HW2 Proof Method

1. Fill in the blanks: given n = 2k for even and n = 2k+1 for odd
   - 4 is even because 4=2(2). Here, __4__ plays the role of n and __2__ plays the role of k .
   - 5 is odd because 5=2(2)+1. Here, __5__ plays the role of n and __2__ plays the role of k .
   - −4 is even because −4=2(-2). Here, __-4__ plays the role of n and __-2__ plays the role of k .
   - −5 is odd because −5=2(-2)+1. Here, __-5__ plays the role of n and __-2__ plays the role of k .
   - 0 is even because 0=2(0). Here, __0__ plays the role of n and __0__ plays the role of k .

2. Fill in the blanks Use a **direct proof** to show that If n1 and n2 are even, then n1+n2 is even.

Suppose that n1 and n2 are even. By definition, there exist an integer __n1__

such that n1=2(__k__) and there exists an integer __n2__ such that n2=2(__k__) . Consider the sum

of n1 and n2 : n1+n2= __2k + 2k__ = 2(__2k__).

Let __2k__= __2(2k)__ Thus n1+n2=2(__2k__) for some integer __k__ . By definition, we

know that n1+n2 is even.

3. Use **direct proof** to show the following theorem: If n is even, then n^2 is even.

Suppose that n is even then n = 2k for some integer k.

Consider. n^2 $= (2k)^2$
$= 4k^2$
$= 2(2k^2)$

Thus $n^2 = 2(2k^2)$ for some integer k. By definition, we know that n is even.

4. Use **direct proof** to show the following theorem: If n is odd, then $3n^2+5n+18$ is even.

Suppose that n is odd then n = 2k+1 for some integer k.

Consider. $3n^2 + 5n + 18 = 3(2k+1)^2 + 5(2k+1) + 18$
$= 3(4k^2 + 4k +1) + 10k + 5 + 18$
$= 12k^2 + 12k +3 + 10k +5 + 18$
$= 12k^2 + 22k + 26$
$= 2(6k^2 + 11k + 13)$

Thus $3n^2 + 5n + 18 = 2(6k^2 + 11k + 13)$ is even. for some integer k. By definition, we know that n is odd.

5. Fill in the blanks. Use a **<u>contrapositive proof</u>** For any integer n, if 5n+3 is even, then n is odd.

Proof. To construct a proof by contrapositive, $\neg Q \rightarrow \neg P$

If n is not odd, 5n+3 is not even

Suppose that n is not odd. We know that n is <u>even</u> . By definition, there exists an integer such that n=2(<u>k</u>) . Consider:

5n+3= <u>5(2k)+3</u> = <u>10k+3</u> =2(<u>5k + 1</u>)+1.

Let <u>2k</u> = <u>2(5k+1)+1</u> . Thus 5n+3=2(<u>5k+1</u>)+1 for some integer <u>k</u> . By definition, we know that 5n+3 is odd. Hence 5n+3 is not even.

6. Use a **<u>contrapositive proof</u>** If $3n^2+5n+18$ is not even, then n is an not even.

Proof. To construct a proof by contrapositive, $\neg Q \rightarrow \neg p$

If n is even, then $3n^2+5n+18$ is even.

Suppose that n is not even. We know that n is <u>even</u> . By definition, there exists an integer such that n = 2k. Consider:

$$
\begin{aligned}
3n^2+5n+18 &= 3(2k)^2+5(2k)+18 \\
&= 12k^2 + 10k + 18 \\
&= 2(6k^2 + 5k + 9)
\end{aligned}
$$

Thus $3n^2+5n+18 = 2(6k^2 + 5k + 9)$ for some integer <u>k</u>. By definition, we know that $3n^2+5n+18$ is even. Hence $3n^2+5n+18$ is even.

7. Use a **<u>contrapositive proof</u>** If n^2 is even, then n is even.

Proof. To construct a proof by contrapositive, $\neg Q \rightarrow \neg p$

If n is not even, then n^2 is not even.
Suppose that n is not even. We know that n is <u>odd</u> . By definition, there exists an integer such that n=2k+1 . Consider:

$$
\begin{aligned}
n^2 &= (2k+1)^2 \\
&= 4k^2 + 4k + 1 \\
&= 2(2k^2 + 2k) + 1
\end{aligned}
$$

Thus $n^2 = 2(2k^2 + 2k) + 1$ for some integer <u>k</u>. By definition, we know that n^2 is odd. Hence n^2 is not even.

8. Fill in the blanks. **Use proof by contradiction**

Let n be an integer. If $n^2 + 5$ is odd, then n is even.

assume both p and $\neg q$ are true

Proof. Suppose, for the sake of contradiction, that $n^2 + 5$ is___odd___ and n is also ___odd___.

By definition, then, there exists integers k and l so that $n^2 + 5$ = ___2k+1___ and n = ___2k+1___.

Hence, we have

$$2k + 1 = n^2 + 5$$
$$= (\_\_2k+1\_\_)^2 + 5$$
$$= \_\_4k^2+4k+1\_\_ + 5$$
$$= 2(\_2k^2 + 2k + 3\_\_).$$

Therefore, 2k + 1 is _$2(2k^2 + 2k + 3)$__. This is clearly impossible, and hence we cannot have that n^2 + 5 is odd and n is also odd.

Therefore, if that $n^2 + 5$ is odd, we must have n is even.

9. Use **proof by contradiction** to show that if n is an integer, and n^3+5 is odd, then n is even

assume both p and $\neg q$ are true

Proof. Suppose, for the sake of contradiction, that $n^3 + 5$ is___odd___ and n is also ___odd___.

By definition, then, there exists integers k so that $n^2 + 5$ = ___2k+1___ and n = ___2k+1___.

Hence, we have

$$2k + 1 = n^3 + 5$$
$$= (2k + 1)^3 + 5$$
$$= (8k^3 + 12k^2 + 6k + 1) + 5$$
$$= 8k^3 + 12k^2 + 6k + 6$$
$$= 2(4k^3 + 6k^2 + 3k + 3)$$

Therefore, 2k+1 is $2(4k^3 + 6k^2 + 3k + 3)$. This is clearly impossible, and hence we cannot have that $n^3 + 5$ is odd and n is also odd.

Therefore, if that $3n^2+5n+18$ is odd, we must have n is even.

10. Use **proof by contradiction** to show that if $3n^2+5n+18$ is not even, then n is not even

assume both p and $\neg q$ are true

Proof. Suppose, for the sake of contradiction, that $3n^2+5n+18$ is odd and n is even.

By definition, then, there exists integers k so that $3n^2+5n+18 = 2k+1$ and $n = 2k$.

Hence, we have

$$2k+1 = 3n^2+5n+18$$

$$= 3(2k)^2 + 5(2k) + 18$$

$$= 3(4k^2) + 5(2k) + 18$$

$$= 12k^2 + 10k + 18$$

$$= 2(6k^2 + 5k + 9)$$

Therefore, $2k+1$ is $2(6k^2 + 5k + 9)$. This is clearly impossible, and hence we cannot have that $3n^2+5n+18$ is odd and n is even.

Therefore, if that $3n^2+5n+18$ is odd, we must have n is not even.