# ECEN 325 Lab 2: Second Order Circuits

## 1 Objectives

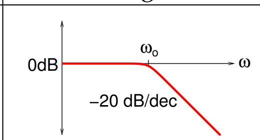
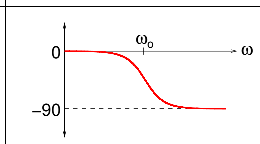
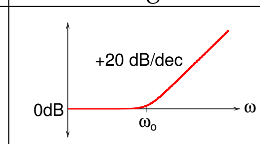
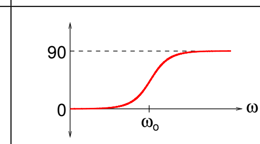
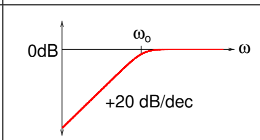
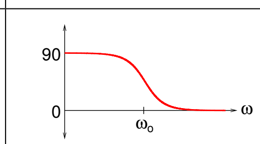
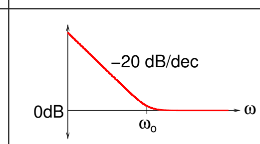
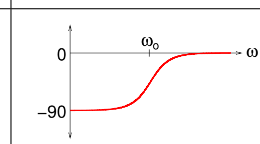
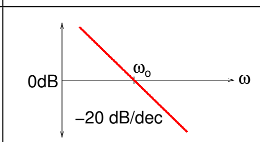
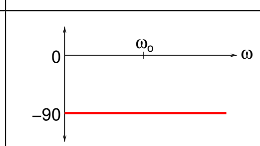
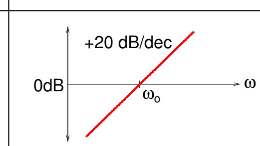
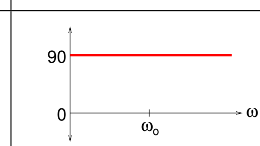
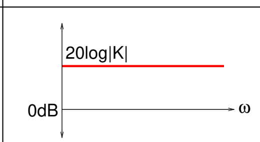
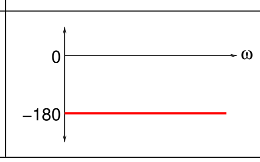
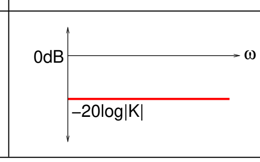
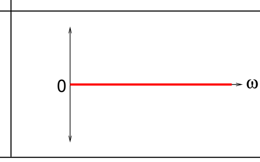
The purpose of the lab is to investigate the frequency response of second order circuits and further practice circuit design and analysis techniques in the frequency domain.

## 2 Introduction

It is useful to format a transfer function as a multiplication of known functions so that its frequency response can be easily sketched without the need for complex tools. Since the y-axis of Bode plots are in dB-scale, Bode plot of the overall transfer function can be simply obtained by graphically adding Bode plots of individual first or second order sections on the same frequency axis.

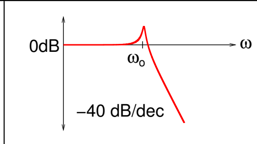
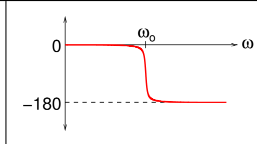
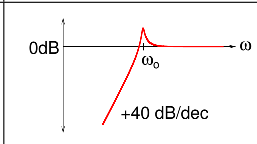
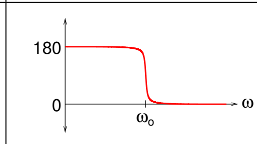
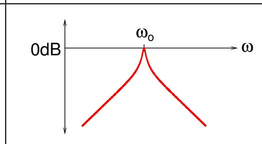
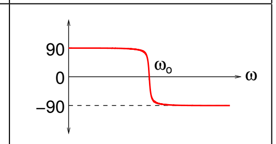
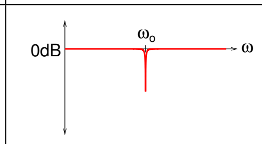
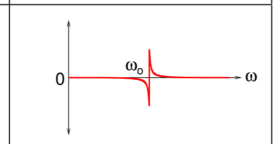
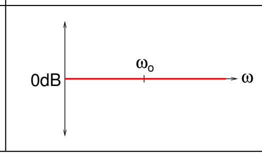
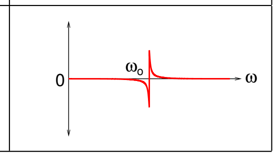
In case the derivation of $H(s)$ already yields multiplication of known functions, then it is best to keep them in these formats. However, if the derivation yields multiplication of transfer functions with high order numerator or denominator polynomials, then you may need to decompose them into smaller known sections. Table 1 shows first order functions and their magnitude and phase responses.

Table 1: First Order Functions

| $H(s)$ | Bode Magnitude | Bode Phase | $H(s)$ | Bode Magnitude | Bode Phase |
|---|---|---|---|---|---|
| $\dfrac{1}{1+\frac{s}{\omega_o}}$ |  |  | $1+\dfrac{s}{\omega_o}$ |  |  |
| $\dfrac{s}{s+\omega_o}$ |  |  | $\dfrac{s+\omega_o}{s}$ |  |  |
| $\dfrac{\omega_o}{s}$ |  |  | $\dfrac{s}{\omega_o}$ |  |  |
| $-K$ |  |  | $\dfrac{1}{K}$ |  |  |

In case any section of the transfer function has a second order denominator with complex poles, then that section cannot be decomposed into first order functions. Table 2 shows the second order functions with complex poles and their magnitude and phase responses. Remember that second order polynomials with real poles can always be expressed as a multiplication of two first-order polynomials, so that a combination of first order functions in Table 1 can still be used.

Table 2: Second Order Functions

| $H(s)$ | Bode Magnitude | Bode Phase |
|---|---|---|
| $\dfrac{\omega_0^2}{s^2 + \frac{\omega_0}{Q}s + \omega_0^2}$ |   |   |
| $\dfrac{s^2}{s^2 + \frac{\omega_0}{Q}s + \omega_0^2}$ |   |   |
| $\dfrac{\frac{\omega_0}{Q}s}{s^2 + \frac{\omega_0}{Q}s + \omega_0^2}$ |   |   |
| $\dfrac{s^2 + \omega_0^2}{s^2 + \frac{\omega_0}{Q}s + \omega_0^2}$ |   |   |
| $\dfrac{s^2 - \frac{\omega_0}{Q}s + \omega_0^2}{s^2 + \frac{\omega_0}{Q}s + \omega_0^2}$ |   |   |

# Calculations

**1.** Derive the transfer functions for the circuits shown in Figs. 1(a), 1(b) and 1(c):

$$H_{LP}(s) = \frac{V_{LP}}{V_i}(s) \qquad H_{HP}(s) = \frac{V_{HP}}{V_i}(s) \qquad H_{BP}(s) = \frac{V_{BP}}{V_i}(s)$$

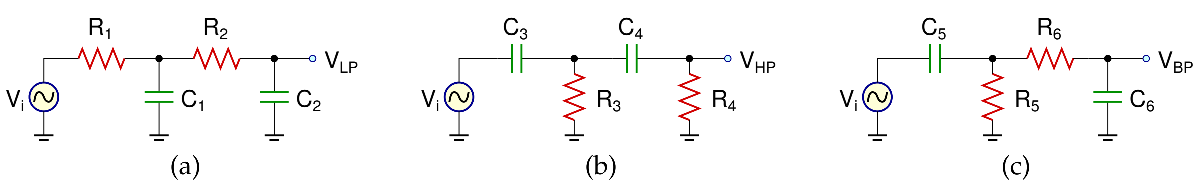



Figure 1: Second order (a) lowpass filter (b) highpass filter (c) bandpass filter

Express the transfer functions as

$$H(s) = \frac{P(s)}{Q(s)} \tag{1}$$

where $P(s)$ and $Q(s)$ are polynomials of $s$ with coefficients in terms of resistors and capacitors. Remember that negative powers of $s$ are not allowed in polynomials, so all the terms in $P(s)$ and $Q(s)$ must contain $s^n$ where $n \geq 0$.

2. Find the resistor and capacitor values such that the transfer functions can be formatted as follows:

$$H_{LP}(s) = \frac{1}{1+\frac{s}{2\pi f_1}} \frac{1}{1+\frac{s}{2\pi f_2}}, \quad f_1 = 4kHz, \ f_2 = 8kHz \tag{2}$$

$$H_{HP}(s) = \frac{s}{s+2\pi f_3} \frac{s}{s+2\pi f_4}, \quad f_3 = 4kHz, \ f_4 = 8kHz \tag{3}$$

$$H_{BP}(s) = \frac{s}{s+2\pi f_5} \frac{1}{1+\frac{s}{2\pi f_6}}, \quad f_5 = 4kHz, \ f_6 = 8kHz \tag{4}$$

When calculating component values, you can make reasonable approximations. For example, $1+x \approx 1$ if $x \ll 1$, which typically requires $x < 0.1$. Similarly, $1+x \approx x$ if $x \gg 1$, which requires $x > 10$.

3. Sketch the magnitude and phase Bode plots for $H_{LP}(s)$, $H_{HP}(s)$, and $H_{BP}(s)$.

4. Calculate the output voltages $V_{LP}(t)$, $V_{HP}(t)$, and $V_{HP}(t)$ for $V_i(t) = 0.5\sin(2\pi 6000t)$.

# Simulations

**For all simulations, provide screenshots showing the schematics and the plots with the simulated values properly labeled.**

Draw the schematics for the circuits in Fig. 1 with the calculated component values. Perform the following simulations for each circuit:

1. Obtain the magnitude and phase **Bode plots** of the transfer function using **AC simulation**, and measure the 3-dB frequencies and passband gains. Also measure the magnitude and phase of the transfer function at 6kHz.

2. Apply the input $V_i(t) = 0.5\sin(2\pi 6000t)$ and obtain the **time-domain waveforms** for the input and the output voltage using **transient simulation**. Measure the magnitudes of the input and the output voltages, and the phase difference between them.

# Measurements

**For all measurements, provide screenshots showing the plots with the measured values properly labeled.**

Build the circuits in Fig. 1 with the simulated component values. Perform the following measurements for each circuit:

1. Obtain the magnitude and phase **Bode plots** of the transfer function using the **network analyzer**, and measure the 3-dB frequency and passband gain. Also measure the magnitude and phase of the transfer function at 6kHz.

2. Apply the input $V_i(t) = 0.5\sin(2\pi 6000t)$ and obtain the **time-domain waveforms** for the input and the output voltage using the **scope**. Measure the magnitudes of the input and the output voltages, and the phase difference between them.

# Report

1. Include calculations, schematics, simulation plots, and measurement plots.

2. Prepare a table showing calculated, simulated and measured results.

3. Compare the results and comment on the differences.

4. The same transfer functions can be obtained using different combinations of resistors and capacitors. Explain your reasoning for your selection. What are the trade-offs if you change your selection of components to realize the same transfer functions?

## Demonstration

1. Build the circuits in Fig. 1(a), (b) and (c) on your breadboard and bring it to your lab session.
2. Your name and UIN must be written on the side of your breadboard.
3. Submit your report to your TA at the beginning of your lab session.
4. For the lowpass filter in Fig. 1(a):
   - Show the frequency response using the network analyzer.
   - Measure and verify -40dB/dec slope at the stopband.
5. For the highpass filter in Fig. 1(b):
   - Show the frequency response using the network analyzer.
   - Measure and verify +40dB/dec slope at the stopband.
6. For the bandpass filter in Fig. 1(c):
   - Show the frequency response using the network analyzer.
     - Measure the low and high 3-dB frequencies.
     - Measure the magnitude and phase at a passband frequency $f_x$ determined by your TA.
   - Show the time-domain input and output waveforms using the scope at the frequency $f_x$.
     - Measure the gain.
     - Measure phase difference between the input and the output.