HW2 Proof Method

1. Fill in the blanks: given n = 2k for even and n = 2k+1 for odd
   - 4 is even because 4=2(_2_). Here, __4__ plays the role of n and __2__ plays the role of k .
   - 5 is odd because 5=2(__2__)+1. Here, _5___ plays the role of n and __2__ plays the role of k .
   - −4 is even because −4=2(__-2__). Here, __-4__plays the role of n and __-2__ plays the role of k .
   - −5 is odd because −5=2(__-2_)+1. Here, __-5_ plays the role of n and __-2__ plays the role of k .
   - 0 is even because 0=2(_0_). Here, _2___ plays the role of n and __0__ plays the role of k .

2. Fill in the blanks Use a **direct proof** to show that If n1 and n2 are even, then n1+n2 is even.

Suppose that n1 and n2 are even. By definition, there exist an integer ________n1_____

such that n1=2(_____k_______) and there exists an integer ____n2__ such that n2=2(_k_) .

Consider the sum of n1 and n2 : n1+n2= _______2k+2k___ =2(___2k___).

Let ____2k____= ____2k____ Thus n1+n2=2(_2k__) for some integer ___k__ . By definition, we know

that n1+n2 is even.

3. Use **direct proof** to show the following theorem: If n is even, then n^2 is even.

n is even then n = 2k for some integer k.
Consider. n2 = (2k)2
= 4k2
= 2(2k2)
Thus n2 = 2(2k2) for some integer k. By definition, we know that n is even.

4. Use **direct proof** to show the following theorem: If n is an even, then $3n^2+5n+18$ is even.

   n is odd then n = 2k+1 for some integer k.
   Consider. 3n2 + 5n + 18 = 3(2k+1)2 + 5(2k+1) + 18
   = 3(4k2 + 4k +1) + 10k + 5 + 18
   = 12k2 + 12k +3 + 10k +5 + 18
   = 12k2 + 22k + 26
   = 2(6k2 + 11k + 13)
   Thus 3n2 + 5n + 18 = 2(6k2 + 11k + 13) is even. for some integer k. By definition, we know that n is

5. Fill in the blanks. Use a **contrapositive proof** For any integer n, if 5n+3 is even, then n is odd.
   If n is not odd, 5n+3 is not even
   Suppose that n is not odd. We know that n is ___even____ . By definition, there exists an integer
   such that n=2(__k____) . Consider:
   5n+3=____5(5k)+3____=_____10k+3____=2(_____5k+1_____)+1.
   Let ____2k_____= ______2(5k+1)+1______. Thus 5n+3=2(__5k+1___)+1
   for some integer __k________ . By definition, we know that 5n+3 is odd. Hence 5n+3 is not even.

6. Use a **contrapositive proof** If $3n^2$+5n+18 is even, then n is an even.

   If n is even, then 3n2+5n+18 is even.
   Suppose that n is not even. We know that n is even . By definition, there exists an integer such that n = 2k.
   Consider:
   3n2+5n+18 = 3(2k)2+5(2k)+18
   = 12k2 + 10k +18
   = 2(6k2 + 5k + 9)
   Thus $3n^2$+5n+18 = 2($6k^2$ + 5k + 9) for some integer k.
   By definition, we know that $3n^2$+5n+18 is even.
   Hence $3n^2$+5n+18 is even.

7. Use a **contrapositive proof** If $n^2$is even, then n is even.

   contrapositive, ¬Q → ¬p
   If n is not even, then n2 is not even.
   Suppose that n is not even. We know that n is odd .
   By definition, there exists an integer such that n=2k+1 .
   Consider: n^2 = (2k+1)2
   = 4k2 + 4k +1
   = 2(2k2 + 2k) +1
   Thus n2 = 2(2k2 + 2k) +1 for some integer k. By definition, we know that n2 is odd. Hence n2 is not even.

8. Fill in the blanks. **Use proof by contradiction**

Both p and ¬q are true

Proof. Suppose, for the sake of contradiction, that n^2 + 5 is_____odd_______ and n is also ___odd_____.

By definition, then, there exists integers k and l so that $n^2 + 5 =$ 2k+1 and n = 2k+1.

Hence, we have

$$2k + 1 = n^2 + 5$$
$$= (2k+1)^2 + 5$$
$$= 4k^2+4k+1 + 5$$
$$= 2(2k^2+2k+3).$$

Therefore, 2k + 1 is $(2k^2 + 2k + 3)$. This is clearly impossible, and hence we cannot have that $n^2 + 5$ is odd and n is also odd.

Therefore, if that $n^2 + 5$ is odd, we must have n is even.

9. Use **proof by contradiction** to show that if n is an integer, and n^3+5 is odd, then n is even

Both Q and ¬Q are true

Suppose, for the sake of contradiction, that n3 + 5 is odd and n is also odd .

By definition, then, there exists integers k so that $n^2 + 5 =$ 2k+1 and n = 2k+1 . Hence, we have

2k + 1 = n3 + 5

$= (2k + 1) + 5$

$= (8k^3 + 12k^2 + 6k + 1) + 5$

$= 8k^3 + 12k^2 + 6k + 6$

$= 2(4k^3 + 6k^2 + 3k + 3)$

Therefore, 2k+1 is $2(4k^3 + 6k^2 + 3k + 3)$. This is clearly impossible, and

hence we cannot have that $n^3 + 5$ is odd and n is also odd.

Therefore, if that $3n^2+5n+18$ is odd, we must have n is even.

10. Use **proof by contradiction** to show that if $3n^2+5n+18$ is even, then n is an even

Both Q and ¬Q are true

Proof. Suppose, for the sake of contradiction, that $3n^2+5n+18$ is odd and n is even.

By definition, then, there exists integers k so that $3n^2+5n+18 = 2k+1$ and $n = 2k$.

Hence, we have

$2k+1 = 3n^2+5n+18$

$= 3(2k)2 + 5(2k) + 18$

$= 3(4k^2) + 5(2k) + 18$

$= 12k^2 + 10k + 18$

$= 2(6k^2 + 5k +9)$

Therefore, 2k+1 is 2(6k2 + 5k + 9). This is clearly impossible, and hence we cannot have that 3n2 +5n+18 is odd and n is even.

Therefore, if that $3n^2+5n+18$ is odd, we must have n is not even