Chapter 1

Part 3
Fundamental and Elements of Logic

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Why Are We Studying Logic?

Some of the reasons:

- Logic is the foundation for computer operation
- Logical conditions are common in programs

  Example:

  Selection: if (score <= max) { ... }

  Iteration: while (i<limit && list[i]!=sentinel) ...

- All manner of structures in computing have properties that need to be proven (and proofs that need to be understood).

  Example: Trees, Graphs, Recursive Algorithms, . . .

- Programs can be proven correct.
- Computational linguistics must represent and reason about human language, and language represents thought (and thus also logic).

PROPOSITION

A **statement** or a **proposition**, is a declarative sentence that is **either TRUE or FALSE, but not both.**

Example:

- 4 is less than 3. *false*
- 7 is an even integer. *false*
- Washington, DC, is the capital of United State. *True*

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Example

i)   Why do we study mathematics?
ii) Study logic.
iii) What is your name?
iv) Quiet, please.

The above sentences are not propositions. Why ?

(i) & (iii) : is question, not a statement.
(ii)& (iv) : is a command.

Example

i) The temperature on the surface of the planet Venus is 800 F.

ii) The sun will come out tomorrow.

Propositions? Why?

- Is a statement since it is either true or false, but not both.
- However, we do not know at this time to determine whether it is true or false.

CONJUNCTIONS

- Compound propositions formed in English with the word "**and**"

- Formed in logic with the caret symbol **(" ∧ ")** / &&

- **True** only when **both participating propositions are true**.

TRUTH TABLE: This tables aid in the evaluation of **compound propositions**.

p	q	p∧q
T	T	T
T	F	F
F	T	F
F	F	F

True (T), False (F)

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p : 2 is an even integer
q : 3 is an odd number

propositions

p ∧ q

symbols

: 2 is an even integer and 3 is an odd number

statements

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p : today is Monday
q : it is hot

p ∧ q: today is Monday and it is hot

Example

Proposition

p : 2 divides 4

q : 2 divides 6

Symbol & Statement

p ∧ q: 2 divides 4 and 2 divides 6.

or,

p ∧ q: 2 divides both 4 and 6.

Proposition

p : 5 is an integer

q : 5 is not an odd integer

Symbol & Statement

p ∧ q: 5 is an integer and 5 is not an odd integer.

or,

p ∧ q: 5 is an integer but 5 is not an odd integer.

DISJUNCTION

- Compound propositions formed in English with the word "**or**",

- Formed in logic with the caret symbol (" ∨ ")

- **True** when **one or both** participating propositions are true.

The truth table for $p \lor q$

p	q	p ∨ q
T	T	T
T	F	T
F	T	T
F	F	F

Example

p: 2 is an integer
q: 3 is greater than 5

p ∨ *q* : 2 is an integer or 3 is greater than 5

p : 1+1=3
q : A decade is 10 years

p ∨ *q* : 1+1=3 or a decade is 10 years

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Example

p : 3 is an even integer
q : 3 is an odd integer

p ∨ q
3 is an even integer or 3 is an odd integer
or
3 is an even integer or an odd integer

NEGATION

Negating a proposition simply flips its value. Symbols representing negation include:

$$\neg x,\ \bar{x},\ \sim x,\ x'\ \ \text{(NOT)}$$

Let p be a proposition. The negation of p, written ¬ p is the statement obtained by negating statement p.

The truth table of ¬ p

p	¬p
T	F
F	T

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Example

p : 2 is positive

¬ $p :$ 2 is not positive.

p : 4 is less than 3

¬ $p :$ 4 is not less than 3.

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Let *p and q* be propositions*.*

*"*if *p, then q"*

is a statement called a **conditional proposition**, written as

$$p \rightarrow q$$

The truth table of **p → q**
=> *Cause and effect relationship*

FALSE if
p = True
and q
=false

TRUE if
both
true or
p=false
for any
value of
q

P ∧ q   R   (P ∧ q)

p	q	p→ q
T	T	T
T	F	F
F	T	T
F	F	T

Example

p : today is Sunday

q : I will go for a walk

$p \rightarrow q$: If today is Sunday, then I will go for a walk.

p : I get a bonus

q : I will buy a new car

$p \rightarrow q$: If I get a bonus, then I will buy a new car

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Example

p : x/2 is an integer.
q : x is an even integer.

p → q : if x/2 is an integer, then x is an even integer.

Let *p* *and* *q* *be propositions.*

"*p* *if and only if* *q*"

is a statement called a **bicondititional proposition**, written as

$$p \longleftrightarrow q \quad \text{or} \quad p \text{ iff } q$$

The truth table of *p ↔ q:*

p	*q*	*p ↔ q*
T	T	T
T	F	F
F	T	F
F	F	T

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Example

p *:* my program will compile

q *:* it has no syntax error.

p $\leftrightarrow$ *q* : My program will compile if and only if it has no syntax error.

p : x is divisible by 3

q : x is divisible by 9

p $\leftrightarrow$ *q* : x is divisible by 3 if and only if x is divisible by 9.

LOGICAL EQUIVALENCE

- The compound propositions *Q* and *R* are made up of the propositions $p_1, ..., p_n$.

- *Q* and *R* are logically equivalent and write,

  $Q \equiv R$

  provided that given any truth values of $p_1, ..., p_n$, either *Q* and *R* are **both true** or *Q* and *R* are **both false**.

Example

$Q = p \rightarrow q$ $R = \neg q \rightarrow \neg p$

Show that, $Q \equiv R$

The truth table shows that, $Q \equiv R$

Shown

p	q	p →q	¬q →¬p	¬q	¬p
T	T	T	T	F	F
T	F	F	F	T	F
F	T	T	T	F	T
F	F	T	T	T	T

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Example

Show that, $\neg (p \rightarrow q) \equiv p \wedge \neg q$

The truth table shows that, $\neg (p \rightarrow q) \equiv p \wedge \neg q$

p	q	¬(p → q)	p ∧¬q	¬ q
T	T	F ✓	F ✓	F
T	F	T ✓	T ✓	T
F	T	F ✓	F ✓	F
F	F	F ✓	F ✓	T

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Precedence of logical connectives is as follows:

not	¬		Highest
and	∧		
or	∨		
If...then	→	↑	
If and only if	↔		Lowest

Example

Construct the truth table for, $A = \neg(p \vee q) \to (q \wedge p)$

Solution

p	q	(p∨q)	¬(p∨q)	(q∧p)	A
T	T	T	F	T	T
T	F	T	F	F	T
F	T	T	F	F	T
F	F	F	T	F	F

Logic and set theory go very well togather. The previous definitions can be made very succinct:

$$x \notin A \text{ if and only if } \neg(x \in A)$$
$$A \subseteq B \text{ if and only if } (x \in A \rightarrow x \in B) \text{ is True}$$
$$x \in (A \cap B) \text{ if and only if } (x \in A \wedge x \in B)$$
$$x \in (A \cup B) \text{ if and only if } (x \in A \vee x \in B)$$
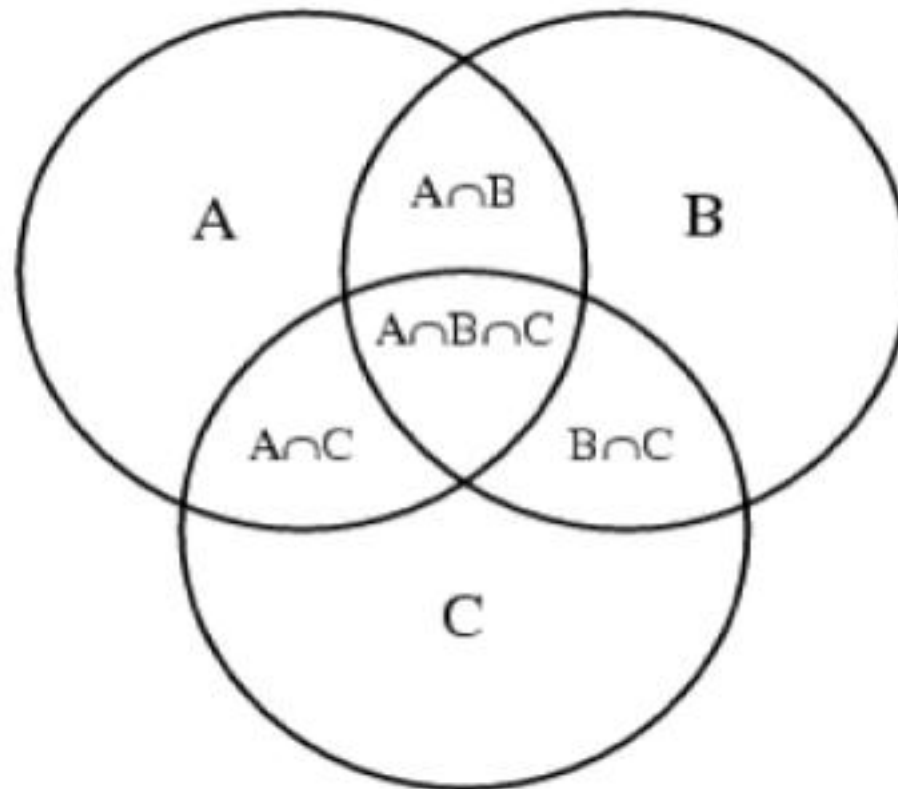$$x \in A{-}B \text{ if and only if } (x \in A \wedge x \notin B)$$
$$x \in A \triangle B \text{ if and only if } (x \in A \wedge x \notin B) \vee (x \in B \wedge x \notin A)$$
$$x \in A' \text{ if and only if } \neg(x \in A)$$
$$X \in P(A) \text{ if and only if } X \subseteq A$$

Venn Diagrams

Venn Diagrams are used to depict the various unions, subsets, complements, intersections etc. of sets.

Logic and Sets
are closely related

Tautology

$p \vee q \leftrightarrow q \vee p$

$p \wedge q \leftrightarrow q \wedge p$

$p \vee (q \vee r) \leftrightarrow (p \vee q) \vee r$

$p \wedge (q \wedge r) \leftrightarrow (p \wedge q) \wedge r$

$p \vee (q \wedge r) \leftrightarrow (p \vee q) \wedge (p \vee r)$

$p \wedge (q \vee r) \leftrightarrow (p \wedge q) \vee (p \wedge r)$

$p \wedge \neg q \leftrightarrow p \wedge \neg (p \wedge q)$

$p \wedge \neg (q \vee r) \leftrightarrow (p \wedge \neg q) \wedge (p \wedge \neg r)$

$p \wedge \neg (q \wedge r) \leftrightarrow (p \wedge \neg q) \vee (p \wedge \neg r)$

$p \wedge (q \wedge \neg r) \leftrightarrow (p \wedge q) \wedge \neg (p \wedge \neg r)$

$p \vee (q \wedge \neg r) \leftrightarrow (p \vee q) \wedge \neg (r \wedge \neg p)$

$p \wedge \neg \vee (q \wedge \neg r) \leftrightarrow (p \wedge \neg q) \vee (p \wedge r)$

Set Operation Identity

$A \cup B = B \cup A$

$A \cap B = B \cap A$

$A \cup (B \cup C) = (A \cup B) \cup C$

$A \cap (B \cap C) = (A \cap B) \cap C$

$A \cup (B \cap C) = (A \cup B) \cap (A \cup C)$

$A \cap (B \cup C) = (A \cap B) \cup (A \cap C)$

$A - B = A - (A \cap B)$

$A - (B \cap C) = (A - B) \cup (A - C)$

$A - (B \cup C) = (A - B) \cap (A - C)$

$A \cap (B - C) = (A \cap B) - (A \cap C)$

$A \cup (B - C) = (A \cup B) - (C - A)$

$A - (B - C) = (A - B) \cup (A \cap C)$

The above identities serve as the basis for an "algebra of sets".

Logic and Sets
are closely related

Tautology

$$p \wedge p \leftrightarrow p$$

$$p \vee p \leftrightarrow p$$

$$p \wedge \neg(q \wedge \neg q) \leftrightarrow p$$

$$p \vee \neg(q \wedge \neg q) \leftrightarrow p$$

Set Operation Identity

$$A \cap A = A$$

$$A \cup A = A$$

$$A - \varnothing = A$$

$$A \cup \varnothing = A$$

Contradiction

$$p \wedge \neg p$$

$$p \wedge (q \wedge \neg q)$$

$$p \wedge \neg p$$

Set Operation Identity

$$A - A = \varnothing$$

$$A \cap \varnothing = \varnothing$$

$$A - A = \varnothing$$

The above identities serve as the basis for an "algebra of sets".

Theorem for Logic

Let *p, q* and *r* be propositions.

Idempotent laws:

$$p \land p \equiv p$$
$$p \lor p \equiv p$$

Truth table

p	$p \land p$	$p \lor p$
T	T	T
F	F	F

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Theorem for Logic

Double negation law:

$$\neg\,\neg\,p \equiv p$$

Commutative laws:

$$p \land q \equiv q \land p$$
$$p \lor q \equiv q \lor p$$

Associative laws:

$$(p \land q) \land r \equiv p \land (q \land r)$$
$$(p \lor q) \lor r \equiv p \lor (q \lor r)$$

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Theorem for Logic

Distributive laws:

$$p \lor (q \land r) \equiv (p \lor q) \land (p \lor r)$$
$$p \land (q \lor r) \equiv (p \land q) \lor (p \land r)$$

PROVE

p	q	r	$p \lor (q \land r)$	$(p \lor q) \land (p \lor r)$
T	T	T	T	T
T	T	F	T	T
T	F	T	T	T
T	F	F	T	T
F	T	T	T	T
F	T	F	F	F
F	F	T	F	F
F	F	F	F	F

Theorem for Logic

Absorption laws:

$$p \land (p \lor q) \equiv p$$
$$p \lor (p \land q) \equiv p$$

PROVE

p	q	$p \land (p \lor q)$	$p \lor (p \land q)$	$p \land q$	$p \lor q$
T	T	T	T	T	T
T	F	T	T	T	F
F	T	F	F	T	F
F	F	F	F	F	F

Shown

Theorem for Logic

De Morgan's laws:

$$\neg(p \wedge q) \equiv (\neg p) \vee (\neg q)$$
$$\neg(p \vee q) \equiv (\neg p) \wedge (\neg q)$$

The truth table for $\neg(p \vee q) \equiv (\neg p) \wedge (\neg q)$

p	q	$\neg(p \vee q)$	$\neg p \wedge \neg q$
T	T	F	F
T	F	F	F
F	T	F	F
F	F	T	T

Thank You

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