Linear mixed models in R
Day 5

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Random structure: correlations between intercept and slope

(1 + Context | Subject)

Random intercept

Random slopes

Variable name for coding levels of random factor

| - random intercept and random slope are correlated with each other

|| - no correlations between random effects

(Context || Subject)

This expands to:

(0 + Context | Subject) + (1|Subject)

Random structure: correlations between intercept and slope

$y_{iCond} = \beta_0 + (\beta_{Cond} + u_{1i})x_{iCond} + \epsilon_{Cond} + u_{0i}$

Between-subject variability in the intercept:

$u_0 \sim Normal(0, \sigma_{u0})$

Between-subject variability in the slope:

$u_1 \sim Normal(0, \sigma_{u1})$

Within-subject variability:

$\epsilon \sim Normal(0, \sigma)$

```
model_Large8 = lmer(RT ~ Group+Context + (1
+Context|Subject) +(1| ItemNr), data=PN_Data)

## Random effects:
##  Groups   Name        Variance Std.Dev. Corr
##  ItemNr   (Intercept) 23832    154.4
##  Subject  (Intercept) 19854    140.9
##           ContextUK   16898    130.0    -0.44
##  Residual             56540    237.8
```

Random structure: correlations between intercept and slope

```
(1 +Context|Subject)

## Random effects:
##  Groups   Name        Variance Std.Dev. Corr
##  Subject  (Intercept) 19854    140.9
##           ContextUK   16898    130.0    -0.44
##  Residual             56540    237.8
```

```
(1 +Context||Subject)

## Random effects:
##  Groups    Name        Variance Std.Dev. Corr
##  Subject   ContextPL    3843     61.99
##            ContextUK    4623     68.00   -1.00
##  Subject.1 (Intercept) 16011    126.53
##  Residual              56540    237.78
```

Random structure: correlations between intercept and slope

Correlations between intercepts and slopes are frequently assumed in research

- Group with higher intercept will also have a higher slope
- If they are present, they can be used to calculate better estimates
- Even low correlations can contribute to better estimates

But they make the model more complex

Correlations of 1 between slope and intercept can prevent the model from converging

- 1 or very close to one, usually means that the model failed to calculate the correlation
- Usually because there was not enough data
- Removing correlations by using || allows model to converge
- But it is recommended to also switch to a simpler model

Summary-Output of an LME

- You prepared your final model
- You checked the assumptions
- You understood and interpreted the outcome

```
## Linear mixed model fit by REML ['lmerMod']
## Formula: RT ~ Group + Context + Group:Context + Age + AoA + Trial + (1 +
##     Context | Subject) + (1 + Context | ItemNr)
##    Data: PN_Data
##
## Random effects:
##  Groups   Name        Variance Std.Dev. Corr
##  ItemNr   (Intercept) 24021    154.99
##           ContextUK    3449     58.73   -0.15
##  Subject  (Intercept) 19522    139.72
##           ContextUK   16426    128.16   -0.41
##  Residual             55688    235.98
## Number of obs: 7227, groups:  ItemNr, 210; Subject, 74
##
## Fixed effects:
##                              Estimate Std. Error t value
## (Intercept)                 1019.6555    79.0877  12.893
## GroupExperimental            -59.6657    37.2576  -1.601
## ContextUK                    -11.2579    23.2922  -0.483
## Age                           -1.9501     2.4100  -0.809
## AoA                            3.2835     3.9845   0.824
## Trial                          0.4629     0.2875   1.610
## GroupExperimental:ContextUK   45.7348    32.1411   1.423
##
## Correlation of Fixed Effects:
##              (Intr) GrpExp CntxUK Age    AoA    Trial
## GrpExprmntl  0.196
## ContextUK   -0.127  0.290
## Age         -0.810 -0.309 -0.012
## AoA         -0.269 -0.217  0.001 -0.234
## Trial       -0.110 -0.001 -0.006  0.000  0.001
## GrpExpr:CUK  0.088 -0.404 -0.704  0.010 -0.001  0.000
```

Reporting linear mixed-effects models

“A one-way ANOVA demonstrated that the effect of leadership style was significant for employee engagement, $F(2, 78) = 4.58$, $p = .013$.”

There are no standardized guidelines for LMEs yet

The complexity, large structure (and a lack of p-values) make reporting LMEs more effortful

- Especially for short-form texts (i.e. abstracts)

Kim, Dedrick, Cao, Ferron (2008) Multilevel Factor Analysis: Reporting Guidelines and a Review of Reporting Practices

Guidelines and checklist for reporting statistical models

- Multilevel factor analysis, not LMEs
- Might still be helpful to remind you about reporting different aspects of your statistical tests

Reporting LME results

As many details as possible

All relevant statistical measures:

- Data structure and size
- Variable transformations and contrast coding
- Maximal model structure
- Final model results for all fixed and random effects
- Post-hoc tests for relevant effects

Reporting as a table

Effect	Estimate	SE	t	by-Picture SD	by-Participant SD
Intercept	−1.14	0.03	−38.00***	0.15	0.18
Group	0.00	0.05	−0.02		
Context	0.02	0.04	0.65	−0.05	−0.11
Word-lexical frequency	0.00	0.01	−0.08	–	
Age	−0.03	0.02	−1.30		–
Age of L2 acquisition	0.03	0.02	1.15		–
log (Trial number)	0.00	0.01	0.78		
Group:Context	0.03	0.07	0.48		–
Group:Word-lexical frequency	0.00	0.00	−0.28	–	–
Control Group:Context:Word-lexical frequency	0.00	0.01	−0.11	–	–
Mig. Group:Context:Word-lexical frequency	0.01	0.01	1.94′	–	–

predict_response()

ggeffects package

Integrated with the ggplots package

Calculates marginal means

Provides them in easily usable format for plots

predict_response()

```
model_Large8 %>%
  predict_response(c("Group", "Context"))

## # Predicted values of RT


## Context: PL

## Group        | Predicted |          95% CI

## ----------------------------------------------

## Control      |    994.54 | 945.32, 1043.76

## Experimental |    955.29 | 906.37, 1004.20

## Context: UK

## Group        | Predicted |          95% CI

## ----------------------------------------------

## Control      |   1006.62 | 957.25, 1055.99

## Experimental |    967.37 | 918.04, 1016.69


## Adjusted for:

## * Subject = 0 (population-level)

## *  ItemNr = 0 (population-level)
```

predict_response()

X - the values of the first term, x-values

group - levels of the second term

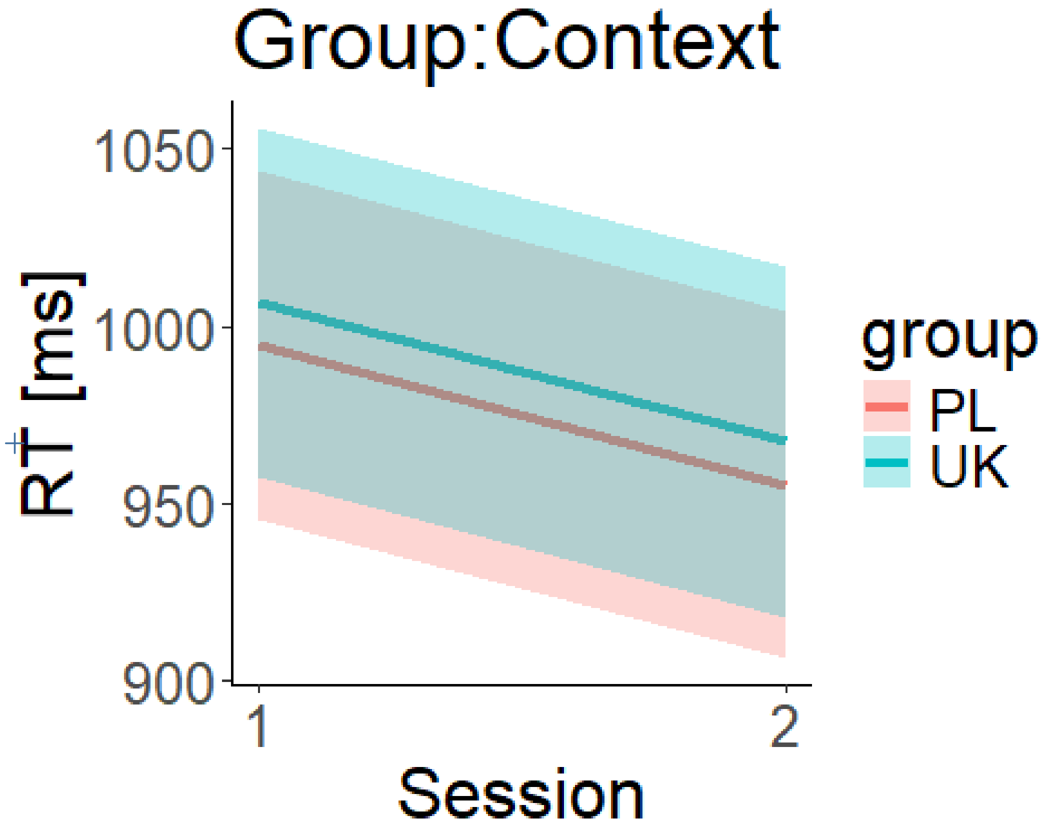
facet – levels of third term

predicted - the predicted y-values

conf.low and conf.high

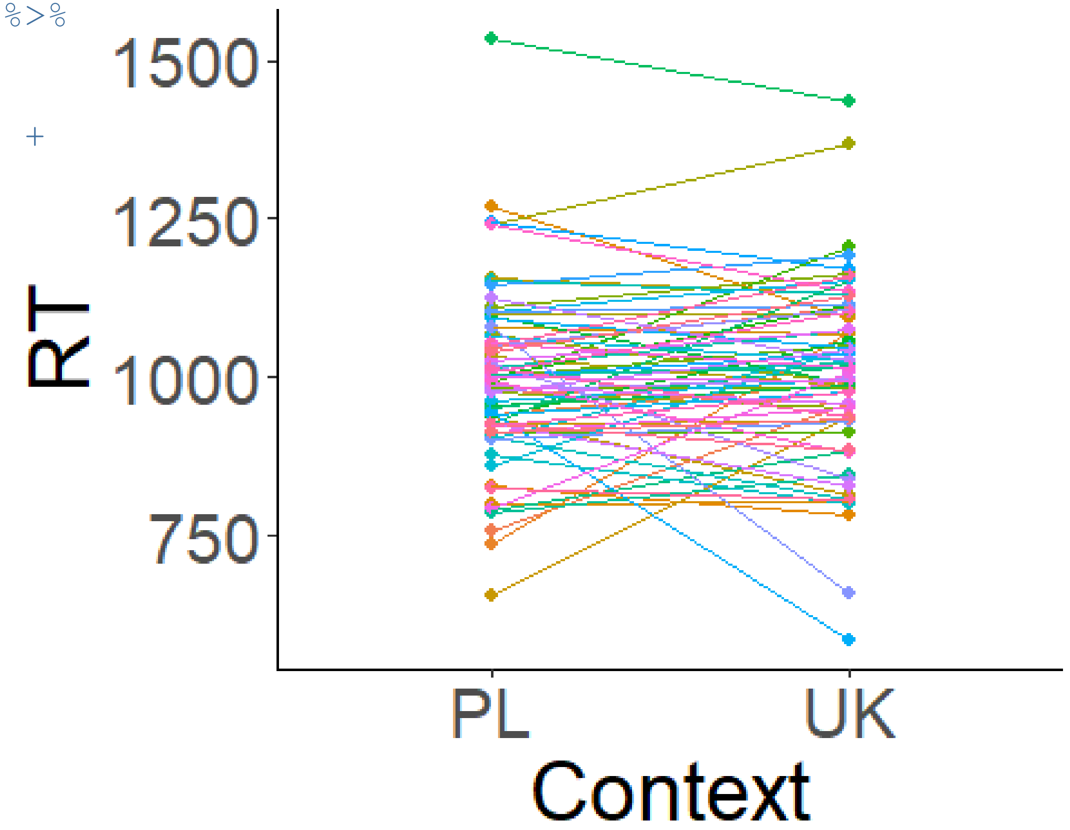
predict_response() for fixed effects

```
model_Large8 %>%
  predict_response(c("Group", "Context")) %>%
  ggplot(aes(x=as.numeric(x), y=predicted,
       color=group, fill=group)) +
  geom_line(linewidth=1.7) +
  geom_ribbon(aes(ymin=conf.low, ymax=conf.high),
       alpha = 0.3, colour = NA) +
  scale_x_continuous(name =
"Session",breaks=c(1,2)) +
  scale_y_continuous(name = "RT [ms]") +
  ggtitle("Predicted effects for Group:Context") +
  theme_classic() +
  theme(text = element_text(size=24),
element_line(size = 2))
```



predict_response() for random effects

```
model_Large8 %>%
  ggpredict(c("Context", "Subject"), type = "re") %>%
    ggplot(aes(x=x, y=predicted, group=group)) +
    geom_point(size=1.7, aes(color=group)) +
    geom_line(aes(color=group)) +  theme_classic() +
    scale_color_discrete(guide="none") +
    scale_y_continuous(name="RT") +
    scale_x_discrete(labels = c("PL",
"UK"),name="Context")+
    theme(text = element_text(size=28),
element_line(size = 2))
```



Power analysis

Statistical tool that calculates the minimum sample size for a given study and analysis

It needs four primary components:

- Statistical power: the likelihood that a test will detect an effect of a certain size if there is one, usually set at 80% or higher
- Sample size: the minimum number of observations needed to observe an effect of a certain size with a given power level
- Alpha: usually 0.05
- Expected effect size

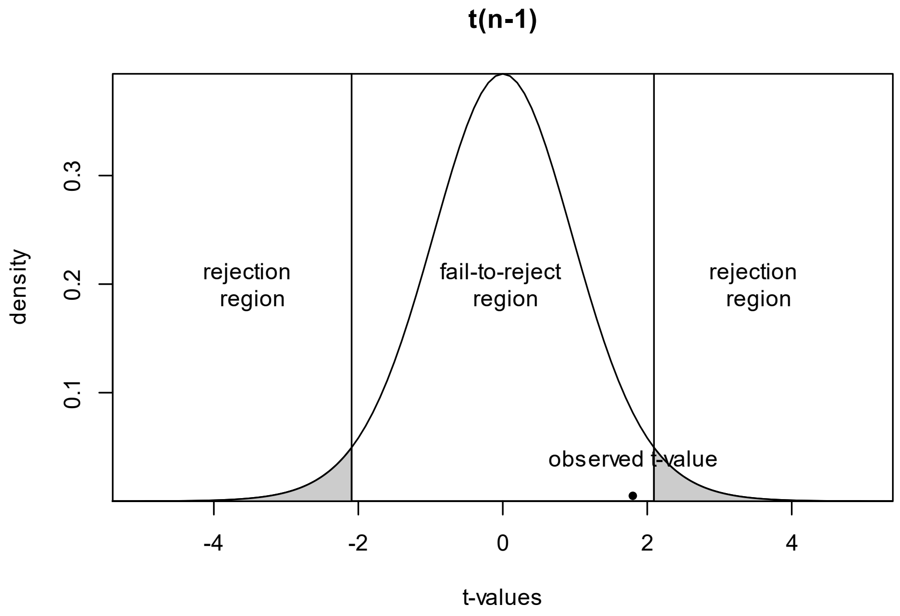
If you have three of those components you can calculate the fourth

- Determine power from existing sample
- Determine necessary sample size to reach a given power

Statistical power

Probability of not rejecting H1, if H1 is true

	$If\ H_0\ is\ True$	$If\ H_1\ is\ True$
$Probability\ to\ reject\ H0$	α	$1-\alpha$
$Probability\ to\ not\ reject\ H_0$	$1-\beta\ (power)$	β



Get data for power analysis

Generate your own data

$y_{iCond} = \beta_0 + (\beta_{Cond} + u_{1i})x_{iCond} + \epsilon_{Cond} + u_{0i}$

$$u_0 \sim Normal(0, \sigma_{u0})$$
$$u_1 \sim Normal(0, \sigma_{u1})$$
$$\epsilon \sim Normal(0, \sigma)$$

Known formula:

```
rnorm(10,mean=430,sd=124)

##  [1] 520.5332 789.1068 120.7319 354.2828 304.0824 370.6565 309.9614 695.6549

##  [9] 434.9770 547.4512

rnorm(10,mean=0,sd=80)

##  [1]    6.447877  -65.969985   35.417299   -5.445993    2.426756  -74.131268

##  [7] -132.068257  -23.546198  178.982205   84.780615
```

Calculate power of a given sample

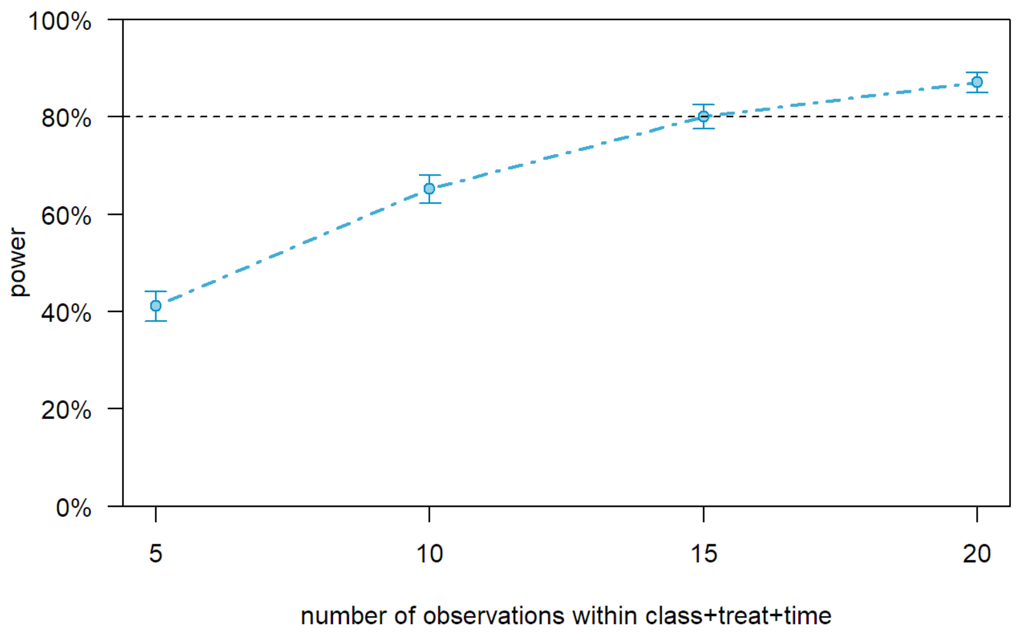
```r
library(simr)

powerSim(model_Large8, nsim=50, test = fcompare(RT~Context), progress = FALSE)


## Power for model comparison, (95% confidence interval):

##       28.00% (16.23, 42.49)

##

## Test: Likelihood ratio

##       Comparison to RT ~ Context + [re]

##

## Based on 50 simulations, (0 warnings, 0 errors)

## alpha = 0.05, nrow = 7227

##

## Time elapsed: 0 h 0 m 51 s
```

Calculate power of a given sample

```
Model_11 <- extend(model_Large8, within="Subject+Context", n=10)

powerSim(Model_11, nsim=50, test = fcompare(RT~Context), progress = FALSE)


## Power for model comparison, (95% confidence interval):

##       18.00% ( 8.58, 31.44)

##

## Test: Likelihood ratio

##       Comparison to RT ~ Context + [re]

##

## Based on 50 simulations, (0 warnings, 0 errors)

## alpha = 0.05, nrow = 1467

##

## Time elapsed: 0 h 0 m 11 s
```

Calculate necessary sample size to get a desired result

```
p_curve_treat <- powerCurve(model_ext_subj, test=fcompare(y~time),within="class+treat+time", breaks=c(5,10,15,20))

plot(p_curve_treat)
```



LME to-do list

1 Hypotheses

- Run power analysis

2 Prepare your data

3 Build your model

4 Analyse your model

5 Report your results

- Report the entire maximal model structure
- Report the results of the final model

Thank you for your attention!