ALGEBRAIC CURVES AND MODULI PROBLEMS

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1. Preliminaries

Left to be added!

2. Algebraic Curves

Let $C \subseteq \mathbb{P}^n$ be an algebraic curves. It is well known result that there are one-to-one correspondence between following three objects

$$\{\textbf{line bundles over } C\} \longleftrightarrow \{\textbf{invertable sheaves over } C\}/\sim \longleftrightarrow \{\textbf{divisors over } C\}/\sim$$

We need to use these correspondences to prove following result

Theorem 2.1. *Pic* $(C) \simeq H^1(C, \mathcal{O}_C^*)$

At the very beginning, we need some knowledge about sheaf cohomology. Suppose $\mathcal{F}$ be a sheaf of $\mathcal{O}_C$-modules over C.

Definition 2.1 (Čech resolution)**.** Let $\mathcal{U} = \{U_i\}$ be open cover of X, i.e. every $U_i \to X$ is open immersion, where X is a scheme, and $\mathcal{F}$ be a $\mathcal{O}_X$-module. Given an finite index set I, we have a open subset $\bigcap_{i \in I} U_i$ of X. Set

$$C_k(\mathcal{U}, \mathcal{F}) = \prod_{|I|=k} \mathcal{F}(\bigcap_{i \in I} U_i)$$

$C_*(\mathcal{U}, \mathcal{F})$ is called Čech complex of open cover $\mathcal{U}$ and sheaf $\mathcal{F}$.