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# CS 321 HW - 3

Assignment:

1. (8 pts) Prove that the following languages are not regular.

a) $L = \{0^n : n=2^k \text{ for some } k > 1\}$

Answer:

According with Pumping Lemma $w_i = xy^iz$, $i = 0,1,2\ldots$; $|w| \geq m$;

We can write $w = O^{2^m}$; $|xy| \leq m$; $y = O^a;\ a\ \geq 1;\ m \geq a \geq 1;$

I. Deleted y according i = 0;

$xy^0z = xz = O^{2^m} / O^a = O^{2^m - a} \in L;$

II. If we add, when i = 2, or more.

$xyyz = O^{2^m} \cdot O^a = O^{2^m + a};$

$L = O^{2^{m+1}} \rightarrow 2^{m+1} = 2^m + 2^{m+1} > 2^m + a;\ 2^m > a\ , because\ m \geq a \geq 1;$

Therefore L $\{0^n : n=2^k\}$ – non regular language.

b) $L = \{ w : n_a(w) \neq n_b(w), w \in \{a, b\}^* \}$

Answer:

According with Pumping Lemma $w_i = xy^iz$, $i = 0,1,2\ldots$; $w = a^n b^n = xyz$;

We can write $|w| \geq m$ ; $w = O^{2^m}$; $y = O^k;\ m \geq k\ \geq 1;$

$w_0 = xz = a^{m-k} b^m$;

Because $m\text{-}k \neq m$, it is conflict.

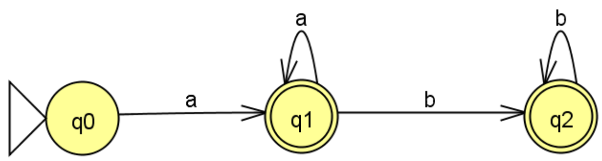
Therefore, $\bar{L} = \{ w : n_a(w) \neq n_b(w), w \in \{a, b\}^* \}$ is non regular language

2. (12 pts) Determine whether or not the following languages are regular. If the language is regular, then give an NFA or regular expression for the language. Otherwise, use the pumping lemma for regular languages to prove the language is not regular.

a) $L = \{ a^n b^n : n > 0\} \cup \{ a^k b^m : k > 0, m > 0\}$

Answer:

If the language is regular, we can create a NFA for that.




In result we proved that L is regular language.

b) $L = \{ a^n b^m : n \le m \le 2n\}$

We can`t to make NFA.

According with Pumping Lemma $w_i = xy^iz$, $i = 0,1,2\ldots$; $|w| \ge m$;

$|xy| \le m$ ; $y = a^k$; $m \ge k \ge 1$; $w_0 = a^n b^m$ ;

Delete: $a^{n-k} b^m$ ; $n \ge k \ge 1$ in result: $n-1 \ge n-k \ge 0$;

From our original Language $\{ a^n b^m : n \le m \le 2n\}$, if $n-k = 0$

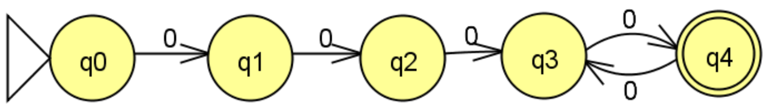
n should be 0, however the $(n-1) \ne 0$ it is conflict.

Therefore, $\bar{L} = \{ a^n b^m : n \le m \le 2n\}$ is non regular language.

c) $L = \{ 0^n : n=2k \text{ for some } k > 1\}$

Answer:

If the language is regular, we can create a NFA for that.




In result we proved that L is regular language

3. (3 pts) Prove or disprove the following statement: If $L_1$ and $L_2$ are non-regular languages, then $L_1 \cup L_2$ is also non-regular. A counter example is sufficient to disprove the statement.

Answer:

As the first (b) question we could know there are two non-regular languages:

1. L = { w : $n_a(w) \neq n_b(w)$, w$\in$ {a, b}* }
2. $\overline{L}$ = { w : $n_a(w) \neq n_b(w)$, w$\in$ {a, b}* }

When we union them, we can get the L = {w: w $\in$ (a,b)*}

It could be represented as regular expressions = (a,b)(a,b)*

Therefore, this is a counter to disprove "if $L_1$ and $L_2$ are non-regular languages, then $L_1 \cup L_2$ is also non-regular"

4. (2 pts) The symmetric difference of two sets S1 and S2 is defined as:

$S_1 \ominus S_2 = \{ x: x \in S_1 \text{ or } x \in S_2$, but $x$ is not in both $S_1$ and $S_2 \}$

Show that the family of regular languages are closed under symmetric difference.

Answer:

$$\mathrm{S1} \ominus \mathrm{S2} = (S_1 \cup S_2) - (S_1 \cap S_2)$$

Regular languages could be closed under:

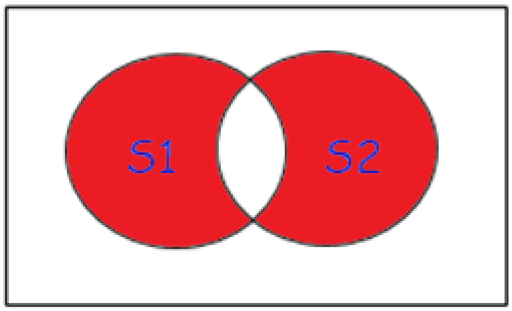
1) Union = $S_1 \cup S_2$ ;
2) Complement = $\bar{S}_1$;
3) Intersection = $S_1 \cap S_2 = \overline{\bar{S}_1 \cup \bar{S}_2}$ ;

4) Difference = $S_1 - S_2 = S_1 \cap \bar{S}_2 = \overline{S_1 \cup \bar{S}_2}$ ;

Use the above operations, we can conclude

$$S1 \ominus S2 = (S_1 \cup S_2) - (S_1 \cap S_2) = (S_1 \cup S_2) - (\overline{\bar{S}_1 \cup \bar{S}_2})$$

$$= \overline{(\overline{S_1 \cup S_2}) \cup (\overline{\bar{S}_1 \cup \bar{S}_2})}$$




Therefore, regular languages are closed under symmetric difference.