Reliability Engineering and System Safety 207 (2021) 107361

Contents lists available at ScienceDirect

Reliability Engineering and System Safe

journal homepage: www.elsevier.com/locate/ress

Remaining useful life prediction for multi-phase deteriorating process based on Wiener process

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ARTICLE INFO

Keywords:
Bayesian approach
Expectation maximization (EM) algorithm
Multi-phase model
Modified information criterion (MIC)
Remaining useful life (RUL)

ABSTRACT

Owing to the environmental stress and internal materials, the degradation signals show multiple phases characteristics, which have frequently been encountered in practice. In this paper, a multi-phase degradation model with jumps based on Wiener process is formulated to describe the multi-phase degradation pattern. The modified information criterion is adopted to determine the change-point number, and a simple yet effective algorithm is proposed for obtaining the change-point locations, which are critical for remaining useful life prediction. In the proposed model, to take into account the unit heterogeneity, all model parameters are assumed to be random variables. A Bayesian approach is used for integrating historical data and real-time data, which involves two stages, the off-line stage and on-line stage. Meanwhile, by treating the drift parameter and the diffusion parameter of each phase as latent parameters, the corresponding hyper-parameters are estimated based on the expectation maximization (EM) algorithm. The model parameters are updated under Bayesian rule at the on-line stage. Then, considering the multiple random change points and the corresponding jumps, the expressions of remaining useful life are derived under the concept of first passage time. Finally, a numerical simulation and a practical case study are provided for demonstrating the effectiveness.

1. Introduction

Prognostic and health management (PHM) is critical for preventing unexpected accidents, improving equipment safety, decreasing operating risks, and minimizing costs [1]. Lifetime prediction is the key part of PHM, and accurate prognostic is the foundation of maintenance support activities, such as condition-based maintenance and spare parts management. With the fast development of signal processing and feature extraction technologies, condition monitoring (CM) data collected by dedicated sensors can provide the information about the system's state of health [2]. The condition monitoring data, also known as degradation data, has been widely used to evaluate the future reliability of a product and predict remaining useful life (RUL) of in-service systems [3]. Degradation data-based methods attain the required degradation data based on the actual situation, which overcome the problems caused by the lack of failure data.

In the past few decades, data-driven methods have been widely investigated. In the existing literature, two kinds of most popular data-driven methods are general path-based methods [2,4,5] and stochastic process-based approaches [6–8]. The basic idea of general path-based approaches is using parametric regression to capture how the degradation signal evolves over time. In the general path-based approaches, the inherent degradation path is deterministic once the regression parameters are known. However, due to the random influence from internal and external environments, the degradation process is uncertain over time. The stochastic nature of the stochastic processes is effective in dealing with the unexplained randomness of the degradation over time, as discussed in [3,9–11]. Compared with general path-based methods, the stochastic process-based methods could effectively capture the uncertainty of degradation process that is caused by failure mechanism and operating environment. Thus, the stochastic process-based approaches have been widely investigated and applied to describe the degradation processes of systems [12]. As to the stochastic process-based approaches, the lifetime or RUL is usually defined as the first passage time (FPT) that the degradation value hits the preset threshold [13]. Nowadays, the primary stochastic process models include Gamma process [13,14], inverse Gaussian process [7,15], and Wiener process [16,17]. There are all the independent increments for the mentioned three models. However, Gamma process and inverse Gaussian process are only suitable for modeling the monotonous degradation process. Except for the monotonous degradation process, Wiener process based approaches is also suitable for the non-monotonous degradation process, which is caused by the environmental stress, internal materials, minor repair, or self-healing [18]. Especially, Wiener process has an independent gaussian increment so that it can effectively

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https://doi.org/10.1016/j.ress.2020.107361
Received 5 April 2020; Received in revised form 16 October 2020; Accepted 16 October 2020
Available online 27 November 2020

Notation

$X(t)$	Degradation observation at time t
k	Change-point number
I	Number of units in the historical data
τ_j	jth change time
τ_j^-	Left limit of τ_j
$x_{\tau_j^-}$	Observation value at time τ_j^-
$g_{\tau_j^-}(x_{\tau_j^-}\|x_{\tau_{j-1}})$	Probability density function of $x_{\tau_j^-}$ conditioning on the $x_{\tau_{j-1}}$
c_j	Location of jth change point
μ_j	Drift coefficient of jth phase
$\sigma_j^2 = 1/\omega_j$	Diffusion coefficient of jth phase
γ_j	Abrupt jump of the jth change points
n_j	Number of observation in the jth phase
L_m	Remaining useful life at time t_m
$\lambda_{1j}, \lambda_{2j}$	Hyper-parameter of jth change time τ_j
ϕ_{1j}, ϕ_{2j}	Hyper-parameter of jth jump γ_j
α_j, β_j	Hyper-parameter of diffusion coefficient ω_j
ν_j, φ_j	Hyper-parameter of drift coefficient μ_j

characterize the uncertainty of degradation trajectory and flexibly solve the mathematical expression of FPT. Meanwhile, it is convenient for parameter estimation and analytical solution of RUL's distribution.

The Wiener process is widely concerned by scholars and a series of effective researches have been carried out. Tsui et al. provided a concise review of data-driven lifetime prognostic approaches in the PHM framework, where Wiener process is included as a key part of the independent increment process-based models [19]. Zhang et al. systematically reviewed the methods of degradation data analysis and RUL estimation based on Wiener process [2].

From a practical point of view, due to the change of internal mechanism, environmental stress as well as operating state, some degradation processes consist of different phases, and each phase is controlled by different mechanism, such as laser degradation, plasma and fluorescent display aging test, organic light-emitting diode aging test, bearing degradation, as well as battery capacity change [10]. In all these cases, single-phase models are often not adequate to capture the degradation path evolution.

To deal with the issue, attempts have been made toward the degradation process with two-phase in recent years. Son et al. proposed the joint prognostic model considering a change point (JPM-C), and detected the change point through consistency correlation coefficient (CCC) to predict the RUL [20]. The hierarchical Bayesian method was applied for the two-phase degradation process based on the change-point regression [21]. Under the Bayesian framework, the change-point Wiener process (CPWP) model was proposed to analyze degradation data with two-phase, and the parameters were estimated by hierarchical Bayesian method [4]. Gao et al. proposed a change point degradation model with random jump and considered a random change point that was influenced by the accumulative effect of a shock process [22].

In practice, except for the above degradation processes with two-phase, there are multiple phases for many degradation processes, such as the Li-ion battery [10], high-performance capacitor, bearing as well as imperfect maintenance activity. Many papers proposed the degradation models subject to multiple change points. Feng et al. established a multi-phase Wiener process model for the high-performance capacitor, and utilized maximum likelihood method for parameter estimation [23]. In their work, both change-point locations and the change-point number are assumed to be deterministic for all units. Hu et al. proposed a new degradation model taking the influence of imperfect maintenance activities on both the degradation level and the degradation rate into account [24]. When the number of the imperfect maintenance reaches the specified n, the equipment will no longer be maintained, which is similar to the case that the change-point number is fixed as n in [24]. However, this is often not true in many applications. Indeed, the change-point number is not a fixed value at times, and the change-point locations often vary from unit to unit, showing significant heterogeneity. Furthermore, the change-point number should be determined before the change-point locations are detected. Differently, the multi-phase degradation model with abrupt jumps was utilized to describe this kind of degradation trajectories, and the degradation state at the change point was treated as a random variable [25,26]. In their work, there is no interpretable way to determine the change-point number and the diffusion coefficients are not treated as the random variables. Subject to multiple change points, a degradation model using Wiener process was proposed for the multi-phase degradation signals and the change point was analyzed from the discrete perspective [10]. The leave-one-out cross-validation approach was applied for selecting the change-point number in [10], which would have an influence on real-time forecasting. Meanwhile, the change-point locations are all obtained by the enumeration method in [10,26]. However, as the number of degradation data and change point increases, the enumeration method is time-consuming, which is not an effective method.

It is noted that the change points, degradation rates and diffusion parameters exist difference for the devices of the same batch [10,25,26]. Furthermore, random jumps exist at the change points for some degradation processes [22,26] so that the degradation state at the change point is a random variable until the new change point appears. Many degradation models subject to multiple change points have been proposed and the mathematical properties are analyzed. However, to the best of our knowledge, very limited work is available to solve all of the above issues in the existing literature.

Motivated by these practical issues, in this paper, a multiple change-point degradation model based on Wiener process is proposed for characterizing the multi-phase degradation trajectory. The major contribution of this paper lies in the following aspects:

(1) The modified information criterion is adopted to determine the change-point number. Meanwhile, a simple yet effective algorithm is proposed for obtaining the change-point locations, which greatly simplifies the computation and improves the operation speed comparing with the enumeration method.

(2) Taking the unit heterogeneity into account, the drift coefficients and diffusion coefficients are all defined as random variables to reflect the unit-to-unit variability. For facilitating practical usage, both off-line method and on-line method are utilized for parameter identification. Specifically, the EM algorithm is used for off-line model parameters estimation, and the Bayesian rule is used for model online updating.

(3) Considering the random change points as well as the uncertainty of the degradation state at the change point, the expressions of RUL is derived under the concept of FPT. Finally, a numerical simulation and a practical case about degradation data of NASA Lithium-ion battery are used to verify the effectiveness and practicability of the proposed model.

The remainder of this paper is organized as follows. In Section 2, a multi-phase degradation model based on Wiener process is established. Section 3 illustrates the modified information criterion for change-point detection and the EM algorithm to estimate the model parameters. Model parameters are updated based on Bayesian rule, and the corresponding expressions of RUL are obtained in Section 4. Section 5 demonstrates the effectiveness and accuracy of the proposed method through a numerical simulation and a real case study. The conclusion and discussion are given in Section 6.

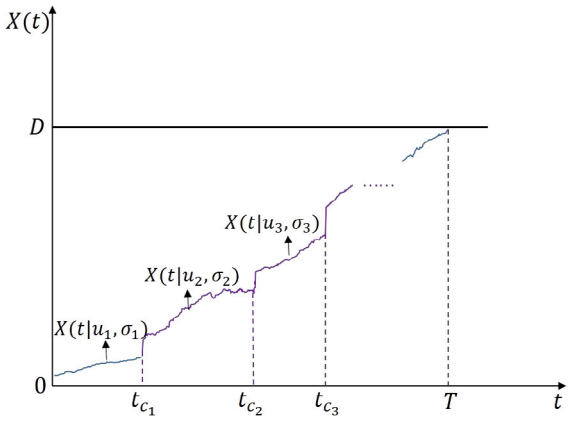

Fig. 1. Degradation trajectory of the multi-phase degradation system.

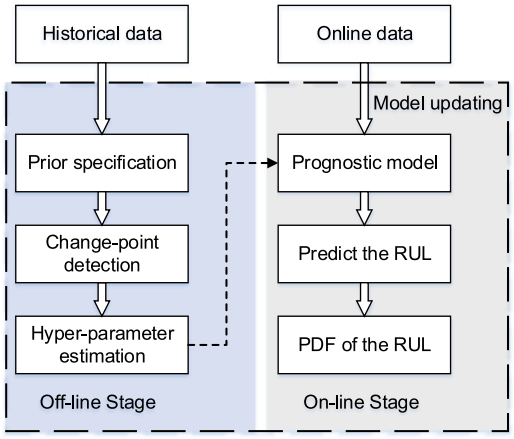

Fig. 2. Illustration of the proposed prognostic framework.

2. Multi-phase degradation model

Single-phase models are often not adequate to capture the degradation trajectory with two or even more distinct phases, which would seriously affect the accuracy of RUL prediction. In this paper, a multi-phase degradation model based on Wiener process is proposed to characterize the degradation trajectory with multi-phase. To take into account the unit heterogeneity, all model parameters are assumed to be random variables, including the change-point locations, the random jumps at the change points, the drift parameter and diffusion parameter of each phase.

Suppose that the unit is inspected at time $t_0, t_1, \ldots, t_n$ with degradation observations $X_0 = X(t_0), X_1 = X(t_1), \cdots, X_n = X(t_n)$, where n is the number of degradation observations. Suppose there are k change points with index locations $c_1, c_2, \ldots, c_k$ $(0 = c_0 < c_1 < c_2 < \cdots < c_k < c_{k+1} = n)$, the degradation trajectory is divided into $k+1$ phases and the change point is assumed to be exactly on the inspection epochs for simplicity as shown in Fig. 1. The multi-phase degradation process based on Wiener process can be expressed piecewise as [10]

$$X(t) = \begin{cases} if \quad t_0 < t \le t_{c_1} = \tau_1, \\ \quad X(t_0) + \mu_1 (t - t_0) + \sigma_1 B(t - t_0) \\ if \quad \tau_1 < t \le t_{c_2} = \tau_2, \\ \quad X(\tau_1) + \mu_2 (t - \tau_1) + \sigma_2 B(t - \tau_1) \\ \qquad \vdots \\ if \quad \tau_k < t \le t_n = \tau_{k+1}, \\ \quad X(\tau_k) + \mu_{k+1} (t - \tau_k) + \sigma_{k+1} B(t - \tau_k) \end{cases} \tag{1}$$

where $t_{c_j} = \tau_j$ is the jth change time and $t_{c_j}^- = \tau_j^-$ denotes its left limit, $X(\tau_j) = X(\tau_j^-) + \gamma_j$ is the initial degradation observation of $(j+1)$-th phase and γ_j is the abrupt jump of the jth change point, $j = 0, 1, 2, \ldots, k$. $\boldsymbol{\mu} = (\mu_1, \mu_2, \ldots, \mu_{k+1})^T$ and $\boldsymbol{\sigma^2} = (\sigma_1^2, \sigma_2^2, \ldots, \sigma_{k+1}^2)^T$ represent the drift coefficient and diffusion coefficient at each phase. $\boldsymbol{c} = (c_1, c_2, \ldots, c_k)^T$ is the change-point locations, $\boldsymbol{\gamma} = (\gamma_1, \gamma_2, \ldots, \gamma_k)^T$ is the jumps at the change points, and $B(t)$ is a standard Brownian motion that captures stochastic dynamics of the degradation process. It is worth nothing that in the above model, the degradation trajectory is discontinuous at the change point. For simplicity, we assume that the above parameters are independent of phases, and all phases are independent [10].

To take into account the unit heterogeneity and real-time RUL prediction, Bayesian approach is utilized to integrate the current available data with historical data [10,25]. Under the Bayesian framework, the prognostic involves two stages, i.e., the off-line stage and the on-line stage. Historical data can provide the prior information, which play a critical role in the posterior inference of a new unit, especially when there are not sufficient observations. At the off-line stage, the prior distributions of the random model parameters are specified, and the corresponding hyper-parameters are estimated through the historical data. At the on-line stage, based on the prior information and the degradation data of the specific unit, the posterior distributions of drift rate and diffusion parameter can be updated for RUL prediction. Meanwhile, the location and the number of the latest change point occurred are updated. The overall prognostic framework is illustrated in Fig. 2.

3. Off-line method

Prior information plays a unique and important role in the Bayesian framework which effectively integrates domain knowledge or empirical knowledge with new CM data. When the number of operating unit's observations is limited, accurate prior information can greatly improve the accuracy of RUL prediction. The key step in the Bayesian method is to determine prior distributions the of parameters, i.e., hyper-parameters. In practice, prior distribution is determined by domain knowledge or expert knowledge, and can also be estimated by historical data. In this paper, the hyper-parameters are estimated from the historical degradation data of the defunct units.

3.1. Change-point detection

In the framework of the multi-phase Wiener process, the parameters about the change points are important for accurate degradation model and accurate RUL prediction, including the change-point number, the change-point locations, and the corresponding abrupt jumps at the change points. Given the change-point number k, the change-point locations and the corresponding jumps at change points are obtained by using two-stage empirical Bayesian method [10], which is an easy and efficient method. Although some biases may be introduced by the two-stage empirical Bayesian method, it is often negligible according to the comparison study [27]. At the first stage, the modified information criterion (MIC) is adopted to determine the change-point number and detect the change-point locations as well as the corresponding jumps at the change points [28,29]. Then, by treating the estimated value of the first stage as the observation value, the hyper-parameters about the change points are estimated through the maximum likelihood (ML) estimation at the second stage.

For simplicity, all phases are independent. As mentioned, the degradation trajectory is divided into $k+1$ phases by k change points and the abrupt jump appear at the change point. As discussed in [10,25], the change time τ_j and the corresponding abrupt jump γ_j are assumed to

be random, which follow the normal distribution, i.e., $\tau_j \sim N\left(\lambda_{1j}, \lambda_{2j}\right)$, $\gamma_j \sim N\left(\phi_{1j}, \phi_{2j}\right)$, $j = 1, \ldots, k$.

Considering the increments of $X(t)$ are Gaussian, stationary, and independent, define the increments of observations as $\Delta X_1 = X_1 - X_0, \Delta X_2 = X_2 - X_1, \ldots, \Delta X_n = X_n - X_{n-1}$, and time increments as $\Delta t_1 = t_1 - t_0, \Delta t_2 = t_2 - t_1, \ldots, \Delta t_n = t_n - t_{n-1}$. Then conditioning on $\boldsymbol{M} = \left(\boldsymbol{\mu}, \boldsymbol{\sigma}^2, \boldsymbol{c}\right)$., the increments $\boldsymbol{\Delta X} = (\Delta X_1, \Delta X_2, \ldots, \Delta X_n)^T$ follow independent normal distribution given by 观测值服从以下正态分布

$$f\left(\boldsymbol{\Delta X} \mid \boldsymbol{M}, k\right) = \prod_{j=1}^{k+1} \prod_{s=1}^{n_j} \left[\left(2\pi\sigma_j^2 \Delta t_{c_{j-1}+s}\right)^{-\frac{1}{2}} \times \exp\left(-\frac{\left(\Delta X_{c_{j-1}+s} - \mu_j \Delta t_{c_{j-1}+s}\right)^2}{2\sigma_j^2 \Delta t_{c_{j-1}+s}} \right) \right] \tag{2}$$

where n_j is the number of observations in the jth phase, $n_j = c_j - c_{j-1}$, $j = 1, 2, \ldots, k+1$. The log-likelihood function can thus be expressed by 对数似然函数表示为

$$l\left(\boldsymbol{M} \mid \boldsymbol{\Delta X}, k\right) = \sum_{j=1}^{k+1} \left[-\frac{n_j}{2} \ln\left(2\pi\sigma_j^2\right) - \frac{1}{2} \sum_{s=1}^{n_j} \ln \Delta t_{c_{j-1}+s} - \frac{1}{2\sigma_j^2} \sum_{s=1}^{n_j} \frac{\left(\Delta X_{c_{j-1}+s} - \mu_j \Delta t_{c_{j-1}+s}\right)^2}{\Delta t_{c_{j-1}+s}} \right] \tag{3}$$

Given the change-point locations, the drift parameter and diffusion parameter that maximize Eq. (3) can be obtained as 给定变化点的位置，使方程（3）最大化的漂移参数和扩散参数

$$\hat{\mu}_j = \frac{\sum_{s=1}^{n_j} \Delta X_{c_{j-1}+s}}{\sum_{s=1}^{n_j} \Delta t_{c_{j-1}+s}}, \widehat{\sigma_j^2} = \frac{1}{n_j} \sum_{s=1}^{n_j} \frac{\left(\Delta X_{c_{j-1}+s} - \hat{\mu}_j \Delta t_{c_{j-1}+s}\right)^2}{\Delta t_{c_{j-1}+s}} \tag{4}$$

Plug in Eq. (4) into Eq. (3), a likelihood function only $\boldsymbol{c}$ can be obtained

$$l\left(\boldsymbol{c}, \hat{\boldsymbol{\mu}}(\boldsymbol{c}), \widehat{\boldsymbol{\sigma}^2}(\boldsymbol{c}) \mid \boldsymbol{\Delta X}, k\right) = \sum_{j=1}^{k+1} \left[-\frac{n_j}{2} \ln\left(2\pi\widehat{\sigma_j^2}\right) - \frac{1}{2}\left(\sum_{s=1}^{n_j} \ln \Delta t_{c_{j-1}+s}\right) - \frac{n_j}{2} \right] \tag{5}$$

The optimal change-point locations can be efficiently obtained by **Algorithm 1**:

$$\hat{\boldsymbol{c}} = \arg\max_{\boldsymbol{c}} l\left(\boldsymbol{c}, \hat{\boldsymbol{\mu}}(\boldsymbol{c}), \widehat{\boldsymbol{\sigma}^2}(\boldsymbol{c}) \mid \boldsymbol{\Delta X}, k\right) \tag{6}$$

Algorithm 1 The change-point locations searching procedure

1: The initial value $c_1, c_2, \cdots, c_k$, where $1 < c_1 < c_2 \cdots < c_k < n$;
2: Define the sum of first two terms in Eq. (5) as W. Fixed c_2, adjusting c_1 to maximize W in the range of $1 < c_1 < c_2$ and denoting c_1 that maximizes W as c_1^*. This is equivalent to the case of one-change-point, and the total number of observations is c_2.
3: Similarly, considering the 2nd and 3rd terms in Eq. (5), c_2^* is obtained.
4: And so on, $c_1^*, c_2^*, \cdots, c_k^*$ are obtained which are new initial value. Through multiple iterations, the estimators of the change-point locations are obtained.

Given the change-point number k, the change-point locations $\boldsymbol{c}$ are quickly yet efficiently estimated through **Algorithm 1**, and the change-point locations of all the units can be obtained in turn. Meanwhile, corresponding jumps at the change points are estimated, i.e., $\hat{\boldsymbol{\gamma}} = (\Delta X_{c_1+1}, \Delta X_{c_2+1}, \ldots, \Delta X_{c_k+1})^T$.

It is important to select the change-point number of each unit first in the above change-point locations estimation. Although increasing the change-point number may improve the model fitting accuracy, it may not result in better prognostic accuracy and will introduce extra uncertainties in RUL prediction, such as uncertainty of future change-point locations [10]. According to the modified information criterion, the change-point number can be determined by **Algorithm 2** [28].

Algorithm 2 Determining the change-point number

1: For a fixed value k of the change-point number, estimate the unknown change-point locations that maximize Eq. (5), that is: $M(k) = l(\hat{c}, \hat{\boldsymbol{\mu}}(\hat{c}), \widehat{\boldsymbol{\sigma}^2}(\hat{c}) \mid \boldsymbol{\Delta X}, k)$.
2: Evaluate the following modified information criterion

$$MIC(k) = -2M(k) + \left[2(k+1) + C \sum_{j=1}^{k+1} \left(\frac{\hat{c}_j - \hat{c}_{j-1}}{n} - \frac{1}{2(k+1)} \right) \right] \log n$$

subject to $k \le K_U$ and a constant C, where K_U is the given upper bound for the change-point number.
3: If $T_{k-1,k} = MIC(k-1) - MIC(k) + \log n \ge \chi_\alpha^2(1)$, where χ^2 represents the Chi-square distribution, increase k to $k+1$ and repeat step 1–2 until $T_{k,k+1} < \chi_\alpha^2(1)$ for $k = 1, 2, \cdots, K_U$. Then, terminate the steps and estimate the change-point number as k.

Suppose there are I units in the historical data, the change-point locations and corresponding jumps estimated are expressed as $\hat{\boldsymbol{c}}^i, \hat{\boldsymbol{\gamma}}^i$ for unit i. Then, by treating the above estimated value $\hat{\boldsymbol{c}}, \hat{\boldsymbol{\gamma}}$ as the observations, the hyper-parameters of the change time and corresponding abrupt jump are estimated through the maximum likelihood estimation:

$$\begin{aligned} \lambda_{1j} &= \frac{1}{I} \sum_{i=1}^{I} \hat{\tau}_j^i, \lambda_{2j} = \frac{1}{I} \sum_{i=1}^{I} \left(\hat{\tau}_j^i - \lambda_{1j}\right)^2 \\ \phi_{1j} &= \frac{1}{I} \sum_{i=1}^{I} \hat{\gamma}_j^i, \phi_{2l} = \frac{1}{I} \sum_{i=1}^{I} \left(\hat{\gamma}_j^i - \phi_{1j}\right)^2 \end{aligned} \tag{7}$$

It is worth nothing that the change-point number of each unit may be random in a batch of units. Therefore, for the above equations, the units that have the same change-point number could be treated as the same category when the maximum likelihood estimators are calculated.

3.2. Prior specification

Suppose the drift coefficient and diffusion coefficient of each phase are random and independent of each other. $\mu_j^i, \left(\sigma_j^i\right)^2$ are regarded as the observations of the random variables μ_j, σ_j^2, respectively, where $i = 1, \ldots, I$, $j = 1, 2, \ldots, k+1$.

After determining the change-point number for each unit and obtaining the estimators of the change-point locations as well as the corresponding jumps, the joint prior distribution of the random variables μ_j, σ_j^2 can be formulated as 随机变量的联合先验分布为

$$\pi\left(\mu_j, \sigma_j^2\right) = \pi\left(\mu_j \mid \sigma_j^2\right) \pi\left(\sigma_j^2\right) \tag{8}$$

Considering the conjugate prior of parameters [30], σ_j^2 is assumed to be followed a inverse Gamma distribution, ω_j follows a Gamma distribution, where $\omega_j = 1/\sigma_j^2$, then $\left(\mu_j \mid \omega_j\right)$ follows a normal distribution, i.e., $\omega_j \sim \Gamma\left(\alpha_j, \beta_j\right)$, $\mu_j \mid \omega_j \sim N\left(\nu_j, {\omega_j}^{-1} \varphi_j\right)$.

Model parameters $\boldsymbol{\mu}^i = (\mu_1^i, \mu_2^i, \ldots, \mu_{k+1}^i)^T$, and $\boldsymbol{\omega}^i = (\omega_1^i, \omega_2^i, \ldots, \omega_{k+1}^i)^T$ are regarded as latent variables. In addition, let $\boldsymbol{Z}^i = (\boldsymbol{\mu}^{iT}, \boldsymbol{\omega}^{iT})^T$ and $\boldsymbol{\theta} = (\alpha_1, \ldots, \alpha_{k+1}, \beta_1, \cdots, \beta_{k+1}, \nu_1, \ldots, \nu_{k+1}, \varphi_1, \ldots, \varphi_{k+1})$, represent the latent variables of the unit i and the corresponding hyper-parameters, respectively.

3.3. Parameters estimation

The diffusion coefficient of each phase is assumed to be followed the inverse Gamma distribution, so that the analytical solution of likelihood function cannot be obtained. As such, it is difficult to estimate the parameters via the traditional maximum likelihood estimation (MLE). The EM algorithm converts a maximization problem involving a complicated likelihood into a sequence of problems with simpler likelihood functions, where at each step the updated parameter estimation could be obtained in a closed form, or a straightforward manner [4]. The EM algorithm provides an iterative approach for the hyper-parameter estimation, which is simple and stable. According to the EM algorithm, the complete log-likelihood function can be formulated as

完整的对数似然函数

$$\ln L(\boldsymbol{\theta}|\boldsymbol{\Delta X}, \boldsymbol{Z}) = \ln p(\boldsymbol{\Delta X}, \boldsymbol{Z}|\boldsymbol{\theta}) = \ln \prod_{i=1}^{I} p\left(\boldsymbol{\Delta X}^i|\boldsymbol{Z}^i, \boldsymbol{\theta}\right) p\left(\boldsymbol{Z}^i|\boldsymbol{\theta}\right) \tag{9}$$

Let $\boldsymbol{\theta}^{(p)}$ denote the estimators in the pth step, then we can attain the Q-function (see Appendix). For obtaining the $\hat{\boldsymbol{\theta}} = \arg\max_{\boldsymbol{\theta}} Q(\boldsymbol{\theta}|\boldsymbol{\theta}^{(p)})$, the next iteration $\boldsymbol{\theta}^{(p+1)}$ could be obtained by solving the equation as follows,

M步

$$\frac{\partial Q\left(\boldsymbol{\theta}\left|\boldsymbol{\theta}^{(p)}\right.\right)}{\partial \boldsymbol{\theta}} = 0 \tag{10}$$

$\alpha_j^{(p+1)}$ is solved by the following equation,

$$\begin{aligned}\psi\left(\alpha_j^{(p+1)}\right) - \ln \alpha_j^{(p+1)} &= \frac{1}{I}\sum_{i=1}^{I} E\left[\ln\left(\omega_j^i\right)|\boldsymbol{\Delta X}^i, \boldsymbol{\theta}^{(p)}\right] \\ &\quad + \ln I - \ln \sum_{i=1}^{I} E\left[\omega_j^i|\boldsymbol{\Delta X}^i, \boldsymbol{\theta}^{(p)}\right] \\ \beta_j^{(p+1)} &= \frac{I \times \alpha_j^{(p+1)}}{\sum_{i=1}^{I} E\left[\omega_j^i|\boldsymbol{\Delta X}^i, \boldsymbol{\theta}^{(p)}\right]} \\ \nu_j^{(p+1)} &= \frac{\sum_{i=1}^{I} E\left[\mu_j^i\left|\omega_j^i, \boldsymbol{\Delta X}^i, \boldsymbol{\theta}^{(p)}\right.\right] \times E\left[\omega_j^i|\boldsymbol{\Delta X}^i, \boldsymbol{\theta}^{(p)}\right]}{\sum_{i=1}^{I} E\left[\omega_j^i|\boldsymbol{\Delta X}^i, \boldsymbol{\theta}^{(p)}\right]} \\ \varphi_j^{(p+1)} &= \frac{1}{I}\sum_{i=1}^{I} E\left[\omega_j^i|\boldsymbol{\Delta X}^i, \boldsymbol{\theta}^{(p)}\right] \times \left(E\left[\left(\mu_j^i\right)^2\left|\omega_j^i, \boldsymbol{\Delta X}^i, \boldsymbol{\theta}^{(p)}\right.\right]\right. \\ &\quad \left. -2\nu_j^{(p+1)} E\left[\mu_j^i\left|\omega_j^i, \boldsymbol{\Delta X}^i, \boldsymbol{\theta}^{(p)}\right.\right] + \left(\nu_j^{(p+1)}\right)^2\right)\end{aligned} \tag{11}$$

where

$$\begin{aligned}E\left[\mu_j^i\left|\omega_j^i, \boldsymbol{\Delta X}^i, \boldsymbol{\theta}^{(p)}\right.\right] &= \frac{\nu_j^{(p)} + \varphi_j^{(p)}\sum_{s=1}^{n_j^i} \Delta X^i_{c_{j-1}+s}}{1 + \varphi_{jp}^{(p)}\sum_{s=1}^{n_j^i} \Delta t^i_{c_{j-1}+s}}, \\ E\left[\left(\mu_j^i\right)^2\left|\omega_j^i, \boldsymbol{\Delta X}^i, \boldsymbol{\theta}^{(p)}\right.\right] &= \left(\frac{\nu_j^{(p)} + \varphi_j^{(p)}\sum_{s=1}^{n_j^i} \Delta X^i_{c_{j-1}+s}}{\varphi_j^{(p)}\sum_{s=1}^{n_j^i} \Delta t^i_{c_{j-1}+s} + 1}\right)^2 \\ &\quad + \frac{\left(\omega_j^i\right)^{-1}\varphi_j^{(p)}}{\varphi_j^{(p)}\sum_{s=1}^{n_j^i} \Delta t^i_{c_{j-1}+s} + 1}, \\ E\left[\omega_j^i|\boldsymbol{\Delta X}^i, \boldsymbol{\theta}^{(p)}\right] &= \frac{\alpha_j^{i*}}{\beta_j^{i*}}, \quad E\left[\ln\left(\omega_j^i\right)|\boldsymbol{\Delta X}^i, \boldsymbol{\theta}^{(p)}\right] = \psi\left(\alpha_j{}^{i*}\right) - \ln\left(\beta_j{}^{i*}\right), \\ \beta_j{}^{i*} &= \beta_j^{(p)} + \frac{\left(\nu_j^{(p)}\right)^2}{2\varphi_j^{(p)}} + \sum_{s=1}^{n_j^i}\frac{\left(\Delta X^i_{c_{j-1}+s}\right)^2}{2\Delta t^i_{c_{j-1}+s}} - \frac{\left(\nu_j^{(p)} + \varphi_j^{(p)}\sum_{s=1}^{n_j^i} \Delta X^i_{c_{j-1}+s}\right)^2}{2\varphi_j^{(p)}\left(1 + \varphi_j^{(p)}\sum_{s=1}^{n_j^i} \Delta t^i_{c_{j-1}+s}\right)} \\ \alpha_j{}^{i*} &= \alpha_j^{(k)} + \frac{n_j^i}{2}, \quad \psi(x) = \frac{d\ln\Gamma(x)}{dx}\end{aligned} \tag{12}$$

$\Gamma(x)$ is the Gamma function.

4. Online updating and RUL distribution

As mentioned above, the prior information is obtained based on the historical data at the off-line stage, then the RUL prediction of a specific operating unit could be accomplished. It is necessary to update the posterior distributions of key parameters for the model accuracy, such as the latest change-point location, the corresponding jump at change point, the change-point number occurred, the drift parameter and diffusion parameter of the current phase. In this section, the key points are how to update the model, and then how to derive the distribution of the RUL.

4.1. Model updating

For a operating unit, if the current time is t_m, the degradation data is denoted as $\boldsymbol{X}_{0:m} = \{X_0, X_1, \ldots, X_m\}$ and time increments are denoted as $\Delta t_1 = t_1 - t_0, \Delta t_2 = t_2 - t_1, \ldots, \Delta t_m = t_m - t_{m-1}$. For simplicity, the interval between inspection epochs are assumed to be constant, i.e., $\Delta t_1 = \Delta t_2 = \cdots = \Delta t_m = \Delta t$.

It is worth mentioning that when the change point does not appear, i.e., $t_m < \tau_1$, only the parameters in the first phase need to be updated and the parameters of the other phase are obtained by the prior information. If the change point occurs, the corresponding parameters should be updated according to the change-point number occurred. For example, if only one change point occurs, only the parameters in the second phase need to be updated.

Due to the change-point number and the change-point locations of the new equipment are unknown, the change-point number, the change-point locations and corresponding abrupt jumps at the change points need to be determined by using the change-point detection method mentioned at the off-line stage.

Let $\alpha_{j0}, \beta_{j0}, \nu_{j0}, \varphi_{j0}$ denote the prior value α_j, β_j, ν_j, φ_j obtained at the off-line stage, respectively. As discussed before, when the change point does not appear, all observations $\mathbf{X}_{0:m} = \{X_0, X_1, \ldots, X_m\}$ could be utilized to update model. According to the Bayesian rule,

$$p\left(\mu_1, \omega_1|\boldsymbol{X}_{0:m}\right) \propto p\left(\boldsymbol{X}_{0:m}|\mu_1, \omega_1\right) p\left(\mu_1, \omega_1\right) \tag{13}$$

where $\omega_1 = 1/\sigma_1^2$ is the diffusion coefficient in the first phase, and

$$\begin{aligned}p\left(\boldsymbol{X}_{0:m}|\mu_1, \omega_1\right) &= \prod_{i=1}^{m}\frac{1}{\sqrt{2\pi\sigma_1^2\Delta t}}\exp\left(-\frac{\left(\Delta X_i - \mu_1\Delta t\right)^2}{2\sigma_1^2\Delta t}\right), \\ p\left(\mu_1, \omega_1\right) &= \frac{\left(\beta_{10}\right)^{\alpha_{10}}}{\Gamma\left(\alpha_{10}\right)}\left(\omega_1\right)^{\alpha_{10}-1}\exp\left(-\beta_{10}\omega_1\right) \\ &\quad \times \frac{1}{\sqrt{2\pi\sigma_1^2\varphi_{10}}}\exp\left(-\frac{\left(\mu_1 - \nu_{10}\right)^2}{2\sigma_1^2\varphi_{10}}\right)\end{aligned} \tag{14}$$

Considering the conjugate prior of parameters, the posterior distribution could be attained,

$$\begin{aligned}p\left(\mu_1, \omega_1|\boldsymbol{X}_{0:m}\right) &\propto \left(\omega_1\right)^{\alpha_1-1}\exp\left(-\omega_1\beta_1\right) \\ &\quad \times \frac{1}{\sqrt{2\pi\left(\omega_1\right)^{-1}\varphi_1}}\exp\left(-\frac{\left(\mu_1 - \nu_1\right)^2}{2\left(\omega_1\right)^{-1}\varphi_1}\right) \\ &\propto \Gamma\left(\alpha_1, \beta_1\right) N\left(\nu_1, \left(\omega_1\right)^{-1}\varphi_1\right)\end{aligned} \tag{15}$$

with

$$\begin{aligned}\alpha_1 &= \alpha_{10} + \frac{m}{2}, \quad \beta_1 = \beta_{10} + \frac{\left(\nu_{10}\right)^2}{2\varphi_{10}} \\ &\quad + \sum_{i=1}^{m}\frac{\left(\Delta X_i\right)^2}{2\Delta t} - \frac{\left(\nu_{10} + \varphi_{10}\sum_{i=1}^{m}\Delta X_i\right)^2}{2\varphi_{10}\left(1 + \varphi_{10}\sum_{i=1}^{m}\Delta t\right)}, \\ \nu_1 &= \frac{\nu_{10} + \varphi_{10}\sum_{i=1}^{m}\Delta X_i}{1 + \varphi_{10}\sum_{i=1}^{m}\Delta t}, \quad \varphi_1 = \frac{\varphi_{10}}{1 + \varphi_{10}\sum_{i=1}^{m}\Delta t}\end{aligned} \tag{16}$$

Similarly, when the change-point number occurred is $j-1$, the parameters in the jth phase need to be updated,

$$p\left(\mu_j,\omega_j\left|\boldsymbol{X}_{c_{j-1}:m}\right.\right)\propto\left(\omega_j\right)^{\alpha_j-1}\exp\left(-\omega_j\beta_j\right)\times\frac{1}{\sqrt{2\pi\left(\omega_j\right)^{-1}\varphi_j}}\exp\left(-\frac{\left(\mu_j-\nu_j\right)^2}{2\left(\omega_j\right)^{-1}\gamma_{jp}}\right)\propto\Gamma\left(\alpha_j,\beta_j\right)N\left(\nu_j,\left(\omega_j\right)^{-1}\varphi_j\right)\tag{17}$$

where

$$\alpha_j=\alpha_{j0}+\frac{m-c_{j-1}}{2},$$

$$\beta_j=\beta_{j0}+\frac{\left(\nu_{j0}\right)^2}{2\varphi_{j0}}+\sum_{i=c_{j-1}}^{m}\frac{\left(\Delta X_i\right)^2}{2\Delta t}-\frac{\left(\nu_{j0}+\varphi_{j0}\sum_{i=c_{j-1}}^{m}\Delta X_i\right)^2}{2\varphi_{j0}\left(1+\varphi_{j0}\sum_{i=c_{j-1}}^{m}\Delta t\right)},\tag{18}$$

$$\nu_j=\frac{\nu_{j0}+\varphi_{j0}\sum_{i=c_{j-1}}^{m}\Delta X_i}{1+\varphi_{j0}\sum_{i=c_{j-1}}^{m}\Delta t},\quad\varphi_j=\frac{\varphi_{j0}}{1+\varphi_{j0}\sum_{i=c_{j-1}}^{m}\Delta t}$$

4.2. RUL distribution

After updating the model, the RUL of the operating unit could be predict. Under the concept of the first passage time (FPT) [31], the lifetime is a random variable and it is usually defined as $T=inf\{t:X(t)\geq D|X(0)<D\}$, where D represents the failure threshold, which is determined by practical engineering condition and usually defined as a constant value. In the traditional Wiener process, if the observation is $X\left(t_m\right)$ at time t_m, for $t\geq t_m$,

$$X(t)=X\left(t_m\right)+\mu\left(t-t_m\right)+\sigma B\left(t-t_m\right)\tag{19}$$

Remaining useful lifetime (RUL) under the concept of FPT is usually defined as follows,

$$L_m=\inf\left\{l:X\left(t_m+l\right)\geq D\left|X\left(t_m\right)<D\right.\right\}\tag{20}$$

If t is lifetime of the degradation process, $t-t_m$ is the RUL at time t_m. Let $l_m=t-t_m, X\left(l_m+t_m\right)-X\left(t_m\right)=\mu\left(l_m\right)+\sigma B\left(l_m\right)$.

First, the case is discussed that the change point does not appear, i.e., $0<t_m<\tau_1$, where the uncertainty of future change points is largest and it is most complicated.

As to the multi-phase model, the RUL is divided into several parts according to the change-point number. We firstly consider the simplest case, i.e., RUL belong to $0<l<\tau_1-t_m$, the RUL distribution is just determined by the first phase and holds the following form [31],

$$f_{L_m}\left(l\left|\boldsymbol{X}_{0:m}\right.\right)=\int_0^{\infty}\int_{-\infty}^{\infty}\left[\frac{D-X_m}{\sqrt{2\pi{\sigma_1}^2l^3}}\exp\left(-\frac{\left(D-X_m-\mu_1l\right)^2}{2{\sigma_1}^2l}\right)\frac{\left(\beta_1\right)^{\alpha_1}}{\Gamma\left(\alpha_1\right)}\left(\omega_1\right)^{\alpha_1-1}\times\exp\left(-\beta_1\omega_1\right)\frac{1}{\sqrt{2\pi\sigma_1^2\varphi_1}}\times\exp\left(-\frac{\left(\mu_1-\nu_1\right)^2}{2\sigma_1^2\varphi_1}\right)\right]d\mu_1d\omega_1\tag{21}$$

The solution of the above equation is intractable. In the paper, the point estimations of drift coefficient and diffusion coefficient are attained, then they are substituted into the equation of the RUL distribution to predict RUL.

The posterior edge density functions of μ_1,ω_1 in the first phase are obtained as follows,

$$p\left(\mu_1\left|\boldsymbol{X}_{0:m}\right.\right)=\int_0^{+\infty}p\left(\mu_1,\omega_1\left|\boldsymbol{X}_{0:m}\right.\right)d\omega_1\tag{22}$$

$$=\int_0^{+\infty}\left[\frac{\left(\beta_1\right)^{\alpha_1}}{\Gamma\left(\alpha_1\right)}\left(\omega_1\right)^{\alpha_1-1}\exp\left(-\omega_1\beta_1\right)\sqrt{\frac{\omega_1}{2\pi\varphi_1}}\exp\left(-\frac{\left(\mu_1-\nu_1\right)^2}{2\left(\omega_1\right)^{-1}\varphi_1}\right)\right]d\omega_1$$

$$p\left(\omega_1\left|\boldsymbol{X}_{0:m}\right.\right)=\int_0^{+\infty}p\left(\mu_1,\omega_1\left|\boldsymbol{X}_{0:m}\right.\right)d\mu_1\tag{23}$$

In the case of the square loss function, $\hat{\mu}_1,\hat{\omega}_1$ represent the point estimation of μ_1,ω_1, respectively.

$$\hat{\mu}_1=E\left[\mu_1\left|\boldsymbol{X}_{0:m}\right.\right]=\int_0^{+\infty}\mu_1p\left(\mu_1\left|\boldsymbol{X}_{0:m}\right.\right)d\mu_1=\int_0^{+\infty}\mu_1\int_0^{+\infty}p\left(\mu_1,\omega_1\left|\boldsymbol{X}_{0:m}\right.\right)d\omega_1d\mu_1\tag{24}$$

$$\hat{\omega}_1=E\left[\omega_1\left|\boldsymbol{X}_{0:m}\right.\right]=\int_0^{+\infty}\omega_1p\left(\omega_1\left|\boldsymbol{X}_{0:m}\right.\right)d\mu_1=\int_0^{+\infty}\omega_1\int_0^{+\infty}p\left(\mu_1,\omega_1\left|\boldsymbol{X}_{0:m}\right.\right)d\omega_1d\mu_1\tag{25}$$

Plug in Eqs. (24), (25) into Eq. (21), the above RUL distribution can be rewritten for $0<l<\tau_1-t_m$,

$$f_{L_m}(l\left|\boldsymbol{X}_{0:m}\right.)=\frac{\sqrt{\hat{\omega}_1}\left(D-X_m\right)}{\sqrt{2\pi l^3}}\exp\left(-\frac{\hat{\omega}_1(D-X_m-\hat{\mu}_1l)^2}{2l}\right)\tag{26}$$

Because the change point c_j does not appear, the value of $x_{\tau_j^-}$ should be a random variable rather than a fixed value, where τ_j^- denotes the left limit of τ_j. Besides, it is determined by the degradation model at the previous phase and the previous change point. To obtain the RUL under the concept of FPT, the expression of $x_{\tau_j^-}$ should be derived under the concept of FPT, i.e., $\tau_j<l+t_m<\tau_{j+1}$. It is defined that $g_{\tau_j^-}(x_{\tau_j^-}|x_{\tau_{j-1}})$ denotes the PDF of $x_{\tau_j^-}$ conditioning on the $x_{\tau_{j-1}}$, and the following result could be obtained,

$$g_{\tau_j^-}\left(x_{\tau_j^-}|x_{\tau_{j-1}}\right)=\int_{-\infty}^{\infty}\int_0^{\infty}\int_{-\infty}^{\infty}g_{\tau_j^-}\left(x_{\tau_j^-}|x_{\tau_{j-1}},\mu_j,\omega_j,\Delta\tau_j\right)\times p\left(\mu_j,\omega_j\right)p\left(\Delta\tau_j\right)d\mu_jd\omega_jd\Delta\tau_j\tag{27}$$

where $\Delta\tau_j=\tau_j-\tau_{j-1},\omega_j=1/\sigma_j^2,j=1,2,\ldots,k$,

$$g_{\tau_j^-}(x_{\tau_j^-}|x_{\tau_{j-1}},\mu_j,\omega_j,\Delta\tau_j)=\sqrt{\frac{\omega_j}{2\pi\Delta\tau_j}}\exp\left[-\frac{\left(x_{\tau_j^-}-x_{\tau_{j-1}}-\mu_j\Delta\tau_j\right)^2}{2\sigma_j^2\Delta\tau_j}\right]-\sqrt{\frac{\omega_j}{2\pi\Delta\tau_j}}\times\exp\left(\frac{2\mu_j\left(D-x_{\tau_{j-1}}\right)}{\sigma_j^2}\right)\exp\left[-\frac{\left(x_{\tau_j^-}+x_{\tau_{j-1}}-2D-\mu_j\Delta\tau_j\right)^2}{2\sigma_j^2\Delta\tau_j}\right]$$

$$p\left(\mu_j,\omega_j\right)=\frac{\left(\beta_j\right)^{\alpha_j}}{\Gamma\left(\alpha_j\right)}\left(\omega_j\right)^{\alpha_j-1}\exp\left(-\beta_j\omega_j\right)\frac{1}{\sqrt{2\pi\sigma_j^2\varphi_j}}\exp\left[-\frac{\left(\mu_j-\nu_j\right)^2}{2\sigma_j^2\varphi_j}\right]\tag{28}$$

$$p(\Delta\tau_j)=\frac{1}{\sqrt{2\pi(\lambda_{2j}-\lambda_{2,j-1})}}\exp\left[-\frac{\left(\Delta\tau_j-(\lambda_{1j}-\lambda_{1,j-1})\right)^2}{2(\lambda_{2j}-\lambda_{2,j-1})}\right]$$

$\lambda_{1j},\lambda_{2j}$ are the hyper-parameters of the change time τ_j, which are estimated at the off-line stage.

Then, according to $x_{\tau_j}=x_{\tau_j^-}+\gamma_j$, we can further obtain the PDF of x_{τ_j} conditioning on the $x_{\tau_{j-1}}$ in terms of the convolution formulation,

$$g_{\tau_j}\left(x_{\tau_j}|x_{\tau_{j-1}}\right)=\int_{-\infty}^{D}g_{\tau_j^-}\left(x_{\tau_j^-}|x_{\tau_{j-1}}\right)g_{\gamma_j}\left(x_{\tau_j}-x_{\tau_j^-}\right)dx_{\tau_j^-}\tag{29}$$

In this way, $g_{\tau_j^-}(x_{\tau_j^-})$ could be obtained with a multi-integral form as follows,

$$g_{\tau_j^-}(x_{\tau_j^-})=\int_{-\infty}^{D}\ldots\int_{-\infty}^{D}\int_{-\infty}^{D}g_{\tau_j^-}(x_{\tau_j^-}|x_{\tau_{j-1}})g_{\tau_{j-1}}\left(x_{\tau_{j-1}}|x_{\tau_{j-2}}\right)\ldots\times g_{\tau_2}\left(x_{\tau_2}|x_{\tau_1}\right)g_{\tau_1}\left(x_{\tau_1}\right)dx_{\tau_{j-1}}\ldots dx_{\tau_2}dx_{\tau_1}\tag{30}$$

$$g_{\tau_j}(x_{\tau_j})=\int_{-\infty}^{D}g_{\tau_j^-}(x_{\tau_j^-})g_{\gamma_j}\left(x_{\tau_j}-x_{\tau_j^-}\right)dx_{\tau_j^-}\tag{31}$$

where $\gamma_j\sim N\left(\phi_{1j},\phi_{2j}\right)$, ϕ_{1j},ϕ_{2j} are estimated at the off-line stage.

The probability that the system RUL equals τ_j-t_m is

$$\Pr(l+t_m=\tau_j)=\Pr\left(x_{\tau_j}=x_{\tau_j^-}+\gamma_j\geq D,x_{\tau_j^-}<D\right)=\int_D^{+\infty}g_{\tau_j}\left(x_{\tau_j}\right)dx_{\tau_j}\tag{32}$$

系统RUL等于 的概率

The PDF of the RUL belong to $\tau_{j-1} < l + t_m < \tau_j$ can be calculated,

$$f_{L_m}(l) = \int_{-\infty}^{+\infty}\int_{-\infty}^{+\infty}\int_{-\infty}^{D} \frac{D - x_{\tau_{j-1}}}{\sqrt{2\pi\sigma_j^2\left(l + t_m - \tau_{j-1}\right)^3}} \times \exp\left[-\frac{\left(D - x_{\tau_{j-1}} - \mu_i\left(l + t_m - \tau_{j-1}\right)\right)^2}{2\sigma_j^2\left(l + t_m - \tau_{j-1}\right)}\right] \times g_{\tau_{j-1}}\left(x_{\tau_{j-1}}\right) p\left(\mu_j\right) p\left(\omega_j\right) dx_{\tau_{i-1}} d\mu_j d\omega_j \tag{33}$$

The solutions of the above equations are intractable. Similarly, the point estimations of parameters are attained for simplifying integration process.

Once obtaining the probability density function of RUL that may be located in each interval, the average RUL can be attained for representing the RUL. Due to the change-point location is a random variable, the average RUL has the following form,

平均RUL具有以下形式

$$\begin{aligned} E\left[L_m\right] &= \int_0^{\infty} l \times f_{L_m}(l)\, dl + \sum_{j=1}^{k}\int_0^{\infty} (\tau_j - t_m)\Pr(l + t_m = \tau_j) p(\tau_j) d\tau_j \\ &= \int_{t_m}^{\infty}\int_0^{\tau_1 - t_m} l \times f_{L_m}(l) p(\tau_1) dl d\tau_1 \\ &\quad + \int_{t_m}^{\infty}\int_{t_m}^{\infty}\int_{\tau_1 - t_m}^{\tau_2 - t_m} l \times f_{L_m}(l) p(\tau_1) p(\tau_2) dl d\tau_1 d\tau_2 + \cdots \\ &\quad + \int_{t_m}^{\infty}\int_{t_m}^{\infty}\int_{\tau_{k-1} - t_m}^{\tau_k - t_m} l \times f_{L_m}(l)\, p(\tau_{k-1}) p(\tau_k) dl d\tau_{k-1} d\tau_k \\ &\quad + \int_{t_m}^{\infty}\int_{\tau_k - t_m}^{\infty} l \times f_{L_m}(l)\, p(\tau_k) dl d\tau_k \\ &\quad + \sum_{j=1}^{k}\int_{t_m}^{\infty} (\tau_j - t_m)\Pr(l + t_m = \tau_j) p(\tau_j) d\tau_j \end{aligned} \tag{34}$$

where $p(\tau_j) = \frac{1}{\sqrt{2\pi(\lambda_{2j})}} \exp\left[-\frac{(\tau_j - \lambda_{1j})^2}{2(\lambda_{2j})}\right], j = 1, 2, \ldots, k.$

It is worth noting that due to the effect of the abrupt jump at the change point, the degradation process may pass the given failure threshold suddenly. That is to say, the distribution of the RUL is not continuous at the change time. Meanwhile, the point estimation method and Monte Carlo simulation approach are used to address the issue that the RUL is intractable due to multiple complex integrations.

As above discussed, the RUL prediction has been accomplished in the case that the change point does not appear. If the change point c_j have occurred, i.e., $\tau_j < t_m < \tau_{j+1}$, the RUL distribution could be obtained, which is similar to the case of $0 < t_m < \tau_1$. Due to the limited space, for the case of $\tau_j < t_m < \tau_{j+1}$, the detailed expression of RUL distribution is omitted in this paper.

5. Case studies

In this section, a numerical simulation and an actual case about degradation data of NASA Lithium-ion battery are used to verify the effectiveness and practicability of the proposed model.

5.1. Simulation study

In this subsection, the effectiveness of the proposed approach is verified, including the change-point detection and the parameter identification. For simplicity, only two-change-point and three-change-point categories are considered in the simulation model. The hyper-parameters are specified in Table 1.

The failure threshold and the inspection interval are set as $D = 40, \Delta t = 0.2$, respectively. For two categories, a total of 2000 degradation signals are generated as training dataset. For two-change-point and three-change-point, the number of degradation signals is all 1000. Another 10 signals are generated as testing dataset as shown in Fig. 3.

Table 1
Hyper-parameters for simulation.

Variables	Two-change-point	Three-change-point
τ_j	$\lambda_{11} = 50, \lambda_{21} = 16$ $\lambda_{12} = 100, \lambda_{22} = 25$	$\lambda_{11} = 30, \lambda_{21} = 9$ $\lambda_{12} = 70, \lambda_{22} = 25$ $\lambda_{13} = 100, \lambda_{23} = 4$
γ_j	$\phi_{11} = 2.5, \phi_{21} = 0.01$ $\phi_{12} = 4, \phi_{22} = 0.16$	$\phi_{11} = 2, \phi_{21} = 0.01$ $\phi_{12} = 3, \phi_{22} = 0.09$ $\phi_{13} = 4, \phi_{23} = 0.0625$
ω_j $(= 1/\sigma_j^2)$	$\alpha_1 = 3, \beta_1 = 0.03$ $\alpha_2 = 4, \beta_2 = 0.05$ $\alpha_3 = 3.5, \beta_3 = 0.06$	$\alpha_1 = 3, \beta_1 = 0.02$ $\alpha_2 = 3, \beta_2 = 0.04$ $\alpha_3 = 2.4, \beta_3 = 0.06$ $\alpha_4 = 3, \beta_4 = 0.04$
μ_j	$\nu_1 = 0.18, \varphi_1 = 1 \times 10^{-3}$ $\nu_2 = 0.26, \varphi_2 = 1 \times 10^{-3}$ $\nu_3 = 0.36, \varphi_3 = 4 \times 10^{-3}$	$\nu_1 = 0.15, \varphi_1 = 4 \times 10^{-3}$ $\nu_2 = 0.18, \varphi_2 = 4 \times 10^{-3}$ $\nu_3 = 0.24, \varphi_3 = 4 \times 10^{-3}$ $\nu_4 = 0.36, \varphi_4 = 1.6 \times 10^{-3}$

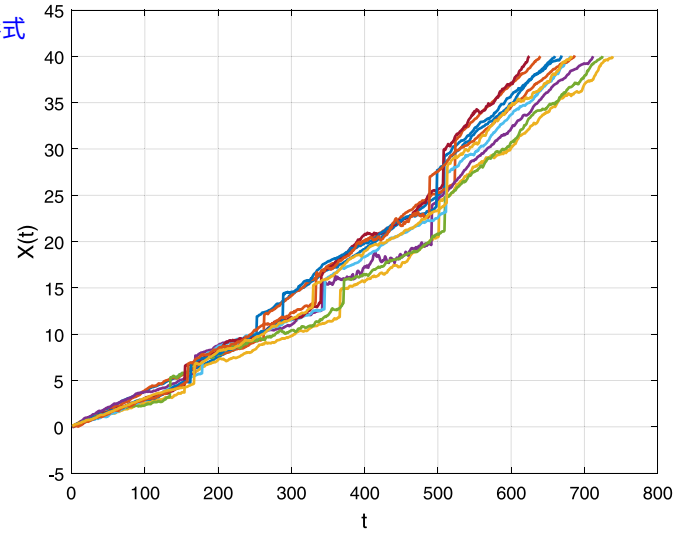

Fig. 3. Degradation trajectory of testing signals.
测试信号的退化轨迹

Through the setting of hyper-parameters about the change-point locations, the degradation trajectory will change around time 50, 100 for the two-change-point case, and the degradation trajectory will change around time 30, 70, 100 for the three-change-point case. Using the modified information criterion, the identified change-point number and corresponding change-point locations are equivalent to the true value for each degradation (see Fig. 4). Fig. 4 demonstrates the effectiveness of the proposed approach for change-point detection. Due to the case of three-change-point is similar to the case of two-change-point, it is omitted in Fig. 4.

For the two-change-point case and the three-change-point, the MIC values and the estimated locations of the change points of the change-point detection are presented in Table 2. Based on the simulation results by Pan and Chen [29] who examined the effect of constant C over a wide range of C, the value of C is set to 1. For two-change-point case, $T_{0,1} = MIC(k = 0) - MIC(k = 1) + \log(664) = 641.19$, and $T_{1,2} = MIC(k = 1) - MIC(k = 2) + \log(664) = 433.75$ are all much greater than $\chi^2_{0.99}(1) = 6.6349$ with a significance level of $\alpha = 0.01$, indicating that there are at least two change points. $T_{2,3} = MIC(k = 2) - MIC(k = 3) + \log(664) = 0.98$ is less than $\chi^2_{0.99}(1)$, hence there are two change points for the two-change-point case. In this way, it is determined that there are three change points for the three-change-point case.

Based on the proposed approach, the change-point number is accurately identified and the corresponding locations are rapidly obtained for each degradation signal by Algorithm 1 as well as Algorithm 2. The hyper-parameters of the change-point location and the corresponding

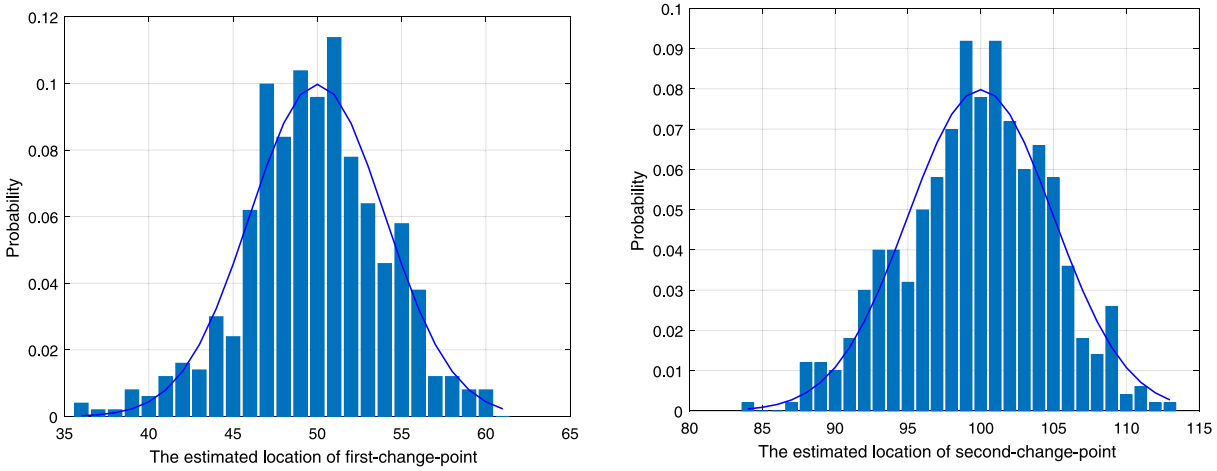

Fig. 4. The change-point location for the case of two-change-point. (a) First-change-point; (b) Second-change-point.

两个突变点情况下的突变点位置

Table 2
Change-point (CP) detection results.

变化点（CP）检测结果

Case	CP number	MIC(k)	CP locations	$T_{k-1,k}$
2	0	−130.74	–	–
	1	−765.51	107.8	641.19
	2	-1.193×10^{3}	47.4, 107.8	433.75
	3	-1.187×10^{3}	45.8, 47.4, 107.8	0.98
	4	−736.18	107.8, 111.4, 116, 121.4	−444.81
3	0	−66.28	–	–
	1	−683.48	102.4	623.73
	2	-1.217×10^{3}	78.6, 102.4	540.21
	3	-1.532×10^{3}	31, 78.6, 102.4	321.47
	4	-1.530×10^{3}	31, 78.6, 102.4, 109.4	4.06

Table 3
The estimated hyper-parameters about the change point (CP).

Variables	Two-change-point	Three-change-point
τ_j	$\lambda_{11}=49.96, \lambda_{21}=15.03$ $\lambda_{12}=100.31, \lambda_{22}=25.46$	$\lambda_{11}=30.03, \lambda_{21}=10.35$ $\lambda_{12}=70.04, \lambda_{22}=28.01$ $\lambda_{13}=100.18, \lambda_{23}=4.18$
γ_j	$\phi_{11}=2.50, \phi_{21}=0.01$ $\phi_{12}=4.00, \phi_{22}=0.15$	$\phi_{11}=2.00, \phi_{21}=0.021$ $\phi_{12}=3.01, \phi_{22}=0.094$ $\phi_{13}=4.00, \phi_{23}=0.066$

abrupt jump at change point are obtained as shown in Table 3, which approximate the true value.

Due to the multi-phase characteristics of degradation data, the hyper-parameters of each phase are estimated respectively according to the expectation maximization (EM) algorithm. The parameter estimations are obtained as shown in following Table 4. As observed in Table 4, some estimators are still biased because of the influence of sample size, simulation step and simulation precision, including the scale parameter β_j of the Gamma distribution. In the Wiener process, the drift coefficient has a great influence on the degradation trajectory, on the contrary, the diffusion coefficient has no obvious influence on the degradation trajectory and it is mainly used to represent the randomness in the degradation process. Therefore, the estimations of the above hyper-parameters could approach the real value, and the basis is acceptable.

For the ten testing signals, RUL is predicted at time 26, 60 and 110. Due to too many monitoring points, $\alpha_1=\alpha_{10}+n/2$ is too large, which

Table 4
Estimated parameters for each phase.

Variables	Two-change-point	Three-change-point
ω_j	$\alpha_1=2.95, \beta_1=0.03$ $\alpha_2=4.15, \beta_2=0.05$ $\alpha_3=3.19, \beta_3=0.06$	$\alpha_1=2.83, \beta_1=0.019$ $\alpha_2=3.03, \beta_2=0.041$ $\alpha_3=2.45, \beta_3=0.061$ $\alpha_4=3.10, \beta_4=0.042$
μ_j	$\nu_1=0.18, \varphi_1=0.002$ $\nu_2=0.26, \varphi_2=0.002$ $\nu_3=0.36, \varphi_3=0.005$	$\nu_1=0.15, \varphi_1=0.005$ $\nu_2=0.18, \varphi_2=0.004$ $\nu_3=0.24, \varphi_3=0.004$ $\nu_4=0.36, \varphi_4=0.0018$

makes the integration process of solving Bayesian estimation too difficult. In this paper, the method of expanding monitoring interval is used to process data, which has no influence on parameter identification. After obtaining the estimations of model parameters in each phase, the RUL is predicted considering two-change-point and three-change-point as shown in following Fig. 5.

Fig. 5 shows the prediction of the ten testing signals at three different time, i.e., 26, 60, 110. As expected, the more observations collected, the more accurate the prediction becomes for all signals. Because the prior information has a great influence on the prediction and the characteristics of the equipment itself are not fully considered, the prediction results have a large error in the early phase. With the more observations collected, the prior has a less influence on the model updating and the posterior updating of the model parameters is accomplished by using the degradation data obtained, which make the prediction becomes more accurate.

5.2. Practical case

In this subsection, the degradation data of Li-ion battery collected by the Prognostic Center of Excellence (PCoE) at NASA Ames are used to verify our approach [32]. Here, four batteries are chosen as shown in Fig. 6, i.e., B0005, B0006, B0007, B0018. Four Li-ion batteries were run through 3 different operational profiles (charge, discharge and impedance) at room temperature. Charging was carried out in a constant current (CC) mode at 1.5 A until the battery voltage reached 4.2 V and then continued in a constant voltage (CV) mode until the charge current dropped to 20 mA. Discharge was carried out at a constant current (CC) level of 2A until the battery voltage fell to 2.7 V, 2.5 V, 2.2 V and 2.5 V for batteries 5, 6, 7 and 18 respectively. Impedance measurement was carried out through an electrochemical impedance spectroscopy (EIS) frequency sweep from 0.1 Hz to 5 kHz. The data

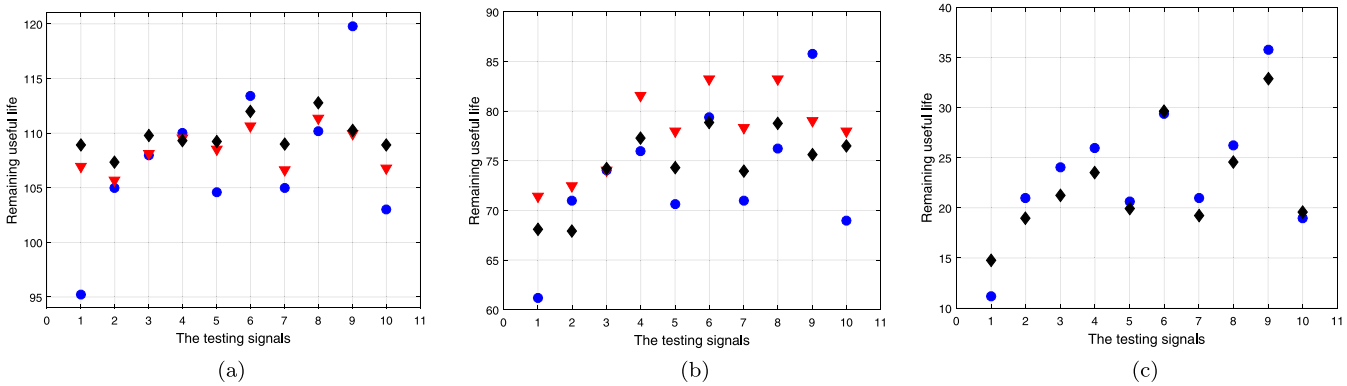

Fig. 5. The RUL prediction of the ten testing signals at time 26, 60, 110. (a–c) The RUL prediction at time 26, 60, 110, respectively. The • denotes the actual RUL; ▼ represents the RUL of two-change-point; ♦ represents the RUL of three-change-point.

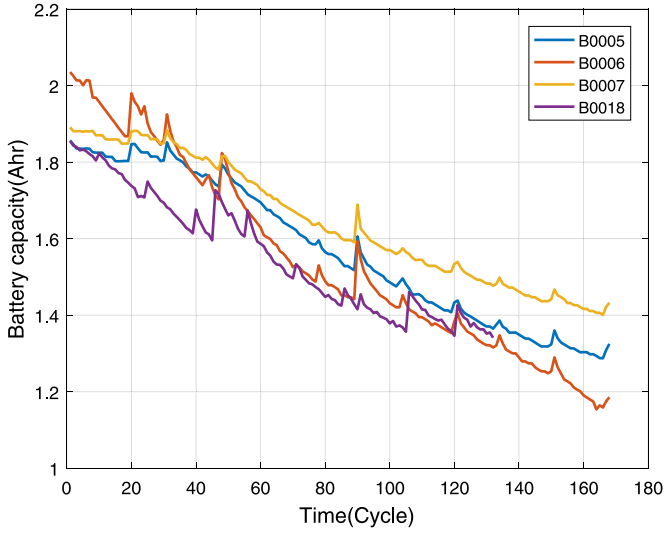

Fig. 6. The degradation trajectories of battery capacity.

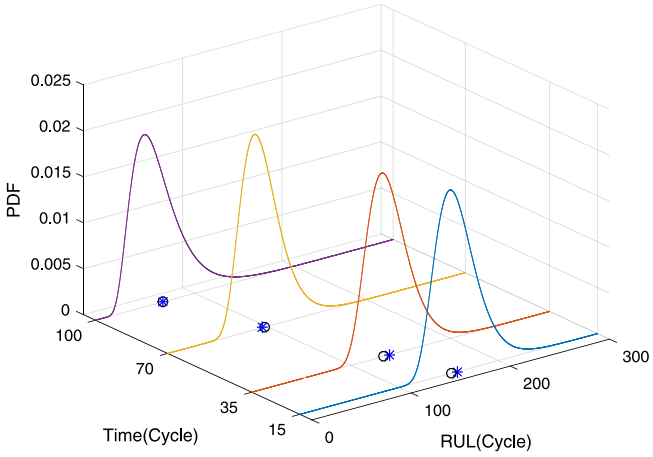

Fig. 7. The PDFs of the estimated RUL for B7. The "∗" represents the estimated RUL; the "∘" denotes the actual RUL.

of B0005, B0006 and B0018 are used at the off-line stage, and the degradation data of B0007 is adopted to update the model and predict the RUL. It is noted that the regeneration phenomenon does exist in the degradation process, which makes the degradation trajectory of the battery capacity multi-phase [33]. Meanwhile, It is worth nothing that the experiments are stopped when the battery capacity reached end-of-life (EOL) criteria, which was a 26% fade in rated capacity (from 2 Ahr to 1.48 Ahr).

Detecting the change points of B5 and B6 according to the proposed approach, for B5, $T_{0,1} = MIC(k=0) - MIC(k=1) + \log(168) = 38.44$, $T_{1,2} = MIC(k=1) - MIC(k=2) + \log(168) = 57.21$, and $T_{2,3} = MIC(k=2) - MIC(k=3) + \log(168) = 24.54$ are all much greater than $\chi^2_{0.99}(1) = 6.6349$ with a significance level of $\alpha = 0.01$, indicating that there are at least three change points. $T_{3,4} = MIC(k=3) - MIC(k=4) + \log(168) = 5.64$ is less than $\chi^2_{0.99}(1)$, hence there are three change points for B5. Similarly, for B18, $T_{0,1} = MIC(k=0) - MIC(k=1) + \log(132) = 30.0593$, and $T_{1,2} = MIC(k=1) - MIC(k=2) + \log(132) = 24.4833$ are all much greater than $\chi^2_{0.99}(1) = 6.6349$ with a significance level of $\alpha = 0.01$, indicating that there are at least two change points. $T_{2,3} = MIC(k=2) - MIC(k=3) + \log(132) = 0.9724$ is less than $\chi^2_{0.99}(1)$, hence there are two change points for B18.

For B5 and B6, the change-point locations estimated are at cycle time 19, 47, and 89. For B18, the change-point locations estimated are at cycle time 45, and 105.

Table 5
Estimated hyper-parameters for practical case.

Variables	Two-change-point	Three-change-point
ω_j	$\alpha_1 = 49.8843, \beta_1 = 307.06$	$\alpha_1 = 1.5747, \beta_1 = 3.9077$
	$\alpha_2 = 50.2218, \beta_2 = 236.12$	$\alpha_2 = 2.7213, \beta_2 = 12.1607$
	$\alpha_3 = 43.6313, \beta_3 = 146.90$	$\alpha_3 = 3.9236, \beta_3 = 14.5317$
		$\alpha_4 = 27.6139, \beta_4 = 35.4258$
μ_j	$\nu_1 = 0.0029, \varphi_1 = 0.0001$	$\nu_1 = 0.0010, \varphi_1 = 0.0004$
	$\nu_2 = 0.0045, \varphi_2 = 0.0009$	$\nu_2 = 0.0028, \varphi_2 = 0.0003$
	$\nu_3 = 0.0046, \varphi_3 = 0.0006$	$\nu_3 = 0.0048, \varphi_3 = 0.0003$
		$\nu_4 = 0.0043, \varphi_4 = 0.0003$

The degradation trajectory are divided into four phases by three change points. Similarly, the degradation trajectory are divided into three phases by two change points. The hyper-parameters are estimated according to the EM algorithm as shown in Table 5.

At the prediction stage, the RUL of B7 is predicted at cycle time 15, 35, 70, and 100. Fig. 7 shows the estimated RUL's distribution considering three change points. The figure of estimated RUL's distribution is omitted considering two change points.

From Fig. 7, with the more observations collected, the change-point locations are determined and the uncertainty of future change points is reduced. Thus, the uncertainty in RUL prediction is reduced and the prediction is more accurate, which proves the effectiveness of the multi-phase degradation model based on Wiener process.

For comparison purpose, single-phase model and multi-phase model are applied to the degradation data of B7. Two criteria, relative error (RE) and mean square error (MSE) of all points, are adopted to evaluate

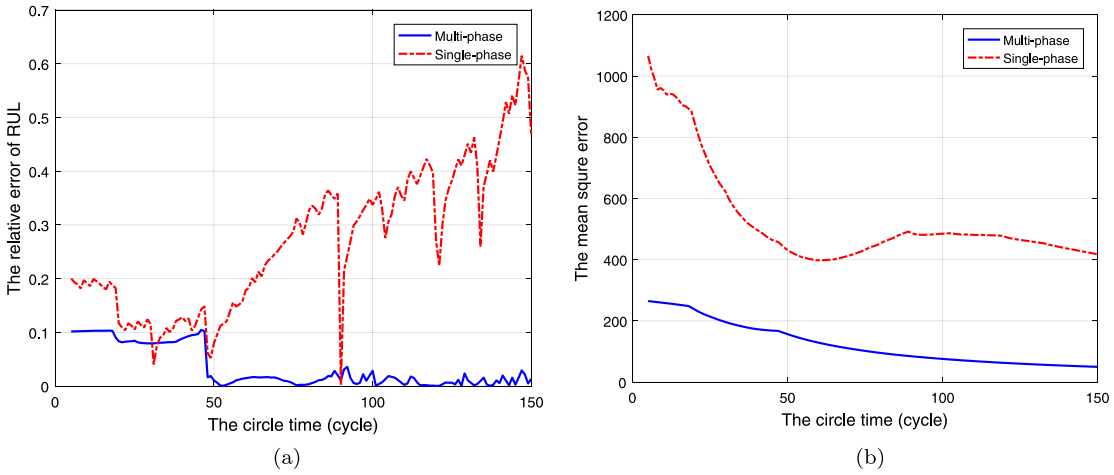

Fig. 8. Comparison of RUL estimation for two models. (a) The relative error; (b) The mean square error.

the prediction performance of different methods.

$$RE = \frac{|R_i - R_{i,ture}|}{R_{i,ture}}, \tag{35}$$

$$MSE = \frac{1}{m}\sum_{i=1}^{m}\left(R_i - R_{i,true}\right)^2 \tag{36}$$

As shown in Fig. 8, the proposed multi-phase model provides smaller RE and MSE than the single-phase model, which indicates the overall prediction of the proposed multi-phase model is more accurate.

6. Conclusion and discussion

In this paper, a multi-phase degradation model is proposed for characterizing the multi-phase characteristics of the degradation signals based on Wiener process. All model parameters are assumed to be random variables for the unit heterogeneity, including the change-point locations, corresponding abrupt jumps at the change points, drift parameter and diffusion parameter of each phase. Then, the Bayesian approach is utilized to integrate the current available data with historical data. Under the Bayesian framework, the prognostic involves two stages, the off-line stage and the on-line stage. At the off-line stage, the modified information criterion (MIC) is adopted to detect the change-point locations and determine the change-point number. For detail, the Algorithm 1 is used to obtain the change-point locations, which greatly simplifies the computation and improves the operation speed comparing with the enumeration method. In addition, by treating the drift parameter and diffusion parameter of each phase as the latent variables, the EM algorithm is used for parameter identification, which is simple and stable. At the on-line stage, the model parameters are updated under Bayesian rule. Furthermore, considering the uncertainties of the change points and the abrupt jumps, the probability density function of RUL is obtained for predicting RUL. Finally, a numerical simulation study and a practical case about Li-ion battery demonstrate that the proposed approach can effectively predict RUL and improve the prediction accuracy.

Although the proposed approach can effectively describe the degradation process with multi-phase features and provide accurate RUL prediction, there are still some problems that need to be investigated in the future. First, the point estimation method and Monte Carlo simulation approach are used for the multi-integral. How to efficiently and accurately calculate the multi-integral is a challenge. Second, in practice, all phases may not be independent, even related to each other. We currently are looking into this issue. In addition, the change-point locations and corresponding jumps are all assumed to follow the normal distribution. The other forms of them are worth further investigating. These issues will be left to our future work.

CRediT authorship contribution statement

Guobo Liao: Conceptualization, Methodology, Writing - original draft. **Hongpeng Yin:** Conceptualization, Supervision. **Min Chen:** Investigation. **Zheng Lin:** Methodology.

Declaration of competing interest

The authors declare that they have no known competing financial interests or personal relationships that could have appeared to influence the work reported in this paper.

Acknowledgments

This work is jointly supported by Fundamental Research Funds for the Central Universities, China (2019CDCGZDH336), China Central Universities foundation (2019CDYGZD001), Graduate Research and Innovation Foundation of Chongqing, China (CYS19072), and Scientific Reserve Talent Programs of Chongqing University, China (cqu2018CDHB1B04).

Appendix. Q-function

Model parameters $\boldsymbol{\mu}^i = (\mu_1^i, \mu_2^i, \ldots, \mu_{k+1}^i)^T$, and $\boldsymbol{\omega}^i = (\omega_1^i, \omega_2^i, \ldots, \omega_{k+1}^i)^T$ are regarded as latent variables. In addition, let $\boldsymbol{Z}^i = (\boldsymbol{\mu}^{iT}, \boldsymbol{\omega}^{iT})^T$ and $\boldsymbol{\theta} = (\alpha_1, \ldots, \alpha_{k+1}, \beta_1, \cdots, \beta_{k+1}, \nu_1, \ldots, \nu_{k+1}, \varphi_1, \ldots, \varphi_{k+1})$, represent the latent variables of the unit i and the corresponding hyper-parameters, respectively.

$$\begin{aligned}
Q(\boldsymbol{\theta}|\boldsymbol{\theta}^{(p)}) &= E_{\boldsymbol{Z}|\boldsymbol{\Delta X},\boldsymbol{\theta}^{(p)}} \ln \prod_{i=1}^{I} p\left(\boldsymbol{\Delta X}^i, \boldsymbol{Z}^i|\boldsymbol{\theta}\right) \\
&= \sum_{i=1}^{I} E_{\boldsymbol{Z}^i|\boldsymbol{\Delta X}^i,\boldsymbol{\theta}^{(p)}} \ln\left[p\left(\boldsymbol{\Delta X}^i|\boldsymbol{Z}^i,\boldsymbol{\theta}\right)p\left(\boldsymbol{Z}^i|\boldsymbol{\theta}\right)\right] \\
&= \sum_{i=1}^{I} E_{\boldsymbol{Z}^i|\boldsymbol{\Delta X}^i,\boldsymbol{\theta}^{(p)}} \ln\left[\prod_{j=1}^{k+1}\frac{(\beta_j)^{\alpha_j}}{\Gamma(\alpha_j)}\left(\omega_j^i\right)^{\alpha_j-1}\exp\left(-\omega_j^i\beta_j\right)\sqrt{\frac{\omega_j^i}{2\pi\varphi_j}}\right. \\
&\quad \left.\times\exp\left(-\frac{\left(\mu_j^i-\nu_j\right)^2}{2\left(\omega_j^i\right)^{-1}\varphi_j}\right)\prod_{s=1}^{n_j^i}\sqrt{\frac{\omega_j^i}{2\pi\Delta t_{c_{j-1}+s}^i}}\exp\left(-\frac{\left(\Delta X_{c_{j-1}+s}^i-\mu_j^i\Delta t_{c_{j-1}+s}^i\right)^2}{2\left(\sigma_j^i\right)^2\Delta t_{c_{j-1}+s}^i}\right)\right]
\end{aligned}$$

$$
\begin{aligned}
&= \sum_{i=1}^{I}\sum_{j=1}^{k+1}\left[\ln\frac{(\beta_j)^{\alpha_j}}{\Gamma(\alpha_j)} + (\alpha_j+1)\,E\left[\ln\left(\omega_j^i\right)|\boldsymbol{\Delta X}^i,\boldsymbol{\theta}^{(p)}\right] - \beta_j E\left[\omega_j^i|\boldsymbol{\Delta X}^i,\boldsymbol{\theta}^{(p)}\right]\right] \\
&\quad + \ln\frac{1}{\sqrt{2\pi\varphi_j}} + \frac{1}{2}E\left[\ln\left(\omega_j^i\right)|\boldsymbol{\Delta X}^i,\boldsymbol{\theta}^{(p)}\right] - \sum_{i=1}^{I}\sum_{j=1}^{k+1}\left[\left(E\left[\left(\mu_j^i\right)^2\left|\omega_j^i,\boldsymbol{\Delta X}^i,\boldsymbol{\theta}^{(p)}\right.\right]\right.\right. \\
&\quad \left.\left. - 2\nu_j E\left[\mu_j^i\left|\omega_j^i,\boldsymbol{\Delta X}^i,\boldsymbol{\theta}^{(p)}\right.\right] + (\nu_j)^2\right)\times\frac{1}{2\varphi_j}E\left[\omega_j^i|\boldsymbol{\Delta X}^i,\boldsymbol{\theta}^{(p)}\right]\right] \\
&\quad - \sum_{i=1}^{I}\sum_{j=1}^{k+1}\sum_{s=1}^{n_j^i}\left[\left(\left(\Delta X_{c_{j-1}+s}^i\right)^2 - 2\Delta X_{c_{j-1}+s}^i\Delta t_{c_{j-1}+s}^i E\left[\mu_j^i\left|\omega_j^i,\boldsymbol{\Delta X}^i,\boldsymbol{\theta}^{(p)}\right.\right]\right.\right. \\
&\quad \left.\left. + \left(\Delta t_{c_{j-1}+s}^i\right)^2 E\left[\left(\mu_j^i\right)^2\left|\omega_j^i,\boldsymbol{\Delta X}^i,\boldsymbol{\theta}^{(p)}\right.\right]\right)\times\frac{1}{2\Delta t_{c_{j-1}+s}^i}E\left[\omega_j^i|\boldsymbol{\Delta X}^i,\boldsymbol{\theta}^{(p)}\right]\right] \\
&\quad + \sum_{i=1}^{I}\sum_{j=1}^{k+1}\sum_{s=1}^{n_j^i}\left[\ln\frac{1}{\sqrt{2\pi\Delta t_{c_{j-1}+s}^i}} + \frac{1}{2}E\left[\ln\left(\omega_j^i\right)|\boldsymbol{\Delta X}^i,\boldsymbol{\theta}^{(p)}\right]\right]
\end{aligned}
\tag{37}
$$

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