# SEMINAR NOTES: COMMUTATIVE ALGEBRAS

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## 1. Rings and Ideals

**Definition 1.1.** A **ring** $R$ is a set with two maps (addition) $+ : R \times R \to R$, (mutiplication)$\times : R \times R \to R$( denote $+(x, y)$ by $x + y$ and $\times(x, y)$ by $x \times y$)that satisfy following properties

(1) R is a abelian group with respect to addtion, its identity is denoted by 0;
(2) R is a monoid with identity $1 \in R$ with respect to multiplication;
(3) $z \times (x + y) = z \times x + z \times y$ and $(x + y) \times z = x \times z + y \times z$ for any given $x, y, z$.

We typically write $xy$ for $x \times y$.

In a ring $R$, if $1 = 0$, then $R$ has only one elements, it is trival and called **zero ring**. Denoted zero ring by 0.

**Definition 1.2.** Let $A$ and $B$ be two rings. $1_A$ and $1_B$ are their identities. A ring homomorphism from $A$ to $B$ is a map $f : A \to B$, which preserves both addition and multipication structure, that means, for any $x, y \in A$

$$f(x + y) = f(x) + f(y)$$
$$f(xy) = f(x)f(y)$$
$$f(1_A) = 1_B$$