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Inverted Pendulum Modeling, Control and Demo

Workshop

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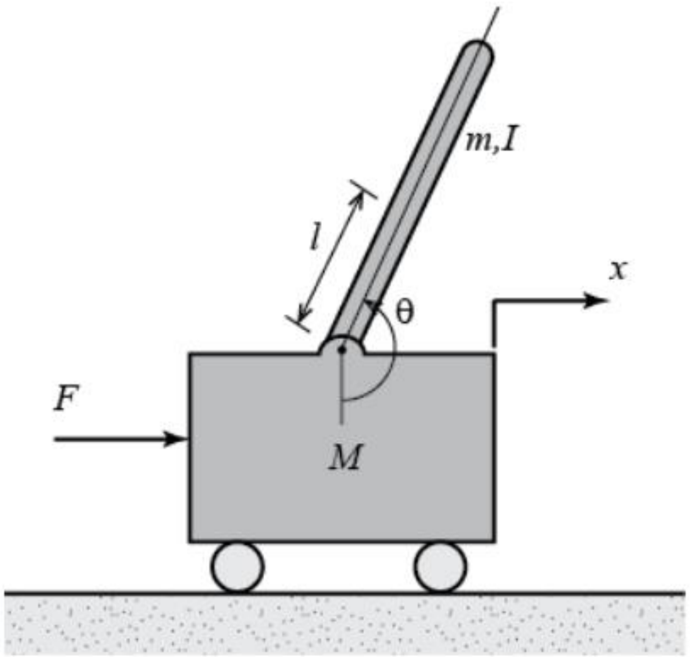
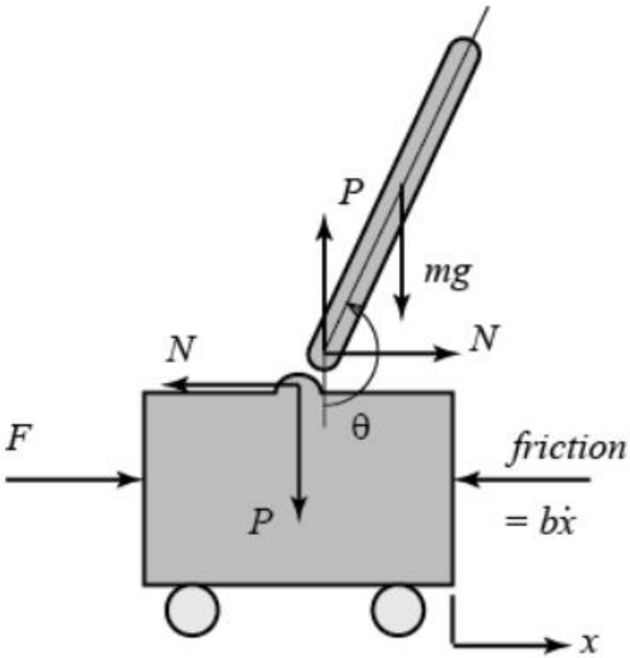
Overview

- System Modeling
- Various Controllers
- PD Control Demo



System Modeling



$$M\ddot{x} + b\dot{x} + N = F$$
$$N = m\ddot{x} + ml\ddot{\theta}\cos\theta - ml\dot{\theta}^2\sin\theta$$
$$(M+m)\ddot{x} + b\dot{x} + ml\ddot{\theta}\cos\theta - ml\dot{\theta}^2\sin\theta = F \qquad (1)$$

$$P\sin\theta + N\cos\theta - mg\sin\theta = ml\ddot{\theta} + m\ddot{x}\cos\theta$$
$$-Pl\sin\theta - Nl\cos\theta = I\ddot{\theta}$$
$$(I+ml^2)\ddot{\theta} + mgl\sin\theta = -ml\ddot{x}\cos\theta \qquad (2)$$

$$\cos\theta = \cos(\pi+\phi) \approx -1$$
$$\sin\theta = \sin(\pi+\phi) \approx -\phi$$
$$\dot{\theta}^2 = \dot{\phi}^2 \approx 0$$

Linearize

$$(I+ml^2)\ddot{\phi} - mgl\phi = ml\ddot{x}$$
$$(M+m)\ddot{x} + b\dot{x} - ml\ddot{\phi} = u$$

Linear EOM

Transfer Function

$$(I+ml^2)\ddot{\phi} - mgl\phi = ml\ddot{x}$$
$$(M+m)\ddot{x} + b\dot{x} - ml\ddot{\phi} = u$$

$$(I+ml^2)\Phi(s)s^2 - mgl\Phi(s) = mlX(s)s^2$$
$$(M+m)X(s)s^2 + bX(s)s - ml\Phi(s)s^2 = U(s)$$

$$P_{pend}(s) = \frac{\Phi(s)}{U(s)} = \frac{\frac{ml}{q}s}{s^3 + \frac{b(I+ml^2)}{q}s^2 - \frac{(M+m)mgl}{q}s - \frac{bmgl}{q}} \qquad \left[\frac{rad}{N}\right]$$

$$P_{cart}(s) = \frac{X(s)}{U(s)} = \frac{\frac{(I+ml^2)s^2 - gml}{q}}{s^4 + \frac{b(I+ml^2)}{q}s^3 - \frac{(M+m)mgl}{q}s^2 - \frac{bmgl}{q}s} \qquad \left[\frac{m}{N}\right]$$

Transfer Function

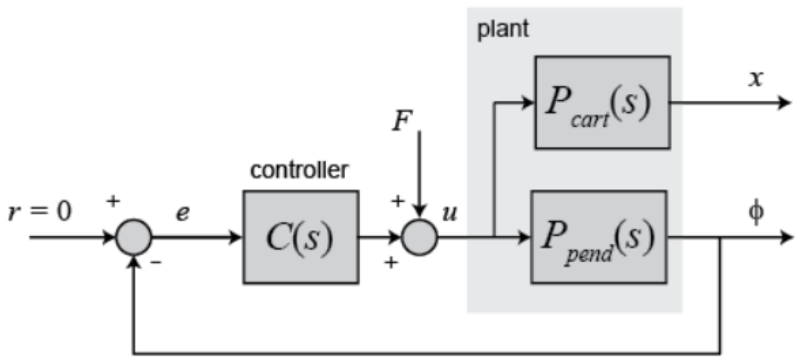
$$\begin{bmatrix} \dot{x} \\ \ddot{x} \\ \dot{\phi} \\ \ddot{\phi} \end{bmatrix} = \begin{bmatrix} 0 & 1 & 0 & 0 \\ 0 & \frac{-(I+ml^2)b}{I(M+m)+Mml^2} & \frac{m^2gl^2}{I(M+m)+Mml^2} & 0 \\ 0 & 0 & 0 & 1 \\ 0 & \frac{-mlb}{I(M+m)+Mml^2} & \frac{mgl(M+m)}{I(M+m)+Mml^2} & 0 \end{bmatrix} \begin{bmatrix} x \\ \dot{x} \\ \phi \\ \dot{\phi} \end{bmatrix} + \begin{bmatrix} 0 \\ \frac{I+ml^2}{I(M+m)+Mml^2} \\ 0 \\ \frac{ml}{I(M+m)+Mml^2} \end{bmatrix} u$$

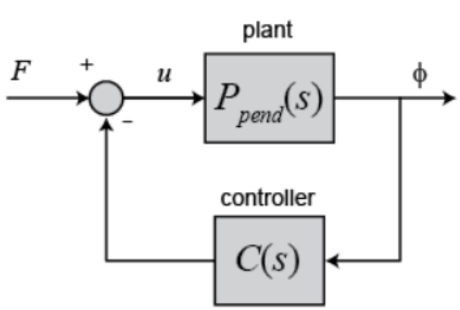
State Space

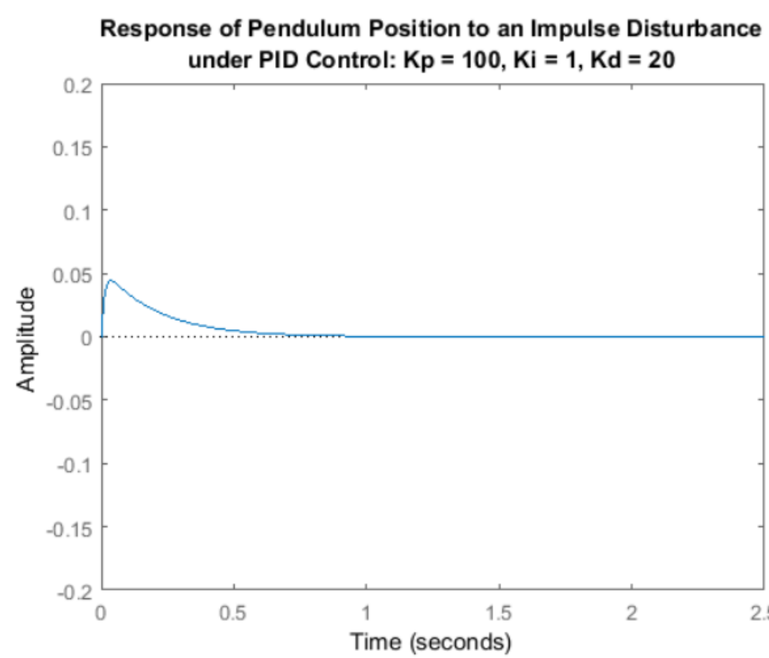
$$\mathbf{y} = \begin{bmatrix} 1 & 0 & 0 & 0 \\ 0 & 0 & 1 & 0 \end{bmatrix} \begin{bmatrix} x \\ \dot{x} \\ \phi \\ \dot{\phi} \end{bmatrix} + \begin{bmatrix} 0 \\ 0 \end{bmatrix} u$$

PID Control for Angle

- Angle PID control can stabilize pendulum but not cart
- Analyze for reference equal to 0 and found impulse response

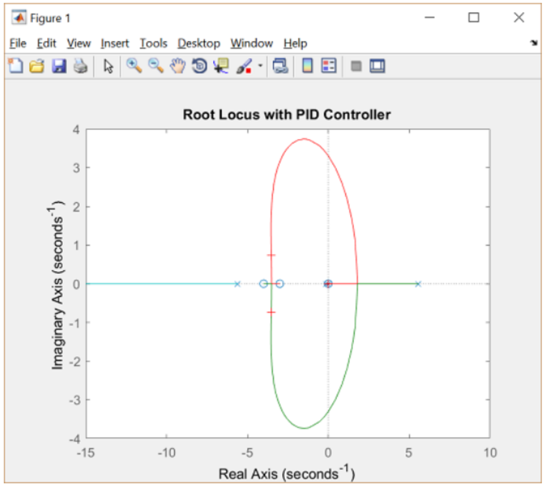


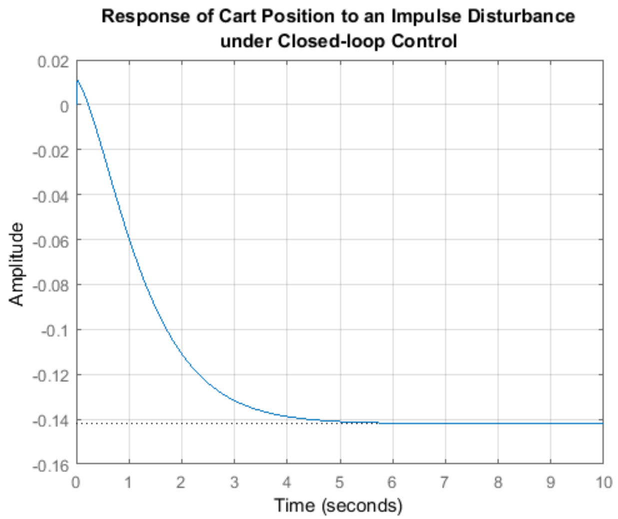


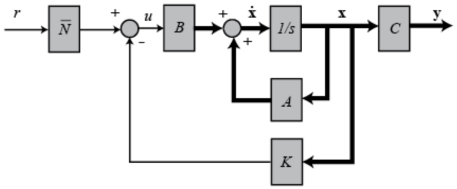


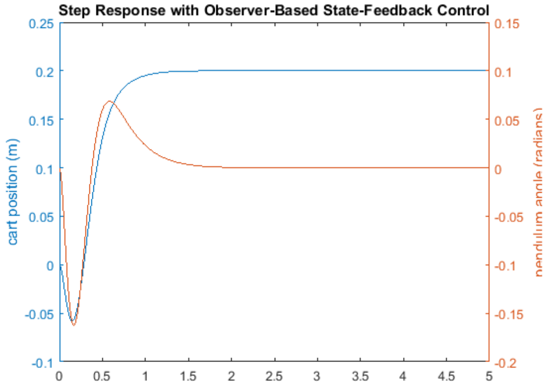
Other Control Techniques

- Root locus for PID design can be used for pole placement
- Frequency domain loop shaping can luckily control cart position
- State space design technique can stabilize both variable



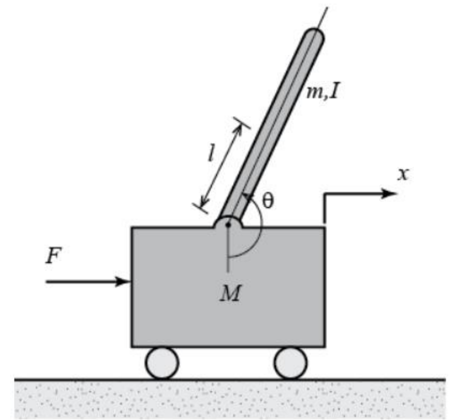




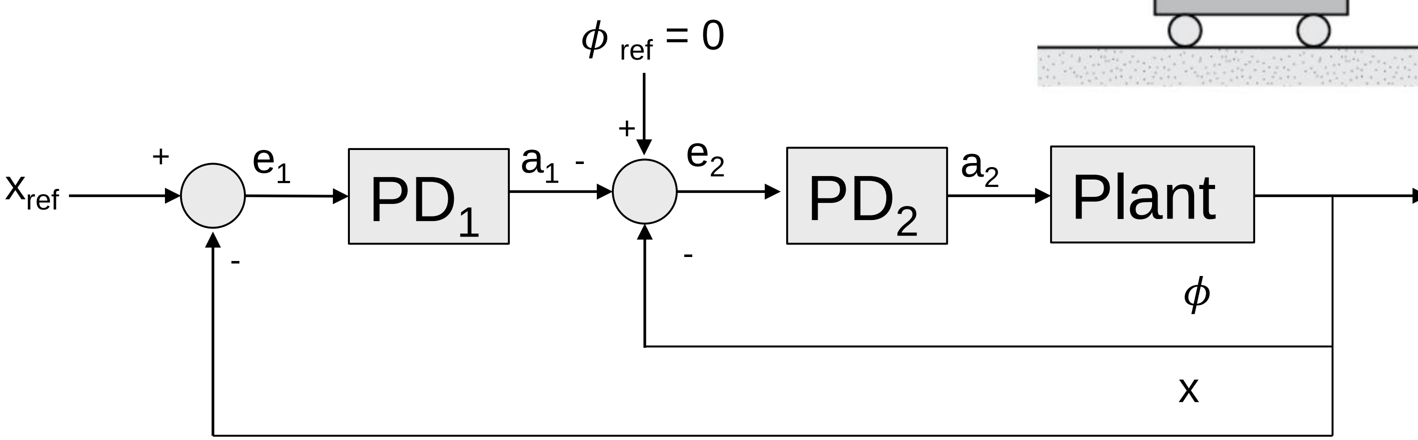


Double PD Control

- Inner and Outer layer PD control
- Negative feedback from position
- Less intuitive due to 2 layer controller

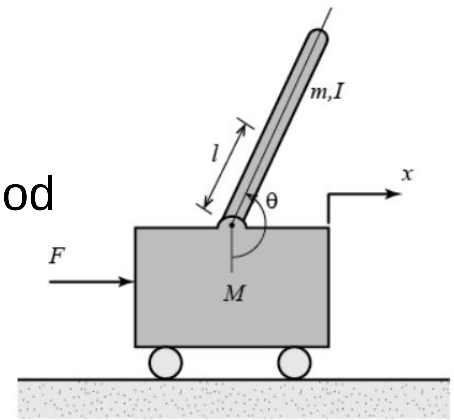


$$a_1 = k_{p1} e_1 + k_{d1} \dot{e}_1 \qquad a_2 = k_{p2} e_2 + k_{d2} \dot{e}_2$$

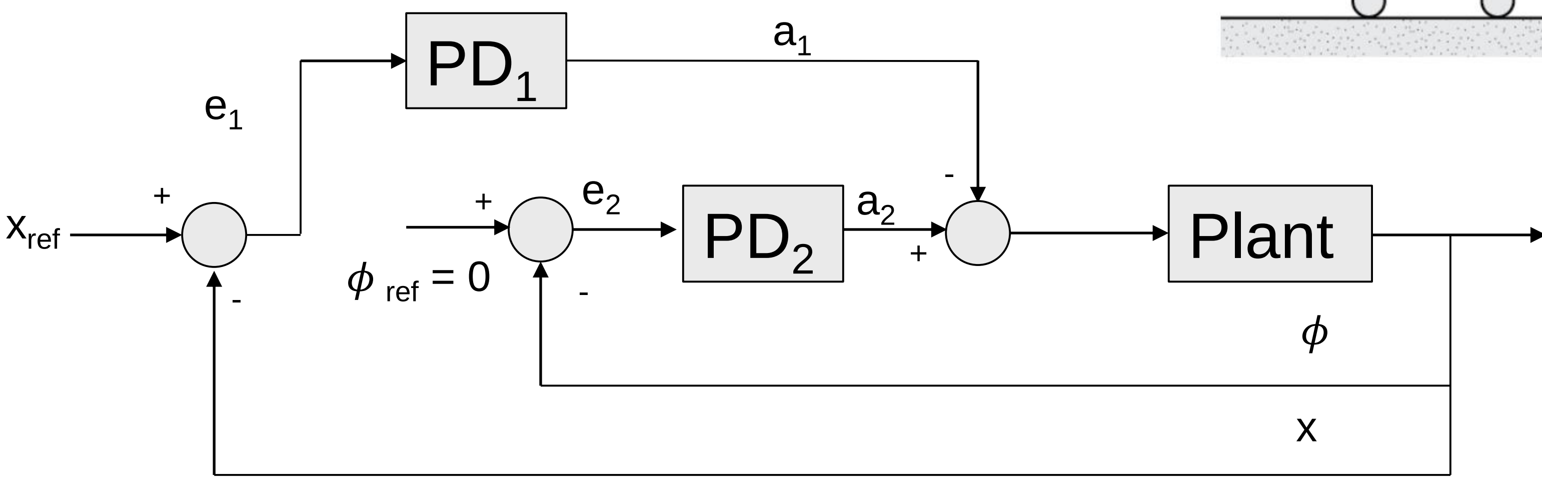


Parallel PD Control Used in Demo

- Use parallel architecture to simplify system
- No need for double PD on position input
- OK performance but not as good as SS method



$$a_1 = k_{p1}e_1 + k_{d1}\dot{e_1} \qquad a_2 = k_{p2}e_2 + k_{d2}\dot{e_2}$$



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Thank You!