Automotive Engineering

A.Y. 2024/2025

Populsion systems and their application to Vehicles

Applied lectures reports

Students:

Walter Maggio (S343988)

Giuseppe Maria Marchese (S348145)

Contents

Report 1

Flywheel dimensioning

1.1 Introduction

The engine flywheel dimensioning is the subject of this report. Its aim is to regularize the crankshaft speed while not affecting the engine torque or power. It is designed based on the forces acting on the piston and the consequent turning moment acting on the crankshaft.

In this report the analysis starts considering a steady-state engine operating point at full load in an SI naturally aspirated engine with port fuel injection (fuel and air are inducted together into the cylinder through the intake valve, during the intake phase).

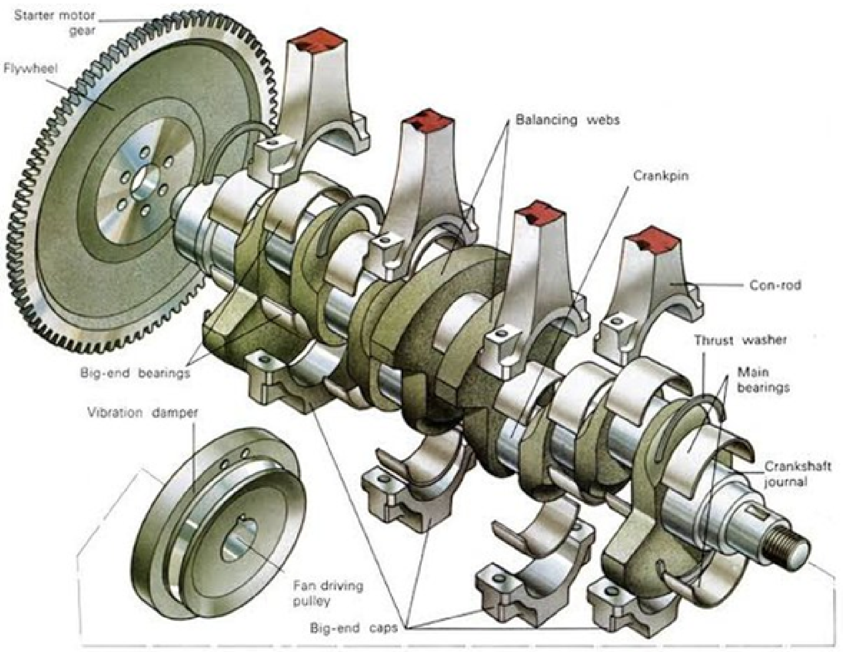

Figure 1.1: Flywheel

The calculation will be implemented firstly for one cylinder, then the dissertation will be expanded for a 4-cylinder engine.

This report comprehend 6 steps:

1. analysis of the thermodynamic cycle;
2. calculation of IMEP and crankshaft moment;
3. calculation of the flywheel diameter (1 cylinder);
4. calculation of the instantaneous crankshaft speed (4 cylinders);
5. calculation of the shaft and resistant moment (4 cylinders);
6. calculation of the flywheel diameter (4 cylinders).

The requirements are the following:

1. numerical results for pressure and temperature of points 1 to 4, lower heating value at temperature T2, IMEP, dynamic irregularity and flywheel diameter for the single-cylinder as well as for the multi-cylinder engine;
2. required charts (both for the single-cylinder engine and for the multicylinder engine):
   - in-cylinder pressure vs swept volume (a-f cycle on p-V diagram);
   - effective pressure $p_{eff}(\theta)$ as a function of the crank angle;
   - a-f cycle with inertia pressure (properly over-imposed);
   - tangential pressure $p_t(\theta)$ as a function of the crank angle;
   - tangential pressure $p_t(\theta)$ and resistant pressure $t_r(\theta)$, shaft work $\left(\frac{W_s}{V_d}\right)$ and resistant work $\left(\frac{W_r}{V_d}\right)$ as a function of crank angle (all in one graph);
   - instantaneous speed.

1.2 Analysis of the thermodynamic cycle

An air-fuel cycle with some modifications, to take into account losses, is considered.

The fundamental hypothesis are:

- intake and exhaust valves instantaneously open and close at dead centers.
- Compression and expansion are described as polytropic transformations.
- Combustion is diabatic with presence of dissociation.
- Combustion and blowdown occur at constant volume.
- There are no leakages.
- C_p and C_p' have constant value.

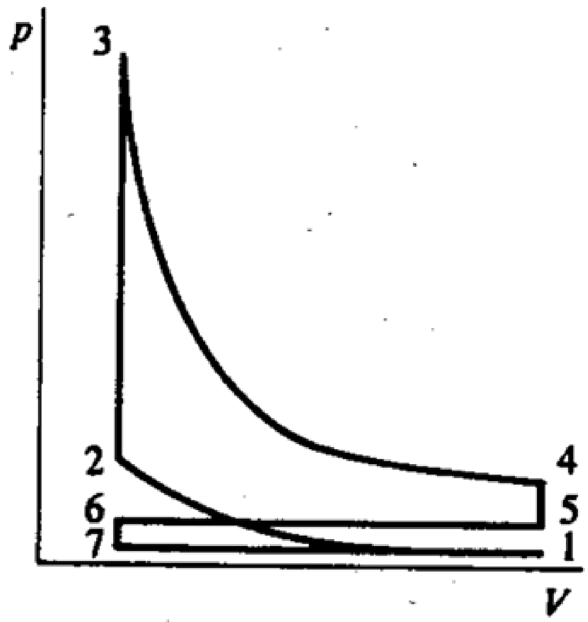

Figure 1.2: air-fuel cycle

From now on, the temperature and the pressure for point 1 to 4 of the thermodynamic cycle will be calculated, with the computation of the lower heating value on point 2.

1.2.1 Point 1

The mass in the system is considered constant, so that is possible to apply the first principle of thermodynamics in Lagrangian form for closed systems.

$$Q = W + \Delta U^* + \Delta E \tag{1.1}$$

The variation of mechanical energy is circa equal to zero for two reasons: the gravitational energy is negligible, because the gas has low density, and the variation of kinetic energy is also negligible between points 6 and 1 of the cycle.

$$\Delta E = 0$$

The work done by the system through the control surface is the sum of the work done by the piston plus the (negative) work done by the external environment

$$W = \int_6^1 p \cdot dV = \int_6^1 p_{ext} \cdot dV + \int_6^1 p_{piston} \cdot dV$$

$$W = -p_{ext} \cdot (V_a + V_f) + p_{int} \cdot (V_1 - V_c)$$

$$V_1 = V_d + V_c = V_{a,1} + V_{f,1} + V_{r,1}$$

The resultant work equation is:

$$W = -p_a \cdot (m_a \cdot v_a + m_f \cdot v_f) + p_1 \cdot (m_a v_{a,1} + m_f \cdot v_{f,1} + m_r \cdot v_{r,1} - m_r \cdot V_{r,1}) \quad (1.2)$$

The variation of internal energy is made only by the variation of temperature, because between point 6 and 1 there is no combustion.

$$u^* = u_t + u_{ch} \approx u_t$$

$$\Delta U^* = \Delta U_t = U(1) - U(6)$$

$$U(1) = m_a \cdot u_{a,1} + m_f \cdot u_{f,1} + m_r \cdot u_{r,1}; \quad U(6) = m_a \cdot u_a + m_f \cdot u_f + m_r \cdot u_r$$

The resultant equation of the variation of the internal energy is:

$$\Delta U^* = m_a \cdot (u_{a,1} - u_a) + m_f \cdot (u_{f,1} - u_f) + m_r \cdot (u_{r,1} - u_r) \quad (1.3)$$

The sum of 1.2 and 1.3 is:

$$\Delta U^* + W = m_a \cdot (u_{a,1} - u_a) + m_f \cdot (u_{f,1} - u_f) + m_r \cdot (u_{r,1} - u_r) - p_a \cdot (m_a \cdot v_a + m_f \cdot v_f) + p_1 \cdot (m_a v_{a,1} + m_f \cdot v_{f,1} + m_r \cdot v_{r,1} - m_r \cdot V_{r,1})$$

$$\Delta U^* + W = m_a \cdot [(u_{a,1} + p_1 \cdot v_{a,1})] - (u_a + p_a \cdot v_a)] + m_f \cdot [(u_{f,1} + p_1 \cdot v_{f,1}) - u_f) - (u_f - p_a \cdot v_f)] + m_r \cdot [(u_{r,1} + p_1 \cdot v_{r,1}) - (u_r + p_1 \cdot v_r)] \tag{1.4}$$

Considering the definition of enthalpy:

$$h \doteq u + p \cdot v$$

all the products can be substituted with enthalpy, except for $p_1 \cdot v_r$. For simplicity:

$$\frac{m_r}{m_1} \approx 5\% \implies p_1 \approx p_r$$

So equation 1.4 is equal to:

$$\Delta U^* + W = m_a \cdot (h_{a,1} - h_a) + m_f \cdot (h_{f,1} - h_f) + m_r \cdot (h_{r,1} - h_r)$$

where $h_{r,1} - h_r$ is the contribution of the fuel, which sees vaporization. For this reason, the contribution of latent heat (r) has to be taken into account.

$$\Delta U^* + W = m_a \cdot c_p \cdot (T_1 - T_a) + m_f \cdot c_f \cdot (T_1 - T_f) + m_r \cdot c_p' \cdot (T_1 - T_r) + m_f \cdot x \cdot r \tag{1.5}$$

Now the left hand side of the equation 1.1 is considered:

$$Q = m_a \cdot c_{p,a} \cdot \Delta T + m_f \cdot c_{p,f} \cdot \Delta T + m_r \cdot c_p' \cdot \Delta T \tag{1.6}$$

Equalling the left (eq: 1.6) to the right hand side (eq: 1.5) of the equation 1.1, the following equation is obtained:

$$T_1 = \frac{(m_a \cdot c_{p,a} + m_f \cdot c_{p,f}) \cdot T_a + m_r \cdot c_p' \cdot T_r - m_f \cdot x \cdot r}{m_a \cdot c_{p,a} + m_f \cdot c_{p,f} + m_r \cdot c_p'} + \Delta T \tag{1.7}$$

At this stage, it is essential to determine how to substitute the mass, as it has not been provided in the given data. The following equation can be considered:

$$\frac{m_a}{m_f} \doteq \alpha = \lambda \cdot \alpha_{st}; \qquad \frac{m_r}{m_f} \doteq \alpha' \tag{1.8}$$

So the equation 1.7 becomes:

$$T_1 = \frac{(\alpha \cdot c_{p,a} + c_{p,f}) \cdot T_a + \alpha' \cdot c'_p \cdot T_r - x \cdot r}{\alpha \cdot c_{p,a} + c_{p,f} + \alpha' \cdot c'_p} + \Delta T \tag{1.9}$$

The next step is to obtain a formula to evaluate α'. Considering equations 1.8 and using the definition of λ:

$$\alpha' = \frac{m_r}{m_f} = \frac{m_r}{m_a} \cdot \frac{m_a}{m_f} = \frac{m_r}{m_a} \cdot \alpha = \frac{\frac{p_r \cdot V_r}{R' \cdot T_r}}{m_a} \cdot \alpha \tag{1.10}$$

$$\lambda_v \doteq \frac{m_{a,real}}{m_{a,ideal}} = \frac{m_{a,real}}{\frac{P_a \cdot V_d}{R \cdot T_a}} \tag{1.11}$$

$$\alpha' = \alpha \cdot \frac{1}{\lambda_v} \cdot \frac{P_r}{P_a} \cdot \frac{T_a}{T_r} \cdot \frac{R}{R'} \cdot \frac{V_c}{V_d} \tag{1.12}$$

where

$$r_c = \frac{V_1}{V_c} = \frac{V_c + V_d}{V_c} \implies \frac{V_c}{V_d} = \frac{1}{r_c - 1} \tag{1.13}$$

At this point, with equations 1.9, 1.12 and 1.13 is possible to evaluate T_1. Now the steps for the calculations of p_1 starts.

Considering the conservation of mass and the gas law:

$$m_1 = m_a + m_f + m_r \implies \frac{p_1 \cdot V_1}{R_1 \cdot T_1} = m_a \cdot (1 + \frac{m_f}{m_a}) + \frac{P_r \cdot V_c}{R' \cdot T_r}$$

where, considering equations 1.10 and 1.11

$$m_a = \lambda_v \cdot \frac{p_a \cdot V_d}{R_a \cdot T_a} \quad and \quad V_c + V_d = r_c \cdot V_c$$

with some computation is possible to obtain:

$$p_1 = p_a \cdot \left[\lambda_v \cdot (r_c - 1)(\frac{1+\alpha}{\alpha}) \cdot \frac{1}{R \cdot T_a} + \frac{p_r}{p_A} \cdot \frac{1}{R' \cdot T_r} \right] \cdot \frac{R_1 \cdot T_1}{r_c} \tag{1.14}$$

The results of the calculations previously described are now reported in the following table.

Parameter	Value	Units
α	13.965	
α'	0.636	
T_1	323.05	K
p_1	0.866	bar

Table 1.1: Results for point 1 of air-fuel cycle

1.2.2 Point 2

For the second point, some considerations are made. The real transformation can be approximated using a polytropic transformation (neglecting leakages) defined by the relationship $pV^m = constant$, provided that its final point (2_{pol}) coincides with the real final point (2). This approximation is achieved by appropriately selecting the polytropic exponent m.

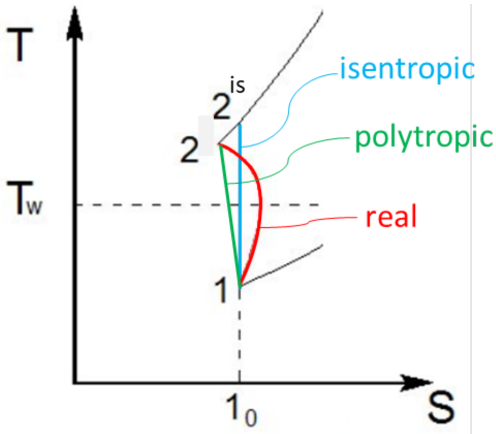

Figure 1.3: 1-2 transformation

The transformation cannot be considered isentropic because heat is exchanged between the cylinder walls and the gas, meaning $m \neq \gamma = \frac{c_p}{c_v}$.

From point 1 to point 2, the entropy decreases ($ds < 0$). Consequently, the heat transfer (δQ) along the chosen polytropic transformation must be negative. Since the temperature difference is positive ($dT > 0$, $T_2 > T_1$), the heat capacity c is negative. Therefore, the polytropic exponent m must be lower than the isentropic exponent γ, satisfying the condition $1 < m < \gamma$.

$$p \cdot v^n = constant \quad \Longrightarrow \quad p_1 \cdot v_1^n = p_2 \cdot v_2^n$$

considering the conservation of mass between point 1 and 2 ($M_1 = M_2$):

$$p_1 \cdot V_1^m = p_2 \cdot V_2^m \quad \Longrightarrow \quad p_2 = p_1 \cdot (\frac{V_1}{V_2})^m = p_1 \cdot r_c^m$$

So p_2 is obtained. Using he polytropic law, it is possible to calculate T_2, too.

$$T \cdot V^{1-m} = constant \quad \Longrightarrow \quad T_2 = T_1 \cdot r_c^{m-1}$$

To calculate the lower heating value in point 2 ($Q_{LHV}(T_2)$) it is useful to observe figure 1.4 in order to understand how to apply the first principle of thermodynamics.

Taken into consideration that it is possible to evaluate the LHV for point 0 ($Q_{LHV}(T_0)$), using the curves it is possible to write the following equations:

$$Q = W + \Delta U^* + \Delta E \quad \Longrightarrow \quad W = 0, \quad \Delta E = 0$$

$$m_f \cdot Q_{LHV}(T_0) = \Delta U^* = (m_a + m_f) \cdot \Delta u^*$$

Considering that it is considered a transformation at constant temperature, it is possible to write the internal energy as:

$$\Delta u^* = \Delta u_{ch} + \Delta u_t, \quad \Delta u_t = 0 \implies \Delta u^* = \big[u_{ch}(x_b = 0) - u_{ch}(x_b = 1)\big]$$

where $u_{ch}(x_b = 1) = 0$ because all fuel is burnt.

So, the LHV at t_0 is:

$$Q_{LHV}(T_0) = (\alpha + 1) \cdot u_{ch_u} \tag{1.15}$$

where u_{ch_u} is the chemical contribution of the unburned gas.

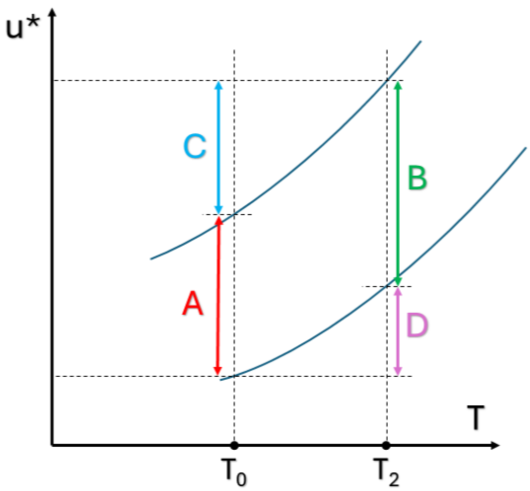

Figure 1.4: 0-2 transformation

$$A + C = B + D \implies \frac{Q_{LHV}(T_0)}{\alpha + 1} + c_v \cdot (T_2 - T_0) = \frac{Q_{LHV}(T_2)}{\alpha + 1} + c_v' \cdot (T_2 - T_0)$$

and so it is possible to compute the LHV for T_2

$$(Q_{LHV})_{T_2} = (Q_{LHV})_{T_0} + (c_v - c_v') \cdot (T_2 - T_0) \cdot (\alpha + 1) \tag{1.16}$$

where $\alpha = \alpha_{st}$ (stoichiometric) because the LHV is measured in stoichiometric conditions.
In the following table the results for point 2 are reported.

Parameter	**Value**	**Units**
T_2	747.76	K
p_2	22.06	bar
$(Q_{LHV})_{T_2}$	4342.3	kJ/kg

Table 1.2: Results for point 2 of air-fuel cycle

1.2.3 Point 3

The first principle of thermodynamics in Lagrangian form is applied to the transformation 2-3, in which occurs the constant volume combustion with heat loss.

Some considerations are made:

- Presence of residuals (α').
- Dissociation.
- Reduction of the heat, transferred by combustion to the burned gases.
- A fuel-rich mixture is used.

Starting from ideal conditions (complete combustion, adiabatic and without dissociation):

$$Q = 0, W = 0, \Delta E = 0 \quad \implies \quad \Delta U^* = 0 \tag{1.17}$$

$$\frac{m_f}{m_1} \cdot Q_{LHV}(T_2) = c_v' \cdot (T_3^{ID} - T_2)$$

Considering the conservation of mass ($m_1 = m_a + m_f + m_r$):

$$\frac{Q_{LHV}(T_2)}{\frac{m_a}{m_f} + \frac{m_f}{m_f} + \frac{m_r}{m_f}} = c_v' \cdot (T_3^{ID} - T_2) \quad \implies \quad \frac{Q_{LHV}(T_2)}{\alpha + 1 + \alpha'} = c_v' \cdot (T_3^{ID} - T_2) \tag{1.18}$$

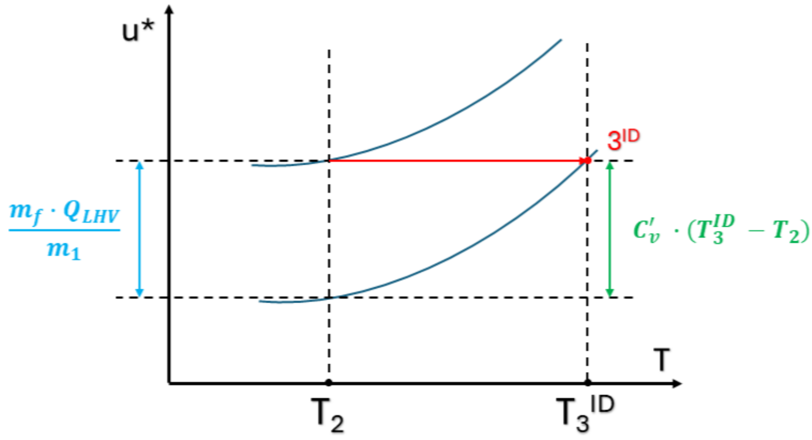

Figure 1.5: 2-3 ideal transformation

From now on, real combustion considerations are added. The diabatic contribution Q and the dissociation contribution Q_d are shown in figure 1.6. This is added to the incomplete combustion effect in a single term, δ_A.

At $T_3 > T* = 1950K$ dissociation occurs, and the contribution, which is taken into account only if $T_3 > T*$, has been experimentally evaluated as follows:

$$Q_d = dq \cdot (T_3^{dd} - T^*)^2 \tag{1.19}$$

So the equation 1.18 becomes:

$$(1 - \delta_A) \cdot \frac{Q_{LHV}(T_2)}{\alpha + 1 + \alpha'} = c_v' \cdot (T_3^{ID} - T_2) + dq \cdot (T_3^{dd} - T^*)^2 \tag{1.20}$$

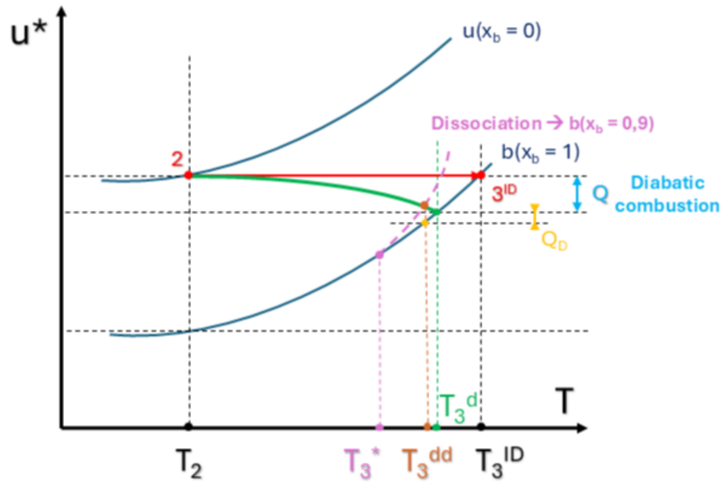

Figure 1.6: 2-3 diabatic transformation with dissociation

The last contribution is the rich-fuel mixture: $m_f = m_{f,st} + m_{f_{eccess}}$. The equation 1.20 is adjusted as follows:

$$(1 - \delta_A) \cdot \frac{Q_{LHV}(T_2)}{1 + \alpha + \alpha'} \cdot \frac{\alpha}{\alpha_{st}} = c'_v \cdot (T_3^{ID} - T_2) + dq \cdot (T_3^{dd} - T^*)^2 \tag{1.21}$$

At this point T_3 can be evaluated with the equation 1.21, extracting the square roots as follows:

$$A \cdot T_3^2 + B \cdot T_3 + C = 0$$

If $T_3 > 1850K$ the result is succesful, otherwise the calculation has to be repeated while removing the term $dq \cdot (T_3 - T^*)$

To calculate p_3 it's sufficient to use the gas and the polytropic laws as follows:

$$p_2 \cdot V_2 = M_2 \cdot R_2 \cdot T_2 \quad \Longrightarrow \quad V_2 = V_c, \quad R_2 = R_1$$

$$p_3 \cdot V_3 = M_3 \cdot R_3 \cdot T_3 \quad \Longrightarrow \quad V_3 = V_c, \quad R_3 = R'$$

Considering the conservation of mass ($M_2 = M_3$) the following equation is obtained:

$$p_3 = p_2 \cdot \frac{R'}{R_2} \cdot \frac{T_3}{T_2} \tag{1.22}$$

The following table reports the results of the calculations previously described.

Parameter	Value	Units
$c_{v,burned}$	1040	$kJ/(kg \cdot K)$
T_3	2732.9	K
p_3	85.68	bar

Table 1.3: Results for point 3 of air-fuel cycle

1.2.4 Point 4

The transformation between point 3-4 is the expansion, which can be approximated by a polytropic law, considering that the real point 4 and the polytropic one coincide.

$$p_3 \cdot V_3^{m'} = p_4 \cdot V_4^{m'} \tag{1.23}$$

$$\frac{V_4}{V_3} = r_c \qquad \Longrightarrow \qquad p_4 = p_3 \cdot \left(\frac{1}{r_c}\right)^{m'} \tag{1.24}$$

$$T_3 \cdot V_3^{m'-1} = T_4 \cdot V_4^{m'-1} \qquad \Longrightarrow \qquad T_4 = T_3 \cdot (r_c)^{1-m'} \tag{1.25}$$

Results are now reported in the table 1.4

Parameter	Value	Units
T_4	1430.4	K
p_4	4.08	bar

Table 1.4: Results for point 4 of air-fuel cycle

1.2.5 Full thermodynamic cycle

The thermodynamic cycle plot is realized using as reference the figure 1.7, where the 4 stroke are expressed in function of the crank angle (°CA). Starting from the expansion stroke at the previously calculated point 3, the pressure initially has the highest value and then it began to lower, while the piston expands the combustion chamber volume until it reaches point 4. Between exhaust and intake the pressure is taken constant, with two steps which corresponds to the opening and closure of the valves in which the pressure value is lowered a bit. Then in the compression stroke starting from point 1 reaches a higher value in point 2 to end the cycle.

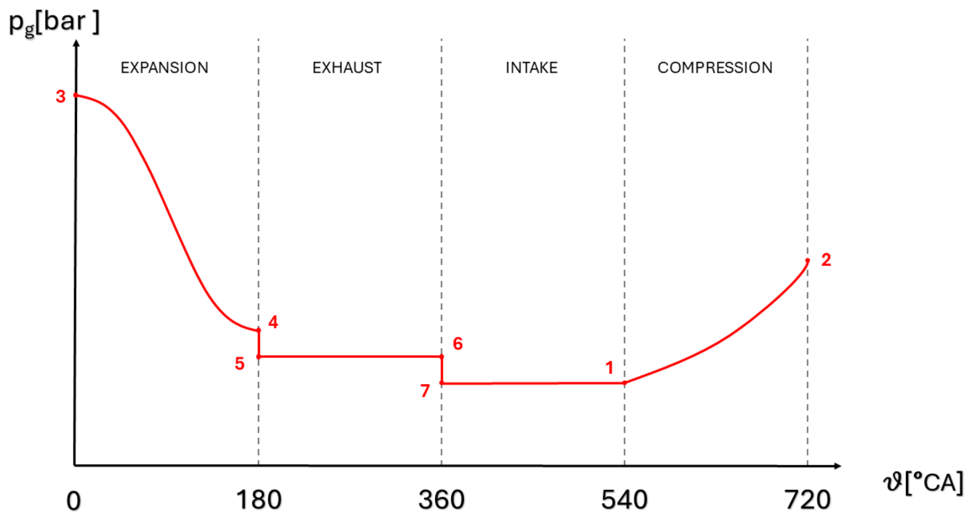

Figure 1.7: Cycle graphic

Between point 3-4 and 1-2 the line is plotted using the polytropic transformation, while for the other points the pressure is kept constant, except for the steps. The result is shown in figure 1.8.

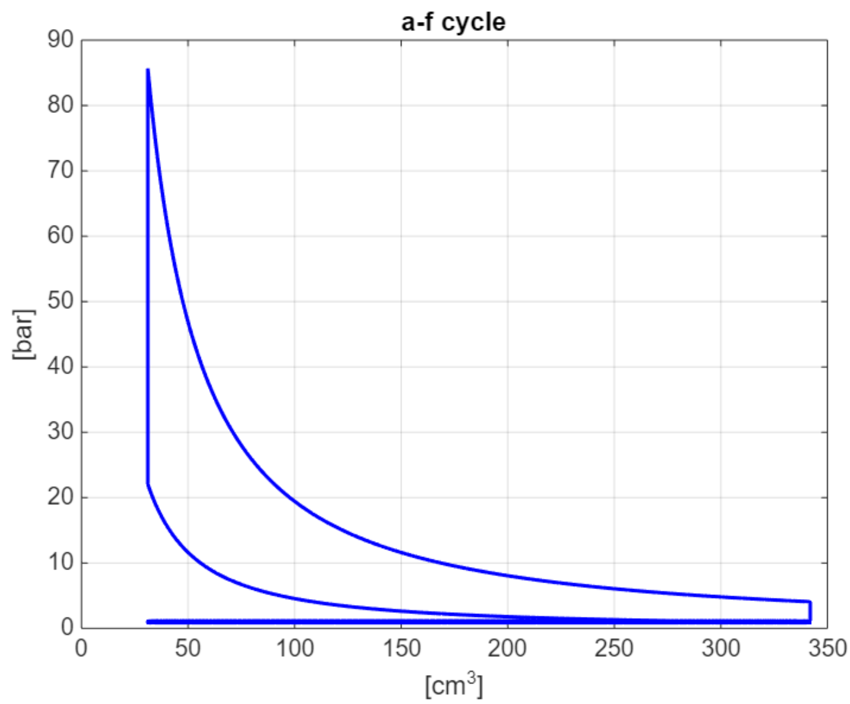

Figure 1.8: Air-fuel cycle

1.3 Calculation of IMEP and crankshaft moment

The IMEP formula is the following one:

$$IMEP = \frac{W}{V_d} = \frac{\int p \cdot V}{V_d} \tag{1.26}$$

Where

$$V_d = 3.1082 \cdot 10^5 \; mm^3 \tag{1.27}$$

Applying equation 1.26 at the air fuel cycle, it becomes:

$$IMEP = \frac{1}{V_d} \cdot \left[\int_1^2 p \cdot dV + \int_2^3 p \cdot dV + ... + \int_6^7 p \cdot dV + \int_7^1 p \cdot dV \right]$$

The terms between 2-3, 4-5 and 6-7 are equal to zero, while in the others integral the previous considerations are used: for 1-2 and 3-4 the polytropic laws is used ($p = p_i \cdot (V_i/V)^m$, for the last to terms we consider constant the pressure. So the following is obtained.

$$IMEP = \frac{1}{V_d} \cdot \left[\int_1^2 p_1 \cdot V_1^m \cdot V^{-m} \cdot dV + \int_3^4 p_3 \cdot V_3^m \cdot V^{-m} \cdot dV + p_r \int_{V_5 = V_d + V_c}^{V_6 = V_c} dV + p_1 \int_{V_7 = V_c}^{V_1 = V_d + V_c} dV \right]$$

The analytical solution of the IMEP is:

$$IMEP = p_1 \cdot \frac{r_c}{r_c - 1} \cdot \frac{1}{1-m} \cdot (r_c^{m-1} - 1) + p_3 \cdot \frac{1}{r_c - 1} \cdot \frac{1}{1-m'} \cdot (r_c^{1-m'} - 1) + (p_1 - p_r) \tag{1.28}$$

The calculated value of $IMEP$ is equal to:

$$IMEP = 11.261 \; bar$$

Successively, the inertia pressure (p_i) and the effective pressure (p_{eff}) are the next targets. They are shown in figure 1.9.

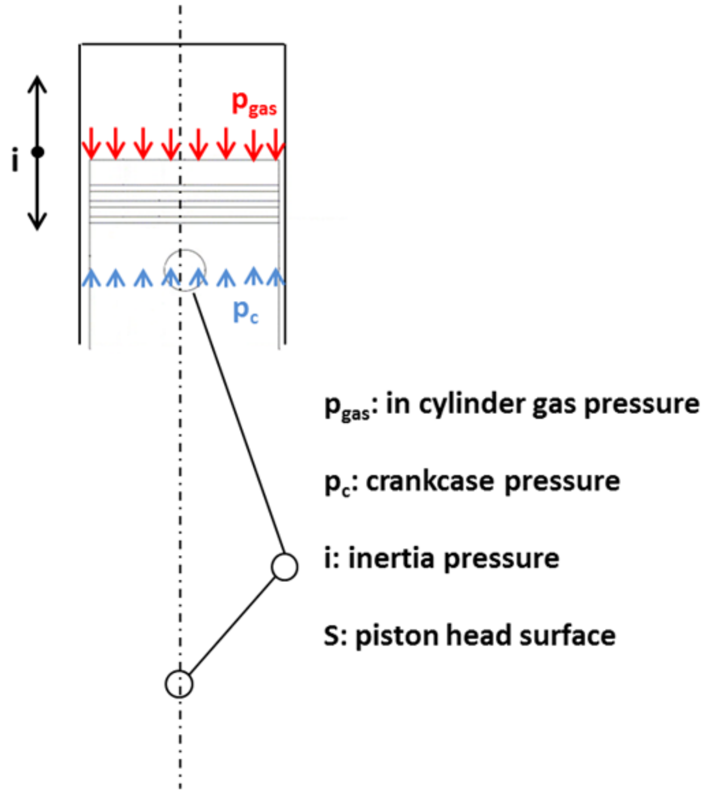

Figure 1.9: Piston pressures

$$p_{eff} = p_{gas} - p_c \pm p_i$$

The pressure in the crankcase is almost the ambient pressure and the inertia pressure is evaluable as the ratio between the inertia forces and the piston area.

$$p_c \approx p_a; \quad p_i = \frac{F_i}{A_p} \tag{1.29}$$

$$F_i = -m_{rec} \cdot a_p$$

where m_{rec} is the mass of the reciprocating elements and a_p is the piston acceleration.

$$a_p = \ddot{x} = \omega^2 \cdot r \cdot \left(\cos\theta + \Lambda \cdot \frac{\cos 2\theta}{\cos\beta} \right) \tag{1.30}$$

$$\begin{aligned} F_i = -m_{rec} \cdot a_p = -\left(\frac{m_{rec}}{V_d}\right) \cdot V_d \cdot \omega^2 \cdot r \cdot \left(\cos\theta + \Lambda \cdot \frac{\cos 2\theta}{\cos\beta} \right) = \\ = -\left(\frac{m_{rec}}{V_d}\right) \cdot \frac{\pi \cdot B^2}{4} \cdot S \cdot \frac{4 \cdot \pi^2 \cdot n^2 \cdot S}{2} \cdot \left(\cos\theta + \Lambda \cdot \frac{\cos 2\theta}{\cos\beta} \right) \end{aligned}$$

Considering that $4 \cdot n^2 \cdot S^2 = (2 \cdot n \cdot S)^2 = u^2$ and $\frac{\pi^2 \cdot B^2}{4} = A_p$.

$$F_i = -\left(\frac{m_{rec}}{V_d}\right) \cdot A_p \cdot \frac{pi^2 \cdot u^2}{2} \cdot \left(\cos\theta + \Lambda \cdot \frac{\cos 2\theta}{\cos\beta}\right) \tag{1.31}$$

And so, considering the equations 1.29 and 1.31, the inertia pressure is computed as follows:

$$p_i = -\left(\frac{m_{rec}}{V_d}\right) \cdot \frac{pi^2 \cdot u^2}{2} \cdot \left(\cos\theta + \Lambda \cdot \frac{\cos 2\theta}{\cos\beta}\right) \tag{1.32}$$

The inertia pressure plot is reported in figure 1.10.

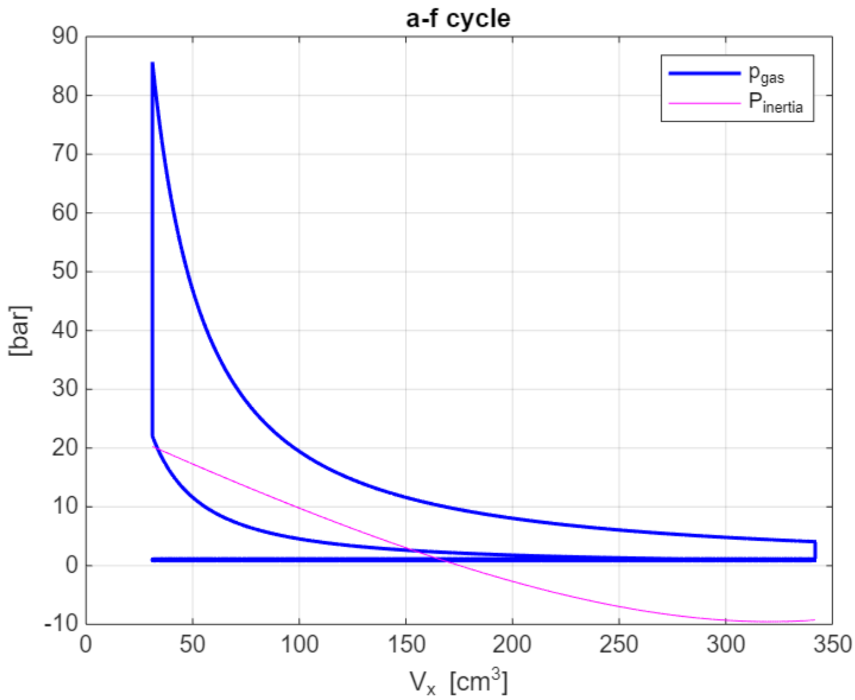

Figure 1.10: Ideal cycle with inertia pressure trend

The effective pressure is computed with the following formula:

$$p_{eff} = p_{gas} - p_a - p_i \tag{1.33}$$

The effective pressure plot is shown in figure 1.11

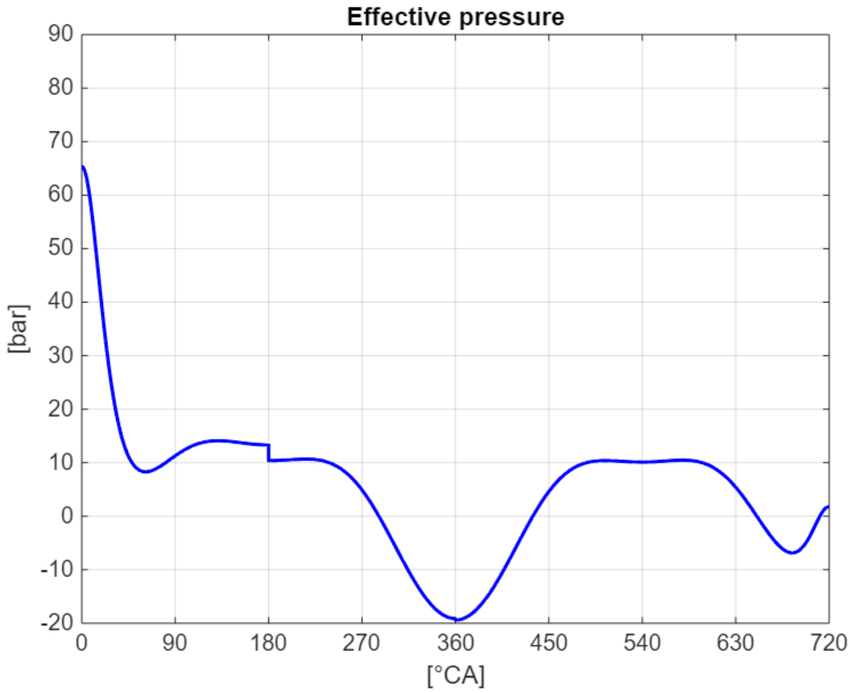

Figure 1.11: Effective pressure

Finally, all the pressure contributions all over the cycle are plotted in function of the crank angle in figure 1.12.

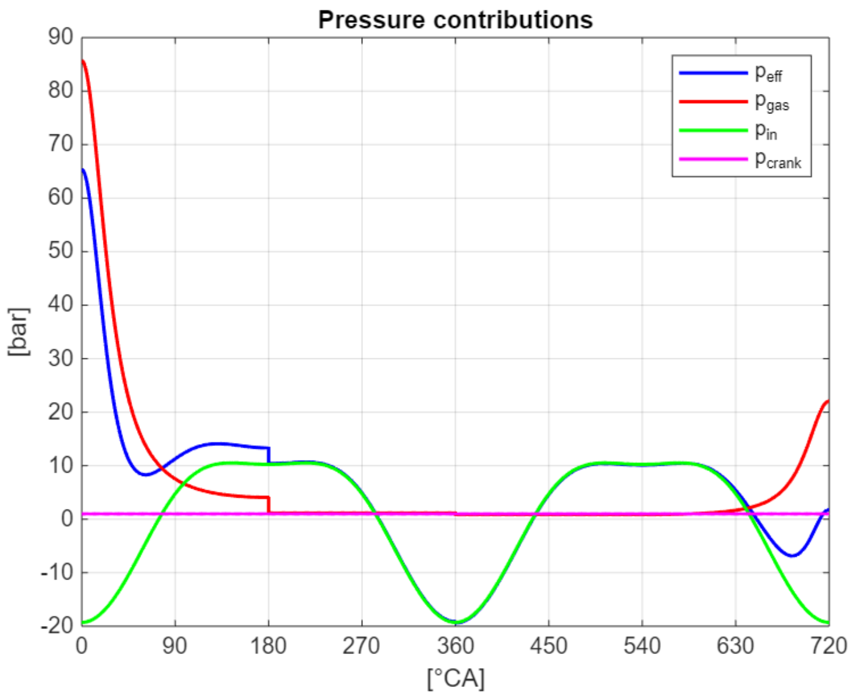

Figure 1.12: Different pressure contributions over the entire cycle

The next step is to plot the tangential pressure. It is evaluated as follows:

$$p_t(\theta) = p_{eff} \cdot \frac{\sin(\alpha + \theta)}{\cos\beta} \tag{1.34}$$

It is equal to zero ad dead centers, so every 180 crank angle degrees. Considering the discontinuity in p_{eff} shown in figure 1.11, every 180 degrees a change of slope in the plot is expected, except for 540 °CA where the function is continuous (point 1 of the cycle). The result is shown in figure 1.13.

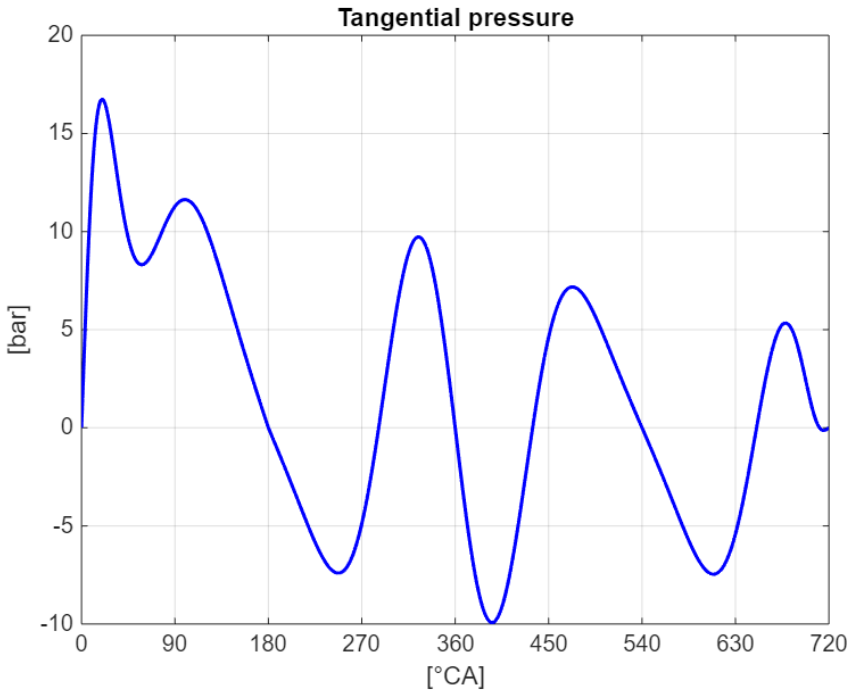

Figure 1.13: Tangential pressure plot

To compute the resistant moment and pressure, the equilibrium equation of the shaft and its moment of inertia are considered:

$$M_s(\theta) - M_r(\theta) = J \cdot \frac{d^2\theta}{dt^2} \tag{1.35}$$

$$J = J_{eng} + J_{fly} + J_{tran} + J_{user} \tag{1.36}$$

but, considering the engine as disconnected from the transmission, the inertia of the transmission and the user are negligible.

$$\int_0^{720} M_s(\theta) \cdot d\theta - \int_0^{720} M_r(\theta) \cdot d\theta = \int_0^{720} J \cdot \frac{d\omega}{dt} \cdot d\theta = \int_0^{720} J \cdot \omega \cdot d\omega \tag{1.37}$$

Considering that the integration is between 0 and $720\deg CA$, the angular velocity ω in that instant is equal to zero because the piston is at a dead center. So the equation 1.37 becomes:

$$W_s - W_r = 0 \quad \Longrightarrow \quad W_s = W_r = IMEP \cdot V_d$$

Keeping in mind that M_r is constant:

$$W_r = \int_0^{720} M_r(\theta) \cdot d\theta = M_r \cdot 4\pi \quad \Longrightarrow \quad M_r = \frac{IMEP \cdot V_d}{4\pi} \tag{1.38}$$

So the resistant pressure is computed as follows:

$$p_r = \frac{F_r}{Ap} = \frac{M_r}{r} \cdot \frac{1}{A_p} = \frac{M_r}{A_p \cdot S/2} = \frac{M_r}{V_d/2}$$

Substituting equation 1.38 into the previous one the following is obtained.

$$p_r = \frac{IMEP}{2\pi} = constant \tag{1.39}$$

$$p_r = 1.7923\ bar$$

The plot with tangential pressure (p_t), resistant pressure (p_r), shaft work (W_s/V_d) and resistant work (W_r/V_d) as a function of crank angle is shown in figure 1.14.

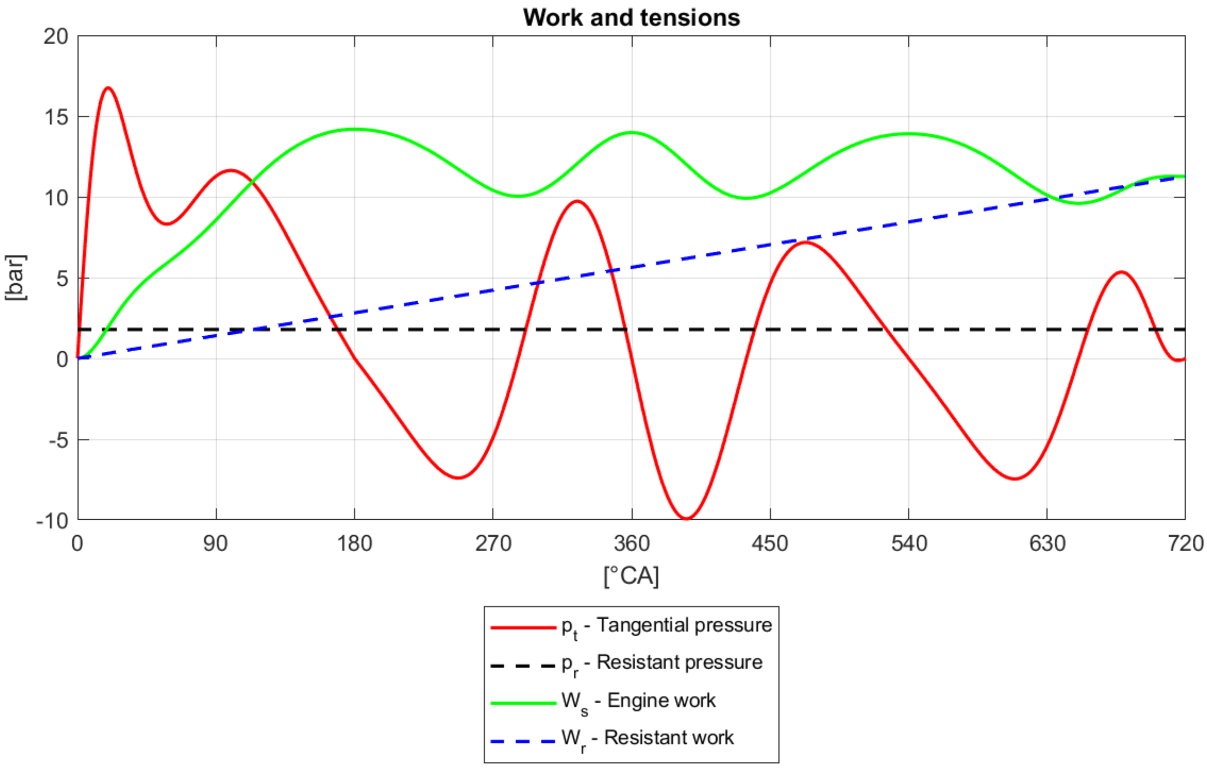

Figure 1.14: Shaft pressures and works

As expected, the engine and the resistant works coincide at 0 and $720°CA$ ($W_s - W_r = 0$).

1.4 Flywheel dimensioning

The maximum speed fluctuation has been limited setting the kinematic irregularity:

$$\delta = \frac{\omega_{max} - \omega_{min}}{\omega_{avg}} = 1\% = 0,01 \tag{1.40}$$

This has been done in order to get a dynamic irregularity (eq. 1.41) which fits in the target.

$$\xi = \frac{\Delta W_{max} + |\Delta W_{min}|}{W(4\pi)} \quad \rightarrow \quad \xi = \frac{\Delta W_{tot}}{IMEP \cdot V_d} \tag{1.41}$$

$$\Delta W_{tot} = 336.68\ J$$

where ΔW_{tot} is calculated as the maximum and minimum value of:

$$\Delta W_{tot} = W_s(\theta) - W_r(\theta) \tag{1.42}$$

where the shaft and resistant work are computed as integral.

$$\int_0^\theta dW_s(\theta) = M_s(\theta) \cdot d\theta; \quad \int_0^\theta dW_r(\theta) = M_r(\theta) \cdot d\theta \tag{1.43}$$

At this point it is necessary to evaluate the inertia moment in order to evaluate the dimensions and eventually the geometry of the flywheel.

$$J = J_{eng} + J_{flyw} \quad \Longleftarrow \quad J_{eng} = \left(\frac{m_{tot}}{V_d}\right) \cdot V_d \cdot r^2 \tag{1.44}$$

where m_{tot}/V_d is a given data.

$$J_{eng} = 4.117 \cdot 10^{-4}\ kg \cdot m^2$$

To calculate J_{flyw} it is necessary to evaluate J (total inertia).

$$\int_0^{\theta_{\omega,max}} M_s \cdot d\theta - \int_0^{\theta_{\omega,max}} M_r \cdot d\theta = \int_0^{\theta_{\omega,max}} J \cdot \frac{d\omega}{dt} \cdot d\theta = \int_0^{\theta_{\omega,max}} J \cdot \omega d\omega = J\frac{\omega_{max}^2 - \omega(0)^2}{2}$$

$$\int_0^{\theta_{\omega,min}} M_s \cdot d\theta - \int_0^{\theta_{\omega,min}} M_r \cdot d\theta = \int_0^{\theta_{\omega,min}} J \cdot \frac{d\omega}{dt} \cdot d\theta = \int_0^{\theta_{\omega,min}} J \cdot \omega d\omega = J\frac{\omega_{min}^2 - \omega(0)^2}{2}$$

$$\Delta W_{tot} = \Delta W_{max} - |\Delta W_{min}| = J \cdot \frac{\omega_{max}^2 - \omega(0)^2}{2} - J\frac{\omega_{min}^2 - \omega(0)^2}{2}$$

$$\Delta W_{tot} = J \cdot \frac{\omega_{max} + \omega_{min}}{2} \cdot \frac{\omega_{max} - \omega_{min}}{2} \tag{1.45}$$

Combining the equations 1.40, 1.45 and $(\omega_{max} - \omega_{min})/2 = \omega_{avg}$, the following is obtained:

$$\Delta W_{tot} = J \cdot \delta \cdot \omega_{avg}^2 \tag{1.46}$$

Finally, combining 1.46 and 1.41, J is computed:

$$J = \frac{\xi \cdot IMEP \cdot V_d}{\delta \cdot \omega_{avg}^2} \tag{1.47}$$

$$J = 0.0853\ kg \cdot m^2$$

The moment of inertia can be calculated with the equations 1.44 and 1.47.

$$J_{flyw} = 0.0849\ kg \cdot m^2$$

The next step is to set the dimensions of the flywheel. The moment of inertia of a disc is considered:

$$J_{flyw} = \int_0^{D_f/2} r^2 \cdot dm = \int_0^{D_f/2} r^3 \cdot \rho_f \cdot 2\pi \cdot w_f \cdot dr \tag{1.48}$$

where D_f, ρ_f and w_f are the diameter, the density and the width of the flywheel. Solving the integral the following formula is obtained.

$$J_{flyw} = \frac{1}{32} \cdot \pi \cdot \rho_f \cdot w_f \cdot D_f^4 \tag{1.49}$$

To solve the equation a practical constraint is imposed: $2 \cdot S < D_f < 5 \cdot S$. In order to do this, another hypothesis is made:

$$w_f = \frac{D_f}{10} \tag{1.50}$$

If the first condition is not verified, the value 10 can be slightly changed. The diameter formula becomes:

$$D_f = \sqrt[5]{\frac{320 \cdot J_f}{\rho : f \cdot \pi}} \tag{1.51}$$

In this case the value is inside the pre-imposed limits:

$$D_f = 257\ mm$$

1.5 Calculation of the instantaneous crankshaft speed

From the equation 1.35 the following one is derived:

$$W_s(\theta) - W_r(\theta) = J \cdot \frac{\omega(\theta)^2 - \omega(0)^2}{2} \tag{1.52}$$

So the equation of ω in function of the crank angle θ is obtained:

$$\omega(\theta) = \sqrt{\omega_0^2 + \frac{2}{J} \cdot \Big[W_s(\theta) - W_r(\theta)\Big]} \tag{1.53}$$

In order to get the value for every angle, some iterations are made, starting with the following imposition:

$$\omega_0 \approx \omega_{avg}$$

Then for the first value the equation becomes:

$$\omega_I(\theta) = \sqrt{\omega_{avg}^2 + \frac{2}{J} \cdot \Big[W_s(\theta) - W_r(\theta)\Big]}$$

After that, ω_{avg} is substituted with $\omega_{avg,I}$ that is computed as following:

$$\omega_{avg,I} = \frac{1}{4\pi} \cdot \int_0^{4\pi} \omega_I(\theta) \cdot d\theta$$

The last iteration is made to shift the instantaneous speed graphic:

$$\omega(\theta) = \omega_I(\theta) - S$$

where S is the shift and its calculated as follows:

$$S = \omega_{avg,I} - \omega_{avg} \tag{1.54}$$

The final plot of the instantaneous speed is reported in figure 1.15.

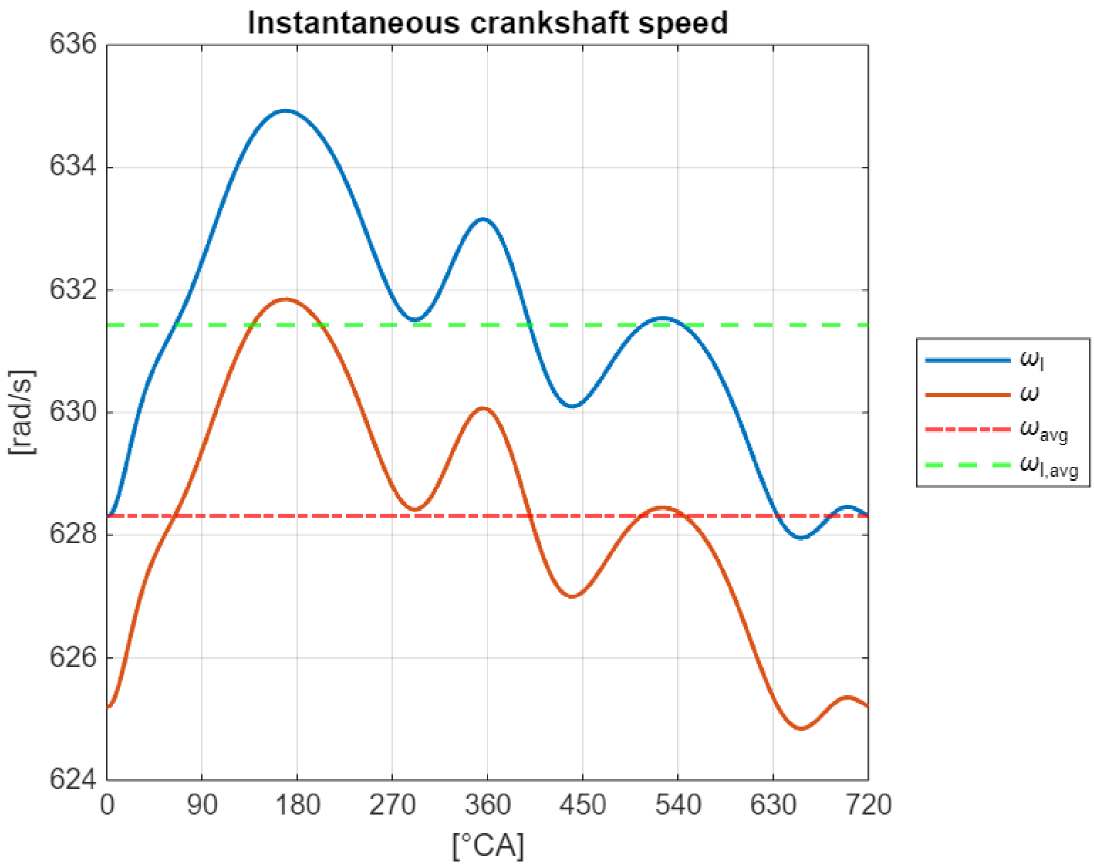

Figure 1.15: Instantaneous crankshaft speed

1.6 Shaft and resistant moment for multi-cylinder engine

In this case, a 4-cylinders engine is chosen to enlarge the dissertation to the multi-cylinders engine.

Considering the firing order 1-3-2-4 the phase shift between the cylinders is defined as:

$$\Delta\phi = \frac{m \cdot 360}{i} \tag{1.55}$$

$$\Delta\phi = 180°$$

where m is the number of revolution on cycle and i is the number of cylinders.

The phase shift for every cylinder is evaluated as:

$$\phi_j = \Delta\phi(j-1) \tag{1.56}$$

The shaft moment and the tangential pressure are computed with the following equations.

$$M_{s,multi}(\theta) = \sum_{j=1}^{i} M_{s,j}(\theta - \phi_j) \tag{1.57}$$

$$p_{t,multi}(\theta) = \frac{M_{s,multi}(\theta)}{V/2} \tag{1.58}$$

After implementing it on Matlab, in figure 1.16 the plot of the tangential pressure for the multi-cylinders case is reported.

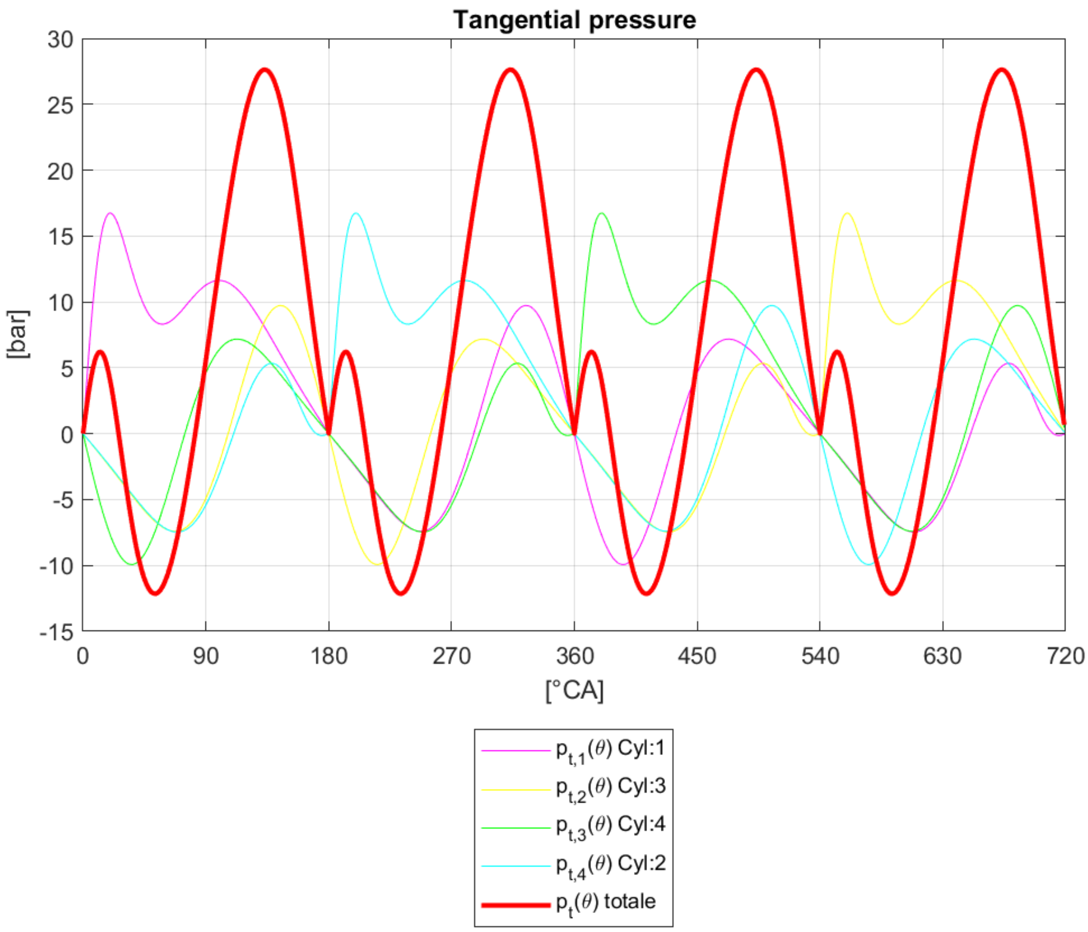

Figure 1.16: Tangential pressure, multi-cylinders

To calculate the shaft dynamic equilibrium the next formula is used.

$$M_{s,multi}(\theta) - M_{r,multi}(\theta) = J \cdot \frac{d^2\theta}{dt^2} = J \cdot \frac{d\omega}{dt} \tag{1.59}$$

By integrating the equation 1.59 over the entire engine cycle (4π) and taking into account the previous considerations, the resistant moment is obtained:

$$\int_0^{4\pi} M_{s,multi}(\theta) \cdot d\theta = IMEP \cdot i \cdot V_d \tag{1.60}$$

where i is the number of cylinders and V_d is the volume displacement of a single cylinder.

Then, as already done before, the resistant pressure is computed as:

$$p_{r,multi}(\theta) = \frac{M_{r,multi(\theta)}}{V_d/2} \tag{1.61}$$

As already done in equation 1.38, by integrating the resistant and shaft moments the resistant and shaft works can be computed. This is plotted in figure 1.17, where the period is equal to the phase shift.

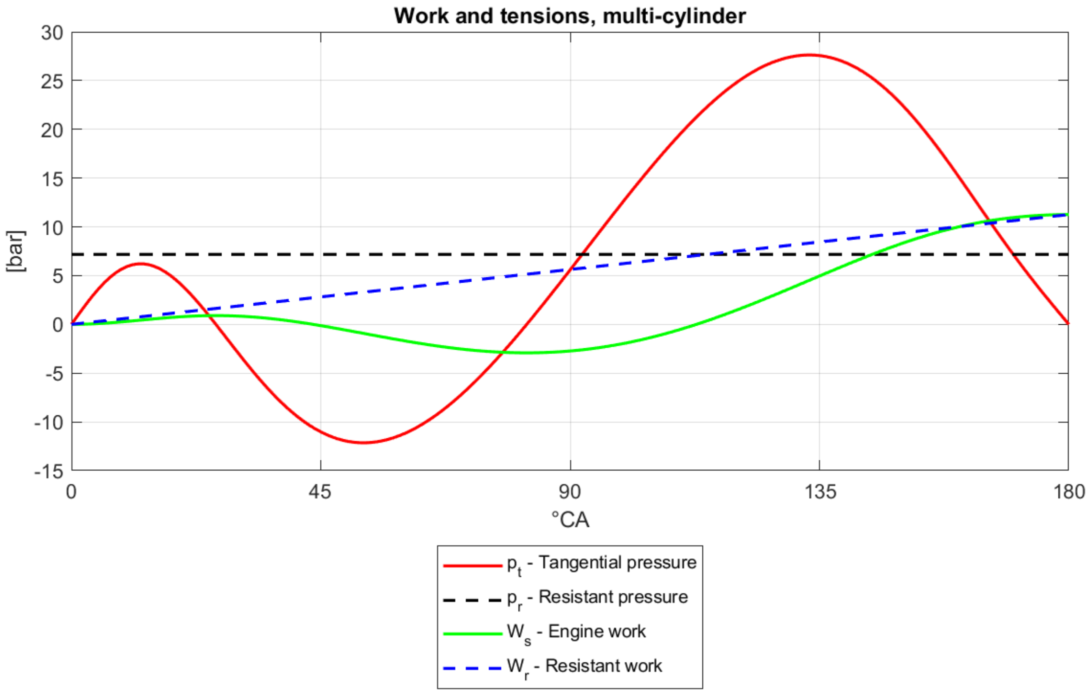

Figure 1.17: Work and tensions, multi-cylinder

1.7 Flywheel dimensioning for multi-cylinder engine

As for the single cylinder case, the dynamic irregularity is calculated to obtain the flywheel moment of inertia.

$$\xi_{multi} = \frac{\Delta W_{max,multi} + |\Delta W_{min,multi}}{W_{multi}(4\pi)} = \frac{\Delta W_{tot,multi}}{IMEP \cdot i \cdot V_d} \tag{1.62}$$

$$\xi_{multi} = 0.193$$

ξ is expected to be equal to the one calculated for the single cylinder case or respect the following condition:

$$\xi_{multi} < \frac{\xi_{single}}{i}$$

which means that the engine torque is more regular.

Afterwards it is possible to calculate the flywheel diameter, considering the same constraint applied in the single cylinder case.

The resultant diameter of the computation is reported.

$$D_{multi-cyl} = 256\ mm \tag{1.63}$$

1.8 Instantaneous crankshaft speed for multi-cylinder engine

Applying the same procedure of the single-cylinder case, the instantaneous crankshaft speed is calculated. In this case the evaluation is made in a period equal to the phase shift.

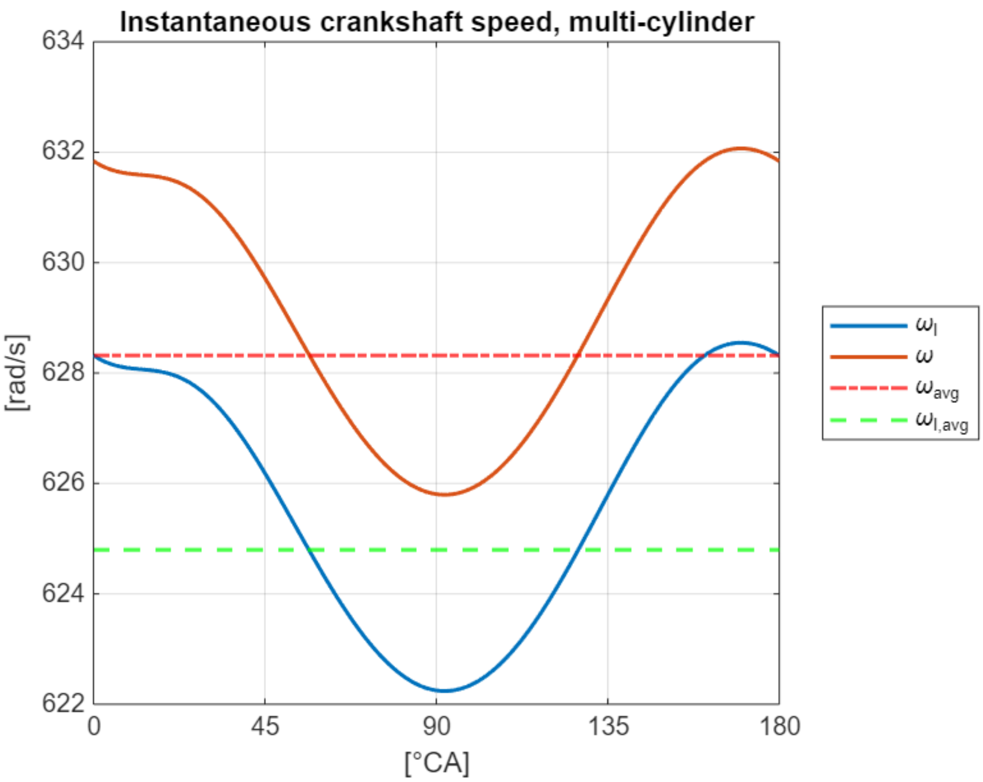

Figure 1.18: Instantaneous crankshaft speed, multi-cylinder

Report 2

Engine test evaluation and HRR analysis

2.1 Introduction

2.1.1 General informations

This report is subdivided in two parts: the engine tests evaluation and computation of a heat release rate (HRR) analysis.

The evaluation concerns a turbocharged combustion ignition engine.

The engine test has been done in an appropriate engine testbed, to characterize the engine and understand the effect of the design and the calibration parameters on performance and emissions. The outputs of the test are stored in a Matlab file, where several parameters have been extracted in function of the working conditions.

The testing outcomes should include mechanical power, thermal efficiency, exhaust emissions and durability evaluation.

The essential components of an engine testbed are:

- a dynamometer (dyno) to counteract and measure the torque;
- multiple sensors to measure the target parameters;
- a control unit for the user, to monitor and control the test.

An example is shown in the next figure.

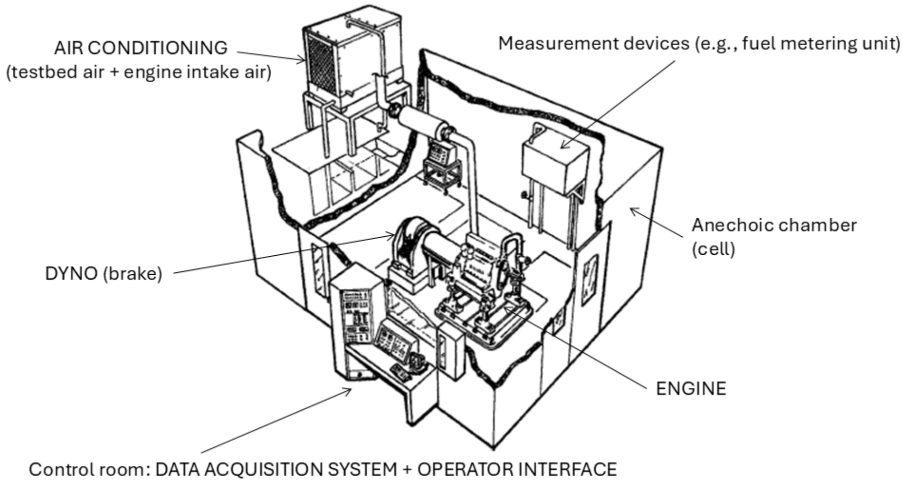

Figure 2.1: Engine testbed

For the heat release rate (HRR) analysis a dataset is given.
The given data contains:

- main variables measured during many steady-state tests;
- a set of measured in-cylinder cycles for one of the steady-state tests performed.

The requirements of the report are:

1. numerical results for corrected power and torque, fuel conversion efficiency, bsfc and volumetric efficiency;
2. plots of fuel conversion efficiency, bsfc and volumetric efficiency as a function of the corrected torque;
3. numerical results for Start-of-Injection, Start-of- Combustion, ignition delay (angular interval), MFB50 (crank angle which corresponds to xb=0.5);
4. plots representing the measured in-cylinder pressure and the “motored” in-cyl pressure (absence of combustion event), the mass fraction burned as a function of the crank angle.
5. Evaluation of MFB10 and MFB90.

2.1.2 Dataset

The engine parameters are reported in table 2.1.

Parameter	Value	Unit
m	2	-
i	4	-
S	104	$[mm]$
r	158	$[mm]$
r_c	17.5	$[mm]$

Table 2.1: Engine parameters

where m is the stroke coefficient, i the number of cylinders, S the stroke, r the connecting rod and r_c the compression ratio.

The air and fuel characteristics are reported in table 2.2.

Parameter	Value	Unit
$p_{0,dry}$	99	$[kPa]$
$T_{0,dry}$	298	$[K]$
R_{air}	287.05	$[J \cdot K/kg]$
γ_{EGR}	1.4	-
$c_{p,EGR}$	1038	$[J/kg/K]$
$Q_{LHV,fuel}$	42.5	$[MJ/kg]$
R_{fuel}	417	$[J \cdot K/kg]$

Table 2.2: Thermodynamic parameters

To compute the full load characteristic at least 60 tests are executed for each point that are successively averaged. The test conditions and results are reported in a given data-sheet and are now reported in the following table.

Engine parameters					Ambient conditions			Int. Man.	
Engine speed $[rpm]$	$\mathbf{T_{dyno}}$ $[N \cdot m]$	$\mathbf{\dot{m}_{fuel}}$ $[kg/h]$	$\mathbf{\dot{m}_{air}}$ $[kg/h]$	$\mathbf{\dot{m}_{EGR}}$ $[kg/h]$	$\mathbf{p_{baro}}$ $[mbar]$	$\mathbf{T_{amb}}$ $[°C]$	$\mathbf{H_{rel}}$ $[\%]$	$\mathbf{T_{int}}$ $[°C]$	$\mathbf{p_{int}}$ $[mbar]$
850	233.25	4.701	86.35	0.65	1009.91	25.2	48.82	31.0	139.1
1000	253.25	5.757	108.71	2.72	1009.96	25.1	48.82	30.7	209.4
1200	351.21	9.509	159.74	0.61	1010.47	25.5	48.03	28.1	504.5
1400	421.82	12.810	212.02	3.36	1010.45	25.0	45.27	29.3	825.8
1600	429.50	14.762	247.39	16.62	1010.20	25.3	49.83	35.0	991.1
1800	431.95	16.577	280.07	26.43	1010.23	25.4	49.68	40.0	1097.1
2000	433.08	18.468	316.73	33.92	1009.33	25.3	49.84	45.5	1192.4
2250	435.44	21.081	370.86	38.72	1009.31	25.4	49.54	49.8	1285.6
2500	429.94	23.449	428.87	40.66	1008.78	25.0	43.53	53.6	1343.9
2750	426.85	26.217	495.21	27.17	1008.71	25.4	50.28	49.7	1393.1
3000	402.56	27.318	537.29	16.35	1008.82	25.4	50.94	49.7	1380.4
3250	364.96	27.357	561.97	12.40	1008.82	25.2	52.03	45.2	1316.2
3500	339.77	28.006	585.47	5.91	1008.84	25.2	50.70	48.2	1267.3
3850	221.99	21.508	532.16	5.33	1008.87	25.1	50.17	41.2	864.1

Table 2.3: Steady state test results

2.2 Steady state test analysis

The first step is to correct the power and the torque. It is done considering the ISO standard 1585, which is performed using a correction factor μ_c, which depends on the engine.

$$\mu_c = (f_a)^{f_m} \tag{2.1}$$

So the corrected power and torque are calculated as:

$$P_0 = \mu_c \cdot P \qquad\qquad T_0 = \mu_c \cdot T \tag{2.2}$$

For a turbocharged CI engine f_a is evaluated as:

$$f_{a,TC} = \left(\frac{p_{0,dry}}{p_{a,dry}}\right)^{0.7} \cdot \left(\frac{T_a}{T_0}\right)^{1.2} \tag{2.3}$$

where T_0 and p_0 are the reference conditions, while T_a and $p_{a,dry}$ are the ambient conditions.

$p_{a,dry}$ is calculated as follows:

$$p_{a,dry} = p_a - \chi \cdot p_{saturation,H_2O} \tag{2.4}$$

where $p_{saturation,H_2O}$ is the saturation pressure, used to eliminate the contribution of the humidity pressure and it is evaluated as:

$$p_{saturation,H_2O} = a_0 + a_1 \cdot T_a + a_2 \cdot T_a^2 + a_3 \cdot T_a^3 + a_4 \cdot T_a^4 \tag{2.5}$$

f_m is computed as follows:

$$if \quad 37.2 \leq m_c \leq 65 \quad \Longrightarrow \quad f_m = 0.036 \cdot q_c - 1.14$$

$$if \quad m_c < 37.2 \quad \Longrightarrow \quad f_m = 0.2$$

$$if \quad m_c > 65 \quad \Longrightarrow \quad f_m = 1.2$$

where m_c is the fuel delivery parameter and is calculated as follows:

$$m_c = \frac{q_f}{p_{manifold}/p_a} \tag{2.6}$$

and m_{fuel} is the fuel flow-rate in $[mg/(l \cdot cycle)]$ and is given by the dataset (table 2.3). It has to be converted considering the following equation:

$$\dot{m}_f[kg/h] = \frac{\dot{m}_f \cdot 10^6}{3600 \cdot \frac{n}{m} \cdot \frac{1}{60} \cdot i \cdot V_d}$$

The displacement volume is calculated as:

$$V_d = \frac{\pi \cdot B^2}{4} \cdot S$$

and it is converted from $[mm^3]$ to $[dm^3] = [l]$.

Considering that the test bench only measure the torque with the dyno, the power is calculated as:

$$P = T \cdot n \cdot \frac{\pi}{30}$$

where the last term is used to convert the engine speed from $[rpm]$ to [rad/s].

The results of the corrections are:

Engine speed $[rpm]$	$\mathbf{P_{corr}}$ $[kW]$	$\mathbf{T_{corr}}$ $[Nm]$
850	20.74	232.95
1000	26.48	252.84
1200	44.10	350.95
1400	61.64	420.46
1600	71.91	429.16
1800	81.39	431.77
2000	90.68	432.96
2250	102.59	435.42
2500	112.40	429.35
2750	122.95	426.95
3000	126.50	402.66
3250	124.21	364.95
3500	124.52	339.73
3850	89.48	221.94

Table 2.4: Corrected power and torque

The mechanical characteristic is plotted in figure 2.2.

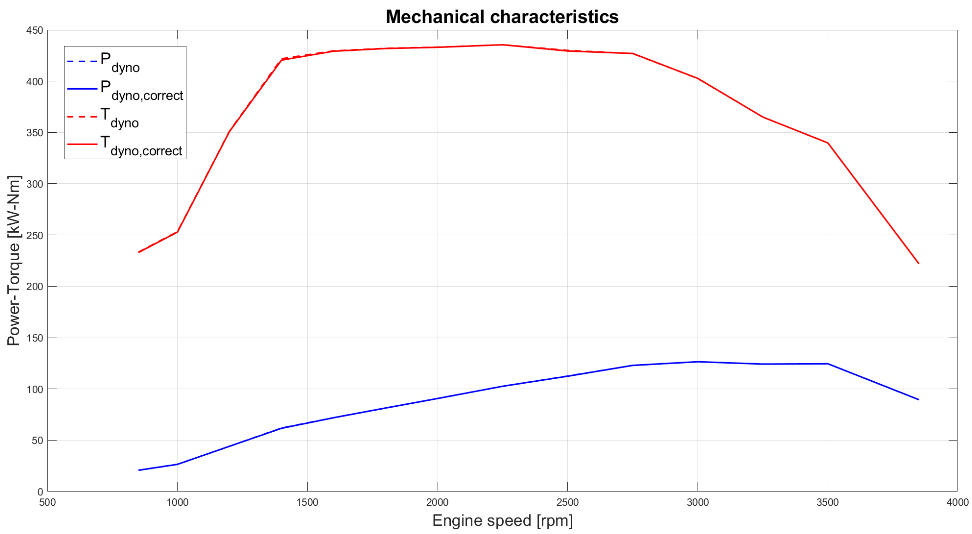

Figure 2.2: Mechanical characteristic

The brake specific fuel consumption (bsfc) is evaluated:

$$bsfc = \frac{\dot{m}_f \cdot 10^3}{P_corr} \tag{2.7}$$

The bsfc plot is shown in figure 2.3.

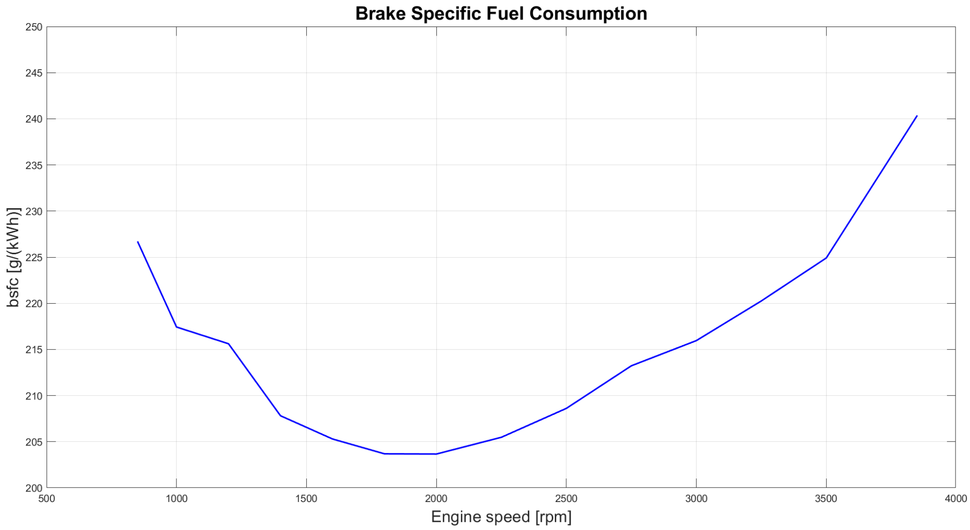

Figure 2.3: bsfc map

The fuel conversion efficiency (η_f) is computed:

$$\eta_f = \frac{1}{bsfc \cdot 2.777 \cdot 10^{-10} \cdot Q_{LHV}} \tag{2.8}$$

where $2.777 \cdot 10^{-10}$ is to convert from g/kWh to kg/J.

The η_f plot is shown in the following figure (2.4).

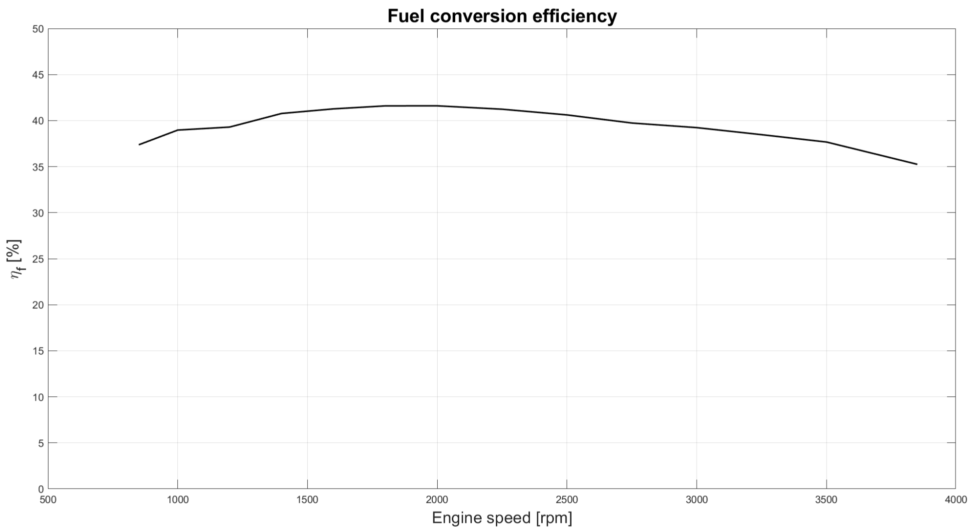

Figure 2.4: Fuel conversion efficiency

Finally, to compute the volumetric efficiency the following equation is considered:

$$\lambda_v = \frac{m_{int}}{m_{int,ref}} = \frac{m_a + m_{EGR}}{\rho_{int} \cdot V_d} = \frac{m_a + m_{EGR}}{V_d} \cdot \frac{R_{mix} \cdot T_{int}}{p_{int}} \tag{2.9}$$

m_a and m_{EGR} are the actual masses of air and exhaust gas recirculated and are calculated as follows:

$$m_a = \frac{\dot{m_a}}{3600} \cdot \frac{1}{i \cdot (n/(2 \cdot 60))} \qquad m_{EGR} = \frac{m_{\dot{E}GR}}{3600} \cdot \frac{1}{i \cdot (n/(2 \cdot 60))} \tag{2.10}$$

ρ_{int} is the density of the intake manifold air and is obtained using the ideal gas law. R_{mix} is the mixture of air and EGR elasticity constant and is calculated as follows.

$$R_{mix} = \frac{\dot{m_a} \cdot R_{air} + m_{\dot{E}GR} \cdot R_{air}}{\dot{m_a} + m_{\dot{E}GR}} \tag{2.11}$$

The result is reported:

Engine speed $[rpm]$	λ_v [%]
850	86.5
1000	88.7
1200	84.8
1400	80.9
1600	81.2
1800	81.0
2000	81.2
2250	82
2500	83.5
2750	81.6
3000	79.3
3250	77.3
3500	76.2
3850	74.8

Table 2.5: Volumetric efficiency values

The volumetric efficiency is plotted in figure 2.5.

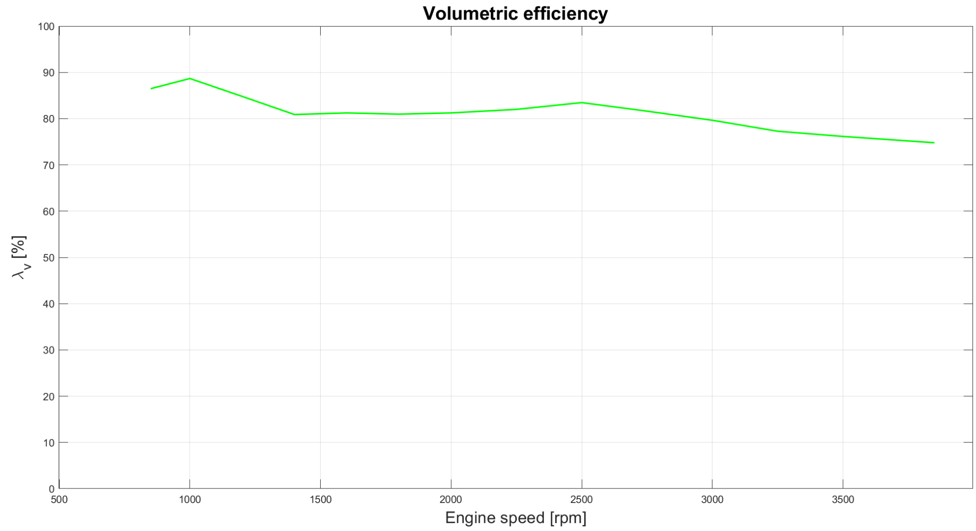

Figure 2.5: Volumetric efficiency

2.3 HRR analysis and combustion diagnostic

In the excel file with the dataset, the in cylinder pressure raw signal is given. The first target is to shift and reference it with respect to $p_{manifold}$ and then to filter it to eliminate some eventual disturbance.

Considering that data are given for 100 cycles, an average is done to obtain a single value for every point of the curve.

After getting the data, to reference them the manifold pressure is used. At $180°$ an average between -5° and +50° is extracted for both pressures, and the the difference between them is done to calculate the real shift.

$$p_{raw,mean} = p_{raw}(185°) - p_{raw}(175°) \qquad p_{man,mean} = p_{man}(185°) - p_{man}(175°)$$

$$S = p_{raw,mean} - p_{man,mean} \tag{2.12}$$

At this point, to get the referenced pressure it is sufficient to sum punctually the raw pressure with the shift.

$$p_{ref} = p_{raw} + S \tag{2.13}$$

The result is shown.

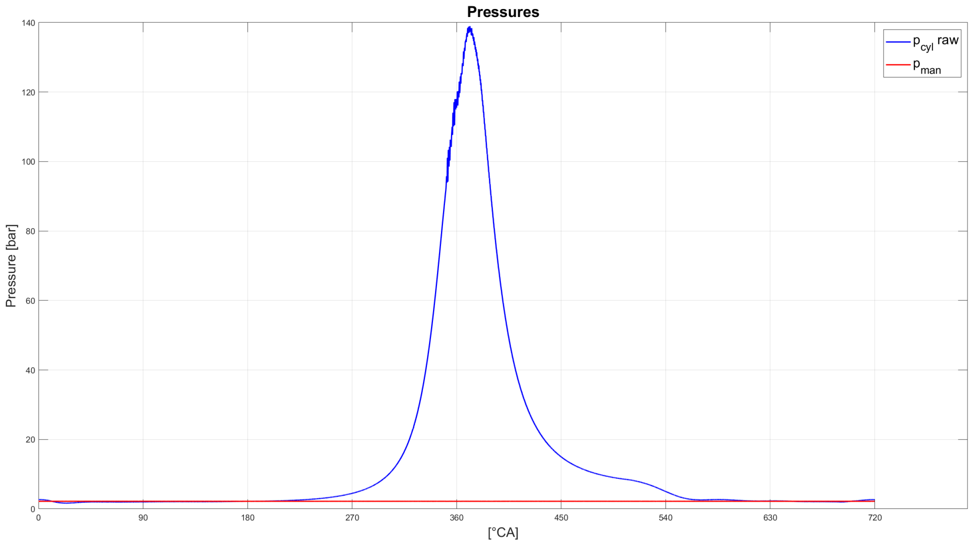

Figure 2.6: Pressure inside the cylinder

To filter the signal a Butterworth filter is used. The Matlab implementation is reported.

```
% Butterworth filter
res = 0.1;

fs_filt = n/60*360/res; %sampling frequency
fc = 4000; %cutoff frequency [Hz]
Wn = fc/(fs_filt/2); %ratio between cutoff and Nyquist frequency
n_filt = 2; % order of the filter
[b,a] = butter(n_filt,Wn);
p_cyl_filt=filtfilt(b,a,p_cyl_ref);

```

The result is shown in the following figure.

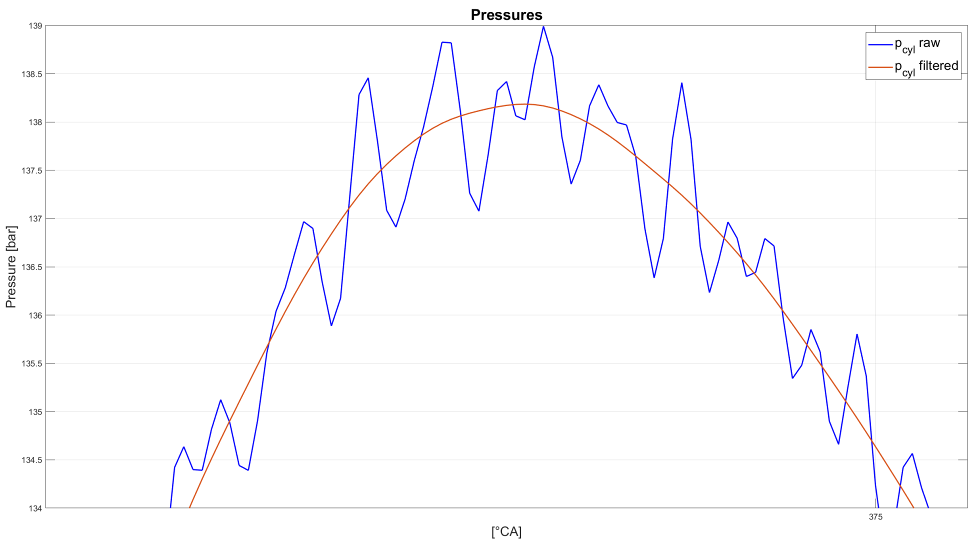

Figure 2.7: Filtered pressure signal

The next target is to calculate the net heat release rate, evaluate the injection current to get the values of start of ignition (SOI), start of combustion (SOC) and evaluate the mass fraction burned (x_b).

The first step is to calculate the net heat (Q_n). To to this, some steps are done:

$$T = \frac{p \cdot V}{m_{mix} \cdot R_{mix}} \tag{2.14}$$

where R_{mix} has been evaluated in equation 2.11 and V is obtained as follows:

$$V(x) = V_c + V_d(x) = \frac{V_{d,max}}{r_c - 1} + x \cdot \left(\frac{\pi \cdot B^2}{4}\right) \tag{2.15}$$

and $V_{d,max} = S \cdot \left(\frac{\pi \cdot B^2}{4}\right)$

$$dQ_n = \frac{\gamma}{\gamma - 1} \cdot p \cdot dV + \frac{1}{\gamma - 1} \cdot V \cdot dP \tag{2.16}$$

where γ for the EGR is 1.4 and γ for the real in-cylinder transformation is calculated as follows:

$$\gamma = 1.338 - 6 \cdot 10^{-5} + 1 \cdot 10^{-8} \cdot T^2 \tag{2.17}$$

Afterwards, the dQ_n and the HRR are calculated:

$$HRR = \frac{dQ_n}{d\theta} \tag{2.18}$$

where $d\theta = 0.1°CA$

The HRR is plotted in the following figure.

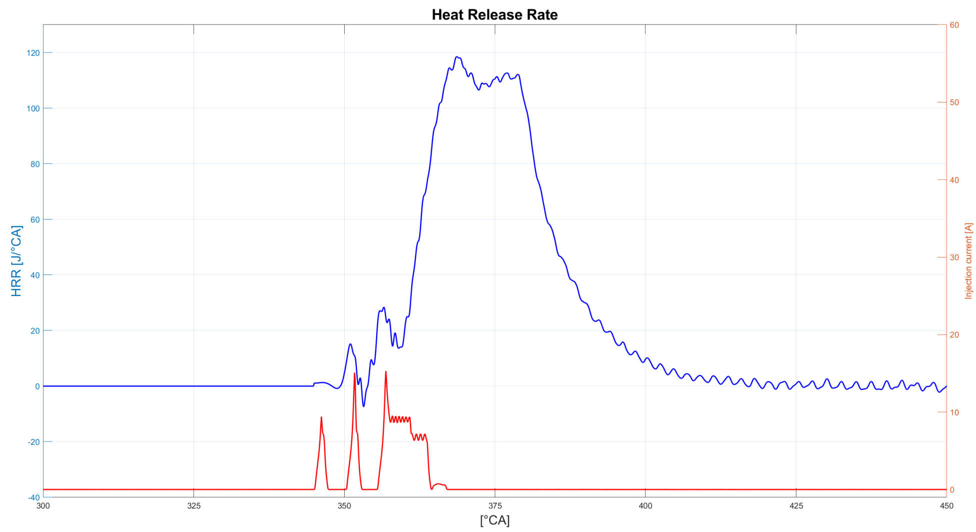

Figure 2.8: Plot of Heat Release Rate

Now, the next target is the evaluation of SOI and SOC.

To determine the start of injection, the injection current data is analyzed. A minimum current average value of $0.5\,A$ is established to indicate the onset of injection, minimizing the likelihood of random errors.

For identifying the start of combustion, the net Heat Release Rate (HRR) is assessed. A threshold value of $0.1\,J/°\mathrm{CA}$ is set as the derivative HRR difference between two crank angles. This threshold represents a realistic increase in HRR derivative, helping to avoid disturbances or random errors.

The results of this evaluation are reported:

$$SOI = 345.1°CA \qquad\qquad SOC = 349.2°CA$$

The ignition delay is also computed:

$$ID = SOC - SOI = 4.1°CA$$

At this point, the mass fraction burned is studied. To calculate it the following formulas are used:

$$Q_{n,cum} = \sum_{i=0.1}^{720} dQ_{n,i} \tag{2.19}$$

$$x_{b.i} = \frac{Q_{n,cum,i}}{Q_{n,cum,max}} \tag{2.20}$$

The mass fraction burned plot is reported:

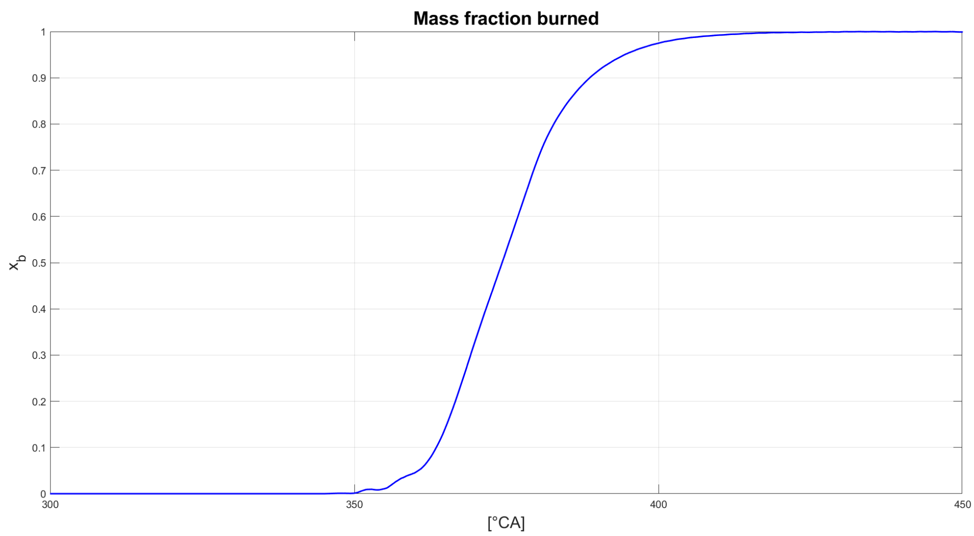

Figure 2.9: Mass fraction burned

The next step is to evaluate the position in terms of crank angles of the mass fraction burned at 0.1, 0.5 and 0.9 percent. The results are shown in table 2.6 and in figure 2.10.

X_b	$[°CA]$
MFB10	363.5
MFB50	374.3
MFB90	388.7

Table 2.6: Mass fraction burned at 10, 50 and 90 %

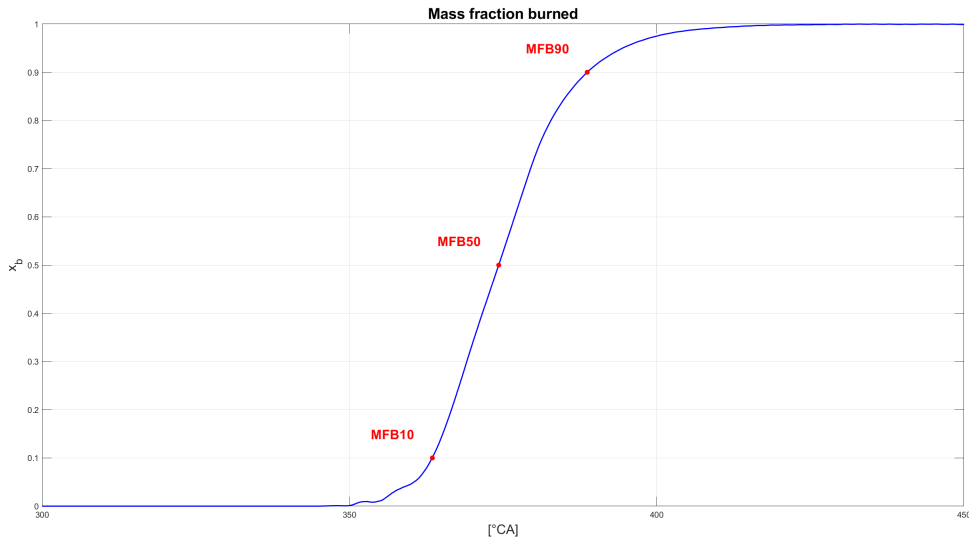

Figure 2.10: Mass fraction burned at 10, 50 and 90 %

The last step is to plot the motored pressure. Before injection, the motored pressure does not differ from the in-cylinder measurement. After that point it is calculated considering an isentropic (or polytropic) compression:

$$p \cdot V^{\gamma} = const$$

where γ has been previously calculated for every crank angle with the equation 2.17.

The result is reported in figure 2.11.

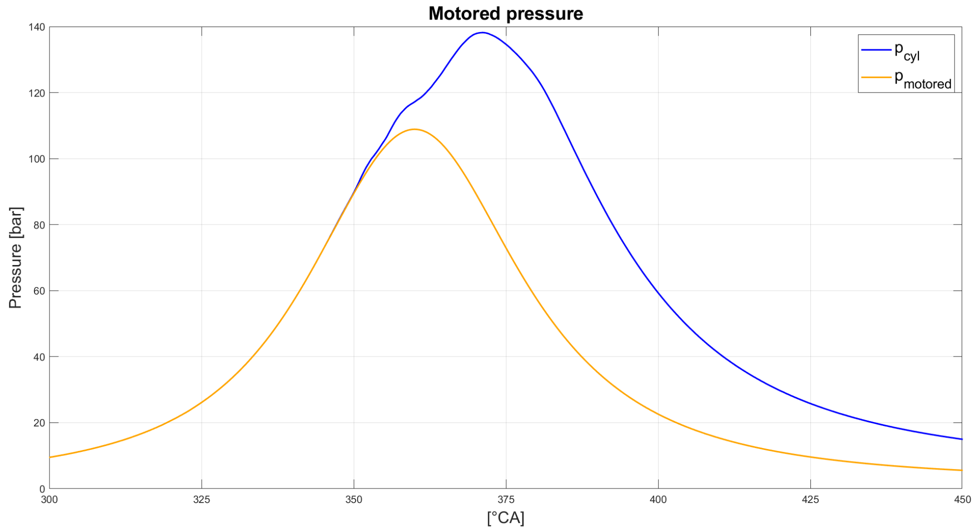

Figure 2.11: Motored pressure