Chapter 5

Hashing

散列

- **散列函数(Hash function)**
  - **Address=hash(key)**

散列

- **散列函数 (Hash function)**
  - **Address=hash(key)**
- **散列表 (Hash Table)的设计目标**

5.1 General Idea

- Sequention search : O(n)
- Binary search: $O(\log_2 n)$
- hashing method: O(C)

Address=hash(key)

also called : name-address function

5.1 General Idea

- **Example:**

H(x) = x % 7

x1 = 8849 H(x1) = 8849 % 7 = 1

x2 = 8850 H(x2) = 8850 % 7 = 2

x3 = 8851 H(x3) = 8851 % 7 = 3

beta = 66698465 H(beta) = 66698465 % 7 = 1

y2 = 8950 H(y2) = 8950 % 7 = 4

	name	type	address	link
0				
1	x1	float	1000	
2	x2	float	1004	
3	x3	float	1008	
4	y2	int	2000	
5				
6				

beta	int	2002	∧

5.1 General Idea

- problems

Find a proper hash function

How to solve a collision

Select a suitable <u>load factor α.</u>

$$\underline{\alpha=n/b}$$

<u>n</u> is number of elements in the hash table

<u>b</u> is the number of buckets in the hash table

$\alpha > 1$ 碰撞频率大

$\alpha < 1$ 碰撞频率小

5.2 Hash Function

1. 取余法

H(Key) = Key % M

其中 ：M <= 基本区长度的最大质数

基本区长	M
8	7
16	13
2048	2039

为什么取最大质数?

1) 若取偶数，如 10，100，…., 2, 4, …, 冲突率是比较大的；

2）若取含有质因子的M，如 M=21 （3*7） 含质因子3和7，对下面的例子：

key:	28	35	63	77	105
则	7	14	0	14	0

关键码中含质因子7的哈希值均为7的倍数。

2.平方取中法

H(Key) = Key^2 的中间部分，其长度取决于表的大小。

设表长 = 2^9 = $(512)_{10}$　　地址 000~777(八进制）

$(2061)_8$	4$\underline{310}$541
$(2062)_8$	4$\underline{314}$704
$(2161)_8$	4$\underline{734}$741
$(2162)_8$	4$\underline{741}$304
$(1100)_8$	1$\underline{210}$000

5.2 Hash Function

3. 乘法杂凑函数

$$H(Key) = \lfloor M * ((\varphi * Key) \% 1) \rfloor$$

例：设表长 = 2^9 = $(512)_{10}$　　地址 000~777(八进制），则

$$H(1) = \lfloor 2^9 * (0.618)_{10} \rfloor = \lfloor 2^9 * (0.4743\ldots)_8 \rfloor = 474$$

5.2 Hash Function

有些书中的

1. Hash1:

to add up the ASCII(or Unicode) value of the characters in the string.

```
public static int hash( String Key, int tableSize )
{   int hashVal = 0;
    for( int i = 0; i < Key.length( ); i++ )
        hashVal += Key.charAt( i );
    return hashVal % tableSize;
}
```

Example:

Suppose TableSize = 10007,

Suppose all the keys are eight or fewer characters long, 8*127=1016

hash function typically can only assume value between 0~1016 引起浪费

5.2 Hash Function

2. Hash2:

$$h_{key} = k_0 + 37k_1 + 37^2k_2 + \ldots..$$

```
public static int hash( String key, int tableSize ) // good  hash fanction
{   int hashVal = 0;
    for( int i = key.length( )-1; i > =0; i-- )
       hashVal = 37 * hashVal + key.charAt( i );

    hashVal %= tableSize;
    if( hashVal < 0 )  //  函数允许溢出，这可能会引进负数
       hashVal += tableSize;
    return hashVal;
}
```

5.3 how to solve a collision
—— linear Probing

碰撞的两个(或多个)关键码称为同义词，即H(k1) = H(k2)，k1不等于k2

1. Open Addressing

1) linear Probing

If hash(key)=d and the bucket is already occupied then we will examine successive buckets d+1, d+2,……m-1, 0, 1, 2, ……d-1, in the array

5.3 how to solve a collision —— linear Probing

<u>Example 1:</u> a hash table with 11 buckets,

$H(k) = k \% 11$,

Then 80, 40, 65, 24, 58, 35

$H(80) = 3$, $H(40) = 7$, $H(65) = 10$,

$H(24) = 2$, $H(58) = 3$, $H(35) = 2$

5.3 how to solve a collision

——linear Probing

				80				40			65
Ht	0	1	2	3	4	5	6	7	8	9	10

			24	80	58			40			65
Ht	0	1	2	3	4	5	6	7	8	9	10

			24	80	58	35		40			65
Ht	0	1	2	3	4	5	6	7	8	9	10

5.3 how to solve a collision
——linear Probing

Performance analysis

the adding sequence is 80,40,65,24,58,35

$ASL_{succ}=(1+1+1+1+2+4)/6=10/6$

5.3 how to solve a collision
——linear Probing

example 2 :

keys : Burke, Ekers, Broad, Blum, Attlee, Hecht, Alton, Ederly

hash(key) = ord(x) – ord('A')

x为取key第一个字母在字母表中的位置。例如：

hash(Attlee) = 0

H(Burke) = 1 , H(Ekers) = 4 , H(Broad) = 1, H(Blum) = 1,
H(Attlee) = 0 , H(Hecht) = 7 , H(Alton) = 0 , H(Ederly) = 4,

设散列表长 m = 26 (0~25)

0	1	2	3	4	5	6	7	8		25
Attlee	Burke	Broad	Blum	Ekers	Alton	Ederly	Hecht	……	……	
1	1	2	3	1	6	3	1			

分析比较次数：

搜索成功的平均搜索长度

(1 + 1 + 2 + 3 + 1 + 1 + 6 + 3)*1/8 = 18/8

5.3 how to solve a collision
linear Probing

<u>example 3 :</u>

hash(key) = key % 10 ;

{ 89, 18, 49, 58, 69 }

0	1	2	3	4	5	6	7	8	9
49	58	69						18	89
2	4	4						1	1

"clustering problem" 堆积----指不同的同义词表合为一张了。从而增加了插入，查找的时间。

5.3 how to solve a collision

linear Probing

H(k) = k % 11,

Then 80, 40, 65, 24, 58, 35

H(80) = 3, H(40) = 7, H(65) = 10,

H(24) = 2, H(58) = 3, H(35) = 2

Ht	0	1	2	3	4	5	6	7	8	9	10
			24	80	58	35		40			65

要删除58，如果真的删了，则后面要查找35就找不到了。

5.3 how to solve a collision
——linear Probing

C++ Implementation

- Assume that each element to be stored in the hash table is of type <u>E</u> and has a field key of type <u>k</u>.
- the hash table is implemented using two arrays: <u>ht</u> and <u>empty</u> .
- <u>empty[i]</u> is true iff <u>ht[i]</u> does not have an element in it. It is defined for the deletion operation

5.3 how to solve a collision
——linear Probing

```
template<class E,class K>class HashTable
{  public:
      HashTable(int divisor =11);
       ~HashTable() {delete[]ht;  delete [] empty;}
       bool Search(const K&k ,E& e)const;
       HashTable<E,K>& Insert(const E&e);
    private:
       int hSearch(const K& k)const;
       int D;   //hash function divisor
       E *ht ; //hash table array
       bool *empty ; //1D array
};
```

5.3 how to solve a collision ——linear Probing

- Constructor for hashtable

```
template<class E,class K>
HashTable<E,K>::HashTable(int divisor)
{  D=divisor;
   ht=new E[D];
   empty= new bool[D];
   for(int i=0;i<D;i++)
       empty[i]=true;
}
```

5.3 how to solve a collision
——linear Probing

```
template<class E,class K>
int HashTable<E,K>::hSearch(const K&k)const
{  int i=k%D;  //home bucket，做hash
    int j=i;  //start at home bucket
   do
     {  if(empty[j] || ht[j].key = =k) return j; //fit
         j=(j+1)%D; //next bucket
     } while(j!=i);  //returned to home?
   return j; //table full;
}
```

5.3 how to solve a collision
——linear Probing

- Search function

```
Template<class E,class K>
 bool HashTable<E,K>::Search(const
   K&k,E&e)const
 {//put element that matches k in e.
  //return false if no match.
   int b=hSearch(k) ;
   if(empty[b]|| ht[b].key!=k)return false;
   e=ht[b];
   return true;
  }
```

5.3 how to solve a collision
——linear Probing

- **Insertion into a hash table**

```
template<class E,class K>
HashTable<E,K>& HashTable<E,K>::Insert(const E& e)
{  K k=e.key;  //extract key
   int b=hSearch(k);
   if(empty[b]){empty[b]=false;  ht[b]=e;
                     return *this;}
   if(ht[b]= =k)throw BadInput();//duplicate
   throw NoMem();  //table full
}
```

5.3 how to solve a collision

——Quadratic probing

2) Quadratic probing

If hash(k)=d and the bucket is already occupied then we will examine successive buckets d+1, $d+2^2$, $d+3^2$……, in the array

example :

(89,18, 49, 58, 69) [个位数为key]

hash(k) = k % 10;

0	1	2	3	4	5	6	7	8	9
49		58	69					18	89
2		3	3					1	1

ASLsucc=(1+1+2+3+3)/5

5.3 how to solve a collision

——Quadratic probing

- **Java Implementation**

HashEntry

element	isActive

第一种情况： null

第二种情况： 非null且该项是活动的，isActive为true

第三种情况：非null 且该项标记为被删除， isActive为false

```
 public interface Hashable
{   int hash( int tableSize );
}
class HashEntry
{   Hashable element;
    boolean isActive;

    public HashEntry( Hashable e ) { this( e, true ) ; }
    public HashEntry( Hashable e, boolean i )
    {   element = e;
        isActive = i;
    }
}
```

5.3 how to solve a collision

——Quadratic probing

```
public class QuadraticProbingHashTable
{    public QuadraticProbingHashable( )
     public QuadraticProbingHashable( int size )
     public void makeEmpty( )

     public Hashable find( Hashable x )
     public void insert( Hashable x )
     public void remove( Hashable x )

     public static int hash( String key, int tableSize )
     private static final int DEFAULT_TABLE_SIZE = 11;

     protected HashEntry [ ] array;
     private int currentSize;
```

5.3 how to solve a collision

——Quadratic probing

```
private void allocateArray(int arraySize )
private boolean isActive( int currentPos )
private int findPos( Hashable x )
private void rehash( )
private static int nextPrime( int n )
private static boolean isPrime( int n )
}
```

5.3 how to solve a collision

——Quadratic probing

- **Constractor**

```
public QuadraticProbingHashTable( )
{   this( DEFAULT_TABLE_SIZE );
}
public QuadraticProbingHashTable( int size )
{   allocateArray( size );
    makeEmpty( );
}
```

5.3 how to solve a collision
——Quadratic probing

- <u>Some other function</u>

```
private void allocateArray( int arraySize )
{   array = new HashEntry[ arraySize ];
}

public void makeEmpty( )
{   currentSize = 0;
    for( int i = 0; i < array.length; i++ )
        array[ i ] = null;
}
```

5.3 how to solve a collision
——Quadratic probing

- <u>Find function</u>

```
public Hashable find( Hashable x )
 {   int currentPos = findPos( x );
     return isActive( currentPos ) ? array[ currentPos ].element : null ;
 }
 private int findPos( hashable x )
 {   int collisionNum = 0;
     int currentPos = x.hash( array.length );
     while( array[ currentPos ] != null &&
                          !array[ currentPos ].element.equals( x ) )
     {    currentPos += 2 * ++collisionNum – 1;
          if( currentPos >= array . length )
              currentPos -= array . length;
      }
      return currentPos;
 }
```

5.3 how to solve a collision

——Quadratic probing

```
private boolean isActive( int currentPos )
{   return array[ currentPos ] != null &&
   array[ currentPos ].isActive;
}
```

- Insert function

```
public void insert( Hashable x )
{   int currentPos = findPos( x );
    if( isActive( currentPos ) ) return;
    array[ currentPos ] = new HashEntry( x, true );
    if( ++currentSize > array . length / 2 )
        rehash( );
}
```

5.3 how to solve a collision

——Quadratic probing

- **Remove function**

```
public final void remove( Hashable x )
{   int currentPos = findPos( x );
    if( isActive( currentPos) )
        array[ currentPos ] . isActive = false;
}
```

5.3 how to solve a collision
——Double Hashing

3) Double Hashing

If $hash_1(k)=d$ and the bucket is already occupied then we will counting $hash_2(k)=c$, examine successive buckets d+c, d+2c, d+3c……, in the array

example:

hash1(k) = k % 10;

hash2(k) = R – (k % R);

其中 R < TableSize的质数

(89, 18, 49, 58, 69)

hash2(49)=7-49%7=7 (R=7)

hash2(58)=7-58%7=5

hash2(69)=7-69%7=1

0	1	2	3	4	5	6	7	8	9
69			58			49		18	89

rehashing

example:

{ 13, 15, 24, 6 }　　23

h(x) = x % 7;　　23 % 7 = 2

13 % 7 = 6

15 % 7 = 1

24 % 7 = 3

6 % 7 = 6

0	1	2	3	4	5	6
6	15	23	24			13

当表项数 > 表的70% 时,可再散列.

即, 取比(2*原表长=14)大的质数17再散列.

6%17=6,　15%17=15,　23%17=6,　24%17=7,　13%17=13

rehashing

0	6
1	15
2	23
3	24
4	
5	
6	13

0	
1	
2	
3	
4	
5	
6	6
7	23
8	24
9	
10	
11	
12	
13	13
14	
15	15

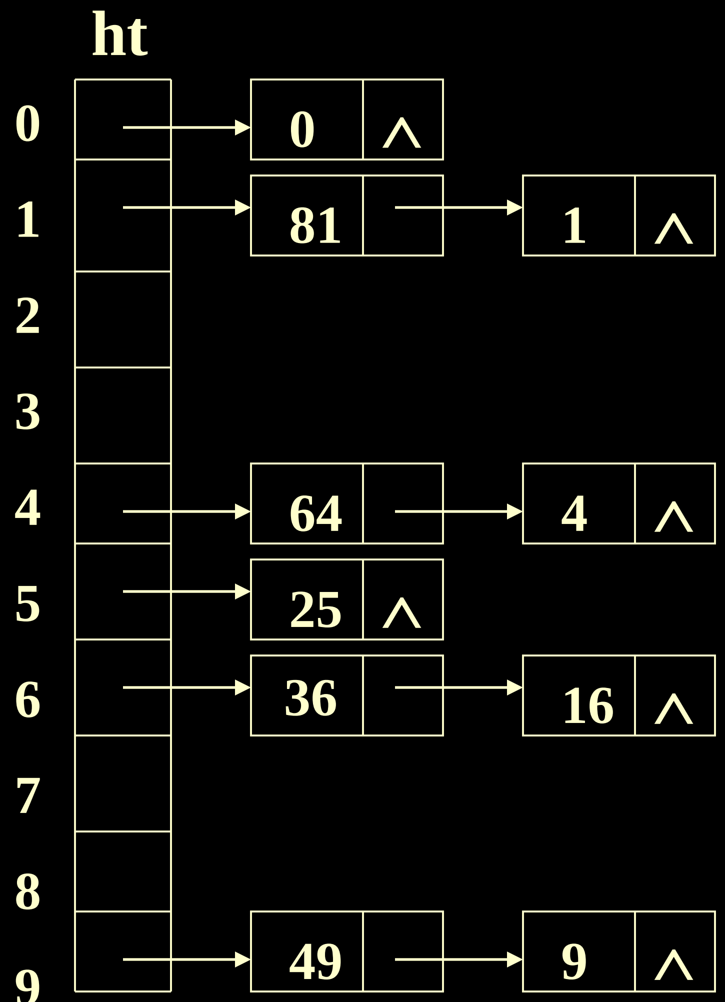
rehashing

```
private void rehash( )
{   HashEntry [ ] oldArray = array ;

    allocateArray(  nextPrime( 2* oldArray.length ) );
    currentSize = 0;

    for( int i = 0; i < oldArray.length;  i++ )
        if( oldArray[ i ] != null && oldArray[ i ] . isActive )
            insert( oldArray[ i ] . Element );
}
```

5.4 how to solve a collision

—— Separate Chaining

2. Separate Chaining 链地址法



0, 1, 4, 9, 16, 25, 36, 49, 64, 81

Hash(x) = x % 10

5.4 how to solve a collision

——Separate Chaining

```
public class SeparateChainingHashTable
{   public  SeparateChainingHashTable( )
    public SeparateChainingHashTable( int size )
    public void insert( Hashable x )
    public void remove( Hashable x )
    public Hashable find( Hashable x )
    public void makeEmpty( )
    public static int hash( String key, int tableSize )

    private static final int DEFAULT_TABLE_SIZE = 101;

    private LinkedList [ ] theLists;

    private static int nextPrime( int n )
    private static boolean isPrime( int n )
 }
```

5.4 how to solve a collision
——Separate Chaining

```
public interface Hashable
{   int hash( int tableSize );
}

public class Employee implements Hashable
{   public int hash( int tableSize )
      {  return SeparateChainingHashTable.hash( name, tableSize ); }
    public boolean equals( object rhs )
      {  return name.equals( ( Employee) rhs ).name ); }

    private String name;
    private double salary;
    private int seniority;
}
```

5.4 how to solve a collision

——Separate Chaining

```
public SeparateChainingHashTable( )
{   this( DEFAULT_TABLE_SIZE );
}

public SeparateChainingHashTable( int size )
{  theLists = new LinkedList[ nextPrime( size ) ];
   for( int i = 0; i < theLists.length; i++ )
        theLists[ i ] = new LinkedList( );
}
public void makeEmpty( )
{   for( int i = 0; i < theLists.length; i++ )
         theLists[ i ].makeEmpty( );
}
```

5.4 how to solve a collision

——Separate Chaining

```
public void remove( Hashable x )
{  theLists[ x.hash( theLists.length ) ].remove( x );
}
public Hashable find( Hashable x )
{  return ( Hashable ) theLists[ x.hash( theLists.length ) ]. Find( x ).
   Retrieve( );
}

public void insert( Hashable x )
{   LinkedList whichList = theLists[ x.hash( theLists.length ) ];
    LinkedListItr itr = whichList.find( x );

    if( itr.isPastEnd( ) )
       whichList.insert( x, whichList.zeroth( ) );
}
```

Chapter 5

exercises:

1. Given input { 4371, 1323, 6173, 4199, 4344, 9679, 1989 }and a hash function h(x) = x (mod 10), show the resulting:

a. Separate chaining hash table.

b. Hash table using linear probing.

c. Hash table using quadratic probing.

d. Hash table with second hash function h_2 (x) = 7- (x mod 7).

2. 设散列表为HT[13]，散列函数为

H(key) = key % 13 。用线性开地址法解决冲突，对下列关键码序列 12,23,45,57,20,03,78,31,15,36：

1) 画出其散列表。

2）计算等概率下搜索成功的平均搜索长度。

3）如果采用链表散列解决冲突，画出该链表。