HW1 Propositional Equivalence

Truth Table

1. Construct the truth table for each of the following formulae:

a. $(q \wedge r) \vee (\neg q \vee \neg r)$

q	r	$q \wedge r$	$\neg q$	$\neg r$	$(\neg q \vee \neg r)$	$(q \wedge r) \vee (\neg q \vee \neg r)$
T	T	T	F	F	F	T
T	F	F	F	T	T	T
F	T	F	T	F	T	T
F	F	F	T	T	T	T

b. $(p \vee \neg p) \wedge (\neg q \wedge r)$

p	q	r	$\neg p$	$p \vee \neg p$	$\neg q$	$\neg q \wedge r$	$(p \vee \neg p) \wedge (\neg q \wedge r)$
T	T	T	F	T	F	F	F
T	T	F	F	T	F	F	F
T	F	T	F	T	T	T	T
T	F	F	F	T	T	F	F
F	T	T	T	T	F	F	F
F	T	F	T	T	F	F	F
F	F	T	T	T	T	T	T
F	F	F	T	T	T	F	F

c. $q \rightarrow (r \rightarrow (\neg q \wedge p))$

p	q	r	$\neg q$	$\neg q \wedge p$	$r \rightarrow (\neg q \wedge p)$	$q \rightarrow (r \rightarrow (\neg q \wedge p))$
T	T	T	F	F	F	F
T	T	F	F	F	T	T
T	F	T	T	T	T	T
T	F	F	T	T	T	T
F	T	T	F	F	F	F
F	T	F	F	F	T	F
F	F	T	T	F	F	T
F	F	F	T	F	T	F

d. $r \wedge ((p \rightarrow \neg r) \wedge (\neg p \rightarrow \neg r))$

p	r	$\neg r$	$p \rightarrow \neg r$	$\neg p$	$\neg p \rightarrow \neg r$	$((p \rightarrow \neg r) \wedge (\neg p \rightarrow \neg r))$	$r \wedge ((p \rightarrow \neg r) \wedge (\neg p \rightarrow \neg r))$
T	T	F	F	F	T	F	F
T	F	T	T	F	T	T	F
F	T	F	T	T	F	F	F
F	F	T	T	T	T	T	F

Tautology

1. Show that each of these conditional statement is a tautology **by using truth table** and **by Proof**

a. $(p \wedge q) \rightarrow p$

p	q	$(p \wedge q)$	$(p \wedge q) \rightarrow p$
T	T	T	T
T	F	F	T
F	T	F	T
F	F	F	T

$(p \wedge q) \rightarrow p$

1. $\neg(p \wedge q) \vee p$
2. $(\neg p \vee \neg q) \vee p$
3. $(\neg p \vee p) \vee \neg q$
4. $T \vee \neg q$
5. T

b. $(p \wedge q) \rightarrow (p \rightarrow q)$

p	q	$(p \wedge q)$	$(p \rightarrow q)$	$(p \wedge q) \rightarrow (p \rightarrow q)$
T	T	T	T	T
T	F	F	F	T
F	T	F	T	T
F	F	F	T	T

$(p \wedge q) \rightarrow (p \rightarrow q)$

1. $\neg(p \wedge q) \vee (\neg p \vee q)$
2. $(\neg p \vee \neg q) \vee (\neg p \vee q)$
3. $(\neg p \vee \neg p) \vee (\neg q \vee q)$
4. $\neg p \vee T$
5. T

Logically equivalent

1. Show that each of these conditional statements are logically equivalent **by using truth table** and **by Proof**

a) $\neg(p \rightarrow q) \equiv p \wedge \neg q$

p	q	$(p \rightarrow q)$	$\neg(p \rightarrow q)$	$\neg q$	$p \wedge \neg q$
T	T	T	F	F	F
T	F	F	T	T	T
F	T	T	F	F	F
F	F	T	F	T	F

$\neg(p \rightarrow q)$

1. $\neg(\neg p \vee q)$
2. $\neg p \wedge \neg q$
3. $p \wedge \neg q$

b) $\neg(p \vee (\neg p \wedge q)) \equiv \neg p \wedge \neg q$

p	q	$\neg p$	$\neg q$	$(\neg p \wedge q)$	$(p \vee (\neg p \wedge q))$	$\neg(p \vee (\neg p \wedge q)$	$\neg p \wedge \neg q$
T	T	F	F	F	T	F	F
T	F	F	T	F	T	F	F
F	T	T	F	T	T	F	F
F	F	T	T	F	F	T	T

$\neg(p \vee (\neg p \wedge q))$

1. $\neg p \wedge \neg (\neg p \wedge q)$
2. $\neg p \wedge (\neg\neg p \vee \neg q)$
3. $\neg p \wedge (p \vee \neg q)$
4. $(\neg p \wedge p) \vee (\neg p \wedge \neg q)$
5. $F \vee (\neg p \wedge \neg q)$
6. $\neg p \wedge \neg q$

Inference Rules

1. "If I am sick, there will be no lecture today;" "either there will be a lecture today, or all the students will be happy;" "the students are not happy."
   - Translate into logic as: $p \rightarrow \neg q$, $r \rightarrow q$, p, $r \vee s$, $t \rightarrow u$.

Step	**Reason**
1. $p \rightarrow \neg q$	Hypothesis
2. $\neg p \vee$	Conditional using (1)
3. $q \vee r$	Hypothesis
4. $\neg p \vee r$	Resolution using (2) and (3)
5. $\neg r$	Hypothesis
6. $\neg p$	Disjunctive syllogism using (4) and (5)

The answer is I'm not sick today

2. In the back of an old cupboard you discover a note signed by a pirate famous for his bizarre sense of humor and love of logical puzzles. In the note he wrote that he had hidden treasure somewhere on the property. He listed 5 true statement and challenged the reader to use them to figure out the location of the treasure
   a) If this house is next to the lake, then the treasure is not in the kitchen
   b) If the tree in the front yard is an elm, then the treasure is in the kitchen
   c) This house is next to a lake
   d) The tree in the front yard is an elm or the treasure is buried under the flagpole
   e) If the tree in the back yard is an oak, then the treasure is in the garage

Where is the treasure hidden?

- Translate into logic as: $p \rightarrow \neg q$, $r \rightarrow q$, p, $r \vee s$, $t \rightarrow u$.

Step	**Reason**
1. p	Hypothesis
2. $p \rightarrow \neg q$	Hypothesis
3. $\neg q$	Modus ponens using (1) and (2)
4. $r \rightarrow q$	Hypothesis
5. $\neg r$	Modus tollens using (3) and (4)
6. $r \vee s$	Hypothesis
7. s	Disjunctive syllogism using (5) and (6)
8. $s \vee t$	Addition (7)
9. $\neg s \rightarrow t$	Conditional using (8)
10. $t \rightarrow u$	Hypothesis
11. $\neg s \rightarrow u$	Hypothetical syllogism using (9) and (10)

Where is the treasure ? if the treasure is not buried under the flagpole then the treasure is in the garage .

Proof by using Inference rule

1. $\neg p \to t$, $q \to s$, $r \to q$, $\neg(q \vee t)$ $\therefore p$

Step	Reason
1. $\neg(q \vee t)$	Hypothesis
2. $\neg q \wedge \neg t$	De morgen's laws using (1)
3. $\neg t$	Simplification using (2)
4. $\neg p \to t$	Hypothesis
5. $\neg(\neg p)$	Modus tollens using (3) and (4)
6. p	Double negation law using (5)

2. p, $s \to \neg r$, $q \vee r$, $q \to \neg p$, $\therefore \neg s$

Step	Reason
1. $q \to p$	Hypothesis
2. $p \to \neg q$	Conditional statement using (1)
3. p	Hypothesis
4. $\neg q$	Modus ponens using (2) and (3)
5. $q \vee r$	Hypothesis
6. r	Disjunctive using (4) and (5)
7. $s \to \neg r$	Hypothesis
8. $\neg r \to \neg s$	Conditional statement using (7)
9. $\neg s$	Modus ponens using (6) and (8)

3. $(p \vee q) \to r$, $\neg r \vee s$, $p \to \neg s$ $\therefore \neg p$

Step	Reason
1. $p \to \neg s$	Hypothesis
2. $\neg p \to \neg s$	Conditional statement using (1)
3. $\neg r \vee s$	Hypothesis
4. $\neg p \vee \neg r$	Resolution using (2) and (3)
5. $(p \vee q) \to r$	Hypothesis
6. $\neg(p \vee q) \vee r$	Conditional statement using (5)
7. $(\neg p \wedge \neg q) \vee r$	De morgen's laws using (6)
8. $\neg p \vee (\neg p \wedge \neg q)$	Resolution using (4) and (7)
9. $\neg p$	Absorption laws using (8)