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CS 321 HW - 6

1. Show that the family of context-free languages is closed under reversal.

Answer:

According with Chomsky Normal Form $L(S)$

If a language is L = $\{a^n b^n\}$ the production of grammar P will be:

S→ AB;

$A \rightarrow a$;

B→ b;

And grammar $G = \{V, T, P, S\}$, where $V = \{S, A, B\}$, $T = \{a, b\}, S = \{S\}$;

After reversal operation we will get the language $L^R = \{b^n a^n\}$;

The grammar will be: $G' = \{V', T', P', S'\}$, where $V' = \{S, A, B\}$, $T' = \{a, b\}, S' = \{S^R\}$;

With the production P':

S→ BA;

$A \rightarrow a$;

B→ b;

In result the family of CFL closed under reversal.

2. Show that the family of context-free languages is not closed under difference.

Answer:

If we have L_1 and L_2 languages and $\overline{\overline{L_1} \cup L_2} = L_3$;

Since the context-free language is closed under union. If the difference is context-free L_3, is the context-free. Then we can write is as $\overline{L_1} \cup L_2 = L_3$.

However, the context-free language is not under compliment.

We need to prove that complement is not context-free:

- $L_i = a^n b^n c^m$ & $L_j = a^m b^m c^n$;
- $\overline{\overline{L_i} \cup \overline{L_j}} = L_i \cap L_j = a^n b^n c^n$;

$L_1 - L_2 = \overline{L}_3$ - is not necessary context-free!

For problems 3-5, use the pumping lemma for context-free languages to prove that L is not a CFL.

3. $L_1 = \{a^n b^m : n = 2^m\}$.

Answer:

$w = a^{2^m} b^m$; $w = u v^i x y^i z$; $|vy| \geq 1$; $|vxy| \leq m$;

$uv^ixy^iz \in L$ for all $i \geq 0$;

Case 1: vxy is within a^k; $w_2 = a^{2^m+k} b^m$;

Since: $2^m + k < 2^{m+1}$ $w_2 \notin L$;

Case 2: $vxy = b^k; w_2 = a^{2^m} b^{m+k}$;

Since: $2^m < (2^{m+1} = 2 \cdot 2^m)$ $w_2 \notin L$;

Case 3: $v = a^k; y = b^t;\ k, t \geq 1, w_2 = a^{2^m+k} b^{m+t}$;

Since: $(2^m + k) < (2^{m+1}), w_2 \notin L$;

Because $2^{m+1} = 2^m + 2^m$, $and\ 2^m\ large\ than\ k$;

Case 4: Either v or y contain both a`s and b`s.

W_2 = the a`s and b`s are “out of order”,

In all cases we obtained a contradiction. Therefore $L_1 = \{a^n b^m : n = 2^m\}$ is context-free must be wrong.

4. $L_2 = \{ a^n b^n c^j : n \leq j \}$.

Answer:

$w = a^m b^m c^{m+1}$; $w = u v^i x y^i z$; $|vy| \geq 1$; $|vxy| \leq m$;

$uv^ixy^iz \in L$ for all $i \geq 0$;

Case 1: vxy is within a^k; $w_2 = a^{m+k} b^m c^{m+1}$;

Since: $m + k > m$ $w_2 \notin L$;

Case 2: $vxy = b^k; w_2 = a^m b^{m+k} c^{m+1}$;

Since: $m + k > m$ $w_2 \notin L$;

Case 3: $vxy = c^k; w_0 = a^m b^m c^{m+1-k}$;

$n_c \geq n_b$ $or\, n_a$; $n_b = n_a$;

If $n_c = n_a$ or $n_b -$ is not a context $-$ free language because L_2 will contain more than 2 depended loops.

If $n_c < n_a$ or n_b, j < n since $m + 1 - k$, $w_0 \notin L$;

Case 4: $v = a^k; y = b^t;\ k, t \geq 1, w_2 = a^{m+k} b^{m+t} c^{m+1}$; ;

If $k = t, n_a$ or $n_b \geq n_c$, $m + k \geq m + 1$;

If $k \neq t, n_a \neq n_b$;

Case 5: $v = b^k; y = c^t;\ k, t \geq 1, w_2 = a^m b^{m+k} c^{m+t+1}$;

$n_a < n_b$, Since m + k > m;

In all cases we obtained a contradiction. Therefore $L_2 = \{ a^n b^n c^j : n \leq j \}$. Is context-free must be wrong.

5. $L_3 = \{ w: w \in \{a,b,c\}^*$ and $n_a(w) < n_b(w) < n_c(w) \}$.

Answer:

Assume that $L_3 = \{ w: w \in \{a,b,c\}^*$ and $n_a(w) < n_b(w) < n_c(w) \}$. Is context-free language

$w = a^{m-1} b^m c^{m+1}$; $w = u v^i x y^i z$; $|vy| \geq 1$; $|vxy| \leq m$;

$uv^ixy^iz \in L$ for all $i \geq 0$;

Case 1: vxy is within a^k; $w_2 = a^{m-1+k} b^m c^{m+1}$;

Since: $n_a > n_b$, $w_2 \notin L$;

Case 2: $vxy = b^k; w_2 = a^{m-1} b^{m+k} c^{m+1}$;

Since: $m + k \geq m + 1$, $w_2 \notin L$;

Case 3: $vxy = c^k; w_0 = a^{m-1} b^m c^{m+1-k}$;

$m + 1 - k \leq m$, $w_0 \notin L$;

Case 4: $v = a^k; y = b^t;\ k, t \geq 1, w_2 = a^{m-1+k} b^{m+t} c^{m+1}$; ;

$n_b \geq n_c$, $m + t \geq m + 1$;

Case 5: $v = b^k; y = c^t;\ k, t \geq 1, w_0 = a^{m-1} b^{m-k} c^{m-t+1}$;

$n_a > n_b$, $m - k \leq m - 1$; $w_0 \notin L$;

There are no different cases to consider.

Since |vxy| $\leq m$, string vxy can`t overlap $a^m, b^m,\ c^m$ at the same time.

In all cases we obtained a contradiction. Therefore L_3 = { w: w $\in$ {a,b,c}* and $n_a(w) < n_b(w) < n_c(w)$ }. Is context-free must be wrong.