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Electronics Overview

Workshop

TiM

THE BEAVER

Department of Mechanical Engineering
Massachusetts Institute of Technology

Outline

- Electronics Overview
- Op-Amp Circuits
- Analog Filters
- Analog to Digital
- Electrical Lab Instruments
- Project Introduction RLC Circuit Control

Electronics Overview

Electrical Units

- Voltage: volts (µv → kv)
- Current: amperes (amps), milliampere (ma 10^{-3}), microampere (µa 10^{-6})
- Resistance: ohms Ω, k-ohms (k 10^{3}), meg ohms (m 10^{6})
- Capacitance: farad, microfarad (µf 10^{-6}), nanofarad (nf 10^{-9}), picofarad (pf 10^{-12})
- Inductance: henry, millihenry, microhenry
- Frequency: mhz, ghz 10^{9}

Electronics Overview

Fingertip Facts

volts ⇔ kohms ⇔ milliamperes

khz 10^3 ⇔ 10^{-3} = milliseconds

mhz 10^6 ⇔ 10^{-6} = microsecond

ghz 10^9 ⇔ 10^{-9} = nanosecond

60hz ⇔ 16.3 millisecond period

capacitance ⇔ µf 10^{-6}, nf 10^{-9}, pf 10^{-12} (pico), 10^{-15} (femtofarad)

seconds = mohm (resistance) x µf (capacitance)

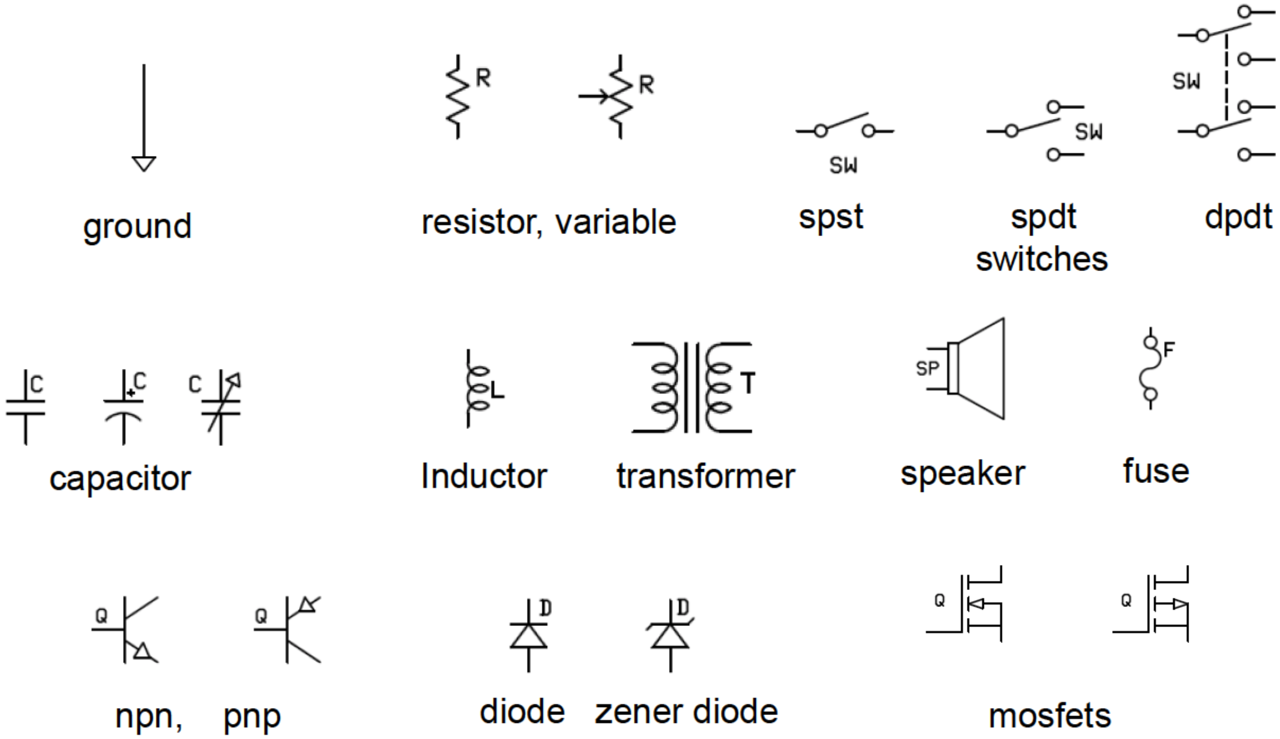
2^{10} ⇔ 1023

$\omega = 2\pi f$

3dB ⇔ half power point $dB = 20\log\left(\frac{V_o}{V_i}\right)$ $dB = 10\log\left(\frac{P_o}{P_i}\right)$

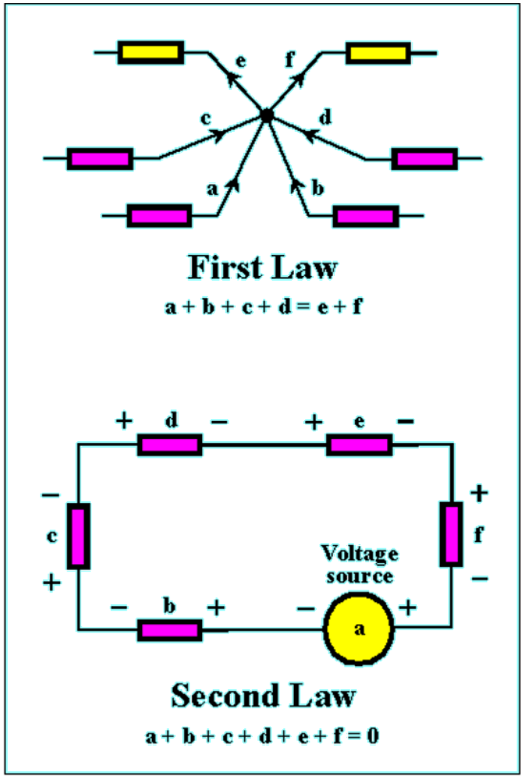
Electronics Overview

Electrical Symbols



Electronics Overview

Kirchhoff's Laws



1st Kirchhoff's Law (KCL)

$$\sum I_{in} = \sum I_{out}$$

2nd Kirchhoff's Law (KVL)

$$\sum \xi = \sum IR$$

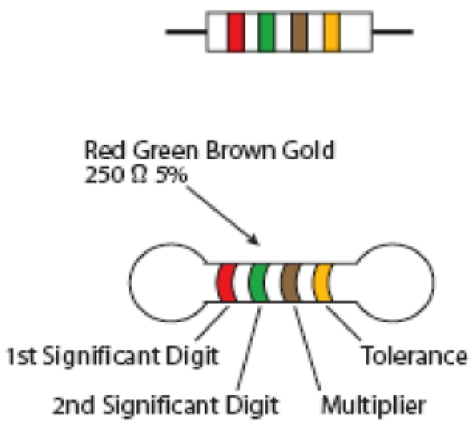
Electronics Overview

Basic Electrical Components: Resistors

- V=IR
- Resistor parameters: resistance, tolerance and power rating.
- Variable resistors: pots
- Resistors are color coded
- Standard values (10%)
  - 10 12 15 18 22 27 33 39 47 56 68 82
- Common tolerance: ±5%, ±2%, ±1%
- Series/parallel combination
- Why is high value used in power lines?

Electronics Overview

Basic Electrical Components: Resistors



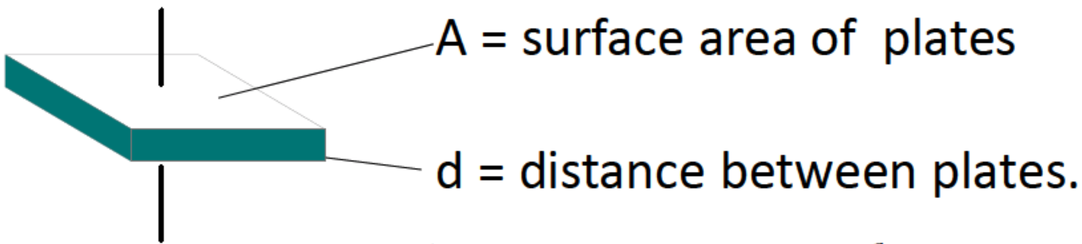
Color	1st-band Digit	2nd-band Digit	3rd-band Digit	4th-band Digit
Black	0	0	10^0 - 1	
Brown	1	1	10^1 - 10	1%
Red	2	2	10^2 - 100	2%
Orange	3	3	10^3 - 1000	3%
Yellow	4	4	10^4 - 10000	4%
Green	5	5	10^5 - 100000	
Blue	6	6	10^6 - 1000000	
Violet	7	7	10^7 - 10000000	
Gray	0	0	10^8 - 100000000	
White	9	9	10^9 - 1000000000	
Gold				5%
Silver				10%
None				20%

Figure by MIT OpenCourseWare.

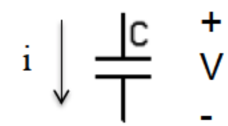
red green brown gold
2 5 0 Ω 5%

Electronics Overview

Basic Electrical Components: Capacitors



$$c = \frac{K\varepsilon_0 A}{d} \qquad Z = \frac{1}{sC} \qquad i = C\frac{dv}{dt}$$



Standard Capacitance Values: 10 12 15 18 22 27 33 39 47 56 68 82

Examples: 100pf, 180pf, 270pf,… 1µf , 2.2µf , 4.7µf ,…

Capacitor marking: 104 = 10×10^4 pf = $10^5 \times 10^{-12}$ f = 10^{-7}f = 0.1 µf

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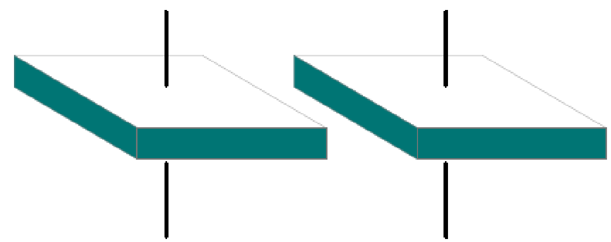
Electronics Overview

Basic Electrical Components: Capacitors

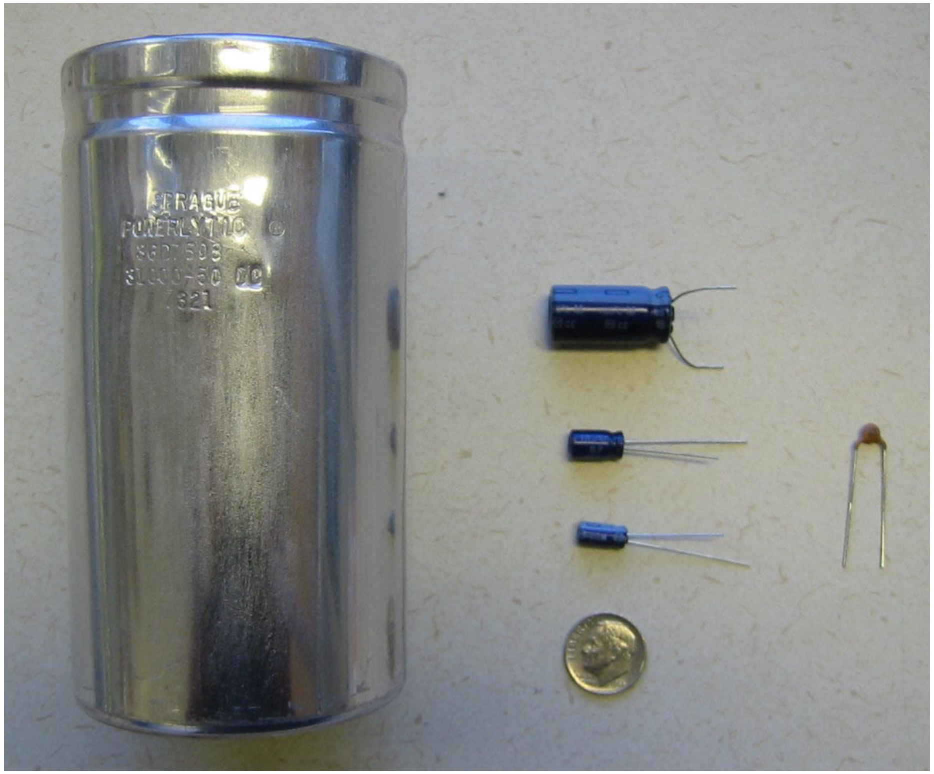
- Parallel / Series combination
  Think!



- Capacitors range for 1 pf (10^{-12}) to 100,000 µf (10^{-1})
- Typically capacitors larger than 1µf are polarized. Non polarized units are marked NP (non-polar) or BP (bipolar).
- All capacitors have maximum voltage ratings.

Electronics Overview

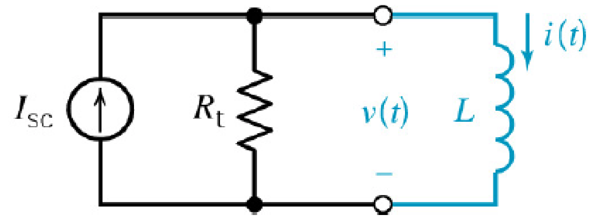
Basic Electrical Components: Capacitors



Electronics Overview

Basic Electrical Components: Inductors

$$v(t) = L\frac{d}{dt}i(t) \qquad Z = sL$$



- Inductors are used in tuned circuits, switching power supplies, voltage converters, light dimmers, GFI.
- Inductors vary from a few µh (etched on a pcb) to henries.

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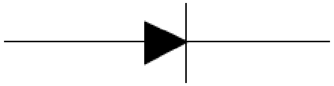
AsiaUS Business Connection

Electronics Overview

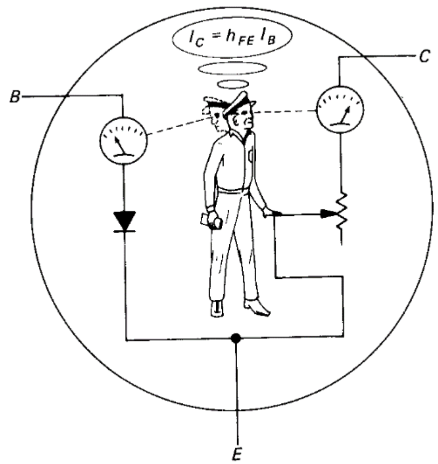
Basic Electrical Components: Diode

- Diodes allow current to flow in the direction of the arrow.
- Can be modeled as an open circuit in one direction and a short circuit in the other (with a 0.6 volt drop)
- Diode parameters: max current, reverse breakdown voltage, reverse recovery time.



Electronics Overview

Basic Electrical Components: Transistor



Thank You, Transistor Man!

A low base current (gate voltage) controls a much larger collector (drain) current

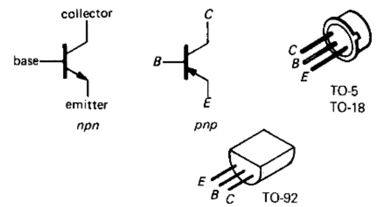


Figure 2.1. Transistor symbols, and small transistor packages.

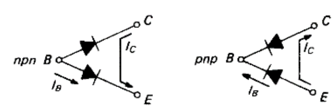
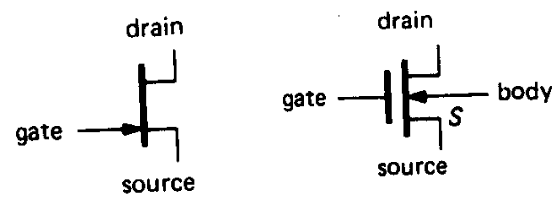


Figure 2.2. An ohmmeter's view of a transistor's terminals.

$$I_C = h_{fe}\, i_B = \beta i_B$$



$$I_s = g_m\, V_{gs}$$

Transconductance

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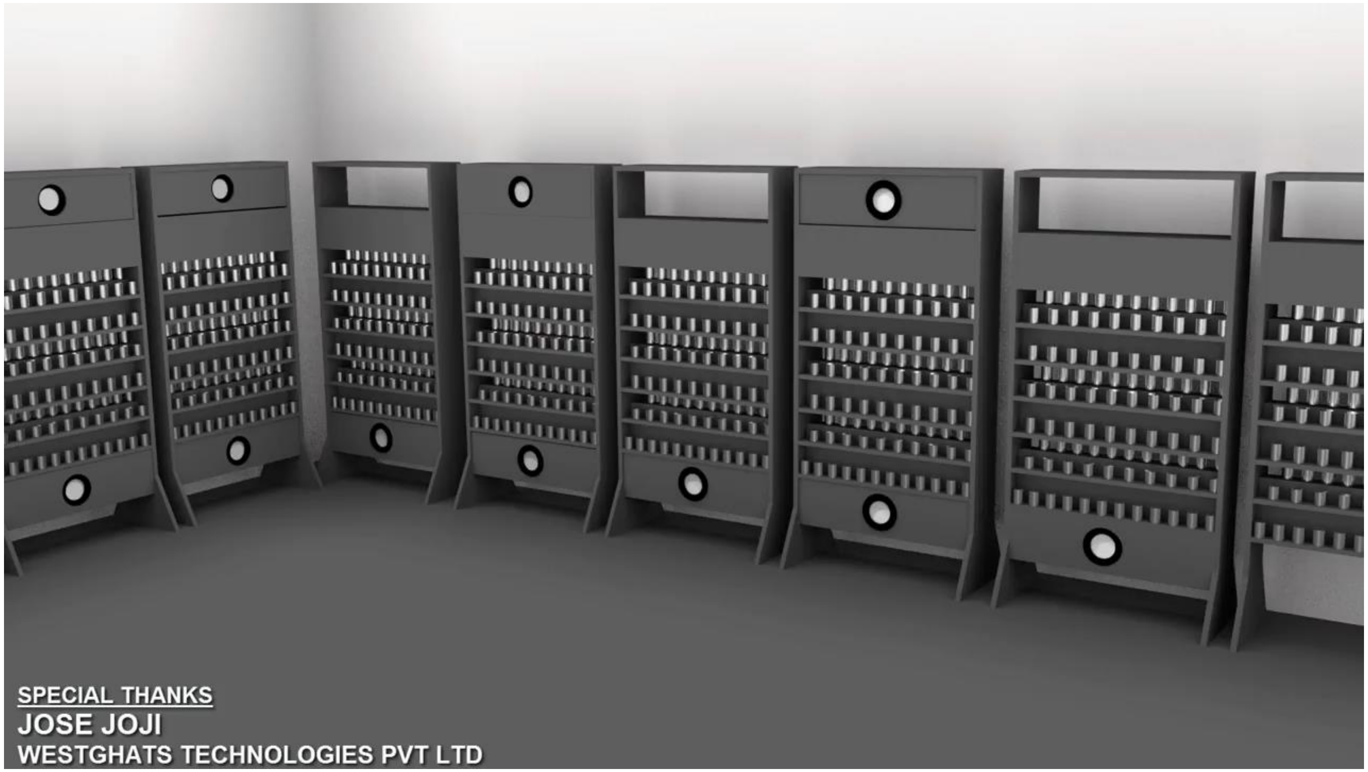
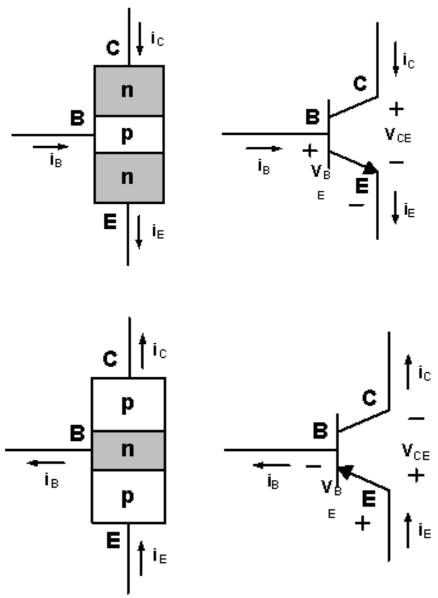
Electronics Overview

Diode and Transistor

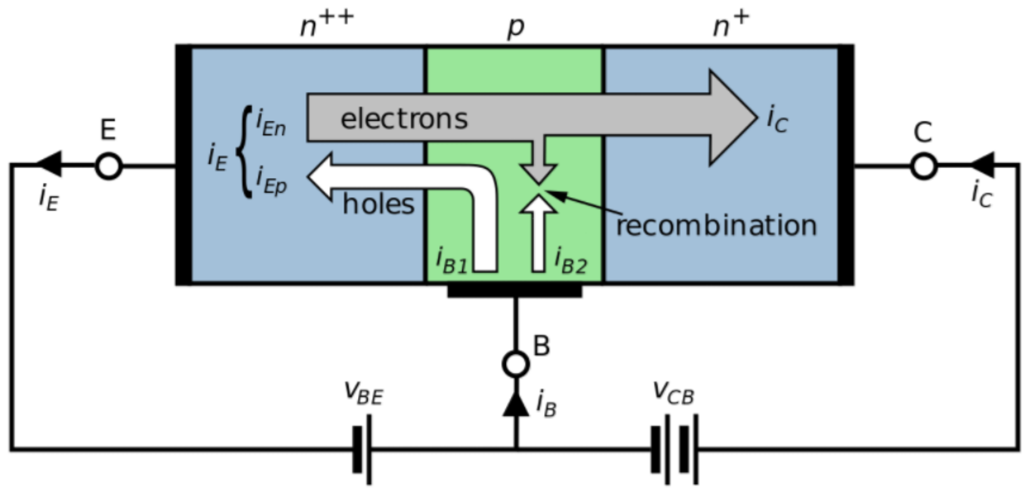


Electronics Overview

Bipolar Junction Transistor



Emitter Base Collector



Emitter and base has forward bias
Thin base layer for minority carrier

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Electronics Overview

Field Effect Transistor

Op-Amp Circuits

"Ideal" OpAmp Model

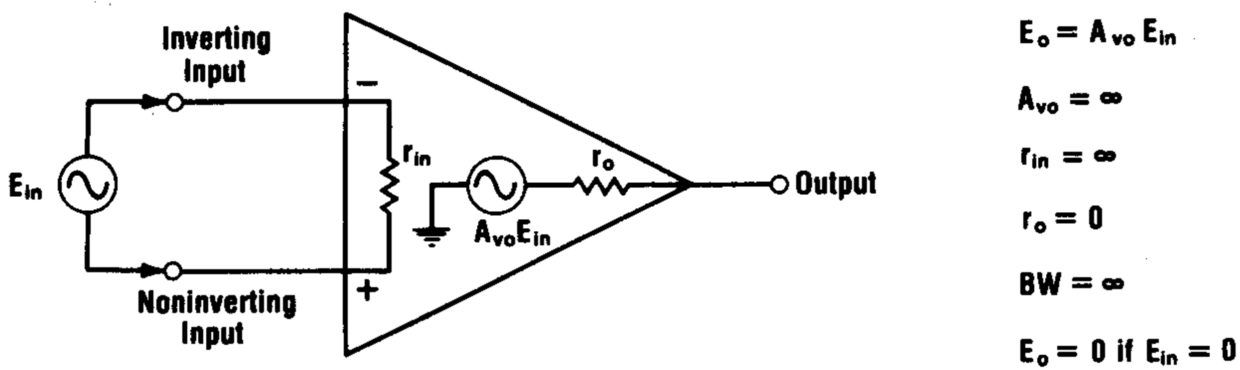
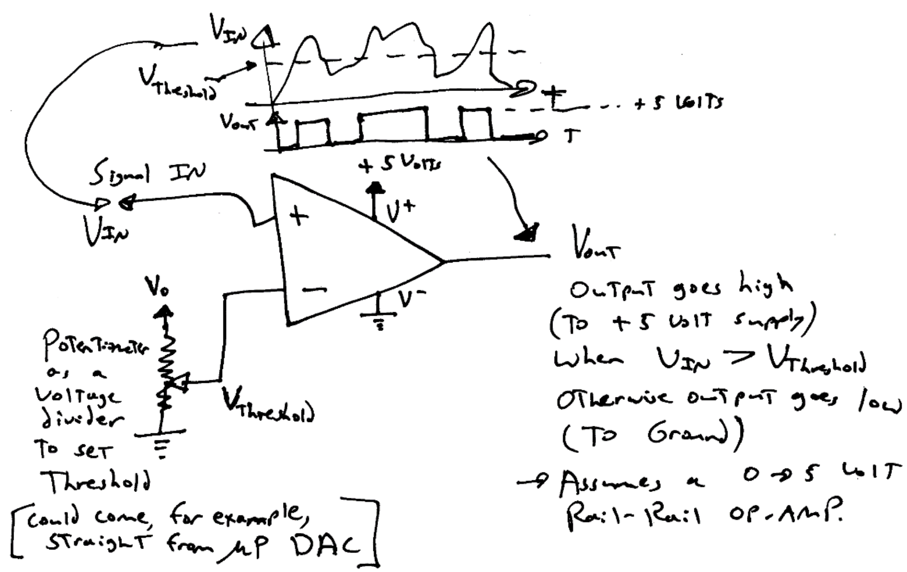


Fig. 1-1. Equivalent circuit for an ideal operational amplifier.

1. The voltage gain is infinite—$A_{vo} = \infty$.
2. The input resistance is infinite—$r_{in} = \infty$.
3. The output resistance is zero—$r_o = 0$.
4. The bandwidth is infinite—$BW = \infty$.
5. There is zero input offset voltage—$E_o = 0$ if $E_{in} = 0$.

Op-Amp Circuits

Comparator

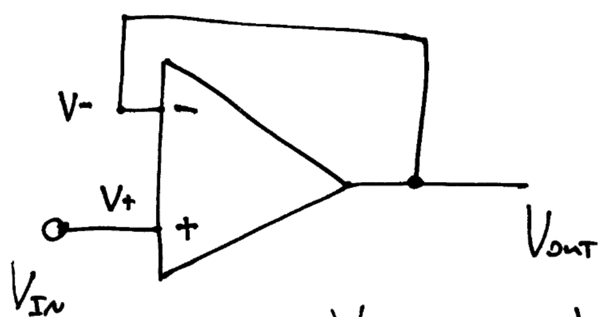


Makes an analog signal into a 1-bit digital signal

- Directly drives logic pin on microprocessor
- Detects when signal is above threshold

Op-Amp Circuits

Voltage-Follower/Buffer



Negative Feedback forces $V- = V+$

$\Rightarrow V_{OUT} = V_{IN}$

V_{OUT} gives low impedance drive from potentially very high impedance V_{IN} sources

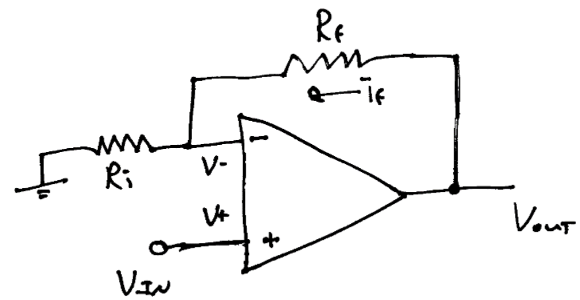
A unity-gain buffer to enable high-impedance sources to drive low-impedance loads

Op-Amp Circuits

Non-Inverting Amplifier

Like voltage follower, but gives voltage gain

- Gain can be adjusted from unity upward via resistor ratio
- High-Z input is good for conditioning High-Z sensors



Again, negative feedback means $V_- = V_+$ when The Op Amp is working.

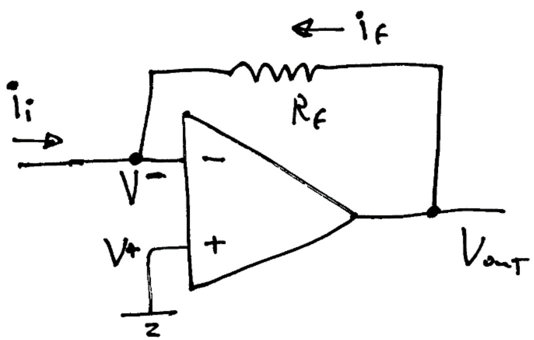
$$V_- = V_{OUT}\underbrace{\left(\frac{R_i}{R_f + R_i}\right)}_{\text{Voltage Divider!}} = V_+ = V_{IN}$$

$$\therefore V_{OUT} = V_{IN}\left(\frac{R_f + R_i}{R_i}\right) = V_{IN}\underbrace{\left(1 + R_f/R_i\right)}_{\text{Gain}}$$

Non-inverting OP-AMP gives Low-Z V_{OUT} from very high-Z V_{IN}, But also offers Voltage Gain $\Rightarrow$ $1 + R_f/R_i$

Op-Amp Circuits

Transimpedance Amplifier



Vout Gives good low-Z drive (low impedance)

$Z_{in} \to \infty$ means

$$i_i = -i_f = -\frac{V_{out}}{R_f}$$

Negative Feedback forces

$$V^- = V^+$$

$$\therefore\ V_{out} = -i_i R_f$$

Converts a current into a voltage

- Generates a proportional (w. R_f) voltage from an input current
- Produces a low-impedance output that can drive a microcomputer's A-D converter, for example

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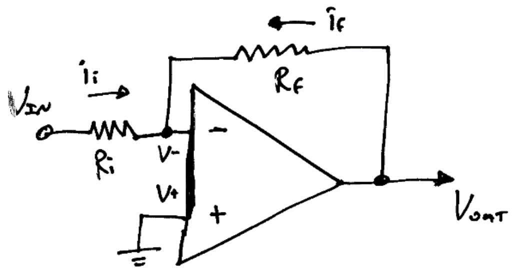
Op-Amp Circuits

Inverting Amplifier



$i_f = -i_i$, since $Z_{in} \to \infty$

$V+ = V- = 0$, because of negative feedback and grounded $V+$

$$\Rightarrow i_f = \frac{V_{OUT}}{R_f} = -i_i = \frac{V_{IN}}{R_i}$$

$$V_{OUT} = V_{IN}\underbrace{\left(-\frac{R_f}{R_i}\right)}_{\text{Gain}}$$

Inverting Op Amp produces negative voltage gain: $-R_f/R_i$

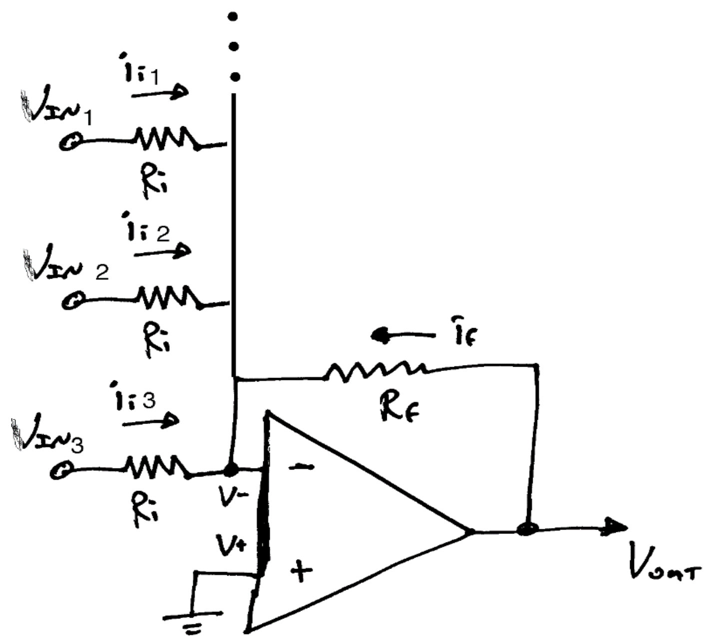
Input Impedance = R_i (not huge, as with inverting Op Amp)

Inverts signal, voltage gain varies from zero upward with the ratio of two resistors

- Extension to summer is trivial with additional R_i's
- Input impedance is not infinite: $Z_{in} = R_i$

Op-Amp Circuits

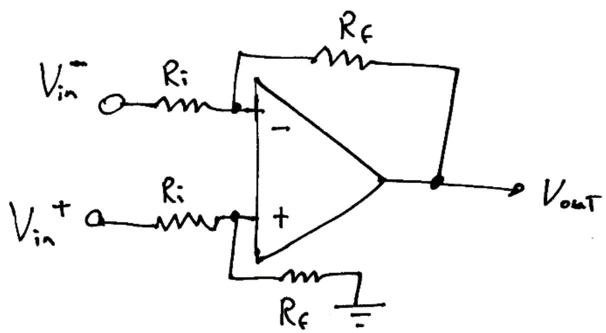
Adder



No crosstalk between inputs because of virtual ground

Op-Amp Circuits

Differential Amplifier



Voltage Gain

$$V_{OUT} = (V_{in}^+ - V_{in}^-)\, R_f/R_i$$

$$Z_{in}^- = R_i$$

$$Z_{in}^+ = R_i + R_f$$

$\Rightarrow$ Unequal Z_{in} $\rightarrow$ Unbalanced

Subtracts two input signals

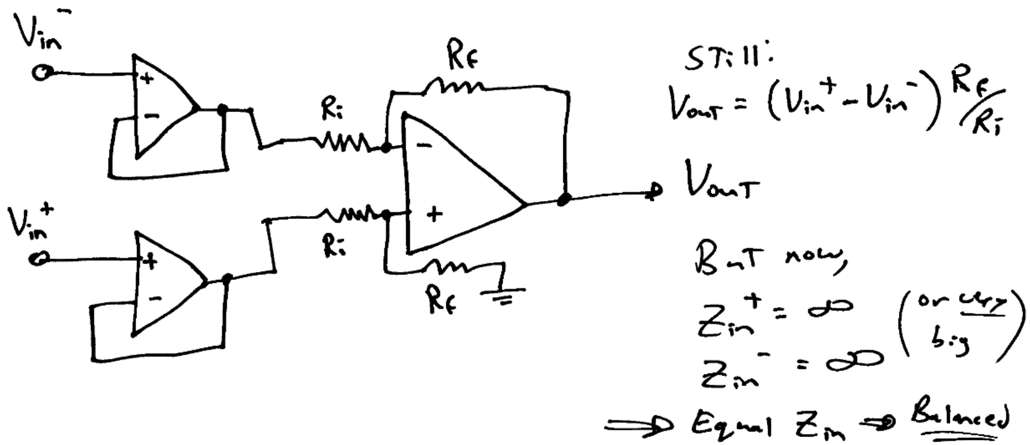
- Input resistors must be equal, feedback and shunt resistors must be equal
- Provides voltage gain

The input impedances aren’t equal, however

- The amplifier is *unbalanced!*
  - A high-impedance sensor will produce common-mode errors (e.g., the system will be sensitive to the common voltage)
  - Differential sensors will be more sensitive to induced pickup signals (which tend to be high impedance)

Op-Amp Circuits

Basic Instrumentation Amplifier



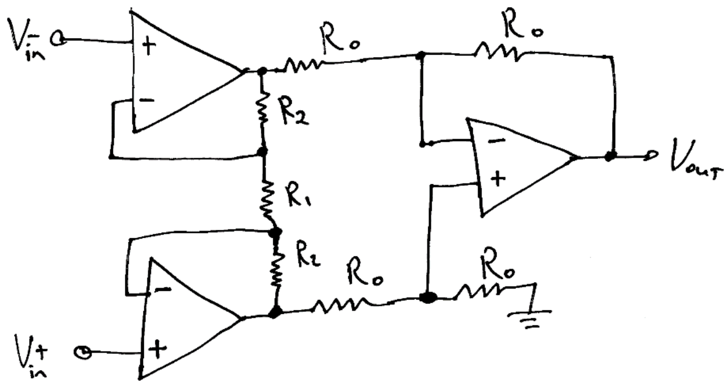
Buffer each leg of the differential amplifier by a voltage follower

- Impedance is now extremely high at both inputs
- Impedance can be set by a shunt resistor across inputs
- This is a *balanced* “instrumentation” amplifier

Op-Amp Circuits

Three-Op-Amp Instrumentation Amplifier

$$V_{OUT} = (V_{in}^{+} - V_{in}^{-})\overbrace{\left(1 + \frac{2R_2}{R_1}\right)}^{\text{Gain}}$$



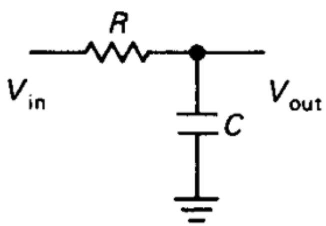
Gain is varied by changing only one resistor, R_1

- No need to re-trim other components for a gain change
- Gain at first stages is better for signal/noise
- This is the instrumentation amplifier of choice

Analog Filters

Passive RC Filters

- Passive LP Filter: RC network: $f_c = 1/(2\pi RC)$



-3dB = 0.707

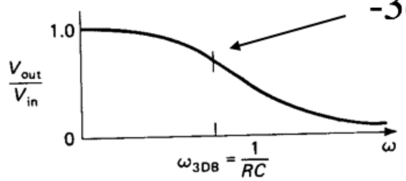
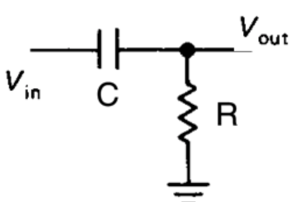


Figure 1.59. Frequency response of low-pass filter.

- Passive HP filter: RC network: $f_c = 1/(2\pi RC)$



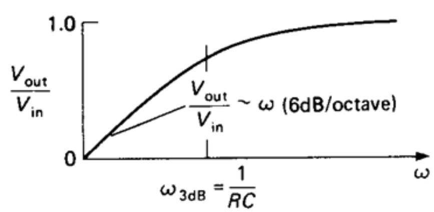


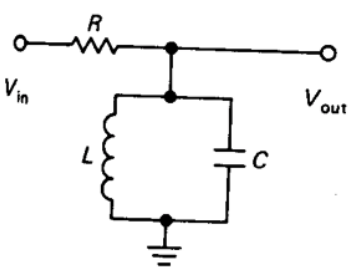
Figure 1.55. Frequency response of high-pass filter.

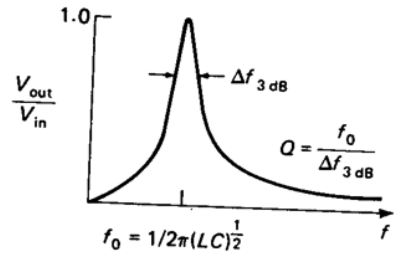
Analog Filters

Passive RLC Filters

- Resonant parallel RLC bandpass filters

$$\boxed{\begin{aligned} Q &= \omega_0 RC \\ &= f_0/\Delta f_{3dB} \end{aligned}}$$

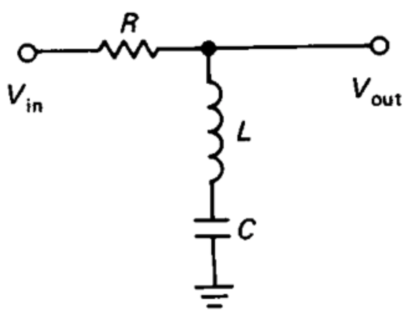


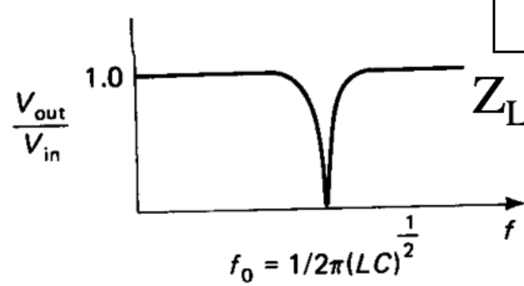


$Z_{LC} \rightarrow \infty$ @ f_0

- Resonant series RLC notch filters

$$\boxed{\begin{aligned} Q &= \omega_0 (L/R) \\ &= f_0/\Delta f_{3dB} \end{aligned}}$$



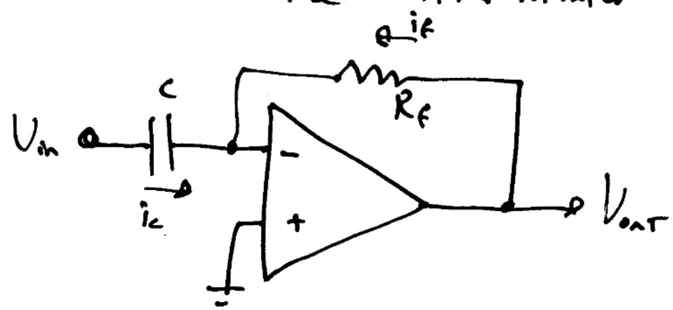


$Z_{LC} \rightarrow 0$ @ f_0

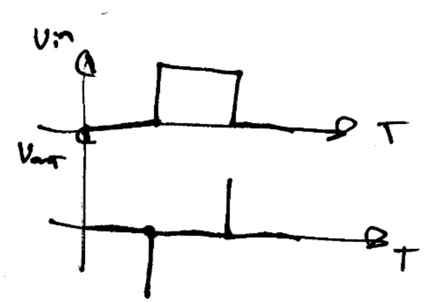
Analog Filters

Active Filters: Differentiator

The Differentiator



$$i_f = -i_c$$

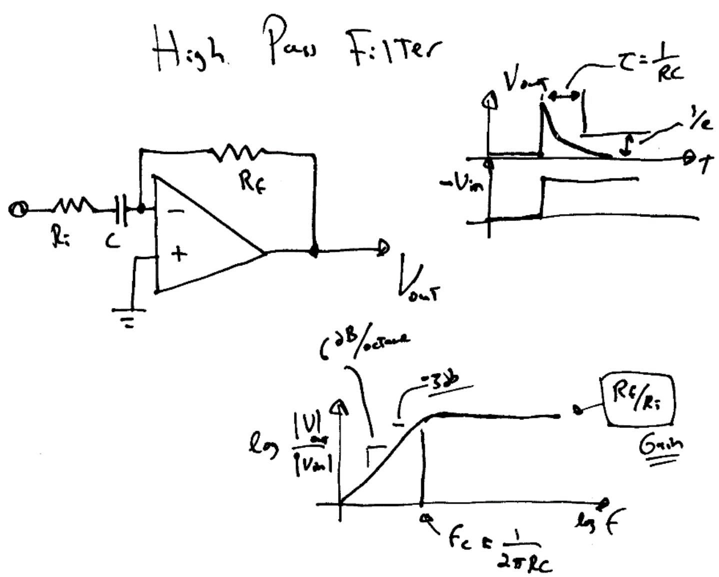


$$\frac{V_{out}}{R_f} = -C\,\frac{dV_{in}}{dT}$$

$$\boxed{V_{out} = -R_f C\,\frac{dV_{in}}{dT}}$$

Analog Filters

Active Filters: First-order Active High Pass Filter

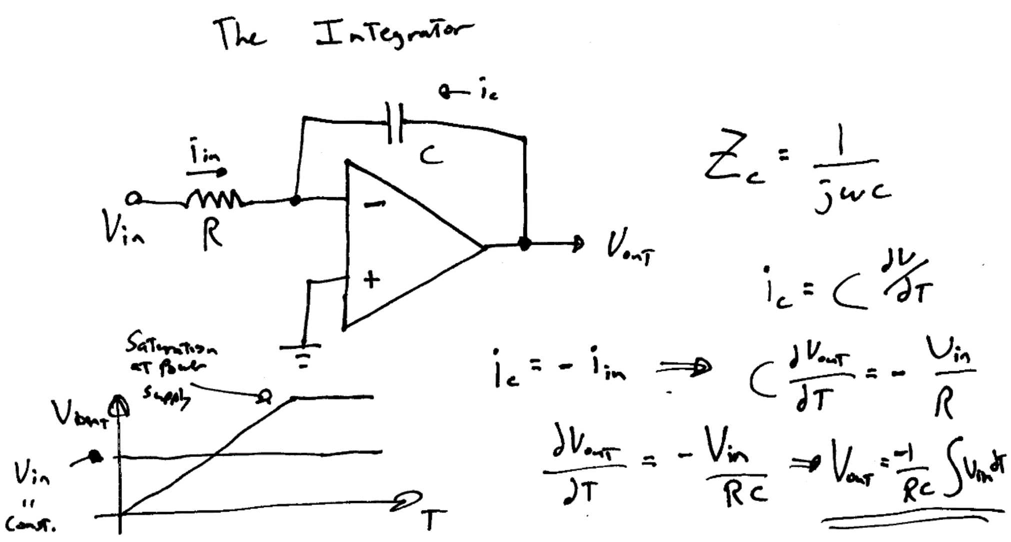


Low impedance drive

Voltage gain via R_f/R_i

Analog Filters

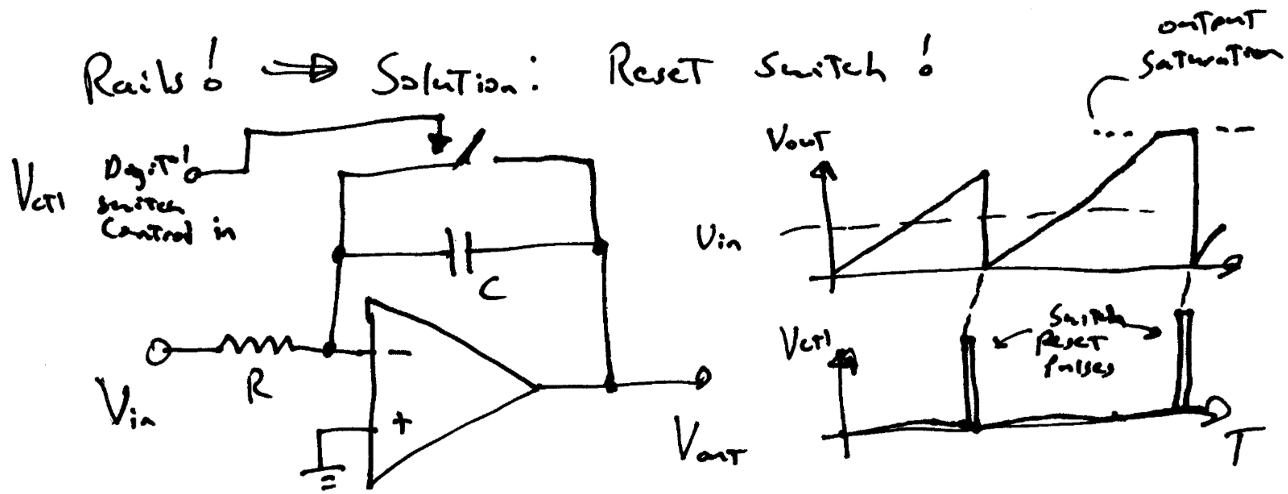
Active Filters: Integrator



Saturates at rail!!

Analog Filters

Active Filters: Integrator with Reset Switch

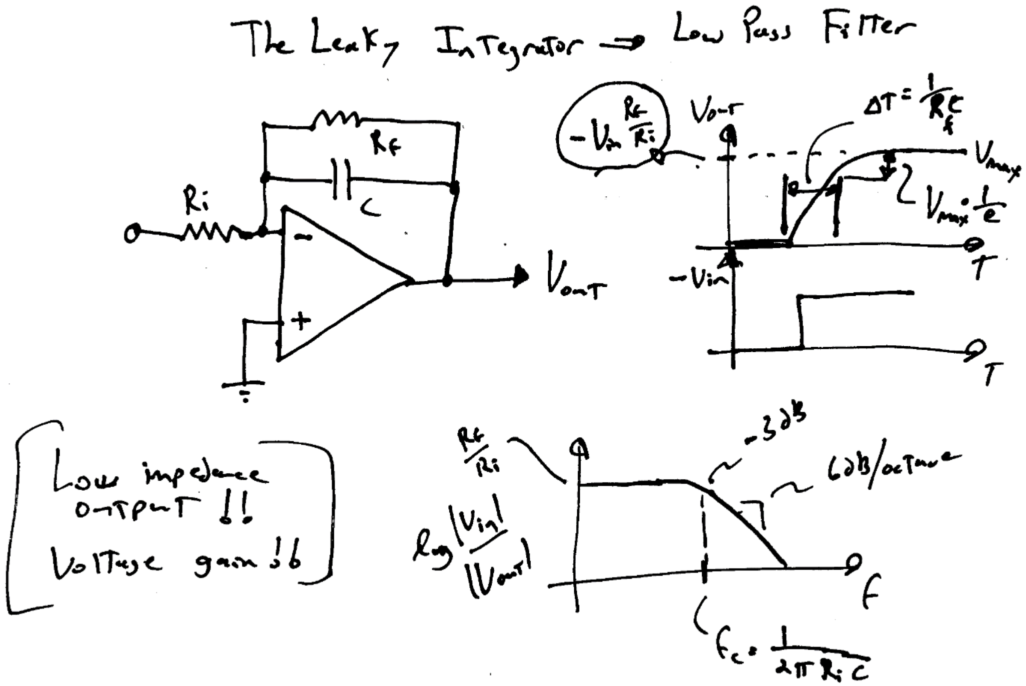


Electronic switch in feedback forces output to ground when closed

- Discharges capacitor
  - Resets Integrator!

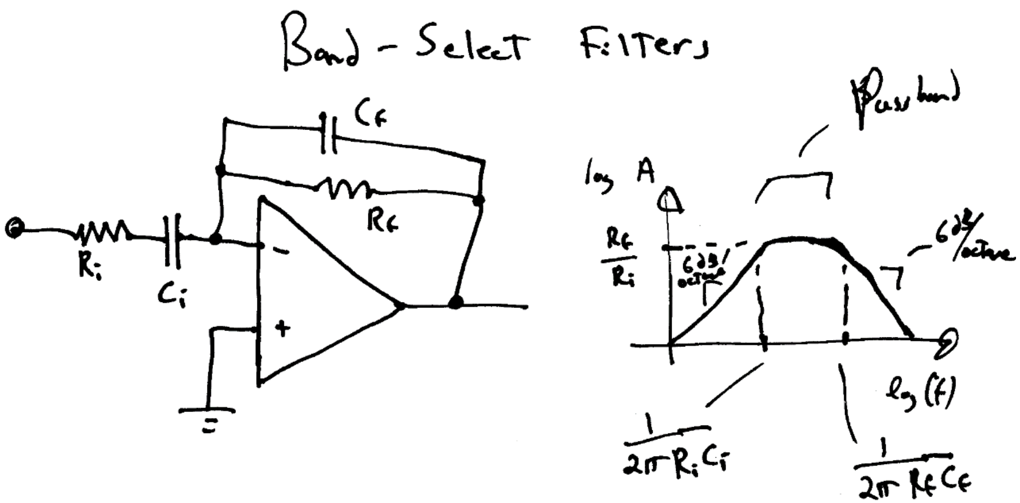
Analog Filters

Active Filters: First-Order Active Low Pass Filter



Analog Filters

Active Filters: First-Order Active Band Pass Filter



Cascaded high and low pass filters

- Always follow high-pass with low-pass (noise)
  - Low-Pass cutoff needs to be below high-pass cutoff!
- No Q, first-order rolloffs

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Analog Filters

Active Filters: Second-Order Filters (Sallen-Key Filters)

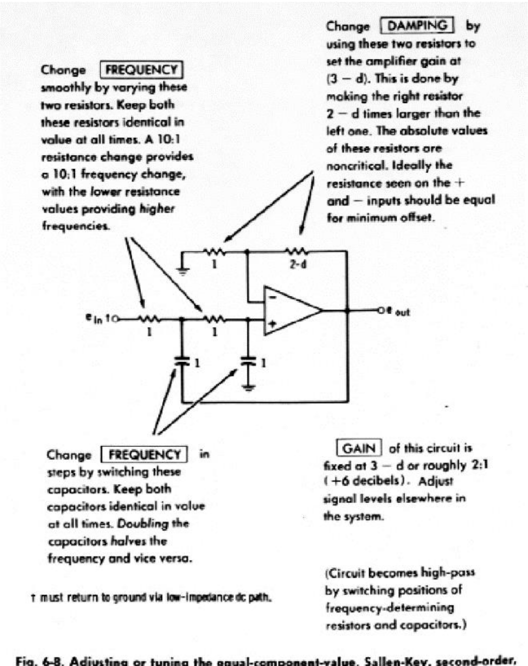


Fig. 6-8. Adjusting or tuning the equal-component-value, Sallen-Key, second-order, low-pass section.

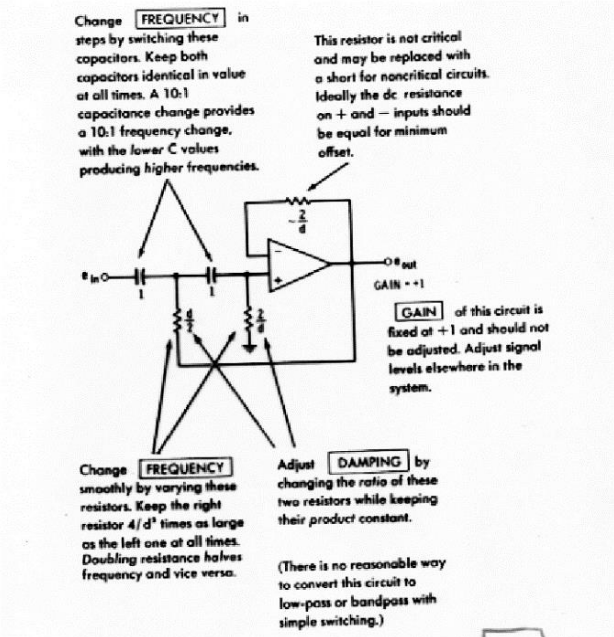
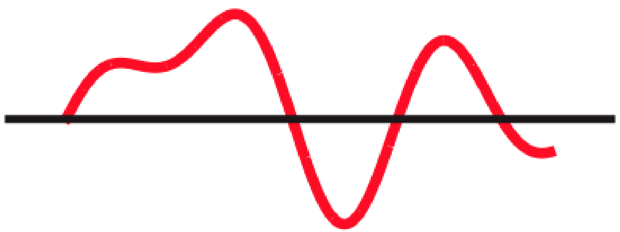


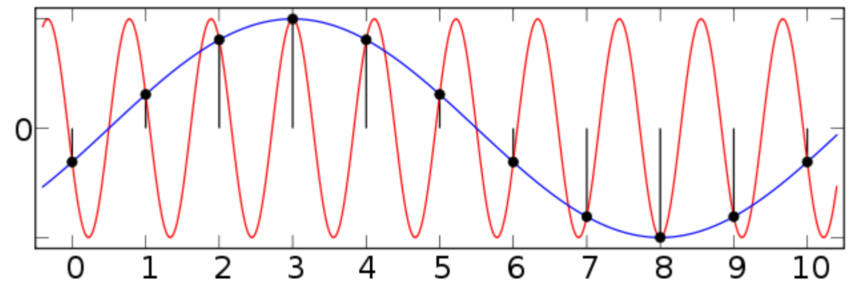
Fig. 8-5. Adjusting or tuning the unity-gain, Sallen-Key, second-order high-pass section.

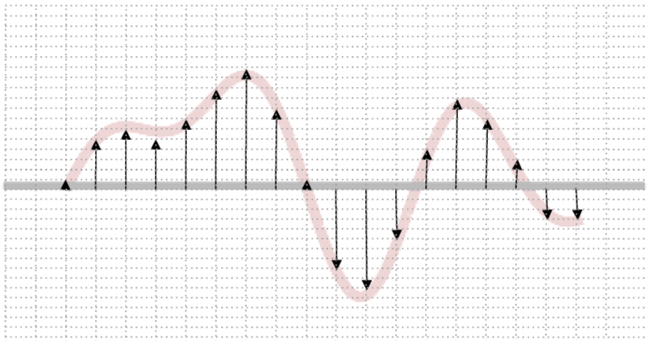
VCVS Filters

Analog to Digital

Sampling







Nyquist Sampling Theorem

$$f_{in} < f_s/2$$

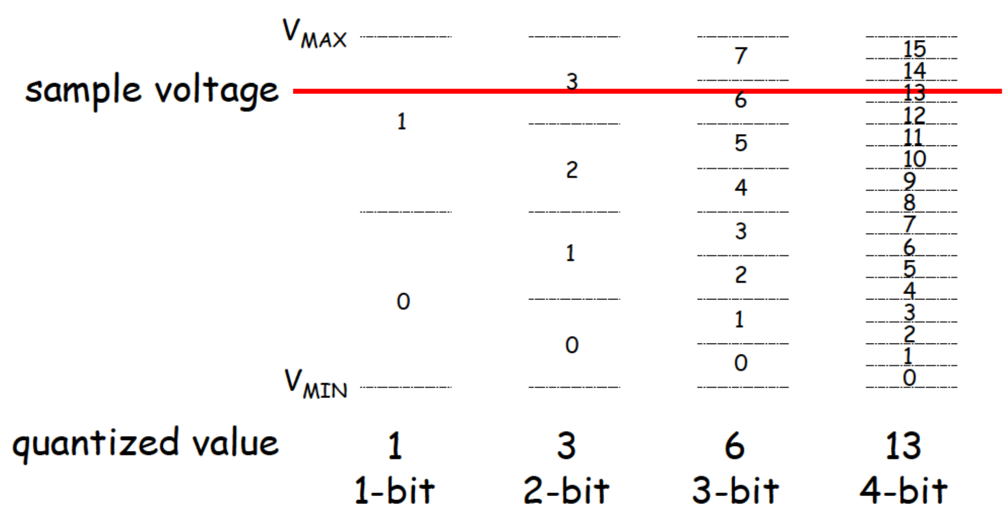
$$2f_{in} < f_s$$

Analog to Digital

Quantization

If we use N bits to encode the magnitude of one of the discrete-time samples, we can capture 2^N possible values.

So we'll divide up the range of possible sample values into 2^N intervals and choose the index of the enclosing interval as the encoding for the sample value.



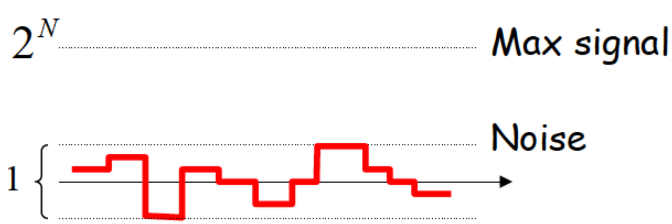
Analog to Digital

Signal-to-Noise Ratio (SNR)

$$SNR = 10\log_{10}\left(\frac{P_{SIGNAL}}{P_{NOISE}}\right) = 10\log_{10}\left(\frac{A^2_{SIGNAL}}{A^2_{NOISE}}\right) = 20\log_{10}\left(\frac{A_{SIGNAL}}{A_{NOISE}}\right)$$

RMS amplitude

SNR is measured in decibels (dB). Note that it's a logarithmic scale: if SNR increases by 3dB the ratio has increased by a factor 2. When applied to audible sounds: the ratio of normal speech levels to the faintest audible sound is 60-70 dB.

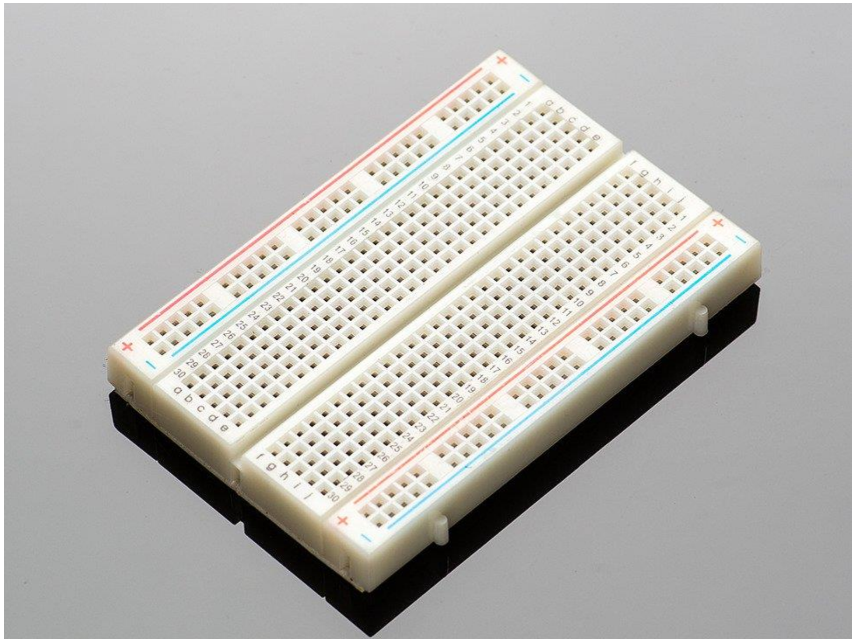


$$SNR = 20\log_{10}\left(\frac{A_{signal}}{A_{noise}}\right) \approx 20\log_{10}(2^N)$$

$$\approx N \cdot 6.02dB$$

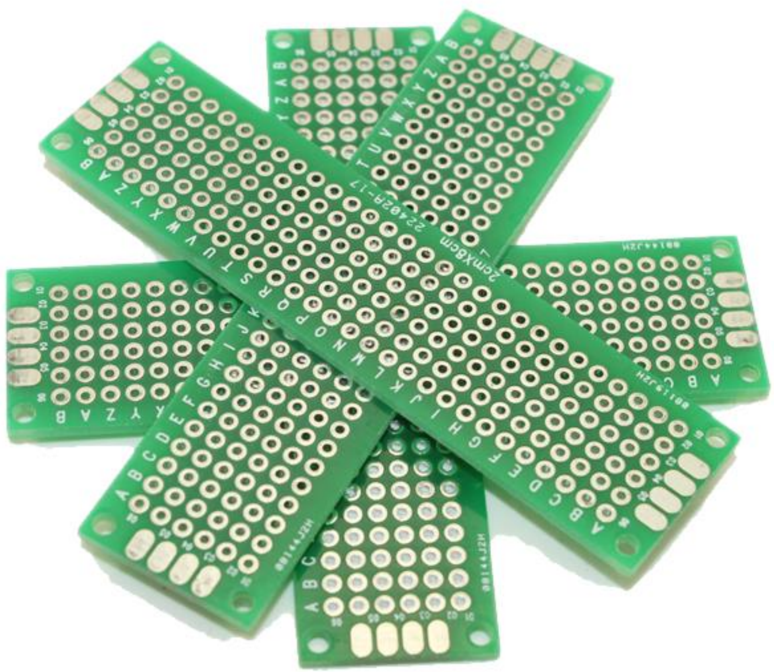
Electrical Lab Instruments

Breadboard



Electrical Lab Instruments

Protoboard

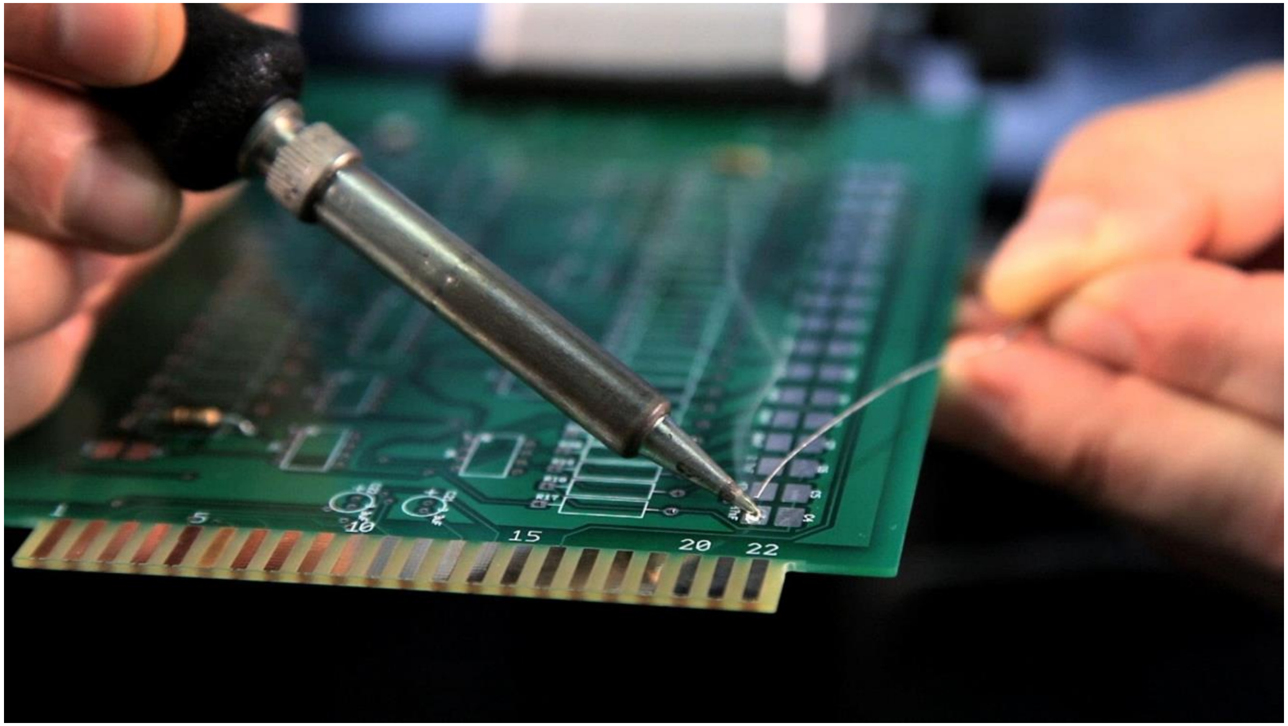


Electrical Lab Instruments

Multimeter

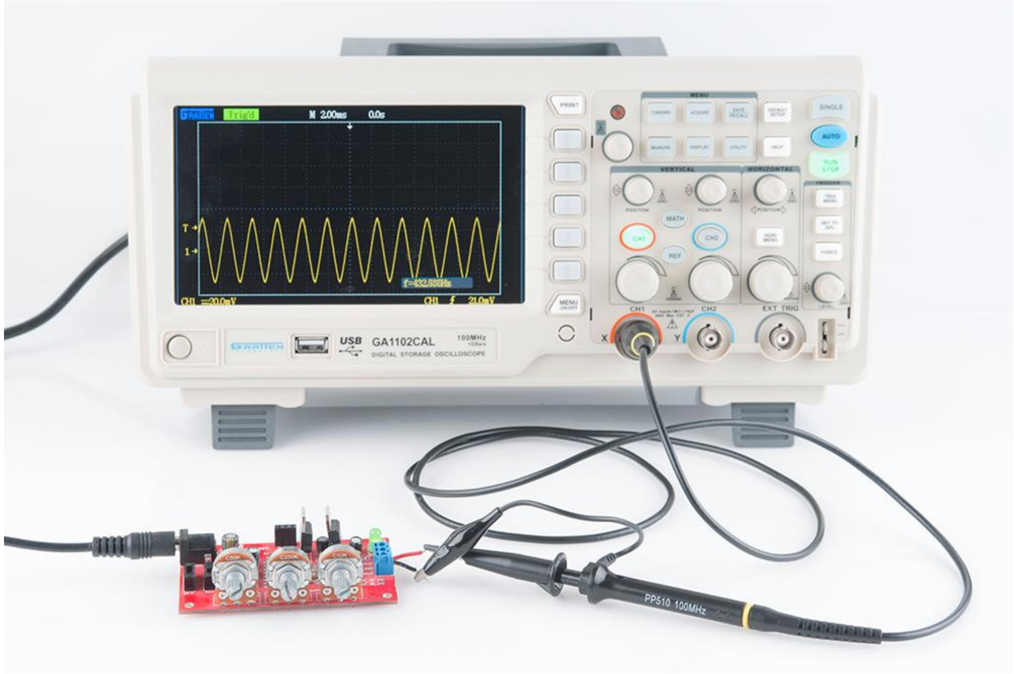
Electrical Lab Instruments

Soldering



Electrical Lab Instruments

Oscilloscope



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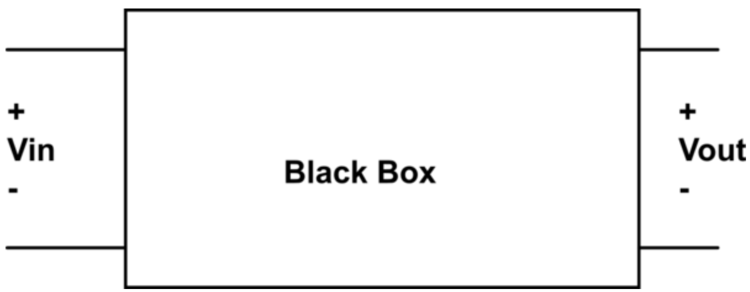
Caution!

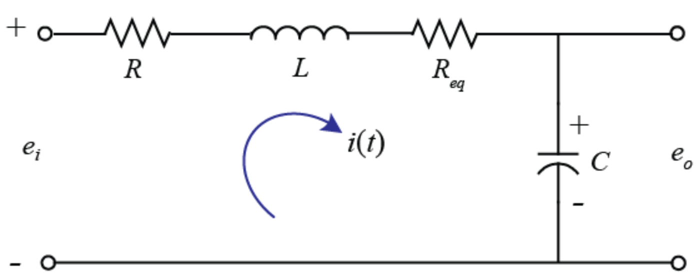
A big wave of mathematical definition and manipulation is coming.

System Identification

Goal of System Identification

- Obtain system characteristics when isolated component measurement is not possible
- Usually presented as frequency response for design purpose





$$e_i - iR - L\frac{di}{dt} - iR_{eq} - \frac{1}{C}\int i\,dt = 0$$

$$E_i(s) - I(s)R - LsI(s) - I(s)R_{eq} - \frac{1}{Cs}I(s) = 0$$

$$I(s) = \frac{E_i(s)}{R + Ls + R_{eq} + \frac{1}{Cs}}$$

$$\frac{E_i(s)}{R + Ls + R_{eq} + \frac{1}{Cs}} = \frac{E_o(s)}{\frac{1}{Cs}}$$

$$G(s) = \frac{E_o(s)}{E_i(s)} = \frac{\frac{1}{Cs}}{R + Ls + R_{eq} + \frac{1}{Cs}}$$

$$G(s) = \frac{\frac{1}{CL}}{s^2 + \frac{R+R_{eq}}{L}s + \frac{1}{CL}} = \frac{\omega_n^2}{s^2 + 2\zeta\omega_n s + \omega_n^2}$$

$$\omega_n = \sqrt{\frac{1}{CL}} \qquad \sigma = \zeta\omega_n = \frac{R + R_{eq}}{2L} \qquad \zeta = \frac{\sigma}{\omega_n} = \frac{R + R_{eq}}{2}\sqrt{\frac{C}{L}}$$

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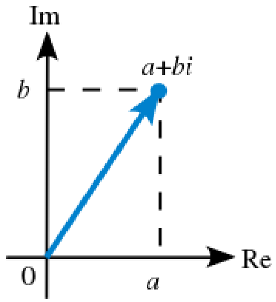
System Identification

Complex Number and Taylor Series Review

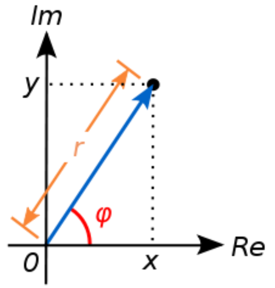
- Real and imaginary parts
- Euler's formula

$$e^{ix} = \cos x + i \sin x$$



- Can be expressed with magnitude and phase

$$z = r(\cos\varphi + i\sin\varphi)$$



- Rules of multiplication and division

$$z_1 z_2 = r_1 r_2 (\cos(\varphi_1 + \varphi_2) + i\sin(\varphi_1 + \varphi_2))$$

$$\frac{z_1}{z_2} = \frac{r_1}{r_2}(\cos(\varphi_1 - \varphi_2) + i\sin(\varphi_1 - \varphi_2))$$

- Taylor series definition

$$\sum_{n=0}^{\infty} \frac{f^{(n)}(a)}{n!}(x-a)^n = f(a) + \frac{f'(a)}{1!}(x-a) + \frac{f''(a)}{2!}(x-a)^2 + \frac{f'''(a)}{3!}(x-a)^3 + \cdots$$

System Identification

Complex Number and Taylor Series Review

- Real and imaginary parts
- Euler's formula

$$e^{ix} = \cos x + i \sin x$$

- Can be expressed with magnitude and phase

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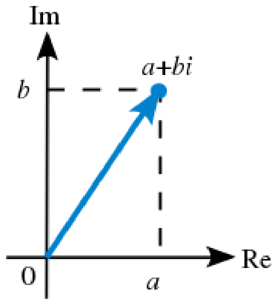
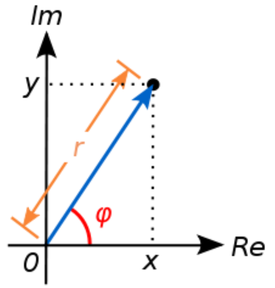
$$z_1 z_2 = r_1 r_2 (\cos(\varphi_1 + \varphi_2) + i \sin(\varphi_1 + \varphi_2))$$

$$\frac{z_1}{z_2} = \frac{r_1}{r_2}(\cos(\varphi_1 - \varphi_2) + i \sin(\varphi_1 - \varphi_2))$$





- Taylor series definition

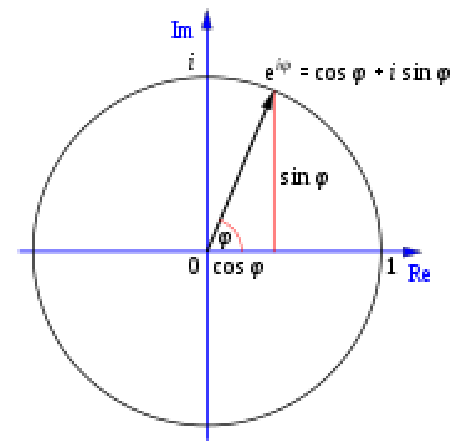
$$\sum_{n=0}^{\infty} \frac{f^{(n)}(a)}{n!}(x-a)^n = f(a) + \frac{f'(a)}{1!}(x-a) + \frac{f''(a)}{2!}(x-a)^2 + \frac{f'''(a)}{3!}(x-a)^3 + \cdots$$

System Identification

Derivation of Euler's Formula with Taylor Series

$$z = x + iy = |z|(\cos\varphi + i\sin\varphi) = re^{i\varphi}$$

$$\begin{aligned} e^{ix} &= 1 + ix + \frac{(ix)^2}{2!} + \frac{(ix)^3}{3!} + \frac{(ix)^4}{4!} + \frac{(ix)^5}{5!} + \frac{(ix)^6}{6!} + \frac{(ix)^7}{7!} + \frac{(ix)^8}{8!} + \cdots \\ &= 1 + ix - \frac{x^2}{2!} - \frac{ix^3}{3!} + \frac{x^4}{4!} + \frac{ix^5}{5!} - \frac{x^6}{6!} - \frac{ix^7}{7!} + \frac{x^8}{8!} + \cdots \\ &= \left(1 - \frac{x^2}{2!} + \frac{x^4}{4!} - \frac{x^6}{6!} + \frac{x^8}{8!} - \cdots\right) + i\left(x - \frac{x^3}{3!} + \frac{x^5}{5!} - \frac{x^7}{7!} + \cdots\right) \\ &= \cos x + i\sin x\ . \end{aligned}$$



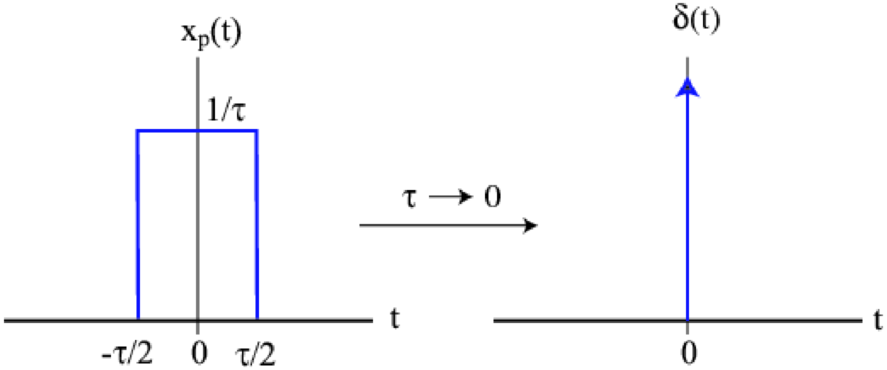
$$e^{i\pi} + 1 = 0$$

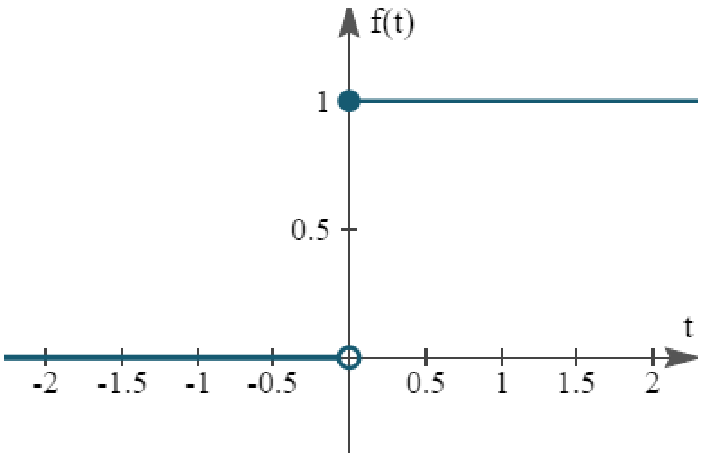
System Identification

Impulse and Step Function

- Idealized function for analysis
- Unit impulse function (Dirac Delta function) denoted as δ(t-a)
- Unit step function (Heaviside step function) denoted as u(t-a)
- The derivative of u(t-a) is δ(t-a)

$$\int_{-\infty}^{\infty} \delta(t)dt = \lim_{\Delta \to 0}[\Delta \cdot 1/\Delta] = 1$$





System Identification

Impulse Response and Convolution

- System response to an impulse function
- Any signal can be decomposed by impulse signal
- Convolution of input yields actual response

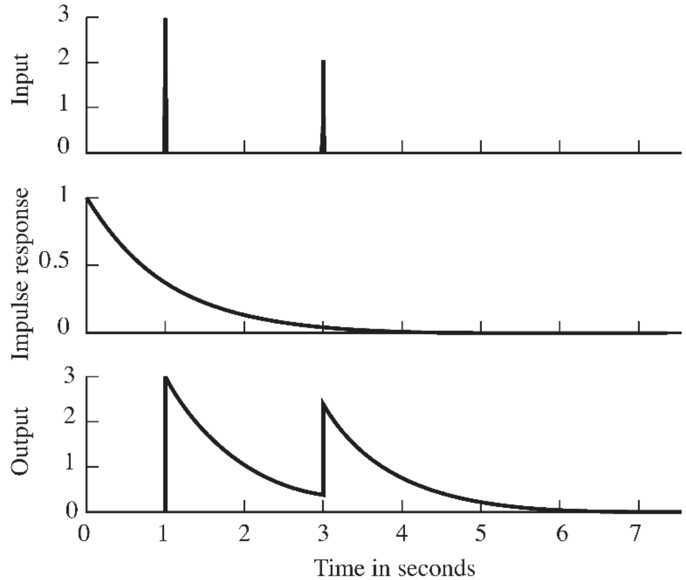
$$\delta_\Delta(t) = \begin{cases} 1/\Delta & 0 \le t < \Delta \\ 0 & \text{else} \end{cases} \qquad \delta(t) \triangleq \lim_{\Delta \to 0} \delta_\Delta(t) = \begin{cases} \infty & t = 0 \\ 0 & \text{else} \end{cases}$$

$$x(t) \approx \sum_{m=-\infty}^{\infty} x(m\Delta)\delta_\Delta(t - m\Delta)\Delta$$

$$x(t) = \lim_{\Delta \to 0} \sum_{m=-\infty}^{\infty} x(m\Delta)\delta_\Delta(t - m\Delta)\Delta = \int_{-\infty}^{\infty} x(\tau)\delta(t-\tau)d\tau$$

$$y(t) = \mathcal{O}[x(t)] = \mathcal{O}[\int_{-\infty}^{\infty} x(\tau)\delta(t-\tau)d\tau]$$

$$= \int_{-\infty}^{\infty} x(\tau)\mathcal{O}[\delta(t-\tau)]d\tau = \int_{-\infty}^{\infty} x(\tau)h(t-\tau)]d\tau = h(t) * x(t)$$



System Identification

Fourier and Laplace Transformation

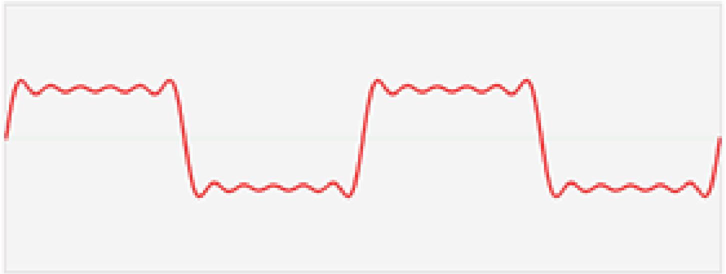
- Fourier transformation: express signal into frequency domain

$$H(\omega) = \mathcal{F}\{f(t)\} = \int_{-\infty}^{\infty} e^{-j\omega t} f(t)dt$$

- Bi-lateral Laplace transformation:

$$G(s) = \mathcal{L}\{f(t)\} = \int_{-\infty}^{\infty} e^{-st} f(t)dt$$

- Use unilateral for casual system
- Use Laplace for simplicity
- Laplace transformation of derivative:



$$\mathcal{L}\left(\frac{df(t)}{dt}\right) = \int_{0^-}^{\infty} \frac{df(t)}{dt} e^{-st}dt$$

$$\int_a^b u \cdot dv = u \cdot v\Big|_a^b - \int_a^b v \cdot du$$

$$du = -s \cdot e^{-st}dt \qquad u = e^{-st}$$

$$v = f(t) \qquad dv = \frac{df(t)}{dt}dt$$

$$\int_{0^-}^{\infty} \frac{df(t)}{dt} e^{-st}dt = \left[e^{-st} \cdot f(t)\right]_{0^-}^{\infty} - \int_{0^-}^{\infty} f(t) \cdot (-s) \cdot e^{-st}dt$$

$$= \left[e^{-st} \cdot f(\infty) - e^{-0^- t} \cdot f(0^-)\right] + s\int_{0^-}^{\infty} f(t) \cdot e^{-st}dt$$

$$\int_{0^-}^{\infty} \frac{df(t)}{dt} e^{-st}dt = \left[\cancel{e^{-st} \cdot f(\infty)} - \cancel{e^{-0^- t}} \cdot f(0^-)\right] + sF(s) = \boxed{sF(s) - f(0^-)}$$

System Identification

The Convolution Theorem

- For causal system, use unilateral Laplace transformation

$$\mathcal{UL}[x(t)] = X(s) \triangleq \int_{-\infty}^{\infty} x(t)u(t)e^{-st}dt = \int_{0^-}^{\infty} x(t)e^{-st}dt$$

- Prove that convolution becomes multiplication

$$L[f * g] = L[f] \cdot L[g] = F \cdot G$$

$$L[f * g] = L[f] \cdot L[g] = F \cdot G$$

Proof: $F(s)\cdot G(s) = F(s)\int_0^\infty e^{-st}g(t)dt = \int_0^\infty F(s)e^{-s\tau}g(\tau)d\tau$

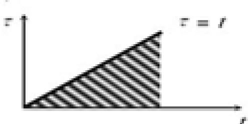
$$e^{-s\tau}F(s) = L[H(t-\tau)f(t-\tau)]$$

$$F(s)\cdot G(s) = \int_0^\infty L[H(t-\tau)f(t-\tau)]g(\tau)d\tau$$

$$= \int_0^\infty \left[\int_0^\infty e^{-st} H(t-\tau) f(t-\tau) dt\right] g(\tau) d\tau$$

$$= \int_0^\infty \int_0^\infty e^{-st} g(\tau) H(t-\tau) f(t-\tau) dt d\tau$$

$$= \int_0^\infty \int_\tau^\infty e^{-st} g(\tau) f(t-\tau) dt d\tau$$



$$= \int_0^\infty \int_0^t e^{-st} g(\tau) f(t-\tau) d\tau dt = \int_0^\infty e^{-st}\left[\int_0^t g(\tau) f(t-\tau) d\tau\right] dt$$

$$= \int_0^\infty e^{-st}(f * g)(t) dt = L[f * g](s)$$

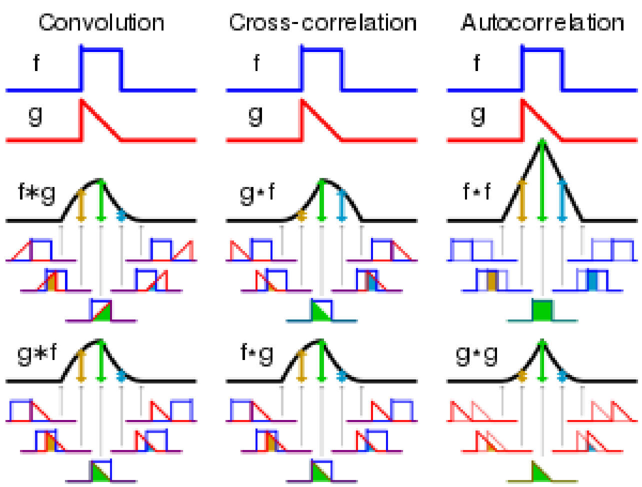
System Identification

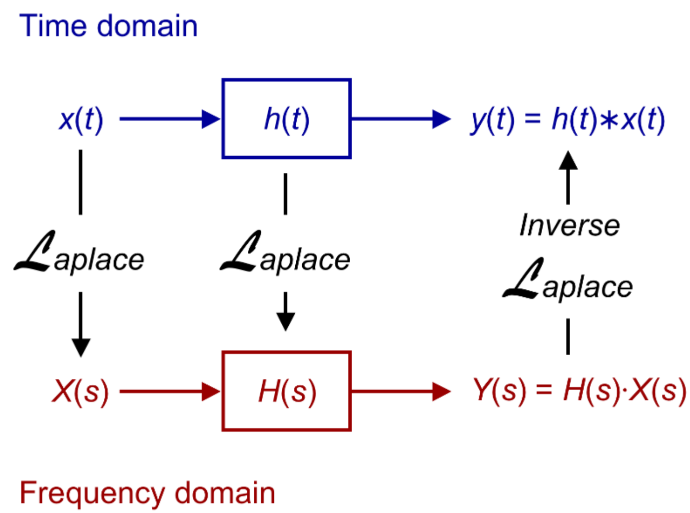
Switching Domain for Analysis

- Signals are convoluted when going through a system

$$(f * g)(t) \stackrel{\text{def}}{=} \int_{-\infty}^{\infty} f(\tau)g(t-\tau)\,d\tau$$

- Multiplications is a lot easier than convolution





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System Identification

Laplace Transformation Table

- Laplace transformation simplifies the analysis process for ODES
- Easy representation
- Solving with partial fraction expansion and lookup tables
- Definition

$$F(s) = \int_0^\infty f(t) e^{-st}\, dt$$

First Derivative	$\frac{df(t)}{dt} \xleftrightarrow{L} sF(s) - f(0^-)$
Second Derivative	$\frac{d^2 f(t)}{dt^2} \xleftrightarrow{L} s^2 F(s) - sf(0^-) - \dot{f}(0^-)$

Common Laplace Transform Pairs

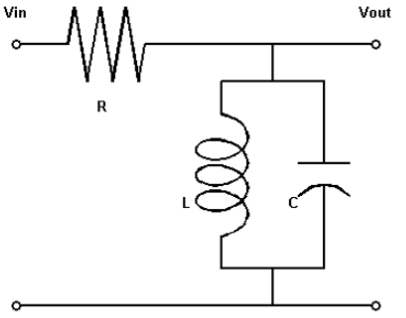
Time Domain Function		Laplace Domain Function
Name	Definition*	
Unit Impulse	$\delta(t)$	1
Unit Step	$\gamma(t)$ †	$\frac{1}{s}$
Unit Ramp	t	$\frac{1}{s^2}$
Parabola	t^2	$\frac{2}{s^3}$
Exponential	e^{-at}	$\frac{1}{s+a}$
Asymptotic Exponential	$\frac{1}{a}(1-e^{-at})$	$\frac{1}{s(s+a)}$
Dual Exponential	$\frac{1}{b-a}\left(e^{-at}-e^{-bt}\right)$	$\frac{1}{(s+a)(s+b)}$
Asymptotic Dual Exponential	$\frac{1}{ab}\left[1+\frac{1}{a-b}\left(be^{-at}-ae^{-bt}\right)\right]$	$\frac{1}{s(s+a)(s+b)}$
Time multiplied Exponential	te^{-at}	$\frac{1}{(s+a)^2}$
Sine	$\sin(\omega_0 t)$	$\frac{\omega_0}{s^2+\omega_0^2}$
Cosine	$\cos(\omega_0 t)$	$\frac{s}{s^2+\omega_0^2}$
Decaying Sine	$e^{-at}\sin(\omega_d t)$	$\frac{\omega_d}{(s+a)^2+\omega_d^2}$
Decaying Cosine	$e^{-at}\cos(\omega_d t)$	$\frac{s+a}{(s+a)^2+\omega_d^2}$
Generic Oscillatory Decay	$e^{-at}\left[B\cos(\omega_d t)+\frac{C-aB}{\omega_d}\sin(\omega_d t)\right]$	$\frac{Bs+C}{(s+a)^2+\omega_d^2}$
Prototype Second Order Lowpass, underdamped	$\frac{\omega_0}{\sqrt{1-\zeta^2}}e^{-\zeta\omega_0 t}\sin\left(\omega_0\sqrt{1-\zeta^2}\,t\right)$	$\frac{\omega_0^2}{s^2+2\zeta\omega_0 s+\omega_0^2}$
Prototype Second Order Lowpass, underdamped - Step Response	$1-\frac{1}{\sqrt{1-\zeta^2}}e^{-\zeta\omega_0 t}\sin\left(\omega_0\sqrt{1-\zeta^2}\,t+\phi\right)$ $\phi=\tan^{-1}\left(\frac{\sqrt{1-\zeta^2}}{\zeta}\right)$	$\frac{\omega_0^2}{s(s^2+2\zeta\omega_0 s+\omega_0^2)}$

System Identification

RLC Circuit Impedance Transfer Function

- Model system using impedance form
- Utilize series and parallel connection
- Obtain the result in transfer function form

$$G(s) = \frac{s/(RC)}{s^2 + s/(RC) + 1/(LC)}$$



- What is a transfer function
  - Linear time-invariant system can be represented by Bode plot
  - Mathematically: Zero initial condition unilateral Laplace transformation

$$F(s) = \int_0^\infty f(t)e^{-st}\,dt$$

  - Characterize frequency response for amplitude and phase

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System Identification

RLC Circuit Bode Plot

- Characterize AC frequency response of system
- Convert to Fourier transform by setting s = jω
- Obtain amplitude and phase

$$G(s) = \mathcal{L}\{f(t)\} = \int_{-\infty}^{\infty} e^{-st} f(t) dt$$

$$H(\omega) = \mathcal{F}\{f(t)\} = \int_{-\infty}^{\infty} e^{-j\omega t} f(t) dt$$

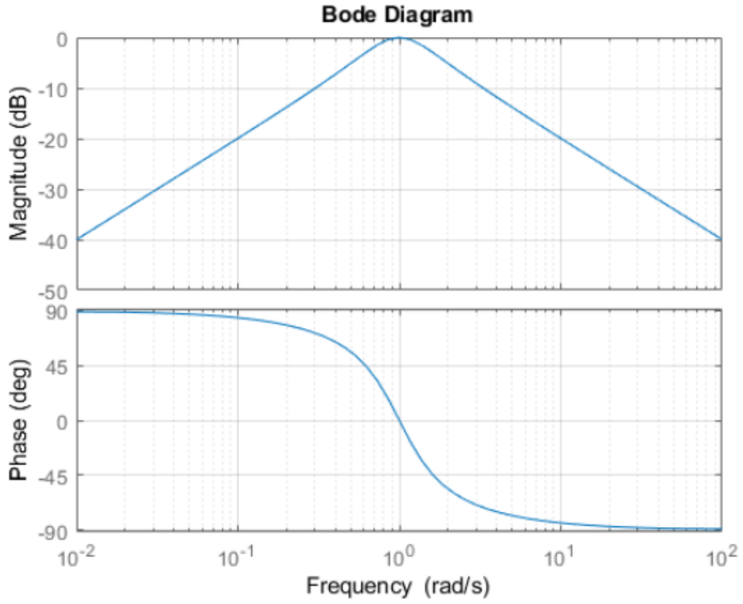
$$u(t) = \sin(\omega t)$$
$$y(t) = y_0 \sin(\omega t + \varphi)$$
$$y_0 = |H(\mathrm{j}\omega)|$$
$$\varphi = \arg H(\mathrm{j}\omega)$$

- Gain expressed in dB

$$G_{\mathrm{dB}} = 20 \log_{10} \left(\frac{V}{V_0} \right) \mathrm{dB}$$

- With R = 1, C = 1, L = 1

$$G(s) = \frac{s/(RC)}{s^2 + s/(RC) + 1/(LC)}$$



Bode plot for RLC Circuit

System Identification

Things to Remember

- A system can be described in many ways
  - Model and parameter values
  - Unit impulse response
  - Unit step response (will appear later)
  - Frequency response (Bode plot)
- A signal can be decomposed with many impulse or sine functions
- Signal going through a system is being convoluted
- We use the unilateral Laplace transformation to make our life easier

$$F(s) = \int_0^\infty f(t)e^{-st}\,dt$$

- We will revisit these concepts in the section of controls

System Identification

Methods for Frequency Response

- Bode plot can be experimentally measured by conducting a frequency sweep with a dynamic signal analyzer, which is a time consuming but reliable procedure.

- Advanced system identification procedure using binary stochastic signal input to simultaneously excite a broad band of signal and obtain bode plot using Fourier transformation and power spectral density functions is also available.

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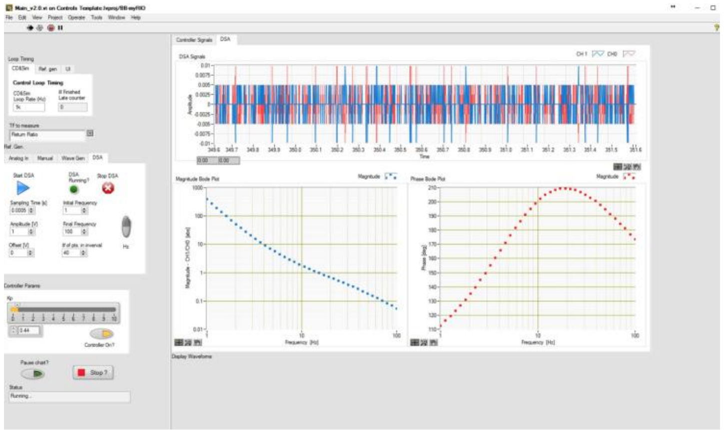
System Identification

Dynamic Signal Analyzer

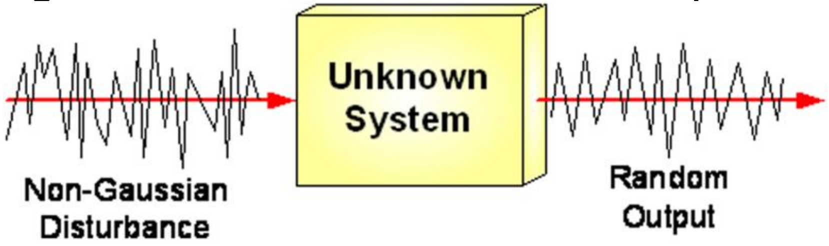
- Conduct frequency sweep automatically to obtain bode plot
- Implementation in LabVIEW available



System Identification

Random Binary Stochastic Signal

- The transfer function of a system can be obtained by the output power spectrum divided by the input power spectrum for amplitude and phase estimation
- The input can be generated by filtering a Gaussian white noise to focus on the frequency of interest
- A Random Binary Stochastic (RBS) signal can be useful for system ID when only digital excitation is available
- RBS signal system ID saves time in general but can be challenging when a controller is in the loop



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Thank You!