Simplex – Big M Method

Maximise $3X_1 + 4X_2$

Subject to
$$
\begin{aligned}
2X_1 + X_2 &<= 600 \\
X_1 + X_2 &<= 225 \\
5X_1 + 4X_2 &<= 1000 \\
X_1 + 2X_2 &>= 150
\end{aligned}
$$

$$X_1, X_2 >= 0$$

Solution:

Standard form:

Maximise $3X_1 + 4X_2$

Subject to
$$
\begin{aligned}
2X_1 + 3X_2 + S_1 &= 600 \\
X_1 + X_2 + S_2 &= 225 \\
5X_1 + 4X_2 + S_3 &= 1000 \\
X_1 + 2X_2 - S_4 &= 150
\end{aligned}
$$

$$X_1, X_2, S_1, S_2, S_3, S_4 >= 0$$

Not in canonical form because there is no basic variable in the fourth equation. Therefore we add an artificial variable to that equation (R_1) and give it a large **negative** coefficient in the objective function, to penalise it:

Maximise $3X_1 + 4X_2$

Subject to
$$
\begin{aligned}
2X_1 + X_2 + S_1 &= 600 \\
X_1 + X_2 + S_2 &= 225 \\
5X_1 + 4X_2 + S_3 &= 1000 \\
X_1 + 2X_2 - S_4 + R_1 &= 150
\end{aligned}
$$

$$X_1, X_2, S_1, S_2, S_3, S_4, R_1 >= 0$$

	X_1	X_2	S_4	S_1	S_2	S_3	R_1	B
Z	-3	-4	0	0	0	0	+M	
S_1	2	3	0	1	0	0	0	600
S_2	1	1	0	0	1	0	0	225
S_3	5	4	0	0	0	1	0	1000
R_1	1	2	-1	0	0	0	1	150

Not in Canonical form because of +M entry on Z row for one basic variable (R_1).
Pivot to replace +M on Z row by zero - Z row – M*R_1 row:

	X_1	X_2	S_4	S_1	S_2	S_3	R_1	b
Z	(-3-M)	(-4-2M)	M	0	0	0	0	-150M
S_1	2	3	0	1	0	0	0	600
S_2	1	1	0	0	1	0	0	225
S_3	5	4	0	0	0	1	0	1000
R_1	1	2	-1	0	0	0	1	150

	X_1	X_2	S_4	S_1	S_2	S_3	R_1	b
Z	-1	0	-2	0	0	0	M	800
S_1	½	0	3/2	1	0	0	-3/2	375
S_2	½	0	½	0	1	0	-½	150
S_3	3	0	2	0	0	1	-2	700
X_2	½	1	-½	0	0	0	½	75

	X_1	X_2	S_4	S_1	S_2	S_3	R_1	b
Z	-1/3	0	0	4/3	0	0	M	800
S_4	1/3	0	1	2/3	0	0	-1	250
S_2	1/3	0	0	-1/3	1	0	0	25
S_3	7/3	0	0	-4/3	0	1	0	200
X_2	2/3	1	0	1/3	0	0	0	200

	X_1	X_2	S_4	S_1	S_2	S_3	R_1	b
Z	0	0	0	1	1	0	M	825
S_4	0	0	1	1	-1	0	-1	225
X_1	1	0	0	-1	3	0	0	75
S_3	0	0	0	1	-7	1	0	25
X_2	0	1	0	1	-2	0	0	250

Optimal tableau: Solution: $X_1^* = 75$ $X_2^* = 150$ $Z^* = 825$