CV Homework1

Siyuan Yin 2100017768

1 Short questions

1.1 Homogeneous coordination

- Find the mathematical relationship between x_i and x'_i.

  Answer:

  $$x'_i = \frac{x_i}{\omega} \quad \forall i \in [N]$$

- Two advantages of homogeneous coordination and why we adopt it in computational photography.

  Answer:

  1. Homogeneous coordinates provide a unified framework for representing both affine and perspective transformations, allowing us to compute transformations, including translation, rotation, scaling and shearing, as matrix multiplications, While in Cartesian coordinates, translations typically require separate handling. This makes computation in homogeneous coordinates more efficient and concise.
  2. In homogeneous coordinates, points at infinity can be handled naturally, making it easier to model and correct for perspective projection. Just set the ω in $(x_1, \cdots, x_N, \omega)$ as zero. But in Cartesian coordinates, this will be more complex.

  Homogeneous coordinates simplifies the representation and combination of complex transformations as unified matrix multiplication. The convenience may be crucial for some computational photography tasks, such as 3D recinstruction from multiple images. Points at infinity remain unaffected by depth of field, so when implemeting depth of field homogeneous coordinates are valuable.

1.2 Dolly zoom

- Analyze how dolly zoom is achieved.

Answer: The dolly zoom effect needs the combination of camera movement and zooming, which creates a special visual perception in the viewer's sense. The dolly zoom is divided into two categories:"Dolly-out & Zoom-in" and "Dolly-in & Zoom-out". The former refers to the camera moving backward while simultaneously zooming in, while the latter is the opposite, where the camera moves forward when zooming out.

2 Camera parameters from the image

- Find the vanishing line and calculate the camera height H.

  Answer: Vinishing line is shown in Figure 1.The position of the edge point A of the hexagonal is(0, 3.74, 0.76) in the world coordinate. Denote the height of point A as h_A, the distance between point A and y-axis as y_A, the distance between vinishing line and y-axis as y_v.I measured the value of y_A is 192 pixels, and the value of y_v 332 pixels. We have:

$$\frac{h_A}{y_A} = \frac{H}{y_v}$$

  So $H = 0.76 \times 332 \div 192 \approx 1.314$ meters.

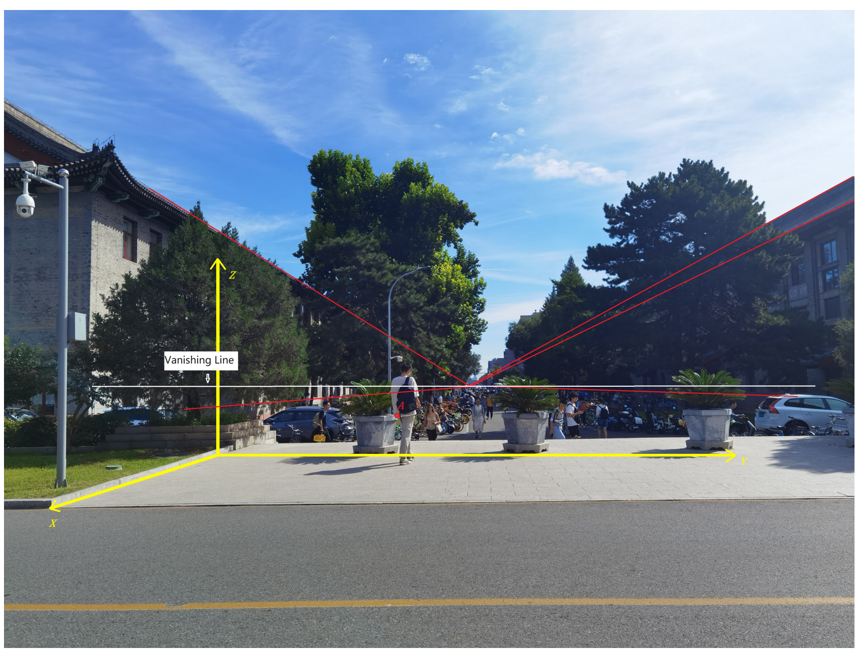

Figure 1: Vanishing line

- Write down the intrinsic parameter matrix and derive the camera focal length f.
  Answer: First, we translate the origin to the coordinate of the camera,(13.4, 4.5, 1.3). So the translation matrix is:

$$\mathbf{T} = \begin{bmatrix} I_{3\times3} & t_{3\times1} \\ 0 & 1 \end{bmatrix} = \begin{bmatrix} 1 & 0 & 0 & -13.4 \\ 0 & 1 & 0 & -4.5 \\ 0 & 0 & 1 & -1.3 \\ 0 & 0 & 0 & 1 \end{bmatrix} \tag{1}$$

  Traditionally, we point the z-axis forward, x to the right, and y to the bottom in the camera coordinate. So the rotation matrix is:

$$\mathbf{R} = \begin{bmatrix} R_{3\times3} & 0 \\ 0 & 1 \end{bmatrix} = \begin{bmatrix} 0 & 1 & 0 & 0 \\ 0 & 0 & -1 & 0 \\ -1 & 0 & 0 & 0 \\ 0 & 0 & 0 & 1 \end{bmatrix} \tag{2}$$

  The projection matrix is:

$$\mathbf{P} = \begin{bmatrix} 1 & 0 & 0 & 0 \\ 0 & 1 & 0 & 0 \\ 0 & 0 & 1 & 0 \end{bmatrix} \tag{3}$$

  The aspect ratio equals 1 and the skew equals zero because the pixels are square. The intrinsics matrix is:

$$\mathbf{In} = \begin{bmatrix} f & 0 & c_x \\ 0 & f & c_y \\ 0 & 0 & 1 \end{bmatrix} \tag{4}$$

  So the final transition matrix is:

$$\Pi = \begin{bmatrix} f & 0 & c_x \\ 0 & f & c_y \\ 0 & 0 & 1 \end{bmatrix} \begin{bmatrix} 1 & 0 & 0 & 0 \\ 0 & 1 & 0 & 0 \\ 0 & 0 & 1 & 0 \end{bmatrix} \begin{bmatrix} 0 & 1 & 0 & 0 \\ 0 & 0 & -1 & 0 \\ -1 & 0 & 0 & 0 \\ 0 & 0 & 0 & 1 \end{bmatrix} \begin{bmatrix} 1 & 0 & 0 & -13.4 \\ 0 & 1 & 0 & -4.5 \\ 0 & 0 & 1 & -1.3 \\ 0 & 0 & 0 & 1 \end{bmatrix} \tag{5}$$

  Denote the coordinate A in world coordinate as $X_A = (0, 3.74, 0.76)$, in image coordinate as $X'_A = (u, v, 1)$. k represents k meters per pixel in image. (k $= 0.76/192 = 0.0039$, $c_x = 2048, c_y = 1536$).We have:

$$X'_A = k\Pi X_A$$

  Then we can calculate the value of f which is the only unknown variable.