LISTA DE EXERCICIOS 05

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QUESTÃO 01

a) $\lim\limits_{(x,y)\to(1,2)} (5x^3 - x^2y^2) = 5 - 1 \cdot 4 = 1$

b) $\lim\limits_{(x,y)\to(1,-1)} e^{-xy} \cdot \cos(x+y) = e$

c) $\lim\limits_{(x,y)\to(2,1)} \frac{4 - x \cdot y}{x^2 + 3y^2} = \frac{2}{7}$

d) $\lim\limits_{(x,y)\to(1,0)} \ln\left(\frac{1+y^2}{x^2+xy}\right) = \ln(1) = 0$

e) $\lim\limits_{(x,y)\to(0,0)} \frac{x^4 - 4y^4}{x^2 + 2y^2} = \lim\limits_{(x,y)\to(0,0)} (x^2 - 2y^2) = 0$

f) $\lim\limits_{(x,y)\to(0,0)} \frac{x^2 + \operatorname{sen}^2 y}{2x^2 + y^2} = L$

i) pela reta $y = 0$: $L = \lim\limits_{x\to0} \frac{x^2}{2x^2} = \frac{1}{2}$

ii) pela reta $x = 0$: $L = \lim\limits_{y\to0} \frac{\operatorname{sen}^2 y}{y^2} = 1$ $\longrightarrow$ Não existe L

g) $\lim\limits_{(x,y)\to(0,0)} \frac{x \cdot y \cdot \cos y}{3x^2 + y^2} = \lim\limits_{(x,y)\to(0,0)} \frac{x \cdot y}{3x^2 + y^2} \cdot \lim\limits_{(x,y)\to(0,0)} \cos y = \lim\limits_{(x,y)\to(0,0)} \frac{x \cdot y}{3x^2 + y^2}$ $\nexists$

h) $\lim\limits_{(x,y)\to(1,0)} \frac{x \cdot y - y}{(x-1)^2 + y^2} = \lim\limits_{(u,y)\to(0,0)} \frac{u \cdot y}{u^2 + y^2}$ $\nexists$

i) $\lim\limits_{(x,y)\to(0,0)} \frac{x \cdot y}{\sqrt{x^2 + y^2}} = 0$

Prova: Seja $\varepsilon > 0$. Tome $\delta = \varepsilon$

Para $\sqrt{x^2+y^2} < \delta \Leftrightarrow x^2 + y^2 < \delta^2$, temos

$\left|\frac{x \cdot y}{\sqrt{x^2+y^2}}\right| = |y| \cdot \left|\frac{x}{\sqrt{x^2+y^2}}\right| \leq |y| < \delta = \varepsilon$ ∎

j) $\lim\limits_{(x,y)\to(0,0)} \frac{x^4 - y^4}{x^2 + y^2} = \lim\limits_{(x,y)\to(0,0)} (x^2 - y^2) = 0$

k) $\lim\limits_{(x,y)\to(0,0)} \frac{x^2 y \cdot e^y}{x^4 + 4y^4} = \lim\limits_{(x,y)\to(0,0)} \frac{x^2 \cdot y}{x^4 + 4y^4} \cdot \lim\limits_{(x,y)\to(0,0)} e^y = \lim\limits_{(x,y)\to(0,0)} \frac{x^2 y}{x^4 + 4y^4}$ $\nexists$

l) $I = \lim\limits_{(x,y)\to(0,0)} \dfrac{x^2 \cdot \operatorname{sen}^2 y}{x^2+2y^2} = 0$

para $y=0 \Rightarrow I=0$, para $x=0 \Rightarrow I=0$, para $y=x$ $I=0$

Prova: Seja $\varepsilon>0$. Tomemos $\delta=\sqrt{\varepsilon}$.

Para $\sqrt{x^2+y^2}<\delta \Rightarrow |x|<\delta$ e $|y|<\delta$.

$$\left|\frac{x^2 \cdot \operatorname{sen}^2 y}{x^2+2y^2}\right| = |\operatorname{sen} y|^2 \cdot \left|\frac{x^2}{x^2+2y^2}\right|$$

$$y^2 \geq 0 \Leftrightarrow 2y^2 \geq 0 \Leftrightarrow x^2+2y^2 \geq x^2 \overset{(x,y)\neq(0,0)}{\Longleftrightarrow} \frac{x^2}{x^2+2y^2} \leq 1$$

$$\Rightarrow \left|\frac{x^2 \operatorname{sen}^2 y}{x^2+2y^2}\right| \leq |\operatorname{sen} y|^2 \cdot 1 \leq |y|^2 < \delta^2 = \varepsilon \ \blacksquare$$

m) $\lim\limits_{(x,y)\to(0,0)} \dfrac{x^2+y^2}{\sqrt{x^2+y^2+1}-1} = \lim\limits_{(x,y)\to(0,0)} \left(\sqrt{x^2+y^2+1}+1\right) = 2$

n) $\lim\limits_{(x,y)\to(0,0)} \dfrac{x \cdot y^4}{x^2+y^8}$ É diferente para $x=0$ e $x=y^2 \Rightarrow$ não Existe

o) $\lim\limits_{(x,y,z)\to(\pi,0,1)} e^{y^2} \cdot tg(xz) = e^{0^2} \cdot tg(1 \cdot \pi) = 0$

p) $\lim\limits_{(x,y,z)\to(0,0,0)} \dfrac{xy+yz}{x^2+y^2+z^2} = I$ não existe

Para $y=0 \Rightarrow I=0$; para a reta $y=x=z \Rightarrow I = 2/3$

q) Análogo a p) $\Rightarrow$ não Existe

r) Análogo a p) $\Rightarrow$ não Existe

QUESTÃO 02 | a) $f(x,y) = \dfrac{x \cdot y}{1+e^{x-y}}$

i) $D_f = \mathbb{R}^2$. Seja $(x_0, y_0) \in \mathbb{R}^2$.

$\lim\limits_{(x,y)\to(x_0,y_0)} f(x,y) = \lim\limits_{(x,y)\to(x_0,y_0)} \dfrac{x \cdot y}{1+e^{x-y}} = \dfrac{x_0 \cdot y_0}{1+e^{x_0-y_0}} = f(x_0,y_0) \Rightarrow f$ é contínua em $\mathbb{R}^2$

b) $f(x,y) = \cos(\sqrt{1+x-y})$; $D_f = \{(x,y) \in \mathbb{R}^2 \,/\, y \leq x+1\}$

Seja $(x_0,y_0) \in D_f \Rightarrow \lim\limits_{(x,y)\to(x_0,y_0)} f(x,y) = f(x_0,y_0) \Rightarrow f$ é contínua em D_f

c) $f(x,y) = \dfrac{1+x^2+y^2}{1-x^2-y^2} \Rightarrow D_f = \{(x,y) \in \mathbb{R}^2 \,/\, x^2+y^2 \neq 1\}$

d) $f(x,y) = \dfrac{e^x+e^y}{e^{xy}-1} \Rightarrow D_f = \{(x,y) \in \mathbb{R}^2 \,/\, x \cdot y \neq 0\}$

e) $f(x,y) = \ln(x^2+y^2-4) \Rightarrow D_f = \{(x,y) \in \mathbb{R}^2 \,/\, x^2+y^2 > 4\}$

f) $f(x,y) = \operatorname{arctg}\left(\dfrac{1}{(x+y)^2}\right) \Rightarrow D_f = \{(x,y) \in \mathbb{R}^2 \,/\, x+y \neq 0\}$

g) $f(x,y,z) = \operatorname{arcsen}(x^2+y^2+z^2) \Rightarrow D_f = \{(x,y,z) \in \mathbb{R}^3 \,/\, x^2+y^2+z^2 \leq 1\}$

h) $f(x,y,z) = (\ln z) \cdot \sqrt{y-x^2} \Rightarrow D_f = \{(x,y,z) \in \mathbb{R}^3 \,/\, y \geq x^2$ e $z > 0\}$

QUESTÃO 03 | a) Como $\lim\limits_{(x,y)\to(0,0)} f(x,y) = 0 \Rightarrow$ Conjunto $= \mathbb{R}^2 - \{(0,0)\}$

b) f é contínua em $\mathbb{R}^2 - \{(0,0)\}$

QUESTÃO 04 | a) $\lim\limits_{(x,y)\to(0,0)} f(x,y) = 0 = m - 2 \Leftrightarrow m = 2$

b) $\lim\limits_{(x,y)\to(0,0)} \dfrac{x^3}{x^2+y^2} = 0 \Rightarrow \dfrac{2m+1}{m+3} = 1 \Leftrightarrow m = 2$