



Chapter 1

Introduction

The purpose and contents of the course



- Introduce most used data structures and algorithms
- Prerequisite of other courses
- Introduce algorithm analysis
- Review Java and C++

The purpose and contents of the course

1. Introduce most used data structures and algorithms.

Use proper data structures to solve different problems.

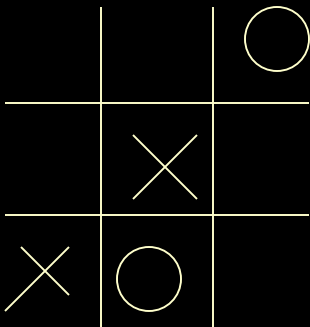
Example :

- Game problem
- Management of library catalogue by computer
- Management of the traffic lights in intersections
- The book : selection problem
solve a popular word puzzle

The purpose and contents of the course

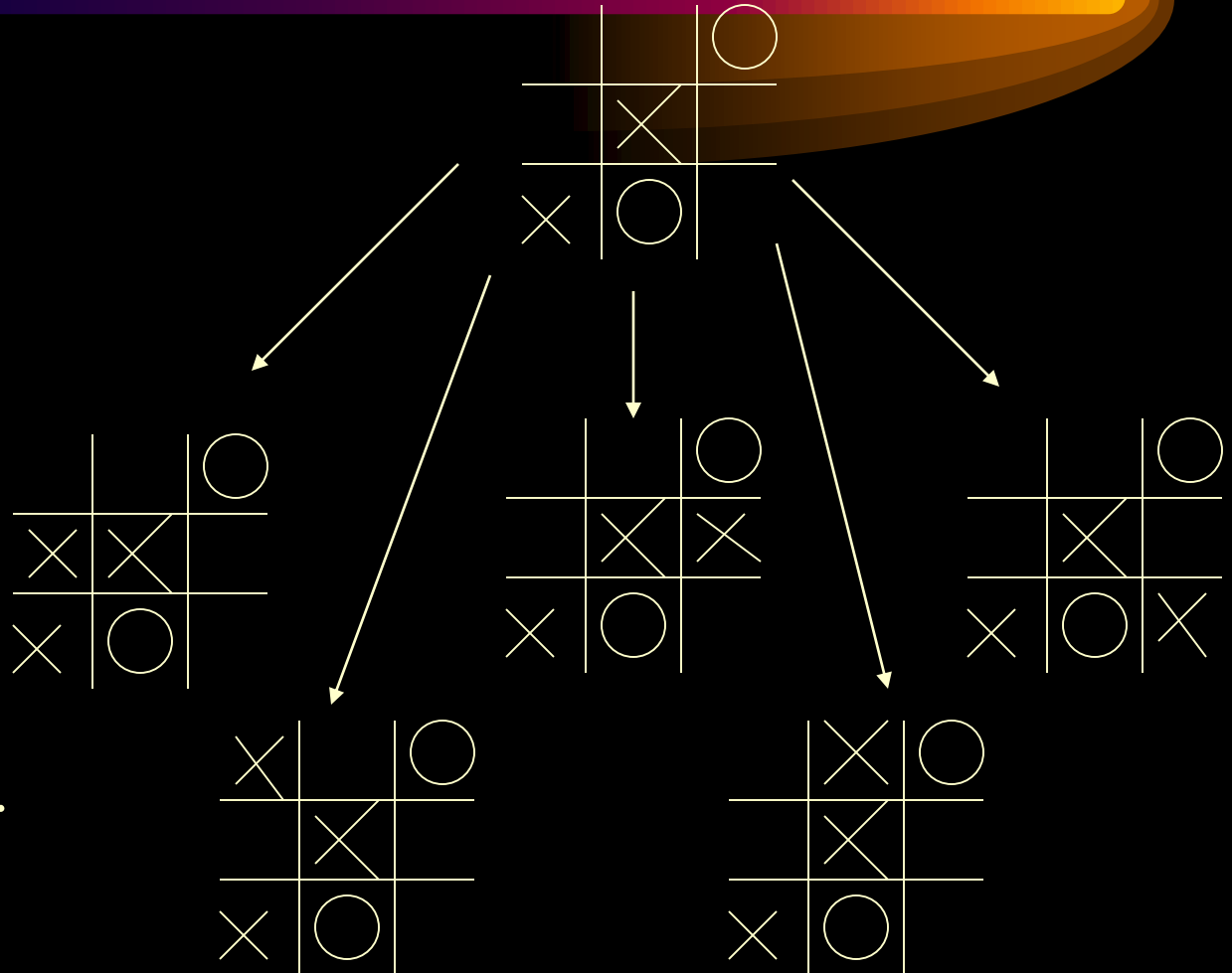
Example 1:

Game
problem:



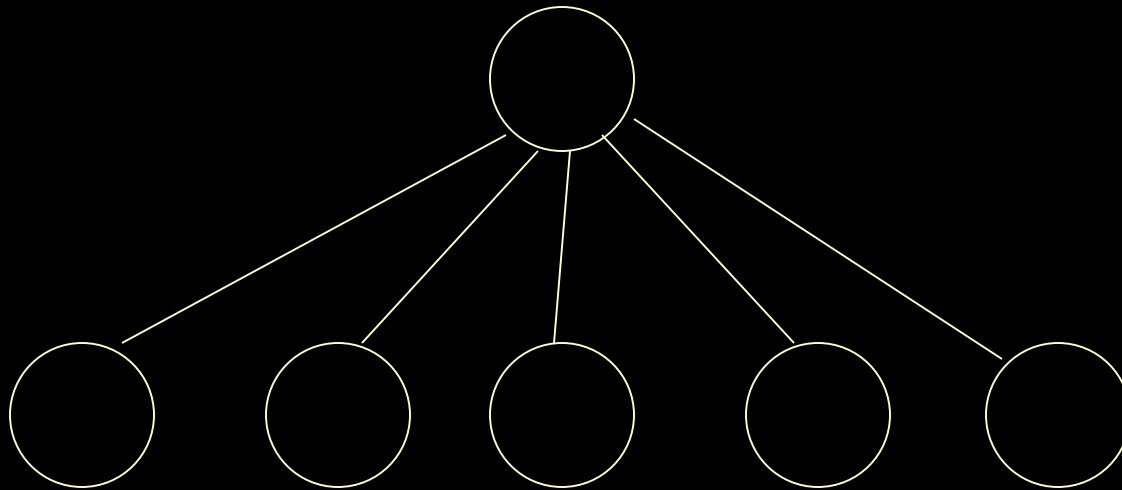
Next step:

X has five choices.



The purpose and contents of the course

Example 1 uses a tree structure.



The purpose and contents of the course

Example 2: Management of library catalogue by computer

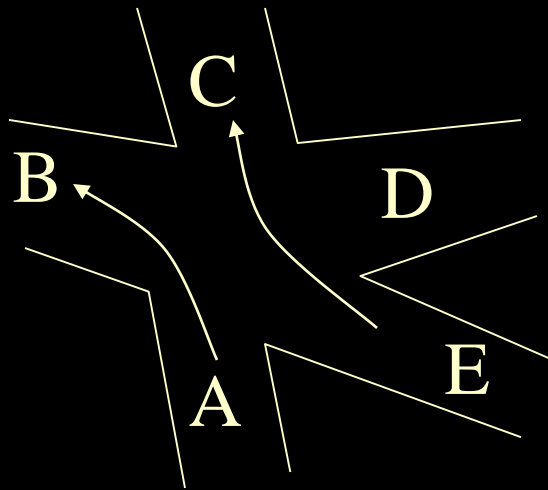
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| D.S. | Sartaj Sahni | 000001 | computer | 2000.1 |

It is a linear list.

The purpose and contents of the course

Example 3:

Management of the traffic lights in intersections



This is a graph

C, E are one-way road, there are 13 path to go.

Can go at the same time :

$A \rightarrow B$ $E \rightarrow C$

Cannot go at the same time:

$E \rightarrow B$ $A \rightarrow D$

The purpose and contents of the course

2 . Prerequisite of other courses:

- **Principles of compiling** : use **stack** to compute expression and implement recursive procedure
- **Operating System**: use **queue** to implement job scheduling
- **Database**: use **B-tree, B⁺ tree** to organize, store and load massive data in the hard memory.

...

The purpose and contents of the course

3. Basic methods of algorithm analysis

standards of the performance of an algorithm:

time complexity, space complexity,

and accuracy

example: sorting

$$a_1, a_2, a_3, \dots, a_{n-1}, a_n$$

$$n-1+n-2+\dots+2+1 = n(n-1)/2 = (n^2-n)/2$$

$$O(n^2)$$

$$O(n \cdot \log_2 n)$$

4. Review Java and C++

What is Data Structure



1. Data

is the carrier of information.

Data is a set of **numbers** , **characters**, and **other symbols**

- that can be used to describe the objective things.
- These symbols can be input into computers , identified and processed by the computer program.

What is Data Structure

Data can divided into two classes:

- { numerical data: int, float, complex,.....
- { non-numerical data: character, string,
graph, voice...

What is Data Structure

2 . Data structure

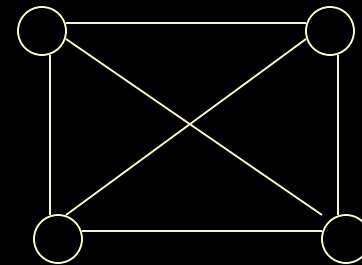
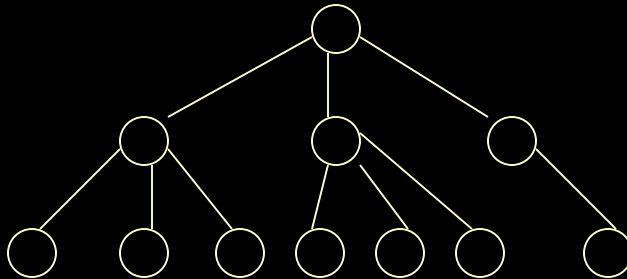
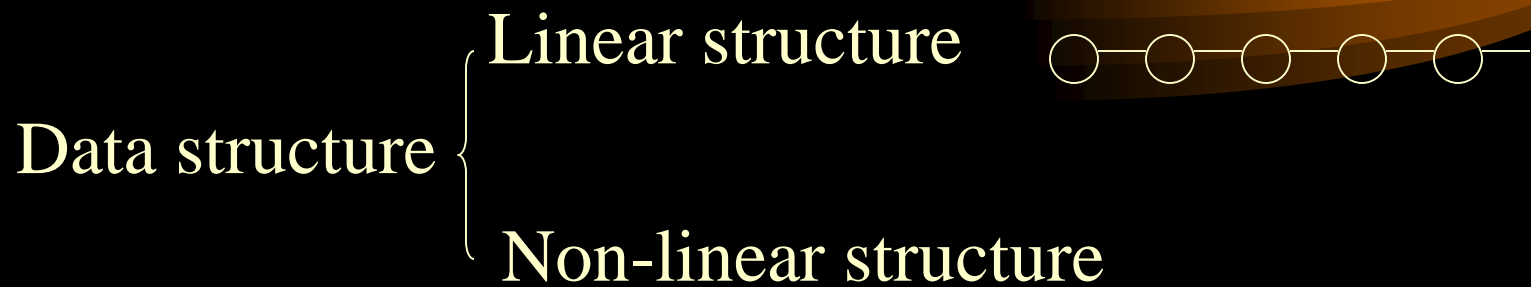
A data structure is a data object together with the relationships among the data members that compose the object

$$\text{Data_Structure} = \{D, R\}$$

D is a data object,

R is a limited set of relationships of all the data members in D.

What is Data Structure



What is Data Structurem

数据结构涉及三个方面:

数据的逻辑结构-----从用户视图看, 是面向问题的。

数据的物理结构-----从具体实现视图看, 是面向计算机的。

相关的操作及其实现。

Example:

学生表: 逻辑结构-----线性表

物理结构-----数组, 链表

操作-----插入, 删除, 查找

ADT and OO

1. Data type

Definition: is a set of values together with a operation set that operate on these values.

Example:

int type

value set: $\{\dots, -2, -1, 0, 1, 2, \dots\}$

operation set: $\{+, -, *, /, \%, \dots\}$

ADT and OO



Most of the programming languages provide a group of predefined data type.

- { Atom data type — int, float, double.....
- { Structure data type—array, struct,.....

ADT and OO

2. ADTs: Abstract Data Types

是将类型和与这个类型有关的操作集合封装在一起的数据模型。

Abstract: is a method used to hide the information.

Example:

```
int x ;
```

```
:
```

```
x=735;
```

```
:
```

abstract of int data type
assignment

```
float x, y ;
```

```
:
```

```
x=x*y+3.0;
```

```
:
```

abstract of float data type
operation

ADT and OO

Abstract data type:

is a new programming method that part the usage and implementation, in order to encapsulate and hide the information.

思想:

将数据类型的使用与它的表示（机内存储）、实现（机内操作的实现）分开。更确切的说，把一个数据类型的表示及在这个类型上的操作实现封装到一个程序模块中，用户不必知道它。

ADT NaturalNumber is

- **objects:** 一个整数的有序子集合,它开始于0,结束于机器能表示的最大整数(MAXINT)。
- **Function:**
 - Zero():NaturalNumber
 - IsZero(x):Boolean
 - Add(x,y):NaturalNumber
 - Equal(x,y):Boolean
 - Successor(x):NaturalNumber
 - Subtract(x,y):NaturalNumber
- **end** NaturalNumber

ADT and OO

3. OO:

object-oriented = object + class + inherit +
communicate

object: attribute values + operates

example: rectangle: 一个几何对象

attribute values : 左上角坐标, 右下角坐标, 边线颜色, 内部颜色

operates : move(x,y);

setEdgeColor(c);

setInterColor(c);

ADT and OO



class: objects of same attributes and operates.

an instance is an object of the class.

different object has different attribute value

ADT and OO

Inherit:

base class—integrate the same part (including attributes and operations) in all the derived classes

derived class—add the specific attributes and operations

Example: base class—polygon

derived class—quadrilateral, triangular

ADT and OO



Communication: each class object communicate with others using messages.

Message: instructions that one class object used in order to require another class object to perform some operation.

Algorithm definition



Algorithm : an operation sequence of solving
a problem

Properties:

1. Input specified
2. Output specified
3. Definiteness
4. Effectiveness
5. Finiteness

Algorithm definition

Program : is written by languages that can be performed by machine.

can't satisfy the finiteness.

For example, OS.

Algorithm: has multiple descriptive methods, such as language, graph, table.

Algorithm definition

```
void selectsort(int a[ ],int n)
{  for (int i = 0; i < n-1; i++)
    {  int k = i;
       for ( int j = i+1; j < n; j++)
           if ( a[j] < a[k]) k = j;
       int temp = a[i]; a[i] = a[k]; a[k] = temp;
    }
}
```

Mathematics Review

1. Exponents

$$X^A X^B = X^{A+B}$$

$$X^A / X^B = X^{A-B}$$

$$(X^A)^B = X^{AB}$$

$$X^N + X^N = 2X^N$$

$$2^N + 2^N = 2^{N+1}$$

Mathematics Review

2. Logarithms(all logarithms are to the base 2)

DEFINITION :

$$X^A = B \quad \text{if and only if} \quad \log_X B = A$$

THEOREM 1.1

$$\log_A B = \log_C B / \log_C A ; \quad A, B, C > 0, A \neq 1$$

THEOREM 1.2

$$\log AB = \log A + \log B; \quad A, B > 0$$

3. Series

$$\sum_{i=0}^N 2^i = 1 + 2^1 + 2^2 + \dots + 2^N = 2^{N+1} - 1$$

$$\sum_{i=1}^N i = 1 + 2 + 3 + \dots + N = (N+1) * N / 2$$

Mathematics Review

4. Modular Arithmetic

We say that A is congruent to B modular N,
written $A \equiv B \pmod{N}$, if N divides A-B.

Example : $81 \equiv 61 \equiv 1 \pmod{10}$

Mathematics Review

5. The P Word (证明方法)

1). Proof by Induction

The first step is proving a **base case**.

the Next step an **inductive hypothesis** is assumed.

theorem is assumed to be true for all cases up to some limit **k**. Using this assumption, the theorem is then shown to be true for the next value, which is typically **$k + 1$** . This proves the theorem(as long as k is finite).

Example: 例如 等比级数的和以及整数平方和的证明等.

Mathematics Review

2). Proof by Contradiction

Proof by contradiction proceeds by assuming that the theorem is false and showing that this assumption implies that some known property is false, and hence the original assumption was erroneous.

Example: proof that there is an infinite number of primes

A Brief Introduction to Recursion

1. Recursive

example:

define a function f , valid on nonnegative integers

$$f(x) = \begin{cases} 0 & x = 0 \\ 2f(x-1) + x^2 & x > 0 \end{cases}$$

$$f(1) = 1, f(2) = 6, f(3) = 21, f(4) = 58$$

```
public static int f ( int x)
{   if ( x == 0) //base case
        return 0;
    else return 2*f(x-1) + x*x; //recursive call
}
```


A Brief Introduction to Recursion

A nonterminating recursive method:

```
public static int bad( int n )  
{  if ( n == 1)           // 不能写成 if(n==0)  
    return 0;  
    else return bad( n/3 + 1) + n-1;  
}
```

two fundamental rules of recursion:

- 1) Base cases.
- 2) Making progress.

A Brief Introduction to Recursion

2. 1) direct recursive

2) indirect recursive

Example 1 factorial function $f(n)=n!$

$$f(n)=\begin{cases} 1 & n \leq 1 \quad (\text{base}) \\ n * f(n-1) & n > 1 \quad (\text{recursive component}) \end{cases}$$

$$f(5)=5f(4)=5*4f(3)=5*4*3f(2)=5*4*3*2f(1)=120$$

static long factorial (int n)

```
{  if ( n <= 1)
    return 1;
    else return n* factorial( n-1 )
}
```

A Brief Introduction to Recursion

```
public class ComputeFactorial
{
    public static void main( String[ ] args )
    {
        System.out. Println( "please enter a nonnegative integer" );
        int n = MyInput.readInt( );

        Syatem.out.println( "Factorial of " + n + " is " + factorial( n ) );
    }
    static long factorial (int n)
    {
        if (n<=1) return 1;
        else return n*factorial(n-1);
    }
}
```

A Brief Introduction to Recursion

Example 2: Compute Fibonacci number

0, 1, 1, 2, 3, 5, 8, 13, 21, 34,

$\left\{ \begin{array}{l} \text{fib}(0) = 0; \\ \text{fib}(1) = 1; \\ \text{fib}(n) = \text{fib}(n-2) + \text{fib}(n-1); \quad n \geq 2 \end{array} \right.$

```
public static long fib(long n)
{   if ( ( n == 0 ) || ( n == 1 ) )
        return n;
    else
        return fib(n-1) + fib(n-2);
}
```

Compute fib(4):

A Brief Introduction to Recursion

```
public class ComputeFibonacci
{
    public static void main( String args[ ])
    {
        System.out.println( "Enter an index for the Fibonacci number ");
        int n = MyInput.readInt( );


        System.out.println( "Fibonacci number at index " + n + " is " +
                            fib(n));
    }
    public static long fib(long n)
    {
        if (( n == 0) || ( n == 1))
            return 1;
        else
            return fib( n-1) + fib(n-2);
    }
}
```

A Brief Introduction to Recursion

Example 3:

computes the sum of the elements $a[0]$ through $a[n-1]$

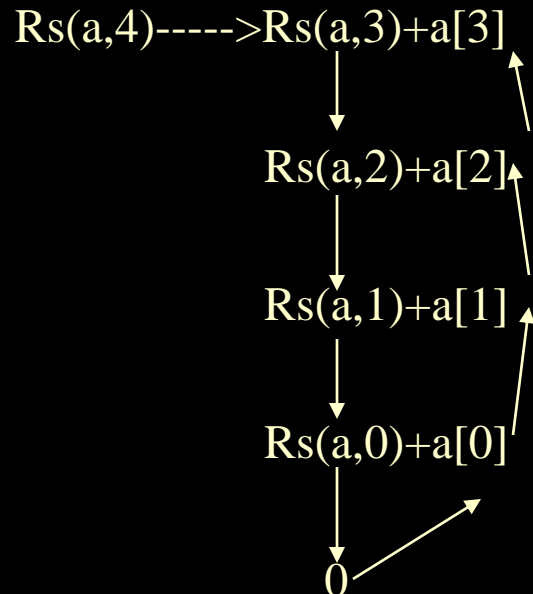
$a[0], a[1], \dots, a[n-2], a[n-1]$



A Brief Introduction to Recursion

```
public static int Rsum(int[] a , int n)
{ if (n>0)
    return Rsum(a,n-1)+a[n-1];
  return 0;
}
```

$a_0 \quad a_1 \quad a_2 \quad a_3$



A Brief Introduction to Recursion

Example 4:

Permutation

$\{a,b,c\}$: abc, acb, bac, bca, cab, cba

permutation of n elements has $n!$

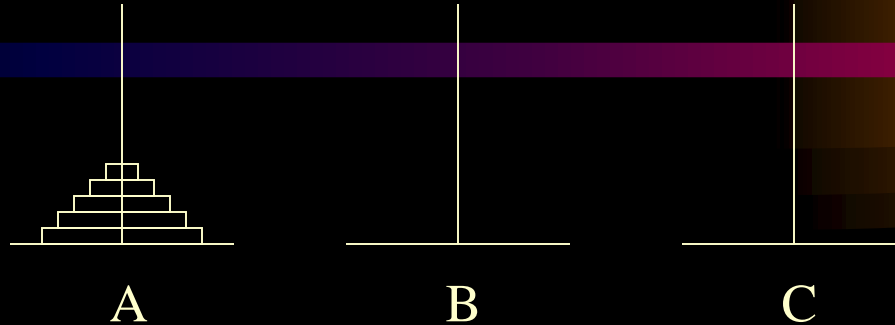
下面在黑板上来分析该问题:

A Brief Introduction to Recursion

```
public static void perm(Char[ ] list , int k, int m)
{   int i;
    if (k==m) { for (i=0; i<=m; i++) cout << list[i];
                cout << endl; }
    else{ for (i=k; i<=m; i++)
           { swap(list[k], list[i]);
             perm(list, k+1, m);
             swap(list[k], list[i]);
           }
    }
}
```

A Brief Introduction to Recursion

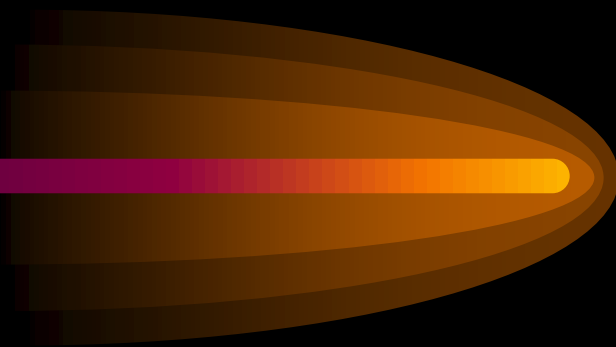
Example 5 : Hanoi tower problem



```
public void moveDISKs(int n, char fromTower, char  
                      toTower, char auxTower)  
  
    if ( n==1)  
        Move disk 1 from the fromTower to the toTower;  
    else  
    { moveDISKs(n-1, fromTower, auxTower, toTower);  
      Move disk n from the fromTower to the toTower;  
      moveDISKs(n-1, auxTower, toTower, fromTower);  
    }
```

A Brief Introduction to Recursion

```
public class TowersOfHanoi {
    public static void main(String[ ] args) {
        System.out.println( "Enter number of disks" );
        int n = MyInput.readInt( );
        System.out.println( "The move are:");
        moveDISKs(n, 'A', 'B', 'C');
    }
    public static void moveDISKs(int n, char fromTower, char
                                toTower, char auxTower) {
        if ( n==1)
            System.out.println( "move disk " + n + "from " +
                                fromTower + " to" + toTower);
        else {
            moveDISKs(n-1, fromTower, auxTower, toTower);
            System.out.println( "Move disk " + n + "from " +
                                fromTower + " to " + toTower );
            moveDISKs(n-1, auxTower, toTower, fromTower);
        }
    }
}
```



Generic Objects in Java

Throughout this text, we will describe algorithms and data structures that are type independent.

{ IntCell — nongeneric class
MemoryCell — generic class

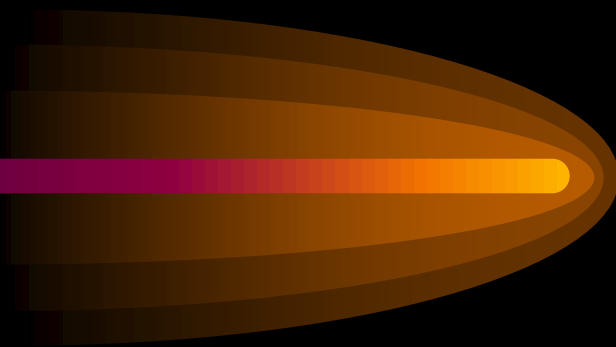
Generic Objects in Java

1. IntCell class

```
public class IntCell
{   public IntCell ( ){ this ( 0 ); }
    public IntCell( int initialValue )
        {   storedValue = initialValue ; }

    public int read ( ) { return storedValue; }
    public void write ( int x ) {   storedValue = x; }

    private int storedValue;
}
```



Generic Objects in Java

1) public and private

member of public : may be accessed by any method in any class .

member of private : may only be accessed by methods in its class.

specifier is omitted (package-friendly visibility) :

a data member whose visibility specifier is omitted is visible to other classes in the same package.

Generic Objects in Java

2) constructor

A class may define methods that describe how an instance of the class is constructed; these are called constructors.

If no constructor is explicitly defined, one that initializes the fields using language defaults is automatically generated.

Generic Objects in Java

3) This

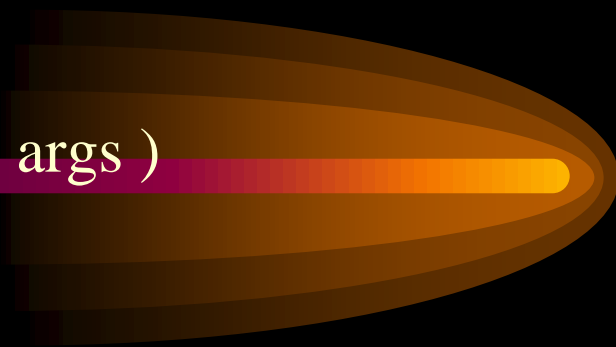
- In many cases, the zero-parameter constructor can be implemented by calling another constructor.

By using **this**, we avoid replicating code logic in separate constructors.

- A different use of **this** is as a reference to the object being acted upon.

Generic Objects in Java

```
public class TestIntCell
{
    public static void main ( String [ ] args )
    {
        IntCell m = new IntCell ( );
        m. write( 5 );
        System.out.println( "Cell contents: " + m.read( ) );
    }
}
```



Generic Objects in Java

1) static and main

The **main** method must be **static**, meaning that it applies to the **TestIntCell** class instead of a single instance of the class.

Each class may declare a **main** method that will be used when the java interpreter is called for that class.

复习一下 **static**

2) Object creation.

Objects are created by using **new**.

Generic Objects in Java

3) Method calls.

```
m.write(5);
```

```
m.read( )
```

```
m.storedValue    //be illegal
```

Generic Objects in Java

2. MemoryCell class

design a class that works for any type of **Object**.

example: sorting of array $a_0, a_1, a_2, a_3, \dots, a_{n-1}$

- C++: use templete

program1.1:

```
int Abc(int a,int b,int c)
{return a+b+b*c+(a+b-c)/(a+b)+4;    }
```

program1.2:

```
float Abc(float a,float b,float c)
{return a+b+b*c+(a+b-c)/(a+b)+4;    }
```

Generic Objects in Java

program 1.1 and 1.2 differ only in the data type of the formal parameters and of the value returned.

We can write a **generic code** in which the data type is a variable whose value is determined by the compiler. This generic code is written using the template statement in program 1.3

Generic Objects in Java

Program 1.3

```
template<class T>  
T Abc(T a,T b,T c)  
{return a+b+b*c+(a+b-c)/(a+b)+4; }
```

From this generic code the compiler can construct 1.1 by substituting **int** for T and 1.2 by substituting **float** for T.

Generic Objects in Java

- Java:
 - 1) use **inheritance**(pre-java 5)
all objects are subclasses of **Object**.

```
public class MemoryCell
{ public Object read( )
    { return storedValue; }
  public void write( Object x )
    { storedValue = x; }

  private Object storedValue;
}
```

Generic Objects in Java

Two problem:

First problem: We must downcast to the correct type

```
public class TestMemoryCell
{
    public static void main( String [ ] args )
    {
        MemoryCell m = new MemoryCell ( );

        m.write( "37 " );
        String val = ( String) m.read( );
        System.out.println( "Contents are : " + val )
    }
}
```

Using the generic MemoryCell class (pre-Jave 5)

the **read** method for **MemoryCell** returns an **Object**.

Generic Objects in Java

Second problem Although all objects are subclasses of **Object**, the primitive types **boolean, short, char, byte, int, long, float, double** are not objects. Thus we cannot directly use **MemoryCell** to store a primitive value.

We must use **wrapper** class provide by Java. Wrapper class store a single primitive value and can be used wherever an **Object** is need.

the wrapper class provides a method that can be used to access the primitive value it stores.

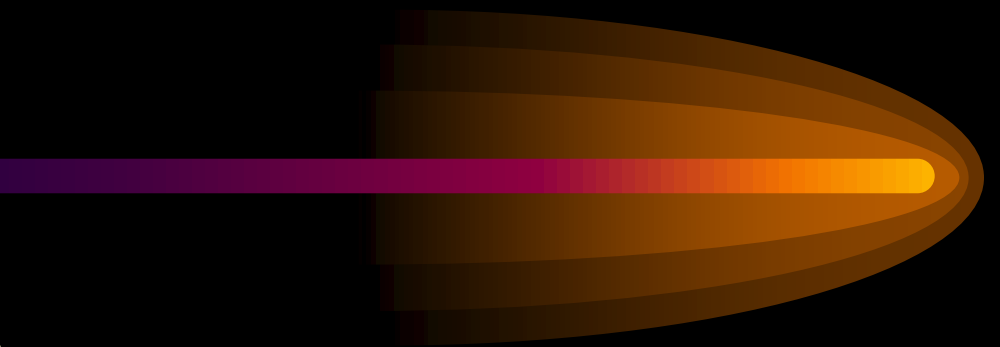
two example:

a) For the **Integer** wrapper, this method is named **intValue**.

Generic Objects in Java

```
public class Integer
{
    public Integer( int x)
    {
        value = x;
    }
    public int intValue( )
    {
        return value;
    }

    private int value;
}
```



Generic Objects in Java

```
public class wrapperdemo
{
    public static void main ( String [ ] args )
    {
        MemoryCell m = new Memorycell ( );

        m.write ( new Integer ( 37 ) );
        Integer wrapperVal = ( Integer ) m . Read ( ) ;
        int val = wrapperVal . intValue ( ) ;
        System . Out . Println ( “ Contents are : “ + val ) ;
    }
}
```

1.7 An illustration of Integer wrapper class

To get the **int** value that is hidden inside the **Integer** object, we must convert the result of **read** back to an **Integer** , and then use method **intValue** access the value . This involves using a type conversion.

Generic Objects in Java

2) Java 5 supports generic classes that are very easy to use

```
public class GenericMemoryCell <Any Type >
{
    public AnyType read( )
    {
        return storedValue; }
    public void write( AnyType x )
    {
        storedvalue = x; }
    private AnyType storedvalue ;
}
```

1.9 Generic implementation of the MemoryCell class

Generic Objects in Java

- When a generic class is specified, the class declaration includes one or more **type parameters** after the class name
- User can create types such as `GenericMemoryCell<String>`
`GenericMemoryCell<Integer>`
but can not create `GenericMemoryCell<int>`
- Inside the `GenericMemoryCell` class declaration, we can declare fields of the generic type and methods that use the generic type as a parameter or return type

Generic Objects in Java

```
class BoxingDemo
{
    public static void main ( String [ ] args )
    {
        GenericMemoryCell<Integer> m =
            new GenericMemoryCell<Integer> ( ) ;

        m . Write ( 37 ) ;
        int val = m . Read ( ) ;

        System . Out . Println ( “ contents are : “ + val ) ;
    }
}
```

insert a call to the Integer constructor behind the scenes

insert a call to the intValue method behind the scenes

1.11 Autoboxing and unboxing

**编译程序在箭头处分别调用Integer构造函数与intValue方法

Generic Objects in Java

b) Implementing Generic findMax

The **MemoryCell** class required no special properties of **Object** ; the only operations performed were reference assignments, which are always available.

Finding the maximum item in an array of items does require a special property; we must be able to compare two items and decide which is larger.

Generic Objects in Java

```
Comparable findMax ( Comparable [ ] a )  
{  
    Comparable maxValue = a[ 0 ];  
    for ( int i = 1; i < a.length; i++ )  
        if ( maxValue.lessThan ( a[i] ) )  
            maxValue = a[i];  
  
    return maxValue;  
}
```

a generic **findMax** algorithm.

Notice that the objects that are manipulated are not **Object**,
But instead are **Comparable**.

Generic Objects in Java

Comparable is an interface.

In java, an interface declares a set of methods that must be implemented.

- In this example, any class that is **Comparable** must provide an implementation of **lessThan**
- A class that is **Comparable** must also declare so by using the **implements** clause.

```
public interface Comparable
{
    boolean lessThan( Comparable rhs );
    ...
}
```

Generic Objects in Java

- Interfaces can also be declared as generic.

prior to Java 5, the Comparable interface was not generic.

In Java 5, the Comparable class is generic.

```
package java.lang;  
  
public interface Comparable<AnyType>  
{  
    public int compareTo ( AnyType other ) ;  
}
```

1.10 Comparable interface, Java 5 version which is generic

Generic Objects in Java

```
Comparable findMax ( Comparable [ ] a )
```

```
{   Comparable maxValue = a[ 0 ];  
    for ( int i = 1; i < a.length; i++ )  
        if ( maxValue.compareTo(a[i])<0 )  
            maxValue = a[i];  
  
    return maxValue;  
}
```

a generic `findMax` algorithm.

Generic Objects in Java

In the book:

Class FindMaxDemo

```
{ public static comparable findMax( Comparable [ ] arr )
{   int maxIndex = 0;
    for ( int i = 1; i < arr.length; i++ )
        if ( arr[ i ] . compareTo( arr[ maxIndex ] ) > 0 )
            maxIndex = i ;
    return arr[ maxIndex ];
}

public static void main ( String [ ] args )
{   Shape [ ] sh1 = { new Circle( 2.0 ), new Square( 3.0 ),
                    new Rectangle( 3.0, 4.0 ) }
    String [ ] sta = { "Joe", " Bob", "Bill", "Zeke" };
    System.out.println( findMax( sh1 ) );
    System.out.println( findMax( st1 ) );
}
}
```

Chapter 1

Exercises:

1. Write a recursive method that returns the number of 1's in the binary representation of N. Use the fact that is equal to the number of 1's in the representation of N/2, plus 1, if N is odd.

2. Write the routines wise the following declarations:

```
public void permute( String str );
```

```
private void permute( char [ ] str, int low, int high )
```

The first routine is a driver that calls the second and prints all the permutations of the characters in String str. If str is “abc”, then the strings that are output are abc, acb, bac, bca, cab, and cba. Use recursion for the second routine.

3. 已知a[n]为整型数组，试写出实现下列运算的递归算法。

1) 求数组a中的最大整数。

2) 求n个整数的平均值。

Chapter 1

4. Write a recursive method that calculates and returns the length of a linked list.
5. Check recursively if the following objects are palindromes:
 - a. A word
 - b. a sentence (ignoring blanks, lower- and uppercase differences, and punctuation marks so that “Madam, I’m Adam” is accepted as a palindrome)

Chapter 1

实习题:

1. 找出从自然数 $1, 2, \dots, n$ 中任取 r 个数的所有组合, 编一个递归算法.

例子: $n = 5$ 1 2 3 4 5

$r = 3$ 5 4 3

5 4 2

5 4 1

5 3 2

5 3 1

5 2 1

4 3 2

4 3 1

4 2 1

3 2 1

2. 实现Hanoi塔