Nikitin Anton

CS 321 HW - 4

Assignments:

1. (9 pts) Give context-free grammars that generate the following languages.

a. $L_1 = \{ w \in \{0, 1\}^* \mid w \text{ contains at least three 1s} \}$

Answer:

G = {V, T, S, P};

V = {S}; T = {0,1}; S = {S}; P={ S-> S1S1S1S| λ | 0S | 1S};

b. $L_2 = \{ w \in \{0, 1\}^* \mid w = w^R \text{ and } |w| \text{ is even} \}$

Answer:

G = {V, T, S, P};

V = {S}; T = {0,1}; S = {S}; P={ S-> 0S0|1S1| λ };

c. $L_3 = \{ a^i b^j c^k \mid i, j, k \geq 0, \text{ and } i = j \text{ or } i = k \}$

Answer:

G = {V, T, S, P};

V = {S,T,Z,Q,A,B}; T = {a,b,c}; S = {S}; P= $\left\{ \begin{array}{c} S \rightarrow T | Z \\ T \rightarrow AB \\ A \rightarrow aAb | \lambda \\ B \rightarrow cB \mid \lambda \\ Z \rightarrow aZc | \mathrm{Q} \\ Q \rightarrow bQ | \lambda \end{array} \right\}$;

2. (5 pts) Consider the set of terminals T = {a, b, (,), +, *, ∅}. Construct a context-free grammar G=(V,T,S,P) that generates all strings in T* that are regular expressions over Σ = {a, b}. Use the grammar to derive the regular expression (a+b)*.

Recall the rules of regular expressions.

1. ∅ and each member of Σ is a regular expression.

2. If r1 and r2 are regular expressions, then so is r1 r2

3. If r1 and r2 are regular expressions, then so is r1+ r2.

4. If r1 is a regular expression, then so is r1*.

5. If r1 is a regular expression, then so is (r1).

6. Nothing else is a regular expression.

Answer:

G = {V, T, S, P};

V = {S}; T = {a,b,c (,), +, *, ∅}; S = {S}; P={ S-> ∅|a|b| S·S| S+S|S*|(S) };

3. (9 pts) Consider the following grammar G = ({S, A}, {a, b}, S, P} where P is defined below

S -> SS | AAA | λ

A -> aA | Aa | b

a. Describe the language generated by this grammar.

L = {w ∈ $\Sigma^* | A_b(w) mod = 3, A_b(w) denote\ the\ number\ of\ b`s\ in\ w$};

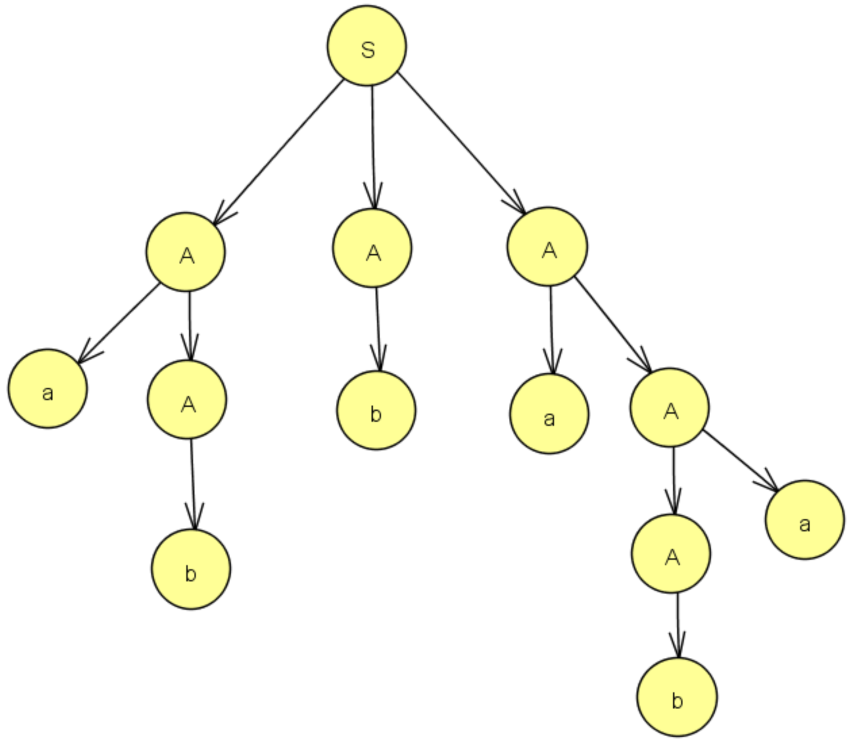
b. Give a left-most derivation for the terminal string abbaba.

Answer:

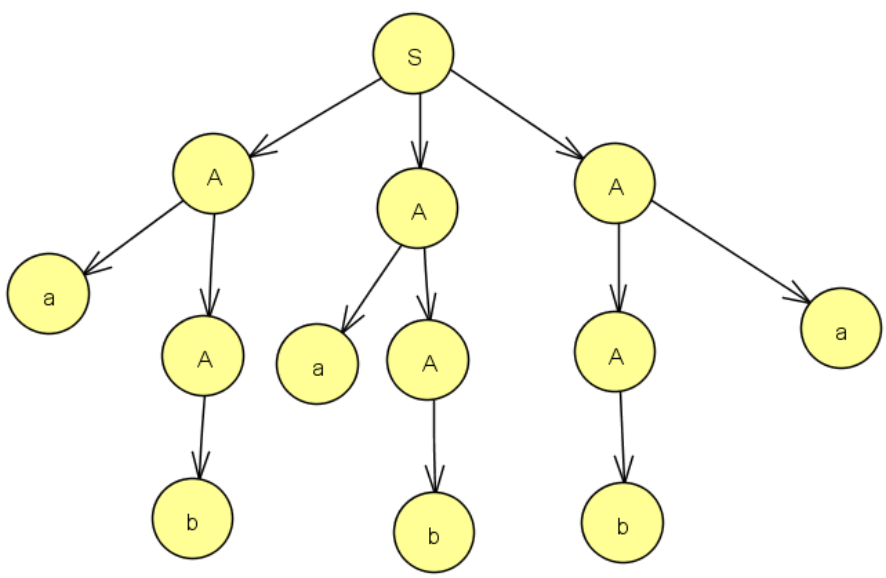
$$S \rightarrow AAA \rightarrow \text{aAAA} \rightarrow \text{abAA} \rightarrow \text{abbA} \rightarrow \text{abbaA} \rightarrow \text{abbaAa} \rightarrow \text{abbaba};$$

c. Show that the grammar is ambiguous by exhibiting two distinct derivation trees for some terminal string.

Leftmost derivation



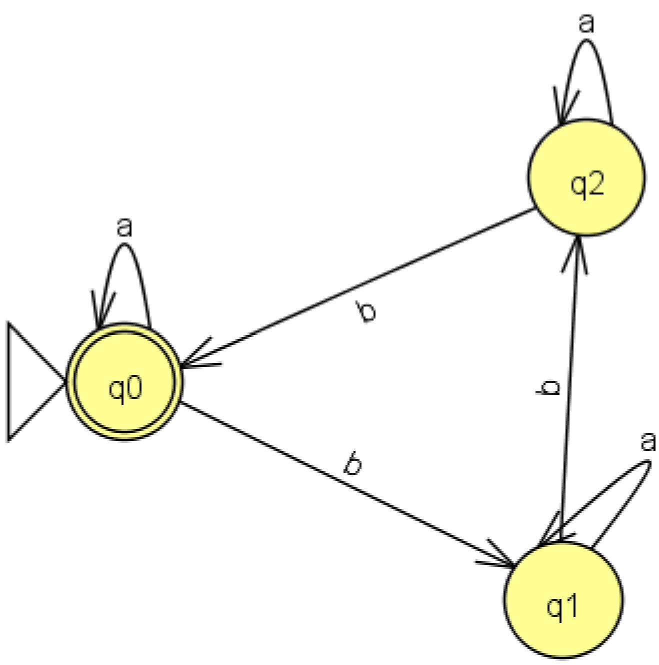
Rightmost derivation



d. If this language is regular, give a regular grammar generating it. If the language is not regular, prove that it is not.

Answer:

We can make NFA for this language, it means that is regular language.



G = {V, T, S, P};

V = { q_0, q_1, q_2}; T = {a,b}; S = {q_0}; P= $\begin{Bmatrix} q_0 \to aq_0 |\ bq_1 | \lambda \\ q_1 \to aq_1 |\ bq_2 \\ q_2 \to q_2 |\ bq_0 \end{Bmatrix}$;

4. (3 pts) Find an s-grammar for L = {$a^n b^{n+1}$: n ≥ 1 }.

Answer:

G = {V, T, S, P};

V = {a,b}; T = {S,A,B}; S = {S}; P= $\begin{Bmatrix} S \to aAB \\ A \to aAB | b \\ B \to b \end{Bmatrix}$;

$S \to aAB \to aaABB \to \text{aaaABBB} \to \text{aaaABBb} \to \text{aaaABbb} \to \text{aaaAbbb}$
$\to \text{aaabbbb};$

5. (4 pts) Let L = { $a^n b^n$: $n \geq 0$ }

a. Show that L^2 is a context-free language.

G` = {V`, T`, S`, P`};

V` = {S,A,B}; T` = {a,b}; S` = {S}; P` = $\begin{Bmatrix} S \to AB \\ A \to aAb|\lambda \\ B \to aBb|\lambda \end{Bmatrix}$

b. Show that L^* is a context-free language.

G` = {V`, T`, S`, P`};

V` = {S}; T` = {a,b}; S` = {S}; P` = { $S \to aSb|\lambda|SS$};