MITMECHE

Closed Loop Control Examples

Workshop
TIM THE BEAVER

Department of Mechanical Engineering
Massachusetts Institute of Technology

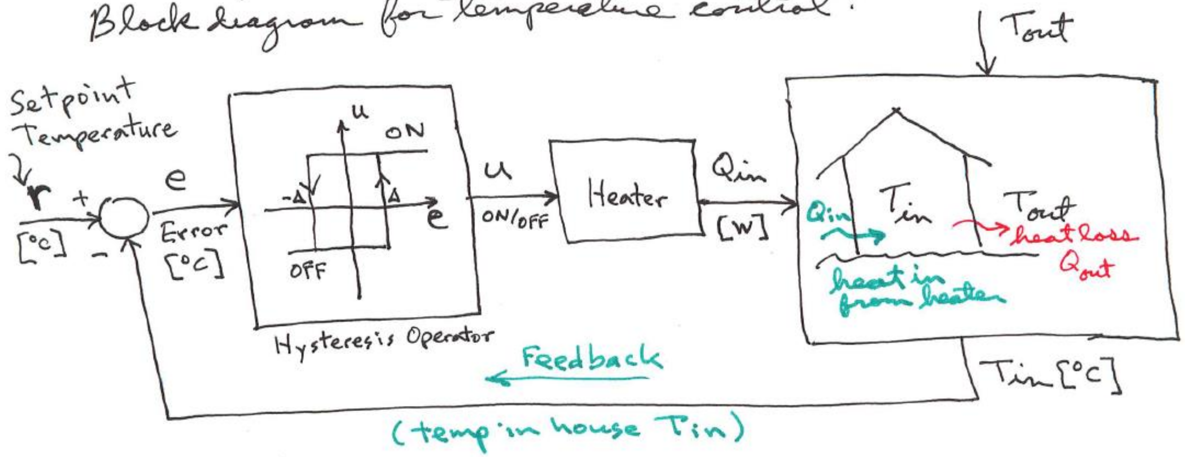
Outline

- Introduction to On-off Control
  - On-off controller concept
  - First order on-off controller example
- Discrete Control Basics
  - Z-transform
  - Mapping from continuous to discrete
- Single Integrator Plant Control
  - Loop shaping design review
  - Single Integrator design example
- Double Integrator Plant Control
- RLC Circuit Control
- Notch Filter

Introduction to On-off Control

- On-off is widely for house temperature control systems for its simplicity

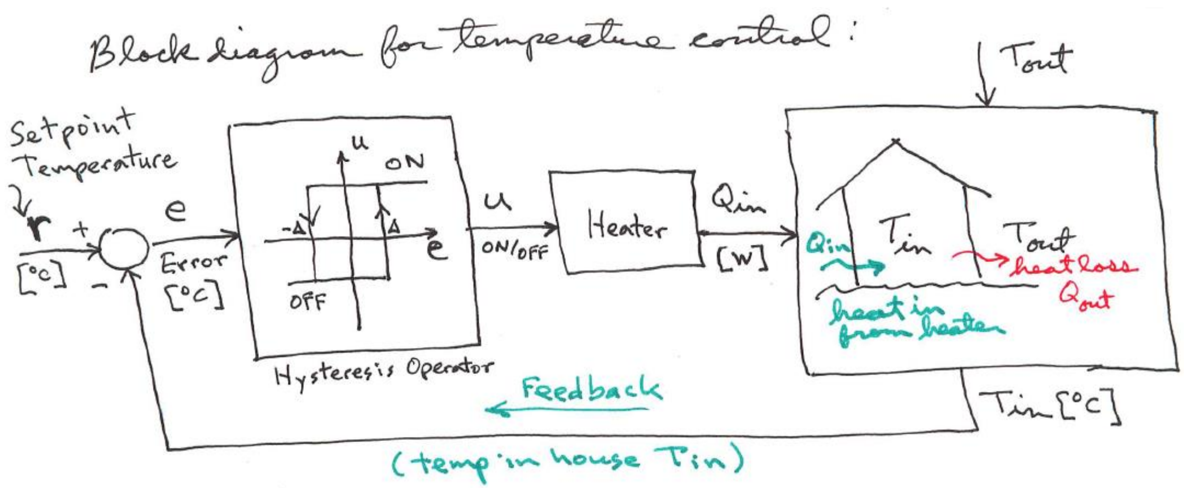
Block diagram for temperature control:



$$Q_{in} = \begin{cases} Q_0\ [W] \text{ if } u = ON & \text{(constant heat into house when heater on)} \\ 0\ [W] \text{ if } u = OFF & \text{(no heat into house when heater off)} \end{cases}$$

Introduction to On-off Control

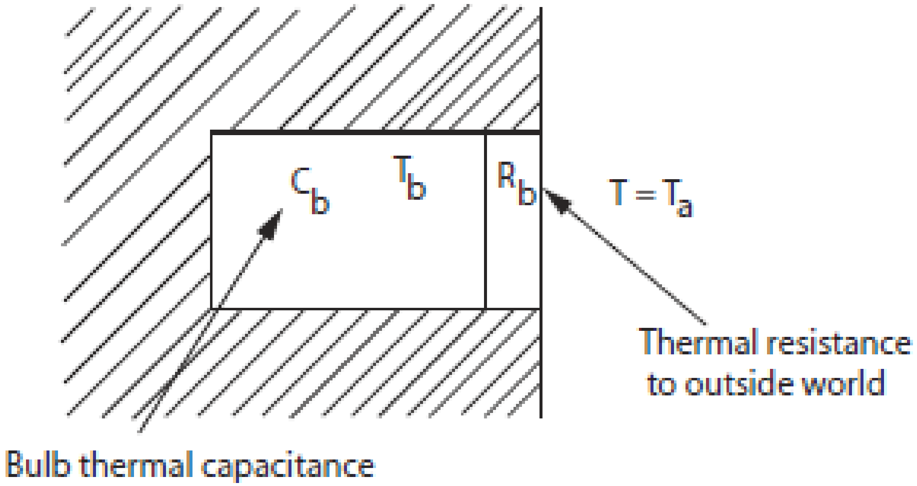
- On-off is widely for house temperature control systems for its simplicity



$$Q_{in} = \begin{cases} Q_0 \text{ [W] if } u = \text{ON} & \text{(constant heat into house when heater on)} \\ 0 \text{ [W] if } u = \text{OFF} & \text{(no heat into house when heater off)} \end{cases}$$

First Order System On-off Control

- On-off is widely for house temperature control systems for its simplicity



$$q_b = \frac{T_a - T_b}{R_b}$$

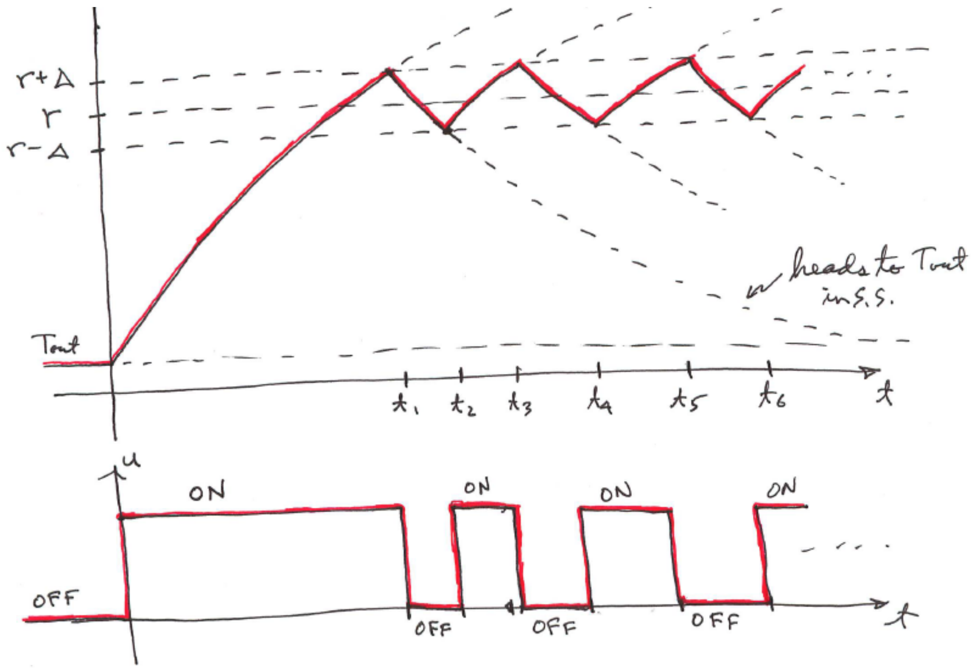
$$C_b \frac{dT_b}{dt} = \frac{T_a - T_b}{R_b}$$

$$R_b C_b \frac{dT}{dt} + T = 0$$

$$T(t) = T_0 e^{-t/R_b C_b} \quad [\mathrm{K}]$$

First Order System On-off Control

- First order system on-off control output



Discrete Control Basics

- Microcontrollers are powerful nowadays and can be very useful for control system implementation for its convenience, especially useful for advanced control algorithms such as state space method, adaptive controller and etc.
- Discretization problems arise when the sampling rate is not very high compared to the frequency range interested
- General procedure for designing controller
  - Design analog controller
  - Discretize the controller using one of the *S* to *Z* transformation
  - Perform simulation to check for performance
  - If performance not good enough, increase sample rate or use different transformation method

Z Transformation

- Z-transformation is the discrete time domain version of the Laplace transformation that simplifies the design and analysis procedure of control systems

$$X(z) = \mathcal{Z}\{x[n]\} = \sum_{n=-\infty}^{\infty} x[n] z^{-n}$$

Z-transformation definition

$$F(s) = \int_0^{\infty} f(t) e^{-st}\, dt$$

Laplace transformation definition

Continuous to Discrete Transformations

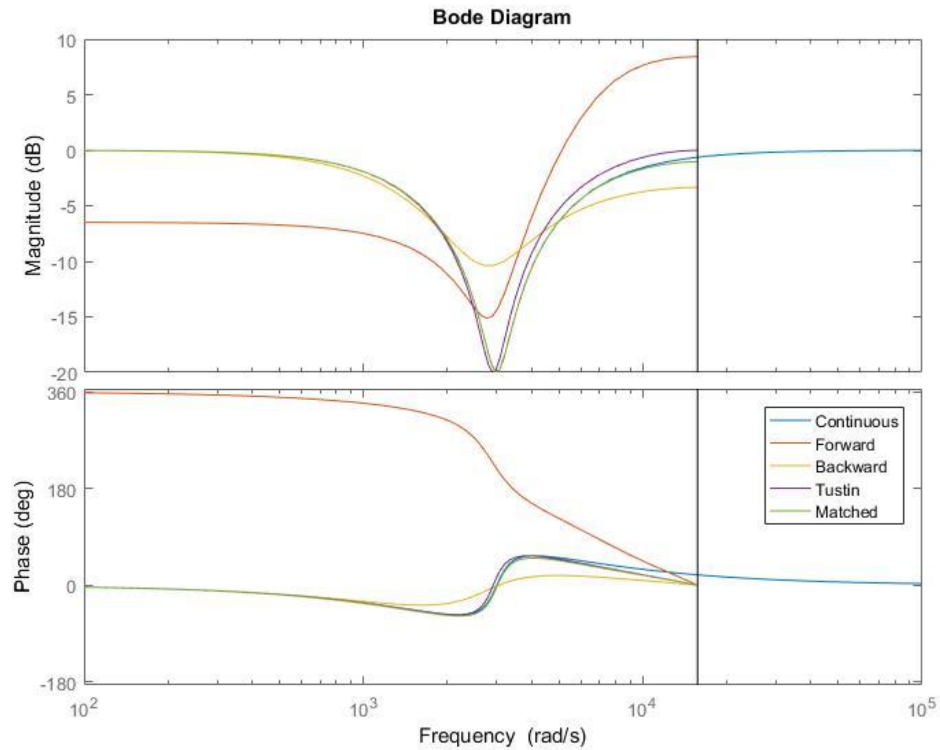
- Backward difference
- Forward difference
- Bilinear (Tustin) transformation

Mapping method	Mapping equation	Equivalent discrete-time filter for $G(s) = \frac{a}{s+a}$
Backward difference method	$s = \frac{1 - z^{-1}}{T}$	$G_D(z) = \frac{a}{\frac{1-z^{-1}}{T} + a}$
Forward difference method	$s = \frac{1 - z^{-1}}{Tz^{-1}}$	This method is not recommended, because the discrete-time equivalent may become unstable.
Bilinear transformation method	$s = \frac{2}{T}\frac{1 - z^{-1}}{1 + z^{-1}}$	$G_D(z) = \frac{a}{\frac{2}{T}\frac{1-z^{-1}}{1+z^{-1}} + a}$

- Matched pole-zero mapping
  - Pole at s = -a is mapped to z = exp(-aT)
  - T is the sampling period

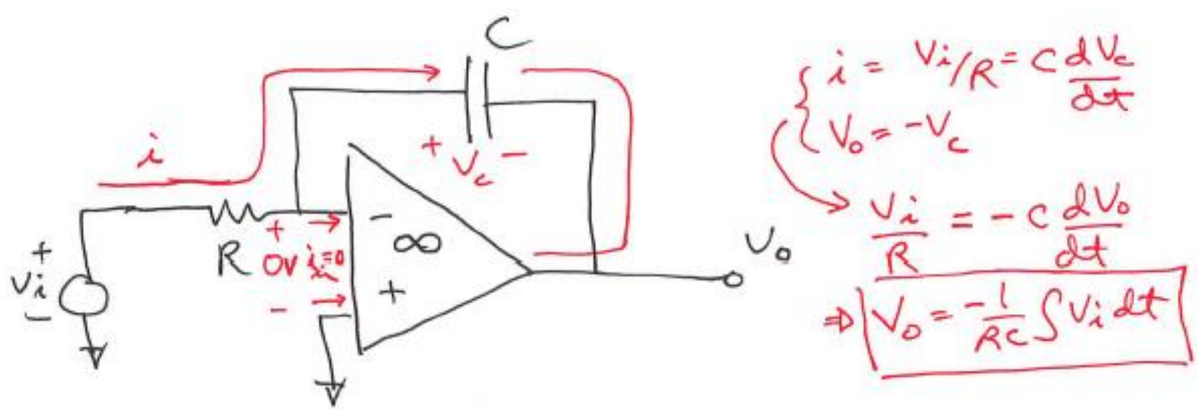
Discrete Control Transformations

- Different methods of transformation yield different approximation results



Single Integrator Plant Control

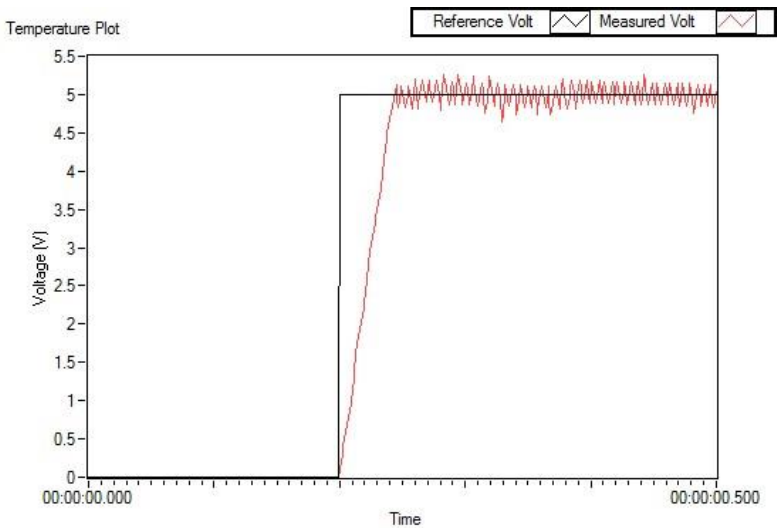
- The diagram for an integrator plant formed by an Op-amp is shown below with R = 100 kΩ and C = 0.1μF

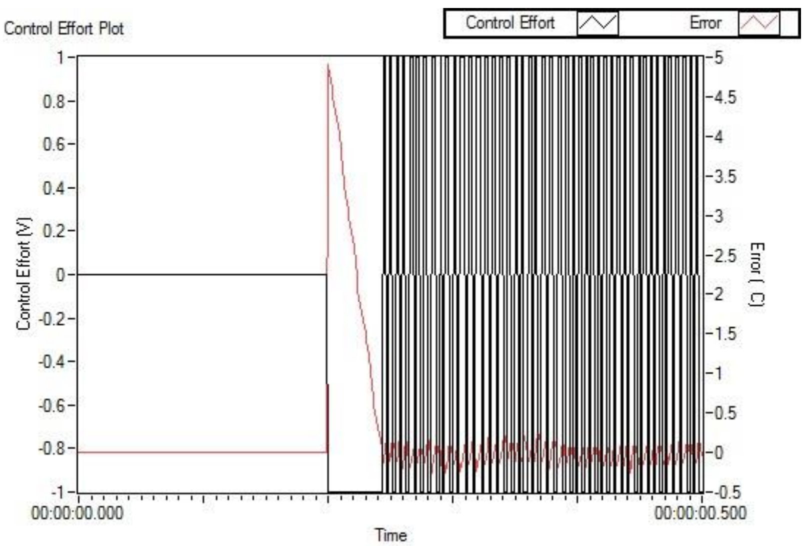


MITMECHE

On-Off Control

- $\Delta V = 0.1$ V and $A = 1$ V yields

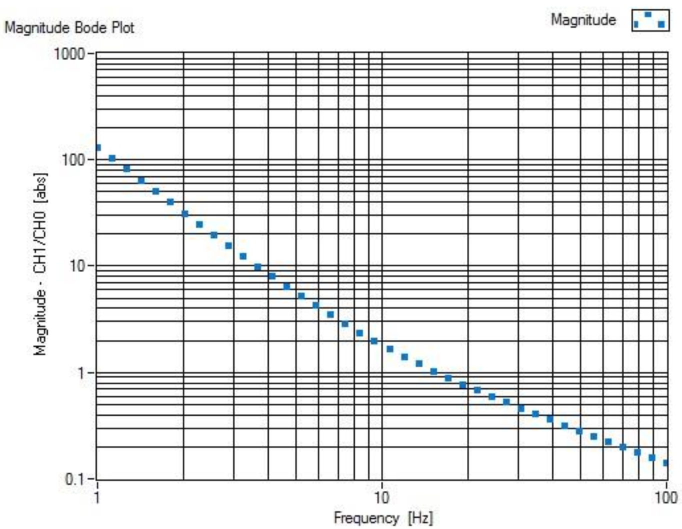


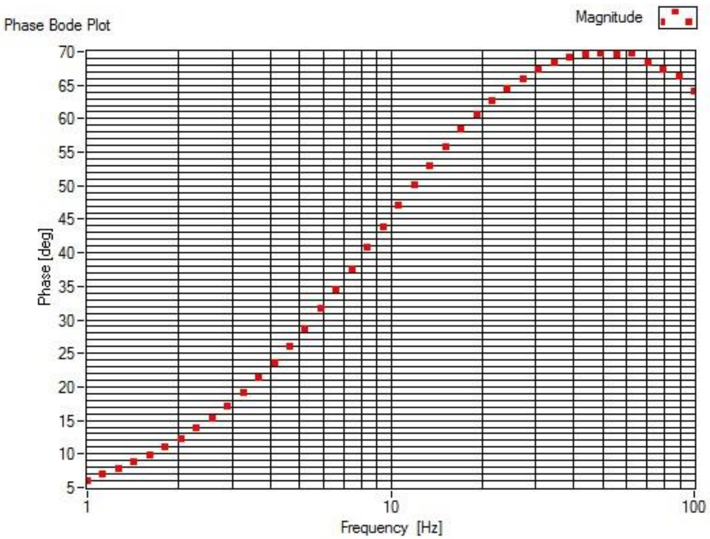


PI Control

- With K_p = 0.866 and K_i = 57.73, the loop transfer function gives a phase margin of 60 degree with transfer function shown below

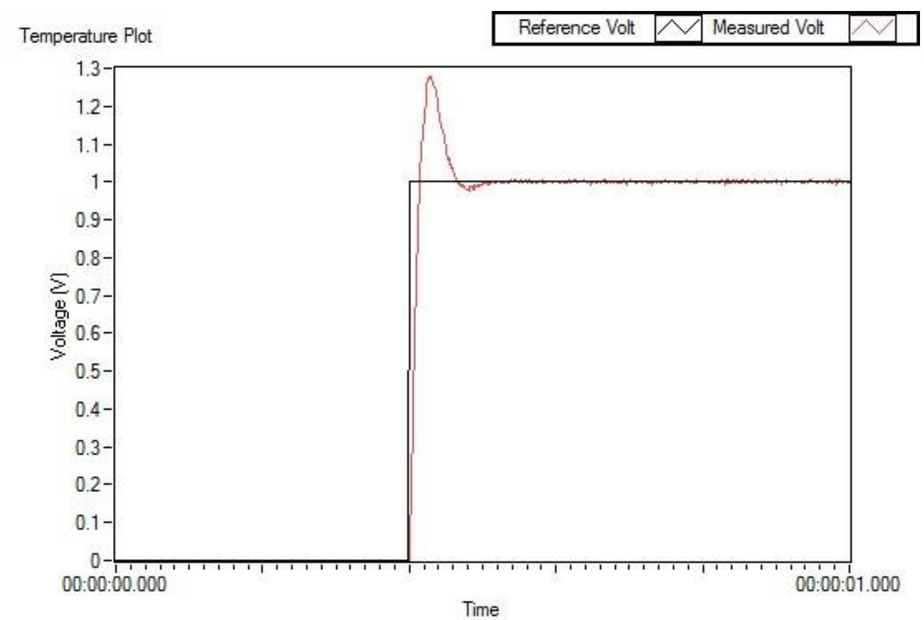
$$LR(s) = 0.866\frac{s + 57.73}{s}\frac{100}{s}$$



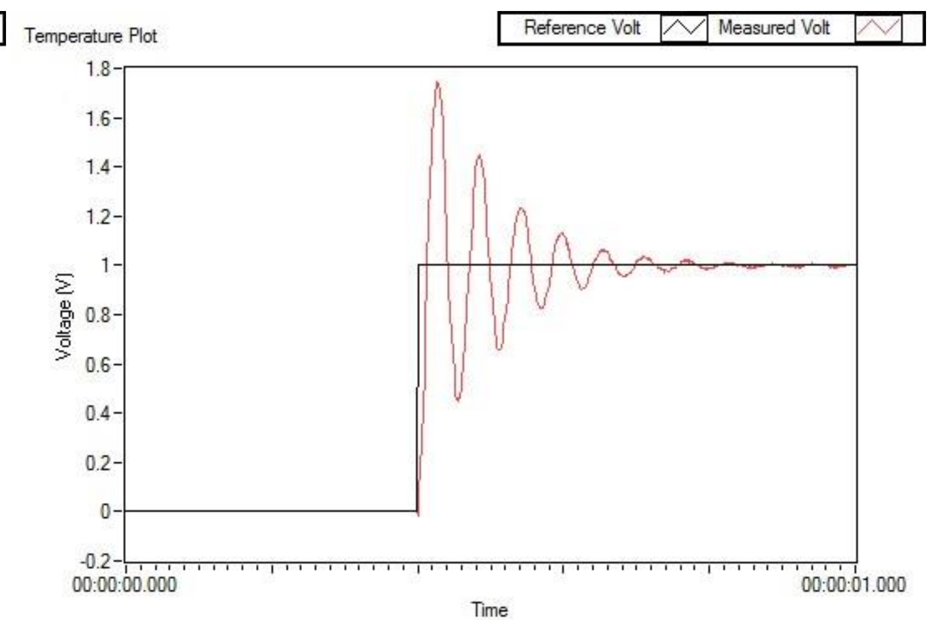


Effect of Phase Margin

- Smaller phase margin yields more oscillation for step response



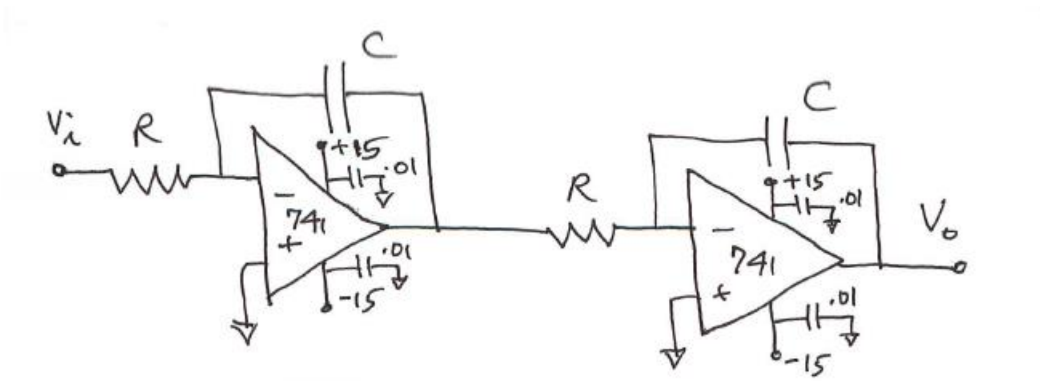
60 ° Phase Margin



15 ° Phase Margin

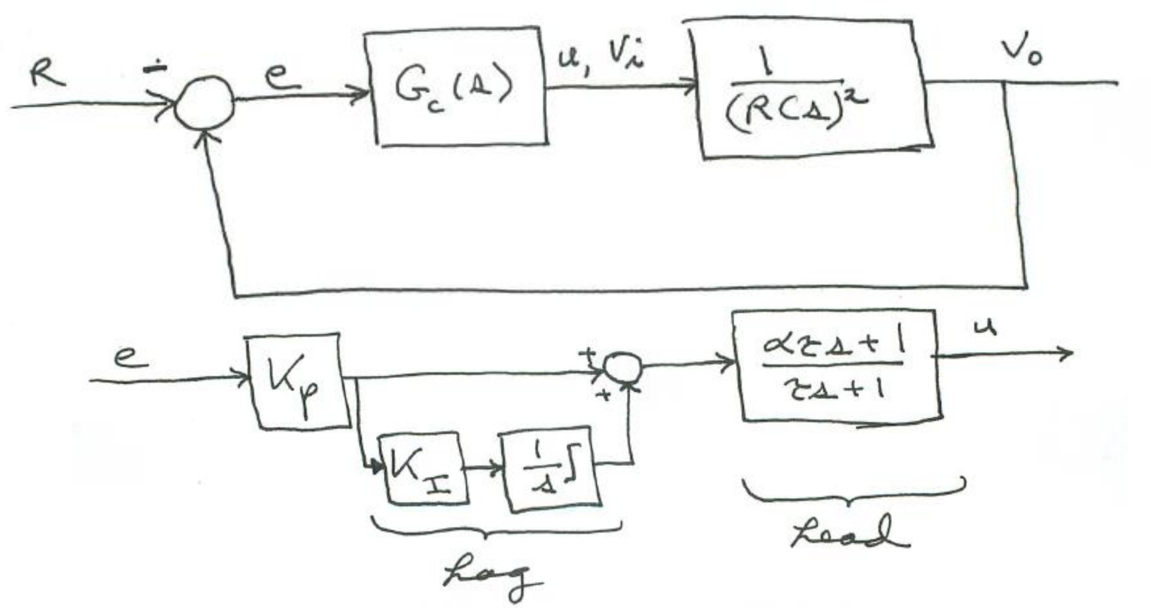
Double Integrator Plant

- The diagram of a double integrator plant is shown below (concatenation of 2 single integrator plants with identical resistor and capacitor values)



Double Integrator Control

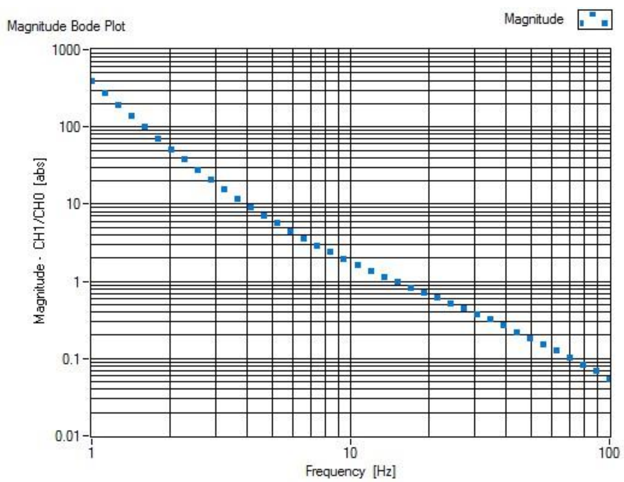
- A PID controller in the form of lead and lag compensator is needed
- The lead compensator adds positive phase margin to the system

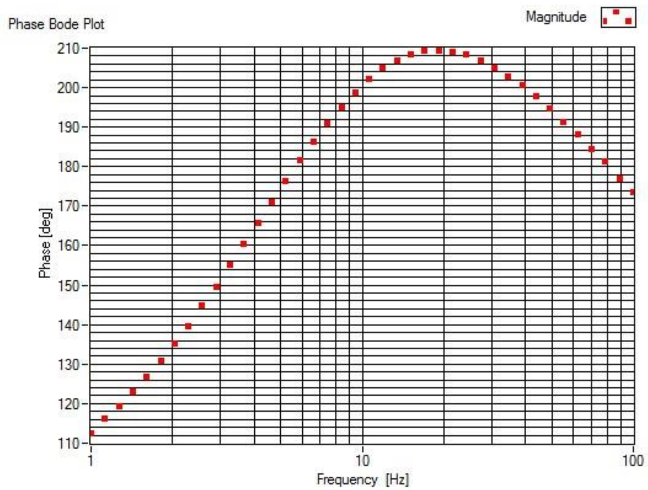


Double Integrator Control

- With $K_p = 0.4437$ and $K_I = 20$, $\alpha = 4.885$ and $\tau = 0.004524$ the loop transfer function gives a phase margin of 30 degree with transfer function shown below

$$LR(s) = 0.4437\frac{s+20}{s}\frac{0.0221s+1}{0.004524s+1}\frac{10000}{s^2}$$





RLC Circuit Control

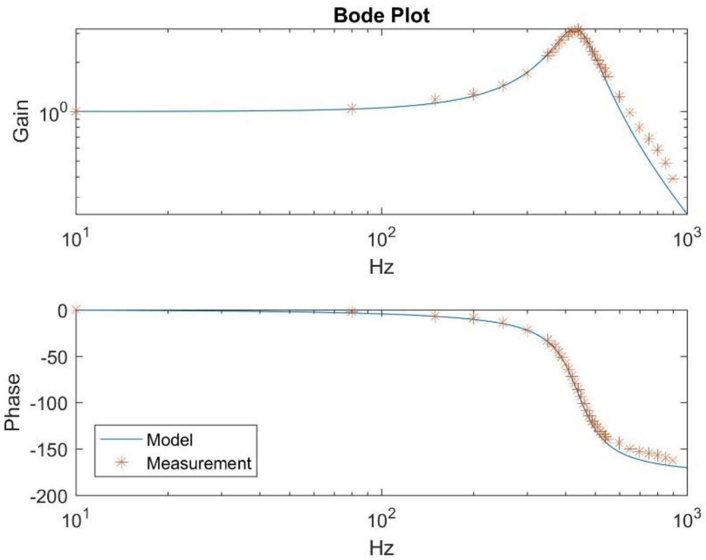
- Transfer function

$$G_p(s) = Y(s)/U(s) = 1/(LCs^2 + RCs + 1)$$

  - R = 80 Ω
  - C = 1.437 μF
  - L = 0.091 H

$$\zeta = \frac{R}{2}\sqrt{\frac{C}{L}} = 0.1590$$

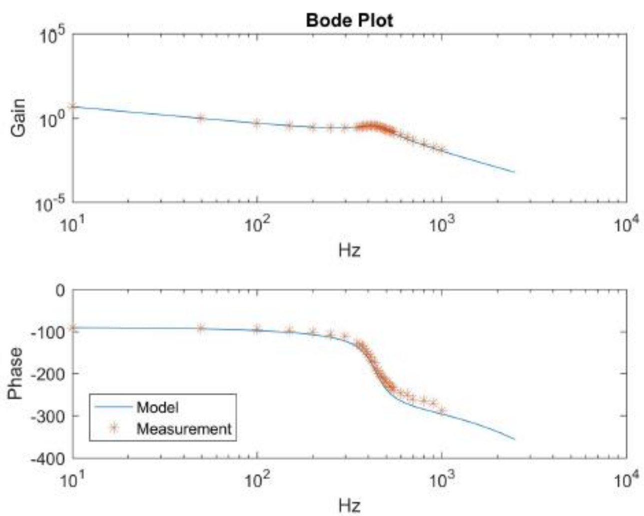
$$\omega_n = \sqrt{\frac{1}{LC}} = 2765.4$$



Integral Control

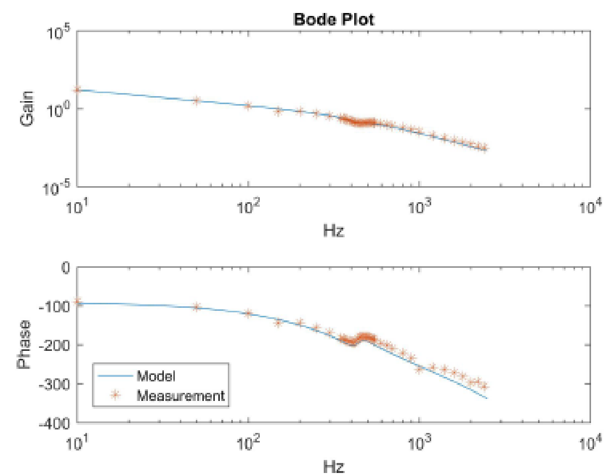
- Integral control alone

$$G_c(s) = -\frac{297}{s}$$



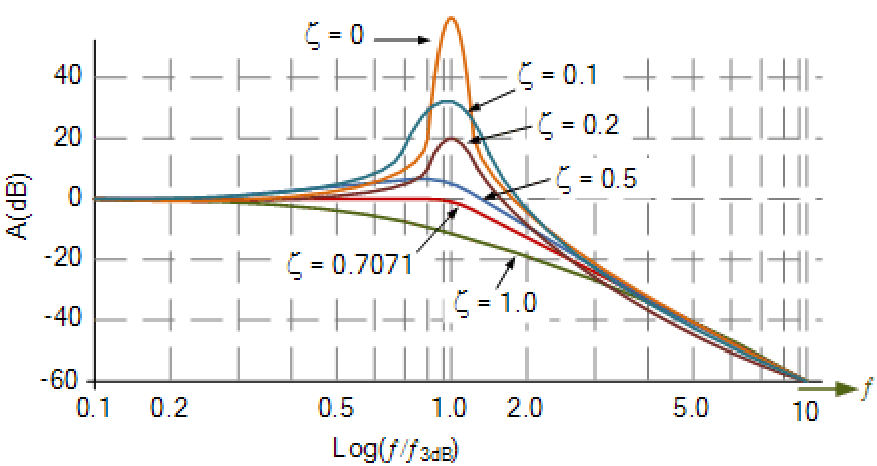
- Integral control with notch filter

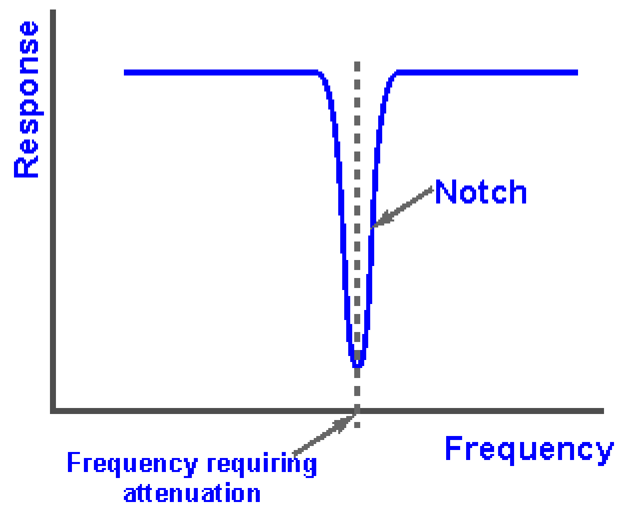
$$G_c(s) = -\frac{1000}{s}\frac{s^2 + 553s + 2765^2}{s^2 + 5530s + 2765^2}$$



Notch Filter

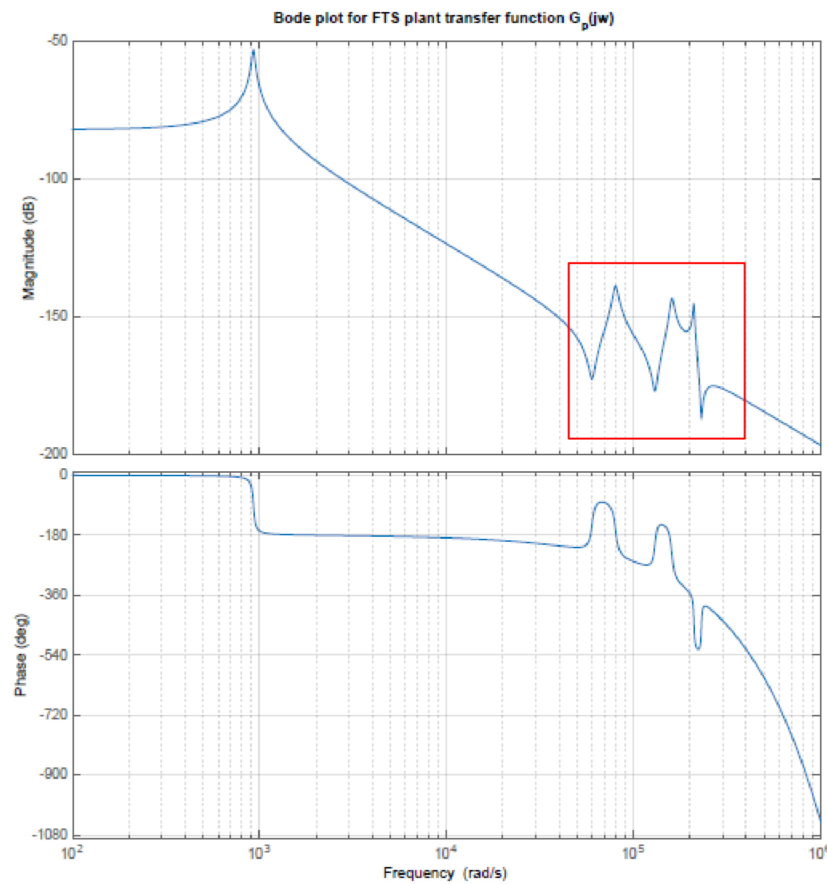
- To compensate for unwanted resonance behavior in second order underdamped system
- Can be helpful to counter-act unwanted high frequency resonance





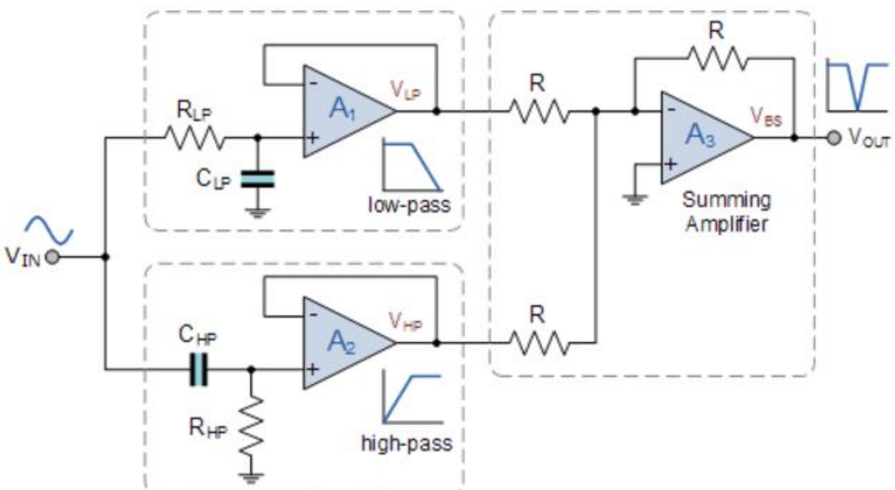
System with High Frequency Resonance

- Unwanted high frequency resonance reduces control system bandwidth
- Resonance can be compensated using notch-filters at specific frequencies



Notch Filter Example

- Different kinds of implementation available
- General second order notch filter formula is given below



$$G_N(s) = \frac{s^2/\omega_z^2 + \frac{2\zeta_z}{\omega_z}s + 1}{s^2/\omega_p^2 + \frac{2\zeta_p}{\omega_p}s + 1}$$

Thank You!