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Robotics Potential Fields

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Abstract. This problem was to calculate the path a robot would take to navigate an obstacle field and get to its goal. Three obstacles were given as negative potential fields which the robot avoided, and a goal was given a positive potential field that attracted the robot. The robot decided each step based on its distance, angle, and influence from every object. After each step, the robot recalculated and determined its next step until it reached its goal. The robot's calculations and steps were simulated with Microsoft Excel.

Keywords. Robotics, Potential Fields, Navigation

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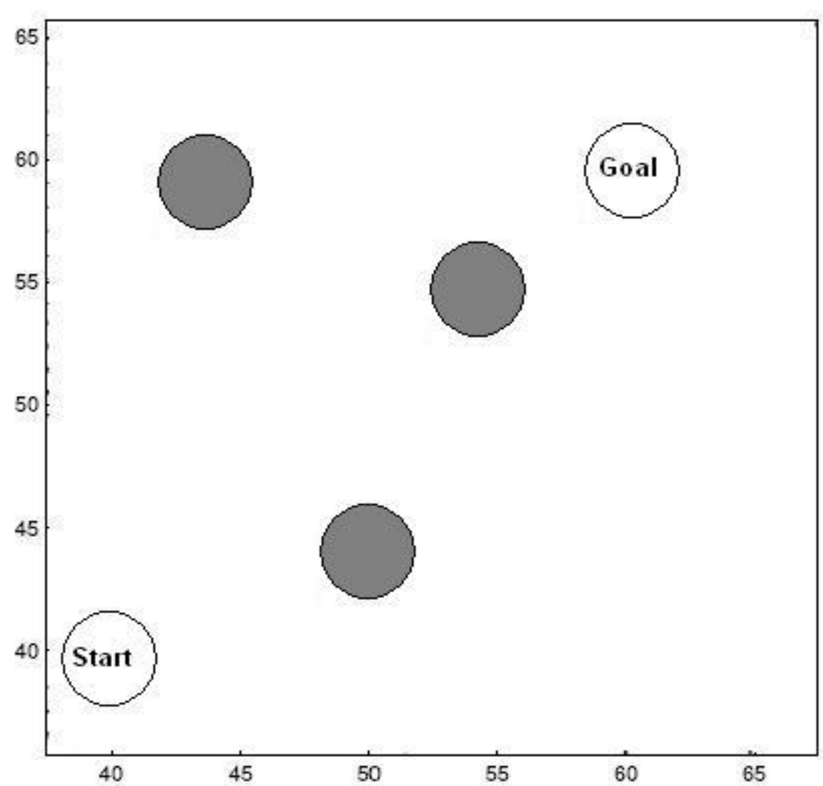
PROBLEM STATEMENT

The path was found of a robot navigating an obstacle course to arrive at a defined goal. Three obstacles were given negative potential fields and the goal was given a positive potential field. Therefore, the robot was attracted to the goal and repelled from the obstacles.

The virtual obstacle course was approximately $65m \times 65m$. All measurements were in meters and positions of objects were described as Cartesian coordinates. The starting position of the robot was at the coordinate $(40, 40)$ and the goal was at $(60, 60)$. The obstacles were located at $(45, 59)$, $(50, 44)$, and $(54, 56)$. The radius of the goal and obstacles was $2m$. The robot was capable of moving in a straight line at a maximum of $5m/step$. The robot's position was also described as a coordinate point in the obstacle course. The robot's position and the value of the potential field were calculated at each step.



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MOTIVATION

The objective of the project was to find the path a robot will take through an obstacle field to get to a goal. This is type of problem and solution is valuable to engineering because in practice, a robot may need to be programmed to decide on a path and avoid obstacles. This is useful in a rescue situation where certain obstacles could be hazardous. A robot can be used in place of a person to reduce risk for rescue personnel. The robot can also be programmed to avoid danger in a rescue operation.

MATHEMATICAL DESCRIPTION AND SOLUTION APPROACH

The following equations and parameters were given for the goal:

- The robot is referred to as the *agent*.
- Let (x_G, y_G) denote the position of the goal.
- Let r denote the radius of the goal.
- Let $\mathbf{v} = [x, y]^T$denote the (x, y) coordinate position of the agent.
- Find the distance (d) between the goal and the agent: $d = \sqrt{(x_G - x)^2 + (y_G - y)^2}$
- Find the angle (θ) between the agent and the goal: $\theta = \tan^{-1}\left(\frac{y_G - y}{x_G - x}\right)$
- The variable s can be assumed as $2m$ and the factor α represents the strength of the attraction potential of the goal.

- Set Δx and Δy according to the following:

$$\begin{cases} \Delta x = \Delta y = 0, & if \quad d < r & (1) \\ \Delta x = \alpha(d-r)(\cos\theta) \text{ and } \Delta y = \alpha(d-r)(\sin\theta), & if \; r \le d \le s+r & (2) \\ \Delta x = \alpha\, s \cos\theta \;\text{ and }\; \Delta y = \alpha\, s \sin\theta\,, & if \quad d > s+r & (3) \end{cases}$$

The following equations and parameters were given for each obstacle:

- Let (x_O, y_O) denote the position of the obstacle.
- Let r denote the radius of the obstacle.
- Let $\mathbf{v} = [x, y]^T$ denote the (x, y) coordinate position of the agent.
- Find the distance (d) between the agent and the obstacle: $d = \sqrt{(x_O - x)^2 + (y_O - y)^2}$
- Find the angle (θ) between the agent and the obstacle: $\theta = \tan^{-1}\left(\frac{y_O - y}{x_O - x}\right)$
- The variable s can be assumed as $2m$. The factor β represents the strength of the repulsion potential of each obstacle.
- Set Δx and Δy according to the following:

$$\begin{cases} \Delta x = -\text{sign}(\cos\theta)\,\infty \;\; and \;\; \Delta y = -\text{sign}(\sin\theta)\,\infty, & if \quad d < r & (4) \\ \Delta x = -\beta(s+r-d)(\cos\theta) \; and \;\; \Delta y = -\beta(s+r-d)(\sin\theta), & if \; r \le d \le s+r & (5) \\ \Delta x = \Delta y = 0, & if \quad d > s+r & (6) \end{cases}$$

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Discussion

The distance and angle between the robot and the goal were calculated first. Note that Δx indicates a change in the x direction and Δy indicates a change in the y direction. Variables s and r were both given as 2 meters. The length $s + r$ was considered as the radius of influence of the potential field of the goal. The strength of the goal's potential field was given as $\alpha = 1$. Equation (1) above described the behavior of the robot once it has reached its goal. If the distance to the goal (d) was less than the radius (r), the robot did not move in the x or y direction because it had arrived at its goal. Equation (2) above described the behavior of the robot when it was near its goal. If d was greater than r and less than the radius of influence of the goal, then the Δx and Δy values were scaled by the factor $(d - r)$. That is, the Δx and Δy values became smaller as the robot approached its goal. Equation (3) above described the behavior of the robot when it was farthest from its goal. The Δx and Δy values were scaled by the fixed factor $s = 2m$.

For each obstacle, the sign of the Δx and Δy values were opposite from the analogous equations with the goal because the robot was repelled from the obstacles rather than attracted. For each obstacle, the strength of the potential field was given as $\beta = 1$. Equation (4) above described the behavior of the robot if it collided with an obstacle. If distance from the obstacle (d) was less than the radius (r), the robot had reached a collision with the obstacle. In this case, the Δx and Δy values were maximized in the direction opposite to the obstacle. This was evaluated as the opposite of the sign (positive or negative) of the cosine or sine of the incident angle between the robot and the obstacle (θ) multiplied by infinity. The maximum step the robot can take was limited to 5 meters. In the event of a collision with an obstacle, the robot would make a maximum step away from the obstacle. Equation (5) above described the behavior of the robot when it is near an obstacle. If d was greater than r and less than the radius of influence of the obstacle, then the Δx and Δy values were scaled by the factor $(s + r - d)$. That is, the Δx and Δy values became greater as the robot approaches an obstacle. Equation (6) above described the behavior of the robot when it was farthest from its goal. When the robot was outside of an obstacle's radius of influence, it was not affected at all.

The robot's calculations and path were simulated using Microsoft Excel. Nested IF statements were used to evaluate the Δx and Δy values with respect to each object. The Δx and Δy values were computed for each obstacle and the goal and then added together to determine the magnitude of the robot's step in both the x and y direction and the coordinates of the robot's new position. The Δx and Δy values may be considered as the influence each object in the obstacle field exerted on the robot. After each step, the calculations are repeated and another step taken until the robot reached its goal.

Conclusion and Recommendations

Using the formulas described above, the path of the robot was plotted. The robot followed a path directly towards the goal for nine consecutive steps. On the 10^{th} step the 3^{rd} obstacle significantly influenced the path directing the robot around it. After correcting the course to navigate around the 3^{rd} obstacle, the path continued on to the goal. The diagram below plots the steps of the robot in the virtual obstacle course. The radius of influence of the potential fields of each obstacle and the goal highlight its diagram. The calculated positions and angles are included in the appendices.

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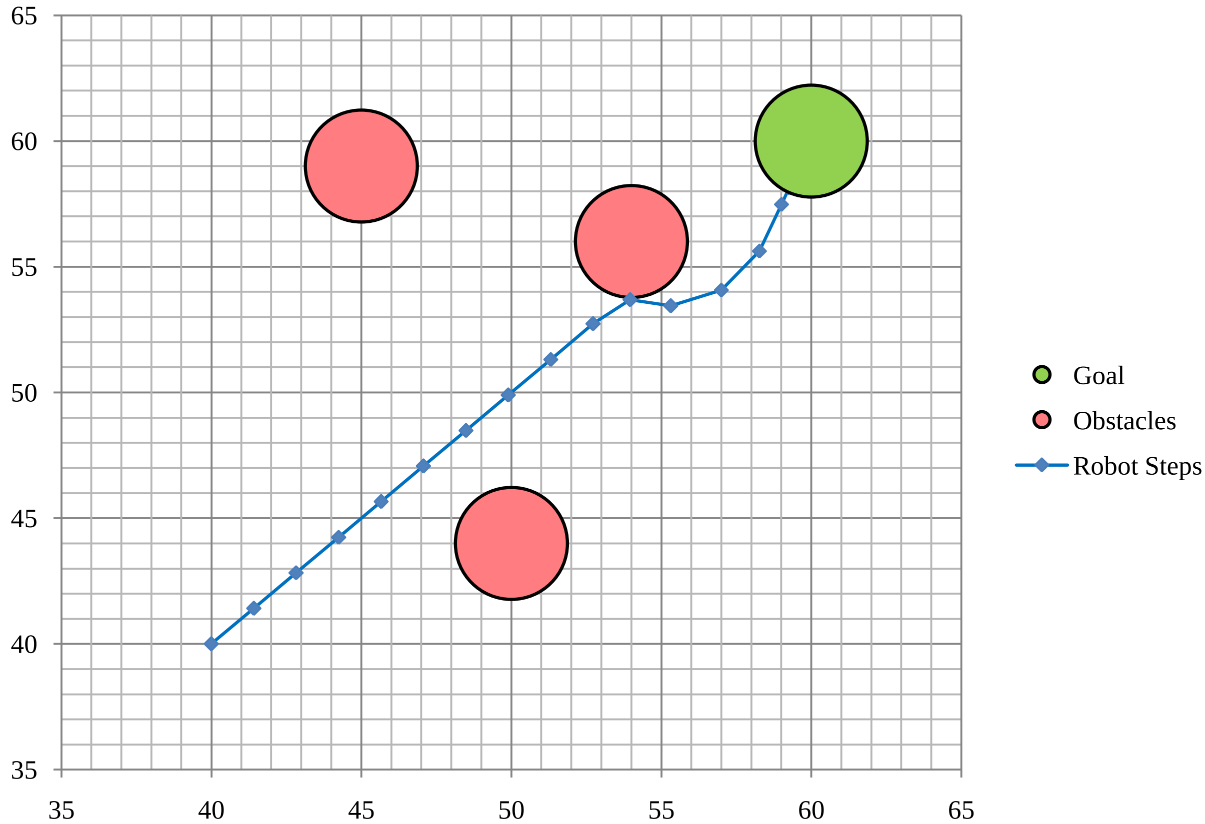
65
60
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Goal
Obstacles
Robot Steps

APPENDICES

		Robot Step Number							
		0	1	2	3	4	5	6	7
	X Coordinate	40.00	41.41	42.83	44.24	45.66	47.07	48.49	49.90
	Y Coordinate	40.00	41.41	42.83	44.24	45.66	47.07	48.49	49.90
Goal	Distance	28.28	26.28	24.28	22.28	20.28	18.28	16.28	14.28
	Angle	0.79	0.79	0.79	0.79	0.79	0.79	0.79	0.79
	ΔX_G	1.41	1.41	1.41	1.41	1.41	1.41	1.41	1.41
	ΔY_G	1.41	1.41	1.41	1.41	1.41	1.41	1.41	1.41
1^{st} Obstacle	Distance	19.65	17.95	16.32	14.78	13.36	12.11	11.08	10.34
	Angle	1.31	1.37	1.44	1.52	1.62	1.74	1.89	2.06
	ΔX_{O1}	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00
	ΔY_{O1}	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00
2^{nd} Obstacle	Distance	10.77	8.97	7.27	5.76	4.65	4.24	4.73	5.90
	Angle	0.38	0.29	0.16	-0.04	-0.36	-0.81	-1.25	-1.55
	ΔX_{O2}	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00
	ΔY_{O2}	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00
3^{rd} Obstacle	Distance	21.26	19.26	17.27	15.28	13.29	11.30	9.32	7.35
	Angle	0.85	0.86	0.87	0.88	0.89	0.91	0.94	0.98
	ΔX_{O3}	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00
	ΔY_{O3}	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00
	Total ΔX	1.41	1.41	1.41	1.41	1.41	1.41	1.41	1.41
	Total ΔY	1.41	1.41	1.41	1.41	1.41	1.41	1.41	1.41

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		Robot Step Number							
		8	9	10	11	12	13	14	15
	X Coordinate	51.31	52.73	53.96	55.32	57.00	58.27	59.01	59.26
	Y Coordinate	51.31	52.73	53.69	53.45	54.08	55.62	57.48	58.14
Goal	Distance	12.28	10.28	8.73	8.05	6.64	4.71	2.71	2.00
	Angle	0.79	0.79	0.81	0.95	1.10	1.19	1.19	1.19
	ΔX_G	1.41	1.41	1.38	1.16	0.90	0.74	0.26	0
	ΔY_G	1.41	1.41	1.45	1.63	1.78	1.86	0.66	0
1st Obstacle	Distance	9.95	9.95	10.42	11.72	12.97	13.69	14.09	14.29
	Angle	2.26	2.46	2.61	2.65	2.75	2.89	3.03	3.08
	ΔX_{O1}	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00
	ΔY_{O1}	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00
2nd Obstacle	Distance	7.43	9.14	10.47	10.84	12.27	14.27	16.21	16.90
	Angle	-1.75	-1.87	-1.96	-2.08	-2.18	-2.19	-2.16	-2.15
	ΔX_{O2}	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00
	ΔY_{O2}	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00
3rd Obstacle	Distance	5.40	3.51	2.31	2.88	3.56	4.29	5.22	5.68
	Angle	1.05	1.20	1.56	2.05	2.57	3.05	-2.85	-2.76
	ΔX_{O3}	0.00	-0.18	-0.03	0.52	0.37	0.00	0.00	0.00
	ΔY_{O3}	0.00	-0.46	-1.69	-1.00	-0.24	0.00	0.00	0.00
	Total ΔX	1.41	1.24	1.36	1.68	1.27	0.74	0.26	0.00
	Total ΔY	1.41	0.96	-0.24	0.23	1.55	1.86	0.66	0.00