DISCRETE STRUCTURE
SECI1013-02
SEMESTER 1

GROUP 10
SECTION 02
ASSIGNMENT 2

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SUBMISSION DATE:
24 NOVEMBER 2025

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SECI1013: DISCRETE STRUCTURES

SESSION 2025/2026 – SEMESTER 1

ASSIGNMENT 2 (CHAPTER 2 – RELATION. FUNCTION & RECURRENCE

INSTRUCTIONS:

a. This assignment must be conducted in a group. Please clearly write the group members' names & matric numbers on the front page of the submission.
b. Solutions for each question must be readable and neatly written on plain A4 paper. Every step or calculation should be properly shown. Failure to do so will result in the rejection of the submission of the assignment.
c. This assignment consist of 7 questions (60 marks), contributing 5% of overall course marks.

Question 1 **[9 marks]**

Let $D = \{1,3,5\}$. Define R on D where $x, y \in D, xRy$ if $3x + y$ is a multiple of 6.

i) Find the element of R.
ii) Determine the domain and range of R.
iii) Draw the digraph of the relation
iv) Determine whether the relation R is assymetric?

Question 2 **[8 marks]**

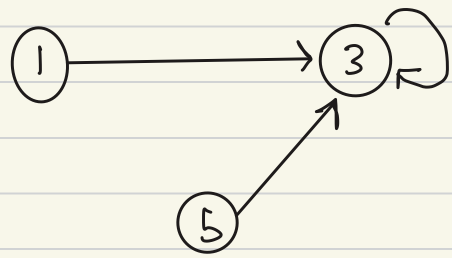
Suppose R is an equivalence relation on the set $A=\{x,y,z\}$. $(x,y) \in$ R and $(y,z) \in$ R. List all possible member of R and justify your answer.

Question 1

i) $R = \{(1,3), (3,3), (5,3)\}$

ii) Domain $= \{1, 3, 5\}$

Range $= \{3\}$

iii) 

iv) $M_R = \begin{pmatrix} 0 & 1 & 0 \\ 0 & 1 & 0 \\ 0 & 1 & 0 \end{pmatrix}$

Since the main diagonal is not all 0s, it is not irreflexsive

For the relation to be assymmetric, it must sastified both irreflexsive and antisymmetric. Since it is not sastified irreflexsive, it is not assymmetric.

Question 2

$A = \{x, y, z\}$

equivalence $\left\{ \begin{array}{l} \text{reflexsive} \\ \text{Symmetry} \\ \text{transitive} \end{array} \right\}$ All 3 is a must

R is reflexsive when it sastified $(x,x) \in R$, for every $x \in A$, so, (x,x), (y,y), (z,z) are element in R

R is symmetry when it sastified for all $x, y \in A$, $(x,y) \in R \to (y,x \in R)$ so, (x,y), (y,x), (y,z), (z,y), (x,z), (z,x) are element in R

R is transitive when it sastified for all $x, y, z \in A$, $(x,y) \in R$ and $(y,z) \in R \rightarrow (x,z) \in R$. so, (x,y), (y,z), (x,z), (z,y) (y,x), (z,x) are the element in R

$$M_R = \begin{pmatrix} 1 & 1 & 1 \\ 1 & 1 & 1 \\ 1 & 1 & 1 \end{pmatrix}$$

Element of R, $R = \{(x,x), (x,y), (x,z), (y,x), (y,y), (y,z), (z,x), (z,y), (z,z)\}$

Question 3 **[15 marks]**

Let B = {*u*, *v*, *w*, *y*} and R={(*u*,*u*), (*u*,*w*), (*v*,*v*), (*v*,*w*), (*w*,*w*), (*w*,*y*), (*y*,*u*), (*y*,*v*), (*y*,*y*)}

i) Construct the matrix of relation, M_R for the relation R on B

ii) List in-degrees and out-degrees of all vertices.

iii) Determine whether the relation R on the set B is a partial order relation. Check all variance Justify for answer.

Question 3

i)

$$M_R = \begin{array}{c} \\ u \\ v \\ w \\ y \end{array} \begin{array}{c} \begin{array}{cccc} u & v & w & y \end{array} \\ \begin{bmatrix} 1 & 0 & 1 & 0 \\ 0 & 1 & 1 & 0 \\ 0 & 0 & 1 & 1 \\ 1 & 1 & 0 & 1 \end{bmatrix} \end{array}$$

ii)

	u	v	w	y
In-degree	2	2	3	2
Out-degree	2	2	2	3

iii)

$$\begin{bmatrix} 1 & 0 & 1 & 0 \\ 0 & 1 & 1 & 0 \\ 0 & 0 & 1 & 1 \\ 1 & 1 & 0 & 1 \end{bmatrix}$$

R is reflexive because M_R has 1's on the main diagonal

$$\begin{bmatrix} 1 & 0 & 1 & 0 \\ 0 & 1 & 1 & 0 \\ 0 & 0 & 1 & 1 \\ 1 & 1 & 0 & 1 \end{bmatrix} \times \begin{bmatrix} 1 & 0 & 1 & 0 \\ 0 & 1 & 1 & 0 \\ 0 & 0 & 1 & 1 \\ 1 & 1 & 0 & 1 \end{bmatrix} = \begin{bmatrix} 1 & 0 & 1 & 1 \\ 0 & 1 & 1 & 1 \\ 1 & 1 & 1 & 1 \\ 1 & 1 & 1 & 1 \end{bmatrix}$$

$M_R \neq M_R^T$

R is not symmetric.

$$\begin{bmatrix} 1 & 0 & 1 & 0 \\ 0 & 1 & 1 & 0 \\ 0 & 0 & 1 & 0 \\ 1 & 1 & 0 & 1 \end{bmatrix}$$

if $i \neq j$, then $m_{ij} = 0$ or $m_{ji} = 0$

$m_{31} = 0$

$(u, w) \in R$ $(w, u) \notin R$

R is antisymmetric

$$\begin{bmatrix} 1 & 0 & 1 & 0 \\ 0 & 1 & 1 & 0 \\ 0 & 0 & 1 & 0 \\ 1 & 1 & 0 & 1 \end{bmatrix} \otimes \begin{bmatrix} 1 & 0 & 1 & 0 \\ 0 & 1 & 1 & 0 \\ 0 & 0 & 1 & 0 \\ 1 & 1 & 0 & 1 \end{bmatrix} = \begin{bmatrix} 1 & 0 & 1 & 1 \\ 0 & 1 & 1 & 1 \\ 1 & 1 & 1 & 1 \\ 1 & 1 & 1 & 1 \end{bmatrix}$$

$\forall i \forall j$, if $(n_{ij} = 1)$ then $(m_{ij} = 1)$

$(n_{43} = 1) \wedge (m_{43} = 0)$

(y, v) and $(v, w) \in R$, $(y, w) \notin R$

Not transitive

R is reflexive and antisymmetric but not transitive, therefore R is not partial order.

Question 4 **[6 marks]**

Let

$$f : [1, \infty) \to [0, \infty), f(x) = (x - 1)^2.$$

Determine whether the function f is **one–one**, **onto**, or **bijective**.
Show full working and justify your answer.

Question 4

Let $f(n_1) = f(n_2)$

$$(n_1 - 1)^2 = (n_2 - 1)^2$$

$$\sqrt{(n_1-1)^2} = \sqrt{(n_2-1)^2}$$

$$|n_1 - 1| = |n_2 - 1|$$

Since the domain $[1, \infty)$, $n \geq 1$

Therefore n_1 and n_2 must be positive, negative values are ignored.

$$n_1 - 1 = n_2 - 1$$

$$n_1 = n_2$$

Since $n_1 = n_2$, the function is one-to-one.

Domain, $X = [1, \infty)$

$$f(1) = (1-1)^2 = 0$$

$$f(\infty) = (\infty - 1)^2 = \infty$$

Codomain, $Y = [0, \infty]$

f is a function from $X = [1, \infty)$ to $[0, \infty]$

Therefore f is onto Y.

Since f is both one-to-one and onto Y, f is a bijection.

Question 4 **[6 marks]**

Let

$$f: [1, \infty) \to [0, \infty), f(x) = (x - 1)^2.$$

Determine whether the function f is **one–one**, **onto**, or **bijective**.
Show full working and justify your answer.

Question 5 **[9 marks]**

Let f and g be functions from the positive integers to the positive integers defined by

$$f(x) = 9x + 4, g(x) = \frac{3}{2}x - 1.$$

a) Find the inverse of $g(x)$.

b) Find the composition $(g \circ f)(x)$.

c) Find the composition $(f \circ g)(x)$.

d) Find the composition $(f \circ g \circ g)(x)$.

Question 6 **[6 marks]**

In a reactor, two intermediates mix to form product P. The initial temperatures are $P_0 = 4.0^\circ F$ and $P_1 = 5.0^\circ F$. Engineers observe that, for $t \geq 2$minutes, the update rule is:

"The new temperature is the previous temperature plus one-quarter of the temperature two minutes ago."

a) Write the recurrence relation that models this.

b) Using your recurrence, list $P_0, P_1, \ldots, P_5$ (exact values preferred).

Question 7 **[7 marks]**

Given the recurrence relation below,

$$s_1 = 2, s_n = s_{n-1}^2 - 1 \text{ for } n \geq 2.$$

a) Write a recursive algorithm to calculate the n^{th} term of the sequence

b) Trace the recursive steps to compute s_4. Show your working in a diagram.

Q5 $f(x) = 9x+4$ $g(x) = \frac{3}{2}x-1$

let $g^{-1}(y) = x$

$y = \frac{3}{2}x - 1$

$x = (y+1)\frac{2}{3}$

$x = \frac{2}{3}y + \frac{2}{3}$

$g^{-1}(y) = \frac{2}{3}y + \frac{2}{3}$

b) $g(f(x))$

$= \frac{3}{2}(9x+4) - 1$

$= \frac{27}{2}x + 5$

c) $f(g(x))$

$= 9\left(\frac{3}{2}x-1\right) + 4$

$= \frac{27}{2}x - 9 + 4$

$= \frac{27}{2}x - 5$

d) $f(g(g(x)))$

$g(x) = \frac{3}{2}x - 1$

$g(g(x)) = \frac{3}{2}\left(\frac{3}{2}x-1\right) - 1$

$= \frac{9}{4}x - \frac{3}{2} - 1$

$= \frac{9}{4}x - \frac{5}{2}$

$f(g(gx)) = 9\left(\frac{9}{4}x - \frac{5}{2}\right) + 4$

$= \frac{81}{4}x - \frac{45}{2} + 4$

$= \frac{81}{4}x - \frac{37}{2}$

$= \frac{1}{2}\left(\frac{81}{2}x - 37\right)$

6)

a) $P_n = P_{n-1} + \frac{1}{4}P_{n-2} \quad n \geq 2$

b) $P_0 = 4.0F$
$P_1 = 5.0F$

$P_2 = 5 + \frac{1}{4}(9)$
$= 6°F$

$P_3 = 6 + \frac{1}{4}(5)$
$= \frac{29}{4}°F$

$P_4 = \frac{29}{4} + \frac{1}{4}(6)$
$= \frac{35}{4}°F$

$P_5 = \frac{35}{4} + \frac{1}{4}\left(\frac{29}{4}\right)$
$= \frac{169}{16}°F$

7)

a)

```
f(n) {
  if(n=1) return 2;
  return (f(n-1) × f(n-1)) - 1
}
```

b)

f(4)
n = 4
because n ≠ 1
return (f(3) × f(3)) − 1

f(3)
n = 3
because n ≠ 1
return (f(2) × f(2)) − 1

f(2)
n = 2
because n ≠ 1
return (f(1) × f(1)) − 1

f(1)
n = 1
because n = 1
return 2

f(1) = 2
return 2

f(2) = (2 × 2) − 1
= 3

f(3) = (3 × 3) − 1
= 8

f(4) = (8 × 8) − 1
= 63

answer = 63