# 2d CFT

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## Week 1

**Exercies 0.0.1.** By the homogeneous relation

$$f(t,h) = b^{-d} f(b^{y_t} t, b^{y_h} h)$$

we have

$$f(t,h) = t^{\frac{d}{y_t}} g(\alpha)$$

where $g(\alpha) = f(1,\alpha)$ and $\alpha = t^{-\frac{y_h}{y_t}} h$. It is easy to see that $\alpha$ is invariance under scaling transformation $x \to x/b$. Hence we have

$$\begin{aligned} C(t,0) &= -T\frac{\partial^2 f}{\partial T^2}\Big|_{h=0} = -\frac{1}{T_c} t^{\frac{d}{y_t}-2} g''(0) \\ M(t,0) &= -\frac{\partial f}{\partial B}\Big|_{h=0} = t^{\frac{d-y_h}{y_t}} g'(0) \\ \chi(t,0) &= \frac{\partial^2 f}{\partial B^2}\Big|_{h=0} = t^{(d-2y_h)/y_t} g''(0) \end{aligned}$$

As function with single variable $h$, $\lim_{t\to 0} M(t,h) \sim h^{\frac{1}{\delta}}$, which implies that $g'(\alpha) \sim \alpha^{\frac{1}{\delta}}$ since $\alpha$ is linear function of $h$. Hence we have

$$\lim_{t\to 0} M = \lim_{t\to 0} t^{(d-y_n-\frac{y_n}{\delta})} h^{1/\delta}$$

since it is non-zero, we have $d - y_n - y_n\frac{1}{\delta} = 0$. Hence we have

$$\delta = \frac{y_h}{d-y_h}$$

**Exercies 0.0.2.** We have following relation

$$G_\sigma(\mathbf{r};t,h) = t^{-2x_\sigma} G_\sigma(\frac{\mathbf{r}}{b}; b^{y_t} t, b^{y_h} h) \tag{1}$$

Let $h = 0, K = b^{y_t} t$,

$$G_\sigma(\mathbf{r};t,0) = t^{2x_\sigma/y_t} G_\sigma(\frac{\mathbf{r}}{Kt^{-1/y_t}}; K, 0)$$

Since $G_\sigma(\mathbf{r}) \sim r^{-\tau} e^{-\frac{r}{\xi}}$, we have $\xi \sim t^{-1/y_t}$. It implies $\nu = 1/y_t$. With relation 1, we have

$$\chi(t,h) = \frac{1}{T}\int d^d\mathbf{r} G_\sigma(\mathbf{r};t,h) = t^{d-2x_\sigma} \chi(b^{y_t} t, b^{y_h} h)$$

So $\gamma = (d-2x_\sigma)/y_t$. But we have $\eta = 2x_\sigma + 2 - d$ for finite limit of $G(r)$ when $t \to 0$ and $h = 0$. Therefore, we get

$$\gamma = \nu(2-\eta)$$

With scaling relations

$$\alpha + 2\beta + \gamma = 2$$
$$\alpha + \beta(1 + \delta) = 2$$

and $\alpha = 2 - d\nu$, we have

$$\beta = \frac{d\nu - 2\nu + \nu\eta}{2}$$
$$\delta = \frac{d - \eta + 2}{d + \eta - 2}$$

**Exercies 0.0.3.** By listed commutation relations, we have, for $r, s > 0$,

$$\begin{aligned}
[D, J_{rs}] = [D, L_{rs}] = \frac{i}{2}[D, [K_r, P_s]] \\
= -\frac{i}{2}\big([P_s, [D, K_r]] + [K_r, [P_s, D]]\big) \\
= \frac{1}{2}[P_s, K_r] - \frac{1}{2}[K_r, P_s] \\
= 0
\end{aligned}$$

For $r = -1, s = 0$, $[D, J_{rs}] = [D, D] = 0$. For $r = -1, s \neq 0$, $[D, J_{-1,s}] = [D, \frac{1}{2}(P_s - K_s)] = \frac{i}{2}(P_s + K_s)$. For $r = 0$, $[D, J_{0s}] = \frac{i}{2}(P_s - K_s)$. Hence (2,25) is satisfied when $(m, n) = (-1, 0)$.

If $(m, n) = (-1, n)$, then we have

$$[J_{mn}, J_{rs}] = \frac{1}{2}[P_n, J_{rs}] - \frac{1}{2}[K_n, J_{rs}]$$

With listed commutation relations, we can easily check it coincides with (2,25) respectively. Similary check in the case of $(m, n) = (0, n)$.