UNIVERSITI TEKNOLOGI MALAYSIA

www.utm.my

4b:

Logic Simplification

Simplification using Boolean Algebra

Simplification using Boolean algebra

- Simplification:
  - To reduce an expression to its simplest form
  - To change a form to a more convenient one for efficient implementation

- We use:-
  - basic laws, rules, theorems of Boolean algebra

- Practice makes perfect!

Example:

Simplify this expression

$$AB + A(B + C) + B(B + C)$$

$$AB + A(B + C) + B(B + C)$$

Step 1: Apply distributive law to the red terms

$$AB + AB + AC + BB + BC$$

Step 2: Apply rule 7 (BB = B) to the green term

$$AB + AB + AC + B + BC$$

Step 3: Apply rule 5 (AB + AB = AB) to the red terms

$$AB + AC + B + BC$$

Step 4: Apply rule 10 (B + BC = B) to the green terms

$$AB + AC + B$$

Step 5: Apply rule 10 (B + AB = B) to the red terms

$$AC + B$$

(Page: 104)

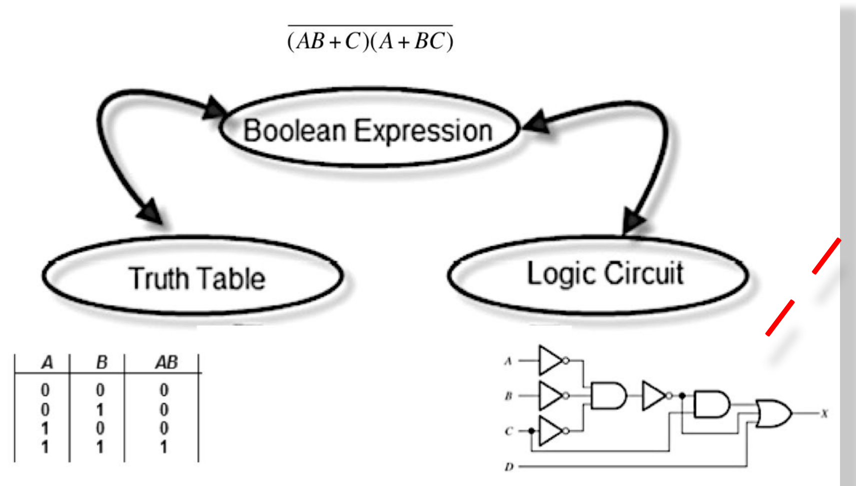
1	$A + 0 = A$
2	$A + 1 = 1$
3	$A \bullet 0 = 0$
4	$A \bullet 1 = A$
5	$A + A = A$
6	$A + \overline{A} = 1$
7	$A \bullet A = A$
8	$A \bullet \overline{A} = 0$
9	$\overline{\overline{A}} = A$
10	$A + AB = A$
11	$A + \overline{A}B = A + B$
12	$(A + B)(A + C) = A + BC$

Exercise 4b.1:

According the example given before, draw the logic circuit for the original expression and the last expression simplified.

$$AB + A(B+C) + B(B+C) \qquad\qquad B + AC$$

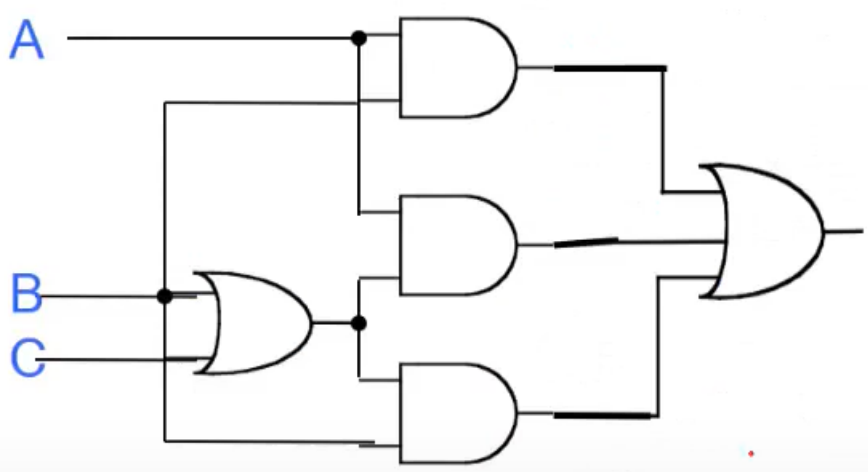
Recall

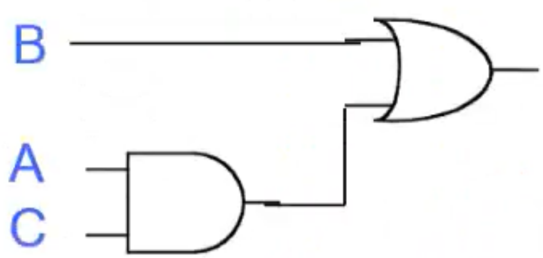


Solution 4b.1:

Before: $AB + A(B+C) + B(B+C)$

After: $B + AC$





Example:

(Page: 104)

1	$A + 0 = A$
2	$A + 1 = 1$
3	$A \bullet 0 = 0$
4	$A \bullet 1 = A$
5	$A + A = A$
6	$A + \overline{A} = 1$
7	$A \bullet A = A$
8	$A \bullet \overline{A} = 0$
9	$\overline{\overline{A}} = A$
10	$A + AB = A$
11	$A + \overline{A}B = A + B$
12	$(A + B)(A + C) = A + BC$

Simplify this expression

$$\overline{A}BC + A\overline{B}\,\overline{C} + \overline{A}\,\overline{B}\,\overline{C} + A\overline{B}C + ABC$$

$$\overline{A}BC + A\overline{B}\,\overline{C} + \overline{A}\,\overline{B}\,\overline{C} + A\overline{B}C + ABC$$

Step 1: Factor BC for the red terms

$$BC(\overline{A} + A) + A\overline{B}\,\overline{C} + \overline{A}\,\overline{B}\,\overline{C} + A\overline{B}C$$

Step 2: Apply rule 6 to the green term and factor the blue term

$$BC\,.\,1 + A\overline{B}(\overline{C} + C) + \overline{A}\,\overline{B}\,\overline{C}$$

Step 3: Apply rule 4 to the red term and rule 6 to the blue term

$$BC + A\overline{B}\,.\,1 + \overline{A}\,\overline{B}\,\overline{C}$$

Step 4: Apply rule 4 to the green term

$$BC + A\overline{B} + \overline{A}\,\overline{B}\,\overline{C}$$

Step 5: Factor the red terms

$$BC + \overline{B}(A + \overline{A}\,\overline{C})$$

Step 6: Apply rule 11 to the blue term

$$BC + \overline{B}(A + \overline{C})$$

Step 7: Use the distributive and commutative laws to get the following expression

$$BC + A\overline{B} + \overline{B}\,\overline{C}$$

Home work

Exercise 4b.2:

According the the example, draw the logic circuit for the original expression and the last expression simplified.

Original expression:

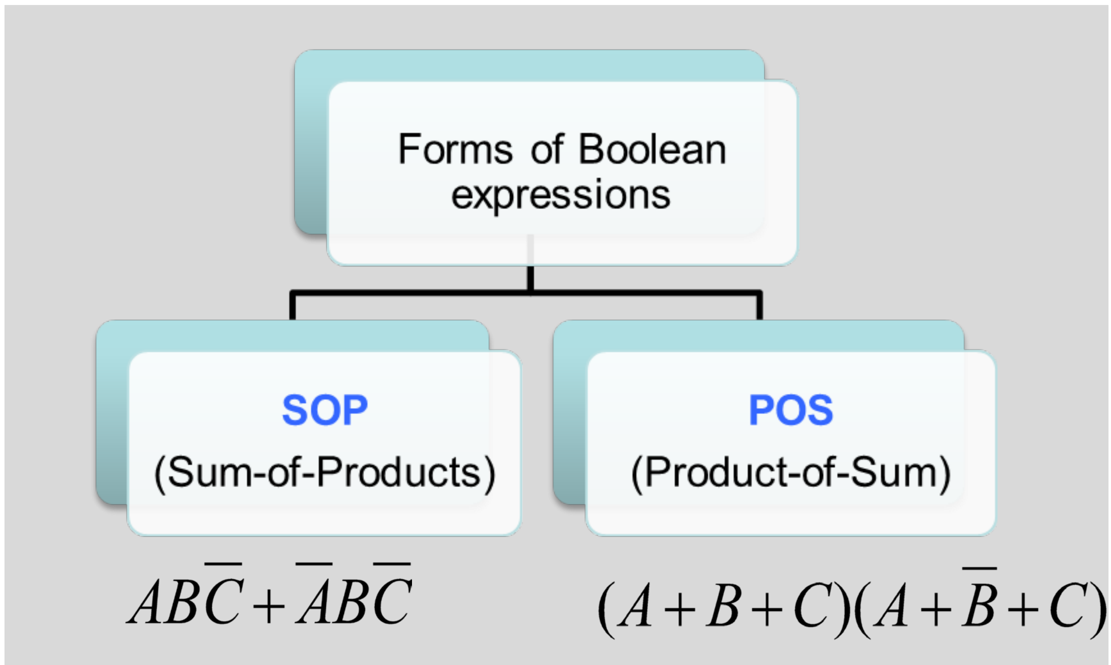
$$\overline{A}BC + A\overline{B}\,\overline{C} + \overline{A}\,\overline{B}\,\overline{C} + A\overline{B}C + ABC$$

Simplified expression: $BC + A\overline{B} + \overline{B}\,\overline{C}$

Forms of
Boolean Expressions

Forms of Boolean expressions

- Boolean expression can be converted into one of 2 forms.



Product term = a term with the product (Boolean multiplication) of literals

Sum term = a term with the sum (Boolean addition) of literals

(a) The Sum-of-Product (SOP) form

- **SOP** = when 2 or more product terms are summed.
- e.g $AB_{P1} + ABC_{P2}$

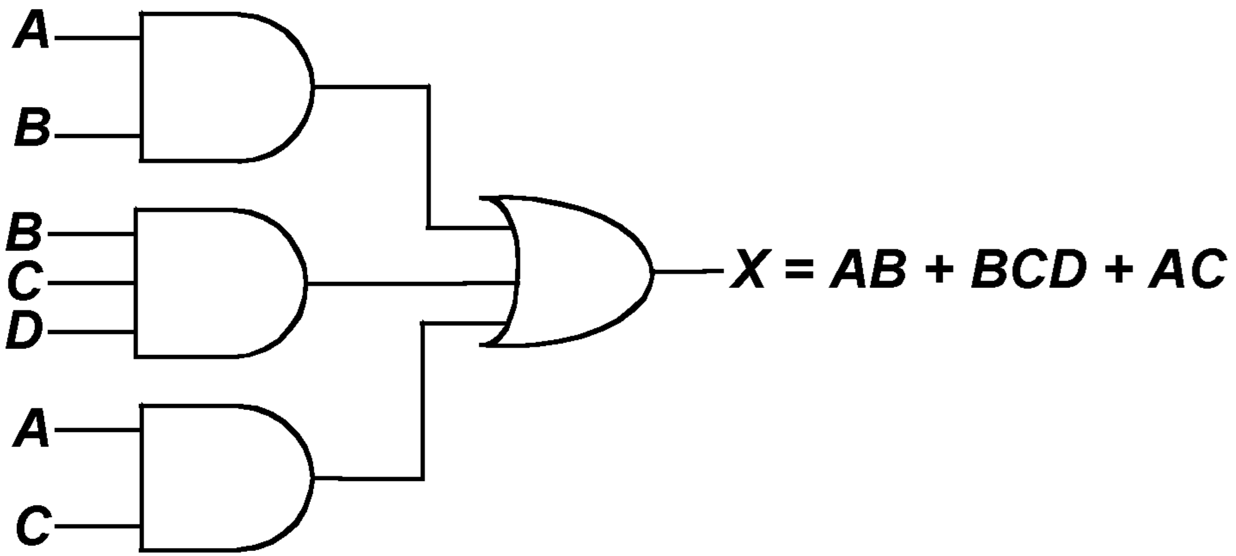
  $ABC_{P1} + CDE_{P2} + BCD_{P3}$
- SOP can also contain a single variable term

> - In SOP a single overbar cannot extend over more than 1 variable, but more than 1 variable can have an overbar.

$\bar{A}\bar{B}\bar{C}$ ☑ $\overline{ABC}$ ☒

- Implementation of the SOP expression

  AB + BCD + AC



Exercise 4b.3:

Convert each of the following Boolean expressions to SOP form:

Home work

(i) $AB + B(CD + EF)$

(ii) $(A + B)(B + C + D)$

(iii) $\overline{(\overline{A + B}) + C}$ =

=

i) AB + B + CDEF

ii) AB + BCD

iii) (A + B) + $\bar{\bar{C}}$

= AB + $\bar{C}$

(b) The Product-of-Sum (POS) form

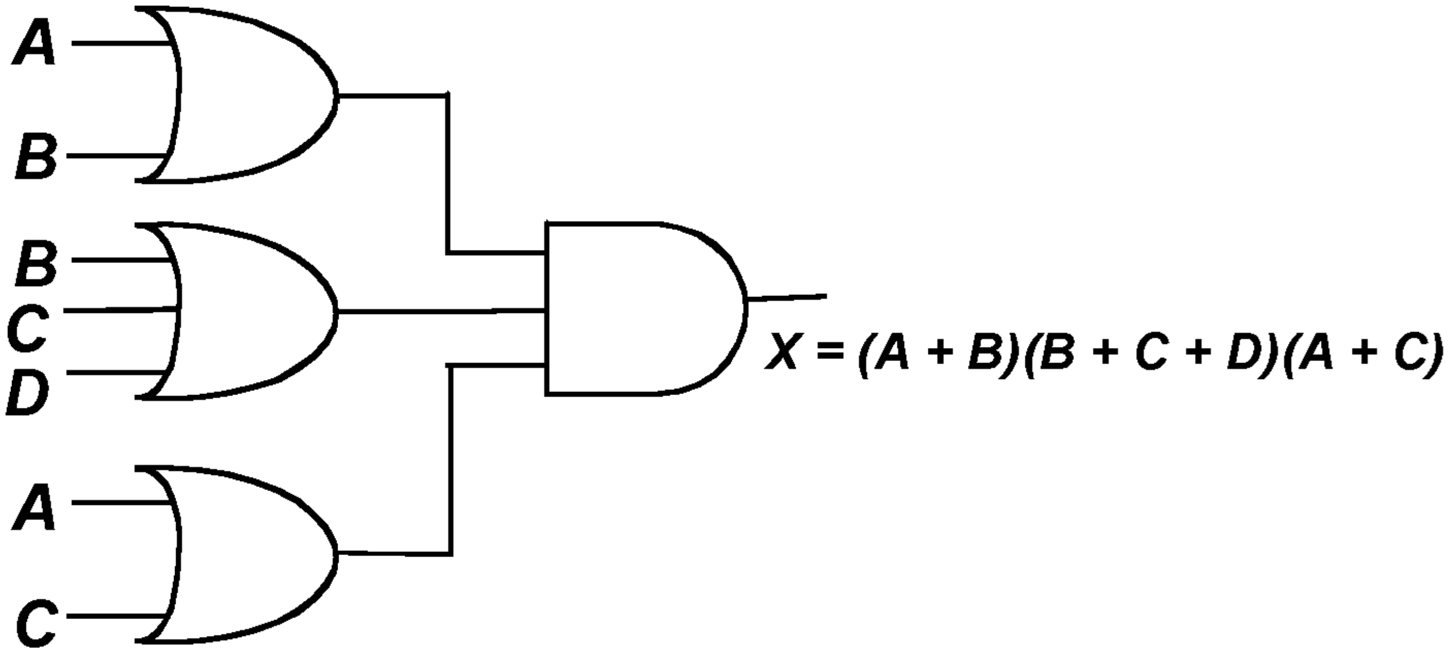
- **POS** = when 2 or more sum terms are multiplied.
  - $(A + B)_{S1}(A + B + C)_{S2}$
  - $(A + B + C)_{S1}(C + D + E)_{S2}(B + C + D)_{S3}$
- Like SOP, POS
  - can also contain a single variable term
  - a single overbar cannot extend over more than 1 variable, but more than 1 variable can have an overbar.

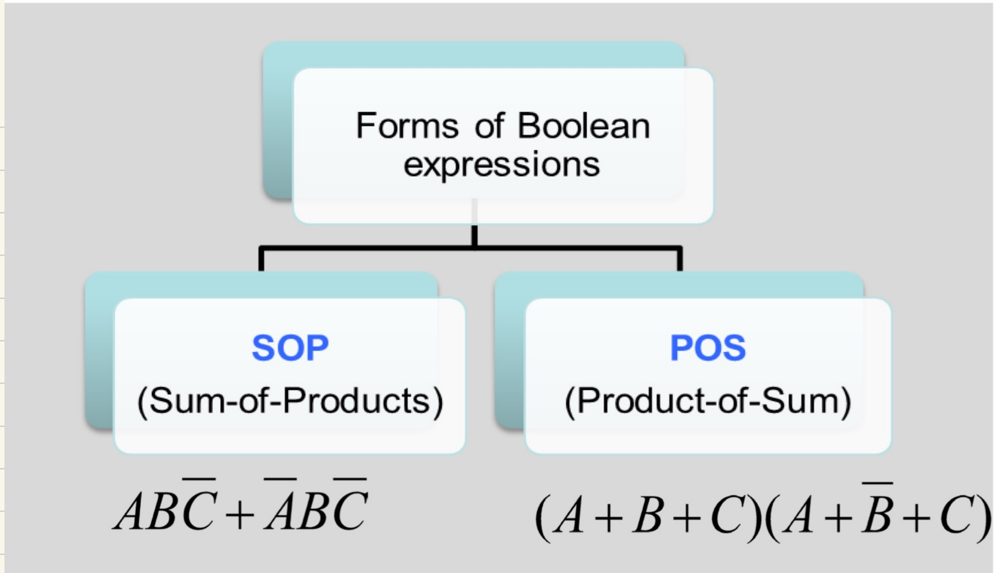
$$\overline{A} + \overline{B} + \overline{C}$$ ☑

$$\overline{A + B + C}$$ ☒

- Implementation of the POS expression

  (A + B)(B + C + D)(A + C)





SOP	POS
$A = 1$	$A = 0$
$\overline{A} = 0$	$\overline{A} = 1$

standard form

SOP

EXP: change to SOP then to Standard form

$= AB + (CD + EF)$

$SOP = AB + CDEF$

$2^{4} = 16$ number of variable missing

term 1 $= AB(C+\overline{C})(D+\overline{D})(E+\overline{E})(F+\overline{F})$

$= ABCDEF + ABCDE\overline{F} + ABCD\overline{E}F + ABC\overline{D}EF + AB\overline{C}DEF$
$ABCD\overline{E}\overline{F} + ABC\overline{D}E\overline{F} + AB\overline{C}DE\overline{F} + ABC\overline{D}\overline{E}F + AB\overline{C}\overline{D}EF$
$AB\overline{C}D\overline{E}F + AB\overline{C}\overline{D}\overline{E}F + ABC\overline{D}\overline{E}\overline{F} + AB\overline{C}D\overline{E}\overline{F} + AB\overline{C}\overline{D}E\overline{F}$
$AB\overline{C}\overline{D}\overline{E}\overline{F}$

Term 2 $= 2^{2} = 4$

$(A+\overline{A})(B+\overline{B})CDEF$

$= ABCDEF + A\overline{B}CDEF +$
$\overline{A}BCDEF + \overline{A}\overline{B}CDEF$

standard SOP = Term 1 + term 2

Standard form

POS

EXP: change to POS then to standard form

$AB + CDEF$

$POS = (A+B)(C+D+E+F)$

> rule 12
>
> $= A + AB$
>
> $= (A+A)(A+B)$

$2^4 = 16$

Term 1 $= (A+B)$

$= (A+B+C\cdot\bar{C}+D\cdot\bar{D}+E\cdot\bar{E}+F\cdot\bar{F})$

$= (A+B+C+D+E+F)(A+B+\bar{C}+D+E+F)(A+B+C+\bar{D}+E+F)(A+B+C+D+\bar{E}+F)$
$(A+B+C+D+E+\bar{F})(A+B+\bar{C}+\bar{D}+E+F)(A+B+\bar{C}+D+\bar{E}+F)(A+B+\bar{C}+D+E+\bar{F})$
$(A+B+C+\bar{D}+\bar{E}+F)(A+B+C+\bar{D}+E+\bar{F})(A+B+C+D+\bar{E}+\bar{F})(A+B+\bar{C}+\bar{D}+\bar{E}+F)$
$(A+B+C+\bar{D}+\bar{E}+\bar{F})(A+B+\bar{C}+D+\bar{E}+\bar{F})(A+B+\bar{C}+\bar{D}+E+\bar{F})(A+B+\bar{C}+\bar{D}+\bar{E}+\bar{F})$

Term 2 $= (C+D+E+F)$

$= (A\cdot\bar{A}+B\cdot\bar{B}+C+D+E+F)$

$= (A+B+C+D+E+F)(A+\bar{B}+C+D+E+F)(\bar{A}+B+C+D+E+F)(\bar{A}+\bar{B}+C+D+E+F)$

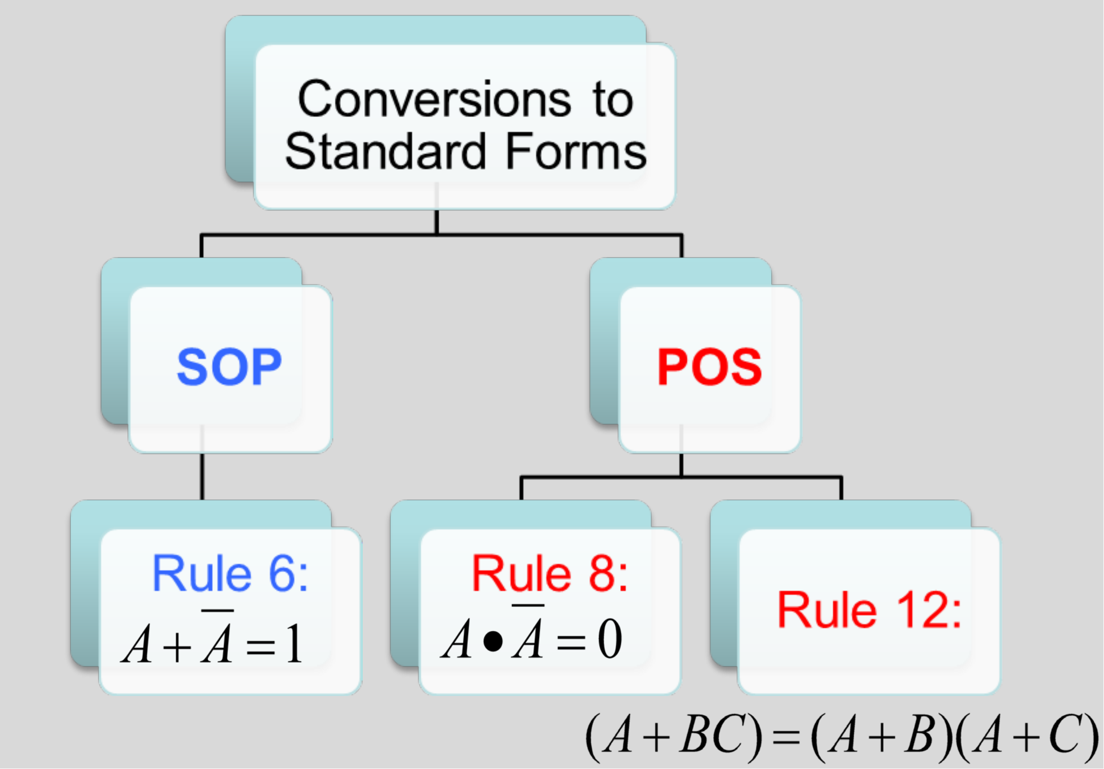
Standard POS = Term 1 + Term 2

Standard of
Boolean Expressions

Standard for of Boolean expressions

- Standardization makes the evaluation, simplification, and implementation of Boolean expressions more systematic and easier.

- A standard SOP / POS form is when ALL the variables appear in each product term / sum term of the expression.



(a) Product term → Standard SOP

- A logic expression can be changed to SOP form using Boolean algebra techniques.
  - $A(B + CD) =$
  - $AB + B(CD + EF) =$

- Standard SOP form is when all the variables appear in each product term in the expression.

$$A\overline{B}CD + \overline{A}\,\overline{B}C\overline{D} + AB\overline{C}\,\overline{D}$$

- To convert product terms to standard SOP
  - Multiply each of the nonstandard term with the missing term using Boolean algebra **Rule 6: $(A + \bar{A})=1$**
  - Repeat until all variables appear in each product term.

Example:

- Convert this Boolean expression to standard SOP form:

$$\underbrace{A\overline{B}C}_{\textit{Term 1}} + \underbrace{\overline{A}\,\overline{B}}_{\textit{Term 2}} + \underbrace{AB\overline{C}D}_{\textit{Term 3}}$$

Solution:

- Variables = A, B, C, D.
- What is missing?
  - Term 1: missing D
  - Term 2: missing C and D
- Complete these terms by applying Boolean rule 6

$$\underbrace{A\overline{B}C}_{\text{Term 1}} + \underbrace{\overline{A}\,\overline{B}}_{\text{Term 2}} + AB\overline{C}D$$

Rule 6: (A + Ā)=1

$$\text{Term 1}: A\overline{B}C = A\overline{B}C(D+\overline{D}) = A\overline{B}CD + A\overline{B}C\overline{D}$$

$$\text{Term 2}: \overline{A}\,\overline{B} = AB(C+\overline{C}) = \overline{A}\,\overline{B}C + \overline{A}\,\overline{B}\,\overline{C}$$

$$= \overline{A}\,\overline{B}C(D+\overline{D}) + \overline{A}\,\overline{B}\,\overline{C}(D+\overline{D}) = \overline{A}\,\overline{B}CD + \overline{A}\,\overline{B}C\overline{D} + \overline{A}\,\overline{B}\,\overline{C}D + \overline{A}\,\overline{B}\,\overline{C}\,\overline{D}$$

- Now we have the standard of SOP expression:

$$A\overline{B}C + \overline{A}\,\overline{B} + AB\overline{C}D$$
$$= A\overline{B}CD + A\overline{B}C\overline{D} + \overline{A}\,\overline{B}CD + \overline{A}\,\overline{B}C\overline{D} + \overline{A}\,\overline{B}\,\overline{C}D + \overline{A}\,\overline{B}\,\overline{C}\,\overline{D} + AB\overline{C}D$$

Exercise 4b.4: Define the variables of SOP expression $A\overline{C}+B\overline{C}$ and convert the expression to standard SOP form.

Solution 4b.4:

$$A\overline{C}+B\overline{C}=A\overline{C}(B+\overline{B})+B\overline{C}(A+\overline{A})$$ (Apply rule 6)

$$=AB\overline{C}+A\overline{B}\,\overline{C}+AB\overline{C}+\overline{A}B\overline{C}$$ (Apply rule 5)

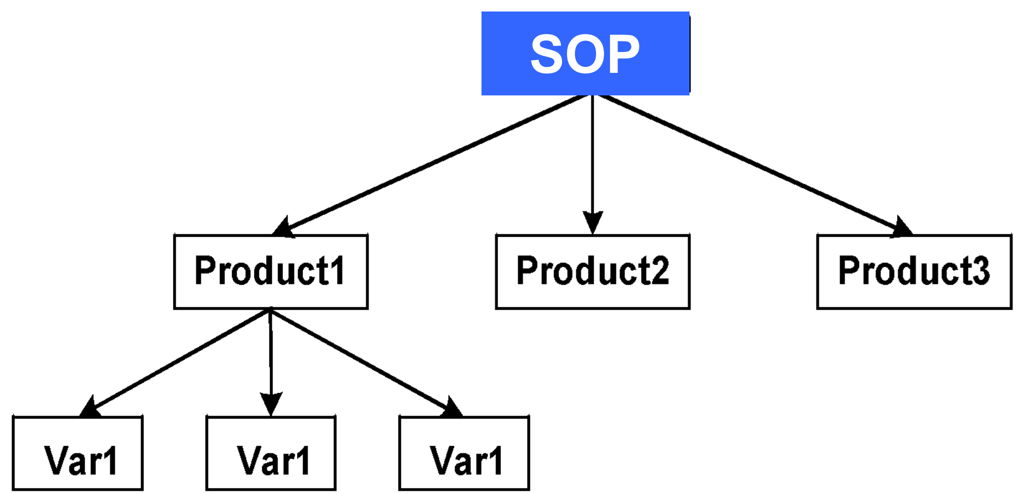
$$=AB\overline{C}+A\overline{B}\,\overline{C}+\overline{A}B\overline{C}$$

Binary representation of a Standard SOP

- A product = 1 only if ALL variables in the term is equal to 1.
  - Remember:
    product = multiplication → 1.1 = 1

$$AB\overline{C} + \overline{A}B\overline{C} = 1$$

- A sum = 1 when ONE or ALL of the variables in the term is equal to 1.
  - Remember:
    sum = addition
    1 + 0 = 1;
    0 + 1 = 1;
    1 + 1 = 1



Example: Determine the binary value for which the following standard SOP expression is equal to 1:

$$ABCD + A\overline{B}\overline{C}D + \overline{A}\overline{B}\overline{C}\overline{D}$$

The term ABCD is equal to 1 when A = 1, B = 1, C = 1, and D = 1

$$ABCD = 1.1.1.1 = 1$$

The term $A\overline{B}\overline{C}D$ is equal to 1 when A = 1, B = 0, C = 0, and D = 1

$$A\overline{B}\overline{C}D = 1.\overline{0}.\overline{0}.1 = 1.1.1.1 = 1$$

The term $\overline{A}\overline{B}\overline{C}\overline{D}$ is equal to 1 when A = 0, B = 0, C = 0, and D = 0

$$\overline{A}\overline{B}\overline{C}\overline{D} = \overline{0}.\overline{0}.\overline{0}.\overline{0} = 1.1.1.1 = 1$$

The SOP expression equals 1 when any or all of the three product terms is 1.

From the standard SOP it is easy to generate the truth table.

$$ABCD + A\bar{B}\bar{C}D + \bar{A}\bar{B}\bar{C}\bar{D}$$

$ABCD$

1 1 1 1

$A\bar{B}\bar{C}D$

1 0 0 1

$\bar{A}\bar{B}\bar{C}\bar{D}$

0 0 0 0

INPUT				OUTPUT
A	B	C	D	F
0	**0**	**0**	**0**	**1**
0	0	0	1	0
0	0	1	0	0
0	0	1	1	0
0	1	0	0	0
0	1	0	1	0
0	1	1	0	0
0	1	1	1	0
1	0	0	0	0
1	**0**	**0**	**1**	**1**
1	0	1	0	0
1	0	1	1	0
1	1	0	0	0
1	1	0	1	0
1	1	1	0	0
1	**1**	**1**	**1**	**1**

(b) Sum term → Standard POS

- Standard POS form = where all the variables appear in each sum term in the expression.
- To convert product terms to standard POS
  - Multiply each of the nonstandard term with the missing term using Boolean algebra **Rule 8: $(A \cdot \bar{A}) = 0$**
  - Apply **Rule 12: $(A + BC) = (A + B)(A + C)$**
  - Repeat until all variables appear in each sum term.

Example:

- Convert this Boolean expression to standard POS form

$$\underbrace{(A+\overline{B}+C)}_{\textit{Term 1}}\underbrace{(\overline{B}+C+\overline{D})}_{\textit{Term 2}}\underbrace{(A+\overline{B}+\overline{C}+D)}_{\textit{Term 3}}$$

Solution:

- Variables = A, B, C, D.
- What is missing?
  - Term 1: missing **D**
  - Term 2: missing **A**

$$\underbrace{(A+\overline{B}+C)}_{\text{Term 1}}\underbrace{(\overline{B}+C+\overline{D})}_{\text{Term 2}}(A+\overline{B}+\overline{C}+D)$$

- Apply rules 8 and 12

Rule 8: (A . Ā) = 0

Rule 12: (A + BC) = (A + B)(A + C)

$$\text{Term 1}: A+\overline{B}+C = A+\overline{B}+C+D\overline{D} = (A+\overline{B}+C+D)(A+\overline{B}+C+\overline{D})$$

$$\text{Term 2}: \overline{B}+C+\overline{D} = \overline{B}+C+\overline{D}+A\overline{A}$$

$$= (A+\overline{B}+C+\overline{D})(\overline{A}+\overline{B}+C+\overline{D})$$

- Now we have the standard of POS expression:

$+ D)$

Exercise 4b.5: Convert the following Boolean expression to standard POS form: $(A+B)(\overline{B}+C)$

Rule 8: (A . Ā) = 0

Rule 12: (A + BC) = (A + B)(A + C)

Solution 4b.5:

$$(A+B)(\overline{B}+C)$$

$$=(A+B+C\cdot\overline{C})(A\cdot\overline{A}+\overline{B}+C)$$

$$=(A+B+C)(A+B+\overline{C})(A+\overline{B}+C)(\overline{A}+\overline{B}+C)$$

- A product = **0** only if ONE or MORE of the sum term is equal to 0.
  - Remember:

    product = multiplication → 1.**0** = **0**

$$(A+B+C)(A+\overline{B}+C)=0$$

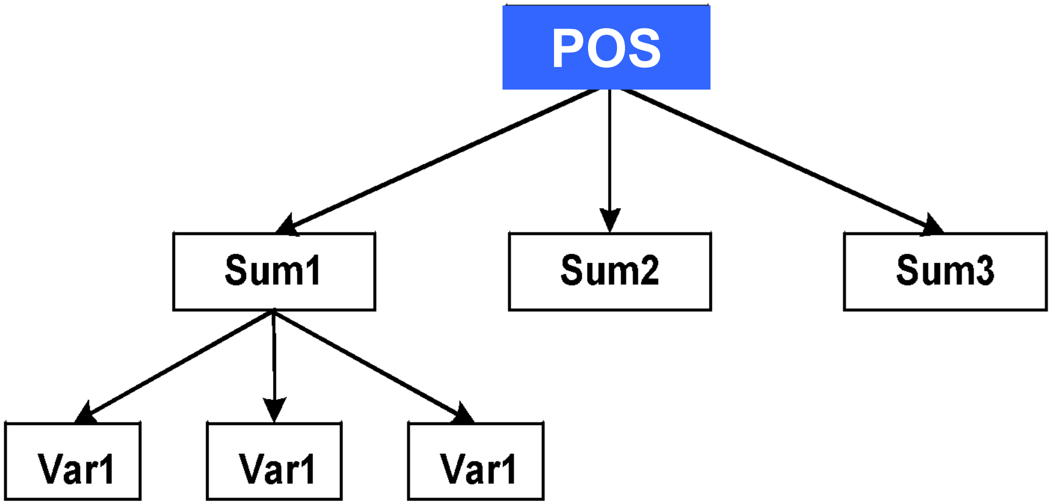
- A sum = 1 when ONE or ALL of the variables in the term is equal to 1.
  - Remember:

    sum = addition

    1 + 0 = 1;

    0 + 1 = 1;

    1 + 1 = 1



Example: Determine the binary value for which following standard POS expression is equal to **0**

$$(A+B+C+D)(A+\overline{B}+\overline{C}+D)(\overline{A}+\overline{B}+\overline{C}+\overline{D})$$

Solution:

$(A + B + C + D) = (0 + 0 + 0 + 0) = 0;$ A=0, B=0, C=0, D=0

$(A + \overline{B} + \overline{C} + D) = (0 + \overline{1} + \overline{1} + 0) = 0;$ A=0, B=1, C=1, D=0

$(\overline{A} + \overline{B} + \overline{C} + \overline{D}) = (\overline{1} + \overline{1} + \overline{1} + \overline{1}) = 0;$ A=1, B=1, C=1, D=1

The **POS** expression equal **0** when ALL of the terms are **0**.

From the standard POS, $(A+B+C+D)(A+\overline{B}+\overline{C}+D)(\overline{A}+\overline{B}+\overline{C}+\overline{D})$
it is easy to generate the truth table.

$(A+B+C+D)$

0 0 0 0

$(A+\overline{B}+\overline{C}+D)$

0 1 1 0

$(\overline{A}+\overline{B}+\overline{C}+\overline{D})$

1 1 1 1

INPUT				OUTPUT
A	B	C	D	F
0	0	0	0	0
0	0	0	1	1
0	0	1	0	1
0	0	1	1	1
0	1	0	0	1
0	1	0	1	1
0	1	1	0	0
0	1	1	1	1
1	0	0	0	1
1	0	0	1	1
1	0	1	0	1
1	0	1	1	1
1	1	0	0	1
1	1	0	1	1
1	1	1	0	1
1	1	1	1	0

Representation of Boolean Expressions

(Refer to module v5 page 122)

Boolean in SOP form can be represented as

1. Boolean expression , e.g. $X = \overline{A}B\overline{C} + A\overline{B}C + ABC$
2. By using a sigma notation , e.g. $X = \sum_{ABC} (2,5,7)$

Determine the terms that 2, 5 and 7 represents?

$\bar{A}B\bar{C} = 010$ $A\bar{B}C = 101$ $ABC = 111$

SOP

$A = 1$

$\overline{A} = 0$

(Refer to module v5 page 122)

Boolean in POS form can be represented as

1. Boolean expression, e.g. $X = (A+B+\overline{C})(\overline{A}+B+C)(\overline{A}+\overline{B}+C)$
2. By using a PI notation, e.g. $X = \prod_{ABC}(1, 4, 6)$

Determine the terms that 1, 4 and 6 represent?

POS

$\overline{A} = 1$

$A = 0$

$001 = (A+B+\overline{C})$ $100 = (\overline{A}+B+C)$ $110 = (\overline{A}+\overline{B}+C)$

Exercise 4b.6: Represent the following Boolean expression:

(i) $AB\overline{C}+A\overline{B}\overline{C}+\overline{A}B\overline{C}$ as a sigma notation

(ii) $(A+B+C)(A+B+\overline{C})(A+\overline{B}+C)(\overline{A}+B+C)$

as a PI notation.

Solution 4b.6(i):

Expression: $AB\overline{C}+A\overline{B}\overline{C}+\overline{A}B\overline{C}$

110 100 010

Sigma notation: $\boxed{\sum_{ABC}(6,4,2)}$

Exercise 4b.6: Represent the following Boolean expression:

(i) $AB\overline{C}+A\overline{B}\overline{C}+\overline{A}B\overline{C}$ as a sigma notation

(ii) $(A+B+C)(A+B+\overline{C})(A+\overline{B}+C)(\overline{A}+B+C)$

as a PI notation.

Solution 4b.6(ii):

Expression: $(A+B+C)(A+B+\overline{C})(A+\overline{B}+C)(\overline{A}+B+C)$ 000 001 010 100

PI notation: $\boxed{\prod_{ABC}(0,1,2,4)}$

Exercise 4b.7:

A Boolean expression is written in sigma notation as X = $\sum_{ABC}(7,4,3)$

Determine the logic level (binary value) for each product term and write whole expression.

Solution 4b.7:

$\sum_{ABC}(7,4,3)$

Logic level: 111 100 011

Expression: (SOP) $ABC + A\overline{B}\,\overline{C} + \overline{A}BC$

Exercise 4b.8:

A Boolean expression is written in PI notation as X = $\prod_{ABC}(7,4,3)$

Determine the logic level (binary value) for each sum term and write whole expression.

Solution 4b.8:

$\prod_{ABC}(7,4,3)$

Logic level: 111 100 011

Expression: (POS)

$$(\overline{A}+\overline{B}+\overline{C})(\overline{A}+B+C)(A+\overline{B}+\overline{C})$$

Representation boolean expression

SOP:

$A = 1$
$\bar{A} = 0$

exp: $AB\bar{C} + A\bar{B}\bar{C} + ABC$

$\begin{array}{ccc} \uparrow & \uparrow & \uparrow \\ 110 & 100 & 111 \\ 6 & 4 & 7 \end{array}$

$\sum_{ABC}(6,4,7)$

product term $= 1$

POS:

$A = 0$
$\bar{A} = 1$

exp: $(A + B + \bar{C})(\bar{A} + \bar{B} + C)$

$\begin{array}{cc} \downarrow & \downarrow \\ 0\ 0\ 1 & 1\ 1\ 0 \end{array}$

$\prod_{ABC}(1,6)$

sum term $= 0$

UTM
UNIVERSITI TEKNOLOGI MALAYSIA
www.utm.my

Converting Standard of Boolean Expressions (SOP ←→ POS)

Converting Standard (SOP ←→ POS)

- **Step 1**: Evaluate each product term in the **SOP** expression → i.e., determine the binary numbers of the **product terms**.

- **Step 2**: Determine all the binary numbers not included in Step 1.

- **Step 3**: Write equivalent **sum term** for each binary number from Step 2 and express in **POS form**.

** Using a similar procedure, to go from POS to SOP

Example: Convert the following SOP expression to an equivalent POS expression.

$$\bar{A}\bar{B}\bar{C} + \bar{A}B\bar{C} + \bar{A}BC + A\bar{B}C + ABC$$

Solution:

- Variables = (A, B, C) = 3. So, 2^3 = 8 possible combinations.

- The SOP has **5** of 8 from the input combination

$$\begin{array}{ccccccccc} \bar{A}\bar{B}\bar{C} & + & \bar{A}B\bar{C} & + & \bar{A}BC & + & A\bar{B}C & + & ABC \\ 2^2\,2^1\,2^0 & & 2^2\,2^1\,2^0 & & 2^2\,2^1\,2^0 & & 2^2\,2^1\,2^0 & & 2^2\,2^1\,2^0 \\ 0\,0\,0 & & 0\,1\,0 & & 0\,1\,1 & & 1\,0\,1 & & 1\,1\,1 \\ 0 & 1 & 2 & & 3 & 4 & 5 & 6 & 7 \end{array}$$

- Convert the above SOP expression to binary equivalent.

A	B	C	Output
0	0	0	1
0	0	1	
0	1	0	1
0	1	1	1
1	0	0	
1	0	1	1
1	1	0	
1	1	1	1

Example: Convert the following SOP expression to an equivalent POS expression.

$$\bar{A}\bar{B}\bar{C} + \bar{A}B\bar{C} + \bar{A}BC + A\bar{B}C + ABC$$

Solution:

A	B	C	Output
0	0	0	1
0	0	1	0
0	1	0	1
0	1	1	1
1	0	0	0
1	0	1	1
1	1	0	0
1	1	1	1

- The POS has the other 3 combinations which are 001, 100, 110; and this make sum term = 0
- These binaries combination (001, 100, 110) can create equivalent POS expression:

$$(A + B + \bar{C})(\bar{A} + B + C)(\bar{A} + \bar{B} + C)$$

0 0 1 1 0 0 1 1 0

1 4 6

Exercise 4b.9: Convert the following SOP expression to an equivalent POS expression:

$A\bar{C} (B + \bar{B}) + B\bar{C} (A + \bar{A})$

$$A\overline{C} + B\overline{C}$$

$AB\bar{C} + A\bar{B}\bar{C} + AB\bar{C} + \bar{A}B\bar{C}$

$AB\bar{C} + A\bar{B}\bar{C} + \bar{A}B\bar{C}$

Solution 4b.9:

- **Step 1**: Need to convert the expression into standard SOP (refer Exercise 4b.4)

$$AB\overline{C} + A\overline{B}\overline{C} + \overline{A}B\overline{C}$$

- **Step 2**: Binary number for each SOP term.

Variables = 3 (A, B, C); 2^3 = 8 possible combinations.

$$AB\overline{C} + A\overline{B}\overline{C} + \overline{A}B\overline{C}$$

1 1 0 1 0 0 0 1 0 (has 3 combinations)

6 4 2

0, 1, 3, 5, 7

… **previous**: $A B \overline{C}+A \overline{B}\,\overline{C}+\overline{A} B \overline{C}$

1 1 0   1 0 0   0 1 0

- **Step 3**: Equivalent sum term for each binary number (i.e., to get POS expression)

0 0 0   0 0 1   0 1 1   1 0 1   1 1 1   (has 5 combinations)

$(A + B + C)(A + B + \overline{C})(A + \overline{B} + \overline{C})(\overline{A} + B + \overline{C})(\overline{A} + \overline{B} + \overline{C})$ the POS expression

$\Updownarrow$

$A \overline{C}+B \overline{C}$

UNIVERSITI TEKNOLOGI MALAYSIA

www.utm.my

Boolean Expressions and Truth Tables

- **Step 1**: determine variables and combinations of binary values $\longrightarrow$ input

- **Step 2**: convert expression to Standard SOP / POS.

- **Step 3**: find the binary values that make the
  product terms = 1 (SOP) or
  sum terms = 0 (POS)

- **Step 4**: the remaining combination will be
  - Equal to 0 (POS)
  - Equal to 1 (SOP)

- Fill in the truth table

Example : Develop a truth table for the standard SOP expression

$$\bar{A}\bar{B}C + A\bar{B}\bar{C} + ABC$$

Solution

- Variables = A, B, C. combinations = $2^3 = 8$
- What binary value makes the product terms = **1**?

$\bar{A}\bar{B}C$
0 0 1 = 1 1 1 = 1

$A\bar{B}\bar{C}$
1 0 0 = 1 1 1 = 1

ABC
1 1 1 = 1 1 1 = 1

- Fill the truth table

INPUTS			OUTPUT	PRODUCT TERM
A	B	C	X	
0	0	0	0	
0	0	1	1	$\bar{A}\bar{B}C$
0	1	0	0	
0	1	1	0	
1	0	0	1	$A\bar{B}\bar{C}$
1	0	1	0	
1	1	0	0	
1	1	1	1	ABC

Example: Develop a truth table for the standard POS expression

$$(A + B + C)(A + \overline{B} + C)(A + \overline{B} + \overline{C})(\overline{A} + B + \overline{C})(\overline{A} + \overline{B} + C)$$

Solution

- Variables = A, B, C. combinations = $2^3 = 8$
- What binary value makes the sum terms = **0**?

$$(A + B + C)(A + \overline{B} + C)(A + \overline{B} + \overline{C})(\overline{A} + B + \overline{C})(\overline{A} + \overline{B} + C)$$

$$0 \quad 0 \quad 0 \quad 0 \quad 1 \quad 0 \quad 0 \quad 1 \quad 1 \quad 1 \quad 0 \quad 1 \quad 1 \quad 1 \quad 0$$

$$\underbrace{0 \quad 0 \quad 0}_{0} \; \underbrace{0 \quad 0 \quad 0}_{0} \; \underbrace{0 \quad 0 \quad 0}_{0} \; \underbrace{0 \quad 0 \quad 0}_{0} \; \underbrace{0 \quad 0 \quad 0}_{0}$$

- Fill the truth table

INPUTS			OUTPUT	PRODUCT TERM
A	B	C	X	
0	0	0	0	$(A + B + C)$
0	0	1	1	
0	1	0	0	$(A + \overline{B} + C)$
0	1	1	0	$(A + \overline{B} + \overline{C})$
1	0	0	1	
1	0	1	0	$(\overline{A} + B + \overline{C})$
1	1	0	0	$(\overline{A} + \overline{B} + C)$
1	1	1	1	

Determining Standard expression from a truth table

- Replace each binary number with its corresponding variable.

  SOP = 1→ var. & 0 → var. complement

  POS = 0→ var. & 1 → var. complement

- Example:

SOP

$$1010 \longrightarrow A\overline{B}C\overline{D}$$

$$A\overline{B}C\overline{D} = 1 \cdot \overline{0} \cdot 1 \cdot \overline{0}$$

$$= 1 \cdot 1 \cdot 1 \cdot 1 = 1$$

POS

$$1001 \longrightarrow \overline{A} + B + C + \overline{D}$$

$$\overline{A} + B + C + \overline{D} = \overline{1} + 0 + 0 + \overline{1}$$

$$= 0 + 0 + 0 + 0 = 0$$

Example:

From the truth table, determine the:

(i) standard SOP expression; and
(ii) equivalent standard POS expression

INPUT			OUTPUT
A	B	C	X
0	0	0	0
0	0	1	0
0	1	0	0
0	1	1	1
1	0	0	1
1	0	1	0
1	1	0	1
1	1	1	1

Solution

Convert the binary values to product terms (SOP).

INPUT			OUTPUT	PRODUCT TERMS
A	B	C	X	
0	0	0	0	
0	0	1	0	
0	1	0	0	
0	1	1	1	$\overline{A}BC$
1	0	0	1	$A\overline{B}\overline{C}$
1	0	1	0	
1	1	0	1	$AB\overline{C}$
1	1	1	1	ABC

Standard SOP expression:

$$\overline{A}BC + A\overline{B}\overline{C} + AB\overline{C} + ABC$$

Solution

Convert the binary values to sum terms (POS).

INPUT			OUTPUT	SUM TERMS
A	B	C	X	
0	0	0	0	$(A + B + C)$
0	0	1	0	$(A + B + \overline{C})$
0	1	0	0	$(A + \overline{B} + C)$
0	1	1	1	
1	0	0	1	
1	0	1	0	$(\overline{A} + B + \overline{C})$
1	1	0	1	
1	1	1	1	

Standard POS expression:

$$(A + B + C)\,(A + B + \overline{C})\,(A + \overline{B} + C)\,(\overline{A} + B + \overline{C})$$

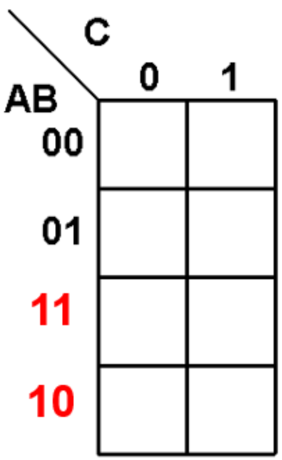
- ✓ Introduction to K-Map (Karnaugh Map)
- ✓ K-Map → SOP expression Simplification
- ✓ K-Map → POS expression Simplification

- ✓ Introduction to K-Map (Karnaugh Map)
- ✓ K-Map → SOP expression Simplification
- ✓ K-Map → POS expression Simplification

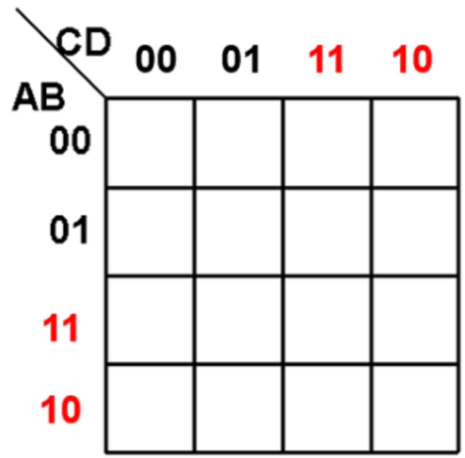
What is Karnaugh Map (K-Map)?

- Special form of a truth table which enables easier pattern recognition
- Pictorial method of simplifying Boolean expressions
- Good for circuit designs with up to 4 variables

- K-Map is similar to the truth table, but it **presents all the possible values of input and output**.
- This is shown in an **array of cells**.
- K-Maps can be used for expressions with **2, 3, 4 or 5** variables.



3 variables K-Map



4 variables K-Map

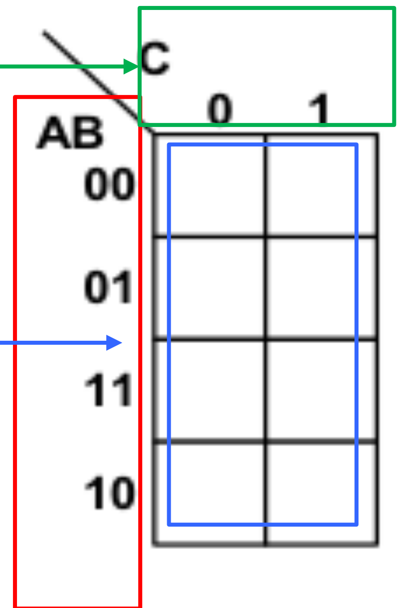
- The number of cells in a K-Map = total number of possible input variable combinations as in truth table

Karnaugh Map (K-Map)

$F = A + B\overline{C}$

A	B	C	F
0	0	0	0
0	0	1	0
0	1	0	1
0	1	1	0
1	0	0	1
1	0	1	1
1	1	0	1
1	1	1	1

- The equivalent K-Map:

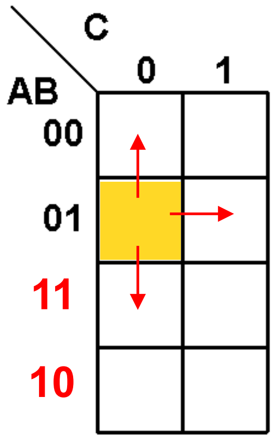


Example:

ABC, → 3 variables = 2^3 = 8 cells.

- Physically, cells that share their walls are **adjacent**
- Cells that **differ by only one variable** are adjacent (*bersebelahan*)

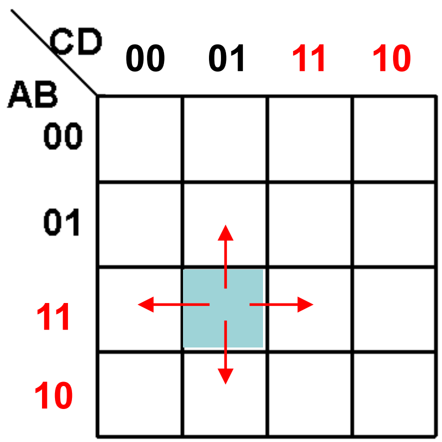
Cell **010** is adjacent to **000, 011** and **110**



AB \ C	0	1
00	$\bar{A}\bar{B}\bar{C}$	$\bar{A}\bar{B}C$
01	$\bar{A}B\bar{C}$	$\bar{A}BC$
11	$AB\bar{C}$	ABC
10	$A\bar{B}\bar{C}$	$A\bar{B}C$

3-Variable Karnaugh Map

Cell **1101** is adjacent to **0101, 1011, 1100** and **1111**

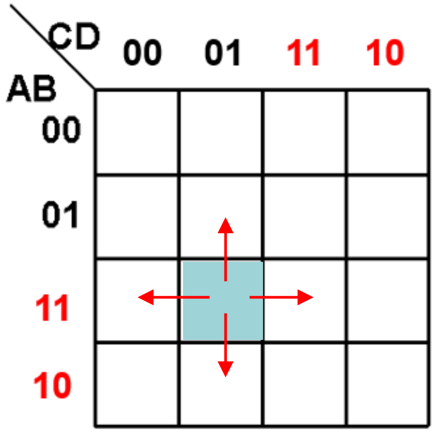


AB \ CD	00	01	11	10
00	$\bar{A}\bar{B}\bar{C}\bar{D}$	$\bar{A}\bar{B}\bar{C}D$	$\bar{A}\bar{B}CD$	$\bar{A}\bar{B}C\bar{D}$
01	$\bar{A}B\bar{C}\bar{D}$	$\bar{A}B\bar{C}D$	$\bar{A}BCD$	$\bar{A}BC\bar{D}$
11	$AB\bar{C}\bar{D}$	$AB\bar{C}D$	$ABCD$	$ABC\bar{D}$
10	$A\bar{B}\bar{C}\bar{D}$	$A\bar{B}\bar{C}D$	$A\bar{B}CD$	$A\bar{B}C\bar{D}$

4-Variable Karnaugh Map

- Physically, cells that share their walls are **adjacent**
- Cells that **differ by only one variable** are adjacent (*bersebelahan*)

Cell **1101** is adjacent to **0101, 1111, 1001** and **1100**

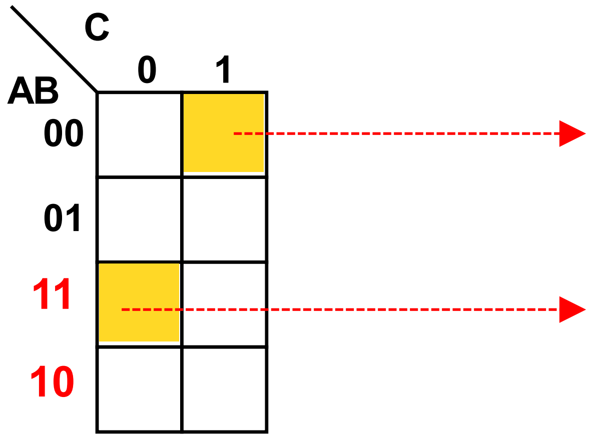


AB \ CD	00	01	11	10
00	$\bar{A}\bar{B}\bar{C}\bar{D}$	$\bar{A}\bar{B}\bar{C}D$	$\bar{A}\bar{B}CD$	$\bar{A}\bar{B}C\bar{D}$
01	$\bar{A}B\bar{C}\bar{D}$	$\bar{A}B\bar{C}D$	$\bar{A}BCD$	$\bar{A}BC\bar{D}$
11	$AB\bar{C}\bar{D}$	$AB\bar{C}D$	$ABCD$	$ABC\bar{D}$
10	$A\bar{B}\bar{C}\bar{D}$	$A\bar{B}\bar{C}D$	$A\bar{B}CD$	$A\bar{B}C\bar{D}$

4-Variable Karnaugh Map

POS:

Get the sum terms for each colored cells for the following K-Map.

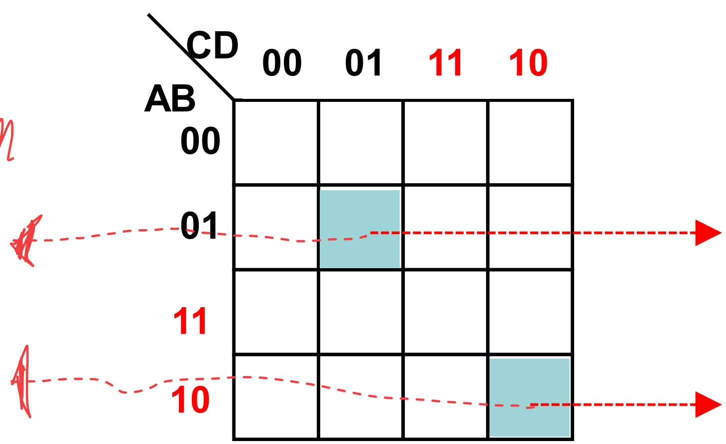


$A+B+\overline{C}$

$\overline{A}+\overline{B}+C$

Product form

Sum form



$\overline{A}B\overline{C}D$

$(A+\overline{B}+C+\overline{D})$

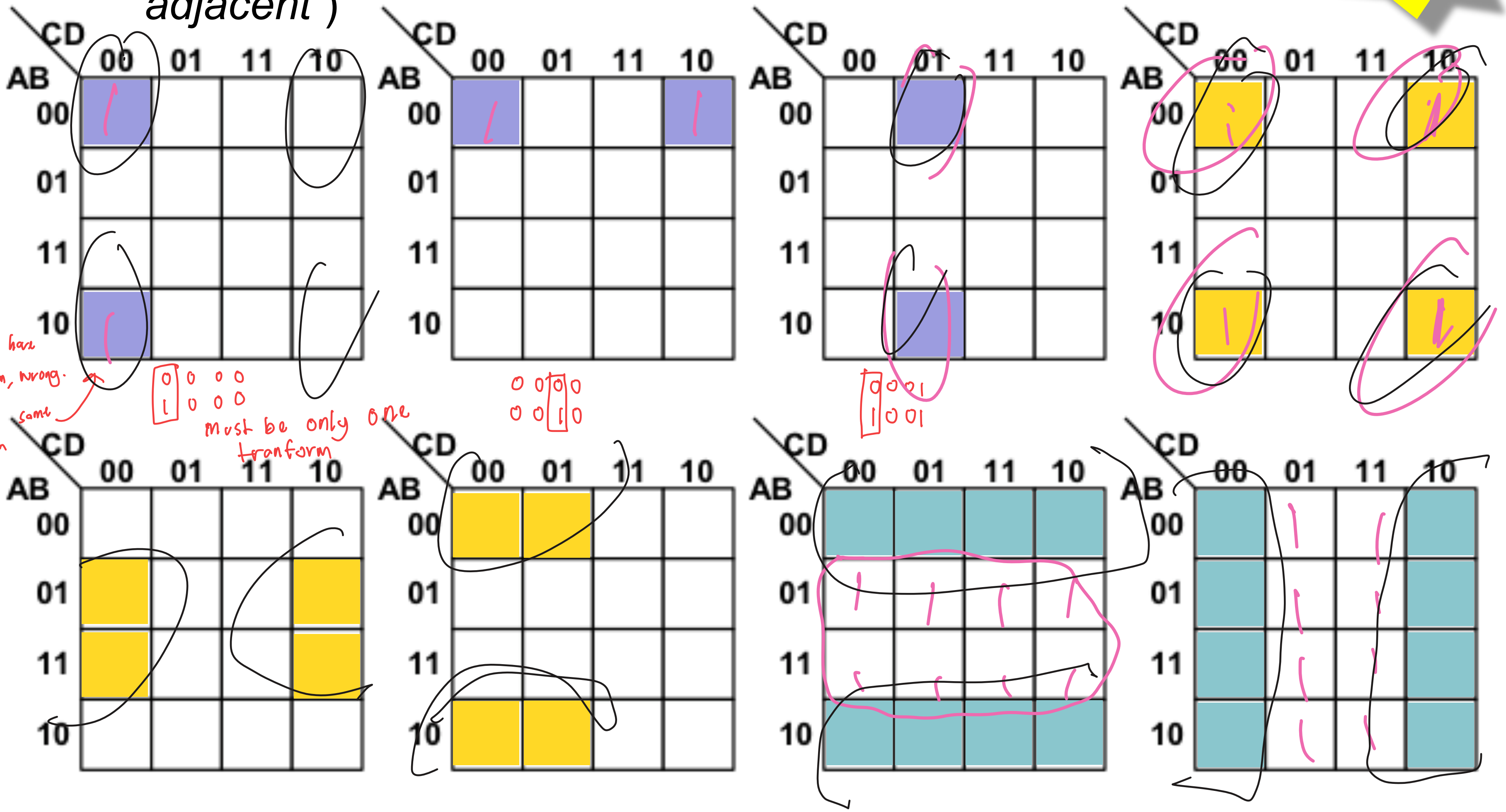
$A\overline{B}C\overline{D}$

$(\overline{A}+B+\overline{C}+D)$

6 4 2 1

Extra

- In a K-map with 4-variable or more, the top-most & bottom-most cells of a column (and row) are adjacent.
  - (*recall → cells that differ by only one variable are adjacent*)



0 0 0 0
1 1 0 0

2 tiles have tranform, wrong.

use the same orientation

2 cells

4 cells

8 cells

Mapping Standard Expression on a K-Map

- K-Map is used to simplify Boolean expressions to their **minimum form**.
- A minimized expression :
  - has the fewest possible term with each term having fewest possible variables.
  - needs fewer logic gates than standard expression.
- To map an SOP / POS expression to a K-map:
  - **Step 1:** determine the binary value of each product / sum term
  - **Step 2**: Place a 1 / 0 in a cell that have the same value as the product / sum term

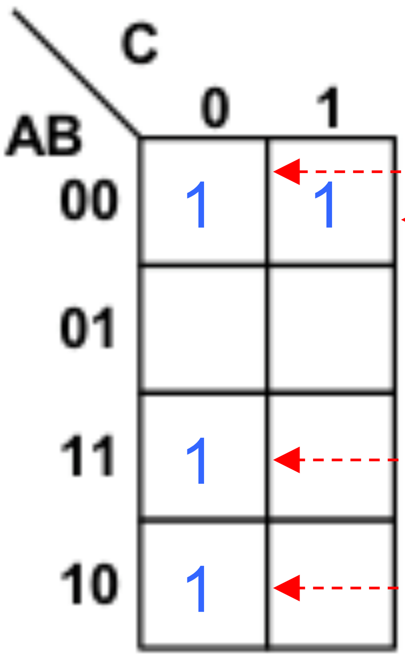
SOP

Example: 3 variables

- **Step 1:** determine the binary value of each product terms

$$\overline{A}\,\overline{B}\,\overline{C} + \overline{A}\,\overline{B}C + AB\overline{C} + A\overline{B}\,\overline{C}$$

0 0 0   0 0 1   1 1 0   1 0 0



- **Step 2:** Place a **1** in a cell that have the same value as the product terms

SOP

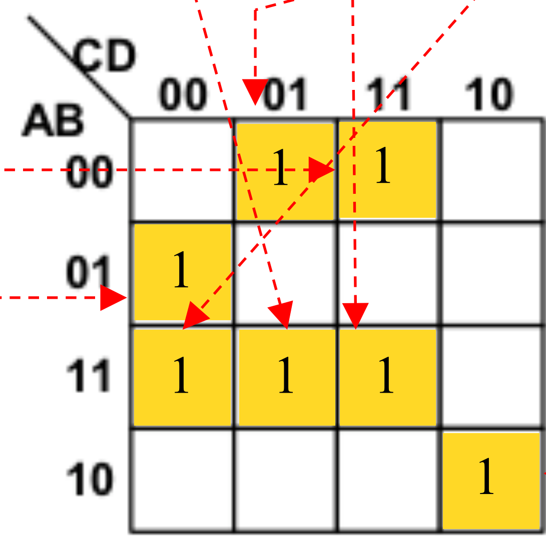
Example: 4 variables

- Map the following expression

$$\overline{A}\,\overline{B}CD+\overline{A}B\overline{C}\,\overline{D}+AB\overline{C}D+ABCD+AB\overline{C}\,\overline{D}+\overline{A}\,\overline{B}\,\overline{C}D+A\overline{B}C\overline{D}$$

0011 0100 1101 1111 1100 0001 1010

- **Step 1:** determine the binary value of each product terms



- **Step 2:** Place a **1** in a cell that have the same value as the product terms

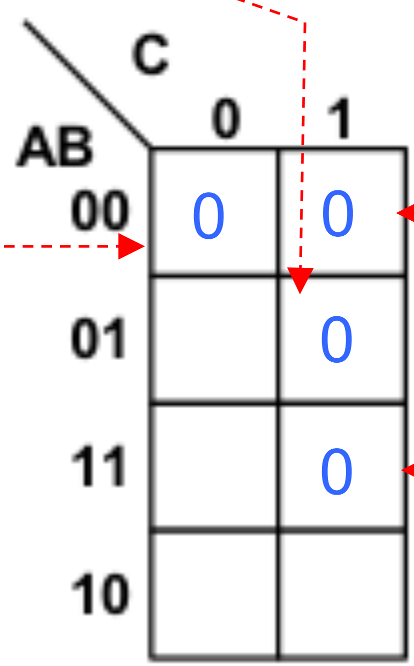
POS

Example: 3 variables

$$(A+B+C)(A+\overline{B}+\overline{C})(A+B+\overline{C})(\overline{A}+\overline{B}+\overline{C})$$

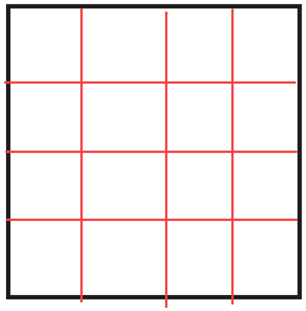
0 0 0 0 1 1 0 0 1 1 1 1

- **Step 1:** determine the binary value of each sum terms
- **Step 2:** Place a **0** in a cell that have the same value as the sum terms



Exercise 4b.12: Map the following expression on a K-map.

$$(A+B+C+D)(A+\overline{B}+\overline{C}+D)(\overline{A}+B+\overline{C}+D)$$



2

1 2

2

1 2

Mapping Non-Standard Expression on a K-Map

- To use K-Maps, expressions must be in standard form.
- For expressions that are not standard, it must be converted to a standard form.

- Recall (SOP) : $A\overline{B} + ABC$

Rule 6

$$A\overline{B}(C+\overline{C}) = A\overline{B}C + A\overline{B}\,\overline{C}$$

$$A\overline{B}C + A\overline{B}\,\overline{C} + ABC$$

1 0 1 1 0 0 1 1 1

Expand numerically to standard form by writing the binary value of given variable & attach possible binary value for missing variables the term.

$$A\overline{B} \quad + \quad ABC$$

1 0 0 1 1 1

1 0 1

Mapping Non-Standard Expression on a K-Map

AB \ C	0	1
00		
01		
11		1
10	1	1

$$A\overline{B}C + A\overline{B}\,\overline{C} + ABC$$

Expand numerically to standard form by writing the binary value of given variable & attach possible binary value for missing variables the term.

$$A\overline{B} \quad + \quad ABC$$

1 0 0   1 1 1

1 0 1

Example: Map the following SOP expression on a K-Map:

$$\bar{B}\bar{C} + A\bar{B} + AB\bar{C} + A\bar{B}C\bar{D} + \bar{A}\bar{B}\bar{C}D + A\bar{B}CD$$

Solution: Expand the terms by including all combinations of the missing variables numerically as follow:

$$(D+\bar{D})(A+\bar{A})\ \bar{B}\bar{C} + A\bar{B} + AB\bar{C} + A\bar{B}C\bar{D} + \bar{A}\bar{B}\bar{C}D + A\bar{B}CD$$

0000	1000	1100	1010	0001	1011
~~0001~~	1001	1101			
~~1000~~	~~1010~~				
~~1001~~	~~1011~~				

(Discard all similar binary values)

Map each binary values by placing 1 in the right cell of the 4-variables K-map

AB \ CD	00	01	11	10
00	1	1		
01				
11	1	1		
10	1	1	1	1

$$\overline{B}\,\overline{C} + A\overline{B} + AB\overline{C} + A\overline{B}C\overline{D} + \overline{A}\,\overline{B}\,\overline{C}D + A\overline{B}CD$$

0000 1000 1100 1010 0001 1011

1001 1101

Example: Map the following POS expression on a K-Map:

$$(A+B+D)(\overline{A}+\overline{B})(\overline{A}+B+C+\overline{D})$$

Solution :

Expand the terms by including all combinations of the missing variables numerically for each sum term.

Example: Map the following POS expression on a K-Map:

$$(A+B+D)(\overline{A}+\overline{B})(\overline{A}+B+C+\overline{D})$$

Solution : $A+B+C+D$

0 0 0 0   0 0 0 0

0 0 1 0   0 0 1 0

Example: Map the following POS expression on a K-Map:

$$(A+B+D)(\overline{A}+\overline{B})(\overline{A}+B+C+\overline{D})$$

Solution :

$$\overline{A}+\overline{B}+C+D$$

0	0	0	0	1	1	0	0
0	0	1	0	1	1	0	1
1	1	0	0	1	1	1	0
1	1	0	1	1	1	1	1
1	1	1	0				
1	1	1	1				

Example: Map the following POS expression on a K-Map:

$$(A+B+D)(\overline{A}+\overline{B})(\overline{A}+B+C+\overline{D})$$

Solution :

1 0 0 1

0 0 0 0
0 0 1 0

1 1 0 0
1 1 0 1
1 1 1 0
1 1 1 1

1 0 0 1

Example: Map the following POS expression on a K-Map:

$$(A+B+D)(\overline{A}+\overline{B})(\overline{A}+B+C+\overline{D})$$

Solution :

0 0 0 0
0 0 1 0

1 1 0 0
1 1 0 1
1 1 1 0
1 1 1 1

1 0 0 1

AB \ CD	00	01	11	10
00	0			0
01				
11	0	0	0	0
10		0		

- ✓ Introduction to K-Map (Karnaugh Map)
- ✓ K-Map → SOP expression Simplification
- ✓ K-Map → POS expression Simplification

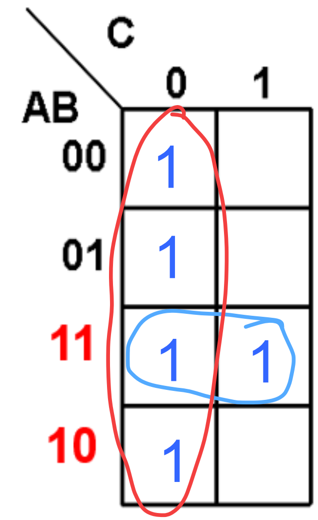
K-Map simplification of SOP expression

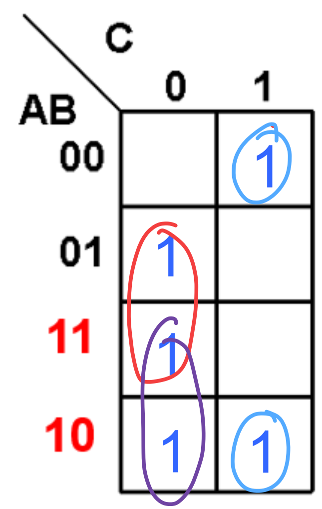
- There are 3 steps to obtain a minimum SOP expression from a K-Map.

1. Grouping the binary **1s**
2. Determine product term for each group
3. Summing the resulting product terms

STEP 1: Grouping the binary 1s

- Group must contain cells in 2^x combination (i.e., 1, 2, 4, 8, 16)
- Each cell must be adjacent to at least 1 other cell in the group, but all cells in a group need not be adjacent
- Try to have the biggest possible group of 1s
- Each binary 1 must be in at least one group.
- The 1s already in a group may be included in another group so long as the overlapping group includes non-common 1s





- Here are some examples:

(a)

AB \ C	0	1
00	1	
01		1
11	1	1
10		

AB \ C	0	1
00	1	
01		1
11	1	1
10		

(b)

AB \ C	0	1
00	1	1
01	1	
11		1
10	1	1

AB \ C	0	1
00	1	1
01	1	
11		1
10	1	1

(c)

AB \ CD	00	01	11	10
00	1	1		
01	1	1	1	1
11				
10		1	1	

AB \ CD	00	01	11	10
00	1	1		
01	1	1	1	1
11				
10		1	1	

(d)

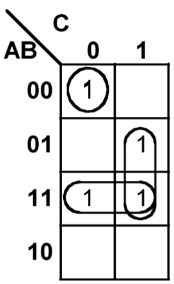
AB \ CD	00	01	11	10
00	1			1
01	1	1		1
11	1	1		1
10	1		1	1

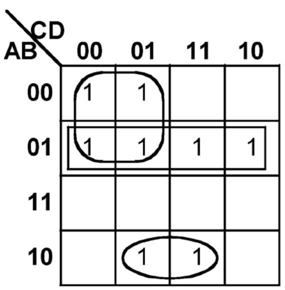
AB \ CD	00	01	11	10
00	1			1
01	1	1		1
11	1	1		1
10	1		1	1

STEP 2: Determining minimum SOP from a K-Map

- ❶ Within each group made, only choose the variables that occur in one form (or remain unchanged): complemented or un-complemented. This is called *contradictory variables*.

- ❷ determine the minimum product term for each group with the *same variables*.



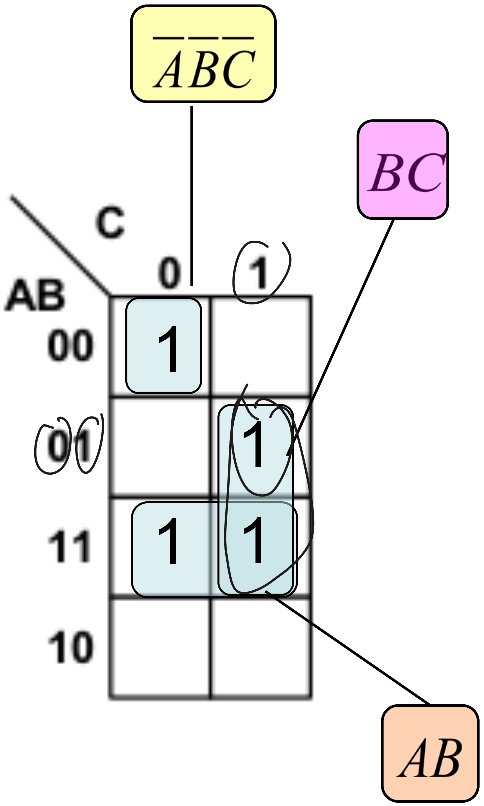


- ❸ When the minimum product terms are derived, sum them to form the minimum SOP

Example:

(Module v5: Page 137)

- Determine the product terms for each of the K-maps and
- write the minimum SOP expression.



A	B	C

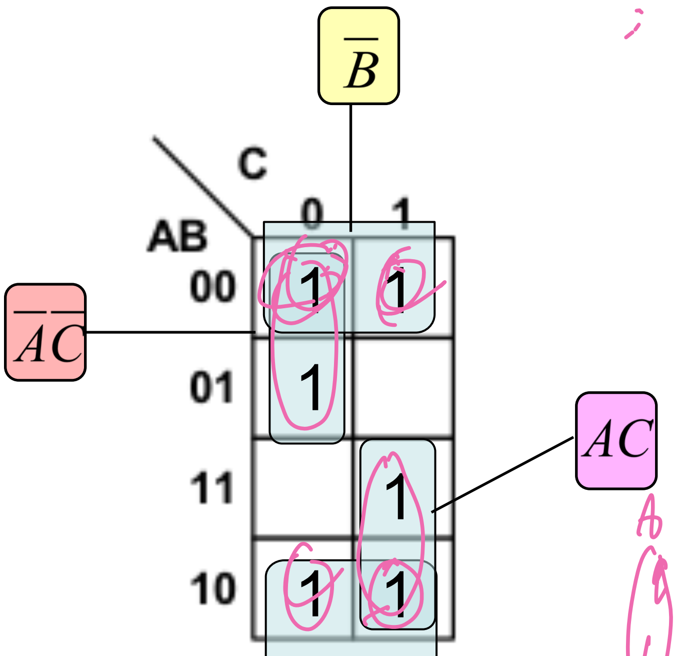
(a) $\overline{A}\overline{B}\overline{C} + AB + BC$

(Module v5: Page 137)

Example:

- Determine the product terms for each of the K-maps and
- write the minimum SOP expression.

when 3 variable
4 group = 1 variable
2 group = 2 variable
1 group = 3 variable



A	B	C

(b) $\overline{A}\,\overline{C}+\overline{B}+AC$

Example:

when 4 variable

(Module v5: Page 137)

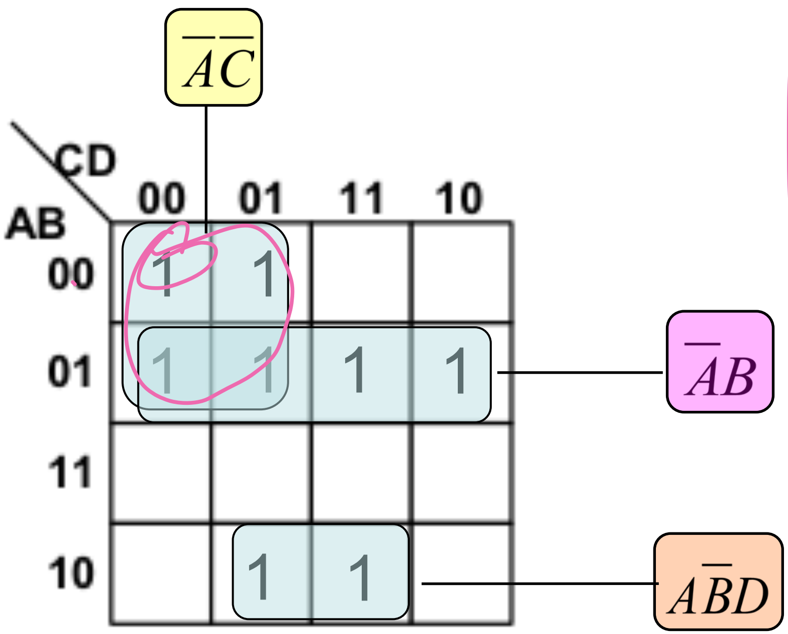
8 group = 1 variable

4 group = 2 variable

2 group = 3 variable

1 group = 4 variable

- Determine the product terms for each of the K-maps and
- write the minimum SOP expression.

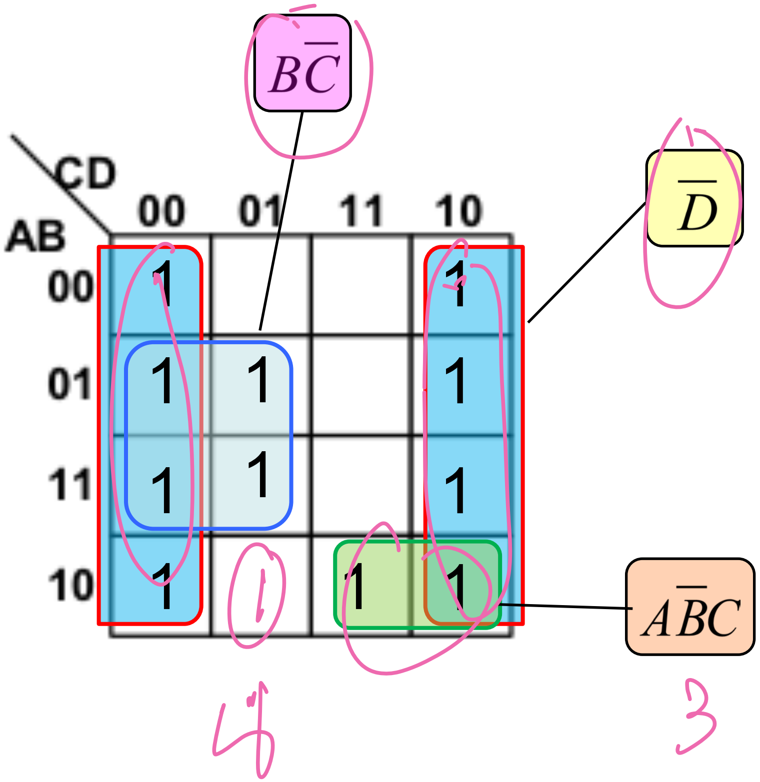


A	B	C	D

(c) $A\bar{B}D + \bar{A}\bar{C} + \bar{A}B$

Example:

- Determine the product terms for each of the K-maps and
- write the minimum SOP expression.



A	B	C	D
0	1	0	0
0	1	0	1
1	1	0	0
1	1	0	1

A	B	C	D

(d) $A\overline{B}C + B\overline{C} + \overline{D}$

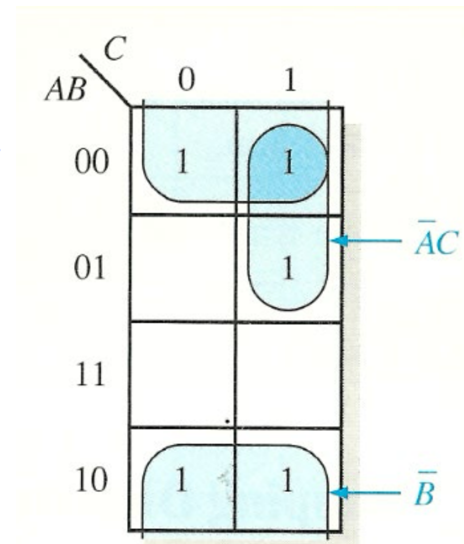
Exercise 4b.13: Use a Karnaugh map to minimize the following standard SOP.

$$A\overline{B}C + \overline{A}BC + \overline{A}\,\overline{B}C + \overline{A}\,\overline{B}\,\overline{C} + A\overline{B}\,\overline{C}$$

Solution 4b.13: The binary values of the expression.

1 0 1 0 1 1 0 0 1 0 0 0 1 0 0

Map the expression and group the cells.



The resulting minimum SOP Expression is =

$$\overline{B} + \overline{A}C$$

Mapping directly K-Map from a truth table

Example:

$$X = \bar{A}\bar{B}\bar{C} + A\bar{B}\bar{C} + AB\bar{C} + ABC$$

INPUTS			OUTPUT
A	B	C	X
0	0	0	1
0	0	1	0
0	1	0	0
0	1	1	0
1	0	0	1
1	0	1	0
1	1	0	1
1	1	1	1

AB \ C	0	1
00	1	
01		
11	1	1
10	1	

‘Don’t Care’ conditions

- Sometime there are certain combinations of input variable that is not allowed.
  - E.g. there are 6 invalid combinations in BCD code (1010, 1011, 1100, 1101, 1110, 1111)

- These states will never occur in an application using BCD code, hence they can be treated as “*don’t care*” terms.

- “*don’t care*” terms may be assigned either a 1 or a 0; it doesn't matter as it will never occur.

Recall Module 2

Binary Coded Decimal (BCD)

- BCD is a way to express each of the decimal digits with a binary code.
- There are only 10 code groups in the BCD system, one for every digit (0000 – 1001)

Decimal	BCD	Decimal	BCD
0	0000	5	0101
1	0001	6	0110
2	0010	7	0111
3	0011	8	1000
4	0100	9	1001

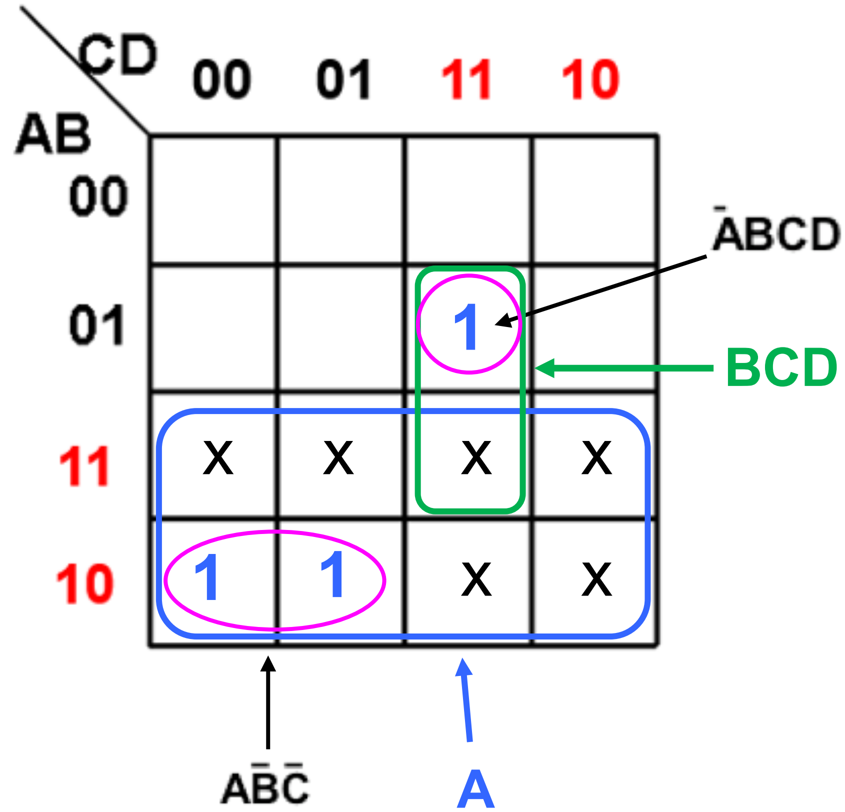
Invalid codes are 1010, 1011, 1100, 1101, 1110, 1111

- "*don't care*" terms can be used with K-maps to produce better results
  - Used as part of a group ➔ the larger the group, the simpler the resulting term will be
  - When grouping 1s (SOP), the Xs can be treated as 1 to make the group larger
  - the Xs is treated as 0s if they cannot be used to advantage.

Example: BCD codes

INPUTS				OUTPUT
A	B	C	D	Y
0	0	0	0	0
0	0	0	1	0
0	0	1	0	0
0	0	1	1	0
0	1	0	0	0
0	1	0	1	0
0	1	1	0	0
0	1	1	1	1
1	0	0	0	1
1	0	0	1	1
1	0	1	0	X
1	0	1	1	X
1	1	0	0	X
1	1	0	1	X
1	1	1	0	X
1	1	1	1	X

Invalid input for BCD



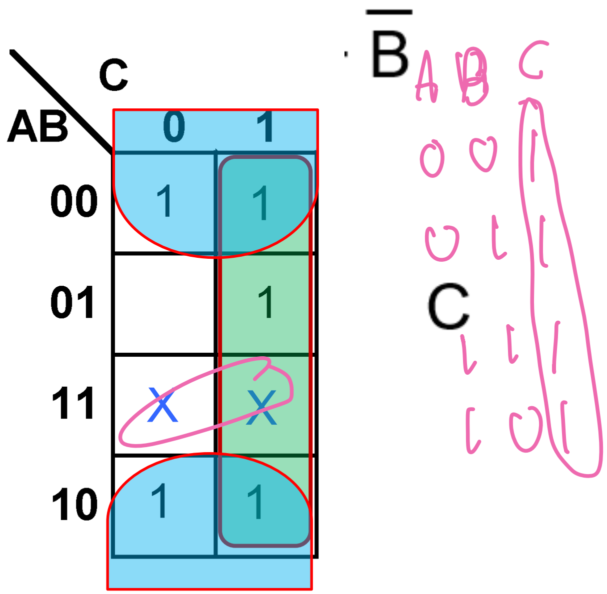
Without "*don't cares*" $Y = A\bar{B}\bar{C} + \bar{A}BCD$

With "don't care" $Y = A + BCD$

Exercise 4b.14: Use a Karnaugh map to minimize a standard SOP expression $A\bar{B}C + \bar{A}BC + \bar{A}\bar{B}C + \bar{A}\bar{B}\bar{C} + A\bar{B}\bar{C}$

(Use "*don't care*" terms, $d(6,7)$ into the K-map to produce better results)

Solution 4b.14:



The binary values of the expression.

1 0 1   0 1 1   0 0 1   0 0 0   1 0 0

The binary value of "don't care" notation.

1 1 0   1 1 1

Map the expression and group the cells.

Expression: $\bar{B} + C$

Exercise 4b.20: Use a Karnaugh map to minimize an expression

$$\sum_{ABCD}(2,5,8,9,11)$$

(Use "*don't care*" terms, $d(0,1,10)$ into the the K-maps to produce better results)

AB \ CD	00	01	11	10
00		1	1	
01	1	X	X	
11	1	X	1	1
10		1	1	

A	B	C	D
0	1	0	0
1	1	0	0

$B\bar{C}$

A	B	C	D
0	0	0	1
0	0	1	1
1	1	1	1
1	0	0	1
1	0	1	1

D

A	B	C	D
1	1	1	1
1	1	1	0
1	1	0	0

AB

Expression $= B\bar{C} + D + AB$

- ✓ Introduction to K-Map (Karnaugh Map)
- ✓ K-Map → SOP expression Simplification
- ✓ K-Map → POS expression Simplification

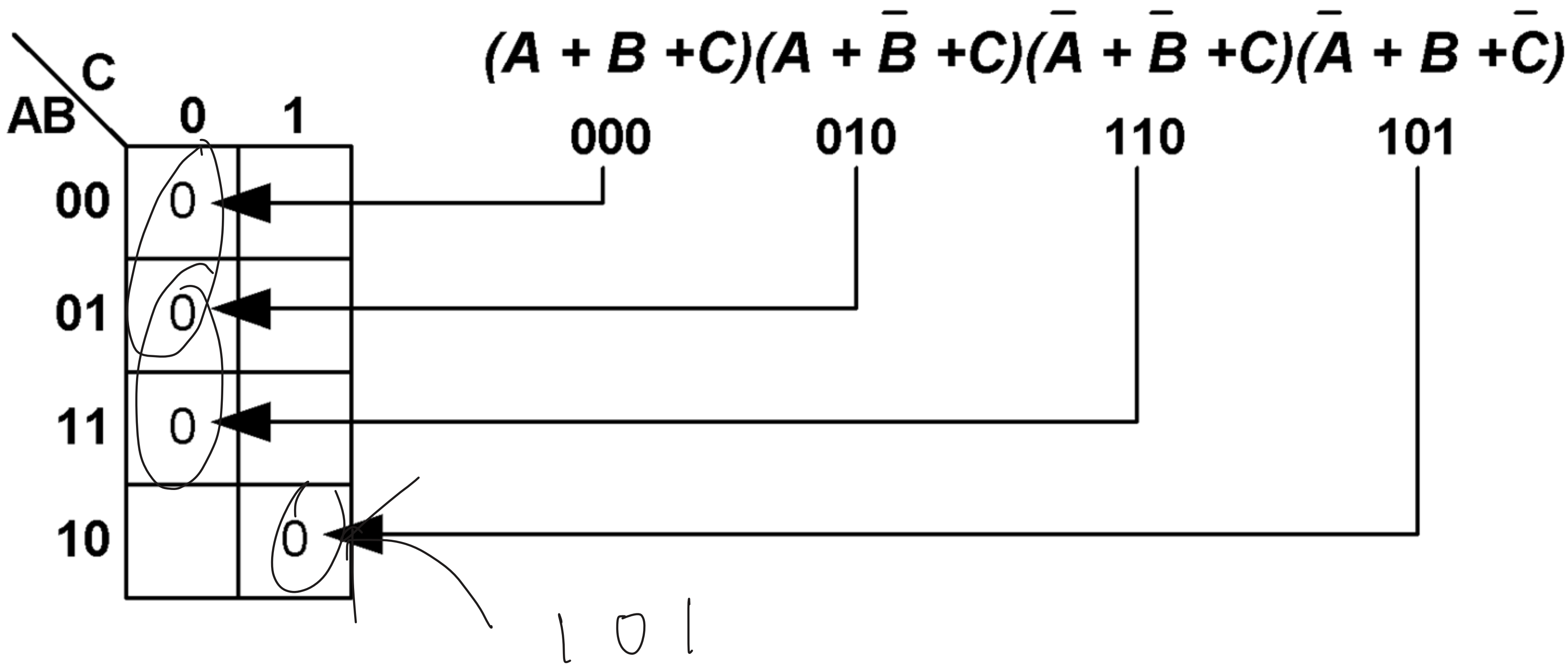
K-Map simplification of POS expression

- For POS expression in standard form, a **0** is put in the K-map for each sum term.
- The method is similar to SOP minimization, except 0 is used.

- To map a Standard POS expression:
  - Step 1: Determine the binary value of each sum term (i.e. that makes the sum term = 0)
  - Step 2: Check result and place a 0 on the corresponding cell in K-map

continue...

Example: Try mapping the following Standard POS expression into the K-Map:

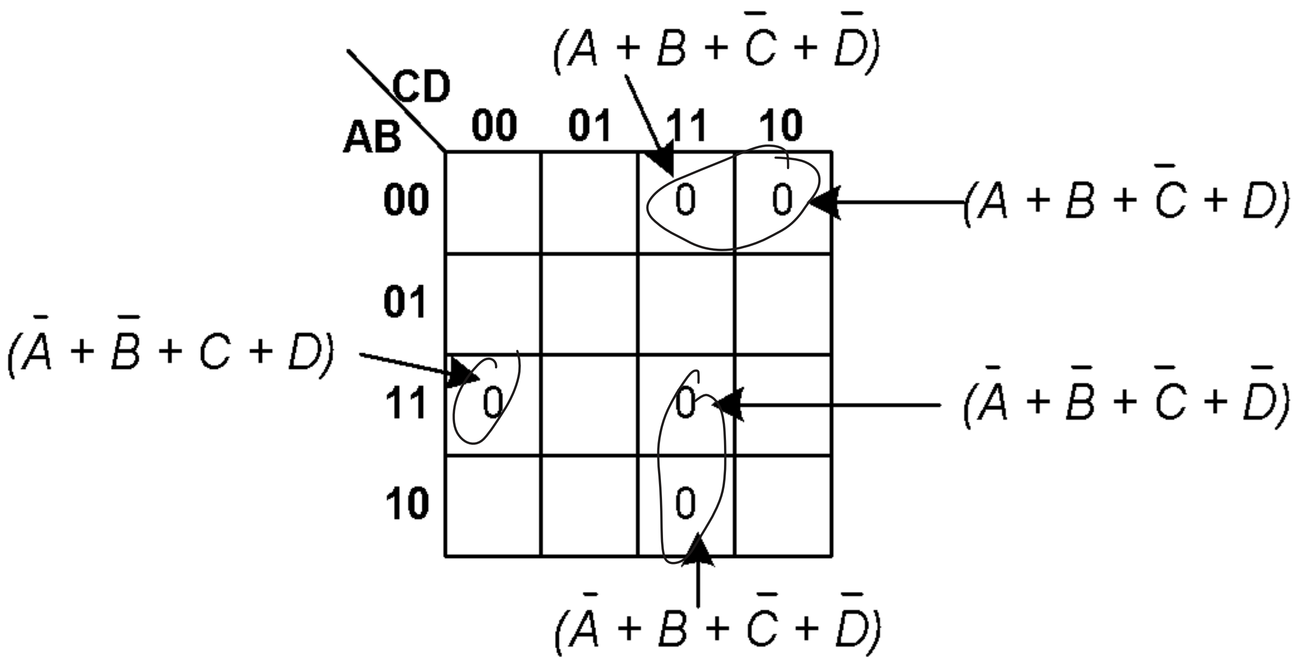


(The method is similar to SOP minimization, except **0** is used for POS)

Example: Try mapping the following Standard POS expression into the K-Map:

$$(\bar{A}+\bar{B}+C+D)(\bar{A}+B+\bar{C}+\bar{D})(A+B+\bar{C}+D)(\bar{A}+\bar{B}+\bar{C}+\bar{D})(A+B+\bar{C}+\bar{D})$$

1100 1011 0010 1111 0011



1111
1011
$\bar{A}$ $\bar{C}$ $\bar{D}$

K-Map simplification of POS expression

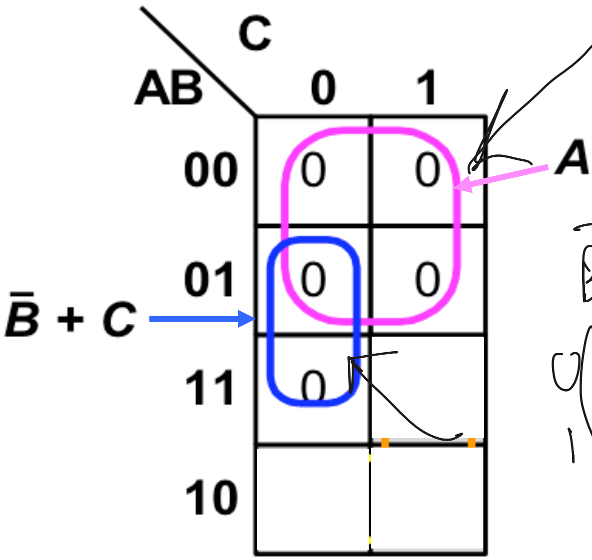
- The process is basically the same as with SOP expressions:
  - Group the 0s instead of 1s
  - The rules of grouping 0s are the same as those for 1s
  - Expression must be in Standard POS form

Example 1: $(A + B + C)(A + B + \bar{C})(A + \bar{B} + C)(A + \bar{B} + \bar{C})(\bar{A} + \bar{B} + C)$

$0+0+0 \quad 0+0+1 \quad 0+1+0 \quad 0+1+1 \quad 1+1+0$

A

0 0 0
0 0 1
0 1 0
0 1 1



$\bar{B} + C$

0 1 0
1 1 0

0

Minimum POS expression = $A(\bar{B} + C)$

Example 2:

Minimize this POS expression using a K-Map:

$$(B + C + D)(A + B + \overline{C} + D)(\overline{A} + B + C + \overline{D})(A + \overline{B} + C + D)(\overline{A} + \overline{B} + C + D)$$

- Change it to standard POS form

$$(A + B + C + D)(\overline{A} + B + C + D)(A + B + \overline{C} + D)(\overline{A} + B + C + \overline{D})(A + \overline{B} + C + D)(\overline{A} + \overline{B} + C + D)$$

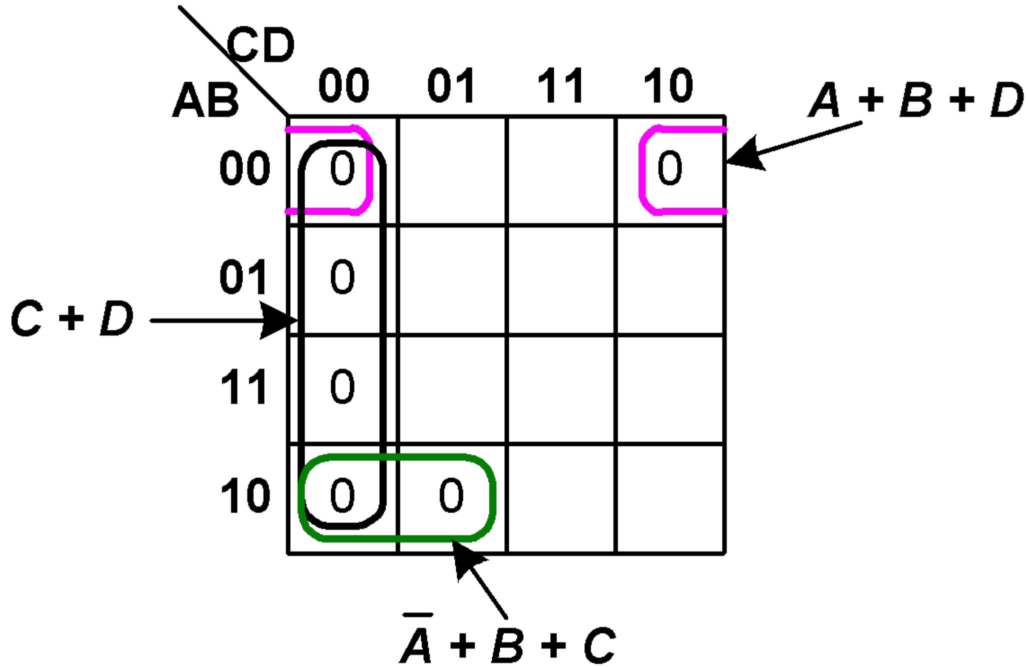
- Get the binary values

(0000)(1000)(0010)(1001)(0100)(1100)

(0000)(1000)(0010)(1001)(0100)(1100)

- Map it, then group the 0s

AB \ CD	00	01	11	10
00	0			0
01	0			
11	0			
10	0	0		



- Minimum POS expression

$$(C + D)(\bar{A} + B + C)(A + B + D)$$

Converting between POS and SOP using K-Map

- A mapped SOP expression can be converted to an equivalent POS expression.
- This is a good way to compare which one can be implemented using fewer gates.
- Given a minimum POS map, the 1s will yield a **standard** SOP expression.
- This SOP expression can then be minimized by grouping the 1s.

Example: Converting between POS and SOP using K-Map

Get the sum terms for POS and write the minimum POS expression.

AB \ CD	00	01	11	10
00		0	0	0
01	0			
11	0			
10		0		

$A + B + \bar{C}$

AB \ CD	00	01	11	10
00		0	0	0
01	0			
11	0			
10		0		

$\bar{B} + C + D$

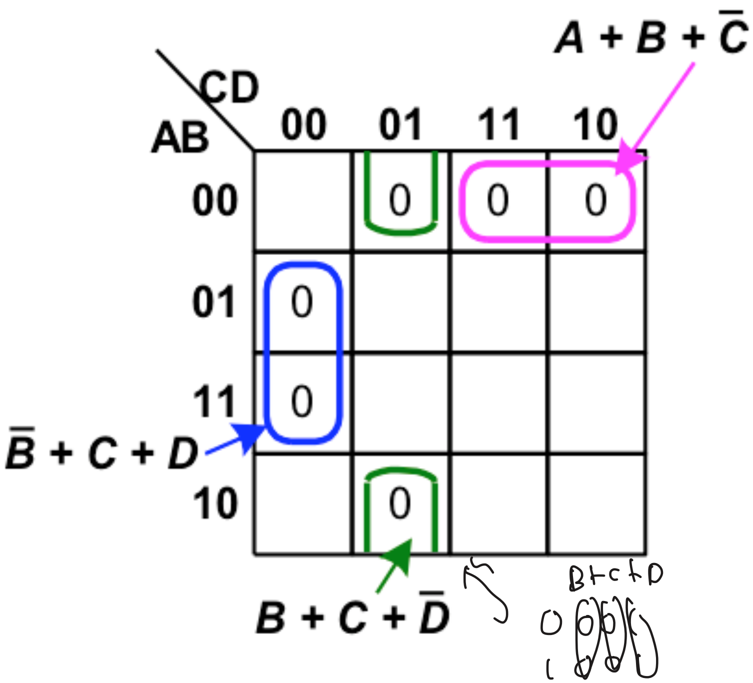
$B + C + \bar{D}$

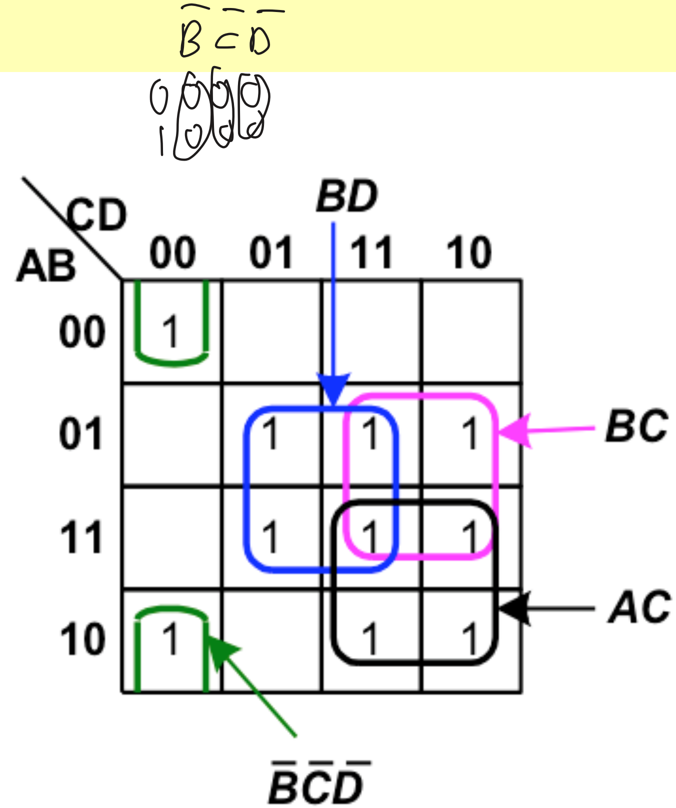
Minimum POS:-

$$(A + B + \bar{C})(\bar{B} + C + D)(B + C + \bar{D})$$

Example: Converting between POS and SOP using K-Map

Get the product terms for SOP and write the minimum SOP expression.





Minimum POS:-

$(A + B + \bar{C})(\bar{B} + C + D)(B + C + \bar{D})$

Minimum SOP: $AC + BC + BD + \bar{B}\bar{C}\bar{D}$

Exercise 4b.16: Transform the following expression into the K-Map. Then, generate the simplified expression from the K-Map.

$$\overline{AB + AC} + \overline{A}\,\overline{B}C$$

Exercise 4b.17: Using the same expression in exercise 4b.16, simplify the expression using the Boolean algebra and laws.

$$\overline{AB + AC} + \overline{A}\,\overline{B}C$$

Exercise 4b.19:

0001 1000 1010

Assume $d(1,5,8,10)$ is a set of don't care used. Generate the new minimized expression from the K-Map.

0101

AB \ CD	00	01	11	10
00	0	X		0
01		X		
11	0	0	0	0
10	X	0		X

$= (\bar{A} + C + \bar{D})(A + B + D)(\bar{A} + \bar{B})$

$\bar{A} + C + \bar{D}$

$A + B + D$

0 0 0 0

0 0 1 0

$\bar{A} + \bar{B}$

1 1 0 0

1 1 0 1

1 1 1 1

1 1 1 0

Simplify a Combinational Logic Circuit

Simplify a Combinational Logic Circuit

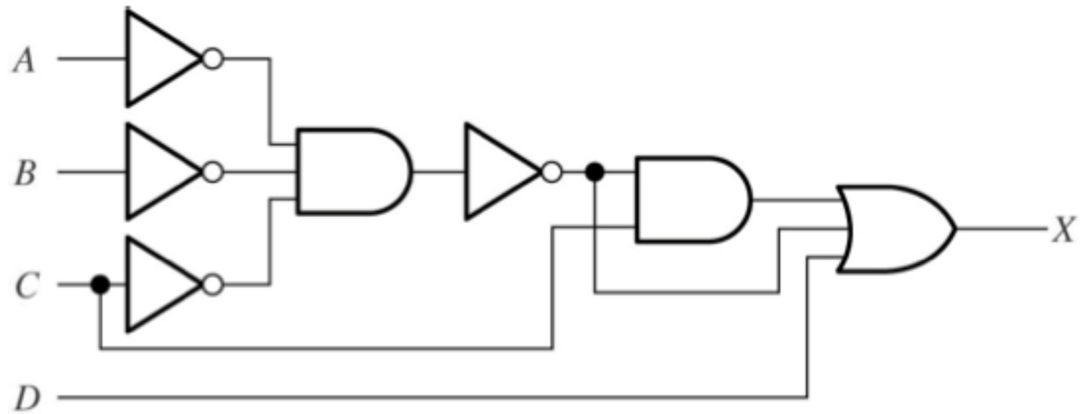
- Simplify a combinational logic circuit will result in lesser gates used
- How to do this?

Step 1 : Determine the input and output variables of each gates in the circuit

Step 2 : Get the final output expression

Step 3 : Apply De Morgan's theorem and Boolean algebra or K-map.

Example 1: Given the logic circuit below, simplify using boolean and DeMorgan's theorem.

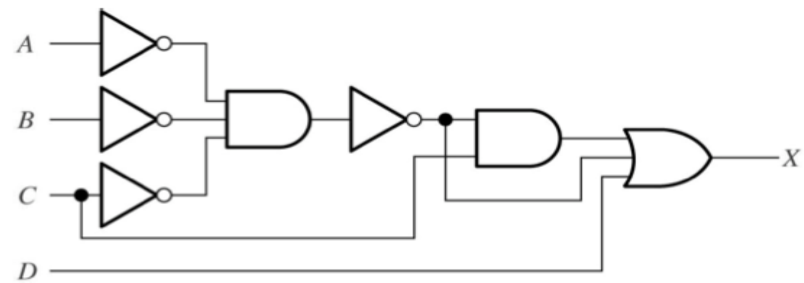


Step1: input variables = A, B, C, D

output variables = X

Step 2: Get the final output expression

$$X = (\overline{\overline{A}\,\overline{B}\,\overline{C}})C + \overline{\overline{A}\,\overline{B}\,\overline{C}} + D$$



Step 3: Apply De Morgan's theorem and Boolean Algebra

$$(\overline{\overline{A}\,\overline{B}\,\overline{C}})C + \overline{\overline{A}\,\overline{B}\,\overline{C}} + D$$

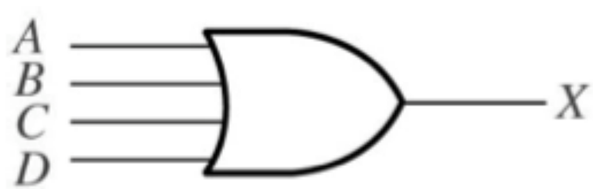
$$= (\overline{\overline{A}} + \overline{\overline{B}} + \overline{\overline{C}})C + (\overline{\overline{A}} + \overline{\overline{B}} + \overline{\overline{C}}) + D \qquad \overline{AB} = \overline{A} + \overline{B}$$

$$= AC + BC + CC + A + B + C + D \qquad \overline{\overline{A}} = A$$

$$= A + AC + B + BC + C + D \qquad C.C = C \qquad C + C = C$$

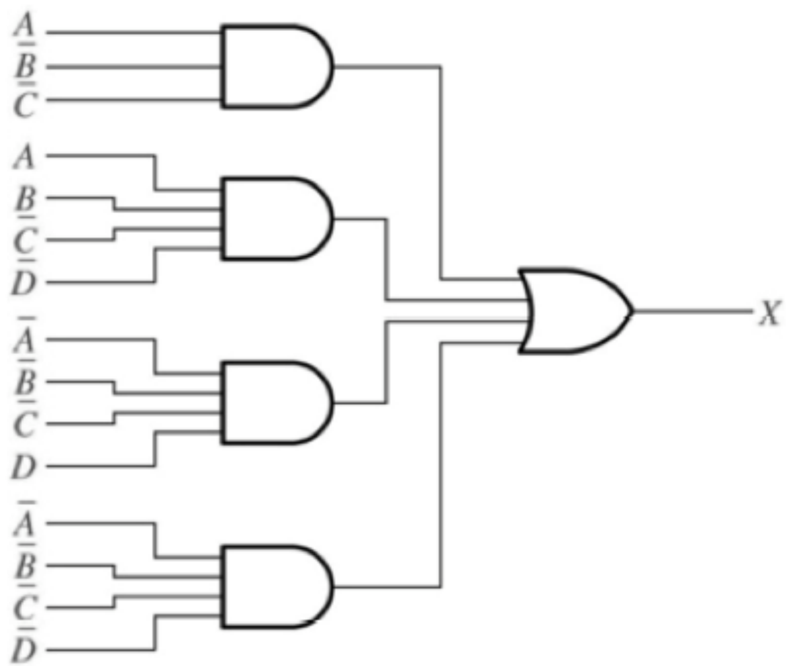
$$= A + B + C + D \qquad A + AC = A \qquad B + BC = B$$

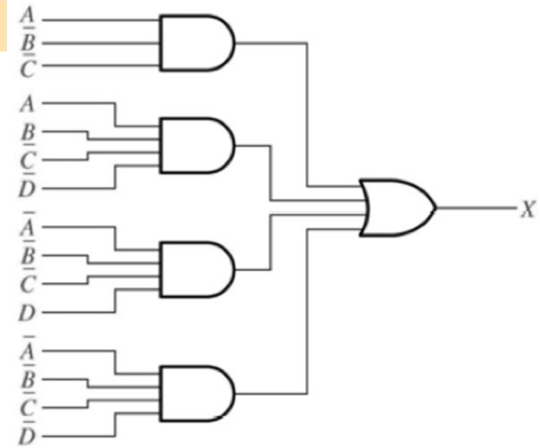
After simplification:



Example 2:

Simplify the combinational logic circuit using K-Map





Step 1 Input variable = A, B, C, D, Output variable = X

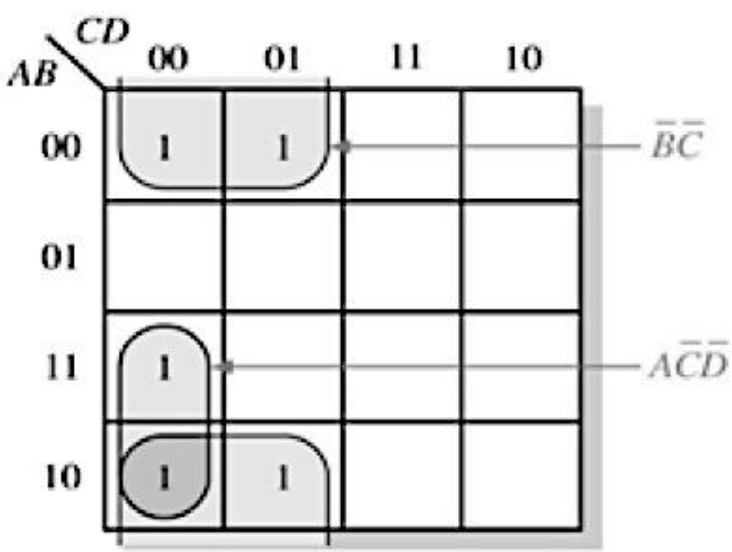
Step 2 Output Expression of the circuit is

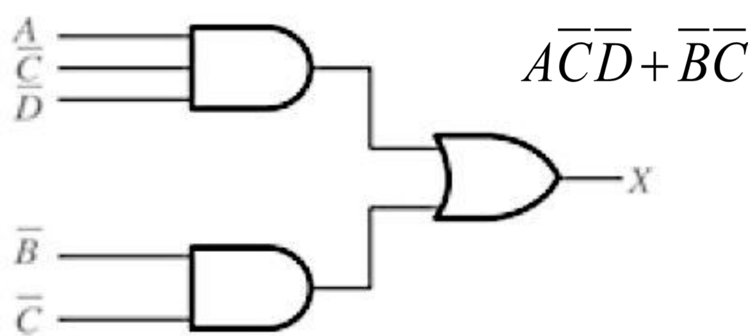
$$X = A\bar{B}\bar{C} + AB\bar{C}\bar{D} + \bar{A}\bar{B}\bar{C}D + \bar{A}\bar{B}\bar{C}\bar{D}$$

Change it to standard SOP form:

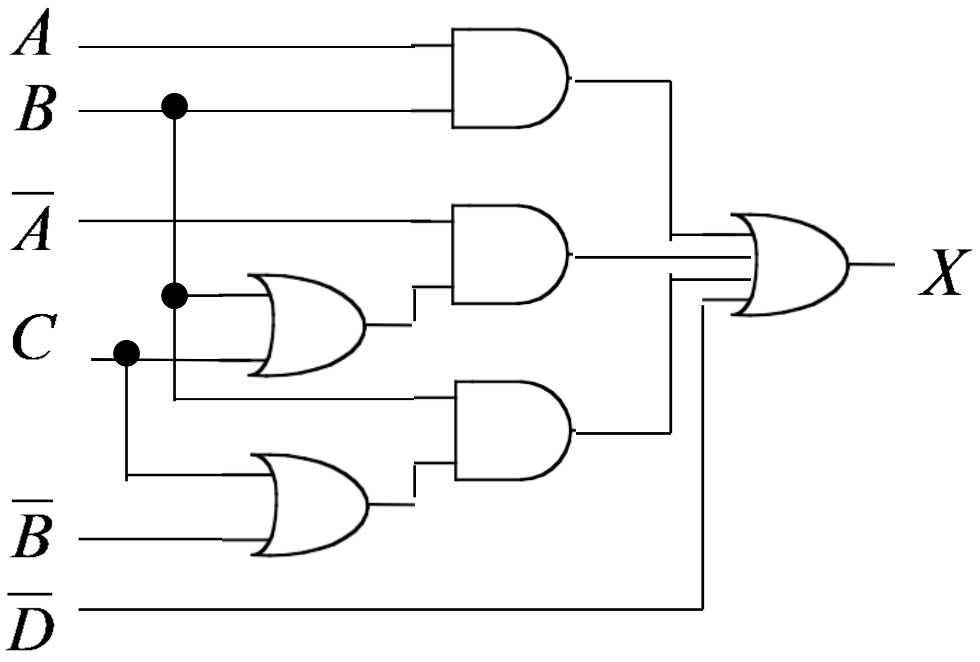
$$X = A\bar{B}\bar{C}\bar{D} + A\bar{B}\bar{C}D + AB\bar{C}\bar{D} + \bar{A}\bar{B}\bar{C}D + \bar{A}\bar{B}\bar{C}\bar{D}$$

Step 3 Use K-map to simplify the circuit





Exercise 4b.21: Simplify the combinational logic circuit using K-map.



Summary

1. $A + A = A$
2. $A + \overline{A} = 1$
3. $A \cdot 1 = 1$
4. $A \cdot 0 = A$
5. $A \cdot A = A$
6. $A \cdot \overline{A} = 0$
7. $A \cdot 1 = A$
8. $A \cdot 0 = 0$
9. $\overline{\overline{A}} = A$
10. $A + AB = A$
11. $A + \overline{A}B = A + B$
12. $(A + B)(A + C) = A + BC$

EXPRESSION

(Example of 2 product terms only)

SOP

Form: $X = \overline{A}B\overline{C} + AB\overline{C}$

Binary: 0 1 0 1 1 0

Notation: $X = \sum_{ABC}(2,6)$

(Example of 2 sum terms only)

POS

Form: $X = (A+\overline{B}+\overline{C})(\overline{A}+B+\overline{C})$

Binary: 0 1 1 1 0 1

Notation: $X = \prod_{ABC}(3,5)$

Standard?

Rule 6: $A + \overline{A} = 1$

Standard?

Rule 8: $A \cdot \overline{A} = 0$

Rule 12: $A + BC = (A+B)(A+C)$

Truth Table

Inputs			Output	Product / Sum Terms
A	B	C	X	
0	1	0	1	$\overline{A}B\overline{C}$
1	1	0	1	$AB\overline{C}$
0	1	1	0	$A+\overline{B}+\overline{C}$
1	0	1	0	$\overline{A}+B+\overline{C}$

(Example of some product / sum terms only)

K-Map

AB \ C	0	1
00		
01	1	0
11	1	
10		0

Don't Care:

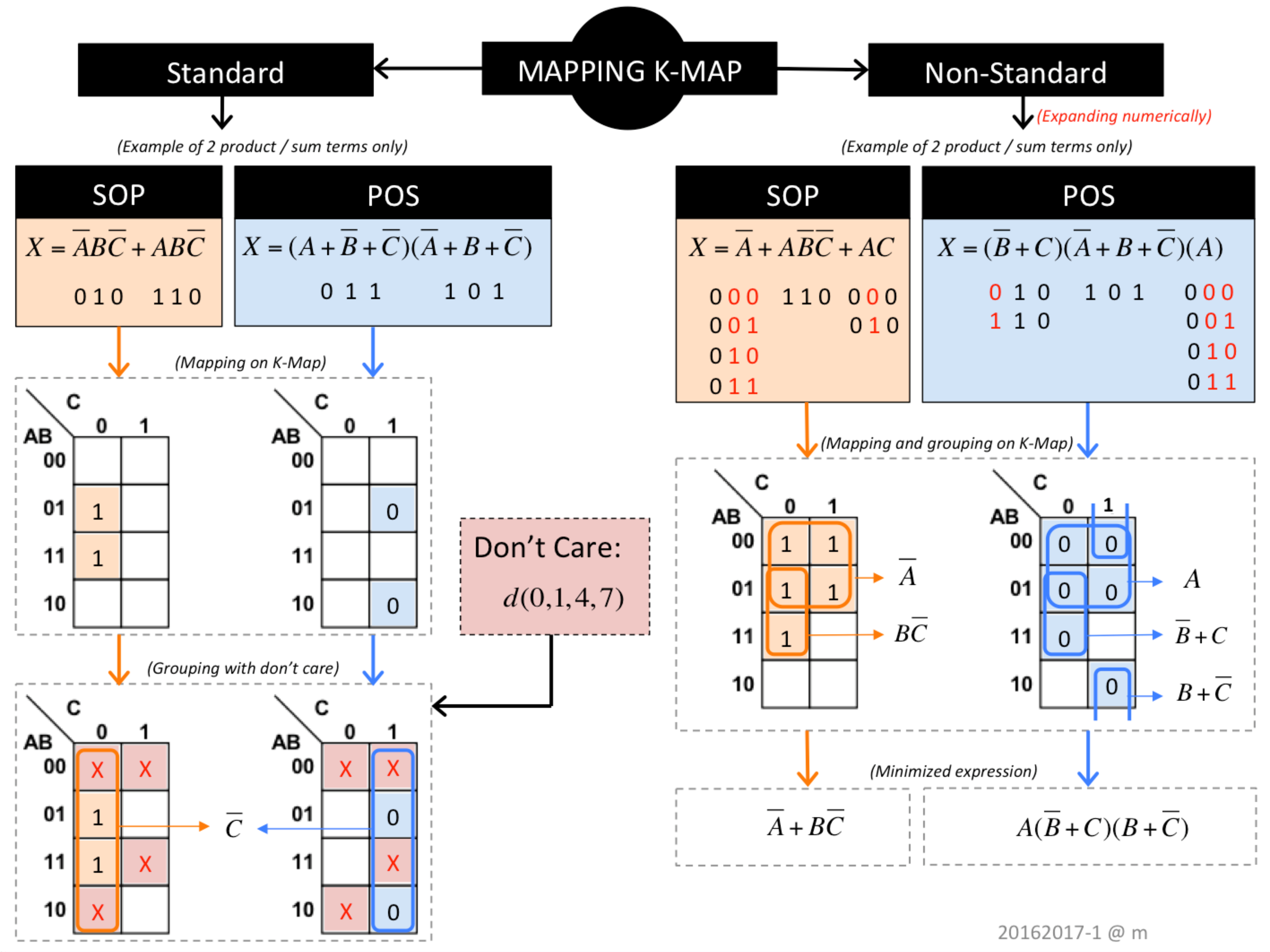
$d(0,1,4,7)$

K-Map

AB \ C	0	1
00	X	X
01	1	0
11	1	X
10	X	0

AB \ C	0	1
00	X	X
01	1	0
11	1	X
10	X	0

$X = \overline{C}$ $X = \overline{C}$



MAPPING K-MAP
Standard
Non-Standard
(Expanding numerically)
(Example of 2 product / sum terms only)
(Example of 2 product / sum terms only)
SOP
$X = \overline{A}B\overline{C} + AB\overline{C}$
0 1 0 1 1 0
POS
$X = (A+\overline{B}+\overline{C})(\overline{A}+B+\overline{C})$
0 1 1 1 0 1
SOP
$X = \overline{A} + A\overline{B}\overline{C} + AC$
0 0 0 1 1 0 0 0 0
0 0 1 0 1 0
0 1 0
0 1 1
POS
$X = (\overline{B}+C)(\overline{A}+B+\overline{C})(A)$
0 1 0 1 0 1 0 0 0
1 1 0 0 0 1
0 1 0
0 1 1
(Mapping on K-Map)
(Grouping with don't care)
Don't Care:
$d(0,1,4,7)$
$\overline{C}$
(Mapping and grouping on K-Map)
$\overline{A}$
$B\overline{C}$
A
$\overline{B}+C$
$B+\overline{C}$
(Minimized expression)
$\overline{A}+B\overline{C}$
$A(\overline{B}+C)(B+\overline{C})$
20162017-1 @ m

Tutorial 4: (Page: 310)

3. Simplify the function using Boolean Algebra

$$Y = AB\overline{C} + A\overline{B} + ABC + \overline{A}\overline{B}$$

4. Develop a truth table for the following expression. From the truth table derive a standard product-of-sums (POS) expression.

$$f = (A + \overline{B})(A + C)(A + B + \overline{C})$$

5. Use a Karnaugh map to reduce the expression to a minimum sum-of-products (SOP) form

$$f = \overline{A}B(C\overline{D} + CD) + ACD$$

$$Y = AB\overline{C} + A\overline{B} + ABC + \overline{A}\,\overline{B}$$

10. Using Karnaugh Map,find the minimum SOP expression for the following function

i)

$$Y = \overline{P}\,\overline{Q}\,\overline{R}\,\overline{S} + \overline{P}\,\overline{Q}\,\overline{R}\,S + \overline{P}\,\overline{Q}\,R\,\overline{S} + \overline{P}Q\,\overline{R}\,\overline{S} + P\,\overline{Q}\,\overline{R}\,\overline{S} + P\overline{Q}\,R\,\overline{S} + P\,Q\,\overline{R}\,\overline{S}$$

ii)

$$G(w, x, y, z) = \sum m(1, 3, 14, 15) + d(0, 2, 6, 8, 13)$$

Tutorial 4: (Page: 312)

10. Using Karnaugh Map,find the minimum SOP expression for the following function

i)

$$Y = \overline{P}\,\overline{Q}\,\overline{R}\,\overline{S} + \overline{P}\,\overline{Q}\,\overline{R}\,S + \overline{P}\,\overline{Q}\,R\,\overline{S} + \overline{P}\,Q\,\overline{R}\,\overline{S} + P\,\overline{Q}\,\overline{R}\,\overline{S} + P\overline{Q}\,R\,\overline{S} + P\,Q\,\overline{R}\,\overline{S}$$

ii)

$$G(w, x, y, z) = \sum m(1, 3, 14, 15) + d(0, 2, 6, 8, 13)$$