ECEN 325 Lab 1: First Order Circuits

Objectives

The purpose of the lab is to investigate the frequency response of first-order circuits and learn the fundamentals about circuit analysis and design in the frequency domain.

Introduction

The frequency response is a representation of the system's response to sinusoidal inputs at varying frequencies; it is defined as the magnitude ratio and phase difference between the input and output signals. If the frequency of the source in a circuit is used as a reference, it is possible to have a complete analysis in the frequency and time domains. Frequency domain analysis is easier than time domain analysis because differential equations used in the time domain are mapped into linear equations that are function of the complex frequency variable s. It is important to obtain the frequency response of a circuit because we can predict its response to any input signal.

In this laboratory experiment we will plot the frequency response of first order RC circuits. We can characterize the circuits by two features of the frequency response:

1. The difference between the magnitude of the output and input signals (given by the amplitude ratio)

2. The time lag or lead between input and output signals (given by the phase shift)

To plot the frequency response, value of the transfer function $H(s) = V_o(s)/V_i(s)$ is calculated within a range of input frequencies. A particularly important method of displaying frequency response is the Bode plot, which is the representation of the magnitude (in decibels) and phase (in degrees) of $H(j\omega)$ as a function of ω or f in logarithmic scale, where ω and f are the frequency variables in radians per second (rad/s) and hertz (Hz), respectively.

As an example, assume that the transfer function is given as

$$H(s) = \frac{V_o}{V_i}(s) = \frac{1}{1 + \frac{s}{2\pi 1000}} \tag{1}$$

$$H(j\omega) = \frac{1}{1 + \frac{j\omega}{2\pi 1000}} \Rightarrow \begin{cases} |H(j\omega)| = \dfrac{1}{\sqrt{1 + \left(\dfrac{\omega}{2\pi 1000}\right)^2}} \\[4mm] \angle H(j\omega) = -\tan^{-1}\dfrac{\omega}{2\pi 1000} \end{cases} \tag{2}$$

If the input signal is $V_i(t) = 0.2\sin(2\pi 3000t)$, which is a sinusoidal signal with 0.2 V amplitude and 3 kHz (or $2\pi 3000$ rad/s) frequency, then the output signal can be calculated as follows:

$$\left. \begin{array}{l} |H(j2\pi 3000)| = \dfrac{1}{\sqrt{1 + \left(\dfrac{2\pi 3000}{2\pi 1000}\right)^2}} = 0.316 \\[8mm] \angle H(j2\pi 3000) = -\tan^{-1}\dfrac{2\pi 3000}{2\pi 1000} = -71.6° = -1.25\ rad \end{array} \right\} \Rightarrow V_o(t) = 0.0632\sin(2\pi 3000t - 1.25) \tag{3}$$

The output voltage $V_o(t)$ calculated above can be verified using either time-domain (transient), or frequency-domain (AC) simulation as follows:
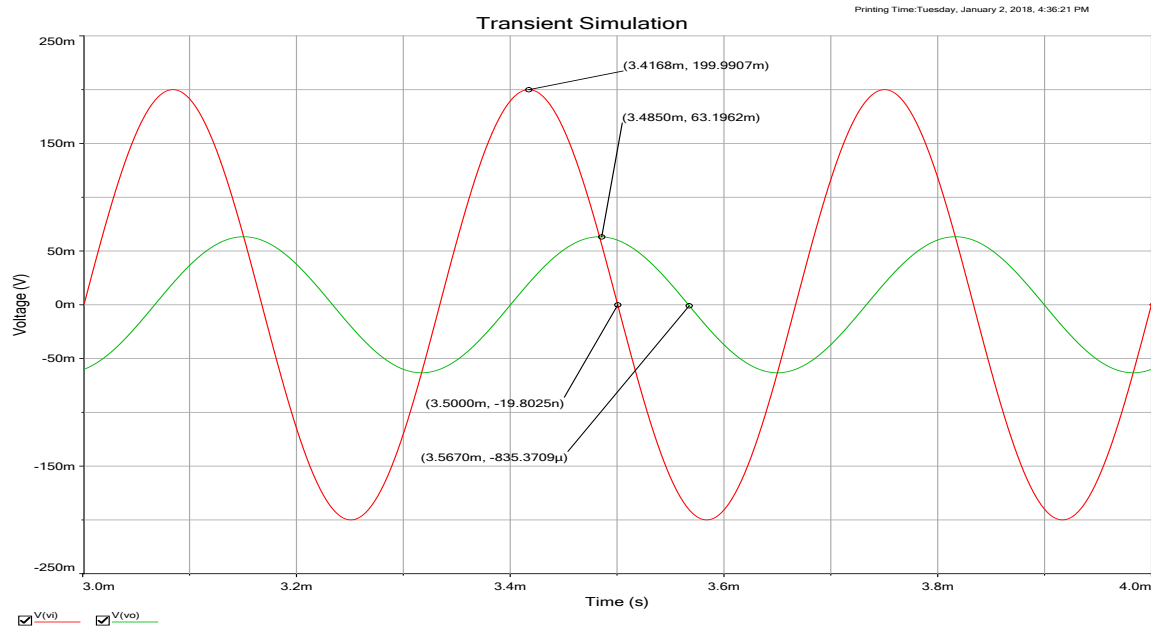

Figure 1: Transient Simulation: $\hat{V}_o = 63.2mV$, $|H| = \dfrac{63.2m}{200m} = 0.316$, $\Delta t = 3.5mS - 3.567mS = -67\mu S$

$T = \dfrac{1}{3000} = 333.3\mu S \Rightarrow \angle H = \dfrac{\Delta t}{T}360° = \dfrac{-67\mu}{333.3\mu}360° = -72.3° = -1.26\ rad$
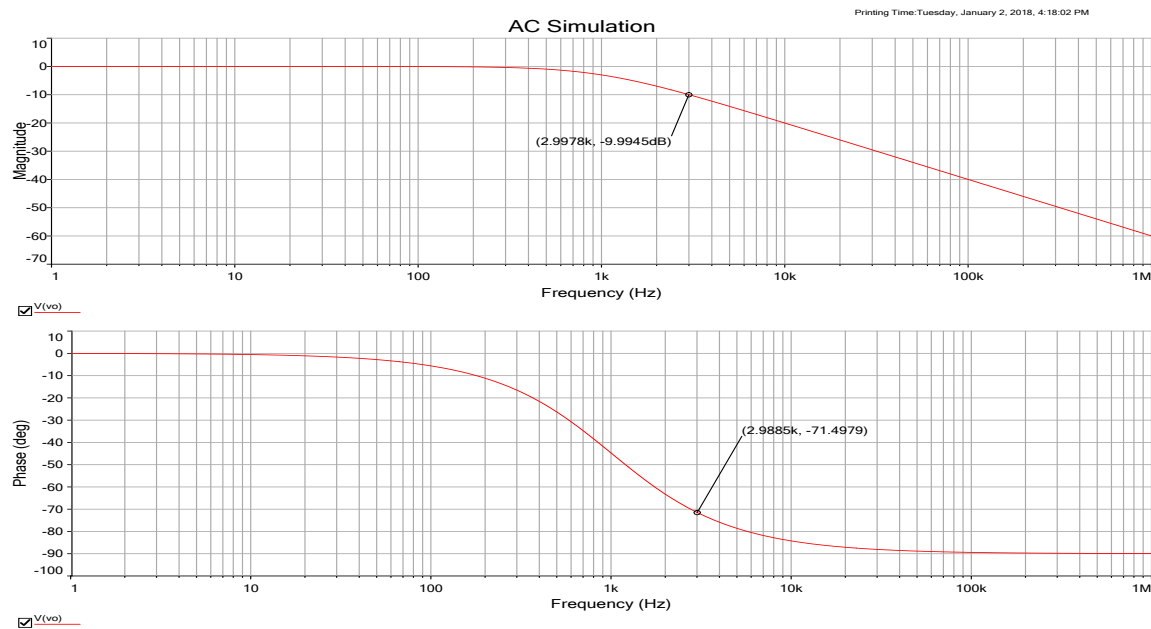

Figure 2: AC Simulation: $|H| = -9.995dB = 10^{-9.995/20} = 0.316$, $\angle H = -71.5° = -1.25\ rad$

As you can see from Figs. 1 and 2, the values of $|H|$ and $\angle H$ can be verified using transient or AC simulation, however AC simulation provides easier comparison and includes data for any input frequency. On the other hand, transient simulation shows the actual voltage output for a specific set of input signals, so it includes more information such as initial transients and nonlinearity, as well as any DC component.

Calculations

1. For the circuits in Figs. 3(a) and 3(b), derive the transfer functions in the following forms:

$$H_{LP}(s) = \frac{V_{LP}}{V_i}(s) = K_L \frac{1}{1 + \dfrac{s}{\omega_L}} \qquad (4)$$

$$H_{HP}(s) = \frac{V_{HP}}{V_i}(s) = K_H \frac{s}{s + \omega_H} \qquad (5)$$

and express ω_L, ω_H, K_L and K_H in terms of resistors and capacitors.
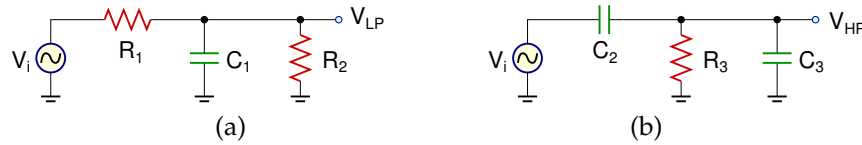

(a)   (b)

Figure 3: First order (a) lowpass filter (b) highpass filter

The frequencies ω_L and ω_H are known as:

- Pole frequency, defined as the root of the denominator of $H(s)$
- Corner frequency, defined as the frequency at which the gain is 0.707 times the passband value
- Half power frequency (compared to passband), since $0.707^2 = 0.5$
- 3dB frequency (compared to passband), since $20 \log_{10}(0.707) = -3dB$

2. Find R_1, R_2, R_3, C_1, C_2, and C_3, such that $f_L = f_H = 5\ kHz$ and $K_L = K_H = 0.5$.

3. Sketch the magnitude and phase Bode plots for $H_{LP}(s)$ and $H_{HP}(s)$.

4. Calculate the output voltages $V_{LP}(t)$ and $V_{HP}(t)$ for $V_i(t) = 0.4 \sin(2\pi 4000t)$.

5. Repeat step 4 for $V_i(t) = 0.3 \sin(2\pi 6000t)$.

Simulations

For all simulations, provide screenshots showing the schematics and the plots with the simulated values properly labeled.

Draw the schematics for the circuits in Fig. 3 with the calculated component values. Perform the following simulations for each circuit:

1. Obtain the magnitude and phase **Bode plots** of the transfer function using **AC simulation**, and measure the 3-dB frequencies and passband gains. Also measure the magnitude and phase of the transfer function at 4kHz and 6kHz.

2. Apply the input $V_i(t) = 0.4 \sin(2\pi 4000t)$ and obtain the **time-domain waveforms** for the input and the output voltages using **transient simulation**. Measure the magnitudes of the input and the output voltages, and the phase difference between them.

3. Repeat step 2 for $V_i(t) = 0.3 \sin(2\pi 6000t)$.

Measurements

For all measurements, provide screenshots showing the plots with the measured values properly labeled.

Build the circuits in Fig. 3 with the with the simulated component values. Perform the following measurements for each circuit:

1. Obtain the magnitude and phase **Bode plots** of the transfer function using the **network analyzer**, and measure the 3-dB frequencies and passband gain. Also measure the magnitude and phase of the transfer function at 4kHz and 6kHz.

2. Apply the input $V_i(t) = 0.4\sin(2\pi 4000t)$ and obtain the **time-domain waveforms** for the input and the output voltages using the **scope**. Measure the magnitudes of the input and the output voltages, and the phase difference between them.

3. Repeat step 2 for $V_i(t) = 0.3\sin(2\pi 6000t)$.

Report

1. Include calculations, schematics, simulation plots, and measurement plots.

2. Prepare a table showing calculated, simulated and measured results.

3. Compare the results and comment on the differences.

Demonstration

1. Build the circuits in Fig. 3(a) and (b) on your breadboard and bring it to your lab session.

2. Your name and UIN must be written on the side of your breadboard.

3. Submit your report to your TA at the beginning of your lab session.

4. For both circuits in Fig. 3(a) and (b):

   - Show the frequency response using the network analyzer.
     - Measure the 3-dB frequency.
     - Measure the magnitude and phase at a frequency f_x determined by your TA.
   - Show the time-domain input and output waveforms using the scope at the frequency f_x.
     - Measure the gain.
     - Measure the phase difference between the input and the output.