

# Chapter 1

## SET THEORY

### [Part 2: Operation on Set]

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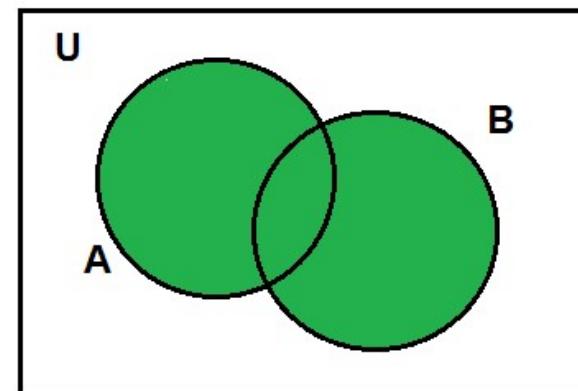
# Union

- The **union** of two sets  $A$  and  $B$ , denoted by  $A \cup B$ , is defined to be the set

$$A \cup B = \{ x \mid x \in A \text{ or } x \in B \}$$

- The union consists of all elements belonging to either  $A$  or  $B$  (or both)

**Venn diagram of  $A \cup B$**



# Example

$A = \{1, 2, 3, 4, 5\}$ ,  $B = \{2, 4, 6\}$  and  $C = \{8, 9\}$

$$A \cup B = \{1, 2, 3, 4, 5, 6\}$$

$$A \cup C = \{1, 2, 3, 4, 5, 8, 9\}$$

$$B \cup C = \{2, 4, 6, 8, 9\}$$

$$A \cup B \cup C = \{1, 2, 3, 4, 5, 6, 8, 9\}$$

If  $A$  and  $B$  are finite sets, the cardinality of  $A \cup B$ ,

$$|A \cup B| = |A| + |B| - |A \cap B|$$

*↖ tolak kerana nhr  
bung data  
yg sama*

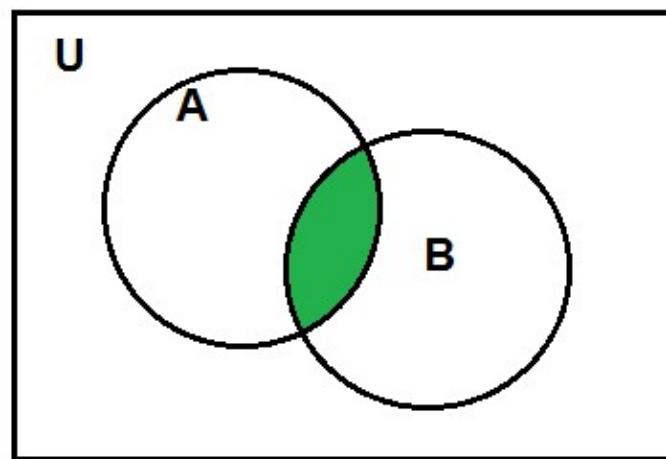
# Intersection

- The **intersection** of two sets  $A$  and  $B$ , denoted by  $A \cap B$ , is defined to be the set

$$A \cap B = \{ x \mid x \in A \text{ and } x \in B \}$$

- The **intersection** consists of all elements belonging to both  $A$  and  $B$ .

**Venn diagram of  $A \cap B$**



# Example

$A=\{1, 2, 3, 4, 5, 6\}$ ,  $B=\{2, 4, 6, 8, 10\}$  and  $C=\{ 1, 2, 8, 10 \}$

$$A \cap B = \{2, 4, 6\}$$

$$A \cap C = \{1, 2\}$$

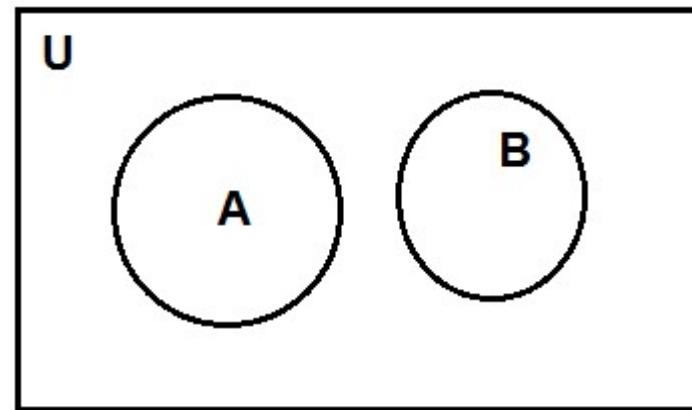
$$C \cap B = \{2, 8, 10\}$$

$$A \cap B \cap C = \{2\}$$

# Disjoint

Two sets  $A$  and  $B$  are said to be **disjoint** if,  $A \cap B = \emptyset$

**Venn diagram,  $A \cap B = \emptyset$**



**Example**

$$A = \{1, 3, 5, 7, 9, 11\}, B = \{2, 4, 6, 8, 10\}$$

$$A \cap B = \emptyset$$

# Difference

The set,

$$A - B = \{x \mid x \in A \text{ and } x \notin B\}$$

is called the difference.

The difference  $A - B$  consists of all elements in  $A$  that are not in  $B$ .

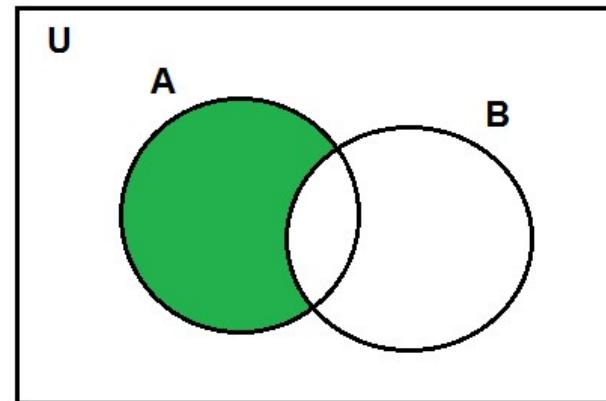
## Venn diagram of $A - B$

### Example

$$A = \{1, 2, 3, 4, 5, 6, 7, 8\}$$

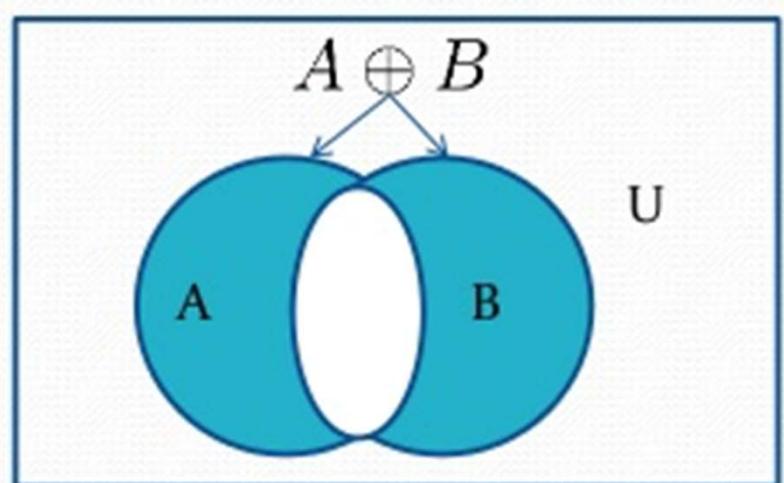
$$B = \{2, 4, 6, 8\}$$

$$A - B = \{1, 3, 5, 7\}$$



# Symmetric Difference

The symmetric difference of set  $A$  and set  $B$ , denoted by  $A \oplus B$  is the set  $(A - B) \cup (B - A)$



Venn Diagram

# Example

$$U = \{0,1,2,3,4,5,6,7,8,9,10\}$$

$$A = \{1, 2, 3, 4, 5\}; B = \{4, 5, 6, 7, 8\}$$

$$A \oplus B = (A - B) \cup (B - A) = \{1, 2, 3, 6, 7, 8\}$$



$$B - A = \{6, 7, 8\}$$

$$A - B = \{1, 2, 3\}$$

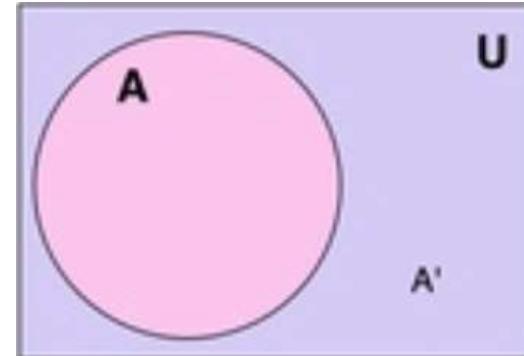
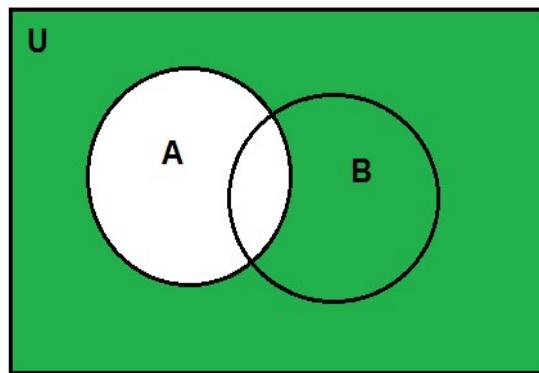
# Complement

The complement of a set  $A$  with respect to a universal set  $U$ , denoted by  $A'$  is defined to be

$$A' = \{x \in U \mid x \notin A\}$$

$$A' = U - A$$

## Venn diagram of $A'$



# Example

Let  $U$  be a universal set,

$$U = \{ 1, 2, 3, 4, 5, 6, 7 \}$$

$$A = \{ 2, 4, 6 \}$$

$$A' = U - A = \{ 1, 3, 5, 7 \}$$

# Exercise

Let,

$$U = \{ a, b, c, d, e, f, g, h, i, j, k, l, m \}$$

$$A = \{ a, c, f, m \}$$

$$B = \{ b, c, g, h, m \}$$

Find:

$$A \cup B, A \cap B, |A \cup B|, A - B \text{ dan } A'.$$

$$A \cup B = \{ a, b, c, f, g, h, m \}$$

$$A \cap B = \{ c, m \}$$

$$|A \cup B| = 7$$

$$A - B = \{ a, f \}$$

$$A' = U - A$$

$$= \{ b, d, e, g, h, i, j, k, l \}$$

# Exercise

Let the universe be the set  $U=\{1, 2, 3, 4, \dots, 10\}$ .

Let  $A=\{1, 4, 7, 10\}$ ,  $B=\{1, 2, 3, 4, 5\}$  and  $C=\{2, 4, 6, 8\}$ .

$$B' = \{6, 7, 8, 9, 10\}$$

List the elements of each set:

$$C-A = \{1, 7, 10\} \cap U' = \emptyset$$

$$b) B' \cap (C-A) = \{7, 10\}$$

$$c) B - A = \{2, 3, 5\}$$

$$d) (A \cup B) \cap (C - B) = \emptyset$$

$$A \cup B = \{1, 2, 3, 4, 5, 7, 10\}$$

$$C - B = \{6, 8\}$$

# Set Identities (Properties of Set)

## ■ Commutative laws

$$A \cap B = B \cap A, \quad A \cup B = B \cup A$$

## ■ Associative laws

$$A \cap (B \cap C) = (A \cap B) \cap C$$

$$A \cup (B \cup C) = (A \cup B) \cup C$$

## ■ Distributive laws

$$A \cup (B \cap C) = (A \cup B) \cap (A \cup C)$$

$$A \cap (B \cup C) = (A \cap B) \cup (A \cap C)$$

# Set Identities (Properties of Set)

## ▪ Absorption laws

$$A \cup (A \cap B) = A$$

$$A \cap (A \cup B) = A$$

## ▪ Idempotent laws

$$A \cap A = A, \quad A \cup A = A$$

## ▪ De Morgan's laws

$$(A \cap B)' = A' \cup B'$$

$$(A \cup B)' = A' \cap B'$$

# Set Identities (Properties of Set)

- **Complement laws**

$$A \cap A' = \emptyset \quad A \cup A' = U$$

- **Double complement laws**

$$(A')' = A$$

- **Complement of  $U$  and  $\emptyset$**

$$\emptyset' = U \quad U' = \emptyset$$

# Set Identities (Properties of Set)

- **Properties of universal set**

$$A \cup U = U \quad A \cap U = A$$

- **Set difference laws**

$$A - B = A \cap B'$$

- **Identity laws**

$$A \cup \emptyset = A \quad A \cap U = A$$

- **Properties of empty set**

$$A \cup \emptyset = A \quad A \cap \emptyset = \emptyset$$

# Example

- Let  $A$ ,  $B$  and  $C$  denote the subsets of a set  $S$  and let  $C'$  denote a complement of  $C$  in  $S$ . ( $S = C \cup C'$ )
- If  $A \cap C = B \cap C$  and  $A \cap C' = B \cap C'$ , then prove that  $A = B$

$$\begin{aligned}
 A &= A \cap S \\
 &= A \cap (C \cup C') \\
 &= (A \cap C) \cup (A \cap C') \quad \text{(distributive)} \\
 &= (B \cap C) \cup (B \cap C') \quad \text{(the given conditions)} \\
 &= B \cap (C \cup C') \quad \text{(distributive)} \\
 &= B \cap S \\
 &= B
 \end{aligned}$$

# Example

By referring to the properties of set operations, show that:

$$A - (A \cap B) = A - B$$

set difference  
 $A - B = A \cap B'$

$$\begin{aligned}
 A - (A \cap B) &= A \cap (A \cap B)' \\
 &= A \cap (A' \cup B') \\
 &= (A \cap A') \cup (A \cap B') \\
 &= \emptyset \cup (A \cap B') \\
 &= (A \cap B') \cup \emptyset \\
 &= A \cap B' \\
 &= A - B
 \end{aligned}$$

[set difference laws]  
 [De Morgan's laws]  
 [distributive laws]  
 [complement laws]  
 [commutative]  
 [Identity laws]

# Exercise

$$\begin{aligned}
 1.) (A \cup (B \cap C))' &= ((A \cup B) \cap (A \cup C))' \\
 &= (A \cup B)' \cup (A \cup C)', \quad \text{Distrib Law} \\
 &= (A' \cap B') \cup (A' \cap C') \quad \text{De Morgan Law} \\
 &= A' \cap (B' \cup C') \quad \text{Distrib Law}
 \end{aligned}$$

shown

1) Let  $A, B$  and  $C$  be sets. Show that

$$(A \cup (B \cap C))' = A' \cap (B' \cup C') \quad 2)$$

2) Let  $A, B$  and  $C$  be sets such that

$$A \cap B = A \cap C \text{ and } A \cup B = A \cup C$$

Prove that  $B = C$

The **union** of a collection of sets is the set that contains those elements that are members of **at least one set** in the collection.

## Notation:

$$\bigcup_{i=1}^n A_i = A_1 \cup A_2 \cup \dots \cup A_n = \{x \in U \mid x \in A_i \text{ for at least one } i = 0, 1, 2, \dots, n\}$$

$$\bigcup_{i=1}^{\infty} A_i = A_1 \cup A_2 \cup \dots \cup A_n = \{x \in U \mid x \in A_i \text{ for at least one nonnegative integer } i\}$$

The ***intersection*** of a collection of sets is the set that contains those elements that are members of **all the sets** in the collection.

## Notation:

$$\bigcap_{i=1}^n A_i = A_1 \cap A_2 \cap \dots \cap A_n = \{x \in U \mid x \in A_i \text{ for all } i = 0, 1, 2, \dots, n\}$$

$$\bigcap_{i=1}^{\infty} A_i = A_1 \cap A_2 \cap \dots \cap A_n = \{x \in U \mid x \in A_i \text{ for all nonnegative integer } i\}$$

# Example

For  $i = 1, 2, \dots$ , let  $A_i = \{i, i+1, i+2, \dots\}$ . Then,

$$\bigcup_{i=1}^n A_i = \bigcup_{i=1}^n \{i, i+1, i+2, \dots\} = \{1, 2, 3, \dots\},$$

and

$$\bigcap_{i=1}^n A_i = \bigcap_{i=1}^n \{i, i+1, i+2, \dots\} = \{n, n+1, n+2, \dots\} = A_n$$

# Cartesian Product

- Let  $A$  and  $B$  be sets. An **ordered pair** of elements  $a \in A$  dan  $b \in B$  written  $(a, b)$  is a listing of the elements  $a$  and  $b$  in a specific order.
- The ordered pair  $(a, b)$  specifies that  $a$  is the first element and  $b$  is the second element.
- An ordered pair  $(a, b)$  is considered distinct from ordered pair  $(b, a)$ , unless  $a=b$ .

**Example**  $(1, 2) \neq (2, 1)$

- The Cartesian product of two sets  $A$  and  $B$ , written  $A \times B$  is the set,

$$A \times B = \{(a, b) \mid a \in A, b \in B\}$$

- For any set  $A$ ,

$$A \times \emptyset = \emptyset \times A = \emptyset$$

## Example

$$A = \{a, b\}, B = \{1, 2\}.$$

$$A \times B = \{(a, 1), (a, 2), (b, 1), (b, 2)\}$$

$$B \times A = \{(1, a), (1, b), (2, a), (2, b)\}$$

- if  $A \neq B$ , then  $A \times B \neq B \times A$ .
- if  $|A| = m$  and  $|B| = n$ , then  $|A \times B| = mn$ .

**Example**  $A = \{1, 3\}$ ,  $B = \{2, 4, 6\}$ .

$$A \times B = \{(1, 2), (1, 4), (1, 6), (3, 2), (3, 4), (3, 6)\}$$

$$B \times A = \{(2, 1), (2, 3), (4, 1), (4, 3), (6, 1), (6, 3)\}$$

$$A \neq B, A \times B \neq B \times A$$
$$|A| = 2, |B| = 3, |A \times B| = 2 \cdot 3 = 6.$$

- The Cartesian product of sets  $A_1, A_2, \dots, A_n$  is defined to be the set of all  $n$ -tuples

$(a_1, a_2, \dots, a_n)$  where  $a_i \in A_i$  for  $i=1, \dots, n$ ;

- It is denoted  $A_1 \times A_2 \times \dots \times A_n$

$|A_1 \times A_2 \times \dots \times A_n| = |A_1| \cdot |A_2| \cdot \dots \cdot |A_n|$

# Example

$$A = \{a, b\}, B = \{1, 2\}, C = \{x, y\}$$

$$A \times B \times C = \{(a, 1, x), (a, 1, y), (a, 2, x), (a, 2, y), (b, 1, x), (b, 1, y), (b, 2, x), (b, 2, y)\}$$

$$|A \times B \times C| = 2 \cdot 2 \cdot 2 = 8$$

# Exercise

$$A \times B = \{(w, 1), (w, 2), (n, 1), (n, 2)\}$$

$|A \times B| = 4$  Let  $A = \{w, x\}$ ,  $B = \{1, 2\}$  and  $C = \{nm, ds, ps\}$ .

$$B \times C = \{(1, nm), (1, ds), (1, ps), (2, nm), (2, ds), (2, ps)\}$$

$|B \times C| = 6$  1) Find  $|A \times B|$ ,  $|B \times C|$ ,  $|A \times C|$ ,  $|A \times B \times C|$ ,  $|B \times C \times A|$ ,  $|A \times B \times A \times C|$

$$A \times C = \{(w, nm), (w, ds), (w, ps), (n, nm), (n, ds), (n, ps)\}$$

2) Determine the following set,

$|A \times C| = 6$  a)  $A \times B$ ,  $B \times C$ ,  $A \times C$

b)  $A \times B \times C$   
c)  $B \times C \times A$

d)  $A \times B \times A \times C$

$|A \times B \times A \times C| = 24$

$$A \times B \times C = \{(w, 1, nm), (w, 1, ds), (w, 1, ps), (w, 2, nm), (w, 2, ds), (w, 2, ps), (n, 1, nm), (n, 1, ds), (n, 1, ps), (n, 2, nm), (n, 2, ds), (n, 2, ps)\}$$

# Exercise

Let  $X = \{1, 2\}$ ,  $Y = \{a\}$  and  $Z = \{b, d\}$ .

List the elements of each set.

a)  $X \times Y = \{(1, a), (2, a)\}$

b)  $Y \times X = \{(a, 1), (a, 2)\}$

c)  $X \times Y \times Z = \{(1, a, b), (1, a, d), (2, a, b), (2, a, d)\}$

d)  $X \times Y \times Y = \{(1, a, a), (2, a, a)\}$

e)  $X \times X \times X = \{(1, 1, 1), (1, 1, 2), (1, 2, 1), (1, 2, 2),$

f)  $Y \times X \times Y \times Z = \{(2, 1, 1), (2, 1, 2), (2, 2, 1), (2, 2, 2)\}$

$= \{(a, 1, a, b), (a, 1, a, d), (a, 2, a, b), (a, 2, a, d)\}$

# Thank You

# Thank You