+m29)+exp(r143+m210)+exp(r158+m211)
+exp(r173+m212)+exp(r188+m213)+exp(r203+m214)+exp(r218+m215));
t89k=exp(r128+m29)/(1+exp(r8+m21)+exp(r23+m22)+exp(r38+m23)+exp(r53+m24)+exp(r68+m25)
+exp(r83+m26)+exp(r98+m27)+exp(r113+m28)+exp(r128+m29)+exp(r143+m210)+exp(r158+m211)
+exp(r173+m212)+exp(r188+m213)+exp(r203+m214)+exp(r218+m215));
t810k=exp(r143+m210)/(1+exp(r8+m21)+exp(r23+m22)+exp(r38+m23)+exp(r53+m24)+exp(r68+m25)
+exp(r83+m26)+exp(r98+m27)+exp(r113+m28)+exp(r128+m29)+exp(r143+m210)+exp(r158+m211)
+exp(r173+m212)+exp(r188+m213)+exp(r203+m214)+exp(r218+m215));
t811k=exp(r158+m211)/(1+exp(r8+m21)+exp(r23+m22)+exp(r38+m23)+exp(r53+m24)+exp(r68+m25)
+exp(r83+m26)+exp(r98+m27)+exp(r113+m28)+exp(r128+m29)+exp(r143+m210)+exp(r158+m211)
+exp(r173+m212)+exp(r188+m213)+exp(r203+m214)+exp(r218+m215));
t812k=exp(r173+m212)/(1+exp(r8+m21)+exp(r23+m22)+exp(r38+m23)+exp(r53+m24)+exp(r68+m25)
+exp(r83+m26)+exp(r98+m27)+exp(r113+m28)+exp(r128+m29)+exp(r143+m210)+exp(r158+m211)
+exp(r173+m212)+exp(r188+m213)+exp(r203+m214)+exp(r218+m215));
t813k=exp(r188+m213)/(1+exp(r8+m21)+exp(r23+m22)+exp(r38+m23)+exp(r53+m24)+exp(r68+m25)
+exp(r83+m26)+exp(r98+m27)+exp(r113+m28)+exp(r128+m29)+exp(r143+m210)+exp(r158+m211)
+exp(r173+m212)+exp(r188+m213)+exp(r203+m214)+exp(r218+m215));
t814k=exp(r203+m214)/(1+exp(r8+m21)+exp(r23+m22)+exp(r38+m23)+exp(r53+m24)+exp(r68+m25)
+exp(r83+m26)+exp(r98+m27)+exp(r113+m28)+exp(r128+m29)+exp(r143+m210)+exp(r158+m211)
+exp(r173+m212)+exp(r188+m213)+exp(r203+m214)+exp(r218+m215));
t815k=exp(r218+m215)/(1+exp(r8+m21)+exp(r23+m22)+exp(r38+m23)+exp(r53+m24)+exp(r68+m25)
+exp(r83+m26)+exp(r98+m27)+exp(r113+m28)+exp(r128+m29)+exp(r143+m210)+exp(r158+m211)+
exp(r173+m212)+exp(r188+m213)+exp(r203+m214)+exp(r218+m215));
t816k=1/(1+exp(r8+m21)+exp(r23+m22)+exp(r38+m23)+exp(r53+m24)+exp(r68+m25)
+exp(r83+m26)+exp(r98+m27)+exp(r113+m28)+exp(r128+m29)+exp(r143+m210)+exp(r158+m211)+
exp(r173+m212)+exp(r188+m213)+exp(r203+m214)+exp(r218+m215));

!row 9
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t91n=exp(r9+m21)/(1+exp(r9+m21)+exp(r24+m22)+exp(r39+m23)+exp(r54+m24)+exp(r69+m25)
+exp(r84+m26)+exp(r99+m27)+exp(r114+m28)+exp(r129+m29)+exp(r144+m210)+exp(r159+m211)
+exp(r174+m212)+exp(r189+m213)+exp(r204+m214)+exp(r219+m215));
t92n=exp(r24+m22)/(1+exp(r9+m21)+exp(r24+m22)+exp(r39+m23)+exp(r54+m24)+exp(r69+m25)
+exp(r84+m26)+exp(r99+m27)+exp(r114+m28)+exp(r129+m29)+exp(r144+m210)+exp(r159+m211)
+exp(r174+m212)+exp(r189+m213)+exp(r204+m214)+exp(r219+m215));
t93n=exp(r39+m23)/(1+exp(r9+m21)+exp(r24+m22)+exp(r39+m23)+exp(r54+m24)+exp(r69+m25)
+exp(r84+m26)+exp(r99+m27)+exp(r114+m28)+exp(r129+m29)+exp(r144+m210)+exp(r159+m211)
+exp(r174+m212)+exp(r189+m213)+exp(r204+m214)+exp(r219+m215));
t94n=exp(r54+m24)/(1+exp(r9+m21)+exp(r24+m22)+exp(r39+m23)+exp(r54+m24)+exp(r69+m25)
+exp(r84+m26)+exp(r99+m27)+exp(r114+m28)+exp(r129+m29)+exp(r144+m210)+exp(r159+m211)
+exp(r174+m212)+exp(r189+m213)+exp(r204+m214)+exp(r219+m215));
t95n=exp(r69+m25)/(1+exp(r9+m21)+exp(r24+m22)+exp(r39+m23)+exp(r54+m24)+exp(r69+m25)
+exp(r84+m26)+exp(r99+m27)+exp(r114+m28)+exp(r129+m29)+exp(r144+m210)+exp(r159+m211)
+exp(r174+m212)+exp(r189+m213)+exp(r204+m214)+exp(r219+m215));
t96n=exp(r84+m26)/(1+exp(r9+m21)+exp(r24+m22)+exp(r39+m23)+exp(r54+m24)+exp(r69+m25)
+exp(r84+m26)+exp(r99+m27)+exp(r114+m28)+exp(r129+m29)+exp(r144+m210)+exp(r159+m211)
+exp(r174+m212)+exp(r189+m213)+exp(r204+m214)+exp(r219+m215));
t97n=exp(r99+m27)/(1+exp(r9+m21)+exp(r24+m22)+exp(r39+m23)+exp(r54+m24)+exp(r69+m25)
+exp(r84+m26)+exp(r99+m27)+exp(r114+m28)+exp(r129+m29)+exp(r144+m210)+exp(r159+m211)
+exp(r174+m212)+exp(r189+m213)+exp(r204+m214)+exp(r219+m215));
t98n=exp(r114+m28)/(1+exp(r9+m21)+exp(r24+m22)+exp(r39+m23)+exp(r54+m24)+exp(r69+m25)
+exp(r84+m26)+exp(r99+m27)+exp(r114+m28)+exp(r129+m29)+exp(r144+m210)+exp(r159+m211)
+exp(r174+m212)+exp(r189+m213)+exp(r204+m214)+exp(r219+m215));
t99n=exp(r129+m29)/(1+exp(r9+m21)+exp(r24+m22)+exp(r39+m23)+exp(r54+m24)+exp(r69+m25)
+exp(r84+m26)+exp(r99+m27)+exp(r114+m28)+exp(r129+m29)+exp(r144+m210)+exp(r159+m211)
+exp(r174+m212)+exp(r189+m213)+exp(r204+m214)+exp(r219+m215));
t910n=exp(r144+m210)/(1+exp(r9+m21)+exp(r24+m22)+exp(r39+m23)+exp(r54+m24)+exp(r69+m25)
+exp(r84+m26)+exp(r99+m27)+exp(r114+m28)+exp(r129+m29)+exp(r144+m210)+exp(r159+m211)
+exp(r174+m212)+exp(r189+m213)+exp(r204+m214)+exp(r219+m215));
t911n=exp(r159+m211)/(1+exp(r9+m21)+exp(r24+m22)+exp(r39+m23)+exp(r54+m24)+exp(r69+m25)
+exp(r84+m26)+exp(r99+m27)+exp(r114+m28)+exp(r129+m29)+exp(r144+m210)+exp(r159+m211)
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+exp(r174+m212)+exp(r189+m213)+exp(r204+m214)+exp(r219+m215));
t912n=exp(r174+m212)/(1+exp(r9+m21)+exp(r24+m22)+exp(r39+m23)+exp(r54+m24)+exp(r69+m25)
+exp(r84+m26)+exp(r99+m27)+exp(r114+m28)+exp(r129+m29)+exp(r144+m210)+exp(r159+m211)
+exp(r174+m212)+exp(r189+m213)+exp(r204+m214)+exp(r219+m215));
t913n=exp(r189+m213)/(1+exp(r9+m21)+exp(r24+m22)+exp(r39+m23)+exp(r54+m24)+exp(r69+m25)
+exp(r84+m26)+exp(r99+m27)+exp(r114+m28)+exp(r129+m29)+exp(r144+m210)+exp(r159+m211)
+exp(r174+m212)+exp(r189+m213)+exp(r204+m214)+exp(r219+m215));
t914n=exp(r204+m214)/(1+exp(r9+m21)+exp(r24+m22)+exp(r39+m23)+exp(r54+m24)+exp(r69+m25)
+exp(r84+m26)+exp(r99+m27)+exp(r114+m28)+exp(r129+m29)+exp(r144+m210)+exp(r159+m211)
+exp(r174+m212)+exp(r189+m213)+exp(r204+m214)+exp(r219+m215));
t915n=exp(r219+m215)/(1+exp(r9+m21)+exp(r24+m22)+exp(r39+m23)+exp(r54+m24)+exp(r69+m25)
+exp(r84+m26)+exp(r99+m27)+exp(r114+m28)+exp(r129+m29)+exp(r144+m210)+exp(r159+m211)+
exp(r174+m212)+exp(r189+m213)+exp(r204+m214)+exp(r219+m215));
t916n=1/(1+exp(r9+m21)+exp(r24+m22)+exp(r39+m23)+exp(r54+m24)+exp(r69+m25)
+exp(r84+m26)+exp(r99+m27)+exp(r114+m28)+exp(r129+m29)+exp(r144+m210)+exp(r159+m211)+
exp(r174+m212)+exp(r189+m213)+exp(r204+m214)+exp(r219+m215));

!row 10
t101o=exp(r10+m21)/(1+exp(r10+m21)+exp(r25+m22)+exp(r40+m23)+exp(r55+m24)+exp(r70+m25)
+exp(r85+m26)+exp(r100+m27)+exp(r115+m28)+exp(r130+m29)+exp(r145+m210)+exp(r160+m211)
+exp(r175+m212)+exp(r190+m213)+exp(r205+m214)+exp(r220+m215));
t102o=exp(r25+m22)/(1+exp(r10+m21)+exp(r25+m22)+exp(r40+m23)+exp(r55+m24)+exp(r70+m25)
+exp(r85+m26)+exp(r100+m27)+exp(r115+m28)+exp(r130+m29)+exp(r145+m210)+exp(r160+m211)
+exp(r175+m212)+exp(r190+m213)+exp(r205+m214)+exp(r220+m215));
t103o=exp(r40+m23)/(1+exp(r10+m21)+exp(r25+m22)+exp(r40+m23)+exp(r55+m24)+exp(r70+m25)
+exp(r85+m26)+exp(r100+m27)+exp(r115+m28)+exp(r130+m29)+exp(r145+m210)+exp(r160+m211)
+exp(r175+m212)+exp(r190+m213)+exp(r205+m214)+exp(r220+m215));
t104o=exp(r55+m24)/(1+exp(r10+m21)+exp(r25+m22)+exp(r40+m23)+exp(r55+m24)+exp(r70+m25)
+exp(r85+m26)+exp(r100+m27)+exp(r115+m28)+exp(r130+m29)+exp(r145+m210)+exp(r160+m211)
+exp(r175+m212)+exp(r190+m213)+exp(r205+m214)+exp(r220+m215));
t105o=exp(r70+m25)/(1+exp(r10+m21)+exp(r25+m22)+exp(r40+m23)+exp(r55+m24)+exp(r70+m25)
+exp(r85+m26)+exp(r100+m27)+exp(r115+m28)+exp(r130+m29)+exp(r145+m210)+exp(r160+m211)
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+exp(r175+m212)+exp(r190+m213)+exp(r205+m214)+exp(r220+m215));
t106o=exp(r85+m26)/(1+exp(r10+m21)+exp(r25+m22)+exp(r40+m23)+exp(r55+m24)+exp(r70+m25)
+exp(r85+m26)+exp(r100+m27)+exp(r115+m28)+exp(r130+m29)+exp(r145+m210)+exp(r160+m211)
+exp(r175+m212)+exp(r190+m213)+exp(r205+m214)+exp(r220+m215));
t107o=exp(r100+m27)/(1+exp(r10+m21)+exp(r25+m22)+exp(r40+m23)+exp(r55+m24)+exp(r70+m25)
+exp(r85+m26)+exp(r100+m27)+exp(r115+m28)+exp(r130+m29)+exp(r145+m210)+exp(r160+m211)+
exp(r175+m212)+exp(r190+m213)+exp(r205+m214)+exp(r220+m215));
t108o=exp(r115+m28)/(1+exp(r10+m21)+exp(r25+m22)+exp(r40+m23)+exp(r55+m24)+exp(r70+m25)
+exp(r85+m26)+exp(r100+m27)+exp(r115+m28)+exp(r130+m29)+exp(r145+m210)+exp(r160+m211)+
exp(r175+m212)+exp(r190+m213)+exp(r205+m214)+exp(r220+m215));
t109o=exp(r130+m29)/(1+exp(r10+m21)+exp(r25+m22)+exp(r40+m23)+exp(r55+m24)+exp(r70+m25)+
exp(r85+m26)+exp(r100+m27)+exp(r115+m28)+exp(r130+m29)+exp(r145+m210)+exp(r160+m211)+
exp(r175+m212)+exp(r190+m213)+exp(r205+m214)+exp(r220+m215));
t1010o=exp(r145+m210)/(1+exp(r10+m21)+exp(r25+m22)+exp(r40+m23)+exp(r55+m24)+exp(r70+m25)
+exp(r85+m26)+exp(r100+m27)+exp(r115+m28)+exp(r130+m29)+exp(r145+m210)+exp(r160+m211)+
exp(r175+m212)+exp(r190+m213)+exp(r205+m214)+exp(r220+m215));
t1011o=exp(r160+m211)/(1+exp(r10+m21)+exp(r25+m22)+exp(r40+m23)+exp(r55+m24)+exp(r70+m25)
+exp(r85+m26)+exp(r100+m27)+exp(r115+m28)+exp(r130+m29)+exp(r145+m210)+exp(r160+m211)+
exp(r175+m212)+exp(r190+m213)+exp(r205+m214)+exp(r220+m215));
t1012o=exp(r175+m212)/(1+exp(r10+m21)+exp(r25+m22)+exp(r40+m23)+exp(r55+m24)+exp(r70+m25)
+exp(r85+m26)+exp(r100+m27)+exp(r115+m28)+exp(r130+m29)+exp(r145+m210)+exp(r160+m211)+
exp(r175+m212)+exp(r190+m213)+exp(r205+m214)+exp(r220+m215));
t1013o=exp(r190+m213)/(1+exp(r10+m21)+exp(r25+m22)+exp(r40+m23)+exp(r55+m24)+exp(r70+m25)
+exp(r85+m26)+exp(r100+m27)+exp(r115+m28)+exp(r130+m29)+exp(r145+m210)+exp(r160+m211)+
exp(r175+m212)+exp(r190+m213)+exp(r205+m214)+exp(r220+m215));
t1014o=exp(r205+m214)/(1+exp(r10+m21)+exp(r25+m22)+exp(r40+m23)+exp(r55+m24)+exp(r70+m25)
+exp(r85+m26)+exp(r100+m27)+exp(r115+m28)+exp(r130+m29)+exp(r145+m210)+exp(r160+m211)+
exp(r175+m212)+exp(r190+m213)+exp(r205+m214)+exp(r220+m215));
t1015o=exp(r220+m215)/(1+exp(r10+m21)+exp(r25+m22)+exp(r40+m23)+exp(r55+m24)+exp(r70+m25)
+exp(r85+m26)+exp(r100+m27)+exp(r115+m28)+exp(r130+m29)+exp(r145+m210)+exp(r160+m211)+
exp(r175+m212)+exp(r190+m213)+exp(r205+m214)+exp(r220+m215));
t1016o=1/(1+exp(r10+m21)+exp(r25+m22)+exp(r40+m23)+exp(r55+m24)+exp(r70+m25)

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+exp(r85+m26)+exp(r100+m27)+exp(r115+m28)+exp(r130+m29)+exp(r145+m210)+exp(r160+m211)+
exp(r175+m212)+exp(r190+m213)+exp(r205+m214)+exp(r220+m215));

!row 11
t111q=exp(r11+m21)/(1+exp(r11+m21)+exp(r26+m22)+exp(r41+m23)+exp(r56+m24)+exp(r71+m25)
+exp(r86+m26)+exp(r101+m27)+exp(r116+m28)+exp(r131+m29)+exp(r146+m210)+exp(r161+m211)+
exp(r176+m212)+exp(r191+m213)+exp(r206+m214)+exp(r221+m215));
t112q=exp(r26+m22)/(1+exp(r11+m21)+exp(r26+m22)+exp(r41+m23)+exp(r56+m24)+exp(r71+m25)
+exp(r86+m26)+exp(r101+m27)+exp(r116+m28)+exp(r131+m29)+exp(r146+m210)+exp(r161+m211)+
exp(r176+m212)+exp(r191+m213)+exp(r206+m214)+exp(r221+m215));
t113q=exp(r41+m23)/(1+exp(r11+m21)+exp(r26+m22)+exp(r41+m23)+exp(r56+m24)+exp(r71+m25)
+exp(r86+m26)+exp(r101+m27)+exp(r116+m28)+exp(r131+m29)+exp(r146+m210)+exp(r161+m211)+
exp(r176+m212)+exp(r191+m213)+exp(r206+m214)+exp(r221+m215));
t114q=exp(r56+m24)/(1+exp(r11+m21)+exp(r26+m22)+exp(r41+m23)+exp(r56+m24)+exp(r71+m25)
+exp(r86+m26)+exp(r101+m27)+exp(r116+m28)+exp(r131+m29)+exp(r146+m210)+exp(r161+m211)+
exp(r176+m212)+exp(r191+m213)+exp(r206+m214)+exp(r221+m215));
t115q=exp(r71+m25)/(1+exp(r11+m21)+exp(r26+m22)+exp(r41+m23)+exp(r56+m24)+exp(r71+m25)
+exp(r86+m26)+exp(r101+m27)+exp(r116+m28)+exp(r131+m29)+exp(r146+m210)+exp(r161+m211)+
exp(r176+m212)+exp(r191+m213)+exp(r206+m214)+exp(r221+m215));
t116q=exp(r86+m26)/(1+exp(r11+m21)+exp(r26+m22)+exp(r41+m23)+exp(r56+m24)+exp(r71+m25)
+exp(r86+m26)+exp(r101+m27)+exp(r116+m28)+exp(r131+m29)+exp(r146+m210)+exp(r161+m211)+
exp(r176+m212)+exp(r191+m213)+exp(r206+m214)+exp(r221+m215));
t117q=exp(r101+m27)/(1+exp(r11+m21)+exp(r26+m22)+exp(r41+m23)+exp(r56+m24)+exp(r71+m25)+
exp(r86+m26)+exp(r101+m27)+exp(r116+m28)+exp(r131+m29)+exp(r146+m210)+exp(r161+m211)+
exp(r176+m212)+exp(r191+m213)+exp(r206+m214)+exp(r221+m215));
t118q=exp(r116+m28)/(1+exp(r11+m21)+exp(r26+m22)+exp(r41+m23)+exp(r56+m24)+exp(r71+m25)+
exp(r86+m26)+exp(r101+m27)+exp(r116+m28)+exp(r131+m29)+exp(r146+m210)+exp(r161+m211)+
exp(r176+m212)+exp(r191+m213)+exp(r206+m214)+exp(r221+m215));
t119q=exp(r131+m29)/(1+exp(r11+m21)+exp(r26+m22)+exp(r41+m23)+exp(r56+m24)+exp(r71+m25)+
exp(r86+m26)+exp(r101+m27)+exp(r116+m28)+exp(r131+m29)+exp(r146+m210)+exp(r161+m211)+
exp(r176+m212)+exp(r191+m213)+exp(r206+m214)+exp(r221+m215));
t1110q=exp(r146+m210)/(1+exp(r11+m21)+exp(r26+m22)+exp(r41+m23)+exp(r56+m24)+exp(r71+m25)+
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exp(r86+m26)+exp(r101+m27)+exp(r116+m28)+exp(r131+m29)+exp(r146+m210)+exp(r161+m211)+
exp(r176+m212)+exp(r191+m213)+exp(r206+m214)+exp(r221+m215));
t1111q=exp(r161+m211)/(1+exp(r11+m21)+exp(r26+m22)+exp(r41+m23)+exp(r56+m24)+exp(r71+m25)
+exp(r86+m26)+exp(r101+m27)+exp(r116+m28)+exp(r131+m29)+exp(r146+m210)+exp(r161+m211)+
exp(r176+m212)+exp(r191+m213)+exp(r206+m214)+exp(r221+m215));
t1112q=exp(r176+m212)/(1+exp(r11+m21)+exp(r26+m22)+exp(r41+m23)+exp(r56+m24)+exp(r71+m25)
+exp(r86+m26)+exp(r101+m27)+exp(r116+m28)+exp(r131+m29)+exp(r146+m210)+exp(r161+m211)+
exp(r176+m212)+exp(r191+m213)+exp(r206+m214)+exp(r221+m215));
t1113q=exp(r191+m213)/(1+exp(r11+m21)+exp(r26+m22)+exp(r41+m23)+exp(r56+m24)+exp(r71+m25)
+exp(r86+m26)+exp(r101+m27)+exp(r116+m28)+exp(r131+m29)+exp(r146+m210)+exp(r161+m211)+
exp(r176+m212)+exp(r191+m213)+exp(r206+m214)+exp(r221+m215));
t1114q=exp(r206+m214)/(1+exp(r11+m21)+exp(r26+m22)+exp(r41+m23)+exp(r56+m24)+exp(r71+m25)
+exp(r86+m26)+exp(r101+m27)+exp(r116+m28)+exp(r131+m29)+exp(r146+m210)+exp(r161+m211)+
exp(r176+m212)+exp(r191+m213)+exp(r206+m214)+exp(r221+m215));
t1115q=exp(r221+m215)/(1+exp(r11+m21)+exp(r26+m22)+exp(r41+m23)+exp(r56+m24)+exp(r71+m25)
+exp(r86+m26)+exp(r101+m27)+exp(r116+m28)+exp(r131+m29)+exp(r146+m210)+exp(r161+m211)+
exp(r176+m212)+exp(r191+m213)+exp(r206+m214)+exp(r221+m215));
t1116q=1/(1+exp(r11+m21)+exp(r26+m22)+exp(r41+m23)+exp(r56+m24)+exp(r71+m25)
+exp(r86+m26)+exp(r101+m27)+exp(r116+m28)+exp(r131+m29)+exp(r146+m210)+exp(r161+m211)+
exp(r176+m212)+exp(r191+m213)+exp(r206+m214)+exp(r221+m215));

!row 12
t121u=exp(r12+m21)/(1+exp(r12+m21)+exp(r27+m22)+exp(r42+m23)+exp(r57+m24)+exp(r72+m25)
+exp(r87+m26)+exp(r102+m27)+exp(r117+m28)+exp(r132+m29)+exp(r147+m210)+exp(r162+m211)
+exp(r177+m212)+exp(r192+m213)+exp(r207+m214)+exp(r222+m215));
t122u=exp(r27+m22)/(1+exp(r12+m21)+exp(r27+m22)+exp(r42+m23)+exp(r57+m24)+exp(r72+m25)
+exp(r87+m26)+exp(r102+m27)+exp(r117+m28)+exp(r132+m29)+exp(r147+m210)+exp(r162+m211)
+exp(r177+m212)+exp(r192+m213)+exp(r207+m214)+exp(r222+m215));
t123u=exp(r42+m23)/(1+exp(r12+m21)+exp(r27+m22)+exp(r42+m23)+exp(r57+m24)+exp(r72+m25)
+exp(r87+m26)+exp(r102+m27)+exp(r117+m28)+exp(r132+m29)+exp(r147+m210)+exp(r162+m211)
+exp(r177+m212)+exp(r192+m213)+exp(r207+m214)+exp(r222+m215));
t124u=exp(r57+m24)/(1+exp(r12+m21)+exp(r27+m22)+exp(r42+m23)+exp(r57+m24)+exp(r72+m25)
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+exp(r87+m26)+exp(r102+m27)+exp(r117+m28)+exp(r132+m29)+exp(r147+m210)+exp(r162+m211)
+exp(r177+m212)+exp(r192+m213)+exp(r207+m214)+exp(r222+m215));
t125u=exp(r72+m25)/(1+exp(r12+m21)+exp(r27+m22)+exp(r42+m23)+exp(r57+m24)+exp(r72+m25)
+exp(r87+m26)+exp(r102+m27)+exp(r117+m28)+exp(r132+m29)+exp(r147+m210)+exp(r162+m211)
+exp(r177+m212)+exp(r192+m213)+exp(r207+m214)+exp(r222+m215));
t126u=exp(r87+m26)/(1+exp(r12+m21)+exp(r27+m22)+exp(r42+m23)+exp(r57+m24)+exp(r72+m25)
+exp(r87+m26)+exp(r102+m27)+exp(r117+m28)+exp(r132+m29)+exp(r147+m210)+exp(r162+m211)
+exp(r177+m212)+exp(r192+m213)+exp(r207+m214)+exp(r222+m215));
t127u=exp(r102+m27)/(1+exp(r12+m21)+exp(r27+m22)+exp(r42+m23)+exp(r57+m24)+exp(r72+m25)
+exp(r87+m26)+exp(r102+m27)+exp(r117+m28)+exp(r132+m29)+exp(r147+m210)+exp(r162+m211)
+exp(r177+m212)+exp(r192+m213)+exp(r207+m214)+exp(r222+m215));
t128u=exp(r117+m28)/(1+exp(r12+m21)+exp(r27+m22)+exp(r42+m23)+exp(r57+m24)+exp(r72+m25)
+exp(r87+m26)+exp(r102+m27)+exp(r117+m28)+exp(r132+m29)+exp(r147+m210)+exp(r162+m211)
+exp(r177+m212)+exp(r192+m213)+exp(r207+m214)+exp(r222+m215));
t129u=exp(r132+m29)/(1+exp(r12+m21)+exp(r27+m22)+exp(r42+m23)+exp(r57+m24)+exp(r72+m25)
+exp(r87+m26)+exp(r102+m27)+exp(r117+m28)+exp(r132+m29)+exp(r147+m210)+exp(r162+m211)
+exp(r177+m212)+exp(r192+m213)+exp(r207+m214)+exp(r222+m215));
t1210u=exp(r147+m210)/(1+exp(r12+m21)+exp(r27+m22)+exp(r42+m23)+exp(r57+m24)+
exp(r72+m25)+exp(r87+m26)+exp(r102+m27)+exp(r117+m28)+exp(r132+m29)+exp(r147+m210)+
exp(r162+m211)+exp(r177+m212)+exp(r192+m213)+exp(r207+m214)+exp(r222+m215));
t1211u=exp(r162+m211)/(1+exp(r12+m21)+exp(r27+m22)+exp(r42+m23)+exp(r57+m24)
+exp(r72+m25)+exp(r87+m26)+exp(r102+m27)+exp(r117+m28)+exp(r132+m29)+exp(r147+m210)
+exp(r162+m211)+exp(r177+m212)+exp(r192+m213)+exp(r207+m214)+exp(r222+m215));
t1212u=exp(r177+m212)/(1+exp(r12+m21)+exp(r27+m22)+exp(r42+m23)+exp(r57+m24)
+exp(r72+m25)+exp(r87+m26)+exp(r102+m27)+exp(r117+m28)+exp(r132+m29)+exp(r147+m210)
+exp(r162+m211)+exp(r177+m212)+exp(r192+m213)+exp(r207+m214)+exp(r222+m215));
t1213u=exp(r192+m213)/(1+exp(r12+m21)+exp(r27+m22)+exp(r42+m23)+exp(r57+m24)
+exp(r72+m25)+exp(r87+m26)+exp(r102+m27)+exp(r117+m28)+exp(r132+m29)+exp(r147+m210)
+exp(r162+m211)+exp(r177+m212)+exp(r192+m213)+exp(r207+m214)+exp(r222+m215));
t1214u=exp(r207+m214)/(1+exp(r12+m21)+exp(r27+m22)+exp(r42+m23)+exp(r57+m24)
+exp(r72+m25)+exp(r87+m26)+exp(r102+m27)+exp(r117+m28)+exp(r132+m29)+exp(r147+m210)
+exp(r162+m211)+exp(r177+m212)+exp(r192+m213)+exp(r207+m214)+exp(r222+m215));

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t1215u=exp(r222+m215)/(1+exp(r12+m21)+exp(r27+m22)+exp(r42+m23)+exp(r57+m24)
+exp(r72+m25)+exp(r87+m26)+exp(r102+m27)+exp(r117+m28)+exp(r132+m29)+exp(r147+m210)
+exp(r162+m211)+exp(r177+m212)+exp(r192+m213)+exp(r207+m214)+exp(r222+m215));
t1216u=1/(1+exp(r12+m21)+exp(r27+m22)+exp(r42+m23)+exp(r57+m24)
+exp(r72+m25)+exp(r87+m26)+exp(r102+m27)+exp(r117+m28)+exp(r132+m29)+exp(r147+m210)
+exp(r162+m211)+exp(r177+m212)+exp(r192+m213)+exp(r207+m214)+exp(r222+m215));

!row 13
t131v=exp(r13+m21)/(1+exp(r13+m21)+exp(r28+m22)+exp(r43+m23)+exp(r58+m24)+exp(r73+m25)
+exp(r88+m26)+exp(r103+m27)+exp(r118+m28)+exp(r133+m29)+exp(r148+m210)+exp(r163+m211)
+exp(r178+m212)+exp(r193+m213)+exp(r208+m214)+exp(r223+m215));
t132v=exp(r28+m22)/(1+exp(r13+m21)+exp(r28+m22)+exp(r43+m23)+exp(r58+m24)+exp(r73+m25)
+exp(r88+m26)+exp(r103+m27)+exp(r118+m28)+exp(r133+m29)+exp(r148+m210)+exp(r163+m211)
+exp(r178+m212)+exp(r193+m213)+exp(r208+m214)+exp(r223+m215));
t133v=exp(r43+m23)/(1+exp(r13+m21)+exp(r28+m22)+exp(r43+m23)+exp(r58+m24)+exp(r73+m25)
+exp(r88+m26)+exp(r103+m27)+exp(r118+m28)+exp(r133+m29)+exp(r148+m210)+exp(r163+m211)
+exp(r178+m212)+exp(r193+m213)+exp(r208+m214)+exp(r223+m215));
t134v=exp(r58+m24)/(1+exp(r13+m21)+exp(r28+m22)+exp(r43+m23)+exp(r58+m24)+exp(r73+m25)
+exp(r88+m26)+exp(r103+m27)+exp(r118+m28)+exp(r133+m29)+exp(r148+m210)+exp(r163+m211)
+exp(r178+m212)+exp(r193+m213)+exp(r208+m214)+exp(r223+m215));
t135v=exp(r73+m25)/(1+exp(r13+m21)+exp(r28+m22)+exp(r43+m23)+exp(r58+m24)+exp(r73+m25)
+exp(r88+m26)+exp(r103+m27)+exp(r118+m28)+exp(r133+m29)+exp(r148+m210)+exp(r163+m211)
+exp(r178+m212)+exp(r193+m213)+exp(r208+m214)+exp(r223+m215));
t136v=exp(r88+m26)/(1+exp(r13+m21)+exp(r28+m22)+exp(r43+m23)+exp(r58+m24)+exp(r73+m25)
+exp(r88+m26)+exp(r103+m27)+exp(r118+m28)+exp(r133+m29)+exp(r148+m210)+exp(r163+m211)
+exp(r178+m212)+exp(r193+m213)+exp(r208+m214)+exp(r223+m215));
t137v=exp(r103+m27)/(1+exp(r13+m21)+exp(r28+m22)+exp(r43+m23)+exp(r58+m24)+exp(r73+m25)
+exp(r88+m26)+exp(r103+m27)+exp(r118+m28)+exp(r133+m29)+exp(r148+m210)+exp(r163+m211)
+exp(r178+m212)+exp(r193+m213)+exp(r208+m214)+exp(r223+m215));
t138v=exp(r118+m28)/(1+exp(r13+m21)+exp(r28+m22)+exp(r43+m23)+exp(r58+m24)+exp(r73+m25)
+exp(r88+m26)+exp(r103+m27)+exp(r118+m28)+exp(r133+m29)+exp(r148+m210)+exp(r163+m211)
+exp(r178+m212)+exp(r193+m213)+exp(r208+m214)+exp(r223+m215));
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t139v=exp(r133+m29)/(1+exp(r13+m21)+exp(r28+m22)+exp(r43+m23)+exp(r58+m24)+exp(r73+m25)
+exp(r88+m26)+exp(r103+m27)+exp(r118+m28)+exp(r133+m29)+exp(r148+m210)+exp(r163+m211)
+exp(r178+m212)+exp(r193+m213)+exp(r208+m214)+exp(r223+m215));
t1310v=exp(r148+m210)/(1+exp(r13+m21)+exp(r28+m22)+exp(r43+m23)+exp(r58+m24)+exp(r73+m25)
+exp(r88+m26)+exp(r103+m27)+exp(r118+m28)+exp(r133+m29)+exp(r148+m210)+exp(r163+m211)
+exp(r178+m212)+exp(r193+m213)+exp(r208+m214)+exp(r223+m215));
t1311v=exp(r163+m211)/(1+exp(r13+m21)+exp(r28+m22)+exp(r43+m23)+exp(r58+m24)+exp(r73+m25)
+exp(r88+m26)+exp(r103+m27)+exp(r118+m28)+exp(r133+m29)+exp(r148+m210)+exp(r163+m211)
+exp(r178+m212)+exp(r193+m213)+exp(r208+m214)+exp(r223+m215));
t1312v=exp(r178+m212)/(1+exp(r13+m21)+exp(r28+m22)+exp(r43+m23)+exp(r58+m24)+exp(r73+m25)
+exp(r88+m26)+exp(r103+m27)+exp(r118+m28)+exp(r133+m29)+exp(r148+m210)+exp(r163+m211)
+exp(r178+m212)+exp(r193+m213)+exp(r208+m214)+exp(r223+m215));
t1313v=exp(r193+m213)/(1+exp(r13+m21)+exp(r28+m22)+exp(r43+m23)+exp(r58+m24)+exp(r73+m25)
+exp(r88+m26)+exp(r103+m27)+exp(r118+m28)+exp(r133+m29)+exp(r148+m210)+exp(r163+m211)
+exp(r178+m212)+exp(r193+m213)+exp(r208+m214)+exp(r223+m215));
t1314v=exp(r208+m214)/(1+exp(r13+m21)+exp(r28+m22)+exp(r43+m23)+exp(r58+m24)+exp(r73+m25)
+exp(r88+m26)+exp(r103+m27)+exp(r118+m28)+exp(r133+m29)+exp(r148+m210)+exp(r163+m211)
+exp(r178+m212)+exp(r193+m213)+exp(r208+m214)+exp(r223+m215));
t1315v=exp(r223+m215)/(1+exp(r13+m21)+exp(r28+m22)+exp(r43+m23)+exp(r58+m24)+exp(r73+m25)
+exp(r88+m26)+exp(r103+m27)+exp(r118+m28)+exp(r133+m29)+exp(r148+m210)+exp(r163+m211)
+exp(r178+m212)+exp(r193+m213)+exp(r208+m214)+exp(r223+m215));
t1316v=1/(1+exp(r13+m21)+exp(r28+m22)+exp(r43+m23)+exp(r58+m24)+exp(r73+m25)
+exp(r88+m26)+exp(r103+m27)+exp(r118+m28)+exp(r133+m29)+exp(r148+m210)+exp(r163+m211)
+exp(r178+m212)+exp(r193+m213)+exp(r208+m214)+exp(r223+m215));

!row 14
t141w=exp(r14+m21)/(1+exp(r14+m21)+exp(r29+m22)+exp(r44+m23)+exp(r59+m24)+exp(r74+m25)
+exp(r89+m26)+exp(r104+m27)+exp(r119+m28)+exp(r134+m29)+exp(r149+m210)+exp(r164+m211)
+exp(r179+m212)+exp(r194+m213)+exp(r209+m214)+exp(r224+m215));
t142w=exp(r29+m22)/(1+exp(r14+m21)+exp(r29+m22)+exp(r44+m23)+exp(r59+m24)+exp(r74+m25)
+exp(r89+m26)+exp(r104+m27)+exp(r119+m28)+exp(r134+m29)+exp(r149+m210)+exp(r164+m211)
+exp(r179+m212)+exp(r194+m213)+exp(r209+m214)+exp(r224+m215));
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t143w=exp(r44+m23)/(1+exp(r14+m21)+exp(r29+m22)+exp(r44+m23)+exp(r59+m24)+exp(r74+m25)
+exp(r89+m26)+exp(r104+m27)+exp(r119+m28)+exp(r134+m29)+exp(r149+m210)+exp(r164+m211)
+exp(r179+m212)+exp(r194+m213)+exp(r209+m214)+exp(r224+m215));
t144w=exp(r59+m24)/(1+exp(r14+m21)+exp(r29+m22)+exp(r44+m23)+exp(r59+m24)+exp(r74+m25)
+exp(r89+m26)+exp(r104+m27)+exp(r119+m28)+exp(r134+m29)+exp(r149+m210)+exp(r164+m211)
+exp(r179+m212)+exp(r194+m213)+exp(r209+m214)+exp(r224+m215));
t145w=exp(r74+m25)/(1+exp(r14+m21)+exp(r29+m22)+exp(r44+m23)+exp(r59+m24)+exp(r74+m25)
+exp(r89+m26)+exp(r104+m27)+exp(r119+m28)+exp(r134+m29)+exp(r149+m210)+exp(r164+m211)
+exp(r179+m212)+exp(r194+m213)+exp(r209+m214)+exp(r224+m215));
t146w=exp(r89+m26)/(1+exp(r14+m21)+exp(r29+m22)+exp(r44+m23)+exp(r59+m24)+exp(r74+m25)
+exp(r89+m26)+exp(r104+m27)+exp(r119+m28)+exp(r134+m29)+exp(r149+m210)+exp(r164+m211)
+exp(r179+m212)+exp(r194+m213)+exp(r209+m214)+exp(r224+m215));
t147w=exp(r104+m27)/(1+exp(r14+m21)+exp(r29+m22)+exp(r44+m23)+exp(r59+m24)+exp(r74+m25)
+exp(r89+m26)+exp(r104+m27)+exp(r119+m28)+exp(r134+m29)+exp(r149+m210)+exp(r164+m211)
+exp(r179+m212)+exp(r194+m213)+exp(r209+m214)+exp(r224+m215));
t148w=exp(r119+m28)/(1+exp(r14+m21)+exp(r29+m22)+exp(r44+m23)+exp(r59+m24)+exp(r74+m25)
+exp(r89+m26)+exp(r104+m27)+exp(r119+m28)+exp(r134+m29)+exp(r149+m210)+exp(r164+m211)
+exp(r179+m212)+exp(r194+m213)+exp(r209+m214)+exp(r224+m215));
t149w=exp(r134+m29)/(1+exp(r14+m21)+exp(r29+m22)+exp(r44+m23)+exp(r59+m24)+exp(r74+m25)
+exp(r89+m26)+exp(r104+m27)+exp(r119+m28)+exp(r134+m29)+exp(r149+m210)+exp(r164+m211)
+exp(r179+m212)+exp(r194+m213)+exp(r209+m214)+exp(r224+m215));
t1410w=exp(r149+m210)/(1+exp(r14+m21)+exp(r29+m22)+exp(r44+m23)+exp(r59+m24)+exp(r74+m25)
+exp(r89+m26)+exp(r104+m27)+exp(r119+m28)+exp(r134+m29)+exp(r149+m210)+exp(r164+m211)
+exp(r179+m212)+exp(r194+m213)+exp(r209+m214)+exp(r224+m215));
t1411w=exp(r164+m211)/(1+exp(r14+m21)+exp(r29+m22)+exp(r44+m23)+exp(r59+m24)+exp(r74+m25)
+exp(r89+m26)+exp(r104+m27)+exp(r119+m28)+exp(r134+m29)+exp(r149+m210)+exp(r164+m211)
+exp(r179+m212)+exp(r194+m213)+exp(r209+m214)+exp(r224+m215));
t1412w=exp(r179+m212)/(1+exp(r14+m21)+exp(r29+m22)+exp(r44+m23)+exp(r59+m24)+exp(r74+m25)
+exp(r89+m26)+exp(r104+m27)+exp(r119+m28)+exp(r134+m29)+exp(r149+m210)+exp(r164+m211)
+exp(r179+m212)+exp(r194+m213)+exp(r209+m214)+exp(r224+m215));
t1413w=exp(r194+m213)/(1+exp(r14+m21)+exp(r29+m22)+exp(r44+m23)+exp(r59+m24)+exp(r74+m25)
+exp(r89+m26)+exp(r104+m27)+exp(r119+m28)+exp(r134+m29)+exp(r149+m210)+exp(r164+m211)
```

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+exp(r179+m212)+exp(r194+m213)+exp(r209+m214)+exp(r224+m215));
t1414w=exp(r209+m214)/(1+exp(r14+m21)+exp(r29+m22)+exp(r44+m23)+exp(r59+m24)+exp(r74+m25)
+exp(r89+m26)+exp(r104+m27)+exp(r119+m28)+exp(r134+m29)+exp(r149+m210)+exp(r164+m211)
+exp(r179+m212)+exp(r194+m213)+exp(r209+m214)+exp(r224+m215));
t1415w=exp(r224+m215)/(1+exp(r14+m21)+exp(r29+m22)+exp(r44+m23)+exp(r59+m24)+exp(r74+m25)
+exp(r89+m26)+exp(r104+m27)+exp(r119+m28)+exp(r134+m29)+exp(r149+m210)+exp(r164+m211)
+exp(r179+m212)+exp(r194+m213)+exp(r209+m214)+exp(r224+m215));
t1416w=1/(1+exp(r14+m21)+exp(r29+m22)+exp(r44+m23)+exp(r59+m24)+exp(r74+m25)
+exp(r89+m26)+exp(r104+m27)+exp(r119+m28)+exp(r134+m29)+exp(r149+m210)+exp(r164+m211)
+exp(r179+m212)+exp(r194+m213)+exp(r209+m214)+exp(r224+m215));

!row 15
t151y=exp(r15+m21)/(1+exp(r15+m21)+exp(r30+m22)+exp(r45+m23)+exp(r60+m24)+exp(r75+m25)
+exp(r90+m26)+exp(r105+m27)+exp(r120+m28)+exp(r135+m29)+exp(r150+m210)+exp(r165+m211)
+exp(r180+m212)+exp(r195+m213)+exp(r210+m214)+exp(r225+m215));
t152y=exp(r30+m22)/(1+exp(r15+m21)+exp(r30+m22)+exp(r45+m23)+exp(r60+m24)+exp(r75+m25)
+exp(r90+m26)+exp(r105+m27)+exp(r120+m28)+exp(r135+m29)+exp(r150+m210)+exp(r165+m211)
+exp(r180+m212)+exp(r195+m213)+exp(r210+m214)+exp(r225+m215));
t153y=exp(r45+m23)/(1+exp(r15+m21)+exp(r30+m22)+exp(r45+m23)+exp(r60+m24)+exp(r75+m25)
+exp(r90+m26)+exp(r105+m27)+exp(r120+m28)+exp(r135+m29)+exp(r150+m210)+exp(r165+m211)
+exp(r180+m212)+exp(r195+m213)+exp(r210+m214)+exp(r225+m215));
t154y=exp(r60+m24)/(1+exp(r15+m21)+exp(r30+m22)+exp(r45+m23)+exp(r60+m24)+exp(r75+m25)
+exp(r90+m26)+exp(r105+m27)+exp(r120+m28)+exp(r135+m29)+exp(r150+m210)+exp(r165+m211)
+exp(r180+m212)+exp(r195+m213)+exp(r210+m214)+exp(r225+m215));
t155y=exp(r75+m25)/(1+exp(r15+m21)+exp(r30+m22)+exp(r45+m23)+exp(r60+m24)+exp(r75+m25)
+exp(r90+m26)+exp(r105+m27)+exp(r120+m28)+exp(r135+m29)+exp(r150+m210)+exp(r165+m211)
+exp(r180+m212)+exp(r195+m213)+exp(r210+m214)+exp(r225+m215));
t156y=exp(r90+m26)/(1+exp(r15+m21)+exp(r30+m22)+exp(r45+m23)+exp(r60+m24)+exp(r75+m25)
+exp(r90+m26)+exp(r105+m27)+exp(r120+m28)+exp(r135+m29)+exp(r150+m210)+exp(r165+m211)
+exp(r180+m212)+exp(r195+m213)+exp(r210+m214)+exp(r225+m215));
t157y=exp(r105+m27)/(1+exp(r15+m21)+exp(r30+m22)+exp(r45+m23)+exp(r60+m24)+exp(r75+m25)
+exp(r90+m26)+exp(r105+m27)+exp(r120+m28)+exp(r135+m29)+exp(r150+m210)+exp(r165+m211)
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+exp(r180+m212)+exp(r195+m213)+exp(r210+m214)+exp(r225+m215));
t158y=exp(r120+m28)/(1+exp(r15+m21)+exp(r30+m22)+exp(r45+m23)+exp(r60+m24)+exp(r75+m25)
+exp(r90+m26)+exp(r105+m27)+exp(r120+m28)+exp(r135+m29)+exp(r150+m210)+exp(r165+m211)
+exp(r180+m212)+exp(r195+m213)+exp(r210+m214)+exp(r225+m215));
t159y=exp(r135+m29)/(1+exp(r15+m21)+exp(r30+m22)+exp(r45+m23)+exp(r60+m24)+exp(r75+m25)
+exp(r90+m26)+exp(r105+m27)+exp(r120+m28)+exp(r135+m29)+exp(r150+m210)+exp(r165+m211)
+exp(r180+m212)+exp(r195+m213)+exp(r210+m214)+exp(r225+m215));
t1510y=exp(r150+m210)/(1+exp(r15+m21)+exp(r30+m22)+exp(r45+m23)+exp(r60+m24)+exp(r75+m25)
+exp(r90+m26)+exp(r105+m27)+exp(r120+m28)+exp(r135+m29)+exp(r150+m210)+exp(r165+m211)
+exp(r180+m212)+exp(r195+m213)+exp(r210+m214)+exp(r225+m215));
t1511y=exp(r165+m211)/(1+exp(r15+m21)+exp(r30+m22)+exp(r45+m23)+exp(r60+m24)+exp(r75+m25)
+exp(r90+m26)+exp(r105+m27)+exp(r120+m28)+exp(r135+m29)+exp(r150+m210)+exp(r165+m211)
+exp(r180+m212)+exp(r195+m213)+exp(r210+m214)+exp(r225+m215));
t1512y=exp(r180+m212)/(1+exp(r15+m21)+exp(r30+m22)+exp(r45+m23)+exp(r60+m24)+exp(r75+m25)
+exp(r90+m26)+exp(r105+m27)+exp(r120+m28)+exp(r135+m29)+exp(r150+m210)+exp(r165+m211)
+exp(r180+m212)+exp(r195+m213)+exp(r210+m214)+exp(r225+m215));
t1513y=exp(r195+m213)/(1+exp(r15+m21)+exp(r30+m22)+exp(r45+m23)+exp(r60+m24)+exp(r75+m25)
+exp(r90+m26)+exp(r105+m27)+exp(r120+m28)+exp(r135+m29)+exp(r150+m210)+exp(r165+m211)
+exp(r180+m212)+exp(r195+m213)+exp(r210+m214)+exp(r225+m215));
t1514y=exp(r210+m214)/(1+exp(r15+m21)+exp(r30+m22)+exp(r45+m23)+exp(r60+m24)+exp(r75+m25)
+exp(r90+m26)+exp(r105+m27)+exp(r120+m28)+exp(r135+m29)+exp(r150+m210)+exp(r165+m211)
+exp(r180+m212)+exp(r195+m213)+exp(r210+m214)+exp(r225+m215));
t1515y=exp(r225+m215)/(1+exp(r15+m21)+exp(r30+m22)+exp(r45+m23)+exp(r60+m24)+exp(r75+m25)
+exp(r90+m26)+exp(r105+m27)+exp(r120+m28)+exp(r135+m29)+exp(r150+m210)+exp(r165+m211)
+exp(r180+m212)+exp(r195+m213)+exp(r210+m214)+exp(r225+m215));
t1516y=1/(1+exp(r15+m21)+exp(r30+m22)+exp(r45+m23)+exp(r60+m24)+exp(r75+m25)
+exp(r90+m26)+exp(r105+m27)+exp(r120+m28)+exp(r135+m29)+exp(r150+m210)+exp(r165+m211)
+exp(r180+m212)+exp(r195+m213)+exp(r210+m214)+exp(r225+m215));

!row 16
t161z=exp(m21)/(1+exp(m21)+exp(m22)+exp(m23)+exp(m24)+exp(m25)+exp(m26)+exp(m27)
+exp(m28)+exp(m29)+exp(m210)+exp(m211)+exp(m212)+exp(m213)+exp(m214)+exp(m215));
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t162z=exp(m22)/(1+exp(m21)+exp(m22)+exp(m23)+exp(m24)+exp(m25)+exp(m26)+exp(m27)
+exp(m28)+exp(m29)+exp(m210)+exp(m211)+exp(m212)+exp(m213)+exp(m214)+exp(m215));
t163z=exp(m23)/(1+exp(m21)+exp(m22)+exp(m23)+exp(m24)+exp(m25)+exp(m26)+exp(m27)
+exp(m28)+exp(m29)+exp(m210)+exp(m211)+exp(m212)+exp(m213)+exp(m214)+exp(m215));
t164z=exp(m24)/(1+exp(m21)+exp(m22)+exp(m23)+exp(m24)+exp(m25)+exp(m26)+exp(m27)
+exp(m28)+exp(m29)+exp(m210)+exp(m211)+exp(m212)+exp(m213)+exp(m214)+exp(m215));
t165z=exp(m25)/(1+exp(m21)+exp(m22)+exp(m23)+exp(m24)+exp(m25)+exp(m26)+exp(m27)
+exp(m28)+exp(m29)+exp(m210)+exp(m211)+exp(m212)+exp(m213)+exp(m214)+exp(m215));
t166z=exp(m26)/(1+exp(m21)+exp(m22)+exp(m23)+exp(m24)+exp(m25)+exp(m26)+exp(m27)
+exp(m28)+exp(m29)+exp(m210)+exp(m211)+exp(m212)+exp(m213)+exp(m214)+exp(m215));
t167z=exp(m27)/(1+exp(m21)+exp(m22)+exp(m23)+exp(m24)+exp(m25)+exp(m26)+exp(m27)
+exp(m28)+exp(m29)+exp(m210)+exp(m211)+exp(m212)+exp(m213)+exp(m214)+exp(m215));
t168z=exp(m28)/(1+exp(m21)+exp(m22)+exp(m23)+exp(m24)+exp(m25)+exp(m26)+exp(m27)
+exp(m28)+exp(m29)+exp(m210)+exp(m211)+exp(m212)+exp(m213)+exp(m214)+exp(m215));
t169z=exp(m29)/(1+exp(m21)+exp(m22)+exp(m23)+exp(m24)+exp(m25)+exp(m26)+exp(m27)
+exp(m28)+exp(m29)+exp(m210)+exp(m211)+exp(m212)+exp(m213)+exp(m214)+exp(m215));
t1610z=exp(m210)/(1+exp(m21)+exp(m22)+exp(m23)+exp(m24)+exp(m25)+exp(m26)+exp(m27)
+exp(m28)+exp(m29)+exp(m210)+exp(m211)+exp(m212)+exp(m213)+exp(m214)+exp(m215));
t1611z=exp(m211)/(1+exp(m21)+exp(m22)+exp(m23)+exp(m24)+exp(m25)+exp(m26)+exp(m27)
+exp(m28)+exp(m29)+exp(m210)+exp(m211)+exp(m212)+exp(m213)+exp(m214)+exp(m215));
t1612z=exp(m212)/(1+exp(m21)+exp(m22)+exp(m23)+exp(m24)+exp(m25)+exp(m26)+exp(m27)
+exp(m28)+exp(m29)+exp(m210)+exp(m211)+exp(m212)+exp(m213)+exp(m214)+exp(m215));
t1613z=exp(m213)/(1+exp(m21)+exp(m22)+exp(m23)+exp(m24)+exp(m25)+exp(m26)+exp(m27)
+exp(m28)+exp(m29)+exp(m210)+exp(m211)+exp(m212)+exp(m213)+exp(m214)+exp(m215));
t1614z=exp(m214)/(1+exp(m21)+exp(m22)+exp(m23)+exp(m24)+exp(m25)+exp(m26)+exp(m27)
+exp(m28)+exp(m29)+exp(m210)+exp(m211)+exp(m212)+exp(m213)+exp(m214)+exp(m215));
t1615z=exp(m215)/(1+exp(m21)+exp(m22)+exp(m23)+exp(m24)+exp(m25)+exp(m26)+exp(m27)
+exp(m28)+exp(m29)+exp(m210)+exp(m211)+exp(m212)+exp(m213)+exp(m214)+exp(m215));
t1616z=1/(1+exp(m21)+exp(m22)+exp(m23)+exp(m24)+exp(m25)+exp(m26)+exp(m27)
+exp(m28)+exp(m29)+exp(m210)+exp(m211)+exp(m212)+exp(m213)+exp(m214)+exp(m215));

!16 post-test attribute profile proportions
```

```
NEW(tpost1 tpost2 tpost3 tpost4 tpost5 tpost6 tpost7 tpost8 tpost9 tpost10 tpost11
tpost12 tpost13 tpost14 tpost15 tpost16);

tpost1=tpre1*t11a+tpre2*t21b+tpre3*t31e+tpre4*t41f+tpre5*t51h+tpre6*t61i+tpre7*t71j+
tpre8*t81k+tpre9*t91n+tpre10*t101o+tpre11*t111q+tpre12*t121u+tpre13*t131v+tpre14*t141w+
tpre15*t151y+tpre16*t161z;

tpost2=tpre1*t12a+tpre2*t22b+tpre3*t32e+tpre4*t42f+tpre5*t52h+tpre6*t62i+tpre7*t72j+
tpre8*t82k+tpre9*t92n+tpre10*t102o+tpre11*t112q+tpre12*t122u+tpre13*t132v+tpre14*t142w+
tpre15*t152y+tpre16*t162z;

tpost3=tpre1*t13a+tpre2*t23b+tpre3*t33e+tpre4*t43f+tpre5*t53h+tpre6*t63i+tpre7*t73j+
tpre8*t83k+tpre9*t93n+tpre10*t103o+tpre11*t113q+tpre12*t123u+tpre13*t133v+tpre14*t143w+
tpre15*t153y+tpre16*t163z;

tpost4=tpre1*t14a+tpre2*t24b+tpre3*t34e+tpre4*t44f+tpre5*t54h+tpre6*t64i+tpre7*t74j+
tpre8*t84k+tpre9*t94n+tpre10*t104o+tpre11*t114q+tpre12*t124u+tpre13*t134v+tpre14*t144w+
tpre15*t154y+tpre16*t164z;

tpost5=tpre1*t15a+tpre2*t25b+tpre3*t35e+tpre4*t45f+tpre5*t55h+tpre6*t65i+tpre7*t75j+
tpre8*t85k+tpre9*t95n+tpre10*t105o+tpre11*t115q+tpre12*t125u+tpre13*t135v+tpre14*t145w+
tpre15*t155y+tpre16*t165z;

tpost6=tpre1*t16a+tpre2*t26b+tpre3*t36e+tpre4*t46f+tpre5*t56h+tpre6*t66i+tpre7*t76j+
tpre8*t86k+tpre9*t96n+tpre10*t106o+tpre11*t116q+tpre12*t126u+tpre13*t136v+tpre14*t146w+
tpre15*t156y+tpre16*t166z;

tpost7=tpre1*t17a+tpre2*t27b+tpre3*t37e+tpre4*t47f+tpre5*t57h+tpre6*t67i+tpre7*t77j+
tpre8*t87k+tpre9*t97n+tpre10*t107o+tpre11*t117q+tpre12*t127u+tpre13*t137v+tpre14*t147w+
tpre15*t157y+tpre16*t167z;

tpost8=tpre1*t18a+tpre2*t28b+tpre3*t38e+tpre4*t48f+tpre5*t58h+tpre6*t68i+tpre7*t78j+
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tpre8*t88k+tpre9*t98n+tpre10*t108o+tpre11*t118q+tpre12*t128u+tpre13*t138v+tpre14*t148w+
tpre15*t158y+tpre16*t168z;

tpost9=tpre1*t19a+tpre2*t29b+tpre3*t39e+tpre4*t49f+tpre5*t59h+tpre6*t69i+tpre7*t79j+
tpre8*t89k+tpre9*t99n+tpre10*t109o+tpre11*t119q+tpre12*t129u+tpre13*t139v+tpre14*t149w+
tpre15*t159y+tpre16*t169z;

tpost10=tpre1*t110a+tpre2*t210b+tpre3*t310e+tpre4*t410f+tpre5*t510h+tpre6*t610i+
tpre7*t710j+tpre8*t810k+tpre9*t910n+tpre10*t1010o+tpre11*t1110q+tpre12*t1210u+
tpre13*t1310v+tpre14*t1410w+tpre15*t1510y+tpre16*t1610z;

tpost11=tpre1*t111a+tpre2*t211b+tpre3*t311e+tpre4*t411f+tpre5*t511h+tpre6*t611i+
tpre7*t711j+tpre8*t811k+tpre9*t911n+tpre10*t1011o+tpre11*t1111q+tpre12*t1211u+
tpre13*t1311v+tpre14*t1411w+tpre15*t1511y+tpre16*t1611z;

tpost12=tpre1*t112a+tpre2*t212b+tpre3*t312e+tpre4*t412f+tpre5*t512h+tpre6*t612i+
tpre7*t712j+tpre8*t812k+tpre9*t912n+tpre10*t1012o+tpre11*t1112q+tpre12*t1212u+
tpre13*t1312v+tpre14*t1412w+tpre15*t1512y+tpre16*t1612z;

tpost13=tpre1*t113a+tpre2*t213b+tpre3*t313e+tpre4*t413f+tpre5*t513h+tpre6*t613i+
tpre7*t713j+tpre8*t813k+tpre9*t913n+tpre10*t1013o+tpre11*t1113q+tpre12*t1213u+
tpre13*t1313v+tpre14*t1413w+tpre15*t1513y+tpre16*t1613z;

tpost14=tpre1*t114a+tpre2*t214b+tpre3*t314e+tpre4*t414f+tpre5*t514h+tpre6*t614i+
tpre7*t714j+tpre8*t814k+tpre9*t914n+tpre10*t1014o+tpre11*t1114q+tpre12*t1214u+
tpre13*t1314v+tpre14*t1414w+tpre15*t1514y+tpre16*t1614z;

tpost15=tpre1*t115a+tpre2*t215b+tpre3*t315e+tpre4*t415f+tpre5*t515h+tpre6*t615i+
tpre7*t715j+tpre8*t815k+tpre9*t915n+tpre10*t1015o+tpre11*t1115q+tpre12*t1215u+
tpre13*t1315v+tpre14*t1415w+tpre15*t1515y+tpre16*t1615z;

tpost16=tpre1*t116a+tpre2*t216b+tpre3*t316e+tpre4*t416f+tpre5*t516h+tpre6*t616i+

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tpre7*t716j+tpre8*t816k+tpre9*t916n+tpre10*t1016o+tpre11*t1116q+tpre12*t1216u+
tpre13*t1316v+tpre14*t1416w+tpre15*t1516y+tpre16*t1616z;

!Wald tests for growth in attribute mastery for treatment group
NEW(tgrowth1 tgrowth2 tgrowth3 tgrowth4);

tgrowth1 = (tpost9+tpost10+tpost11+tpost12+tpost13+tpost14+tpost15+tpost16) -
(tpre9+tpre10+tpre11+tpre12+tpre13+tpre14+tpre15+tpre16);

tgrowth2 = (tpost5+tpost6+tpost7+tpost8+tpost13+tpost14+tpost15+tpost16) -
(tpre5+tpre6+tpre7+tpre8+tpre13+tpre14+tpre15+tpre16);

tgrowth3 = (tpost3+tpost4+tpost7+tpost8+tpost11+tpost12+tpost15+tpost16) -
(tpre3+tpre4+tpre7+tpre8+tpre11+tpre12+tpre15+tpre16);

tgrowth4 = (tpost2+tpost4+tpost6+tpost8+tpost10+tpost12+tpost14+tpost16) -
(tpre2+tpre4+tpre6+tpre8+tpre10+tpre12+tpre14+tpre16);


!!!!!!!!!!!!!!!!!!!!!!!!!!!!!!!!!!!!!!!!!!!!!!!!!!!!!!!!!!!!!!!!!
!!!!! Control group pre-post analysis !!!!!

!16 Pre-test Attribute Profile Proportions
NEW(d1 cpre1 cpre2 cpre3 cpre4 cpre5 cpre6 cpre7 cpre8 cpre9
cpre10 cpre11 cpre12 cpre13 cpre14 cpre15 cpre16);

d1 = 1 + exp(m11+aa1) + exp(m12+aa2) + exp(m13+aa3) + exp(m14+aa4) + exp(m15+aa5) +
exp(m16+aa6) + exp(m17+aa7) + exp(m18+aa8) + exp(m19+aa9) + exp(m110+aa10) +
exp(m111+aa11) + exp(m112+aa12) + exp(m113+aa13) + exp(m114+aa14) + exp(m115+aa15);

cpre1 = exp(m11+aa1)/d1;
cpre2 = exp(m12+aa2)/d1;
```

```
cpre3 = exp(m13+aa3)/d1;
cpre4 = exp(m14+aa4)/d1;
cpre5 = exp(m15+aa5)/d1;
cpre6 = exp(m16+aa6)/d1;
cpre7 = exp(m17+aa7)/d1;
cpre8 = exp(m18+aa8)/d1;
cpre9 = exp(m19+aa9)/d1;
cpre10 = exp(m110+aa10)/d1;
cpre11 = exp(m111+aa11)/d1;
cpre12 = exp(m112+aa12)/d1;
cpre13 = exp(m113+aa13)/d1;
cpre14 = exp(m114+aa14)/d1;
cpre15 = exp(m115+aa15)/d1;
cpre16 = 1/d1;

!attribute profile transition probabilities
New(c11a c12a c13a c14a c15a c16a c17a c18a
c19a c110a c111a c112a c113a c114a c115a c116a
c21b c22b c23b c24b c25b c26b c27b c28b c29b c210b c211b c212b c213b c214b c215b c216b
c31e c32e c33e c34e c35e c36e c37e c38e c39e c310e c311e c312e c313e c314e c315e c316e
c41f c42f c43f c44f c45f c46f c47f c48f c49f c410f c411f c412f c413f c414f c415f c416f
c51h c52h c53h c54h c55h c56h c57h c58h c59h c510h c511h c512h c513h c514h c515h c516h
c61i c62i c63i c64i c65i c66i c67i c68i c69i c610i c611i c612i c613i c614i c615i c616i
c71j c72j c73j c74j c75j c76j c77j c78j c79j c710j c711j c712j c713j c714j c715j c716j
c81k c82k c83k c84k c85k c86k c87k c88k c89k c810k c811k c812k c813k c814k c815k c816k
c91n c92n c93n c94n c95n c96n c97n c98n c99n c910n c911n c912n c913n c914n c915n c916n
c101o c102o c103o c104o c105o c106o c107o c108o c109o
c1010o c1011o c1012o c1013o c1014o c1015o c1016o
c111q c112q c113q c114q c115q c116q c117q c118q c119q
c1110q c1111q c1112q c1113q c1114q c1115q c1116q
c121u c122u c123u c124u c125u c126u c127u c128u c129u
c1210u c1211u c1212u c1213u c1214u c1215u c1216u
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c131v c132v c133v c134v c135v c136v c137v c138v c139v
c1310v c1311v c1312v c1313v c1314v c1315v c1316v
c141w c142w c143w c144w c145w c146w c147w c148w c149w
c1410w c1411w c1412w c1413w c1414w c1415w c1416w
c151y c152y c153y c154y c155y c156y c157y c158y c159y
c1510y c1511y c1512y c1513y c1514y c1515y c1516y
c161z c162z c163z c164z c165z c166z c167z c168z c169z
c1610z c1611z c1612z c1613z c1614z c1615z c1616z);

!row 1
  c11a=exp(bb1+m21+aa16)/(1+exp(bb1+m21+aa16)+exp(bb16+m22+aa17)+exp(bb31+m23+aa18)+
  exp(bb46+m24+aa19)+exp(bb61+m25+aa20)+exp(bb76+m26+aa21)+exp(bb91+m27+aa22)+
  exp(bb106+m28+aa23)
  +exp(bb121+m29+aa24)+exp(bb136+m210+aa25)+exp(bb151+m211+aa26)+exp(bb166+m212+aa27)+
  exp(bb181+m213+aa28)+exp(bb196+m214+aa29)+exp(bb211+m215+aa30));
  c12a=exp(bb16+m22+aa17)/(1+exp(bb1+m21+aa16)+exp(bb16+m22+aa17)+exp(bb31+m23+aa18)+
  exp(bb46+m24+aa19)+exp(bb61+m25+aa20)+exp(bb76+m26+aa21)+exp(bb91+m27+aa22)+
  exp(bb106+m28+aa23)
  +exp(bb121+m29+aa24)+exp(bb136+m210+aa25)+exp(bb151+m211+aa26)+exp(bb166+m212+aa27)+
  exp(bb181+m213+aa28)+exp(bb196+m214+aa29)+exp(bb211+m215+aa30));
  c13a=exp(bb31+m23+aa18)/(1+exp(bb1+m21+aa16)+exp(bb16+m22+aa17)+exp(bb31+m23+aa18)+
  exp(bb46+m24+aa19)+exp(bb61+m25+aa20)+exp(bb76+m26+aa21)+exp(bb91+m27+aa22)+
  exp(bb106+m28+aa23)
  +exp(bb121+m29+aa24)+exp(bb136+m210+aa25)+exp(bb151+m211+aa26)+exp(bb166+m212+aa27)+
  exp(bb181+m213+aa28)+exp(bb196+m214+aa29)+exp(bb211+m215+aa30));
  c14a=exp(bb46+m24+aa19)/(1+exp(bb1+m21+aa16)+exp(bb16+m22+aa17)+exp(bb31+m23+aa18)+
  exp(bb46+m24+aa19)+exp(bb61+m25+aa20)+exp(bb76+m26+aa21)+exp(bb91+m27+aa22)+
  exp(bb106+m28+aa23)
  +exp(bb121+m29+aa24)+exp(bb136+m210+aa25)+exp(bb151+m211+aa26)+exp(bb166+m212+aa27)+
  exp(bb181+m213+aa28)+exp(bb196+m214+aa29)+exp(bb211+m215+aa30));
  c15a=exp(bb61+m25+aa20)/(1+exp(bb1+m21+aa16)+exp(bb16+m22+aa17)+exp(bb31+m23+aa18)+
  exp(bb46+m24+aa19)+exp(bb61+m25+aa20)+exp(bb76+m26+aa21)+exp(bb91+m27+aa22)+
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exp(bb106+m28+aa23)
+exp(bb121+m29+aa24)+exp(bb136+m210+aa25)+exp(bb151+m211+aa26)+exp(bb166+m212+aa27)+
exp(bb181+m213+aa28)+exp(bb196+m214+aa29)+exp(bb211+m215+aa30));
c16a=exp(bb76+m26+aa21)/(1+exp(bb1+m21+aa16)+exp(bb16+m22+aa17)+exp(bb31+m23+aa18)+
exp(bb46+m24+aa19)+exp(bb61+m25+aa20)+exp(bb76+m26+aa21)+exp(bb91+m27+aa22)+
exp(bb106+m28+aa23)
+exp(bb121+m29+aa24)+exp(bb136+m210+aa25)+exp(bb151+m211+aa26)+exp(bb166+m212+aa27)+
exp(bb181+m213+aa28)+exp(bb196+m214+aa29)+exp(bb211+m215+aa30));
c17a=exp(bb91+m27+aa22)/(1+exp(bb1+m21+aa16)+exp(bb16+m22+aa17)+exp(bb31+m23+aa18)+
exp(bb46+m24+aa19)+exp(bb61+m25+aa20)+exp(bb76+m26+aa21)+exp(bb91+m27+aa22)+
exp(bb106+m28+aa23)
+exp(bb121+m29+aa24)+exp(bb136+m210+aa25)+exp(bb151+m211+aa26)+exp(bb166+m212+aa27)+
exp(bb181+m213+aa28)+exp(bb196+m214+aa29)+exp(bb211+m215+aa30));
c18a=exp(bb106+m28+aa23)/(1+exp(bb1+m21+aa16)+exp(bb16+m22+aa17)+exp(bb31+m23+aa18)+
exp(bb46+m24+aa19)+exp(bb61+m25+aa20)+exp(bb76+m26+aa21)+exp(bb91+m27+aa22)+
exp(bb106+m28+aa23)
+exp(bb121+m29+aa24)+exp(bb136+m210+aa25)+exp(bb151+m211+aa26)+exp(bb166+m212+aa27)+
exp(bb181+m213+aa28)+exp(bb196+m214+aa29)+exp(bb211+m215+aa30));
c19a=exp(bb121+m29+aa24)/(1+exp(bb1+m21+aa16)+exp(bb16+m22+aa17)+exp(bb31+m23+aa18)+
exp(bb46+m24+aa19)+exp(bb61+m25+aa20)+exp(bb76+m26+aa21)+exp(bb91+m27+aa22)+
exp(bb106+m28+aa23)
+exp(bb121+m29+aa24)+exp(bb136+m210+aa25)+exp(bb151+m211+aa26)+exp(bb166+m212+aa27)+
exp(bb181+m213+aa28)+exp(bb196+m214+aa29)+exp(bb211+m215+aa30));
c110a=exp(bb136+m210+aa25)/(1+exp(bb1+m21+aa16)+exp(bb16+m22+aa17)+exp(bb31+m23+aa18)+
exp(bb46+m24+aa19)+exp(bb61+m25+aa20)+exp(bb76+m26+aa21)+exp(bb91+m27+aa22)+
exp(bb106+m28+aa23)
+exp(bb121+m29+aa24)+exp(bb136+m210+aa25)+exp(bb151+m211+aa26)+exp(bb166+m212+aa27)+
exp(bb181+m213+aa28)+exp(bb196+m214+aa29)+exp(bb211+m215+aa30));
c111a=exp(bb151+m211+aa26)/(1+exp(bb1+m21+aa16)+exp(bb16+m22+aa17)+exp(bb31+m23+aa18)+
exp(bb46+m24+aa19)+exp(bb61+m25+aa20)+exp(bb76+m26+aa21)+exp(bb91+m27+aa22)+
exp(bb106+m28+aa23)
+exp(bb121+m29+aa24)+exp(bb136+m210+aa25)+exp(bb151+m211+aa26)+exp(bb166+m212+aa27)+

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exp(bb181+m213+aa28)+exp(bb196+m214+aa29)+exp(bb211+m215+aa30));
c112a=exp(bb166+m212+aa27)/(1+exp(bb1+m21+aa16)+exp(bb16+m22+aa17)+exp(bb31+m23+aa18)+
exp(bb46+m24+aa19)+exp(bb61+m25+aa20)+exp(bb76+m26+aa21)+exp(bb91+m27+aa22)+
exp(bb106+m28+aa23)
+exp(bb121+m29+aa24)+exp(bb136+m210+aa25)+exp(bb151+m211+aa26)+exp(bb166+m212+aa27)+
exp(bb181+m213+aa28)+exp(bb196+m214+aa29)+exp(bb211+m215+aa30));
c113a=exp(bb181+m213+aa28)/(1+exp(bb1+m21+aa16)+exp(bb16+m22+aa17)+exp(bb31+m23+aa18)+
exp(bb46+m24+aa19)+exp(bb61+m25+aa20)+exp(bb76+m26+aa21)+exp(bb91+m27+aa22)+
exp(bb106+m28+aa23)
+exp(bb121+m29+aa24)+exp(bb136+m210+aa25)+exp(bb151+m211+aa26)+exp(bb166+m212+aa27)+
exp(bb181+m213+aa28)+exp(bb196+m214+aa29)+exp(bb211+m215+aa30));
c114a=exp(bb196+m214+aa29)/(1+exp(bb1+m21+aa16)+exp(bb16+m22+aa17)+exp(bb31+m23+aa18)+
exp(bb46+m24+aa19)+exp(bb61+m25+aa20)+exp(bb76+m26+aa21)+exp(bb91+m27+aa22)+
exp(bb106+m28+aa23)
+exp(bb121+m29+aa24)+exp(bb136+m210+aa25)+exp(bb151+m211+aa26)+exp(bb166+m212+aa27)+
exp(bb181+m213+aa28)+exp(bb196+m214+aa29)+exp(bb211+m215+aa30));
c115a=exp(bb211+m215+aa30)/(1+exp(bb1+m21+aa16)+exp(bb16+m22+aa17)+exp(bb31+m23+aa18)+
exp(bb46+m24+aa19)+exp(bb61+m25+aa20)+exp(bb76+m26+aa21)+exp(bb91+m27+aa22)+
exp(bb106+m28+aa23)
+exp(bb121+m29+aa24)+exp(bb136+m210+aa25)+exp(bb151+m211+aa26)+exp(bb166+m212+aa27)+
exp(bb181+m213+aa28)+exp(bb196+m214+aa29)+exp(bb211+m215+aa30));
c116a = 1/(1+exp(bb1+m21+aa16)+exp(bb16+m22+aa17)+exp(bb31+m23+aa18)+
exp(bb46+m24+aa19)+exp(bb61+m25+aa20)+exp(bb76+m26+aa21)+exp(bb91+m27+aa22)+
exp(bb106+m28+aa23)
+exp(bb121+m29+aa24)+exp(bb136+m210+aa25)+exp(bb151+m211+aa26)+exp(bb166+m212+aa27)+
exp(bb181+m213+aa28)+exp(bb196+m214+aa29)+exp(bb211+m215+aa30));

!row 2
c21b=exp(bb2+m21+aa16)/(1+exp(bb2+m21+aa16)+exp(bb17+m22+aa17)+exp(bb32+m23+aa18)+
exp(bb47+m24+aa19)+exp(bb62+m25+aa20)+exp(bb77+m26+aa21)+exp(bb92+m27+aa22)+
exp(bb107+m28+aa23)
+exp(bb122+m29+aa24)+exp(bb137+m210+aa25)+exp(bb152+m211+aa26)+exp(bb167+m212+aa27)+
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exp(bb182+m213+aa28)+exp(bb197+m214+aa29)+exp(bb212+m215+aa30));
c22b=exp(bb17+m22+aa17)/(1+exp(bb2+m21+aa16)+exp(bb17+m22+aa17)+exp(bb32+m23+aa18)+
exp(bb47+m24+aa19)+exp(bb62+m25+aa20)+exp(bb77+m26+aa21)+exp(bb92+m27+aa22)+
exp(bb107+m28+aa23)
+exp(bb122+m29+aa24)+exp(bb137+m210+aa25)+exp(bb152+m211+aa26)+exp(bb167+m212+aa27)+
exp(bb182+m213+aa28)+exp(bb197+m214+aa29)+exp(bb212+m215+aa30));
c23b=exp(bb32+m23+aa18)/(1+exp(bb2+m21+aa16)+exp(bb17+m22+aa17)+exp(bb32+m23+aa18)+
exp(bb47+m24+aa19)+exp(bb62+m25+aa20)+exp(bb77+m26+aa21)+exp(bb92+m27+aa22)+
exp(bb107+m28+aa23)
+exp(bb122+m29+aa24)+exp(bb137+m210+aa25)+exp(bb152+m211+aa26)+exp(bb167+m212+aa27)+
exp(bb182+m213+aa28)+exp(bb197+m214+aa29)+exp(bb212+m215+aa30));
c24b=exp(bb47+m24+aa19)/(1+exp(bb2+m21+aa16)+exp(bb17+m22+aa17)+exp(bb32+m23+aa18)+
exp(bb47+m24+aa19)+exp(bb62+m25+aa20)+exp(bb77+m26+aa21)+exp(bb92+m27+aa22)+
exp(bb107+m28+aa23)
+exp(bb122+m29+aa24)+exp(bb137+m210+aa25)+exp(bb152+m211+aa26)+exp(bb167+m212+aa27)+
exp(bb182+m213+aa28)+exp(bb197+m214+aa29)+exp(bb212+m215+aa30));
c25b=exp(bb62+m25+aa20)/(1+exp(bb2+m21+aa16)+exp(bb17+m22+aa17)+exp(bb32+m23+aa18)+
exp(bb47+m24+aa19)+exp(bb62+m25+aa20)+exp(bb77+m26+aa21)+exp(bb92+m27+aa22)+
exp(bb107+m28+aa23)
+exp(bb122+m29+aa24)+exp(bb137+m210+aa25)+exp(bb152+m211+aa26)+exp(bb167+m212+aa27)+
exp(bb182+m213+aa28)+exp(bb197+m214+aa29)+exp(bb212+m215+aa30));
c26b=exp(bb77+m26+aa21)/(1+exp(bb2+m21+aa16)+exp(bb17+m22+aa17)+exp(bb32+m23+aa18)+
exp(bb47+m24+aa19)+exp(bb62+m25+aa20)+exp(bb77+m26+aa21)+exp(bb92+m27+aa22)+
exp(bb107+m28+aa23)
+exp(bb122+m29+aa24)+exp(bb137+m210+aa25)+exp(bb152+m211+aa26)+exp(bb167+m212+aa27)+
exp(bb182+m213+aa28)+exp(bb197+m214+aa29)+exp(bb212+m215+aa30));
c27b=exp(bb92+m27+aa22)/(1+exp(bb2+m21+aa16)+exp(bb17+m22+aa17)+exp(bb32+m23+aa18)+
exp(bb47+m24+aa19)+exp(bb62+m25+aa20)+exp(bb77+m26+aa21)+exp(bb92+m27+aa22)+
exp(bb107+m28+aa23)
+exp(bb122+m29+aa24)+exp(bb137+m210+aa25)+exp(bb152+m211+aa26)+exp(bb167+m212+aa27)+
exp(bb182+m213+aa28)+exp(bb197+m214+aa29)+exp(bb212+m215+aa30));
c28b=exp(bb107+m28+aa23)/(1+exp(bb2+m21+aa16)+exp(bb17+m22+aa17)+exp(bb32+m23+aa18)+

exp(bb47+m24+aa19)+exp(bb62+m25+aa20)+exp(bb77+m26+aa21)+exp(bb92+m27+aa22)+
exp(bb107+m28+aa23)
+exp(bb122+m29+aa24)+exp(bb137+m210+aa25)+exp(bb152+m211+aa26)+exp(bb167+m212+aa27)+
exp(bb182+m213+aa28)+exp(bb197+m214+aa29)+exp(bb212+m215+aa30));
c29b=exp(bb122+m29+aa24)/(1+exp(bb2+m21+aa16)+exp(bb17+m22+aa17)+exp(bb32+m23+aa18)+
exp(bb47+m24+aa19)+exp(bb62+m25+aa20)+exp(bb77+m26+aa21)+exp(bb92+m27+aa22)+
exp(bb107+m28+aa23)
+exp(bb122+m29+aa24)+exp(bb137+m210+aa25)+exp(bb152+m211+aa26)+exp(bb167+m212+aa27)+
exp(bb182+m213+aa28)+exp(bb197+m214+aa29)+exp(bb212+m215+aa30));
c210b=exp(bb137+m210+aa25)/(1+exp(bb2+m21+aa16)+exp(bb17+m22+aa17)+exp(bb32+m23+aa18)+
exp(bb47+m24+aa19)+exp(bb62+m25+aa20)+exp(bb77+m26+aa21)+exp(bb92+m27+aa22)+
exp(bb107+m28+aa23)
+exp(bb122+m29+aa24)+exp(bb137+m210+aa25)+exp(bb152+m211+aa26)+exp(bb167+m212+aa27)+
exp(bb182+m213+aa28)+exp(bb197+m214+aa29)+exp(bb212+m215+aa30));
c211b=exp(bb152+m211+aa26)/(1+exp(bb2+m21+aa16)+exp(bb17+m22+aa17)+exp(bb32+m23+aa18)+
exp(bb47+m24+aa19)+exp(bb62+m25+aa20)+exp(bb77+m26+aa21)+exp(bb92+m27+aa22)+
exp(bb107+m28+aa23)
+exp(bb122+m29+aa24)+exp(bb137+m210+aa25)+exp(bb152+m211+aa26)+exp(bb167+m212+aa27)+
exp(bb182+m213+aa28)+exp(bb197+m214+aa29)+exp(bb212+m215+aa30));
c212b=exp(bb167+m212+aa27)/(1+exp(bb2+m21+aa16)+exp(bb17+m22+aa17)+exp(bb32+m23+aa18)+
exp(bb47+m24+aa19)+exp(bb62+m25+aa20)+exp(bb77+m26+aa21)+exp(bb92+m27+aa22)+
exp(bb107+m28+aa23)
+exp(bb122+m29+aa24)+exp(bb137+m210+aa25)+exp(bb152+m211+aa26)+exp(bb167+m212+aa27)+
exp(bb182+m213+aa28)+exp(bb197+m214+aa29)+exp(bb212+m215+aa30));
c213b=exp(bb182+m213+aa28)/(1+exp(bb2+m21+aa16)+exp(bb17+m22+aa17)+exp(bb32+m23+aa18)+
exp(bb47+m24+aa19)+exp(bb62+m25+aa20)+exp(bb77+m26+aa21)+exp(bb92+m27+aa22)+
exp(bb107+m28+aa23)
+exp(bb122+m29+aa24)+exp(bb137+m210+aa25)+exp(bb152+m211+aa26)+exp(bb167+m212+aa27)+
exp(bb182+m213+aa28)+exp(bb197+m214+aa29)+exp(bb212+m215+aa30));
c214b=exp(bb197+m214+aa29)/(1+exp(bb2+m21+aa16)+exp(bb17+m22+aa17)+exp(bb32+m23+aa18)+
exp(bb47+m24+aa19)+exp(bb62+m25+aa20)+exp(bb77+m26+aa21)+exp(bb92+m27+aa22)+
exp(bb107+m28+aa23)

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 +exp(bb122+m29+aa24)+exp(bb137+m210+aa25)+exp(bb152+m211+aa26)+exp(bb167+m212+aa27)+
 exp(bb182+m213+aa28)+exp(bb197+m214+aa29)+exp(bb212+m215+aa30));
 c215b=exp(bb212+m215+aa30)/(1+exp(bb2+m21+aa16)+exp(bb17+m22+aa17)+exp(bb32+m23+aa18)+
 exp(bb47+m24+aa19)+exp(bb62+m25+aa20)+exp(bb77+m26+aa21)+exp(bb92+m27+aa22)+
 exp(bb107+m28+aa23)
 +exp(bb122+m29+aa24)+exp(bb137+m210+aa25)+exp(bb152+m211+aa26)+exp(bb167+m212+aa27)+
 exp(bb182+m213+aa28)+exp(bb197+m214+aa29)+exp(bb212+m215+aa30));
 c216b=1/(1+exp(bb2+m21+aa16)+exp(bb17+m22+aa17)+exp(bb32+m23+aa18)+
 exp(bb47+m24+aa19)+exp(bb62+m25+aa20)+exp(bb77+m26+aa21)+exp(bb92+m27+aa22)+
 exp(bb107+m28+aa23)
 +exp(bb122+m29+aa24)+exp(bb137+m210+aa25)+exp(bb152+m211+aa26)+exp(bb167+m212+aa27)+
 exp(bb182+m213+aa28)+exp(bb197+m214+aa29)+exp(bb212+m215+aa30));

!row 3
c31e=exp(bb3+m21+aa16)/(1+exp(bb3+m21+aa16)+exp(bb18+m22+aa17)+exp(bb33+m23+aa18)
+exp(bb48+m24+aa19)+exp(bb63+m25+aa20)+exp(bb78+m26+aa21)+exp(bb93+m27+aa22)+
exp(bb108+m28+aa23)
+exp(BB123+m29+AA24)+exp(BB138+m210+AA25)+exp(BB153+m211+AA26)+exp(BB168+m212+AA27)
+exp(bb183+m213+aa28)+exp(bb198+m214+aa29)+exp(bb213+m215+aa30));
c32e=exp(bb18+m22+aa17)/(1+exp(bb3+m21+aa16)+exp(bb18+m22+aa17)+exp(bb33+m23+aa18)
+exp(BB48+m24+AA19)+exp(BB63+m25+AA20)+exp(BB78+m26+AA21)+exp(BB93+m27+AA22)
+exp(BB108+m28+AA23)
+exp(BB123+m29+AA24)+exp(BB138+m210+AA25)+exp(BB153+m211+AA26)+exp(BB168+m212+AA27)
+exp(bb183+m213+aa28)+exp(bb198+m214+aa29)+exp(bb213+m215+aa30));
c33e=exp(bb33+m23+aa18)/(1+exp(bb3+m21+aa16)+exp(bb18+m22+aa17)+exp(bb33+m23+aa18)
+exp(BB48+m24+AA19)+exp(BB63+m25+AA20)+exp(BB78+m26+AA21)+exp(BB93+m27+AA22)
+exp(BB108+m28+AA23)
+exp(BB123+m29+AA24)+exp(BB138+m210+AA25)+exp(BB153+m211+AA26)+exp(BB168+m212+AA27)
+exp(bb183+m213+aa28)+exp(bb198+m214+aa29)+exp(bb213+m215+aa30));
c34e=exp(bb48+m24+aa19)/(1+exp(bb3+m21+aa16)+exp(bb18+m22+aa17)+exp(bb33+m23+aa18)
+exp(BB48+m24+AA19)+exp(BB63+m25+AA20)+exp(BB78+m26+AA21)+exp(BB93+m27+AA22)
+exp(BB108+m28+AA23)
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+exp(BB123+m29+AA24)+exp(BB138+m210+AA25)+exp(BB153+m211+AA26)+exp(BB168+m212+AA27)
+exp(bb183+m213+aa28)+exp(bb198+m214+aa29)+exp(bb213+m215+aa30));
c35e=exp(bb63+m25+aa20)/(1+exp(bb3+m21+aa16)+exp(bb18+m22+aa17)+exp(bb33+m23+aa18)
+exp(BB48+m24+AA19)+exp(BB63+m25+AA20)+exp(BB78+m26+AA21)+exp(BB93+m27+AA22)
+exp(BB108+m28+AA23)
+exp(BB123+m29+AA24)+exp(BB138+m210+AA25)+exp(BB153+m211+AA26)+exp(BB168+m212+AA27)
+exp(bb183+m213+aa28)+exp(bb198+m214+aa29)+exp(bb213+m215+aa30));
c36e=exp(bb78+m26+aa21)/(1+exp(bb3+m21+aa16)+exp(bb18+m22+aa17)+exp(bb33+m23+aa18)
+exp(BB48+m24+AA19)+exp(BB63+m25+AA20)+exp(BB78+m26+AA21)+exp(BB93+m27+AA22)
+exp(BB108+m28+AA23)
+exp(BB123+m29+AA24)+exp(BB138+m210+AA25)+exp(BB153+m211+AA26)+exp(BB168+m212+AA27)
+exp(bb183+m213+aa28)+exp(bb198+m214+aa29)+exp(bb213+m215+aa30));
c37e=exp(bb93+m27+aa22)/(1+exp(bb3+m21+aa16)+exp(bb18+m22+aa17)+exp(bb33+m23+aa18)
+exp(BB48+m24+AA19)+exp(BB63+m25+AA20)+exp(BB78+m26+AA21)+exp(BB93+m27+AA22)
+exp(BB108+m28+AA23)
+exp(BB123+m29+AA24)+exp(BB138+m210+AA25)+exp(BB153+m211+AA26)+exp(BB168+m212+AA27)
+exp(bb183+m213+aa28)+exp(bb198+m214+aa29)+exp(bb213+m215+aa30));
c38e=exp(bb108+m28+aa23)/(1+exp(bb3+m21+aa16)+exp(bb18+m22+aa17)+exp(bb33+m23+aa18)
+exp(BB48+m24+AA19)+exp(BB63+m25+AA20)+exp(BB78+m26+AA21)+exp(BB93+m27+AA22)
+exp(BB108+m28+AA23)
+exp(BB123+m29+AA24)+exp(BB138+m210+AA25)+exp(BB153+m211+AA26)+exp(BB168+m212+AA27)
+exp(bb183+m213+aa28)+exp(bb198+m214+aa29)+exp(bb213+m215+aa30));
c39e=exp(bb123+m29+aa24)/(1+exp(bb3+m21+aa16)+exp(bb18+m22+aa17)+exp(bb33+m23+aa18)
+exp(BB48+m24+AA19)+exp(BB63+m25+AA20)+exp(BB78+m26+AA21)+exp(BB93+m27+AA22)
+exp(BB108+m28+AA23)
+exp(BB123+m29+AA24)+exp(BB138+m210+AA25)+exp(BB153+m211+AA26)+exp(BB168+m212+AA27)
+exp(bb183+m213+aa28)+exp(bb198+m214+aa29)+exp(bb213+m215+aa30));
c310e=exp(bb138+m210+aa25)/(1+exp(bb3+m21+aa16)+exp(bb18+m22+aa17)+exp(bb33+m23+aa18)
+exp(BB48+m24+AA19)+exp(BB63+m25+AA20)+exp(BB78+m26+AA21)+exp(BB93+m27+AA22)
+exp(BB108+m28+AA23)
+exp(BB123+m29+AA24)+exp(BB138+m210+AA25)+exp(BB153+m211+AA26)+exp(BB168+m212+AA27)
+exp(bb183+m213+aa28)+exp(bb198+m214+aa29)+exp(bb213+m215+aa30));
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c311e=exp(bb153+m211+aa26)/(1+exp(bb3+m21+aa16)+exp(bb18+m22+aa17)+exp(bb33+m23+aa18)
+exp(BB48+m24+AA19)+exp(BB63+m25+AA20)+exp(BB78+m26+AA21)+exp(BB93+m27+AA22)
+exp(BB108+m28+AA23)
+exp(BB123+m29+AA24)+exp(BB138+m210+AA25)+exp(BB153+m211+AA26)+exp(BB168+m212+AA27)
+exp(bb183+m213+aa28)+exp(bb198+m214+aa29)+exp(bb213+m215+aa30));
c312e=exp(bb168+m212+aa27)/(1+exp(bb3+m21+aa16)+exp(bb18+m22+aa17)+exp(bb33+m23+aa18)
+exp(BB48+m24+AA19)+exp(BB63+m25+AA20)+exp(BB78+m26+AA21)+exp(BB93+m27+AA22)
+exp(BB108+m28+AA23)
+exp(BB123+m29+AA24)+exp(BB138+m210+AA25)+exp(BB153+m211+AA26)+exp(BB168+m212+AA27)
+exp(bb183+m213+aa28)+exp(bb198+m214+aa29)+exp(bb213+m215+aa30));
c313e=exp(bb183+m213+aa28)/(1+exp(bb3+m21+aa16)+exp(bb18+m22+aa17)+exp(bb33+m23+aa18)
+exp(BB48+m24+AA19)+exp(BB63+m25+AA20)+exp(BB78+m26+AA21)+exp(BB93+m27+AA22)
+exp(BB108+m28+AA23)
+exp(BB123+m29+AA24)+exp(BB138+m210+AA25)+exp(BB153+m211+AA26)+exp(BB168+m212+AA27)
+exp(bb183+m213+aa28)+exp(bb198+m214+aa29)+exp(bb213+m215+aa30));
c314e=exp(bb198+m214+aa29)/(1+exp(bb3+m21+aa16)+exp(bb18+m22+aa17)+exp(bb33+m23+aa18)
+exp(BB48+m24+AA19)+exp(BB63+m25+AA20)+exp(BB78+m26+AA21)+exp(BB93+m27+AA22)
+exp(BB108+m28+AA23)
+exp(BB123+m29+AA24)+exp(BB138+m210+AA25)+exp(BB153+m211+AA26)+exp(BB168+m212+AA27)
+exp(bb183+m213+aa28)+exp(bb198+m214+aa29)+exp(bb213+m215+aa30));
c315e=exp(bb213+m215+aa30)/(1+exp(bb3+m21+aa16)+exp(bb18+m22+aa17)+exp(bb33+m23+aa18)
+exp(BB48+m24+AA19)+exp(BB63+m25+AA20)+exp(BB78+m26+AA21)+exp(BB93+m27+AA22)
+exp(BB108+m28+AA23)
+exp(BB123+m29+AA24)+exp(BB138+m210+AA25)+exp(BB153+m211+AA26)+exp(BB168+m212+AA27)
+exp(bb183+m213+aa28)+exp(bb198+m214+aa29)+exp(bb213+m215+aa30));
c316e=1/(1+exp(bb3+m21+aa16)+exp(bb18+m22+aa17)+exp(bb33+m23+aa18)
+exp(BB48+m24+AA19)+exp(BB63+m25+AA20)+exp(BB78+m26+AA21)+exp(BB93+m27+AA22)
+exp(BB108+m28+AA23)
+exp(BB123+m29+AA24)+exp(BB138+m210+AA25)+exp(BB153+m211+AA26)+exp(BB168+m212+AA27)
+exp(bb183+m213+aa28)+exp(bb198+m214+aa29)+exp(bb213+m215+aa30));

!row 4
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c41f=exp(bb4+m21+aa16)/(1+exp(bb4+m21+aa16)+exp(bb19+m22+aa17)+exp(bb34+m23+aa18)
+exp(BB49+m24+AA19)+exp(BB64+m25+AA20)+exp(BB79+m26+AA21)+exp(BB94+m27+AA22)
+exp(BB109+m28+AA23)
+exp(BB124+m29+AA24)+exp(BB139+m210+AA25)+exp(BB154+m211+AA26)+exp(BB169+m212+AA27)
+exp(bb184+m213+aa28)+exp(bb199+m214+aa29)+exp(bb214+m215+aa30));
c42f=exp(bb19+m22+aa17)/(1+exp(bb4+m21+aa16)+exp(bb19+m22+aa17)+exp(bb34+m23+aa18)
+exp(BB49+m24+AA19)+exp(BB64+m25+AA20)+exp(BB79+m26+AA21)+exp(BB94+m27+AA22)
+exp(BB109+m28+AA23)
+exp(BB124+m29+AA24)+exp(BB139+m210+AA25)+exp(BB154+m211+AA26)+exp(BB169+m212+AA27)
+exp(bb184+m213+aa28)+exp(bb199+m214+aa29)+exp(bb214+m215+aa30));
c43f=exp(bb34+m23+aa18)/(1+exp(bb4+m21+aa16)+exp(bb19+m22+aa17)+exp(bb34+m23+aa18)
+exp(BB49+m24+AA19)+exp(BB64+m25+AA20)+exp(BB79+m26+AA21)+exp(BB94+m27+AA22)
+exp(BB109+m28+AA23)
+exp(BB124+m29+AA24)+exp(BB139+m210+AA25)+exp(BB154+m211+AA26)+exp(BB169+m212+AA27)
+exp(bb184+m213+aa28)+exp(bb199+m214+aa29)+exp(bb214+m215+aa30));
c44f=exp(bb49+m24+aa19)/(1+exp(bb4+m21+aa16)+exp(bb19+m22+aa17)+exp(bb34+m23+aa18)
+exp(BB49+m24+AA19)+exp(BB64+m25+AA20)+exp(BB79+m26+AA21)+exp(BB94+m27+AA22)
+exp(BB109+m28+AA23)
+exp(BB124+m29+AA24)+exp(BB139+m210+AA25)+exp(BB154+m211+AA26)+exp(BB169+m212+AA27)
+exp(bb184+m213+aa28)+exp(bb199+m214+aa29)+exp(bb214+m215+aa30));
c45f=exp(bb64+m25+aa20)/(1+exp(bb4+m21+aa16)+exp(bb19+m22+aa17)+exp(bb34+m23+aa18)
+exp(bb49+m24+aa19)+exp(bb64+m25+aa20)+exp(bb79+m26+aa21)+exp(bb94+m27+aa22)+
exp(bb109+m28+aa23)
+exp(BB124+m29+AA24)+exp(BB139+m210+AA25)+exp(BB154+m211+AA26)+exp(BB169+m212+AA27)
+exp(bb184+m213+aa28)+exp(bb199+m214+aa29)+exp(bb214+m215+aa30));
c46f=exp(bb79+m26+aa21)/(1+exp(bb4+m21+aa16)+exp(bb19+m22+aa17)+exp(bb34+m23+aa18)
+exp(bb49+m24+aa19)+exp(bb64+m25+aa20)+exp(bb79+m26+aa21)+exp(bb94+m27+aa22)+
exp(bb109+m28+aa23)
+exp(BB124+m29+AA24)+exp(BB139+m210+AA25)+exp(BB154+m211+AA26)+exp(BB169+m212+AA27)
+exp(bb184+m213+aa28)+exp(bb199+m214+aa29)+exp(bb214+m215+aa30));
c47f=exp(bb94+m27+aa22)/(1+exp(bb4+m21+aa16)+exp(bb19+m22+aa17)+exp(bb34+m23+aa18)
+exp(bb49+m24+aa19)+exp(bb64+m25+aa20)+exp(bb79+m26+aa21)+exp(bb94+m27+aa22)+

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exp(bb109+m28+aa23)
+exp(BB124+m29+AA24)+exp(BB139+m210+AA25)+exp(BB154+m211+AA26)+exp(BB169+m212+AA27)
+exp(bb184+m213+aa28)+exp(bb199+m214+aa29)+exp(bb214+m215+aa30));
c48f=exp(bb109+m28+aa23)/(1+exp(bb4+m21+aa16)+exp(bb19+m22+aa17)+exp(bb34+m23+aa18)
+exp(bb49+m24+aa19)+exp(bb64+m25+aa20)+exp(bb79+m26+aa21)+exp(bb94+m27+aa22)+
exp(bb109+m28+aa23)
+exp(BB124+m29+AA24)+exp(BB139+m210+AA25)+exp(BB154+m211+AA26)+exp(BB169+m212+AA27)
+exp(bb184+m213+aa28)+exp(bb199+m214+aa29)+exp(bb214+m215+aa30));
c49f=exp(bb124+m29+aa24)/(1+exp(bb4+m21+aa16)+exp(bb19+m22+aa17)+exp(bb34+m23+aa18)
+exp(bb49+m24+aa19)+exp(bb64+m25+aa20)+exp(bb79+m26+aa21)+exp(bb94+m27+aa22)+
exp(bb109+m28+aa23)
+exp(BB124+m29+AA24)+exp(BB139+m210+AA25)+exp(BB154+m211+AA26)+exp(BB169+m212+AA27)
+exp(bb184+m213+aa28)+exp(bb199+m214+aa29)+exp(bb214+m215+aa30));
c410f=exp(bb139+m210+aa25)/(1+exp(bb4+m21+aa16)+exp(bb19+m22+aa17)+exp(bb34+m23+aa18)
+exp(bb49+m24+aa19)+exp(bb64+m25+aa20)+exp(bb79+m26+aa21)+exp(bb94+m27+aa22)+
exp(bb109+m28+aa23)
+exp(BB124+m29+AA24)+exp(BB139+m210+AA25)+exp(BB154+m211+AA26)+exp(BB169+m212+AA27)
+exp(bb184+m213+aa28)+exp(bb199+m214+aa29)+exp(bb214+m215+aa30));
c411f=exp(bb154+m211+aa26)/(1+exp(bb4+m21+aa16)+exp(bb19+m22+aa17)+exp(bb34+m23+aa18)
+exp(bb49+m24+aa19)+exp(bb64+m25+aa20)+exp(bb79+m26+aa21)+exp(bb94+m27+aa22)+
exp(bb109+m28+aa23)
+exp(BB124+m29+AA24)+exp(BB139+m210+AA25)+exp(BB154+m211+AA26)+exp(BB169+m212+AA27)
+exp(bb184+m213+aa28)+exp(bb199+m214+aa29)+exp(bb214+m215+aa30));
c412f=exp(bb169+m212+aa27)/(1+exp(bb4+m21+aa16)+exp(bb19+m22+aa17)+exp(bb34+m23+aa18)
+exp(bb49+m24+aa19)+exp(bb64+m25+aa20)+exp(bb79+m26+aa21)+exp(bb94+m27+aa22)+
exp(bb109+m28+aa23)
+exp(BB124+m29+AA24)+exp(BB139+m210+AA25)+exp(BB154+m211+AA26)+exp(BB169+m212+AA27)
+exp(bb184+m213+aa28)+exp(bb199+m214+aa29)+exp(bb214+m215+aa30));
c413f=exp(bb184+m213+aa28)/(1+exp(bb4+m21+aa16)+exp(bb19+m22+aa17)+exp(bb34+m23+aa18)
+exp(bb49+m24+aa19)+exp(bb64+m25+aa20)+exp(bb79+m26+aa21)+exp(bb94+m27+aa22)+
exp(bb109+m28+aa23)
+exp(BB124+m29+AA24)+exp(BB139+m210+AA25)+exp(BB154+m211+AA26)+exp(BB169+m212+AA27)
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+exp(bb184+m213+aa28)+exp(bb199+m214+aa29)+exp(bb214+m215+aa30));
c414f=exp(bb199+m214+aa29)/(1+exp(bb4+m21+aa16)+exp(bb19+m22+aa17)+exp(bb34+m23+aa18)
+exp(bb49+m24+aa19)+exp(bb64+m25+aa20)+exp(bb79+m26+aa21)+exp(bb94+m27+aa22)+
exp(bb109+m28+aa23)
+exp(BB124+m29+AA24)+exp(BB139+m210+AA25)+exp(BB154+m211+AA26)+exp(BB169+m212+AA27)
+exp(bb184+m213+aa28)+exp(bb199+m214+aa29)+exp(bb214+m215+aa30));
c415f=exp(bb214+m215+aa30)/(1+exp(bb4+m21+aa16)+exp(bb19+m22+aa17)+exp(bb34+m23+aa18)
+exp(bb49+m24+aa19)+exp(bb64+m25+aa20)+exp(bb79+m26+aa21)+exp(bb94+m27+aa22)+
exp(bb109+m28+aa23)
+exp(BB124+m29+AA24)+exp(BB139+m210+AA25)+exp(BB154+m211+AA26)+exp(BB169+m212+AA27)
+exp(bb184+m213+aa28)+exp(bb199+m214+aa29)+exp(bb214+m215+aa30));
c416f=1/(1+exp(bb4+m21+aa16)+exp(bb19+m22+aa17)+exp(bb34+m23+aa18)
+exp(bb49+m24+aa19)+exp(bb64+m25+aa20)+exp(bb79+m26+aa21)+exp(bb94+m27+aa22)+
exp(bb109+m28+aa23)
+exp(BB124+m29+AA24)+exp(BB139+m210+AA25)+exp(BB154+m211+AA26)+exp(BB169+m212+AA27)
+exp(bb184+m213+aa28)+exp(bb199+m214+aa29)+exp(bb214+m215+aa30));

!row 5
c51h=exp(bb5+m21+aa16)/(1+exp(bb5+m21+aa16)+exp(bb20+m22+aa17)+exp(bb35+m23+aa18)
+exp(BB50+m24+AA19)+exp(BB65+m25+AA20)+exp(BB80+m26+AA21)+exp(BB95+m27+AA22)
+exp(BB110+m28+AA23)+exp(BB125+m29+AA24)+exp(BB140+m210+AA25)+exp(BB155+m211+AA26)
+exp(bb170+m212+aa27)+exp(bb185+m213+aa28)+exp(bb200+m214+aa29)+exp(bb215+m215+aa30));
c52h=exp(bb20+m22+aa17)/(1+exp(bb5+m21+aa16)+exp(bb20+m22+aa17)+exp(bb35+m23+aa18)
+exp(BB50+m24+AA19)+exp(BB65+m25+AA20)+exp(BB80+m26+AA21)+exp(BB95+m27+AA22)
+exp(BB110+m28+AA23)+exp(BB125+m29+AA24)+exp(BB140+m210+AA25)+exp(BB155+m211+AA26)
+exp(bb170+m212+aa27)+exp(bb185+m213+aa28)+exp(bb200+m214+aa29)+exp(bb215+m215+aa30));
c53h=exp(bb35+m23+aa18)/(1+exp(bb5+m21+aa16)+exp(bb20+m22+aa17)+exp(bb35+m23+aa18)
+exp(BB50+m24+AA19)+exp(BB65+m25+AA20)+exp(BB80+m26+AA21)+exp(BB95+m27+AA22)
+exp(BB110+m28+AA23)+exp(BB125+m29+AA24)+exp(BB140+m210+AA25)+exp(BB155+m211+AA26)
+exp(bb170+m212+aa27)+exp(bb185+m213+aa28)+exp(bb200+m214+aa29)+exp(bb215+m215+aa30));
c54h=exp(bb50+m24+aa19)/(1+exp(bb5+m21+aa16)+exp(bb20+m22+aa17)+exp(bb35+m23+aa18)
+exp(BB50+m24+AA19)+exp(BB65+m25+AA20)+exp(BB80+m26+AA21)+exp(BB95+m27+AA22)
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+exp(BB110+m28+AA23)+exp(BB125+m29+AA24)+exp(BB140+m210+AA25)+exp(BB155+m211+AA26)
+exp(bb170+m212+aa27)+exp(bb185+m213+aa28)+exp(bb200+m214+aa29)+exp(bb215+m215+aa30));
c55h=exp(bb65+m25+aa20)/(1+exp(bb5+m21+aa16)+exp(bb20+m22+aa17)+exp(bb35+m23+aa18)
+exp(BB50+m24+AA19)+exp(BB65+m25+AA20)+exp(BB80+m26+AA21)+exp(BB95+m27+AA22)
+exp(BB110+m28+AA23)+exp(BB125+m29+AA24)+exp(BB140+m210+AA25)+exp(BB155+m211+AA26)
+exp(bb170+m212+aa27)+exp(bb185+m213+aa28)+exp(bb200+m214+aa29)+exp(bb215+m215+aa30));
c56h=exp(bb80+m26+aa21)/(1+exp(bb5+m21+aa16)+exp(bb20+m22+aa17)+exp(bb35+m23+aa18)
+exp(BB50+m24+AA19)+exp(BB65+m25+AA20)+exp(BB80+m26+AA21)+exp(BB95+m27+AA22)
+exp(BB110+m28+AA23)+exp(BB125+m29+AA24)+exp(BB140+m210+AA25)+exp(BB155+m211+AA26)
+exp(bb170+m212+aa27)+exp(bb185+m213+aa28)+exp(bb200+m214+aa29)+exp(bb215+m215+aa30));
c57h=exp(bb95+m27+aa22)/(1+exp(bb5+m21+aa16)+exp(bb20+m22+aa17)+exp(bb35+m23+aa18)
+exp(BB50+m24+AA19)+exp(BB65+m25+AA20)+exp(BB80+m26+AA21)+exp(BB95+m27+AA22)
+exp(BB110+m28+AA23)+exp(BB125+m29+AA24)+exp(BB140+m210+AA25)+exp(BB155+m211+AA26)
+exp(bb170+m212+aa27)+exp(bb185+m213+aa28)+exp(bb200+m214+aa29)+exp(bb215+m215+aa30));
c58h=exp(bb110+m28+aa23)/(1+exp(bb5+m21+aa16)+exp(bb20+m22+aa17)+exp(bb35+m23+aa18)
+exp(BB50+m24+AA19)+exp(BB65+m25+AA20)+exp(BB80+m26+AA21)+exp(BB95+m27+AA22)
+exp(BB110+m28+AA23)+exp(BB125+m29+AA24)+exp(BB140+m210+AA25)+exp(BB155+m211+AA26)
+exp(bb170+m212+aa27)+exp(bb185+m213+aa28)+exp(bb200+m214+aa29)+exp(bb215+m215+aa30));
c59h=exp(bb125+m29+aa24)/(1+exp(bb5+m21+aa16)+exp(bb20+m22+aa17)+exp(bb35+m23+aa18)
+exp(BB50+m24+AA19)+exp(BB65+m25+AA20)+exp(BB80+m26+AA21)+exp(BB95+m27+AA22)
+exp(BB110+m28+AA23)+exp(BB125+m29+AA24)+exp(BB140+m210+AA25)+exp(BB155+m211+AA26)
+exp(bb170+m212+aa27)+exp(bb185+m213+aa28)+exp(bb200+m214+aa29)+exp(bb215+m215+aa30));
c510h=exp(bb140+m210+aa25)/(1+exp(bb5+m21+aa16)+exp(bb20+m22+aa17)+exp(bb35+m23+aa18)
+exp(BB50+m24+AA19)+exp(BB65+m25+AA20)+exp(BB80+m26+AA21)+exp(BB95+m27+AA22)
+exp(BB110+m28+AA23)+exp(BB125+m29+AA24)+exp(BB140+m210+AA25)+exp(BB155+m211+AA26)
+exp(bb170+m212+aa27)+exp(bb185+m213+aa28)+exp(bb200+m214+aa29)+exp(bb215+m215+aa30));
c511h=exp(bb155+m211+aa26)/(1+exp(bb5+m21+aa16)+exp(bb20+m22+aa17)+exp(bb35+m23+aa18)
+exp(BB50+m24+AA19)+exp(BB65+m25+AA20)+exp(BB80+m26+AA21)+exp(BB95+m27+AA22)
+exp(BB110+m28+AA23)+exp(BB125+m29+AA24)+exp(BB140+m210+AA25)+exp(BB155+m211+AA26)
+exp(bb170+m212+aa27)+exp(bb185+m213+aa28)+exp(bb200+m214+aa29)+exp(bb215+m215+aa30));
c512h=exp(bb170+m212+aa27)/(1+exp(bb5+m21+aa16)+exp(bb20+m22+aa17)+exp(bb35+m23+aa18)
+exp(BB50+m24+AA19)+exp(BB65+m25+AA20)+exp(BB80+m26+AA21)+exp(BB95+m27+AA22)

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+exp(BB110+m28+AA23)+exp(BB125+m29+AA24)+exp(BB140+m210+AA25)+exp(BB155+m211+AA26)
+exp(bb170+m212+aa27)+exp(bb185+m213+aa28)+exp(bb200+m214+aa29)+exp(bb215+m215+aa30));
c513h=exp(bb185+m213+aa28)/(1+exp(bb5+m21+aa16)+exp(bb20+m22+aa17)+exp(bb35+m23+aa18)
+exp(BB50+m24+AA19)+exp(BB65+m25+AA20)+exp(BB80+m26+AA21)+exp(BB95+m27+AA22)
+exp(BB110+m28+AA23)+exp(BB125+m29+AA24)+exp(BB140+m210+AA25)+exp(BB155+m211+AA26)
+exp(bb170+m212+aa27)+exp(bb185+m213+aa28)+exp(bb200+m214+aa29)+exp(bb215+m215+aa30));
c514h=exp(bb200+m214+aa29)/(1+exp(bb5+m21+aa16)+exp(bb20+m22+aa17)+exp(bb35+m23+aa18)
+exp(BB50+m24+AA19)+exp(BB65+m25+AA20)+exp(BB80+m26+AA21)+exp(BB95+m27+AA22)
+exp(BB110+m28+AA23)+exp(BB125+m29+AA24)+exp(BB140+m210+AA25)+exp(BB155+m211+AA26)
+exp(bb170+m212+aa27)+exp(bb185+m213+aa28)+exp(bb200+m214+aa29)+exp(bb215+m215+aa30));
c515h=exp(bb215+m215+aa30)/(1+exp(bb5+m21+aa16)+exp(bb20+m22+aa17)+exp(bb35+m23+aa18)
+exp(BB50+m24+AA19)+exp(BB65+m25+AA20)+exp(BB80+m26+AA21)+exp(BB95+m27+AA22)
+exp(BB110+m28+AA23)+exp(BB125+m29+AA24)+exp(BB140+m210+AA25)+exp(BB155+m211+AA26)
+exp(bb170+m212+aa27)+exp(bb185+m213+aa28)+exp(bb200+m214+aa29)+exp(bb215+m215+aa30));
c516h=1/(1+exp(bb5+m21+aa16)+exp(bb20+m22+aa17)+exp(bb35+m23+aa18)
+exp(BB50+m24+AA19)+exp(BB65+m25+AA20)+exp(BB80+m26+AA21)+exp(BB95+m27+AA22)
+exp(BB110+m28+AA23)+exp(BB125+m29+AA24)+exp(BB140+m210+AA25)+exp(BB155+m211+AA26)
+exp(bb170+m212+aa27)+exp(bb185+m213+aa28)+exp(bb200+m214+aa29)+exp(bb215+m215+aa30));

!row 6
c61i=exp(bb6+m21+aa16)/(1+exp(bb6+m21+aa16)+exp(bb21+m22+aa17)+exp(bb36+m23+aa18)
+exp(BB51+m24+AA19)+exp(BB66+m25+AA20)+exp(BB81+m26+AA21)+exp(BB96+m27+AA22)
+exp(BB111+m28+AA23)+exp(BB126+m29+AA24)+exp(BB141+m210+AA25)+exp(BB156+m211+AA26)
+exp(bb171+m212+aa27)+exp(bb186+m213+aa28)+exp(bb201+m214+aa29)+exp(bb216+m215+aa30));
c62i=exp(bb21+m22+aa17)/(1+exp(bb6+m21+aa16)+exp(bb21+m22+aa17)+exp(bb36+m23+aa18)
+exp(BB51+m24+AA19)+exp(BB66+m25+AA20)+exp(BB81+m26+AA21)+exp(BB96+m27+AA22)
+exp(BB111+m28+AA23)+exp(BB126+m29+AA24)+exp(BB141+m210+AA25)+exp(BB156+m211+AA26)
+exp(bb171+m212+aa27)+exp(bb186+m213+aa28)+exp(bb201+m214+aa29)+exp(bb216+m215+aa30));
c63i=exp(bb36+m23+aa18)/(1+exp(bb6+m21+aa16)+exp(bb21+m22+aa17)+exp(bb36+m23+aa18)
+exp(BB51+m24+AA19)+exp(BB66+m25+AA20)+exp(BB81+m26+AA21)+exp(BB96+m27+AA22)
+exp(BB111+m28+AA23)+exp(BB126+m29+AA24)+exp(BB141+m210+AA25)+exp(BB156+m211+AA26)
+exp(bb171+m212+aa27)+exp(bb186+m213+aa28)+exp(bb201+m214+aa29)+exp(bb216+m215+aa30));
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c64i=exp(bb51+m24+aa19)/(1+exp(bb6+m21+aa16)+exp(bb21+m22+aa17)+exp(bb36+m23+aa18)
+exp(BB51+m24+AA19)+exp(BB66+m25+AA20)+exp(BB81+m26+AA21)+exp(BB96+m27+AA22)
+exp(BB111+m28+AA23)+exp(BB126+m29+AA24)+exp(BB141+m210+AA25)+exp(BB156+m211+AA26)
+exp(bb171+m212+aa27)+exp(bb186+m213+aa28)+exp(bb201+m214+aa29)+exp(bb216+m215+aa30));
c65i=exp(bb66+m25+aa20)/(1+exp(bb6+m21+aa16)+exp(bb21+m22+aa17)+exp(bb36+m23+aa18)
+exp(BB51+m24+AA19)+exp(BB66+m25+AA20)+exp(BB81+m26+AA21)+exp(BB96+m27+AA22)
+exp(BB111+m28+AA23)+exp(BB126+m29+AA24)+exp(BB141+m210+AA25)+exp(BB156+m211+AA26)
+exp(bb171+m212+aa27)+exp(bb186+m213+aa28)+exp(bb201+m214+aa29)+exp(bb216+m215+aa30));
c66i=exp(bb81+m26+aa21)/(1+exp(bb6+m21+aa16)+exp(bb21+m22+aa17)+exp(bb36+m23+aa18)
+exp(BB51+m24+AA19)+exp(BB66+m25+AA20)+exp(BB81+m26+AA21)+exp(BB96+m27+AA22)
+exp(BB111+m28+AA23)+exp(BB126+m29+AA24)+exp(BB141+m210+AA25)+exp(BB156+m211+AA26)
+exp(bb171+m212+aa27)+exp(bb186+m213+aa28)+exp(bb201+m214+aa29)+exp(bb216+m215+aa30));
c67i=exp(bb96+m27+aa22)/(1+exp(bb6+m21+aa16)+exp(bb21+m22+aa17)+exp(bb36+m23+aa18)
+exp(BB51+m24+AA19)+exp(BB66+m25+AA20)+exp(BB81+m26+AA21)+exp(BB96+m27+AA22)
+exp(BB111+m28+AA23)+exp(BB126+m29+AA24)+exp(BB141+m210+AA25)+exp(BB156+m211+AA26)
+exp(bb171+m212+aa27)+exp(bb186+m213+aa28)+exp(bb201+m214+aa29)+exp(bb216+m215+aa30));
c68i=exp(bb111+m28+aa23)/(1+exp(bb6+m21+aa16)+exp(bb21+m22+aa17)+exp(bb36+m23+aa18)
+exp(BB51+m24+AA19)+exp(BB66+m25+AA20)+exp(BB81+m26+AA21)+exp(BB96+m27+AA22)
+exp(BB111+m28+AA23)+exp(BB126+m29+AA24)+exp(BB141+m210+AA25)+exp(BB156+m211+AA26)
+exp(bb171+m212+aa27)+exp(bb186+m213+aa28)+exp(bb201+m214+aa29)+exp(bb216+m215+aa30));
c69i=exp(bb126+m29+aa24)/(1+exp(bb6+m21+aa16)+exp(bb21+m22+aa17)+exp(bb36+m23+aa18)
+exp(BB51+m24+AA19)+exp(BB66+m25+AA20)+exp(BB81+m26+AA21)+exp(BB96+m27+AA22)
+exp(BB111+m28+AA23)+exp(BB126+m29+AA24)+exp(BB141+m210+AA25)+exp(BB156+m211+AA26)
+exp(bb171+m212+aa27)+exp(bb186+m213+aa28)+exp(bb201+m214+aa29)+exp(bb216+m215+aa30));
c610i=exp(bb141+m210+aa25)/(1+exp(bb6+m21+aa16)+exp(bb21+m22+aa17)+exp(bb36+m23+aa18)
+exp(BB51+m24+AA19)+exp(BB66+m25+AA20)+exp(BB81+m26+AA21)+exp(BB96+m27+AA22)
+exp(BB111+m28+AA23)+exp(BB126+m29+AA24)+exp(BB141+m210+AA25)+exp(BB156+m211+AA26)
+exp(bb171+m212+aa27)+exp(bb186+m213+aa28)+exp(bb201+m214+aa29)+exp(bb216+m215+aa30));
c611i=exp(bb156+m211+aa26)/(1+exp(bb6+m21+aa16)+exp(bb21+m22+aa17)+exp(bb36+m23+aa18)
+exp(BB51+m24+AA19)+exp(BB66+m25+AA20)+exp(BB81+m26+AA21)+exp(BB96+m27+AA22)
+exp(BB111+m28+AA23)+exp(BB126+m29+AA24)+exp(BB141+m210+AA25)+exp(BB156+m211+AA26)
+exp(bb171+m212+aa27)+exp(bb186+m213+aa28)+exp(bb201+m214+aa29)+exp(bb216+m215+aa30));
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c612i=exp(bb171+m212+aa27)/(1+exp(bb6+m21+aa16)+exp(bb21+m22+aa17)+exp(bb36+m23+aa18)
+exp(BB51+m24+AA19)+exp(BB66+m25+AA20)+exp(BB81+m26+AA21)+exp(BB96+m27+AA22)
+exp(BB111+m28+AA23)+exp(BB126+m29+AA24)+exp(BB141+m210+AA25)+exp(BB156+m211+AA26)
+exp(bb171+m212+aa27)+exp(bb186+m213+aa28)+exp(bb201+m214+aa29)+exp(bb216+m215+aa30));
c613i=exp(bb186+m213+aa28)/(1+exp(bb6+m21+aa16)+exp(bb21+m22+aa17)+exp(bb36+m23+aa18)
+exp(BB51+m24+AA19)+exp(BB66+m25+AA20)+exp(BB81+m26+AA21)+exp(BB96+m27+AA22)
+exp(BB111+m28+AA23)+exp(BB126+m29+AA24)+exp(BB141+m210+AA25)+exp(BB156+m211+AA26)
+exp(bb171+m212+aa27)+exp(bb186+m213+aa28)+exp(bb201+m214+aa29)+exp(bb216+m215+aa30));
c614i=exp(bb201+m214+aa29)/(1+exp(bb6+m21+aa16)+exp(bb21+m22+aa17)+exp(bb36+m23+aa18)
+exp(BB51+m24+AA19)+exp(BB66+m25+AA20)+exp(BB81+m26+AA21)+exp(BB96+m27+AA22)
+exp(BB111+m28+AA23)+exp(BB126+m29+AA24)+exp(BB141+m210+AA25)+exp(BB156+m211+AA26)
+exp(bb171+m212+aa27)+exp(bb186+m213+aa28)+exp(bb201+m214+aa29)+exp(bb216+m215+aa30));
c615i=exp(bb216+m215+aa30)/(1+exp(bb6+m21+aa16)+exp(bb21+m22+aa17)+exp(bb36+m23+aa18)
+exp(BB51+m24+AA19)+exp(BB66+m25+AA20)+exp(BB81+m26+AA21)+exp(BB96+m27+AA22)
+exp(BB111+m28+AA23)+exp(BB126+m29+AA24)+exp(BB141+m210+AA25)+exp(BB156+m211+AA26)
+exp(bb171+m212+aa27)+exp(bb186+m213+aa28)+exp(bb201+m214+aa29)+exp(bb216+m215+aa30));
c616i=1/(1+exp(bb6+m21+aa16)+exp(bb21+m22+aa17)+exp(bb36+m23+aa18)
+exp(BB51+m24+AA19)+exp(BB66+m25+AA20)+exp(BB81+m26+AA21)+exp(BB96+m27+AA22)
+exp(BB111+m28+AA23)+exp(BB126+m29+AA24)+exp(BB141+m210+AA25)+exp(BB156+m211+AA26)
+exp(bb171+m212+aa27)+exp(bb186+m213+aa28)+exp(bb201+m214+aa29)+exp(bb216+m215+aa30));

!row 7
c71j=exp(bb7+m21+aa16)/(1+exp(bb7+m21+aa16)+exp(bb22+m22+aa17)+exp(bb37+m23+aa18)
+exp(BB52+m24+AA19)+exp(BB67+m25+AA20)+exp(BB82+m26+AA21)+exp(BB97+m27+AA22)
+exp(BB112+m28+AA23)+exp(BB127+m29+AA24)+exp(BB142+m210+AA25)+exp(BB157+m211+AA26)
+exp(bb172+m212+aa27)+exp(bb187+m213+aa28)+exp(bb202+m214+aa29)+exp(bb217+m215+aa30));
c72j=exp(bb22+m22+aa17)/(1+exp(bb7+m21+aa16)+exp(bb22+m22+aa17)+exp(bb37+m23+aa18)
+exp(BB52+m24+AA19)+exp(BB67+m25+AA20)+exp(BB82+m26+AA21)+exp(BB97+m27+AA22)
+exp(BB112+m28+AA23)+exp(BB127+m29+AA24)+exp(BB142+m210+AA25)+exp(BB157+m211+AA26)
+exp(bb172+m212+aa27)+exp(bb187+m213+aa28)+exp(bb202+m214+aa29)+exp(bb217+m215+aa30));
c73j=exp(bb37+m23+aa18)/(1+exp(bb7+m21+aa16)+exp(bb22+m22+aa17)+exp(bb37+m23+aa18)
+exp(BB52+m24+AA19)+exp(BB67+m25+AA20)+exp(BB82+m26+AA21)+exp(BB97+m27+AA22)
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+exp(BB112+m28+AA23)+exp(BB127+m29+AA24)+exp(BB142+m210+AA25)+exp(BB157+m211+AA26)
+exp(bb172+m212+aa27)+exp(bb187+m213+aa28)+exp(bb202+m214+aa29)+exp(bb217+m215+aa30));
c74j=exp(bb52+m24+aa19)/(1+exp(bb7+m21+aa16)+exp(bb22+m22+aa17)+exp(bb37+m23+aa18)
+exp(BB52+m24+AA19)+exp(BB67+m25+AA20)+exp(BB82+m26+AA21)+exp(BB97+m27+AA22)
+exp(BB112+m28+AA23)+exp(BB127+m29+AA24)+exp(BB142+m210+AA25)+exp(BB157+m211+AA26)
+exp(bb172+m212+aa27)+exp(bb187+m213+aa28)+exp(bb202+m214+aa29)+exp(bb217+m215+aa30));
c75j=exp(bb67+m25+aa20)/(1+exp(bb7+m21+aa16)+exp(bb22+m22+aa17)+exp(bb37+m23+aa18)
+exp(BB52+m24+AA19)+exp(BB67+m25+AA20)+exp(BB82+m26+AA21)+exp(BB97+m27+AA22)
+exp(BB112+m28+AA23)+exp(BB127+m29+AA24)+exp(BB142+m210+AA25)+exp(BB157+m211+AA26)
+exp(bb172+m212+aa27)+exp(bb187+m213+aa28)+exp(bb202+m214+aa29)+exp(bb217+m215+aa30));
c76j=exp(bb82+m26+aa21)/(1+exp(bb7+m21+aa16)+exp(bb22+m22+aa17)+exp(bb37+m23+aa18)
+exp(BB52+m24+AA19)+exp(BB67+m25+AA20)+exp(BB82+m26+AA21)+exp(BB97+m27+AA22)
+exp(BB112+m28+AA23)+exp(BB127+m29+AA24)+exp(BB142+m210+AA25)+exp(BB157+m211+AA26)
+exp(bb172+m212+aa27)+exp(bb187+m213+aa28)+exp(bb202+m214+aa29)+exp(bb217+m215+aa30));
c77j=exp(bb97+m27+aa22)/(1+exp(bb7+m21+aa16)+exp(bb22+m22+aa17)+exp(bb37+m23+aa18)
+exp(BB52+m24+AA19)+exp(BB67+m25+AA20)+exp(BB82+m26+AA21)+exp(BB97+m27+AA22)
+exp(BB112+m28+AA23)+exp(BB127+m29+AA24)+exp(BB142+m210+AA25)+exp(BB157+m211+AA26)
+exp(bb172+m212+aa27)+exp(bb187+m213+aa28)+exp(bb202+m214+aa29)+exp(bb217+m215+aa30));
c78j=exp(bb112+m28+aa23)/(1+exp(bb7+m21+aa16)+exp(bb22+m22+aa17)+exp(bb37+m23+aa18)
+exp(BB52+m24+AA19)+exp(BB67+m25+AA20)+exp(BB82+m26+AA21)+exp(BB97+m27+AA22)
+exp(BB112+m28+AA23)+exp(BB127+m29+AA24)+exp(BB142+m210+AA25)+exp(BB157+m211+AA26)
+exp(bb172+m212+aa27)+exp(bb187+m213+aa28)+exp(bb202+m214+aa29)+exp(bb217+m215+aa30));
c79j=exp(bb127+m29+aa24)/(1+exp(bb7+m21+aa16)+exp(bb22+m22+aa17)+exp(bb37+m23+aa18)
+exp(BB52+m24+AA19)+exp(BB67+m25+AA20)+exp(BB82+m26+AA21)+exp(BB97+m27+AA22)
+exp(BB112+m28+AA23)+exp(BB127+m29+AA24)+exp(BB142+m210+AA25)+exp(BB157+m211+AA26)
+exp(bb172+m212+aa27)+exp(bb187+m213+aa28)+exp(bb202+m214+aa29)+exp(bb217+m215+aa30));
c710j=exp(bb142+m210+aa25)/(1+exp(bb7+m21+aa16)+exp(bb22+m22+aa17)+exp(bb37+m23+aa18)
+exp(BB52+m24+AA19)+exp(BB67+m25+AA20)+exp(BB82+m26+AA21)+exp(BB97+m27+AA22)
+exp(BB112+m28+AA23)+exp(BB127+m29+AA24)+exp(BB142+m210+AA25)+exp(BB157+m211+AA26)
+exp(bb172+m212+aa27)+exp(bb187+m213+aa28)+exp(bb202+m214+aa29)+exp(bb217+m215+aa30));
c711j=exp(bb157+m211+aa26)/(1+exp(bb7+m21+aa16)+exp(bb22+m22+aa17)+exp(bb37+m23+aa18)
+exp(BB52+m24+AA19)+exp(BB67+m25+AA20)+exp(BB82+m26+AA21)+exp(BB97+m27+AA22)

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+exp(BB112+m28+AA23)+exp(BB127+m29+AA24)+exp(BB142+m210+AA25)+exp(BB157+m211+AA26)
+exp(bb172+m212+aa27)+exp(bb187+m213+aa28)+exp(bb202+m214+aa29)+exp(bb217+m215+aa30));
c712j=exp(bb172+m212+aa27)/(1+exp(bb7+m21+aa16)+exp(bb22+m22+aa17)+exp(bb37+m23+aa18)
+exp(BB52+m24+AA19)+exp(BB67+m25+AA20)+exp(BB82+m26+AA21)+exp(BB97+m27+AA22)
+exp(BB112+m28+AA23)+exp(BB127+m29+AA24)+exp(BB142+m210+AA25)+exp(BB157+m211+AA26)
+exp(bb172+m212+aa27)+exp(bb187+m213+aa28)+exp(bb202+m214+aa29)+exp(bb217+m215+aa30));
c713j=exp(bb187+m213+aa28)/(1+exp(bb7+m21+aa16)+exp(bb22+m22+aa17)+exp(bb37+m23+aa18)
+exp(BB52+m24+AA19)+exp(BB67+m25+AA20)+exp(BB82+m26+AA21)+exp(BB97+m27+AA22)
+exp(BB112+m28+AA23)+exp(BB127+m29+AA24)+exp(BB142+m210+AA25)+exp(BB157+m211+AA26)
+exp(bb172+m212+aa27)+exp(bb187+m213+aa28)+exp(bb202+m214+aa29)+exp(bb217+m215+aa30));
c714j=exp(bb202+m214+aa29)/(1+exp(bb7+m21+aa16)+exp(bb22+m22+aa17)+exp(bb37+m23+aa18)
+exp(BB52+m24+AA19)+exp(BB67+m25+AA20)+exp(BB82+m26+AA21)+exp(BB97+m27+AA22)
+exp(BB112+m28+AA23)+exp(BB127+m29+AA24)+exp(BB142+m210+AA25)+exp(BB157+m211+AA26)
+exp(bb172+m212+aa27)+exp(bb187+m213+aa28)+exp(bb202+m214+aa29)+exp(bb217+m215+aa30));
c715j=exp(bb217+m215+aa30)/(1+exp(bb7+m21+aa16)+exp(bb22+m22+aa17)+exp(bb37+m23+aa18)
+exp(BB52+m24+AA19)+exp(BB67+m25+AA20)+exp(BB82+m26+AA21)+exp(BB97+m27+AA22)
+exp(BB112+m28+AA23)+exp(BB127+m29+AA24)+exp(BB142+m210+AA25)+exp(BB157+m211+AA26)
+exp(bb172+m212+aa27)+exp(bb187+m213+aa28)+exp(bb202+m214+aa29)+exp(bb217+m215+aa30));
c716j=1/(1+exp(bb7+m21+aa16)+exp(bb22+m22+aa17)+exp(bb37+m23+aa18)
+exp(BB52+m24+AA19)+exp(BB67+m25+AA20)+exp(BB82+m26+AA21)+exp(BB97+m27+AA22)
+exp(BB112+m28+AA23)+exp(BB127+m29+AA24)+exp(BB142+m210+AA25)+exp(BB157+m211+AA26)
+exp(bb172+m212+aa27)+exp(bb187+m213+aa28)+exp(bb202+m214+aa29)+exp(bb217+m215+aa30));

!row 8
c81k=exp(bb8+m21+aa16)/(1+exp(bb8+m21+aa16)+exp(bb23+m22+aa17)+exp(bb38+m23+aa18)
+exp(BB53+m24+AA19)+exp(BB68+m25+AA20)+exp(BB83+m26+AA21)+exp(BB98+m27+AA22)
+exp(BB113+m28+AA23)+exp(BB128+m29+AA24)+exp(BB143+m210+AA25)+exp(BB158+m211+AA26)
+exp(bb173+m212+aa27)+exp(bb188+m213+aa28)+exp(bb203+m214+aa29)+exp(bb218+m215+aa30));
c82k=exp(bb23+m22+aa17)/(1+exp(bb8+m21+aa16)+exp(bb23+m22+aa17)+exp(bb38+m23+aa18)
+exp(BB53+m24+AA19)+exp(BB68+m25+AA20)+exp(BB83+m26+AA21)+exp(BB98+m27+AA22)
+exp(BB113+m28+AA23)+exp(BB128+m29+AA24)+exp(BB143+m210+AA25)+exp(BB158+m211+AA26)
+exp(bb173+m212+aa27)+exp(bb188+m213+aa28)+exp(bb203+m214+aa29)+exp(bb218+m215+aa30));
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c83k=exp(bb38+m23+aa18)/(1+exp(bb8+m21+aa16)+exp(bb23+m22+aa17)+exp(bb38+m23+aa18)
+exp(BB53+m24+AA19)+exp(BB68+m25+AA20)+exp(BB83+m26+AA21)+exp(BB98+m27+AA22)
+exp(BB113+m28+AA23)+exp(BB128+m29+AA24)+exp(BB143+m210+AA25)+exp(BB158+m211+AA26)
+exp(bb173+m212+aa27)+exp(bb188+m213+aa28)+exp(bb203+m214+aa29)+exp(bb218+m215+aa30));
c84k=exp(bb53+m24+aa19)/(1+exp(bb8+m21+aa16)+exp(bb23+m22+aa17)+exp(bb38+m23+aa18)
+exp(BB53+m24+AA19)+exp(BB68+m25+AA20)+exp(BB83+m26+AA21)+exp(BB98+m27+AA22)
+exp(BB113+m28+AA23)+exp(BB128+m29+AA24)+exp(BB143+m210+AA25)+exp(BB158+m211+AA26)
+exp(bb173+m212+aa27)+exp(bb188+m213+aa28)+exp(bb203+m214+aa29)+exp(bb218+m215+aa30));
c85k=exp(bb68+m25+aa20)/(1+exp(bb8+m21+aa16)+exp(bb23+m22+aa17)+exp(bb38+m23+aa18)
+exp(BB53+m24+AA19)+exp(BB68+m25+AA20)+exp(BB83+m26+AA21)+exp(BB98+m27+AA22)
+exp(BB113+m28+AA23)+exp(BB128+m29+AA24)+exp(BB143+m210+AA25)+exp(BB158+m211+AA26)
+exp(bb173+m212+aa27)+exp(bb188+m213+aa28)+exp(bb203+m214+aa29)+exp(bb218+m215+aa30));
c86k=exp(bb83+m26+aa21)/(1+exp(bb8+m21+aa16)+exp(bb23+m22+aa17)+exp(bb38+m23+aa18)
+exp(BB53+m24+AA19)+exp(BB68+m25+AA20)+exp(BB83+m26+AA21)+exp(BB98+m27+AA22)
+exp(BB113+m28+AA23)+exp(BB128+m29+AA24)+exp(BB143+m210+AA25)+exp(BB158+m211+AA26)
+exp(bb173+m212+aa27)+exp(bb188+m213+aa28)+exp(bb203+m214+aa29)+exp(bb218+m215+aa30));
c87k=exp(bb98+m27+aa22)/(1+exp(bb8+m21+aa16)+exp(bb23+m22+aa17)+exp(bb38+m23+aa18)
+exp(BB53+m24+AA19)+exp(BB68+m25+AA20)+exp(BB83+m26+AA21)+exp(BB98+m27+AA22)
+exp(BB113+m28+AA23)+exp(BB128+m29+AA24)+exp(BB143+m210+AA25)+exp(BB158+m211+AA26)
+exp(bb173+m212+aa27)+exp(bb188+m213+aa28)+exp(bb203+m214+aa29)+exp(bb218+m215+aa30));
c88k=exp(bb113+m28+aa23)/(1+exp(bb8+m21+aa16)+exp(bb23+m22+aa17)+exp(bb38+m23+aa18)
+exp(BB53+m24+AA19)+exp(BB68+m25+AA20)+exp(BB83+m26+AA21)+exp(BB98+m27+AA22)
+exp(BB113+m28+AA23)+exp(BB128+m29+AA24)+exp(BB143+m210+AA25)+exp(BB158+m211+AA26)
+exp(bb173+m212+aa27)+exp(bb188+m213+aa28)+exp(bb203+m214+aa29)+exp(bb218+m215+aa30));
c89k=exp(bb128+m29+aa24)/(1+exp(bb8+m21+aa16)+exp(bb23+m22+aa17)+exp(bb38+m23+aa18)
+exp(BB53+m24+AA19)+exp(BB68+m25+AA20)+exp(BB83+m26+AA21)+exp(BB98+m27+AA22)
+exp(BB113+m28+AA23)+exp(BB128+m29+AA24)+exp(BB143+m210+AA25)+exp(BB158+m211+AA26)
+exp(bb173+m212+aa27)+exp(bb188+m213+aa28)+exp(bb203+m214+aa29)+exp(bb218+m215+aa30));
c810k=exp(bb143+m210+aa25)/(1+exp(bb8+m21+aa16)+exp(bb23+m22+aa17)+exp(bb38+m23+aa18)
+exp(BB53+m24+AA19)+exp(BB68+m25+AA20)+exp(BB83+m26+AA21)+exp(BB98+m27+AA22)
+exp(BB113+m28+AA23)+exp(BB128+m29+AA24)+exp(BB143+m210+AA25)+exp(BB158+m211+AA26)
+exp(bb173+m212+aa27)+exp(bb188+m213+aa28)+exp(bb203+m214+aa29)+exp(bb218+m215+aa30));
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c811k=exp(bb158+m211+aa26)/(1+exp(bb8+m21+aa16)+exp(bb23+m22+aa17)+exp(bb38+m23+aa18)
+exp(BB53+m24+AA19)+exp(BB68+m25+AA20)+exp(BB83+m26+AA21)+exp(BB98+m27+AA22)
+exp(BB113+m28+AA23)+exp(BB128+m29+AA24)+exp(BB143+m210+AA25)+exp(BB158+m211+AA26)
+exp(bb173+m212+aa27)+exp(bb188+m213+aa28)+exp(bb203+m214+aa29)+exp(bb218+m215+aa30));
c812k=exp(bb173+m212+aa27)/(1+exp(bb8+m21+aa16)+exp(bb23+m22+aa17)+exp(bb38+m23+aa18)
+exp(BB53+m24+AA19)+exp(BB68+m25+AA20)+exp(BB83+m26+AA21)+exp(BB98+m27+AA22)
+exp(BB113+m28+AA23)+exp(BB128+m29+AA24)+exp(BB143+m210+AA25)+exp(BB158+m211+AA26)
+exp(bb173+m212+aa27)+exp(bb188+m213+aa28)+exp(bb203+m214+aa29)+exp(bb218+m215+aa30));
c813k=exp(bb188+m213+aa28)/(1+exp(bb8+m21+aa16)+exp(bb23+m22+aa17)+exp(bb38+m23+aa18)
+exp(BB53+m24+AA19)+exp(BB68+m25+AA20)+exp(BB83+m26+AA21)+exp(BB98+m27+AA22)
+exp(BB113+m28+AA23)+exp(BB128+m29+AA24)+exp(BB143+m210+AA25)+exp(BB158+m211+AA26)
+exp(bb173+m212+aa27)+exp(bb188+m213+aa28)+exp(bb203+m214+aa29)+exp(bb218+m215+aa30));
c814k=exp(bb203+m214+aa29)/(1+exp(bb8+m21+aa16)+exp(bb23+m22+aa17)+exp(bb38+m23+aa18)
+exp(BB53+m24+AA19)+exp(BB68+m25+AA20)+exp(BB83+m26+AA21)+exp(BB98+m27+AA22)
+exp(BB113+m28+AA23)+exp(BB128+m29+AA24)+exp(BB143+m210+AA25)+exp(BB158+m211+AA26)
+exp(bb173+m212+aa27)+exp(bb188+m213+aa28)+exp(bb203+m214+aa29)+exp(bb218+m215+aa30));
c815k=exp(bb218+m215+aa30)/(1+exp(bb8+m21+aa16)+exp(bb23+m22+aa17)+exp(bb38+m23+aa18)
+exp(BB53+m24+AA19)+exp(BB68+m25+AA20)+exp(BB83+m26+AA21)+exp(BB98+m27+AA22)
+exp(BB113+m28+AA23)+exp(BB128+m29+AA24)+exp(BB143+m210+AA25)+exp(BB158+m211+AA26)
+exp(bb173+m212+aa27)+exp(bb188+m213+aa28)+exp(bb203+m214+aa29)+exp(bb218+m215+aa30));
c816k=1/(1+exp(bb8+m21+aa16)+exp(bb23+m22+aa17)+exp(bb38+m23+aa18)
+exp(BB53+m24+AA19)+exp(BB68+m25+AA20)+exp(BB83+m26+AA21)+exp(BB98+m27+AA22)
+exp(BB113+m28+AA23)+exp(BB128+m29+AA24)+exp(BB143+m210+AA25)+exp(BB158+m211+AA26)
+exp(bb173+m212+aa27)+exp(bb188+m213+aa28)+exp(bb203+m214+aa29)+exp(bb218+m215+aa30));

!row 9
c91n=exp(bb9+m21+aa16)/(1+exp(bb9+m21+aa16)+exp(bb24+m22+aa17)+exp(bb39+m23+aa18)
+exp(BB54+m24+AA19)+exp(BB69+m25+AA20)+exp(BB84+m26+AA21)+exp(BB99+m27+AA22)
+exp(BB114+m28+AA23)+exp(BB129+m29+AA24)+exp(BB144+m210+AA25)+exp(BB159+m211+AA26)
+exp(bb174+m212+aa27)+exp(bb189+m213+aa28)+exp(bb204+m214+aa29)+exp(bb219+m215+aa30));
c92n=exp(bb24+m22+aa17)/(1+exp(bb9+m21+aa16)+exp(bb24+m22+aa17)+exp(bb39+m23+aa18)
+exp(BB54+m24+AA19)+exp(BB69+m25+AA20)+exp(BB84+m26+AA21)+exp(BB99+m27+AA22)
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+exp(BB114+m28+AA23)+exp(BB129+m29+AA24)+exp(BB144+m210+AA25)+exp(BB159+m211+AA26)
+exp(bb174+m212+aa27)+exp(bb189+m213+aa28)+exp(bb204+m214+aa29)+exp(bb219+m215+aa30));
c93n=exp(bb39+m23+aa18)/(1+exp(bb9+m21+aa16)+exp(bb24+m22+aa17)+exp(bb39+m23+aa18)
+exp(BB54+m24+AA19)+exp(BB69+m25+AA20)+exp(BB84+m26+AA21)+exp(BB99+m27+AA22)
+exp(BB114+m28+AA23)+exp(BB129+m29+AA24)+exp(BB144+m210+AA25)+exp(BB159+m211+AA26)
+exp(bb174+m212+aa27)+exp(bb189+m213+aa28)+exp(bb204+m214+aa29)+exp(bb219+m215+aa30));
c94n=exp(bb54+m24+aa19)/(1+exp(bb9+m21+aa16)+exp(bb24+m22+aa17)+exp(bb39+m23+aa18)
+exp(BB54+m24+AA19)+exp(BB69+m25+AA20)+exp(BB84+m26+AA21)+exp(BB99+m27+AA22)
+exp(BB114+m28+AA23)+exp(BB129+m29+AA24)+exp(BB144+m210+AA25)+exp(BB159+m211+AA26)
+exp(bb174+m212+aa27)+exp(bb189+m213+aa28)+exp(bb204+m214+aa29)+exp(bb219+m215+aa30));
c95n=exp(bb69+m25+aa20)/(1+exp(bb9+m21+aa16)+exp(bb24+m22+aa17)+exp(bb39+m23+aa18)
+exp(BB54+m24+AA19)+exp(BB69+m25+AA20)+exp(BB84+m26+AA21)+exp(BB99+m27+AA22)
+exp(BB114+m28+AA23)+exp(BB129+m29+AA24)+exp(BB144+m210+AA25)+exp(BB159+m211+AA26)
+exp(bb174+m212+aa27)+exp(bb189+m213+aa28)+exp(bb204+m214+aa29)+exp(bb219+m215+aa30));
c96n=exp(bb84+m26+aa21)/(1+exp(bb9+m21+aa16)+exp(bb24+m22+aa17)+exp(bb39+m23+aa18)
+exp(BB54+m24+AA19)+exp(BB69+m25+AA20)+exp(BB84+m26+AA21)+exp(BB99+m27+AA22)
+exp(BB114+m28+AA23)+exp(BB129+m29+AA24)+exp(BB144+m210+AA25)+exp(BB159+m211+AA26)
+exp(bb174+m212+aa27)+exp(bb189+m213+aa28)+exp(bb204+m214+aa29)+exp(bb219+m215+aa30));
c97n=exp(bb99+m27+aa22)/(1+exp(bb9+m21+aa16)+exp(bb24+m22+aa17)+exp(bb39+m23+aa18)
+exp(BB54+m24+AA19)+exp(BB69+m25+AA20)+exp(BB84+m26+AA21)+exp(BB99+m27+AA22)
+exp(BB114+m28+AA23)+exp(BB129+m29+AA24)+exp(BB144+m210+AA25)+exp(BB159+m211+AA26)
+exp(bb174+m212+aa27)+exp(bb189+m213+aa28)+exp(bb204+m214+aa29)+exp(bb219+m215+aa30));
c98n=exp(bb114+m28+aa23)/(1+exp(bb9+m21+aa16)+exp(bb24+m22+aa17)+exp(bb39+m23+aa18)
+exp(BB54+m24+AA19)+exp(BB69+m25+AA20)+exp(BB84+m26+AA21)+exp(BB99+m27+AA22)
+exp(BB114+m28+AA23)+exp(BB129+m29+AA24)+exp(BB144+m210+AA25)+exp(BB159+m211+AA26)
+exp(bb174+m212+aa27)+exp(bb189+m213+aa28)+exp(bb204+m214+aa29)+exp(bb219+m215+aa30));
c99n=exp(bb129+m29+aa24)/(1+exp(bb9+m21+aa16)+exp(bb24+m22+aa17)+exp(bb39+m23+aa18)
+exp(BB54+m24+AA19)+exp(BB69+m25+AA20)+exp(BB84+m26+AA21)+exp(BB99+m27+AA22)
+exp(BB114+m28+AA23)+exp(BB129+m29+AA24)+exp(BB144+m210+AA25)+exp(BB159+m211+AA26)
+exp(bb174+m212+aa27)+exp(bb189+m213+aa28)+exp(bb204+m214+aa29)+exp(bb219+m215+aa30));
c910n=exp(bb144+m210+aa25)/(1+exp(bb9+m21+aa16)+exp(bb24+m22+aa17)+exp(bb39+m23+aa18)
+exp(BB54+m24+AA19)+exp(BB69+m25+AA20)+exp(BB84+m26+AA21)+exp(BB99+m27+AA22)

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+exp(BB114+m28+AA23)+exp(BB129+m29+AA24)+exp(BB144+m210+AA25)+exp(BB159+m211+AA26)
+exp(bb174+m212+aa27)+exp(bb189+m213+aa28)+exp(bb204+m214+aa29)+exp(bb219+m215+aa30));
c911n=exp(bb159+m211+aa26)/(1+exp(bb9+m21+aa16)+exp(bb24+m22+aa17)+exp(bb39+m23+aa18)
+exp(BB54+m24+AA19)+exp(BB69+m25+AA20)+exp(BB84+m26+AA21)+exp(BB99+m27+AA22)
+exp(BB114+m28+AA23)+exp(BB129+m29+AA24)+exp(BB144+m210+AA25)+exp(BB159+m211+AA26)
+exp(bb174+m212+aa27)+exp(bb189+m213+aa28)+exp(bb204+m214+aa29)+exp(bb219+m215+aa30));
c912n=exp(bb174+m212+aa27)/(1+exp(bb9+m21+aa16)+exp(bb24+m22+aa17)+exp(bb39+m23+aa18)
+exp(BB54+m24+AA19)+exp(BB69+m25+AA20)+exp(BB84+m26+AA21)+exp(BB99+m27+AA22)
+exp(BB114+m28+AA23)+exp(BB129+m29+AA24)+exp(BB144+m210+AA25)+exp(BB159+m211+AA26)
+exp(bb174+m212+aa27)+exp(bb189+m213+aa28)+exp(bb204+m214+aa29)+exp(bb219+m215+aa30));
c913n=exp(bb189+m213+aa28)/(1+exp(bb9+m21+aa16)+exp(bb24+m22+aa17)+exp(bb39+m23+aa18)
+exp(BB54+m24+AA19)+exp(BB69+m25+AA20)+exp(BB84+m26+AA21)+exp(BB99+m27+AA22)
+exp(BB114+m28+AA23)+exp(BB129+m29+AA24)+exp(BB144+m210+AA25)+exp(BB159+m211+AA26)
+exp(bb174+m212+aa27)+exp(bb189+m213+aa28)+exp(bb204+m214+aa29)+exp(bb219+m215+aa30));
c914n=exp(bb204+m214+aa29)/(1+exp(bb9+m21+aa16)+exp(bb24+m22+aa17)+exp(bb39+m23+aa18)
+exp(BB54+m24+AA19)+exp(BB69+m25+AA20)+exp(BB84+m26+AA21)+exp(BB99+m27+AA22)
+exp(BB114+m28+AA23)+exp(BB129+m29+AA24)+exp(BB144+m210+AA25)+exp(BB159+m211+AA26)
+exp(bb174+m212+aa27)+exp(bb189+m213+aa28)+exp(bb204+m214+aa29)+exp(bb219+m215+aa30));
c915n=exp(bb219+m215+aa30)/(1+exp(bb9+m21+aa16)+exp(bb24+m22+aa17)+exp(bb39+m23+aa18)
+exp(BB54+m24+AA19)+exp(BB69+m25+AA20)+exp(BB84+m26+AA21)+exp(BB99+m27+AA22)
+exp(BB114+m28+AA23)+exp(BB129+m29+AA24)+exp(BB144+m210+AA25)+exp(BB159+m211+AA26)
+exp(bb174+m212+aa27)+exp(bb189+m213+aa28)+exp(bb204+m214+aa29)+exp(bb219+m215+aa30));
c916n=1/(1+exp(bb9+m21+aa16)+exp(bb24+m22+aa17)+exp(bb39+m23+aa18)
+exp(BB54+m24+AA19)+exp(BB69+m25+AA20)+exp(BB84+m26+AA21)+exp(BB99+m27+AA22)
+exp(BB114+m28+AA23)+exp(BB129+m29+AA24)+exp(BB144+m210+AA25)+exp(BB159+m211+AA26)
+exp(bb174+m212+aa27)+exp(bb189+m213+aa28)+exp(bb204+m214+aa29)+exp(bb219+m215+aa30));

!row 10
c101o=exp(bb10+m21+aa16)/(1+exp(bb10+m21+aa16)+exp(bb25+m22+aa17)+exp(bb40+m23+aa18)
+exp(BB55+m24+AA19)+exp(BB70+m25+AA20)+exp(BB85+m26+AA21)+exp(BB100+m27+AA22)
+exp(BB115+m28+AA23)+exp(BB130+m29+AA24)+exp(BB145+m210+AA25)+exp(BB160+m211+AA26)
+exp(bb175+m212+aa27)+exp(bb190+m213+aa28)+exp(bb205+m214+aa29)+exp(bb220+m215+aa30));
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c102o=exp(bb25+m22+aa17)/(1+exp(bb10+m21+aa16)+exp(bb25+m22+aa17)+exp(bb40+m23+aa18)
+exp(BB55+m24+AA19)+exp(BB70+m25+AA20)+exp(BB85+m26+AA21)+exp(BB100+m27+AA22)
+exp(BB115+m28+AA23)+exp(BB130+m29+AA24)+exp(BB145+m210+AA25)+exp(BB160+m211+AA26)
+exp(bb175+m212+aa27)+exp(bb190+m213+aa28)+exp(bb205+m214+aa29)+exp(bb220+m215+aa30));
c103o=exp(bb40+m23+aa18)/(1+exp(bb10+m21+aa16)+exp(bb25+m22+aa17)+exp(bb40+m23+aa18)
+exp(BB55+m24+AA19)+exp(BB70+m25+AA20)+exp(BB85+m26+AA21)+exp(BB100+m27+AA22)
+exp(BB115+m28+AA23)+exp(BB130+m29+AA24)+exp(BB145+m210+AA25)+exp(BB160+m211+AA26)
+exp(bb175+m212+aa27)+exp(bb190+m213+aa28)+exp(bb205+m214+aa29)+exp(bb220+m215+aa30));
c104o=exp(bb55+m24+aa19)/(1+exp(bb10+m21+aa16)+exp(bb25+m22+aa17)+exp(bb40+m23+aa18)
+exp(BB55+m24+AA19)+exp(BB70+m25+AA20)+exp(BB85+m26+AA21)+exp(BB100+m27+AA22)
+exp(BB115+m28+AA23)+exp(BB130+m29+AA24)+exp(BB145+m210+AA25)+exp(BB160+m211+AA26)
+exp(bb175+m212+aa27)+exp(bb190+m213+aa28)+exp(bb205+m214+aa29)+exp(bb220+m215+aa30));
c105o=exp(bb70+m25+aa20)/(1+exp(bb10+m21+aa16)+exp(bb25+m22+aa17)+exp(bb40+m23+aa18)
+exp(BB55+m24+AA19)+exp(BB70+m25+AA20)+exp(BB85+m26+AA21)+exp(BB100+m27+AA22)
+exp(BB115+m28+AA23)+exp(BB130+m29+AA24)+exp(BB145+m210+AA25)+exp(BB160+m211+AA26)
+exp(bb175+m212+aa27)+exp(bb190+m213+aa28)+exp(bb205+m214+aa29)+exp(bb220+m215+aa30));
c106o=exp(bb85+m26+aa21)/(1+exp(bb10+m21+aa16)+exp(bb25+m22+aa17)+exp(bb40+m23+aa18)
+exp(BB55+m24+AA19)+exp(BB70+m25+AA20)+exp(BB85+m26+AA21)+exp(BB100+m27+AA22)
+exp(BB115+m28+AA23)+exp(BB130+m29+AA24)+exp(BB145+m210+AA25)+exp(BB160+m211+AA26)
+exp(bb175+m212+aa27)+exp(bb190+m213+aa28)+exp(bb205+m214+aa29)+exp(bb220+m215+aa30));
c107o=exp(bb100+m27+aa22)/(1+exp(bb10+m21+aa16)+exp(bb25+m22+aa17)+exp(bb40+m23+aa18)
+exp(BB55+m24+AA19)+exp(BB70+m25+AA20)+exp(BB85+m26+AA21)+exp(BB100+m27+AA22)
+exp(BB115+m28+AA23)+exp(BB130+m29+AA24)+exp(BB145+m210+AA25)+exp(BB160+m211+AA26)
+exp(bb175+m212+aa27)+exp(bb190+m213+aa28)+exp(bb205+m214+aa29)+exp(bb220+m215+aa30));
c108o=exp(bb115+m28+aa23)/(1+exp(bb10+m21+aa16)+exp(bb25+m22+aa17)+exp(bb40+m23+aa18)
+exp(BB55+m24+AA19)+exp(BB70+m25+AA20)+exp(BB85+m26+AA21)+exp(BB100+m27+AA22)
+exp(BB115+m28+AA23)+exp(BB130+m29+AA24)+exp(BB145+m210+AA25)+exp(BB160+m211+AA26)
+exp(bb175+m212+aa27)+exp(bb190+m213+aa28)+exp(bb205+m214+aa29)+exp(bb220+m215+aa30));
c109o=exp(bb130+m29+aa24)/(1+exp(bb10+m21+aa16)+exp(bb25+m22+aa17)+exp(bb40+m23+aa18)
+exp(BB55+m24+AA19)+exp(BB70+m25+AA20)+exp(BB85+m26+AA21)+exp(BB100+m27+AA22)
+exp(BB115+m28+AA23)+exp(BB130+m29+AA24)+exp(BB145+m210+AA25)+exp(BB160+m211+AA26)
+exp(bb175+m212+aa27)+exp(bb190+m213+aa28)+exp(bb205+m214+aa29)+exp(bb220+m215+aa30));

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c1010o=exp(bb145+m210+aa25)/(1+exp(bb10+m21+aa16)+exp(bb25+m22+aa17)+exp(bb40+m23+aa18)
+exp(BB55+m24+AA19)+exp(BB70+m25+AA20)+exp(BB85+m26+AA21)+exp(BB100+m27+AA22)
+exp(BB115+m28+AA23)+exp(BB130+m29+AA24)+exp(BB145+m210+AA25)+exp(BB160+m211+AA26)
+exp(bb175+m212+aa27)+exp(bb190+m213+aa28)+exp(bb205+m214+aa29)+exp(bb220+m215+aa30));
c1011o=exp(bb160+m211+aa26)/(1+exp(bb10+m21+aa16)+exp(bb25+m22+aa17)+exp(bb40+m23+aa18)
+exp(BB55+m24+AA19)+exp(BB70+m25+AA20)+exp(BB85+m26+AA21)+exp(BB100+m27+AA22)
+exp(BB115+m28+AA23)+exp(BB130+m29+AA24)+exp(BB145+m210+AA25)+exp(BB160+m211+AA26)
+exp(bb175+m212+aa27)+exp(bb190+m213+aa28)+exp(bb205+m214+aa29)+exp(bb220+m215+aa30));
c1012o=exp(bb175+m212+aa27)/(1+exp(bb10+m21+aa16)+exp(bb25+m22+aa17)+exp(bb40+m23+aa18)
+exp(BB55+m24+AA19)+exp(BB70+m25+AA20)+exp(BB85+m26+AA21)+exp(BB100+m27+AA22)
+exp(BB115+m28+AA23)+exp(BB130+m29+AA24)+exp(BB145+m210+AA25)+exp(BB160+m211+AA26)
+exp(bb175+m212+aa27)+exp(bb190+m213+aa28)+exp(bb205+m214+aa29)+exp(bb220+m215+aa30));
c1013o=exp(bb190+m213+aa28)/(1+exp(bb10+m21+aa16)+exp(bb25+m22+aa17)+exp(bb40+m23+aa18)
+exp(BB55+m24+AA19)+exp(BB70+m25+AA20)+exp(BB85+m26+AA21)+exp(BB100+m27+AA22)
+exp(BB115+m28+AA23)+exp(BB130+m29+AA24)+exp(BB145+m210+AA25)+exp(BB160+m211+AA26)
+exp(bb175+m212+aa27)+exp(bb190+m213+aa28)+exp(bb205+m214+aa29)+exp(bb220+m215+aa30));
c1014o=exp(bb205+m214+aa29)/(1+exp(bb10+m21+aa16)+exp(bb25+m22+aa17)+exp(bb40+m23+aa18)
+exp(BB55+m24+AA19)+exp(BB70+m25+AA20)+exp(BB85+m26+AA21)+exp(BB100+m27+AA22)
+exp(BB115+m28+AA23)+exp(BB130+m29+AA24)+exp(BB145+m210+AA25)+exp(BB160+m211+AA26)
+exp(bb175+m212+aa27)+exp(bb190+m213+aa28)+exp(bb205+m214+aa29)+exp(bb220+m215+aa30));
c1015o=exp(bb220+m215+aa30)/(1+exp(bb10+m21+aa16)+exp(bb25+m22+aa17)+exp(bb40+m23+aa18)
+exp(BB55+m24+AA19)+exp(BB70+m25+AA20)+exp(BB85+m26+AA21)+exp(BB100+m27+AA22)
+exp(BB115+m28+AA23)+exp(BB130+m29+AA24)+exp(BB145+m210+AA25)+exp(BB160+m211+AA26)
+exp(bb175+m212+aa27)+exp(bb190+m213+aa28)+exp(bb205+m214+aa29)+exp(bb220+m215+aa30));
c1016o=1/(1+exp(bb10+m21+aa16)+exp(bb25+m22+aa17)+exp(bb40+m23+aa18)
+exp(BB55+m24+AA19)+exp(BB70+m25+AA20)+exp(BB85+m26+AA21)+exp(BB100+m27+AA22)
+exp(BB115+m28+AA23)+exp(BB130+m29+AA24)+exp(BB145+m210+AA25)+exp(BB160+m211+AA26)
+exp(bb175+m212+aa27)+exp(bb190+m213+aa28)+exp(bb205+m214+aa29)+exp(bb220+m215+aa30));

!row 11
c111q=exp(bb11+m21+aa16)/(1+exp(bb11+m21+aa16)+exp(bb26+m22+aa17)+exp(bb41+m23+aa18)
+exp(BB56+m24+AA19)+exp(BB71+m25+AA20)+exp(BB86+m26+AA21)+exp(BB101+m27+AA22)
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+exp(BB116+m28+AA23)+exp(BB131+m29+AA24)+exp(BB146+m210+AA25)+exp(BB161+m211+AA26)
+exp(bb176+m212+aa27)+exp(bb191+m213+aa28)+exp(bb206+m214+aa29)+exp(bb221+m215+aa30));
c112q=exp(bb26+m22+aa17)/(1+exp(bb11+m21+aa16)+exp(bb26+m22+aa17)+exp(bb41+m23+aa18)
+exp(BB56+m24+AA19)+exp(BB71+m25+AA20)+exp(BB86+m26+AA21)+exp(BB101+m27+AA22)
+exp(BB116+m28+AA23)+exp(BB131+m29+AA24)+exp(BB146+m210+AA25)+exp(BB161+m211+AA26)
+exp(bb176+m212+aa27)+exp(bb191+m213+aa28)+exp(bb206+m214+aa29)+exp(bb221+m215+aa30));
c113q=exp(bb41+m23+aa18)/(1+exp(bb11+m21+aa16)+exp(bb26+m22+aa17)+exp(bb41+m23+aa18)
+exp(BB56+m24+AA19)+exp(BB71+m25+AA20)+exp(BB86+m26+AA21)+exp(BB101+m27+AA22)
+exp(BB116+m28+AA23)+exp(BB131+m29+AA24)+exp(BB146+m210+AA25)+exp(BB161+m211+AA26)
+exp(bb176+m212+aa27)+exp(bb191+m213+aa28)+exp(bb206+m214+aa29)+exp(bb221+m215+aa30));
c114q=exp(bb56+m24+aa19)/(1+exp(bb11+m21+aa16)+exp(bb26+m22+aa17)+exp(bb41+m23+aa18)
+exp(BB56+m24+AA19)+exp(BB71+m25+AA20)+exp(BB86+m26+AA21)+exp(BB101+m27+AA22)
+exp(BB116+m28+AA23)+exp(BB131+m29+AA24)+exp(BB146+m210+AA25)+exp(BB161+m211+AA26)
+exp(bb176+m212+aa27)+exp(bb191+m213+aa28)+exp(bb206+m214+aa29)+exp(bb221+m215+aa30));
c115q=exp(bb71+m25+aa20)/(1+exp(bb11+m21+aa16)+exp(bb26+m22+aa17)+exp(bb41+m23+aa18)
+exp(BB56+m24+AA19)+exp(BB71+m25+AA20)+exp(BB86+m26+AA21)+exp(BB101+m27+AA22)
+exp(BB116+m28+AA23)+exp(BB131+m29+AA24)+exp(BB146+m210+AA25)+exp(BB161+m211+AA26)
+exp(bb176+m212+aa27)+exp(bb191+m213+aa28)+exp(bb206+m214+aa29)+exp(bb221+m215+aa30));
c116q=exp(bb86+m26+aa21)/(1+exp(bb11+m21+aa16)+exp(bb26+m22+aa17)+exp(bb41+m23+aa18)
+exp(BB56+m24+AA19)+exp(BB71+m25+AA20)+exp(BB86+m26+AA21)+exp(BB101+m27+AA22)
+exp(BB116+m28+AA23)+exp(BB131+m29+AA24)+exp(BB146+m210+AA25)+exp(BB161+m211+AA26)
+exp(bb176+m212+aa27)+exp(bb191+m213+aa28)+exp(bb206+m214+aa29)+exp(bb221+m215+aa30));
c117q=exp(bb101+m27+aa22)/(1+exp(bb11+m21+aa16)+exp(bb26+m22+aa17)+exp(bb41+m23+aa18)
+exp(BB56+m24+AA19)+exp(BB71+m25+AA20)+exp(BB86+m26+AA21)+exp(BB101+m27+AA22)
+exp(BB116+m28+AA23)+exp(BB131+m29+AA24)+exp(BB146+m210+AA25)+exp(BB161+m211+AA26)
+exp(bb176+m212+aa27)+exp(bb191+m213+aa28)+exp(bb206+m214+aa29)+exp(bb221+m215+aa30));
c118q=exp(bb116+m28+aa23)/(1+exp(bb11+m21+aa16)+exp(bb26+m22+aa17)+exp(bb41+m23+aa18)
+exp(BB56+m24+AA19)+exp(BB71+m25+AA20)+exp(BB86+m26+AA21)+exp(BB101+m27+AA22)
+exp(BB116+m28+AA23)+exp(BB131+m29+AA24)+exp(BB146+m210+AA25)+exp(BB161+m211+AA26)
+exp(bb176+m212+aa27)+exp(bb191+m213+aa28)+exp(bb206+m214+aa29)+exp(bb221+m215+aa30));
c119q=exp(bb131+m29+aa24)/(1+exp(bb11+m21+aa16)+exp(bb26+m22+aa17)+exp(bb41+m23+aa18)
+exp(BB56+m24+AA19)+exp(BB71+m25+AA20)+exp(BB86+m26+AA21)+exp(BB101+m27+AA22)

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+exp(BB116+m28+AA23)+exp(BB131+m29+AA24)+exp(BB146+m210+AA25)+exp(BB161+m211+AA26)
+exp(bb176+m212+aa27)+exp(bb191+m213+aa28)+exp(bb206+m214+aa29)+exp(bb221+m215+aa30));
c1110q=exp(bb146+m210+aa25)/(1+exp(bb11+m21+aa16)+exp(bb26+m22+aa17)+exp(bb41+m23+aa18)
+exp(BB56+m24+AA19)+exp(BB71+m25+AA20)+exp(BB86+m26+AA21)+exp(BB101+m27+AA22)
+exp(BB116+m28+AA23)+exp(BB131+m29+AA24)+exp(BB146+m210+AA25)+exp(BB161+m211+AA26)
+exp(bb176+m212+aa27)+exp(bb191+m213+aa28)+exp(bb206+m214+aa29)+exp(bb221+m215+aa30));
c1111q=exp(bb161+m211+aa26)/(1+exp(bb11+m21+aa16)+exp(bb26+m22+aa17)+exp(bb41+m23+aa18)
+exp(BB56+m24+AA19)+exp(BB71+m25+AA20)+exp(BB86+m26+AA21)+exp(BB101+m27+AA22)
+exp(BB116+m28+AA23)+exp(BB131+m29+AA24)+exp(BB146+m210+AA25)+exp(BB161+m211+AA26)
+exp(bb176+m212+aa27)+exp(bb191+m213+aa28)+exp(bb206+m214+aa29)+exp(bb221+m215+aa30));
c1112q=exp(bb176+m212+aa27)/(1+exp(bb11+m21+aa16)+exp(bb26+m22+aa17)+exp(bb41+m23+aa18)
+exp(BB56+m24+AA19)+exp(BB71+m25+AA20)+exp(BB86+m26+AA21)+exp(BB101+m27+AA22)
+exp(BB116+m28+AA23)+exp(BB131+m29+AA24)+exp(BB146+m210+AA25)+exp(BB161+m211+AA26)
+exp(bb176+m212+aa27)+exp(bb191+m213+aa28)+exp(bb206+m214+aa29)+exp(bb221+m215+aa30));
c1113q=exp(bb191+m213+aa28)/(1+exp(bb11+m21+aa16)+exp(bb26+m22+aa17)+exp(bb41+m23+aa18)
+exp(BB56+m24+AA19)+exp(BB71+m25+AA20)+exp(BB86+m26+AA21)+exp(BB101+m27+AA22)
+exp(BB116+m28+AA23)+exp(BB131+m29+AA24)+exp(BB146+m210+AA25)+exp(BB161+m211+AA26)
+exp(bb176+m212+aa27)+exp(bb191+m213+aa28)+exp(bb206+m214+aa29)+exp(bb221+m215+aa30));
c1114q=exp(bb206+m214+aa29)/(1+exp(bb11+m21+aa16)+exp(bb26+m22+aa17)+exp(bb41+m23+aa18)
+exp(BB56+m24+AA19)+exp(BB71+m25+AA20)+exp(BB86+m26+AA21)+exp(BB101+m27+AA22)
+exp(BB116+m28+AA23)+exp(BB131+m29+AA24)+exp(BB146+m210+AA25)+exp(BB161+m211+AA26)
+exp(bb176+m212+aa27)+exp(bb191+m213+aa28)+exp(bb206+m214+aa29)+exp(bb221+m215+aa30));
c1115q=exp(bb221+m215+aa30)/(1+exp(bb11+m21+aa16)+exp(bb26+m22+aa17)+exp(bb41+m23+aa18)
+exp(BB56+m24+AA19)+exp(BB71+m25+AA20)+exp(BB86+m26+AA21)+exp(BB101+m27+AA22)
+exp(BB116+m28+AA23)+exp(BB131+m29+AA24)+exp(BB146+m210+AA25)+exp(BB161+m211+AA26)
+exp(bb176+m212+aa27)+exp(bb191+m213+aa28)+exp(bb206+m214+aa29)+exp(bb221+m215+aa30));
c1116q=1/(1+exp(bb11+m21+aa16)+exp(bb26+m22+aa17)+exp(bb41+m23+aa18)
+exp(BB56+m24+AA19)+exp(BB71+m25+AA20)+exp(BB86+m26+AA21)+exp(BB101+m27+AA22)
+exp(BB116+m28+AA23)+exp(BB131+m29+AA24)+exp(BB146+m210+AA25)+exp(BB161+m211+AA26)
+exp(bb176+m212+aa27)+exp(bb191+m213+aa28)+exp(bb206+m214+aa29)+exp(bb221+m215+aa30));

!row 12
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c121u=exp(bb12+m21+aa16)/(1+exp(bb12+m21+aa16)+exp(bb27+m22+aa17)+exp(bb42+m23+aa18)
+exp(BB57+m24+AA19)+exp(BB72+m25+AA20)+exp(BB87+m26+AA21)+exp(BB102+m27+AA22)
+exp(BB117+m28+AA23)+exp(BB132+m29+AA24)+exp(BB147+m210+AA25)+exp(BB162+m211+AA26)
+exp(bb177+m212+aa27)+exp(bb192+m213+aa28)+exp(bb207+m214+aa29)+exp(bb222+m215+aa30));
c122u=exp(bb27+m22+aa17)/(1+exp(bb12+m21+aa16)+exp(bb27+m22+aa17)+exp(bb42+m23+aa18)
+exp(BB57+m24+AA19)+exp(BB72+m25+AA20)+exp(BB87+m26+AA21)+exp(BB102+m27+AA22)
+exp(BB117+m28+AA23)+exp(BB132+m29+AA24)+exp(BB147+m210+AA25)+exp(BB162+m211+AA26)
+exp(bb177+m212+aa27)+exp(bb192+m213+aa28)+exp(bb207+m214+aa29)+exp(bb222+m215+aa30));
c123u=exp(bb42+m23+aa18)/(1+exp(bb12+m21+aa16)+exp(bb27+m22+aa17)+exp(bb42+m23+aa18)
+exp(BB57+m24+AA19)+exp(BB72+m25+AA20)+exp(BB87+m26+AA21)+exp(BB102+m27+AA22)
+exp(BB117+m28+AA23)+exp(BB132+m29+AA24)+exp(BB147+m210+AA25)+exp(BB162+m211+AA26)
+exp(bb177+m212+aa27)+exp(bb192+m213+aa28)+exp(bb207+m214+aa29)+exp(bb222+m215+aa30));
c124u=exp(bb57+m24+aa19)/(1+exp(bb12+m21+aa16)+exp(bb27+m22+aa17)+exp(bb42+m23+aa18)
+exp(BB57+m24+AA19)+exp(BB72+m25+AA20)+exp(BB87+m26+AA21)+exp(BB102+m27+AA22)
+exp(BB117+m28+AA23)+exp(BB132+m29+AA24)+exp(BB147+m210+AA25)+exp(BB162+m211+AA26)
+exp(bb177+m212+aa27)+exp(bb192+m213+aa28)+exp(bb207+m214+aa29)+exp(bb222+m215+aa30));
c125u=exp(bb72+m25+aa20)/(1+exp(bb12+m21+aa16)+exp(bb27+m22+aa17)+exp(bb42+m23+aa18)
+exp(BB57+m24+AA19)+exp(BB72+m25+AA20)+exp(BB87+m26+AA21)+exp(BB102+m27+AA22)
+exp(BB117+m28+AA23)+exp(BB132+m29+AA24)+exp(BB147+m210+AA25)+exp(BB162+m211+AA26)
+exp(bb177+m212+aa27)+exp(bb192+m213+aa28)+exp(bb207+m214+aa29)+exp(bb222+m215+aa30));
c126u=exp(bb87+m26+aa21)/(1+exp(bb12+m21+aa16)+exp(bb27+m22+aa17)+exp(bb42+m23+aa18)
+exp(BB57+m24+AA19)+exp(BB72+m25+AA20)+exp(BB87+m26+AA21)+exp(BB102+m27+AA22)
+exp(BB117+m28+AA23)+exp(BB132+m29+AA24)+exp(BB147+m210+AA25)+exp(BB162+m211+AA26)
+exp(bb177+m212+aa27)+exp(bb192+m213+aa28)+exp(bb207+m214+aa29)+exp(bb222+m215+aa30));
c127u=exp(bb102+m27+aa22)/(1+exp(bb12+m21+aa16)+exp(bb27+m22+aa17)+exp(bb42+m23+aa18)
+exp(BB57+m24+AA19)+exp(BB72+m25+AA20)+exp(BB87+m26+AA21)+exp(BB102+m27+AA22)
+exp(BB117+m28+AA23)+exp(BB132+m29+AA24)+exp(BB147+m210+AA25)+exp(BB162+m211+AA26)
+exp(bb177+m212+aa27)+exp(bb192+m213+aa28)+exp(bb207+m214+aa29)+exp(bb222+m215+aa30));
c128u=exp(bb117+m28+aa23)/(1+exp(bb12+m21+aa16)+exp(bb27+m22+aa17)+exp(bb42+m23+aa18)
+exp(BB57+m24+AA19)+exp(BB72+m25+AA20)+exp(BB87+m26+AA21)+exp(BB102+m27+AA22)
+exp(BB117+m28+AA23)+exp(BB132+m29+AA24)+exp(BB147+m210+AA25)+exp(BB162+m211+AA26)
+exp(bb177+m212+aa27)+exp(bb192+m213+aa28)+exp(bb207+m214+aa29)+exp(bb222+m215+aa30));
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c129u=exp(bb132+m29+aa24)/(1+exp(bb12+m21+aa16)+exp(bb27+m22+aa17)+exp(bb42+m23+aa18)
+exp(BB57+m24+AA19)+exp(BB72+m25+AA20)+exp(BB87+m26+AA21)+exp(BB102+m27+AA22)
+exp(BB117+m28+AA23)+exp(BB132+m29+AA24)+exp(BB147+m210+AA25)+exp(BB162+m211+AA26)
+exp(bb177+m212+aa27)+exp(bb192+m213+aa28)+exp(bb207+m214+aa29)+exp(bb222+m215+aa30));
c1210u=exp(bb147+m210+aa25)/(1+exp(bb12+m21+aa16)+exp(bb27+m22+aa17)+exp(bb42+m23+aa18)
+exp(BB57+m24+AA19)+exp(BB72+m25+AA20)+exp(BB87+m26+AA21)+exp(BB102+m27+AA22)
+exp(BB117+m28+AA23)+exp(BB132+m29+AA24)+exp(BB147+m210+AA25)+exp(BB162+m211+AA26)
+exp(bb177+m212+aa27)+exp(bb192+m213+aa28)+exp(bb207+m214+aa29)+exp(bb222+m215+aa30));
c1211u=exp(bb162+m211+aa26)/(1+exp(bb12+m21+aa16)+exp(bb27+m22+aa17)+exp(bb42+m23+aa18)
+exp(BB57+m24+AA19)+exp(BB72+m25+AA20)+exp(BB87+m26+AA21)+exp(BB102+m27+AA22)
+exp(BB117+m28+AA23)+exp(BB132+m29+AA24)+exp(BB147+m210+AA25)+exp(BB162+m211+AA26)
+exp(bb177+m212+aa27)+exp(bb192+m213+aa28)+exp(bb207+m214+aa29)+exp(bb222+m215+aa30));
c1212u=exp(bb177+m212+aa27)/(1+exp(bb12+m21+aa16)+exp(bb27+m22+aa17)+exp(bb42+m23+aa18)
+exp(BB57+m24+AA19)+exp(BB72+m25+AA20)+exp(BB87+m26+AA21)+exp(BB102+m27+AA22)
+exp(BB117+m28+AA23)+exp(BB132+m29+AA24)+exp(BB147+m210+AA25)+exp(BB162+m211+AA26)
+exp(bb177+m212+aa27)+exp(bb192+m213+aa28)+exp(bb207+m214+aa29)+exp(bb222+m215+aa30));
c1213u=exp(bb192+m213+aa28)/(1+exp(bb12+m21+aa16)+exp(bb27+m22+aa17)+exp(bb42+m23+aa18)
+exp(BB57+m24+AA19)+exp(BB72+m25+AA20)+exp(BB87+m26+AA21)+exp(BB102+m27+AA22)
+exp(BB117+m28+AA23)+exp(BB132+m29+AA24)+exp(BB147+m210+AA25)+exp(BB162+m211+AA26)
+exp(bb177+m212+aa27)+exp(bb192+m213+aa28)+exp(bb207+m214+aa29)+exp(bb222+m215+aa30));
c1214u=exp(bb207+m214+aa29)/(1+exp(bb12+m21+aa16)+exp(bb27+m22+aa17)+exp(bb42+m23+aa18)
+exp(BB57+m24+AA19)+exp(BB72+m25+AA20)+exp(BB87+m26+AA21)+exp(BB102+m27+AA22)
+exp(BB117+m28+AA23)+exp(BB132+m29+AA24)+exp(BB147+m210+AA25)+exp(BB162+m211+AA26)
+exp(bb177+m212+aa27)+exp(bb192+m213+aa28)+exp(bb207+m214+aa29)+exp(bb222+m215+aa30));
c1215u=exp(bb222+m215+aa30)/(1+exp(bb12+m21+aa16)+exp(bb27+m22+aa17)+exp(bb42+m23+aa18)
+exp(BB57+m24+AA19)+exp(BB72+m25+AA20)+exp(BB87+m26+AA21)+exp(BB102+m27+AA22)
+exp(BB117+m28+AA23)+exp(BB132+m29+AA24)+exp(BB147+m210+AA25)+exp(BB162+m211+AA26)
+exp(bb177+m212+aa27)+exp(bb192+m213+aa28)+exp(bb207+m214+aa29)+exp(bb222+m215+aa30));
c1216u=1/(1+exp(bb12+m21+aa16)+exp(bb27+m22+aa17)+exp(bb42+m23+aa18)
+exp(BB57+m24+AA19)+exp(BB72+m25+AA20)+exp(BB87+m26+AA21)+exp(BB102+m27+AA22)
+exp(BB117+m28+AA23)+exp(BB132+m29+AA24)+exp(BB147+m210+AA25)+exp(BB162+m211+AA26)
+exp(bb177+m212+aa27)+exp(bb192+m213+aa28)+exp(bb207+m214+aa29)+exp(bb222+m215+aa30));
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!row 13
c131v=exp(bb13+m21+aa16)/(1+exp(bb13+m21+aa16)+exp(bb28+m22+aa17)+exp(bb43+m23+aa18)
+exp(BB58+m24+AA19)+exp(BB73+m25+AA20)+exp(BB88+m26+AA21)+exp(BB103+m27+AA22)
+exp(BB118+m28+AA23)+exp(BB133+m29+AA24)+exp(BB148+m210+AA25)+exp(BB163+m211+AA26)
+exp(bb178+m212+aa27)+exp(bb193+m213+aa28)+exp(bb208+m214+aa29)+exp(bb223+m215+aa30));
c132v=exp(bb28+m22+aa17)/(1+exp(bb13+m21+aa16)+exp(bb28+m22+aa17)+exp(bb43+m23+aa18)
+exp(BB58+m24+AA19)+exp(BB73+m25+AA20)+exp(BB88+m26+AA21)+exp(BB103+m27+AA22)
+exp(BB118+m28+AA23)+exp(BB133+m29+AA24)+exp(BB148+m210+AA25)+exp(BB163+m211+AA26)
+exp(bb178+m212+aa27)+exp(bb193+m213+aa28)+exp(bb208+m214+aa29)+exp(bb223+m215+aa30));
c133v=exp(bb43+m23+aa18)/(1+exp(bb13+m21+aa16)+exp(bb28+m22+aa17)+exp(bb43+m23+aa18)
+exp(BB58+m24+AA19)+exp(BB73+m25+AA20)+exp(BB88+m26+AA21)+exp(BB103+m27+AA22)
+exp(BB118+m28+AA23)+exp(BB133+m29+AA24)+exp(BB148+m210+AA25)+exp(BB163+m211+AA26)
+exp(bb178+m212+aa27)+exp(bb193+m213+aa28)+exp(bb208+m214+aa29)+exp(bb223+m215+aa30));
c134v=exp(bb58+m24+aa19)/(1+exp(bb13+m21+aa16)+exp(bb28+m22+aa17)+exp(bb43+m23+aa18)
+exp(BB58+m24+AA19)+exp(BB73+m25+AA20)+exp(BB88+m26+AA21)+exp(BB103+m27+AA22)
+exp(BB118+m28+AA23)+exp(BB133+m29+AA24)+exp(BB148+m210+AA25)+exp(BB163+m211+AA26)
+exp(bb178+m212+aa27)+exp(bb193+m213+aa28)+exp(bb208+m214+aa29)+exp(bb223+m215+aa30));
c135v=exp(bb73+m25+aa20)/(1+exp(bb13+m21+aa16)+exp(bb28+m22+aa17)+exp(bb43+m23+aa18)
+exp(BB58+m24+AA19)+exp(BB73+m25+AA20)+exp(BB88+m26+AA21)+exp(BB103+m27+AA22)
+exp(BB118+m28+AA23)+exp(BB133+m29+AA24)+exp(BB148+m210+AA25)+exp(BB163+m211+AA26)
+exp(bb178+m212+aa27)+exp(bb193+m213+aa28)+exp(bb208+m214+aa29)+exp(bb223+m215+aa30));
c136v=exp(bb88+m26+aa21)/(1+exp(bb13+m21+aa16)+exp(bb28+m22+aa17)+exp(bb43+m23+aa18)
+exp(BB58+m24+AA19)+exp(BB73+m25+AA20)+exp(BB88+m26+AA21)+exp(BB103+m27+AA22)
+exp(BB118+m28+AA23)+exp(BB133+m29+AA24)+exp(BB148+m210+AA25)+exp(BB163+m211+AA26)
+exp(bb178+m212+aa27)+exp(bb193+m213+aa28)+exp(bb208+m214+aa29)+exp(bb223+m215+aa30));
c137v=exp(bb103+m27+aa22)/(1+exp(bb13+m21+aa16)+exp(bb28+m22+aa17)+exp(bb43+m23+aa18)
+exp(BB58+m24+AA19)+exp(BB73+m25+AA20)+exp(BB88+m26+AA21)+exp(BB103+m27+AA22)
+exp(BB118+m28+AA23)+exp(BB133+m29+AA24)+exp(BB148+m210+AA25)+exp(BB163+m211+AA26)
+exp(bb178+m212+aa27)+exp(bb193+m213+aa28)+exp(bb208+m214+aa29)+exp(bb223+m215+aa30));
c138v=exp(bb118+m28+aa23)/(1+exp(bb13+m21+aa16)+exp(bb28+m22+aa17)+exp(bb43+m23+aa18)
+exp(BB58+m24+AA19)+exp(BB73+m25+AA20)+exp(BB88+m26+AA21)+exp(BB103+m27+AA22)
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+exp(BB118+m28+AA23)+exp(BB133+m29+AA24)+exp(BB148+m210+AA25)+exp(BB163+m211+AA26)
+exp(bb178+m212+aa27)+exp(bb193+m213+aa28)+exp(bb208+m214+aa29)+exp(bb223+m215+aa30));
c139v=exp(bb133+m29+aa24)/(1+exp(bb13+m21+aa16)+exp(bb28+m22+aa17)+exp(bb43+m23+aa18)
+exp(BB58+m24+AA19)+exp(BB73+m25+AA20)+exp(BB88+m26+AA21)+exp(BB103+m27+AA22)
+exp(BB118+m28+AA23)+exp(BB133+m29+AA24)+exp(BB148+m210+AA25)+exp(BB163+m211+AA26)
+exp(bb178+m212+aa27)+exp(bb193+m213+aa28)+exp(bb208+m214+aa29)+exp(bb223+m215+aa30));
c1310v=exp(bb148+m210+aa25)/(1+exp(bb13+m21+aa16)+exp(bb28+m22+aa17)+exp(bb43+m23+aa18)
+exp(BB58+m24+AA19)+exp(BB73+m25+AA20)+exp(BB88+m26+AA21)+exp(BB103+m27+AA22)
+exp(BB118+m28+AA23)+exp(BB133+m29+AA24)+exp(BB148+m210+AA25)+exp(BB163+m211+AA26)
+exp(bb178+m212+aa27)+exp(bb193+m213+aa28)+exp(bb208+m214+aa29)+exp(bb223+m215+aa30));
c1311v=exp(bb163+m211+aa26)/(1+exp(bb13+m21+aa16)+exp(bb28+m22+aa17)+exp(bb43+m23+aa18)
+exp(BB58+m24+AA19)+exp(BB73+m25+AA20)+exp(BB88+m26+AA21)+exp(BB103+m27+AA22)
+exp(BB118+m28+AA23)+exp(BB133+m29+AA24)+exp(BB148+m210+AA25)+exp(BB163+m211+AA26)
+exp(bb178+m212+aa27)+exp(bb193+m213+aa28)+exp(bb208+m214+aa29)+exp(bb223+m215+aa30));
c1312v=exp(bb178+m212+aa27)/(1+exp(bb13+m21+aa16)+exp(bb28+m22+aa17)+exp(bb43+m23+aa18)
+exp(BB58+m24+AA19)+exp(BB73+m25+AA20)+exp(BB88+m26+AA21)+exp(BB103+m27+AA22)
+exp(BB118+m28+AA23)+exp(BB133+m29+AA24)+exp(BB148+m210+AA25)+exp(BB163+m211+AA26)
+exp(bb178+m212+aa27)+exp(bb193+m213+aa28)+exp(bb208+m214+aa29)+exp(bb223+m215+aa30));
c1313v=exp(bb193+m213+aa28)/(1+exp(bb13+m21+aa16)+exp(bb28+m22+aa17)+exp(bb43+m23+aa18)
+exp(BB58+m24+AA19)+exp(BB73+m25+AA20)+exp(BB88+m26+AA21)+exp(BB103+m27+AA22)
+exp(BB118+m28+AA23)+exp(BB133+m29+AA24)+exp(BB148+m210+AA25)+exp(BB163+m211+AA26)
+exp(bb178+m212+aa27)+exp(bb193+m213+aa28)+exp(bb208+m214+aa29)+exp(bb223+m215+aa30));
c1314v=exp(bb208+m214+aa29)/(1+exp(bb13+m21+aa16)+exp(bb28+m22+aa17)+exp(bb43+m23+aa18)
+exp(BB58+m24+AA19)+exp(BB73+m25+AA20)+exp(BB88+m26+AA21)+exp(BB103+m27+AA22)
+exp(BB118+m28+AA23)+exp(BB133+m29+AA24)+exp(BB148+m210+AA25)+exp(BB163+m211+AA26)
+exp(bb178+m212+aa27)+exp(bb193+m213+aa28)+exp(bb208+m214+aa29)+exp(bb223+m215+aa30));
c1315v=exp(bb223+m215+aa30)/(1+exp(bb13+m21+aa16)+exp(bb28+m22+aa17)+exp(bb43+m23+aa18)
+exp(BB58+m24+AA19)+exp(BB73+m25+AA20)+exp(BB88+m26+AA21)+exp(BB103+m27+AA22)
+exp(BB118+m28+AA23)+exp(BB133+m29+AA24)+exp(BB148+m210+AA25)+exp(BB163+m211+AA26)
+exp(bb178+m212+aa27)+exp(bb193+m213+aa28)+exp(bb208+m214+aa29)+exp(bb223+m215+aa30));
c1316v=1/(1+exp(bb13+m21+aa16)+exp(bb28+m22+aa17)+exp(bb43+m23+aa18)
+exp(BB58+m24+AA19)+exp(BB73+m25+AA20)+exp(BB88+m26+AA21)+exp(BB103+m27+AA22)

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+exp(BB118+m28+AA23)+exp(BB133+m29+AA24)+exp(BB148+m210+AA25)+exp(BB163+m211+AA26)
+exp(bb178+m212+aa27)+exp(bb193+m213+aa28)+exp(bb208+m214+aa29)+exp(bb223+m215+aa30));

!row 14
c141w=exp(bb14+m21+aa16)/(1+exp(bb14+m21+aa16)+exp(bb29+m22+aa17)+exp(bb44+m23+aa18)
+exp(BB59+m24+AA19)+exp(BB74+m25+AA20)+exp(BB89+m26+AA21)+exp(BB104+m27+AA22)
+exp(BB119+m28+AA23)+exp(BB134+m29+AA24)+exp(BB149+m210+AA25)+exp(BB164+m211+AA26)
+exp(bb179+m212+aa27)+exp(bb194+m213+aa28)+exp(bb209+m214+aa29)+exp(bb224+m215+aa30));
c142w=exp(bb29+m22+aa17)/(1+exp(bb14+m21+aa16)+exp(bb29+m22+aa17)+exp(bb44+m23+aa18)
+exp(BB59+m24+AA19)+exp(BB74+m25+AA20)+exp(BB89+m26+AA21)+exp(BB104+m27+AA22)
+exp(BB119+m28+AA23)+exp(BB134+m29+AA24)+exp(BB149+m210+AA25)+exp(BB164+m211+AA26)
+exp(bb179+m212+aa27)+exp(bb194+m213+aa28)+exp(bb209+m214+aa29)+exp(bb224+m215+aa30));
c143w=exp(bb44+m23+aa18)/(1+exp(bb14+m21+aa16)+exp(bb29+m22+aa17)+exp(bb44+m23+aa18)
+exp(BB59+m24+AA19)+exp(BB74+m25+AA20)+exp(BB89+m26+AA21)+exp(BB104+m27+AA22)
+exp(BB119+m28+AA23)+exp(BB134+m29+AA24)+exp(BB149+m210+AA25)+exp(BB164+m211+AA26)
+exp(bb179+m212+aa27)+exp(bb194+m213+aa28)+exp(bb209+m214+aa29)+exp(bb224+m215+aa30));
c144w=exp(bb59+m24+aa19)/(1+exp(bb14+m21+aa16)+exp(bb29+m22+aa17)+exp(bb44+m23+aa18)
+exp(BB59+m24+AA19)+exp(BB74+m25+AA20)+exp(BB89+m26+AA21)+exp(BB104+m27+AA22)
+exp(BB119+m28+AA23)+exp(BB134+m29+AA24)+exp(BB149+m210+AA25)+exp(BB164+m211+AA26)
+exp(bb179+m212+aa27)+exp(bb194+m213+aa28)+exp(bb209+m214+aa29)+exp(bb224+m215+aa30));
c145w=exp(bb74+m25+aa20)/(1+exp(bb14+m21+aa16)+exp(bb29+m22+aa17)+exp(bb44+m23+aa18)
+exp(BB59+m24+AA19)+exp(BB74+m25+AA20)+exp(BB89+m26+AA21)+exp(BB104+m27+AA22)
+exp(BB119+m28+AA23)+exp(BB134+m29+AA24)+exp(BB149+m210+AA25)+exp(BB164+m211+AA26)
+exp(bb179+m212+aa27)+exp(bb194+m213+aa28)+exp(bb209+m214+aa29)+exp(bb224+m215+aa30));
c146w=exp(bb89+m26+aa21)/(1+exp(bb14+m21+aa16)+exp(bb29+m22+aa17)+exp(bb44+m23+aa18)
+exp(BB59+m24+AA19)+exp(BB74+m25+AA20)+exp(BB89+m26+AA21)+exp(BB104+m27+AA22)
+exp(BB119+m28+AA23)+exp(BB134+m29+AA24)+exp(BB149+m210+AA25)+exp(BB164+m211+AA26)
+exp(bb179+m212+aa27)+exp(bb194+m213+aa28)+exp(bb209+m214+aa29)+exp(bb224+m215+aa30));
c147w=exp(bb104+m27+aa22)/(1+exp(bb14+m21+aa16)+exp(bb29+m22+aa17)+exp(bb44+m23+aa18)
+exp(BB59+m24+AA19)+exp(BB74+m25+AA20)+exp(BB89+m26+AA21)+exp(BB104+m27+AA22)
+exp(BB119+m28+AA23)+exp(BB134+m29+AA24)+exp(BB149+m210+AA25)+exp(BB164+m211+AA26)
+exp(bb179+m212+aa27)+exp(bb194+m213+aa28)+exp(bb209+m214+aa29)+exp(bb224+m215+aa30));
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c148w=exp(bb119+m28+aa23)/(1+exp(bb14+m21+aa16)+exp(bb29+m22+aa17)+exp(bb44+m23+aa18)
+exp(BB59+m24+AA19)+exp(BB74+m25+AA20)+exp(BB89+m26+AA21)+exp(BB104+m27+AA22)
+exp(BB119+m28+AA23)+exp(BB134+m29+AA24)+exp(BB149+m210+AA25)+exp(BB164+m211+AA26)
+exp(bb179+m212+aa27)+exp(bb194+m213+aa28)+exp(bb209+m214+aa29)+exp(bb224+m215+aa30));
c149w=exp(bb134+m29+aa24)/(1+exp(bb14+m21+aa16)+exp(bb29+m22+aa17)+exp(bb44+m23+aa18)
+exp(BB59+m24+AA19)+exp(BB74+m25+AA20)+exp(BB89+m26+AA21)+exp(BB104+m27+AA22)
+exp(BB119+m28+AA23)+exp(BB134+m29+AA24)+exp(BB149+m210+AA25)+exp(BB164+m211+AA26)
+exp(bb179+m212+aa27)+exp(bb194+m213+aa28)+exp(bb209+m214+aa29)+exp(bb224+m215+aa30));
c1410w=exp(bb149+m210+aa25)/(1+exp(bb14+m21+aa16)+exp(bb29+m22+aa17)+exp(bb44+m23+aa18)
+exp(BB59+m24+AA19)+exp(BB74+m25+AA20)+exp(BB89+m26+AA21)+exp(BB104+m27+AA22)
+exp(BB119+m28+AA23)+exp(BB134+m29+AA24)+exp(BB149+m210+AA25)+exp(BB164+m211+AA26)
+exp(bb179+m212+aa27)+exp(bb194+m213+aa28)+exp(bb209+m214+aa29)+exp(bb224+m215+aa30));
c1411w=exp(bb164+m211+aa26)/(1+exp(bb14+m21+aa16)+exp(bb29+m22+aa17)+exp(bb44+m23+aa18)
+exp(BB59+m24+AA19)+exp(BB74+m25+AA20)+exp(BB89+m26+AA21)+exp(BB104+m27+AA22)
+exp(BB119+m28+AA23)+exp(BB134+m29+AA24)+exp(BB149+m210+AA25)+exp(BB164+m211+AA26)
+exp(bb179+m212+aa27)+exp(bb194+m213+aa28)+exp(bb209+m214+aa29)+exp(bb224+m215+aa30));
c1412w=exp(bb179+m212+aa27)/(1+exp(bb14+m21+aa16)+exp(bb29+m22+aa17)+exp(bb44+m23+aa18)
+exp(BB59+m24+AA19)+exp(BB74+m25+AA20)+exp(BB89+m26+AA21)+exp(BB104+m27+AA22)
+exp(BB119+m28+AA23)+exp(BB134+m29+AA24)+exp(BB149+m210+AA25)+exp(BB164+m211+AA26)
+exp(bb179+m212+aa27)+exp(bb194+m213+aa28)+exp(bb209+m214+aa29)+exp(bb224+m215+aa30));
c1413w=exp(bb194+m213+aa28)/(1+exp(bb14+m21+aa16)+exp(bb29+m22+aa17)+exp(bb44+m23+aa18)
+exp(BB59+m24+AA19)+exp(BB74+m25+AA20)+exp(BB89+m26+AA21)+exp(BB104+m27+AA22)
+exp(BB119+m28+AA23)+exp(BB134+m29+AA24)+exp(BB149+m210+AA25)+exp(BB164+m211+AA26)
+exp(bb179+m212+aa27)+exp(bb194+m213+aa28)+exp(bb209+m214+aa29)+exp(bb224+m215+aa30));
c1414w=exp(bb209+m214+aa29)/(1+exp(bb14+m21+aa16)+exp(bb29+m22+aa17)+exp(bb44+m23+aa18)
+exp(BB59+m24+AA19)+exp(BB74+m25+AA20)+exp(BB89+m26+AA21)+exp(BB104+m27+AA22)
+exp(BB119+m28+AA23)+exp(BB134+m29+AA24)+exp(BB149+m210+AA25)+exp(BB164+m211+AA26)
+exp(bb179+m212+aa27)+exp(bb194+m213+aa28)+exp(bb209+m214+aa29)+exp(bb224+m215+aa30));
c1415w=exp(bb224+m215+aa30)/(1+exp(bb14+m21+aa16)+exp(bb29+m22+aa17)+exp(bb44+m23+aa18)
+exp(BB59+m24+AA19)+exp(BB74+m25+AA20)+exp(BB89+m26+AA21)+exp(BB104+m27+AA22)
+exp(BB119+m28+AA23)+exp(BB134+m29+AA24)+exp(BB149+m210+AA25)+exp(BB164+m211+AA26)
+exp(bb179+m212+aa27)+exp(bb194+m213+aa28)+exp(bb209+m214+aa29)+exp(bb224+m215+aa30));

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c1416w=1/(1+exp(bb14+m21+aa16)+exp(bb29+m22+aa17)+exp(bb44+m23+aa18)
+exp(BB59+m24+AA19)+exp(BB74+m25+AA20)+exp(BB89+m26+AA21)+exp(BB104+m27+AA22)
+exp(BB119+m28+AA23)+exp(BB134+m29+AA24)+exp(BB149+m210+AA25)+exp(BB164+m211+AA26)
+exp(bb179+m212+aa27)+exp(bb194+m213+aa28)+exp(bb209+m214+aa29)+exp(bb224+m215+aa30));

!row 15
c151y=exp(bb15+m21+aa16)/(1+exp(bb15+m21+aa16)+exp(bb30+m22+aa17)+exp(bb45+m23+aa18)
+exp(BB60+m24+AA19)+exp(BB75+m25+AA20)+exp(BB90+m26+AA21)+exp(BB105+m27+AA22)
+exp(BB120+m28+AA23)+exp(BB135+m29+AA24)+exp(BB150+m210+AA25)+exp(BB165+m211+AA26)
+exp(bb180+m212+aa27)+exp(bb195+m213+aa28)+exp(bb210+m214+aa29)+exp(bb225+m215+aa30));
c152y=exp(bb30+m22+aa17)/(1+exp(bb15+m21+aa16)+exp(bb30+m22+aa17)+exp(bb45+m23+aa18)
+exp(BB60+m24+AA19)+exp(BB75+m25+AA20)+exp(BB90+m26+AA21)+exp(BB105+m27+AA22)
+exp(BB120+m28+AA23)+exp(BB135+m29+AA24)+exp(BB150+m210+AA25)+exp(BB165+m211+AA26)
+exp(bb180+m212+aa27)+exp(bb195+m213+aa28)+exp(bb210+m214+aa29)+exp(bb225+m215+aa30));
c153y=exp(bb45+m23+aa18)/(1+exp(bb15+m21+aa16)+exp(bb30+m22+aa17)+exp(bb45+m23+aa18)
+exp(BB60+m24+AA19)+exp(BB75+m25+AA20)+exp(BB90+m26+AA21)+exp(BB105+m27+AA22)
+exp(BB120+m28+AA23)+exp(BB135+m29+AA24)+exp(BB150+m210+AA25)+exp(BB165+m211+AA26)
+exp(bb180+m212+aa27)+exp(bb195+m213+aa28)+exp(bb210+m214+aa29)+exp(bb225+m215+aa30));
c154y=exp(bb60+m24+aa19)/(1+exp(bb15+m21+aa16)+exp(bb30+m22+aa17)+exp(bb45+m23+aa18)
+exp(BB60+m24+AA19)+exp(BB75+m25+AA20)+exp(BB90+m26+AA21)+exp(BB105+m27+AA22)
+exp(BB120+m28+AA23)+exp(BB135+m29+AA24)+exp(BB150+m210+AA25)+exp(BB165+m211+AA26)
+exp(bb180+m212+aa27)+exp(bb195+m213+aa28)+exp(bb210+m214+aa29)+exp(bb225+m215+aa30));
c155y=exp(bb75+m25+aa20)/(1+exp(bb15+m21+aa16)+exp(bb30+m22+aa17)+exp(bb45+m23+aa18)
+exp(BB60+m24+AA19)+exp(BB75+m25+AA20)+exp(BB90+m26+AA21)+exp(BB105+m27+AA22)
+exp(BB120+m28+AA23)+exp(BB135+m29+AA24)+exp(BB150+m210+AA25)+exp(BB165+m211+AA26)
+exp(bb180+m212+aa27)+exp(bb195+m213+aa28)+exp(bb210+m214+aa29)+exp(bb225+m215+aa30));
c156y=exp(bb90+m26+aa21)/(1+exp(bb15+m21+aa16)+exp(bb30+m22+aa17)+exp(bb45+m23+aa18)
+exp(BB60+m24+AA19)+exp(BB75+m25+AA20)+exp(BB90+m26+AA21)+exp(BB105+m27+AA22)
+exp(BB120+m28+AA23)+exp(BB135+m29+AA24)+exp(BB150+m210+AA25)+exp(BB165+m211+AA26)
+exp(bb180+m212+aa27)+exp(bb195+m213+aa28)+exp(bb210+m214+aa29)+exp(bb225+m215+aa30));
c157y=exp(bb105+m27+aa22)/(1+exp(bb15+m21+aa16)+exp(bb30+m22+aa17)+exp(bb45+m23+aa18)
+exp(BB60+m24+AA19)+exp(BB75+m25+AA20)+exp(BB90+m26+AA21)+exp(BB105+m27+AA22)
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+exp(BB120+m28+AA23)+exp(BB135+m29+AA24)+exp(BB150+m210+AA25)+exp(BB165+m211+AA26)
+exp(bb180+m212+aa27)+exp(bb195+m213+aa28)+exp(bb210+m214+aa29)+exp(bb225+m215+aa30));
c158y=exp(bb120+m28+aa23)/(1+exp(bb15+m21+aa16)+exp(bb30+m22+aa17)+exp(bb45+m23+aa18)
+exp(BB60+m24+AA19)+exp(BB75+m25+AA20)+exp(BB90+m26+AA21)+exp(BB105+m27+AA22)
+exp(BB120+m28+AA23)+exp(BB135+m29+AA24)+exp(BB150+m210+AA25)+exp(BB165+m211+AA26)
+exp(bb180+m212+aa27)+exp(bb195+m213+aa28)+exp(bb210+m214+aa29)+exp(bb225+m215+aa30));
c159y=exp(bb135+m29+aa24)/(1+exp(bb15+m21+aa16)+exp(bb30+m22+aa17)+exp(bb45+m23+aa18)
+exp(BB60+m24+AA19)+exp(BB75+m25+AA20)+exp(BB90+m26+AA21)+exp(BB105+m27+AA22)
+exp(BB120+m28+AA23)+exp(BB135+m29+AA24)+exp(BB150+m210+AA25)+exp(BB165+m211+AA26)
+exp(bb180+m212+aa27)+exp(bb195+m213+aa28)+exp(bb210+m214+aa29)+exp(bb225+m215+aa30));
c1510y=exp(bb150+m210+aa25)/(1+exp(bb15+m21+aa16)+exp(bb30+m22+aa17)+exp(bb45+m23+aa18)
+exp(BB60+m24+AA19)+exp(BB75+m25+AA20)+exp(BB90+m26+AA21)+exp(BB105+m27+AA22)
+exp(BB120+m28+AA23)+exp(BB135+m29+AA24)+exp(BB150+m210+AA25)+exp(BB165+m211+AA26)
+exp(bb180+m212+aa27)+exp(bb195+m213+aa28)+exp(bb210+m214+aa29)+exp(bb225+m215+aa30));
c1511y=exp(bb165+m211+aa26)/(1+exp(bb15+m21+aa16)+exp(bb30+m22+aa17)+exp(bb45+m23+aa18)
+exp(BB60+m24+AA19)+exp(BB75+m25+AA20)+exp(BB90+m26+AA21)+exp(BB105+m27+AA22)
+exp(BB120+m28+AA23)+exp(BB135+m29+AA24)+exp(BB150+m210+AA25)+exp(BB165+m211+AA26)
+exp(bb180+m212+aa27)+exp(bb195+m213+aa28)+exp(bb210+m214+aa29)+exp(bb225+m215+aa30));
c1512y=exp(bb180+m212+aa27)/(1+exp(bb15+m21+aa16)+exp(bb30+m22+aa17)+exp(bb45+m23+aa18)
+exp(BB60+m24+AA19)+exp(BB75+m25+AA20)+exp(BB90+m26+AA21)+exp(BB105+m27+AA22)
+exp(BB120+m28+AA23)+exp(BB135+m29+AA24)+exp(BB150+m210+AA25)+exp(BB165+m211+AA26)
+exp(bb180+m212+aa27)+exp(bb195+m213+aa28)+exp(bb210+m214+aa29)+exp(bb225+m215+aa30));
c1513y=exp(bb195+m213+aa28)/(1+exp(bb15+m21+aa16)+exp(bb30+m22+aa17)+exp(bb45+m23+aa18)
+exp(BB60+m24+AA19)+exp(BB75+m25+AA20)+exp(BB90+m26+AA21)+exp(BB105+m27+AA22)
+exp(BB120+m28+AA23)+exp(BB135+m29+AA24)+exp(BB150+m210+AA25)+exp(BB165+m211+AA26)
+exp(bb180+m212+aa27)+exp(bb195+m213+aa28)+exp(bb210+m214+aa29)+exp(bb225+m215+aa30));
c1514y=exp(bb210+m214+aa29)/(1+exp(bb15+m21+aa16)+exp(bb30+m22+aa17)+exp(bb45+m23+aa18)
+exp(BB60+m24+AA19)+exp(BB75+m25+AA20)+exp(BB90+m26+AA21)+exp(BB105+m27+AA22)
+exp(BB120+m28+AA23)+exp(BB135+m29+AA24)+exp(BB150+m210+AA25)+exp(BB165+m211+AA26)
+exp(bb180+m212+aa27)+exp(bb195+m213+aa28)+exp(bb210+m214+aa29)+exp(bb225+m215+aa30));
c1515y=exp(bb225+m215+aa30)/(1+exp(bb15+m21+aa16)+exp(bb30+m22+aa17)+exp(bb45+m23+aa18)
+exp(BB60+m24+AA19)+exp(BB75+m25+AA20)+exp(BB90+m26+AA21)+exp(BB105+m27+AA22)

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+exp(BB120+m28+AA23)+exp(BB135+m29+AA24)+exp(BB150+m210+AA25)+exp(BB165+m211+AA26)
+exp(bb180+m212+aa27)+exp(bb195+m213+aa28)+exp(bb210+m214+aa29)+exp(bb225+m215+aa30));
c1516y=1/(1+exp(bb15+m21+aa16)+exp(bb30+m22+aa17)+exp(bb45+m23+aa18)
+exp(BB60+m24+AA19)+exp(BB75+m25+AA20)+exp(BB90+m26+AA21)+exp(BB105+m27+AA22)
+exp(BB120+m28+AA23)+exp(BB135+m29+AA24)+exp(BB150+m210+AA25)+exp(BB165+m211+AA26)
+exp(bb180+m212+aa27)+exp(bb195+m213+aa28)+exp(bb210+m214+aa29)+exp(bb225+m215+aa30));

!row 16
c161z=exp(m21+aa16)/(1+exp(m21+aa16)+exp(m22+aa17)+exp(m23+aa18)+exp(m24+aa19)
+exp(m25+AA20)
+exp(m26+AA21)+exp(m27+AA22)+exp(m28+AA23)+exp(m29+AA24)+exp(m210+AA25)+exp(m211+AA26)
+exp(m212+aa27)+exp(m213+aa28)+exp(m214+aa29)+exp(m215+aa30));
c162z=exp(m22+aa17)/(1+exp(m21+aa16)+exp(m22+aa17)+exp(m23+aa18)+exp(m24+aa19)
+exp(m25+AA20)
+exp(m26+AA21)+exp(m27+AA22)+exp(m28+AA23)+exp(m29+AA24)+exp(m210+AA25)+exp(m211+AA26)
+exp(m212+aa27)+exp(m213+aa28)+exp(m214+aa29)+exp(m215+aa30));
c163z=exp(m23+aa18)/(1+exp(m21+aa16)+exp(m22+aa17)+exp(m23+aa18)+exp(m24+aa19)
+exp(m25+AA20)
+exp(m26+AA21)+exp(m27+AA22)+exp(m28+AA23)+exp(m29+AA24)+exp(m210+AA25)+exp(m211+AA26)
+exp(m212+aa27)+exp(m213+aa28)+exp(m214+aa29)+exp(m215+aa30));
c164z=exp(m24+aa19)/(1+exp(m21+aa16)+exp(m22+aa17)+exp(m23+aa18)+exp(m24+aa19)
+exp(m25+AA20)
+exp(m26+AA21)+exp(m27+AA22)+exp(m28+AA23)+exp(m29+AA24)+exp(m210+AA25)+exp(m211+AA26)
+exp(m212+aa27)+exp(m213+aa28)+exp(m214+aa29)+exp(m215+aa30));
c165z=exp(m25+aa20)/(1+exp(m21+aa16)+exp(m22+aa17)+exp(m23+aa18)+exp(m24+aa19)
+exp(m25+AA20)
+exp(m26+AA21)+exp(m27+AA22)+exp(m28+AA23)+exp(m29+AA24)+exp(m210+AA25)+exp(m211+AA26)
+exp(m212+aa27)+exp(m213+aa28)+exp(m214+aa29)+exp(m215+aa30));
c166z=exp(m26+aa21)/(1+exp(m21+aa16)+exp(m22+aa17)+exp(m23+aa18)+exp(m24+aa19)
+exp(m25+AA20)
+exp(m26+AA21)+exp(m27+AA22)+exp(m28+AA23)+exp(m29+AA24)+exp(m210+AA25)+exp(m211+AA26)
+exp(m212+aa27)+exp(m213+aa28)+exp(m214+aa29)+exp(m215+aa30));
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c167z=exp(m27+aa22)/(1+exp(m21+aa16)+exp(m22+aa17)+exp(m23+aa18)+exp(m24+aa19)
+exp(m25+AA20)
+exp(m26+AA21)+exp(m27+AA22)+exp(m28+AA23)+exp(m29+AA24)+exp(m210+AA25)+exp(m211+AA26)
+exp(m212+aa27)+exp(m213+aa28)+exp(m214+aa29)+exp(m215+aa30));
c168z=exp(m28+aa23)/(1+exp(m21+aa16)+exp(m22+aa17)+exp(m23+aa18)+exp(m24+aa19)
+exp(m25+AA20)
+exp(m26+AA21)+exp(m27+AA22)+exp(m28+AA23)+exp(m29+AA24)+exp(m210+AA25)+exp(m211+AA26)
+exp(m212+aa27)+exp(m213+aa28)+exp(m214+aa29)+exp(m215+aa30));
c169z=exp(m29+aa24)/(1+exp(m21+aa16)+exp(m22+aa17)+exp(m23+aa18)+exp(m24+aa19)
+exp(m25+AA20)
+exp(m26+AA21)+exp(m27+AA22)+exp(m28+AA23)+exp(m29+AA24)+exp(m210+AA25)+exp(m211+AA26)
+exp(m212+aa27)+exp(m213+aa28)+exp(m214+aa29)+exp(m215+aa30));
c1610z=exp(m210+aa25)/(1+exp(m21+aa16)+exp(m22+aa17)+exp(m23+aa18)+exp(m24+aa19)
+exp(m25+AA20)
+exp(m26+AA21)+exp(m27+AA22)+exp(m28+AA23)+exp(m29+AA24)+exp(m210+AA25)+exp(m211+AA26)
+exp(m212+aa27)+exp(m213+aa28)+exp(m214+aa29)+exp(m215+aa30));
c1611z=exp(m211+aa26)/(1+exp(m21+aa16)+exp(m22+aa17)+exp(m23+aa18)+exp(m24+aa19)
+exp(m25+AA20)
+exp(m26+AA21)+exp(m27+AA22)+exp(m28+AA23)+exp(m29+AA24)+exp(m210+AA25)+exp(m211+AA26)
+exp(m212+aa27)+exp(m213+aa28)+exp(m214+aa29)+exp(m215+aa30));
c1612z=exp(m212+aa27)/(1+exp(m21+aa16)+exp(m22+aa17)+exp(m23+aa18)+exp(m24+aa19)
+exp(m25+AA20)
+exp(m26+AA21)+exp(m27+AA22)+exp(m28+AA23)+exp(m29+AA24)+exp(m210+AA25)+exp(m211+AA26)
+exp(m212+aa27)+exp(m213+aa28)+exp(m214+aa29)+exp(m215+aa30));
c1613z=exp(m213+aa28)/(1+exp(m21+aa16)+exp(m22+aa17)+exp(m23+aa18)+exp(m24+aa19)
+exp(m25+AA20)
+exp(m26+AA21)+exp(m27+AA22)+exp(m28+AA23)+exp(m29+AA24)+exp(m210+AA25)+exp(m211+AA26)
+exp(m212+aa27)+exp(m213+aa28)+exp(m214+aa29)+exp(m215+aa30));
c1614z=exp(m214+aa29)/(1+exp(m21+aa16)+exp(m22+aa17)+exp(m23+aa18)+exp(m24+aa19)
+exp(m25+AA20)
+exp(m26+AA21)+exp(m27+AA22)+exp(m28+AA23)+exp(m29+AA24)+exp(m210+AA25)+exp(m211+AA26)
+exp(m212+aa27)+exp(m213+aa28)+exp(m214+aa29)+exp(m215+aa30));
```

```
c1615z=exp(m215+aa30)/(1+exp(m21+aa16)+exp(m22+aa17)+exp(m23+aa18)+exp(m24+aa19)
+exp(m25+AA20)
+exp(m26+AA21)+exp(m27+AA22)+exp(m28+AA23)+exp(m29+AA24)+exp(m210+AA25)+exp(m211+AA26)
+exp(m212+aa27)+exp(m213+aa28)+exp(m214+aa29)+exp(m215+aa30));
c1616z=1/(1+exp(m21+aa16)+exp(m22+aa17)+exp(m23+aa18)+exp(m24+aa19)+exp(m25+aa20)
+exp(m26+AA21)+exp(m27+AA22)+exp(m28+AA23)+exp(m29+AA24)+exp(m210+AA25)+exp(m211+AA26)
+exp(m212+aa27)+exp(m213+aa28)+exp(m214+aa29)+exp(m215+aa30));

!16 post-test attribute profile proportions
NEW(cpost1 cpost2 cpost3 cpost4 cpost5 cpost6 cpost7 cpost8 cpost9 cpost10 cpost11
cpost12 cpost13 cpost14 cpost15 cpost16);

cpost1=cpre1*c11a+cpre2*c21b+cpre3*c31e+cpre4*c41f+cpre5*c51h+cpre6*c61i+cpre7*c71j+
cpre8*c81k+cpre9*c91n+cpre10*c101o+cpre11*c111q+cpre12*c121u+cpre13*c131v+cpre14*c141w+
cpre15*c151y+cpre16*c161z;

cpost2=cpre1*c12a+cpre2*c22b+cpre3*c32e+cpre4*c42f+cpre5*c52h+cpre6*c62i+cpre7*c72j+
cpre8*c82k+cpre9*c92n+cpre10*c102o+cpre11*c112q+cpre12*c122u+cpre13*c132v+cpre14*c142w+
cpre15*c152y+cpre16*c162z;

cpost3=cpre1*c13a+cpre2*c23b+cpre3*c33e+cpre4*c43f+cpre5*c53h+cpre6*c63i+cpre7*c73j+
cpre8*c83k+cpre9*c93n+cpre10*c103o+cpre11*c113q+cpre12*c123u+cpre13*c133v+cpre14*c143w+
cpre15*c153y+cpre16*c163z;

cpost4=cpre1*c14a+cpre2*c24b+cpre3*c34e+cpre4*c44f+cpre5*c54h+cpre6*c64i+cpre7*c74j+
cpre8*c84k+cpre9*c94n+cpre10*c104o+cpre11*c114q+cpre12*c124u+cpre13*c134v+cpre14*c144w+
cpre15*c154y+cpre16*c164z;

cpost5=cpre1*c15a+cpre2*c25b+cpre3*c35e+cpre4*c45f+cpre5*c55h+cpre6*c65i+cpre7*c75j+
cpre8*c85k+cpre9*c95n+cpre10*c105o+cpre11*c115q+cpre12*c125u+cpre13*c135v+cpre14*c145w+
cpre15*c155y+cpre16*c165z;
```

cpost6=cpre1*c16a+cpre2*c26b+cpre3*c36e+cpre4*c46f+cpre5*c56h+cpre6*c66i+cpre7*c76j+
cpre8*c86k+cpre9*c96n+cpre10*c106o+cpre11*c116q+cpre12*c126u+cpre13*c136v+cpre14*c146w+
cpre15*c156y+cpre16*c166z;

cpost7=cpre1*c17a+cpre2*c27b+cpre3*c37e+cpre4*c47f+cpre5*c57h+cpre6*c67i+cpre7*c77j+
cpre8*c87k+cpre9*c97n+cpre10*c107o+cpre11*c117q+cpre12*c127u+cpre13*c137v+cpre14*c147w+
cpre15*c157y+cpre16*c167z;

cpost8=cpre1*c18a+cpre2*c28b+cpre3*c38e+cpre4*c48f+cpre5*c58h+cpre6*c68i+cpre7*c78j+
cpre8*c88k+cpre9*c98n+cpre10*c108o+cpre11*c118q+cpre12*c128u+cpre13*c138v+cpre14*c148w+
cpre15*c158y+cpre16*c168z;

cpost9=cpre1*c19a+cpre2*c29b+cpre3*c39e+cpre4*c49f+cpre5*c59h+cpre6*c69i+cpre7*c79j+
cpre8*c89k+cpre9*c99n+cpre10*c109o+cpre11*c119q+cpre12*c129u+cpre13*c139v+cpre14*c149w+
cpre15*c159y+cpre16*c169z;

cpost10=cpre1*c110a+cpre2*c210b+cpre3*c310e+cpre4*c410f+cpre5*c510h+cpre6*c610i+
cpre7*c710j+cpre8*c810k+cpre9*c910n+cpre10*c1010o+cpre11*c1110q+cpre12*c1210u+
cpre13*c1310v+cpre14*c1410w+cpre15*c1510y+cpre16*c1610z;

cpost11=cpre1*c111a+cpre2*c211b+cpre3*c311e+cpre4*c411f+cpre5*c511h+cpre6*c611i
+cpre7*c711j+cpre8*c811k+cpre9*c911n+cpre10*c1011o+cpre11*c1111q+cpre12*c1211u+
cpre13*c1311v+cpre14*c1411w+cpre15*c1511y+cpre16*c1611z;

cpost12=cpre1*c112a+cpre2*c212b+cpre3*c312e+cpre4*c412f+cpre5*c512h+cpre6*c612i
+cpre7*c712j+cpre8*c812k+cpre9*c912n+cpre10*c1012o+cpre11*c1112q+cpre12*c1212u+
cpre13*c1312v+cpre14*c1412w+cpre15*c1512y+cpre16*c1612z;

cpost13=cpre1*c113a+cpre2*c213b+cpre3*c313e+cpre4*c413f+cpre5*c513h+cpre6*c613i
+cpre7*c713j+cpre8*c813k+cpre9*c913n+cpre10*c1013o+cpre11*c1113q+cpre12*c1213u+
cpre13*c1313v+cpre14*c1413w+cpre15*c1513y+cpre16*c1613z;

```
cpost14=cpre1*c114a+cpre2*c214b+cpre3*c314e+cpre4*c414f+cpre5*c514h+cpre6*c614i
+cpre7*c714j+cpre8*c814k+cpre9*c914n+cpre10*c1014o+cpre11*c1114q+cpre12*c1214u+
cpre13*c1314v+cpre14*c1414w+cpre15*c1514y+cpre16*c1614z;

cpost15=cpre1*c115a+cpre2*c215b+cpre3*c315e+cpre4*c415f+cpre5*c515h+cpre6*c615i
+cpre7*c715j+cpre8*c815k+cpre9*c915n+cpre10*c1015o+cpre11*c1115q+cpre12*c1215u
+cpre13*c1315v+cpre14*c1415w+cpre15*c1515y+cpre16*c1615z;

cpost16=cpre1*c116a+cpre2*c216b+cpre3*c316e+cpre4*c416f+cpre5*c516h+cpre6*c616i
+cpre7*c716j+cpre8*c816k+cpre9*c916n+cpre10*c1016o+cpre11*c1116q+cpre12*c1216u+
cpre13*c1316v+cpre14*c1416w+cpre15*c1516y+cpre16*c1616z;

!Wald tests for growth in attribute mastery for control group
NEW(cgrowth1 cgrowth2 cgrowth3 cgrowth4);

cgrowth1 = (cpost9+cpost10+cpost11+cpost12+cpost13+cpost14+cpost15+cpost16) -
(cpre9+cpre10+cpre11+cpre12+cpre13+cpre14+cpre15+cpre16);

cgrowth2 = (cpost5+cpost6+cpost7+cpost8+cpost13+cpost14+cpost15+cpost16) -
(cpre5+cpre6+cpre7+cpre8+cpre13+cpre14+cpre15+cpre16);

cgrowth3 = (cpost3+cpost4+cpost7+cpost8+cpost11+cpost12+cpost15+cpost16) -
(cpre3+cpre4+cpre7+cpre8+cpre11+cpre12+cpre15+cpre16);

cgrowth4 = (cpost2+cpost4+cpost6+cpost8+cpost10+cpost12+cpost14+cpost16) -
(cpre2+cpre4+cpre6+cpre8+cpre10+cpre12+cpre14+cpre16);


!!!!!!!!!!!!!!!!!!!!!!!!!!!!!!!!!!!!!!!!!!!!!!!!!!!!!!!!!!!!!!!!!!!!!!!!!!!!!!!!!!!
!!! Wald test for differential growth: (treatment - control)
NEW(diff1 diff2 diff3 diff4);
diff1 = tgrowth1 - cgrowth1;
```

```
diff2 = tgrowth2 - cgrowth2;
diff3 = tgrowth3 - cgrowth3;
diff4 = tgrowth4 - cgrowth4;


! Specify two-way structural model
!Pre-test
  NEW(G_10 G_111 G_112 G_113 G_114 G_1212 G_1213 G_1214 G_1223 G_1224 G_1234);
  m11 =  - (G_111+G_112+G_113+G_114+G_1212+G_1213+G_1214+G_1223+G_1224+G_1234);
  m12 = G_114 - (G_111+G_112+G_113+G_114+G_1212+G_1213+G_1214+G_1223+G_1224+G_1234);
  m13 = G_113 - (G_111+G_112+G_113+G_114+G_1212+G_1213+G_1214+G_1223+G_1224+G_1234);
  m14 = G_113+G_114+G_1234 - (G_111+G_112+G_113+G_114+G_1212+G_1213+G_1214+G_1223+
  G_1224+G_1234);
  m15 = G_112 - (G_111+G_112+G_113+G_114+G_1212+G_1213+G_1214+G_1223+G_1224+G_1234);
  m16 = G_112+G_114+G_1224 - (G_111+G_112+G_113+G_114+G_1212+G_1213+G_1214+G_1223+
  G_1224+G_1234);
  m17 = G_112+G_113+G_1223 - (G_111+G_112+G_113+G_114+G_1212+G_1213+G_1214+G_1223+
  G_1224+G_1234);
  m18 = G_112+G_113+G_114+G_1223+G_1224+G_1234 - (G_111+G_112+G_113+G_114+G_1212+
  G_1213+G_1214+G_1223+G_1224+G_1234);
  m19 = G_111 - (G_111+G_112+G_113+G_114+G_1212+G_1213+G_1214+G_1223+G_1224+G_1234);
  m110 = G_111+G_114+G_1214 - (G_111+G_112+G_113+G_114+G_1212+G_1213+G_1214+G_1223+
  G_1224+G_1234);
  m111 = G_111+G_113+G_1213 - (G_111+G_112+G_113+G_114+G_1212+G_1213+G_1214+G_1223+
  G_1224+G_1234);
  m112 = G_111+G_113+G_114+G_1213+G_1214+G_1234 - (G_111+G_112+G_113+G_114+G_1212+
  G_1213+G_1214+G_1223+G_1224+G_1234);
  m113 = G_111+G_112+G_1212 - (G_111+G_112+G_113+G_114+G_1212+G_1213+G_1214+G_1223+
  G_1224+G_1234);
  m114 = G_111+G_112+G_114+G_1212+G_1214+G_1224 - (G_111+G_112+G_113+G_114+G_1212+
  G_1213+G_1214+G_1223+G_1224+G_1234);
  m115 = G_111+G_112+G_113+G_1212+G_1213+G_1223 - (G_111+G_112+G_113+G_114+G_1212+
```

```
  G_1213+G_1214+G_1223+G_1224+G_1234);
  G_10 = - (G_111+G_112+G_113+G_114+G_1212+G_1213+G_1214+G_1223+G_1224+G_1234);

!Post-test
  NEW(G_20 G_211 G_212 G_213 G_214 G_2212 G_2213 G_2214 G_2223 G_2224 G_2234);
  m21 =  - (G_211+G_212+G_213+G_214+G_2212+G_2213+G_2214+G_2223+G_2224+G_2234);
  m22 = G_214 - (G_211+G_212+G_213+G_214+G_2212+G_2213+G_2214+G_2223+G_2224+G_2234);
  m23 = G_213 - (G_211+G_212+G_213+G_214+G_2212+G_2213+G_2214+G_2223+G_2224+G_2234);
  m24 = G_213+G_214+G_2234 - (G_211+G_212+G_213+G_214+G_2212+G_2213+G_2214+G_2223+
  G_2224+G_2234);
  m25 = G_212 - (G_211+G_212+G_213+G_214+G_2212+G_2213+G_2214+G_2223+G_2224+G_2234);
  m26 = G_212+G_214+G_2224 - (G_211+G_212+G_213+G_214+G_2212+G_2213+G_2214+G_2223+
  G_2224+G_2234);
  m27 = G_212+G_213+G_2223 - (G_211+G_212+G_213+G_214+G_2212+G_2213+G_2214+G_2223+
  G_2224+G_2234);
  m28 = G_212+G_213+G_214+G_2223+G_2224+G_2234 - (G_211+G_212+G_213+G_214+G_2212+
  G_2213+G_2214+G_2223+G_2224+G_2234);
  m29 = G_211 - (G_211+G_212+G_213+G_214+G_2212+G_2213+G_2214+G_2223+G_2224+G_2234);
  m210 = G_211+G_214+G_2214 - (G_211+G_212+G_213+G_214+G_2212+G_2213+G_2214+G_2223+
  G_2224+G_2234);
  m211 = G_211+G_213+G_2213 - (G_211+G_212+G_213+G_214+G_2212+G_2213+G_2214+G_2223+
  G_2224+G_2234);
  m212 = G_211+G_213+G_214+G_2213+G_2214+G_2234 - (G_211+G_212+G_213+G_214+G_2212+
  G_2213+G_2214+G_2223+G_2224+G_2234);
  m213 = G_211+G_212+G_2212 - (G_211+G_212+G_213+G_214+G_2212+G_2213+G_2214+G_2223+
  G_2224+G_2234);
  m214 = G_211+G_212+G_214+G_2212+G_2214+G_2224 - (G_211+G_212+G_213+G_214+G_2212+
  G_2213+G_2214+G_2223+G_2224+G_2234);
  m215 = G_211+G_212+G_213+G_2212+G_2213+G_2223 - (G_211+G_212+G_213+G_214+G_2212+
  G_2213+G_2214+G_2223+G_2224+G_2234);
  G_20 = - (G_211+G_212+G_213+G_214+G_2212+G_2213+G_2214+G_2223+G_2224+G_2234);
```

```
SAVEDATA: ! Format, name of posterior probabilities of class membership file
FORMAT = F10.5;
FILE = prepost1_out.txt;
SAVE = CPROBABILITIES;
```

APPENDIX G

CHAPTER 3 SUPPLEMENTARY MATERIALS

Power of Wald Test of Group-differential Growth after Multiple Tests Correction in Small Sample Size Condition at $\alpha = .10$

Pre-test Base-rates	Attribute Mastery Growth		Multiple Tests Correction Method											
			Bonferroni				Šidák				Hochberg			
	Control	Treatment	Att 1	Att 2	Att 3	Att 4	Att 1	Att 2	Att 3	Att 4	Att 1	Att 2	Att 3	Att 4
Same	.00	.10	.479	.479	.354	.458	.500	.479	.354	.458	.625	.625	.458	.583
Same	.00	.20	1	.980	.940	.940	1	.980	.940	.940	1	1	.960	.980
Same	.00	.30	1	1	1	1	1	1	1	1	1	1	1	1
Same	.10	.20	.500	.440	.300	.400	.520	.480	.300	.400	.620	.560	.480	.560
Same	.10	.30	1	1	.960	.960	1	1	.960	.960	1	1	.980	.960
Same	.20	.30	.646	.417	.438	.375	.646	.438	.438	.375	.708	.563	.583	.542
Different	.00	.10	.449	.510	.449	.347	.449	.510	.449	.367	.490	.592	.571	.571
Different	.00	.20	.959	.959	.939	.959	.959	.959	.939	.959	1	1	1	1
Different	.00	.30	1	1	1	1	1	1	1	1	1	1	1	1
Different	.10	.20	.429	.571	.429	.408	.429	.592	.469	.408	.571	.633	.571	.571
Different	.10	.30	1	.979	.894	.979	1	.979	.894	.979	1	1	.979	1
Different	.20	.30	.460	.540	.400	.340	.480	.540	.400	.340	.580	.620	.540	.540

Note. Group sample sizes were 50. Att = attribute. In same base-rate conditions, both group had base-rates of {.10,.20,.30,.40}. In different base-rate conditions, Control and Treatment groups had base-rates of {.10, .20, .30, .40} and {.40, .30, .20, .10}, respectively.

Power of Wald Test of Group-differential Growth after Multiple Tests Correction in Medium Sample Size Condition at α = .10

Pre-test Base-rates	Attribute Mastery Growth		Multiple Tests Correction Method											
			Bonferroni				Šidák				Hochberg			
	Control	Treatment	Att 1	Att 2	Att 3	Att 4	Att 1	Att 2	Att 3	Att 4	Att 1	Att 2	Att 3	Att 4
Same	.00	.10	.891	.870	.543	.609	.913	.891	.543	.609	.978	.957	.739	.804
Same	.00	.20	1	1	1	1	1	1	1	1	1	1	1	1
Same	.00	.30	1	1	1	1	1	1	1	1	1	1	1	1
Same	.10	.20	.875	.750	.688	.688	.875	.771	.688	.688	.896	.833	.813	.813
Same	.10	.30	1	1	1	1	1	1	1	1	1	1	1	1
Same	.20	.30	.816	.694	.735	.673	.837	.694	.735	.673	.918	.776	.857	.776
Different	.00	.10	.735	.714	.735	.776	.755	.714	.735	.796	.857	.837	.857	.878
Different	.00	.20	1	1	1	1	1	1	1	1	1	1	1	1
Different	.00	.30	1	1	1	1	1	1	1	1	1	1	1	1
Different	.10	.20	.792	.667	.729	.688	.792	.688	.729	.688	.896	.833	.854	.813
Different	.10	.30	1	1	1	1	1	1	1	1	1	1	1	1
Different	.20	.30	.780	.740	.700	.660	.780	.760	.700	.680	.860	.880	.820	.820

Note. Group sample sizes were 50. Att = attribute. In same base-rate conditions, both group had base-rates of {.10,.20,.30,.40}. In different base-rate conditions, Control and Treatment groups had base-rates of {.10, .20, .30, .40} and {.40, .30, .20, .10}, respectively.

Power of Wald Test of Group-differential Growth after Multiple Tests Correction in Large Sample Size Condition at α = .10

Pre-test Base-rates	Attribute Mastery Growth		Multiple Tests Correction Method											
			Bonferroni				Šidák				Hochberg			
	Control	Treatment	Att 1	Att 2	Att 3	Att 4	Att 1	Att 2	Att 3	Att 4	Att 1	Att 2	Att 3	Att 4
Same	.00	.10	1	.959	1	.959	1	.959	1	.959	1	.980	1	1
Same	.00	.20	1	1	1	1	1	1	1	1	1	1	1	1
Same	.00	.30	1	1	1	1	1	1	1	1	1	1	1	1
Same	.10	.20	1	.958	.979	.917	1	.958	.979	.917	1	.979	1	.979
Same	.10	.30	1	1	1	1	1	1	1	1	1	1	1	1
Same	.20	.30	.980	1	.980	.940	.980	1	.980	.960	.980	1	1	.980
Different	.00	.10	.980	.920	1	.980	.980	.920	1	.980	1	1	1	.980
Different	.00	.20	1	1	1	1	1	1	1	1	1	1	1	1
Different	.00	.30	1	1	1	1	1	1	1	1	1	1	1	1
Different	.10	.20	.980	1	.959	.939	.980	1	.959	.939	1	1	1	.980
Different	.10	.30	1	1	1	1	1	1	1	1	1	1	1	1
Different	.20	.30	.980	.900	.940	.980	.980	.900	.940	.980	1	.980	.980	1

Note. Group sample sizes were 50. Att = attribute. In same base-rate conditions, both group had base-rates of {.10,.20,.30,.40}. In different base-rate conditions, Control and Treatment groups had base-rates of {.10, .20, .30, .40} and {.40, .30, .20, .10}, respectively.