



Centrality Measures and Network Models

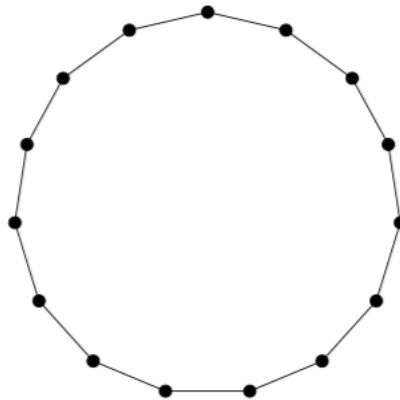
Homework 3

Objective

The student will explain the basic concepts of Networks: Centrality Measures.

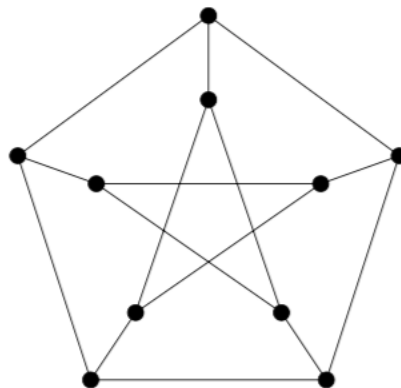
Instructions

1. A network consists of n nodes in a ring, where n is odd:

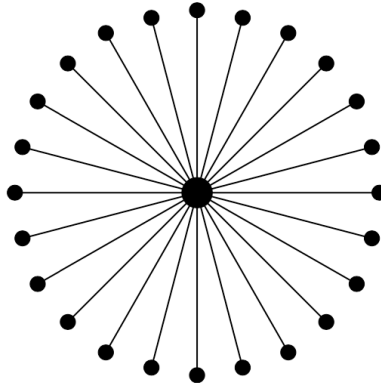


All the nodes have the same closeness centrality. What is it, as a function of n ?

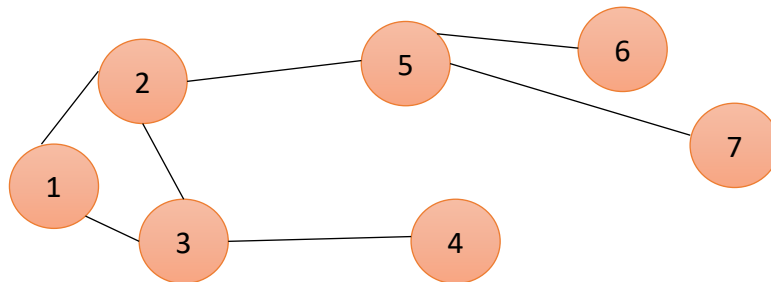
2. Calculate the closeness centrality of each node of the nodes in this network:



3. A “star graph” consists of a single central node and $n - 1$ other nodes connected to it. What is the (unnormalized) betweenness centrality of the central node as a function of n ?



4. Consider the following network. For each question, in case of a tie, answer with all the tied top nodes.
- Which node has the highest degree centrality?
 - Which node has the highest betweenness centrality?
 - Which node has the highest closeness centrality?



5. Consider a social network where a connection represents a sexual relationship. Read the report by Liljeros et al. (2001) about a study of such a network based on a sample of 4781 Swedes. (If you do not have access to the journal through your institution, you can download a preprint of the paper at,

<https://arxiv.org/abs/cond-mat/0106507>

What is the maximum degree in this network? What does it mean? If you consider the subnetworks with nodes corresponding to males and females, respectively, do they have the same degree distribution? Why or why not?



6. Consider the Random Graph Model. Show that the binomial distribution of the degree given by,

$$p_k = \binom{N-1}{k} p^k (1-p)^{N-1-k}$$

follows a Poisson Distribution given by,

$$p_k = e^{-\langle k \rangle} \frac{\langle k \rangle^k}{k!}$$

under the condition $\langle k \rangle \ll N$.

7. Sociologist estimate that a typical person knows about 1,000 individuals on a first name basis. Considering that the society can be modeled with a Random Graph Network of $N = 7 \times 10^9$ of people, obtain the maximum and minimum degree (expression and numerical value).
8. Show that in a Scale Free Network, the maximum degree k_{max} and k_{min} follows the relation given by,

$$k_{max} = k_{min} N^{\frac{1}{\gamma-1}}$$

where N is the number of nodes in the Network and γ is the degree exponent.

Homework Submission

Submit your Homework as a unique PDF document.

Recommended Books

- Reza Zafarani, Mohammad Ali Abbasi, Juan Liu (2014). Social Media Mining: An Introduction. Chapter: Network Measures.
- Menczer, Fortunato and Davis (2020) A First Course in Network Science.
- Mark Newman (2018) Networks. The empirical study of Networks. UK: Oxford University Press. Chapter: Measures and Metrics.