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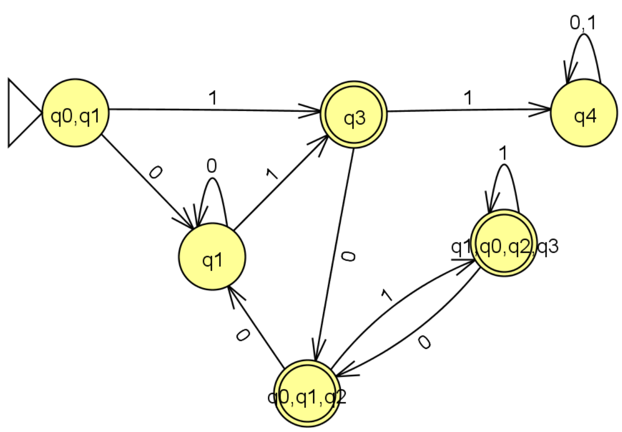
CS 321 HW - 2

Assignment:

1) (4 pts) Convert the following NFA into an equivalent DFA

Answer:



2) (3 pts) Show that the language L = { vwv : v, w ∈ {a,b}*, |v| = 2} is a regular language.

Answer:

Regular expression:

L(M) = ab (a + b)* ab + bb (a + b)* bb + ba (a + b)* ba + aa (a + b)* aa ;

In result we are proof that L is regular language.

3) (4 pts) Prove that if L is regular language then L^R is a regular language.

Answer:

[Definition of reverse]

Let $M = (Q_M, \Sigma_M, \delta, q_M, F_M)$ be a NFA and L = L(M).

The definition of L^R will be:

$$1.\ M^R = \left\{M_R \cup P\{q_s\}, \Sigma_M, \delta_{M^R}, q_s, \{q_M\}\right\} and\ q_s \notin Q_M$$

$$2.\, p \subset \delta_M(q,a) \Leftrightarrow p \subset \delta_{M^R}(p,a), where\ a \subset \Sigma_M\ and\ q,p \subset Q_M$$

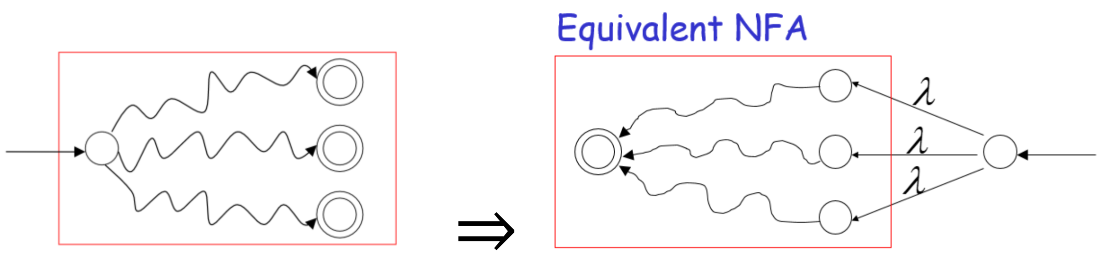
$$3.\ if\ q_M\ equal\ to\ F_M\ in\ L, then\ q_s, we\ wil\ be\ F_M\ in\ L^R$$

[Inductive hypothesis]

Assume for regular expressions r1, that L(r1) and L^R(r1) are regular languages.

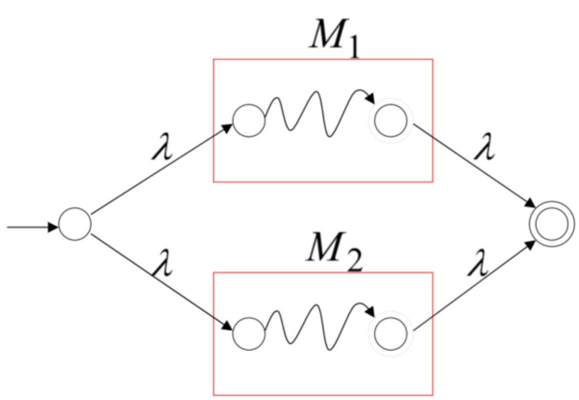
[Inductive step]

If there are multiple final state in L regular language and we need to reverse this. We have to create an equivalent of NFA, where states will connect to new node, which is a initial state, by λ.



As a result, we can prove that the states in L^R will be the union of M^R and q_s.

If we can make some operation like Union, Concatenation, Star or at least one of them, with a reverse language that language is regular.



4) (9 pts) Give regular expressions for the following languages on Σ = {a, b}

a) L1 = { w : n_a(w) mod 3 = 1}.

Answer:

b*a b* (a b*a b*a b*)*

b) L2 = { w : w ends in aa }.

Answer:

(a+b)* aa

c) L3 = all strings containing no more than three a’s.

Answer:

b*ab*ab*ab*+ b* + b*ab* + b*ab* ab*

5) (4 pts) Consider a type of scientific notation for real numbers with the following rules:

a. A number can be preceded by a “+” or “-“ sign or the sign may be absent.

b. Numeric values must be of the form cb1b2…bn where bi is any digit, but c must be nonzero.

c. The number may be followed by an exponent field of the form e”+” y1y2 or e”-“y1y2, where yi can be any digit {0, 1, 2, 3, 4, 5, 6, 7, 8, 9}.

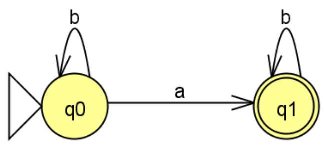
For example the strings -123e+10 and 257 represent real number in this scientific format. Give a regular expression for this scientific notation. Let Σ = { 0, 1, 2, 3, 4, 5, 6, 7, 8, 9, “+”, “-“, e} . (Note: With this convention “+” is the sign associated with the scientific number and + the operator of the regular expression.)

Answer:

(“+” + “-” + λ) (1+2+3+4+5+6+7+8+9) (0+1+2+3+4+5+6+7+8+9)* + (“+” + “-” + λ) (1+2+3+4+5+6+7+8+9) (0+1+2+3+4+5+6+7+8+9)* (“e”) (“+” + “-”) (0+1+2+3+4+5+6+7+8+9) (0+1+2+3+4+5+6+7+8+9)

6) (6 pts) Find a regular grammars for the following languages on Σ={a, b}:

a) L_0 is all strings with exactly one a



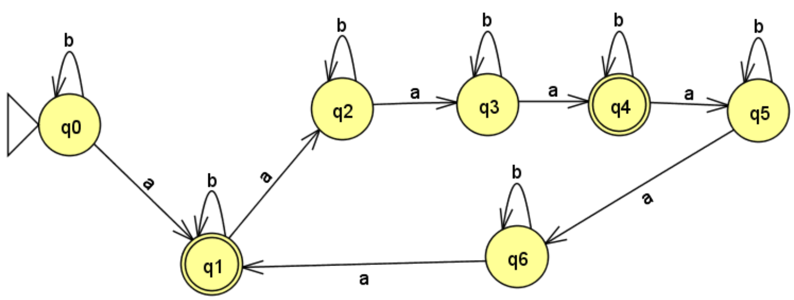
q_0 -> S ; q_1 ->A;

Grammar G:

S-> bS | aA

A - > bA | λ

b) L_1 = { w : $n_a(w)$ mod 3 = 1}.



q_0 -> S ; q_1 ->A; q_2 -> B; q_3 -> C ; q_4 -> D; q_5 -> E; q_6 -> F;

Grammar G:

S -> bS | aA

A -> bA | aB | λ

B -> bB | aC

C -> bC | aD

D -> bD |aE | λ

E -> bE | aF

F -> bF |aA