UNIVERSIDAD POLITÉCNICA DE YUCATÁN

Graph Theory

Homework 2

Objective

The student will explain the basic concepts of Networks including: definitions, characteristics and applications.

Instructions

1. Which word or words from the following list describe each of the five networks below: directed, undirected, cyclic, acyclic, approximately acyclic, planar, approximately planar, tree, approximate tree.

   a) The Internet, at the level of autonomous systems.
   b) A food web.
   c) The stem and branches of a plant.
   d) A spider web.
   e) A complete clique of four nodes.

   Give one real-life example of each of the following types of networks, not including the five examples above:

   f) An acyclic (or approximately acyclic) directed network
   g) A cyclic directed network
   h) A tree (or approximate tree)
   i) A planar (or approximately planar) network
   j) A bipartite network

2. Is it possible to have the following degrees in a graph with 7 nodes?

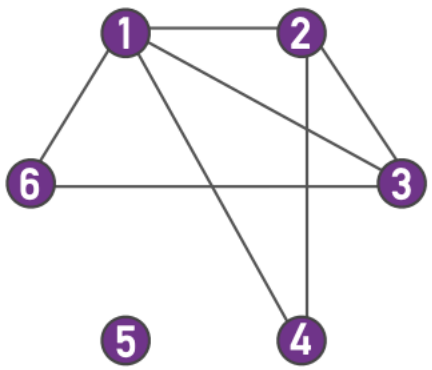
   $$\{4,4,4,3,5,7,2\}$$

   Why?

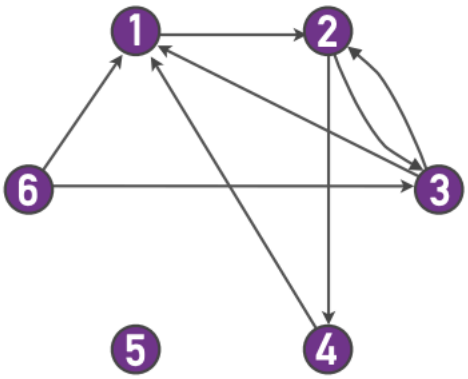
3. A simple network consists of n nodes in a single component. What is the maximum possible number of edges it could have? What is the minimum possible number of edges it could have? Explain briefly how you arrive at your answers.
4. The adjacency matrix is a useful graph representation for many analytical calculations. However, when we need to store a network in a computer, we can save

computer memory by offering the list of links in a Lx2 matrix, whose rows contain the starting and end point i and j of each link.

Construct for the networks (a) and (b) in



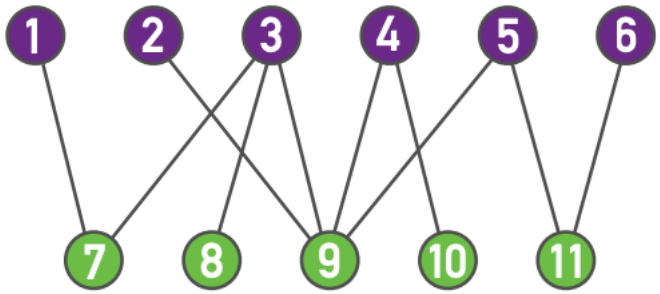
(a)



(b)

a) The corresponding adjacency matrices.
b) The corresponding link lists.
c) Determine the average clustering coefficient of the network show in (a).
d) In the (a) network, how many paths (with possible repetition of nodes and links) of length 3 exist starting from node 1 and ending at node 3? And in (b)?

5. Consider the bipartite network of the following figure.

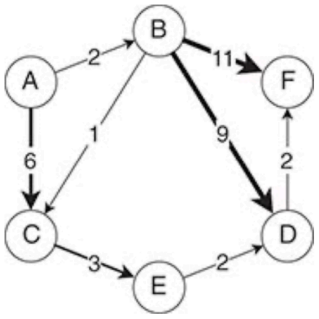


a) Construct its adjacency matrix. Why is it a block-diagonal matrix?
b) Construct the adjacency matrix of its two projections, on the purple and on the green nodes, respectively.
c) Calculate the average degree of the purple nodes and the average degree of the green nodes in the bipartite network.
d) Calculate the average degree in each of the two network projections. Is it surprising that the values are different from those obtained in point (c)?

6. (a) Netflix keeps data on customer preferences using a big bipartite network connecting users to titles they have watched and/or rated. Netflix's movie library contains approximately 100,000 titles if you count streaming and DVD-by-mail. In

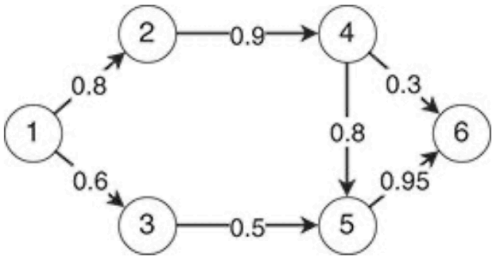
the fourth quarter of 2013, Netflix reported having about 33 million users. Assume the average user's degree in this network is 1000. Approximately how many links are in this network? Would you consider this network sparse or dense? Explain.
(b) Suppose that from 2013 to 2014 Netflix's library has remained the same size, while the number of users has increased. Further suppose that the average user's degree in this network has remained constant. Has the density of this network increased, decreased, or stayed the same?

7. Consider the following weighted directed network:



   a. Which of the following most accurately describes the connectedness of this network?
      i. Strongly connected.
      ii. Weakly connected.
      iii. Disconnected.
      iv. None of the above.
   b. What is the in-strength of node D? What is the out-strength of node C?
   c. How many nodes are in the largest strongly connected component?

8. Consider the following network:



   a. Which of the following most accurately describes the connectedness of this network?
      i. Strongly connected.
      ii. Weakly connected.
      iii. Disconnected.
      iv. None of the above.
   b. When discussing path lengths on a weighted graph, one must first define how the weights are related to the distances. The length of a path between

two nodes is then the sum of the distances of the links in that path. Consider the previous network and assume that the link weights represent distances. Using this distance metric, what is the shortest path between node 1 and 6?

c. A common way to define the distance between two nodes is the inverse (or reciprocal) of the link weight. Consider the previous network and assume that the distance between two adjacent nodes is defined as the reciprocal of the link weight. Using this distance metric, what is the shortest path between node 1 and 6?

Homework Submission

Submit your Homework as a unique PDF document.

Recommended Books

- Reza Zafarani, Mohammad Ali Abbasi, Juan Liu (2014). Social Media Mining: An Introduction.
- Barabási, Albert-László (2016). Network Science.
- Menczer, Fortunato and Davis (2020) A First Course in Network Science.
- Mark Newman (2018) Networks. The empirical study of Networks. UK: Oxford University Press.

Grading Scheme

Check list

	✓
Question 1	12.5 pts
Question 2	6.25 pts
Question 3	6.25 pts
Question 4	18.75 pts
Question 5	18.75 pts
Question 6	12.5 pts
Question 7	12.5 pts
Question 8	12.5 pts

*Partial credit can be assigned to each question if the answer is not completely correct.