For office use only	Team Control Number	For office use only
T1 ________	**67449**	F1 ________
T2 ________		F2 ________
T3 ________	Problem Chosen	F3 ________
T4 ________	**C**	F4 ________

2017 Mathematical Contest in Modeling (MCM) Summary Sheet

(Attach a copy of this page to each copy of your solution paper.)

Summary

In order to analyze the cooperative relationship between self-driving cars and non-self-driving vehicles, we propose two indexes in this paper, called highway density index and the degree of cooperation index. We put those two indexes into consideration attempting to explore the best proportion of self-driving cars by the model. The former index is based on the linear combination of speed which is solved by Particle Swarm Optimization and the number of vehicles in transit. The latter function is based on the car-following theory of physiology and psychology.

First of all, according to the offered data and experience, this paper determines to employ highway density simplifying the building and evaluation of model. We use the proportion of total area of all vehicles on the road to total area of the highway and collaborative information interaction to evaluate model.

Next to calculate two evaluation indexes, we select and calculate the following factors, the safety area of the car which includes the floor area of car and the safety distance between the two cars, total area of road, the number of vehicles and speed of cars, etc.

Finally, we need to calculate changes of the total highway density and degree of cooperation with the increase of the proportion of the use of self-driving cars.

A Model of Analysing the Effect on Traffic Caused by Self-driving

Control Number: #67449

January 24, 2017

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Key Words: self-driving; traffic model; PSO;

Contents

1 A Letter to the Governor's Office

Dear governor, We select two indexes to quantify the relationship between self-driving cars and non-self-driving cars as well as finding the optimal proportion of using self-driving cars, the model is developed to predict the effects on the proportion of self-driving cars in currently traffic.

Highway density index is a simplified and feasible index to measure the degree of congestion which uses a simple formula to quantify the degree of congestion on the road and takes into account the impact of the degree of total collaborative information interaction. The degree of cooperation is another indicator to measure the effect of the proportion of self-driving cars on the traffic which its function is expressed as the effect on the safety distance. Therefore, the index used in this paper is feasible and appropriate for self-driving analysis.

In order to obtain evaluation index, we use basic formula of traffic flow calculating the following parameter, such as safety area of the car, the total area of the road, the number of cars on the road and vehicle speed. Then,we could find the optimum proportion of self-driving through the four parameters that we have mentioned above and select the best processing strategy by calculation.

In brief, the model successfully predicts the relationship between self-driving cars and non-self-driving cars and the corresponding road conditions,including a list() this model is based on previous data and takes into account the effects of multiple factors, so this model is suitable for forecasting and providing assistance for decision making.

We have strongly believe that this model can be effectively used to analyze and predict the relationship between self-driving and non-self-driving, which can determine the optimal proportion of self-driving cars.

percentage	Simulation	Function
0.095	0.6206	0.6479
0.1	0.6244	0.6431
0.105	0.6575	0.6386
0.495	0.4891	0.4960
0.5	0.4506	0.4951
0.505	0.5104	0.4942
0.525	0.4915	0.4906
0.7	0.4440	0.4641
0.895	0.4310	0.4415
0.9	0.4083	0.4410
0.905	0.4553	0.4405

Yours sincerely,

Team #67449

2 Introduction

2.1 Statement of the Problem

With the development of the society, the traffic problem is becoming more and more serious in our daily life, the necessity of quantitative analysis for the traffic problem is increasingly prominent.

Washington has serious problems in traffic, so self-driving cars were proposed as a solution to address the dilemma. The governor of Washington needs a good model to analyze the impact on current traffic due to self-driving and the relationship between self-driving and non-self-driving.

So this paper presents a model that we need to analyze the previous traffic data to give an optimal proportion of using self-driving cars as well as research impact of self-driving on existing traffic and non-self-driving cars.

2.2 Assumptions and Definitions

2.2.1 Assumptions

- 1 The traffic system is consistent with the car-following model of physiology and psychology.
- 2 The traffic system is act as a whole.
- 3 Vehicles are evenly distributed on roads.
- 4 The difference between self-driving and non-self-driving is only related to the degree of information exchange and the degree of tracking of system speed.
- 5 The speed of cars will be affected by inertia, cognition and society.

2.2.2 Definitions

name	definition	denotation	unit
Highway density index	A composite index which depends on ρ_1, ρ_2, ρ_3, ρ_4	HDI	
Road area	Area of road. See the example, as shown in the figure in Appendix A, the red color area is the definition	S_r	m^2
Degree of cooperation	A variable to evaluate the cooperation efficiency of cars	DoC	
Safe distance	A interval distance between two cars, affected by DoC	d	m

The adaptation velocity in peak hours	Adaptable velocity of cars in peak hours	v_1	m/min
The adaptation velocity in non-peak hours	Adaptable velocity of cars in non-peak hours	v_2	m/min
The limited maximum velocity in all roads	In the condition of problem it is 1609.344m/min(60 miles per hours)	v_{max}	m/min
Traffic volume in peak hours in maximum velocity	How many cars per minute in v_{max} in peak hours	N_1	$cars/min$
Traffic volume in peak hours in adaptation velocity	How many cars per minute in v_1 in peak hours	N_2	$cars/min$
Traffic volume in non-peak hours in maximum velocity	How many cars per minute in v_{max} in non-peak hours	N_3	$cars/min$
Traffic volume in non-peak hours in adaptation velocity	How many cars per minute inv_2 in non-peak hours	N_4	$cars/min$
Density of road in peak hours in maximum velocity	product of N_1 multiply area of one car to S_r ratio	ρ_1	
Density of road in peak hours in adaptation velocity	N_2 multiply area of one car to S_r ratio	ρ_2	
Density of road in non-peak hours in maximum velocity	N_3 multiply area of one car to S_r ratio	ρ_3	
Density of road in non-peak hours in adaptation velocity	N_4 multiply area of one car to S_r ratio	ρ_4	
Area of one car	Area of one car	S_{car}	m^2

2.3 Baseline Model

This paper uses two evaluation indexes which are highway density index and degree of cooperation of cars to evaluate the effect on traffic and non-self-driving caused by self-driving.

Based upon analysis of offered data and traffic simulation, we decide to employ highway density and degree of cooperation of cars quantifying the relation between self-driving and non-self-driving and influence to transport.

Main task is that obtaining two indexes by correlated variables.

First step we get degree of cooperation function via traffic simulation and regression, then calculate the area of one car.

Second step, plug v_{max} into equation to get N_1, N_2(weighted by time).

Third step, we adopt PSO model simulating velocity changes of cars and get N_3 and N_4.

Fourth step, Let N_1, N_2, N_3, N_4 respectively multiply the area of single car then those products divide S_r to obtain ρ_1, ρ_2, ρ_3, ρ_4. Previous efforts make it practicable to output the weight of each ρ as well as calculate the highway density index for the later analysis. Furthermore, adjust learning parameters for PSO leads us conducting research about effect from self-driving to non-self-driving.

Finally we find that highway density index changes and interplay between self-driving and non-self-driving as self-driving to vehicle ratio increases by adjusting arguments of model.

2.4 Advantages of Our Model

- 1 Our model considers the problem from the perspective of macro system, which is convenient for further decision making.
- 2 We use a simple model to explore the solution of problem in a straightforward way.
- 3 Our model is innovatively introduced into the PSO algorithm, which is a modern computer intelligence algorithm to explore the global optimal speed of the vehicle.
- 4 We introduce the highway density index which can be easily described the road congestion under different situation.

3 Building the Evaluation Index

3.1 Calculation of Degree of Cooperation and Safe Distance

The model is based on the car-following theory of physiology and psychology in traffic flow. It describes that the safety distance will cause the traffic flow changing and make the road condition become blocked or unimpeded. Meanwhile, the phenomenon of phantom traffic jams shows that self-driving could shorten safety distance through information communication.

According to references and simulation of traffic system, we obtain the degree of cooperation function:

$$DoC = \frac{0.93}{1 + \frac{0.93}{0.12} * e^{-7.1*percentage}} \tag{1}$$

This is a logistic model, it has the following shape

The reason why the model has the lowest value and peak may be due to the fact that even if it is non-self-driving, there are still some certain information interactions

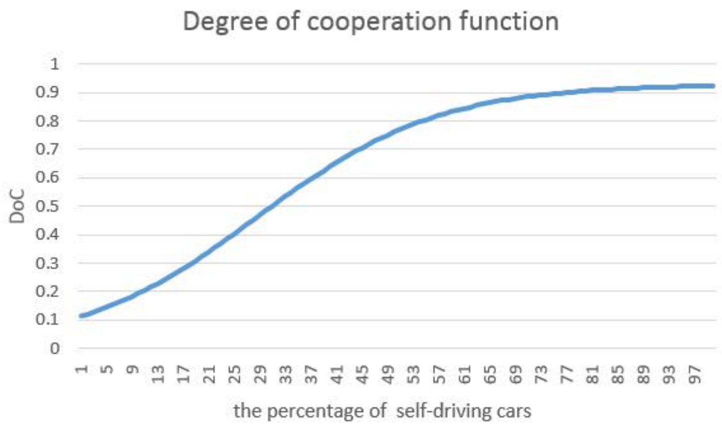

Figure 1: Degree of Cooperation Function

(though not much) between vehicles and vehicles. In the case of self-driving cars, because of the influence of cost, technology and other factors, we can not achieve the perfect information transmission and the varied conditions between vehicles and vehicles cause information asymmetry affecting the effect of information interaction.

After solving this function, we can find that the percentage of self-driving has the highest growth rate at 52.5%, It can achieve a balance point while the percentage of self-driving come to 70% with DoC =0.8825.

Now, we consider the case of self-driving cars at 10%, at the meantime Doc = 0.1933. We use formula to measure the effect of DoC on the safety distance, called d:

$$d = \frac{7.8}{(1 + DoC)^2} \tag{2}$$

The constant 7.8 is a simple and feasible initial safety distance, we put the DoC into the formula to get d = 5.478m. At the same time, we can get the per unit area of the car is 20.478 m^2

$$S_{car} = (d + 5) * 3 \tag{3}$$

Here we assume that the vehicle length is 5 meters and the vehicle width is 3 meters, which is a significant intermediate value that it can provide a feasible starting point for the definition of density.

3.2 Calculation of Density of Road

Next, we take traffic volume per minute into consideration. First of all, we have to identify an indicator that can be used as a reference and then we select v_{max} = 60 $miles/hour$ meaning 1609.334 m/min as the optimal situation. We set congestion time for 1 hours and figure out N_1 (traffic volume in peak hours in maximum velocity) and N_3 (traffic volume in non-peak hours in maximum velocity). They are given by the following formula:

$$N_1 = \frac{S_r}{v_{max}} * \frac{ave * 8\%}{2 * 60} + N_3 \tag{4}$$

$$N_3 = \frac{S_r}{v_{max}} * \frac{ave * 92\%}{24 * 60} \tag{5}$$

ave is average daily traffic volume per road, S_r is obtained by the following formula

$$S_r = (endofmp - startofmp) * thenumberoflane * 3.6576 \tag{6}$$

For example, as for the first section of Interstates 5 along the INCR direction, its area should be $(101.87 - 100.93) * 1609.334 * 3 * 3.6576 = 16599.37m^2$

We also do the same for all the roads, and the images are as follows (number 1 to 224 are the data for increasing direction , number 225-448 are the data for decreasing direction, 1 is corresponding to 255, 2 is corresponding to 226, and so on).

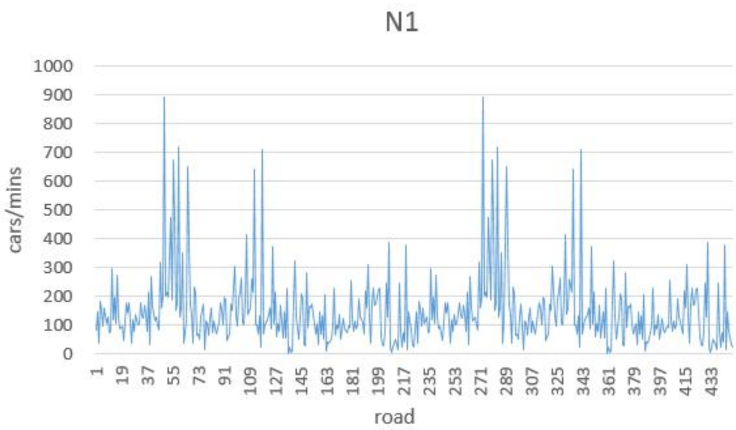

Figure 2: N_1

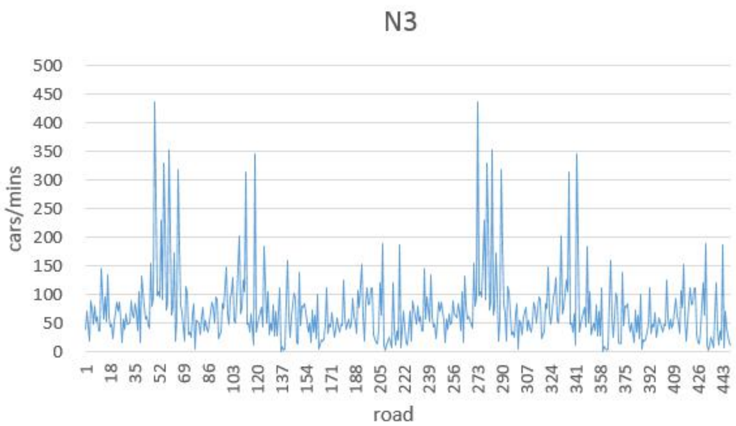

Figure 3: N_3

PSO algorithm is developed by J. Kennedy and R. C. Eberhart, which based on the computational method simulating the migration of birds in search of food. Simply speaking, every individual update itself location through those parameters, such as inertia weight and acceleration constants, (used to describe the cognition part)(used to describe the social part) and seek the optimal value by n times update. In particular situation, only the p information will be conveyed to the rest of the individual, The iteration formula is as follows:

$$v_{id}^{k} = w * v_{id}^{k-1} + c_1 r_1 (pbest_{id} - x_{id}^{k-1}) + c_2 r_2 (gbest_{id} - x_{id}^{k-1}) \tag{7}$$

$$x_{id}^{k} = x_{id}^{k-1} + v_{id}^{k-1} \tag{8}$$

We assume that each vehicle in the traffic system has been seeking to minimize the rate of adaptation by changing the speed of the vehicle , which makes the problem become a single objective linear programming problem. We use PSO to simulate the process of updating each vehicle's speed. By referring to the literature, we use the following parameters:

$$w = 0.5$$
$$c_1 = 2.05$$
$$c_2 = 2.05$$
$$p = 0.75$$
$$n = 60$$

Then we employ simulated v_1, v_2 to replace v_{max} updating formulas of N_1 and N_3 to get N_2 and N_4 as shown in the figures below.

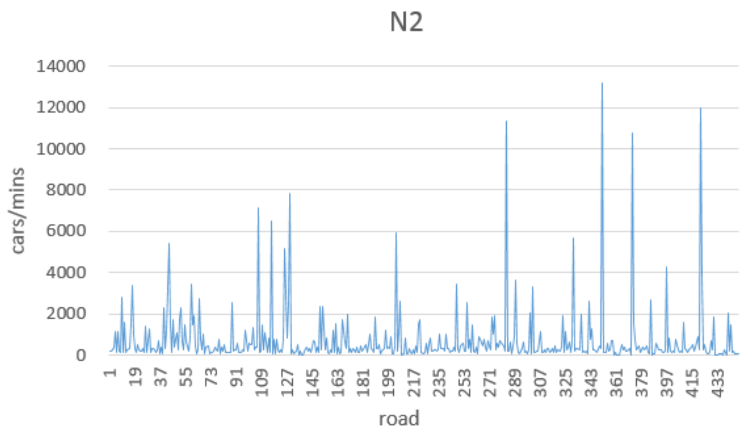

Figure 4: N_2

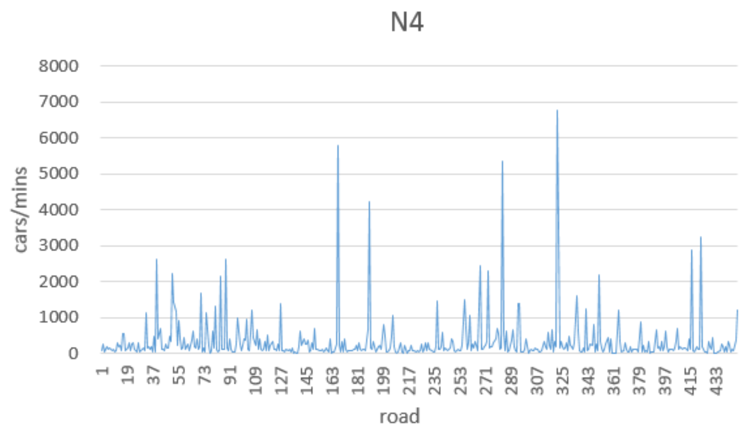

Figure 5: N_4

There are a series of qualitative indicators describing the degree of congestion, but we try to construct a quantitative index to describe the degree of congestion.

We use the density formula in physics:

$$Density = \frac{mass}{volume} \tag{9}$$

And we also use a similar method to define the highway densityand give a formula to calculate it.

$$\rho = N * \frac{S_{car}}{S_r} \tag{10}$$

S_r is obtained by the following formula:

$$S_r = (endMilepost - startMilepost) * 3.6576 * NumberofLanes \tag{11}$$

Since 12 feet = 3.6576m, we are going to plug N_1, N_2, N_3, N_4 into above equation, then we can get ρ_1(Density of road in peak hours in maximum velocity). ρ_2(Density of road in peak hours in adaptation velocity). ρ_3(Density of road in non-peak hours in maximum velocity). ρ_4(Density of road in non-peak hours in adaptation velocity).

The chart is as follows:

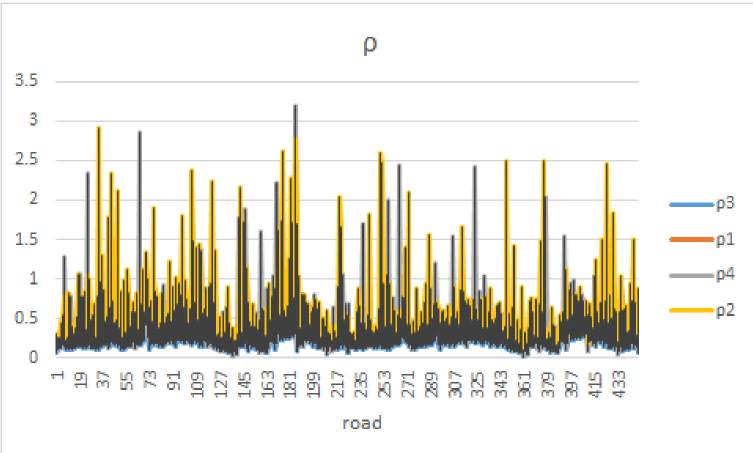

Figure 6: ρ

3.3 Obtaining Weight of Evaluation Index via PCA

We consider the above four cases, and get the corresponding road density ρ_1, ρ_2, ρ_3, ρ_4. But that doesn't make it easier for us to evaluate the condition of the road. In order to obtain a simple and uniform index, we consider a number of methods to extract eigenvalues and finally apply PCA (Principal components analysis). PCA is a method to get eigenvectors and eigenvalues by calculate the empirical mean and deviations from the mean and find the covariance matrix.The result is 1 factor will be retained by the MINEIGEN criterion. The results are as follows: Then we can get a new comprehensive evaluation index, called HDI (Highway Density Index), the following is the formula:

$$HDI = 0.41928 * \rho_1 + 0.17629 * \rho_2 + 0.41928 * \rho_3 + 0.23441 * \rho_4 \tag{12}$$

The SAS System

The FACTOR Procedure
Initial Factor Method: Principal Components

Scoring Coefficients Estimated by Regression

Squared Multiple Correlations of the Variables with Each Factor
Factor1
1.0000000

Standardized Scoring Coefficients	
	Factor1
VAR1	0.41928
VAR2	0.41928
VAR3	0.23441
VAR4	0.17629

Figure 7: The Report of PCA form the SAS System

there is HDI = 0.6244 when self-driving comes to 10%. So far we have constructed a comprehensive index to evaluate the condition of the highway. Next, HDI in 10% will be as the baseline to analyze the situation when 50% and 90% .

4 Determining the Optimal Self-driving to Vehicle Ratio

Then we select appropriate points to find the each point of HDI and fitting the HDI regression function, and generate the following image:

The regression function is Logit Model, the function expression is:

$$HDI = -0.092ln(x) + 0.4313 \tag{13}$$

$$R^2(GoodnessofFit) = 0.8572 \tag{14}$$

We respectivelysimulated 10% .50% .90% and their 0.5% varianceand substituted the regression function to get the following data and its distribution:

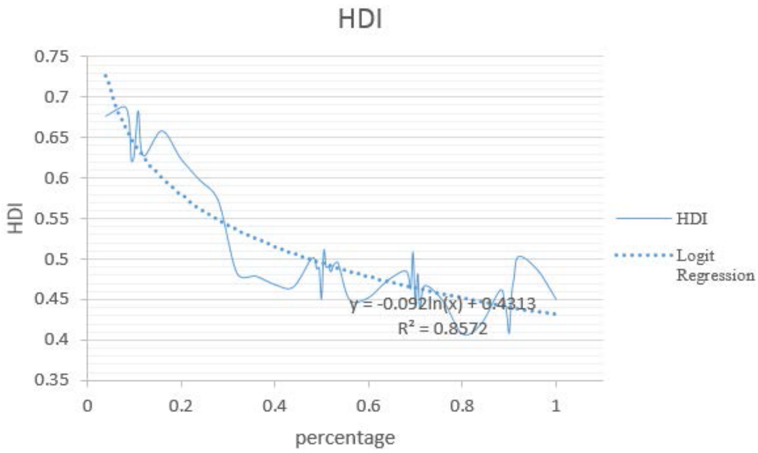

Figure 8: HDI

percentage	Simulation	Function
0.095	0.6206	0.6479
0.1	0.6244	0.6431
0.105	0.6575	0.6386
0.495	0.4891	0.4960
0.5	0.4506	0.4951
0.505	0.5104	0.4942
0.525	0.4915	0.4906
0.7	0.4440	0.4641
0.895	0.4310	0.4415
0.9	0.4083	0.4410
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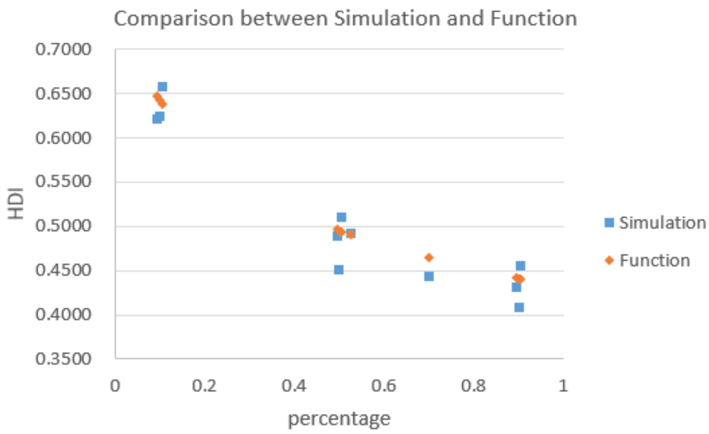

Figure 9: Comparison between Simulation and Function

It is obvious that the results obtained by function regression have better stability, as for small change of reaction is less obvious.So we can draw a conclusion that the index of HDI is stable and effective. Although the DoC gets the maximum growth rate of 42.5% which is not match the function , but 70% of the stability point does reflect the changes in the HDI, this is due to self-driving has smaller d than non-self-driving. (Because it is not affected by physical and psychological factors).

5 Analysis of Relation between Non-self-driving and Self-driving

Next we discuss the relationship between self-driving and non-self-driving, because self-driving cars have a large global information synchronization system and have less self-will, which can get more global information. Here, we modify the PSO parameter from

$$\begin{aligned} w &= 0.5 \\ c_1 &= 2.05 \\ c_2 &= 2.05 \\ p &= 0.75 \\ n &= 60 \end{aligned}$$

to

$$\begin{aligned} w &= 0.3 \\ c_1 &= 2.67 \\ c_2 &= 3.08 \\ p &= 0.95 \\ n &= 60 \end{aligned}$$

Re-simulation and substituted into the above percentage, the results are as follows:

percentage	Non-self-driving car	Self-driving car
0.095	0.6206	0.6137
0.1	0.6244	0.6716
0.105	0.6575	0.6909
0.495	0.4891	0.4620
0.5	0.4506	0.4854
0.505	0.5104	0.4750
0.525	0.4915	0.4635
0.7	0.4440	0.4294
0.895	0.4310	0.4151
0.9	0.4083	0.4221
0.905	0.4553	0.4646

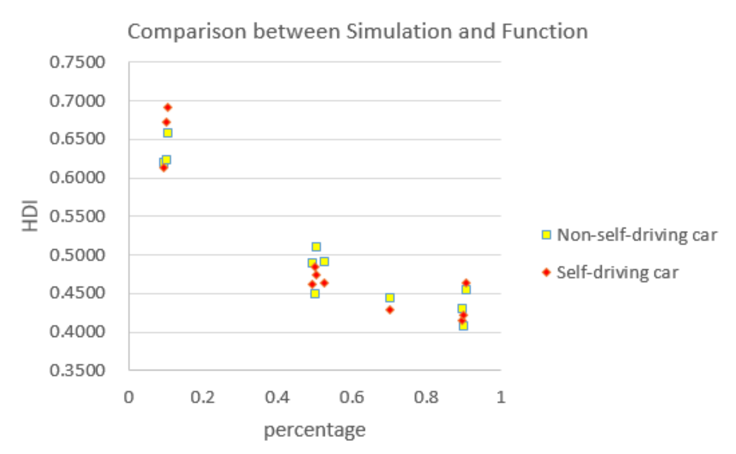

Figure 10: Comparison between Simulation and Function

From the figure, self-driving cars in the low proportion of the positive impact of the time as the impact of non-unmanned vehicles. The positive impact of self-driving cars in the low proportion is less than non-self-driving cars. With the increase of proportion, self-driving cars brings more positive impact than non-self-driving cars. When the percentage in a low level, the self-driving cars with small inertia weight is difficult to get more external information, so the adaptation speed has great disparity to v_{max}. We can get more information under the premise of the larger proportion.Self-driving cars can convergence quickly when they are good at external information learning.

6 Conclusions and Discussion

This paper attempt to construct two index, called DoC and HDI, evaluating the effect of the increase proportion of self-driving cars on traffic conditions.
We get these conclusion:

- 1 $HDI(10\%) = 0.6431$, $HDI(50\%) = 0.4951$, $HDI(90\%) = 0.441$(When $HDI = 0$, means completely smooth, $HDI = 1$ means completely blocked),the result is stable.
- 2 When the percentage of self-driving cars is above 28%, the road becomes smooth and tends to be stable at 70%.
- 3 Because self-driving-cars and non-self-driving-cars are difficult to communicate,so construct special lanes for self-driving-cars is meaningful.
- 4 In summary,we suggest that the government can reduce traffic pressure by:
  - a Constructing special lanes for self-driving-cars.
  - b Increasing percentage of self-driving cars through allowance the self-driving-car buyers.
  - c .Constructing a message switching system to assistant them learn about each other's information.

To simplify the problem,we use a simple model,called Logistic Model to construct DoC-Maybe we could try more complex models.Using PSO to train BPNN(Back Propagation Neural Networks) is an accurate method to simulate the change of speed.

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Appendices

Appendix A Figure of Road Area

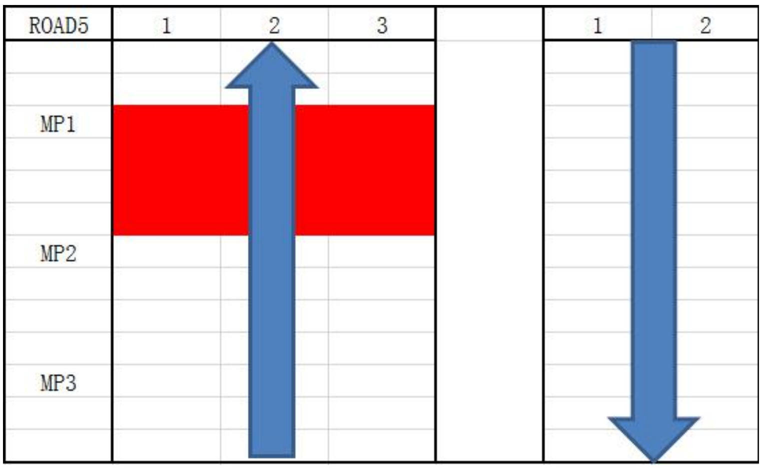

Figure 11: Road area