DISCRETE STRUCTURE | SECI1013

UNIVERSITI TEKNOLOGI MALAYSIA

Counting Methods (Part 1)

CHAPTER 3

SEM 1 20252026

Innovating Sustainable Solutions

Basic Counting Principles

- Counting principle is all about choices we might make given many possibilities.
- It is used to find the number of possible outcomes.
- It provides a basis of computing probabilities of discrete events.

Basic Counting Principles

Some sample of counting problems:

- Problem 1 - How many ways are there to seat *n* couples at a round table, such that each couple sits together?

- Problem 2- How many ways are there to express a positive integer *n* as a sum of positive integers?

Basic Counting Principles

- Problem 3 - There are three boxes containing books. The first box contains 15 mathematics books by different authors, the second box contains 12 chemistry books by different authors, and the third box contains 10 computer science books by different authors. A student wants to take a book from one of the three boxes. In how many ways can the student do this?

$\boxed{15} + \boxed{12} + \boxed{10}$

$= 37$ ways

Basic Counting Principles

- Problem 4 - The department will award a free computer to either a CS student or a CS professor. How many different choices are there, if there are 530 students and 15 professors?

T_1 = number of students , $n_1 = 530$

T_2 = number of prof , $n_2 = 15$

$n_1 + n_2 = 545$ ways

UTM UNIVERSITI TEKNOLOGI MALAYSIA

- Problem 5 - A scholarship is available, and the student to receive this scholarship must be chosen from the Mathematics, Computer Science, or the Engineering Department. How many different choices are there for this student if there are 38 qualified students from the Mathematics Department, 45 qualified students from the Computer Science Department and 27 qualified students from the Engineering Department?

T_1 = Qualified from Math $N_1 = 38$
T_2 = Qualified from CS $N_2 = 95$
T_2 = Qualified from Engi $N_3 = 27$

$N_1 + N_2 + N_3 = 110$ ways

Basic Counting Principles

- There are a number of basic principles that we can use to solve such problems:
  1) **Addition Principle**
  2) **Multiplication Principle**

Addition Principle

- Suppose that tasks $T_1, T_2, \ldots T_k$ can be done in $n_1, n_2, \ldots n_k$ ways, respectively.

- If all these tasks are **independent** of each other, then the number of ways to do one of these tasks is $n_1 + n_2 + \ldots + n_k$

Basic Counting Principles

- If a task can be done in n_1 ways and a second task in n_2 ways, and if these two tasks cannot be done at the same time, then there are $n_1 + n_2$ ways to do either task.

Example 1

We want to find the number of integers between 5 and 50 that end with 1 or 7.

Solution:

T_1 = number of integer 5 – 50 that end with 1

$\Rightarrow$ 11, 21, 31, 41

$N_1 = 4$ ways

T_2 = number of integer 5 – 50 that end with 7

$\Rightarrow$ 7, 17, 27, 37, 47

$N_2 = 5$ ways

$N_1 + N_2 = 4 + 5$

$= 9$ ways

- T_1: find all integers between 5 and 50 that end with 1.

  ➔11, 21, 31, 41 => 4 ways
- T_2: find all integers between 5 and 50 that end with 7.

  ➔7, 17, 27, 37, 47 => 5 ways

.: $n_1 + n_2 = 4 + 5 = 9$ ways.

Example 2

We want to find the number of integers between 4 and 100 that end with 3 or 5.

Solution:

T_1 : number of integer from 4 to 100 that end with 3

=> 13, 23, 33, 43, 53, 63, 73, 83, 93

N_1 = 9 ways

T_2 : number of integer from 4 to 100 that end with 5

=> 5, 15, 25, 35, 45, 55, 65, 75, 85, 95

N_2 = 10 ways

$N_1 + N_2 = 9 + 10$

$= 19$ ways

- T_1: find all integers between 4 and 100 that end with 3.

  => 13, 23, 33, 43, 53, 63, 73, 83, 93

  $\Rightarrow$9 integers

- T_2: find all integers between 4 and 100 that end with 5.

  => 5, 15, 25, 35, 45, 55, 65, 75, 85, 95

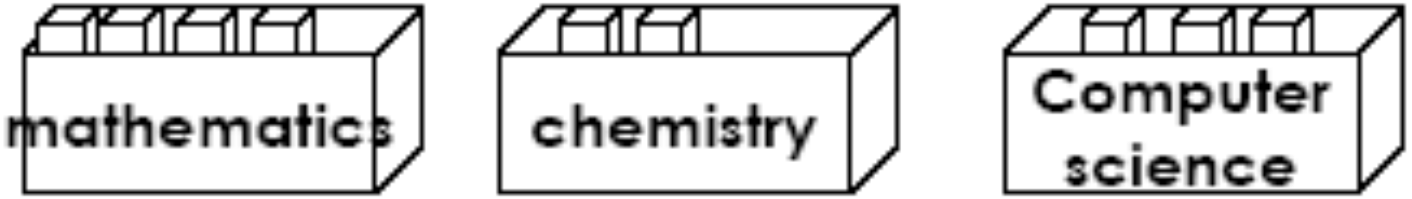
  => 10 integers

- Task T_1 can be done in 9 ways.
- Task T_2 can be done in 10 ways.
- The number of ways to do one of these tasks is,

$$9+10=19$$

- Task T can be completed in 19 ways.

Example 3



4 books | 2 books | 3 books

- A student wants to take a book from one of the three boxes.

T_1 : take a book from math boxes
=> 4 books
$N_1 = 4$ ways

T_2 : take a book from chem boxes
=> 2 books
$N_2 = 2$ ways

T_3 : take a book from cs boxes
=> 3 books
$N_3 = 3$ ways

$N_1 + N_2 + N_3 = 4 + 2 + 3$
$= 9$ ways

- T_1: choose a mathematics book
  - 4 ways
- T_2: choose a chemistry book
  - 2 ways
- T_3: choose a computer science book.
  - 3 ways
- The number of ways to do one of these tasks is,
  - $4+2+3=9$

Multiplication Principle

- A task T can be completed in k successive steps.

  Step 1 can be completed in n_1 different ways.

  Step 2 can be completed in n_2 different ways.

  Step *k* can be completed in n_k different ways.

- Then the task T can be completed in $n_1.n_2.... n_k$ different ways.

Suppose that a procedure can be broken down into two successive tasks. If there are n_1 ways to do the first task and n_2 ways to do the second task after the first task has been done, then there are n_1n_2 ways to do the procedure.

Example 4

- Morgan is a lead actor in a new movie. She needs to shoot a scene in the morning in studio A and an afternoon scene in studio C. She looks at the map and finds that there is no direct route from studio A to studio C. Studio B is located between studios A and C. Morgan's friends Brad and Jennifer are shooting a movie in studio B. There are three roads, say A_1, A_2, and A_3, from studio A to studio B and four roads, say B_1, B_2, B_3, and B_4, from studio B to studio C. In how many ways can Morgan go from studio A to studio C and have lunch with Brad and Jennifer at Studio B?

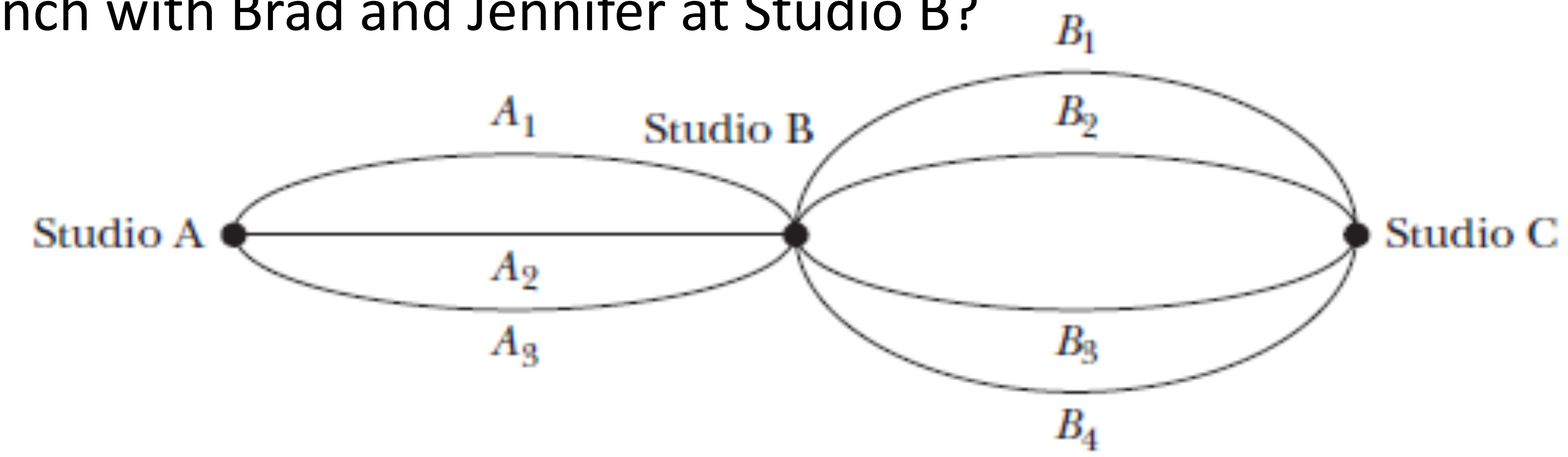


FIGURE 7.1 Routes from studio A to studio C

T_1 = road from A to b

$\Rightarrow A1, A2, A3$

$N_1 = 3$ ways

T_2 = road from B to C

$\Rightarrow B1, B2, B3, B4$

$N_2 = 4$ ways

$N_1 \cdot N_2 = 3 \times 4$

$= 12$ ways

Example 5

- The letters *A*, *B*, *C*, *D*, and *E* are to be used to form strings of length 4.
- How many strings can be formed if we do not allow repetitions?
- For example:
  - *BADE, ACBD, AEBC* ..

UTM UNIVERSITI TEKNOLOGI MALAYSIA

T_1 : first letter without repition $\quad N_1 \cdot N_2 \cdot N_3 \cdot N_4 = 120$ ways

=> 5 letter

$N_1 = 5$ ways

T_2 = second letter without repetition

=> 4 letter

$N_2 = 4$ ways

T_3 = third letter without repetition

=> 3 letter

$N_3 = 3$ ways

T_4 = fourth letter without repetition

=> 2 letter

$N_4 = 2$ ways

Example 6

- The letters *A, B, C, D*, and *E* are to be used to form strings of length 4.
- How many strings can be formed if we allow repetitions?
- For example:
  - *BABB, AABB, ACEE* ..

T_1 : First letter with repetition.

$\Rightarrow$ 5 letter

N_1 = 5 ways

T_2 = second letter with repetition

$\Rightarrow$ 5 letter

N_2 = 5 ways

T_3 = third letter with repetition

$\Rightarrow$ 5 letter

N_3 = 5 ways

T_4 = Fourth letter with repetition

$\Rightarrow$ 5 letter

N_4 = 5 ways

$N_1 \cdot N_2 \cdot N_3 \cdot N_4 = 5 \cdot 5 \cdot 5 \cdot 5$

5 625 ways

Example 7

- The letters *A, B, C, D*, and *E* are to be used to form strings of length 4.
- How many strings begin with *A*, if repetitions are not allowed?
- For example:
  - *ADEC, ACBD, AEBC* ..

UTM UNIVERSITI TEKNOLOGI MALAYSIA

T_1 = A as the first letter

$\Rightarrow$ A

$N_1 = 1$ ways

T_2 = second letter without repetition

$\Rightarrow$ 4 letter

$N_2 = 4$ ways

T_3 = Third letter without repetition

$\Rightarrow$ 3 letter

$N_3 = 3$ ways

T_4 = Fourth letter without repetition

$\Rightarrow$ 2 letter

$N_4 = 2$ ways

$$N_1 \cdot N_2 \cdot N_3 \cdot N_4 = 1 \cdot 4 \cdot 3 \cdot 2$$
$$= 24 \text{ ways}$$

Basic Counting Principles

- The counting problem that we have considered so far involved either the addition principle or the multiplication principle.
- Sometimes, however, we need to use both of these counting principles to solve a particular problem.

Example 8

- How many 8-bit strings begin either 101 or 111?

T_1 : 8-bit string begin with 101

$$\Rightarrow \underset{1}{\underline{1\ 0\ 1}}\ \underset{2}{\_}\ \underset{2}{\_}\ \underset{2}{\_}\ \underset{2}{\_}\ \underset{2}{\_}$$

$N_1 = 1 \cdot 2 \cdot 2 \cdot 2 \cdot 2 \cdot 2$

$= 32$ ways

T_2 : 8-bit string begin with 111

$$\Rightarrow \underset{1}{\underline{1\ 1\ 1}}\ \underset{2}{\_}\ \underset{2}{\_}\ \underset{2}{\_}\ \underset{2}{\_}\ \underset{2}{\_}$$

$N_2 = 1 \cdot 2 \cdot 2 \cdot 2 \cdot 2 \cdot 2$

$= 32$ ways

$N_1 + N_2 = 32 + 32$

$= 64$ ways

Example 9

- The following items are available for breakfast.
  - 4 types of cereal
  - 2 types of juice
  - 3 types of bread
- How many ways a breakfast can be prepared if exactly 2 items are selected from 2 different groups?

T_1 : 2 item from cereal and juice

$\Rightarrow 4 \cdot 2$

$N_1 = 8$ ways

T_2 : 2 item from cereal and bread

$\Rightarrow 4 \cdot 3$

$N_2 = 12$ ways

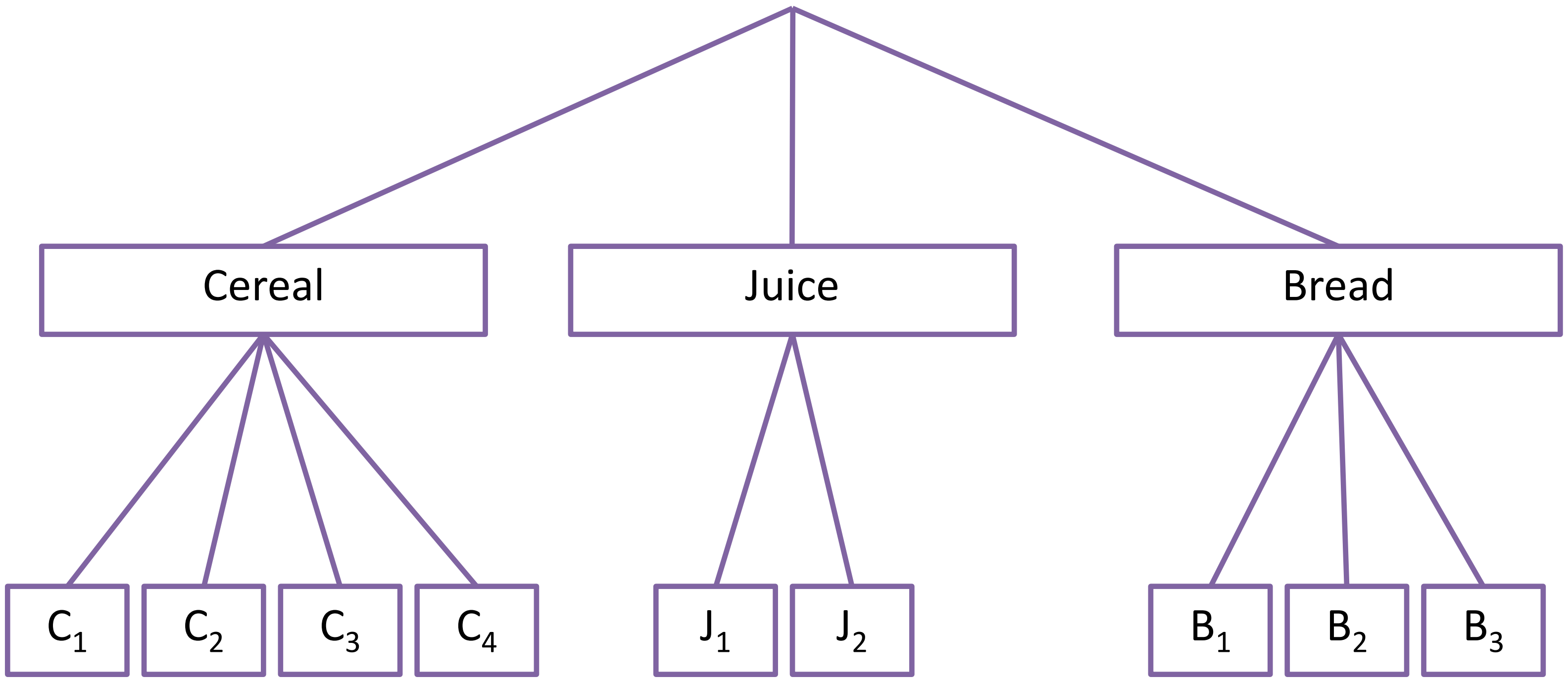
T_3 : 2 item from juice and bread

$\Rightarrow 2 \cdot 3$

$N_3 = 6$ ways

$N_1 + N_2 + N_3$

$= 26$ ways



C_1J_1	C_1J_2
C_2J_1	C_2J_2
C_3J_1	C_3J_2
C_3J_1	C_3J_2

C_1B_1	C_1B_2	C_1B_3
C_2B_1	C_2B_2	C_2B_3
C_3B_1	C_3B_2	C_3B_3
C_4B_1	C_4B_2	C_4B_3

J_1B_1	J_1B_2	J_1B_3
J_2B_1	J_2B_2	J_2B_3

Example 10

A six-person committee composed of Aina, Wan, Chan, Tan, Syed and Helmi is to be selected to hold as a chairperson, secretary, and treasurer.

1 ▪In how many ways can this be done?

2 ▪In how many ways can this be done if either Aina or Wan must be chairperson?

3 ▪In how many ways can this be done if Syed must hold one of the position?

4 ▪In how many ways can this be done if Tan and Helmi must hold any position?

Example 10 - Solution

1.

T_1 : Choosen as chair person

$\Rightarrow$ 6 person

$N_1 = 6$ ways

T_2 : choosen as secretary

$\Rightarrow$ 5 person

$N_2 = 5$ ways

T_3 = choosen as treasury

$\Rightarrow$ 4 person

$N_3 = 4$ ways

$N_1 \cdot N_2 \cdot N_3 = 6 \cdot 5 \cdot 4$

$= 120$ ways

2.

T_1 : Aina or Wan must selected as chair person

$\Rightarrow$ 2 person

$N_1 = 2$ ways

T_2 : choosen as secretary

$\Rightarrow$ 5 person

$N_2 = 5$ ways

T_3 : choosen as treasury

$\Rightarrow$ 4 person

$N_3 = 4$ ways

$N_1 \cdot N_2 \cdot N_3 = 2 \cdot 5 \cdot 4$

$= 40$ ways

3. T_1: Syed choosen as chairperson

$\Rightarrow 1 \cdot 5 \cdot 4$

$N_1 = 20$ ways

T_2: Syed choosen as secretary

$\Rightarrow 5 \cdot 1 \cdot 4$

$N_2 = 20$ ways

T_3: Syed choosen as treasurer

$\Rightarrow 5 \cdot 4 \cdot 1$

$N_3 = 20$ ways

$N_1 + N_2 + N_3 = 20 + 20 + 20$

$= 60$ ways

4.

T_1: Tan and helmi chouse as Choirperson or secretary

$\Rightarrow 2 \cdot 1 \cdot 4$

$N_1 = 8$ ways

T_2: Tan and helmi choosen as chairperson or treasurer

$\Rightarrow 2 \cdot 4 \cdot 1$

$N_2 = 8$ ways

T_3: Tan and helmi choosen as Secretary or treasurer

$\Rightarrow 4 \cdot 2 \cdot 1$

$N_3 = 8$ ways

$N_1 + N_2 + N_3 = 8 + 8 + 8$

$= 24$ ways

- OR
  - ✓Select the chairperson (2 ways)
  - ✓Select the secretary (5 ways)
  - ✓Select the treasurer (4 ways)
  - ✓2. 5. 4 =40

- In how many ways can this be done if Syed must hold one of the position?
  - If Syed is chairperson 5. 4 =20
  - If Syed is secretary 5. 4 =20
  - If Syed is treasurer 5. 4 =20
  - 20+20+20 =60

- OR
    - Assign Syed for any position is 3 ways
    - Fill the highest remaining position is 5 ways
    - Fill the last position is 4 ways
    - 3. 5. 4 =60

- In how many ways can this be done if Tan and Helmi must hold any position?
  - Assign Tan - 3 ways
  - Assign Helmi - 2 ways
  - Fill the remaining position - 4 ways
  - 3. 2. 4 =24

- In how many ways can this be done if Tan and Helmi must hold any position?
  - ➢ Assign Tan - 3 ways
  - ➢ Assign Helmi - 2 ways
  - ➢ Fill the remaining position - 4 ways
  - ➢ 3. 2. 4 =24