

# Chapter 1

## SET THEORY

### [Part 2: Operation on Set]

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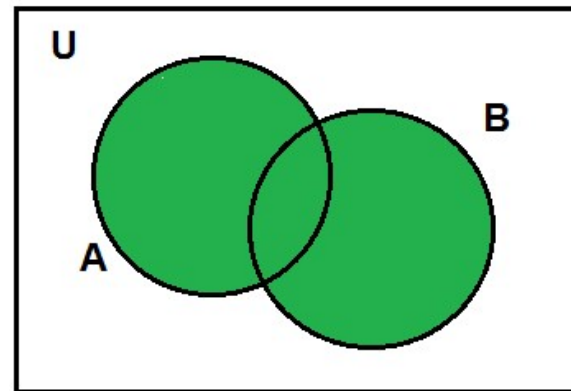
# Union

- The **union** of two sets  $A$  and  $B$ , denoted by  $A \cup B$ , is defined to be the set

$$A \cup B = \{x \mid x \in A \text{ or } x \in B\}$$

- The union consists of all elements belonging to either  $A$  or  $B$  (or both)

**Venn diagram of  $A \cup B$**



# Example

$A = \{1, 2, 3, 4, 5\}$ ,  $B = \{2, 4, 6\}$  and  $C = \{8, 9\}$

$$A \cup B = \{1, 2, 3, 4, 5, 6\}$$

$$A \cup C = \{1, 2, 3, 4, 5, 8, 9\}$$

$$B \cup C = \{2, 4, 6, 8, 9\}$$

$$A \cup B \cup C = \{1, 2, 3, 4, 5, 6, 8, 9\}$$

# Union

If  $A$  and  $B$  are finite sets, the **cardinality** of  $A \cup B$ ,

$$|A \cup B| = |A| + |B| - |A \cap B|$$

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bung data  
yg sama

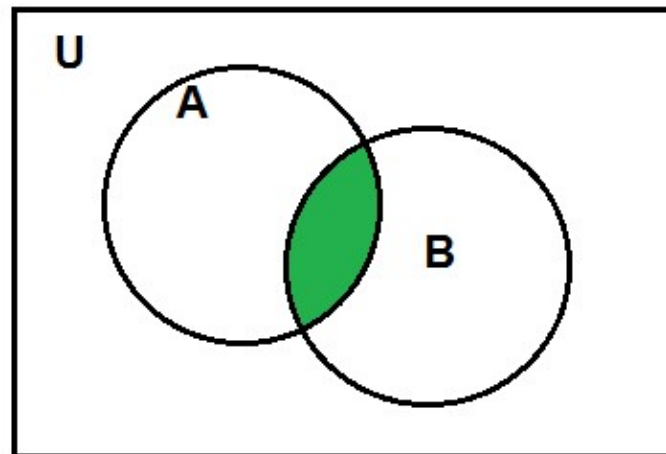
# Intersection

- The **intersection** of two sets  $A$  and  $B$ , denoted by  $A \cap B$ , is defined to be the set

$$A \cap B = \{ x \mid x \in A \text{ and } x \in B \}$$

- The **intersection** consists of all elements belonging to both  $A$  and  $B$ .

**Venn diagram of  $A \cap B$**



# Example

$A = \{1, 2, 3, 4, 5, 6\}$ ,  $B = \{2, 4, 6, 8, 10\}$  and  $C = \{1, 2, 8, 10\}$

$$A \cap B = \{2, 4, 6\}$$

$$A \cap C = \{1, 2\}$$

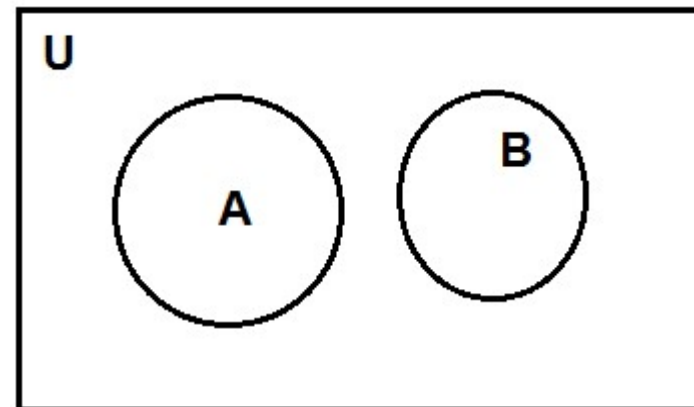
$$C \cap B = \{2, 8, 10\}$$

$$A \cap B \cap C = \{2\}$$

# Disjoint

Two sets  $A$  and  $B$  are said to be **disjoint** if,  $A \cap B = \emptyset$

**Venn diagram,  $A \cap B = \emptyset$**



## Example

$$A = \{1, 3, 5, 7, 9, 11\}, B = \{2, 4, 6, 8, 10\}$$

$$A \cap B = \emptyset$$



# Difference

The set,

$$A - B = \{x \mid x \in A \text{ and } x \notin B\}$$

is called the **difference**.

The difference  $A - B$  consists of all elements in  $A$  that are not in  $B$ .

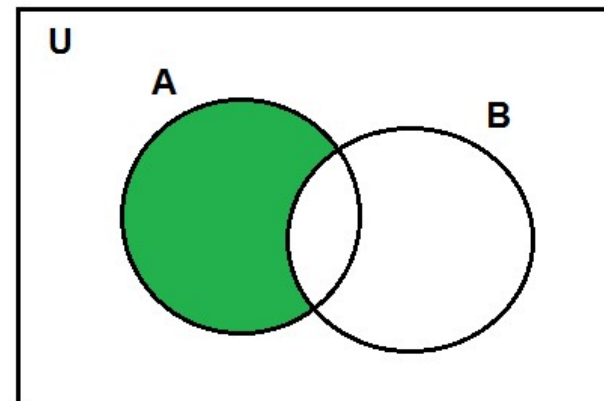
## Venn diagram of $A - B$

### Example

$$A = \{1, 2, 3, 4, 5, 6, 7, 8\}$$

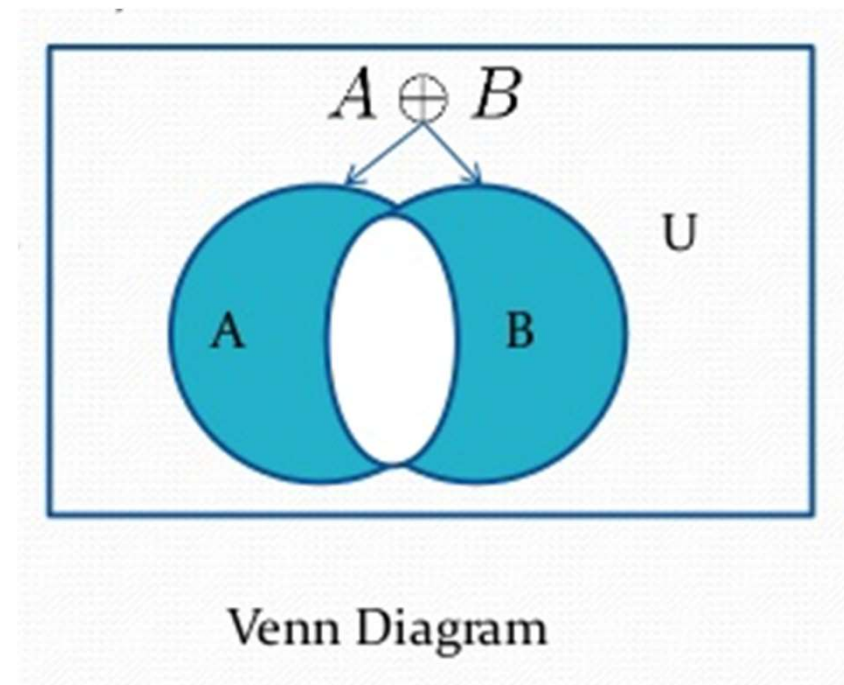
$$B = \{2, 4, 6, 8\}$$

$$A - B = \{1, 3, 5, 7\}$$



# Symmetric Difference

The symmetric difference of set  $A$  and set  $B$ , denoted by  $A \oplus B$  is the set  $(A - B) \cup (B - A)$



## Example

$$U = \{0,1,2,3,4,5,6,7,8,9,10\}$$

$$A = \{1,2,3,4,5\}; B = \{4,5,6,7,8\}$$

$$A \oplus B = (A - B) \cup (B - A) = \{1,2,3,6,7,8\}$$



$$A - B = \{1,2,3\}$$



$$B - A = \{6,7,8\}$$

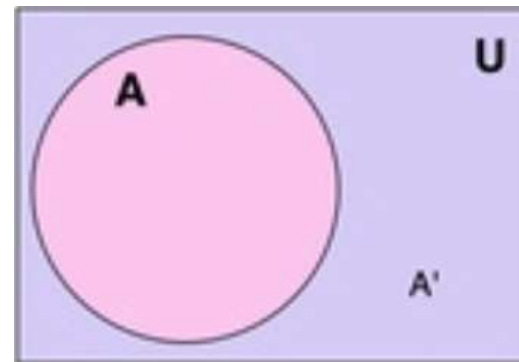
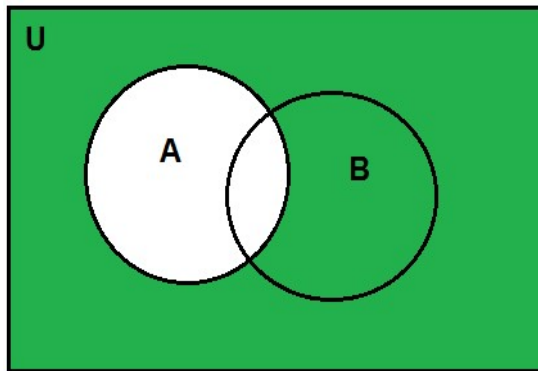
# Complement

The complement of a set  $A$  with respect to a universal set  $U$ , denoted by  $A'$  is defined to be

$$A' = \{x \in U \mid x \notin A\}$$

$$A' = U - A$$

## Venn diagram of $A'$



# Example

Let  $U$  be a universal set,

$$U = \{ 1, 2, 3, 4, 5, 6, 7 \}$$

$$A = \{ 2, 4, 6 \}$$

$$A' = U - A = \{ 1, 3, 5, 7 \}$$

# Exercise

Let,

$$U = \{ a, b, c, d, e, f, g, h, i, j, k, l, m \}$$

$$A = \{ a, c, f, m \}$$

$$B = \{ b, c, g, h, m \}$$

Find:

$$A \cup B, A \cap B, |A \cup B|, A - B \text{ dan } A'.$$

$$A \cup B = \{ a, b, c, f, g, h, m \}$$

$$A - B = \{ a, f \}$$

$$A \cap B = \{ c, m \}$$

$$A' = U - A$$

$$|A \cup B| = 7$$

$$= \{ b, d, e, g, h, i, j, k, l \}$$

# Exercise

Let the universe be the set  $U=\{1, 2, 3, 4, \dots, 10\}$ .

Let  $A=\{1, 4, 7, 10\}$ ,  $B=\{1, 2, 3, 4, 5\}$  and  $C=\{2, 4, 6, 8\}$ .

List the elements of each set:

- a)  $U' = \emptyset$
- b)  $B' \cap (C-A) = \{7, 10\}$
- c)  $B - A = \{2, 3, 5\}$
- d)  $(A \cup B) \cap (C - B) = \emptyset$

$$A \cup B = \{1, 2, 3, 4, 5, 7, 10\}$$

$$C - B = \{6, 8\}$$

# Set Identities (Properties of Set)

## ▪ Commutative laws

$$A \cap B = B \cap A, \quad A \cup B = B \cup A$$

## ▪ Associative laws

$$A \cap (B \cap C) = (A \cap B) \cap C$$

$$A \cup (B \cup C) = (A \cup B) \cup C$$

## ▪ Distributive laws

$$A \cup (B \cap C) = (A \cup B) \cap (A \cup C)$$

$$A \cap (B \cup C) = (A \cap B) \cup (A \cap C)$$



# Set Identities (Properties of Set)

- **Absorption laws**

$$A \cup (A \cap B) = A$$

$$A \cap (A \cup B) = A$$

- **Idempotent laws**

$$A \cap A = A, \quad A \cup A = A$$

- **De Morgan's laws**

$$(A \cap B)' = A' \cup B'$$

$$(A \cup B)' = A' \cap B'$$

# Set Identities (Properties of Set)

- **Complement laws**

$$A \cap A' = \emptyset \quad A \cup A' = U$$

- **Double complement laws**

$$(A')' = A$$

- **Complement of  $U$  and  $\emptyset$**

$$\emptyset' = U \quad U' = \emptyset$$

# Set Identities (Properties of Set)

- **Properties of universal set**

$$A \cup U = U \quad A \cap U = A$$

- **Set difference laws**

$$A - B = A \cap B'$$

- **Identity laws**

$$A \cup \emptyset = A \quad A \cap U = A$$

- **Properties of empty set**

$$A \cup \emptyset = A \quad A \cap \emptyset = \emptyset$$

## Example

- Let  $A$ ,  $B$  and  $C$  denote the subsets of a set  $S$  and let  $C'$  denote a complement of  $C$  in  $S$ . ( $S = (C \cup C')$ )
- If  $A \cap C = B \cap C$  and  $A \cap C' = B \cap C'$ , then prove that  $A = B$

$$\begin{aligned} A &= A \cap S \\ &= A \cap (C \cup C') \\ &= (A \cap C) \cup (A \cap C') && \text{(distributive)} \\ &= (B \cap C) \cup (B \cap C') && \text{(the given conditions)} \\ &= B \cap (C \cup C') && \text{(distributive)} \\ &= B \cap S \\ &= B \end{aligned}$$

## Example

By referring to the properties of set operations, show that:

$$A - (A \cap B) = A - B$$

set difference

$$A - B = A \cap B'$$

$$\begin{aligned} A - (A \cap B) &= A \cap (A \cap B)' \\ &= A \cap (A' \cup B') \\ &= (A \cap A') \cup (A \cap B') \\ &= \emptyset \cup (A \cap B') \\ &= (A \cap B') \cup \emptyset \\ &= A \cap B' \\ &= A - B \end{aligned}$$

[set difference laws]

[De Morgan's laws]

[distributive laws]

[complement laws]

[commutative]

[Identity laws]

## Exercise

$$\begin{aligned}
 1.) (A \cup (B \cap C))' &= ((A \cup B) \cap (A \cup C))' && \text{Distributive Law} \\
 &= (A \cup B)' \cap (A \cup C)' && \text{De Morgan's Law} \\
 &= (A' \cap B') \cap (A' \cap C') && \text{De Morgan's Law} \\
 &= A' \cap (B' \cap C') && \text{Distributive Law}
 \end{aligned}$$

*Handwritten notes: "tutur lepas" (bracket removed), "masuk ke dalam" (go inside), "Distributive Law", "De Morgan's Law", "Distributive Law".*

Shown

1) Let  $A$ ,  $B$  and  $C$  be sets. Show that

$$(A \cup (B \cap C))' = A' \cap (B' \cup C')$$

2) Let  $A$ ,  $B$  and  $C$  be sets such that

$$A \cap B = A \cap C \text{ and } A \cup B = A \cup C$$

Prove that  $B = C$

The ***union*** of a collection of sets is the set that contains those elements that are members of **at least one set** in the collection.

## Notation:

$$\bigcup_{i=1}^n A_i = A_1 \cup A_2 \cup \dots \cup A_n = \{x \in U \mid x \in A_i \text{ for at least one } i = 0, 1, 2, \dots, n\}$$

$$\bigcup_{i=1}^{\infty} A_i = A_1 \cup A_2 \cup \dots \cup A_n = \{x \in U \mid x \in A_i \text{ for at least one nonnegative integer } i\}$$

The ***intersection*** of a collection of sets is the set that contains those elements that are members of **all the sets** in the collection.

## Notation:

$$\bigcap_{i=1}^n A_i = A_1 \cap A_2 \cap \dots \cap A_n = \{x \in U \mid x \in A_i \text{ for all } i = 0, 1, 2, \dots, n\}$$

$$\bigcap_{i=1}^{\infty} A_i = A_1 \cap A_2 \cap \dots \cap A_n = \{x \in U \mid x \in A_i \text{ for all nonnegative integer } i\}$$



# Example

For  $i = 1, 2, \dots$ , let  $A_i = \{i, i+1, i+2, \dots\}$ . Then,

$$\bigcup_{i=1}^n A_i = \bigcup_{i=1}^n \{i, i+1, i+2, \dots\} = \{1, 2, 3, \dots\},$$

and

$$\bigcap_{i=1}^n A_i = \bigcap_{i=1}^n \{i, i+1, i+2, \dots\} = \{n, n+1, n+2, \dots\} = A_n$$

# Cartesian Product

- Let  $A$  and  $B$  be sets. An **ordered pair** of elements  $a \in A$  dan  $b \in B$  written  $(a, b)$  is a listing of the elements  $a$  and  $b$  in a specific order.
- The ordered pair  $(a, b)$  specifies that  $a$  is the first element and  $b$  is the second element.
- An ordered pair  $(a, b)$  is considered distinct from ordered pair  $(b, a)$ , unless  $a=b$ .

**Example**  $(1, 2) \neq (2, 1)$

# Cartesian Product

- The Cartesian product of two sets  $A$  and  $B$ , written  $A \times B$  is the set,

$$A \times B = \{(a, b) \mid a \in A, b \in B\}$$

- For any set  $A$ ,

$$A \times \emptyset = \emptyset \times A = \emptyset$$

## Example

$$A = \{a, b\}, B = \{1, 2\}.$$

$$A \times B = \{(a, 1), (a, 2), (b, 1), (b, 2)\}$$

$$B \times A = \{(1, a), (1, b), (2, a), (2, b)\}$$

# Cartesian Product

- if  $A \neq B$ , then  $A \times B \neq B \times A$ .
- if  $|A| = m$  and  $|B| = n$ , then  $|A \times B| = mn$ .

**Example**      $A = \{1, 3\}$ ,  $B = \{2, 4, 6\}$ .

$$A \times B = \{(1, 2), (1, 4), (1, 6), (3, 2), (3, 4), (3, 6)\}$$

$$B \times A = \{(2, 1), (2, 3), (4, 1), (4, 3), (6, 1), (6, 3)\}$$

$$A \neq B, A \times B \neq B \times A$$

$$|A| = 2, |B| = 3, |A \times B| = 2 \cdot 3 = 6.$$

# Cartesian Product

- The Cartesian product of sets  $A_1, A_2, \dots, A_n$  is defined to be the set of all  $n$ -tuples

$(a_1, a_2, \dots, a_n)$  where  $a_i \in A_i$  for  $i=1, \dots, n$ ;

- It is denoted  $A_1 \times A_2 \times \dots \times A_n$

$$|A_1 \times A_2 \times \dots \times A_n| = |A_1| \cdot |A_2| \cdot \dots \cdot |A_n|$$

# Example

$$A = \{a, b\}, B = \{1, 2\}, C = \{x, y\}$$

$$A \times B \times C = \{(a, 1, x), (a, 1, y), (a, 2, x), (a, 2, y), \\ (b, 1, x), (b, 1, y), (b, 2, x), (b, 2, y)\}$$

$$|A \times B \times C| = 2 \cdot 2 \cdot 2 = 8$$



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## Exercise

$$A \times B = \{(w, 1), (w, 2), (x, 1), (x, 2)\}$$

$|A \times B| = 4$  Let  $A = \{w, x\}$ ,  $B = \{1, 2\}$  and  $C = \{nm, ds, ps\}$ .

$$B \times C = \{(1, nm), (1, ds), (1, ps), (2, nm), (2, ds), (2, ps)\}$$

$|B \times C| = 6$  1) Find  $|A \times B|$ ,  $|B \times C|$ ,  $|A \times C|$ ,  $|A \times B \times C|$ ,  $|B \times C \times A|$ ,  $|A \times B \times A \times C|$

$$A \times C = \{(w, nm), (w, ds), (w, ps), (x, nm), (x, ds), (x, ps)\}$$

2) Determine the following set,

$$|A \times C| = 6$$

a)  $A \times B$ ,  $B \times C$ ,  $A \times C$

$$|A \times B \times C| = 12$$

b)  $A \times B \times C$

c)  $B \times C \times A$

$$|B \times C \times A| = 12$$

d)  $A \times B \times A \times C$

$$|A \times B \times A \times C| = 24$$

$$\begin{aligned} & \{(1, nm, w), (1, nm, x), (1, ds, w), (1, ds, x), (1, ps, w), (1, ps, x), \\ & (2, nm, w), (2, nm, x), (2, ds, w), (2, ds, x), (2, ps, w), (2, ps, x)\} \end{aligned}$$

$$\begin{aligned} A \times B \times C = & \{(w, 1, nm), (w, 1, ds), (w, 1, ps), (w, 2, nm), \\ & (w, 2, ds), (w, 2, ps), (x, 1, nm), (x, 1, ds), \\ & (x, 1, ps), (x, 2, nm), (x, 2, ds), (x, 2, ps)\} \end{aligned}$$

# Exercise

Let  $X = \{1, 2\}$ ,  $Y = \{a\}$  and  $Z = \{b, d\}$ .

List the elements of each set.

a)  $X \times Y = \{(1, a), (2, a)\}$

b)  $Y \times X = \{(a, 1), (a, 2)\}$

c)  $X \times Y \times Z = \{(1, a, b), (1, a, d), (2, a, b), (2, a, d)\}$

d)  $X \times Y \times Y = \{(1, a, a), (2, a, a)\}$

e)  $X \times X \times X = \{(1, 1, 1), (1, 1, 2), (1, 2, 1), (1, 2, 2),$

f)  $Y \times X \times Y \times Z$   
 $(2, 1, 1), (2, 1, 2), (2, 2, 1), (2, 2, 2)$

$= \{(a, 1, a, b), (a, 1, a, d), (a, 2, a, b), (a, 2, a, d)\}$



# Thank You

# Thank You