1 11.6

1.1 06

$\int_0^2 |x^2 - x|\, dx$

Quando a minha função é positiva

$$x^2 - x \geq 0$$
$$x(x-1) \geq 0$$

De 0 até 1: $f(x) < 0$.

$$I = \int_0^1 (x - x^2)dx + \int_1^2 (x^2 - x)dx$$
$$I = \left(\frac{x^2}{2} - \frac{x^3}{3}\right)_0^1 + \left(\frac{x^3}{3} - \frac{x^2}{2}\right)_1^2$$
$$I = \left(\frac{1}{2} - \frac{1}{3}\right) + \left(\frac{8}{3} - \frac{4}{2}\right) - \left(\frac{1}{3} - \frac{1}{2}\right)$$

1.2 07

$\int_{-1}^2 |3 - 2x - x^2|\, dx$

Analisando a função:

$$f(x) = 3 - 2x - x^2$$
$$f(x) = 0$$
$$3 - 2x - x^2 = 0$$
$$x^2 + 2x - 3 = 0$$

$\Delta = 2^2 - 4 \cdot 1 \cdot (-3) = 16$

$$x = \frac{-2 \pm 4}{2}$$
$$x = -3 \text{ ou } x = 1$$

$$I = \int_{-1}^1 (3 - 2x - x^2)dx + \int_1^2 (x^2 + 2x - 3)dx$$

1.3 15

2 12.4

2.1 questão 01. a)

$\int \sqrt{1 - 4x^2} dx$

Queremos: $a^2 - x^2$

Sabemos que: $1 = \frac{4}{4}$

$$\sqrt{1 - 4x^2} = \sqrt{\frac{4}{4} - 4x^2}$$

$$= \sqrt{4\left(\frac{1}{4} - x^2\right)}$$

$$= 2\sqrt{\frac{1}{4} - x^2} = 2\sqrt{\left(\frac{1}{2}\right)^2 - x^2}$$

Faça: $x = \frac{1}{2}\sin u \left(-\frac{\pi}{2} \leq u \leq \frac{\pi}{2}\right)$
$\frac{dx}{du} = \frac{1}{2}\cos u$

$$dx = \frac{1}{2}\cos u du$$

$$I = \int 2\sqrt{\frac{1}{4} - \frac{1}{4}\sin^2 u} \cdot \frac{1}{2}\cos u du$$

$$I = \int \frac{1}{2}\sqrt{1 - \sin^2 u}\cos u du$$

$$I = \frac{1}{2}\int \cos^2 u du$$

Lembrando: $\cos^2 x = \frac{1 + \cos(2x)}{2}$

$$I = \frac{1}{2}\int \left(\frac{1}{2} + \frac{\cos(2u)}{2}\right) du$$

$$I = \frac{1}{4}\int (1 + \cos(2u)) du = \frac{1}{4}\left(\int du + \int \cos(2u) du\right)$$

$$I = \frac{1}{4}\left(u + \frac{\sin(2u)}{2}\right) + C$$

Lembrando: $\sin(2x) = 2\sin x \cos x$

$$I = \frac{1}{4}(u + \sin u \cos u) + C$$

$$I = \frac{1}{4}\left(arcsin(2x) + 2x\sqrt{1 - 4x^2}\right) + C$$

2.2 i)

$\int \frac{1}{x\sqrt{1+x^2}} dx$
Quando temos $a^2 + x^2$ fazemos a substituição $x = a\tan u$

Façamos:

$$x = \tan u \rightarrow dx = \sec^2 u du (**)$$

$$I = \int \frac{1}{\tan u\sqrt{1 + \tan^2 u}} \sec^2 u du$$

$$1 + \tan^2 u = \sec^2 u$$

$$I = \int \frac{1}{\tan u \sec u} \sec^2 u du$$

$$I = \int \frac{\sec u}{\tan u} du$$

$$I = \int \frac{\frac{1}{\cos u}}{\frac{\sin u}{\cos u}} du$$

$$I = \int \frac{1}{\sin u} du = -\ln|cotg(u) + cossec(u)| + C$$

$$I = -\ln\left|\frac{1}{x} + \frac{\sqrt{1+x^2}}{x}\right| + C$$

Lembrando:

$$-\ln x = \ln\frac{1}{x} = \ln x^{-1}$$

2.3 02:

$$4x^2 + y^2 \leq 1$$
$$\frac{x^2}{\left(\frac{1}{2}\right)^2} + \frac{y^2}{1^2} \leq 1$$

Calculando a área da parte superior da elipse:

$$y^2 = 1 - 4x^2$$
$$y = \sqrt{1-4x^2}$$

$$A = 2\int_{-\frac{1}{2}}^{\frac{1}{2}} \sqrt{1-4x^2}dx$$
$$A = 2\frac{1}{4}\left(arcsin(2x) + 2x\sqrt{1-4x^2}\right)_{-\frac{1}{2}}^{\frac{1}{2}}$$
$$A = \frac{1}{2}\cdot(arcsin(1) - arcsin(-1))$$
$$\boxed{A = \frac{\pi}{2}}$$

2.4 04. c)

$$I = \int \frac{1}{1+\sqrt{x}}dx = \int \frac{\sqrt{x}}{\sqrt{x}(1+\sqrt{x})}dx$$

Faça a substituição: $u = \sqrt{x} = x^{\frac{1}{2}}$

$$\frac{du}{dx} = \frac{1}{2}x^{-\frac{1}{2}} = \frac{1}{2\sqrt{x}}$$
$$\frac{dx}{\sqrt{x}} = 2du$$

Logo:

$$I = 2\int \frac{u}{1+u}du$$

Faça: $1 + u = t \rightarrow du = dt$

$$I = 2\int \frac{t-1}{t}dt = 2\int dt - 2\int \frac{1}{t}dt$$

$$I = 2\left(t - \ln|t|\right) + C$$
$$I = 2\left(1 + \sqrt{x} - \ln\left(1+\sqrt{x}\right)\right) + C$$

3 16.2

3.1 01 a)

$f(x) = \ln(1+x)$

i) $f'(x) = \dfrac{1}{1+x} \to f'(x_o) = 1$

ii) $f''(x) = \dfrac{0 \cdot (1+x) - 1(1)}{(1+x)^2} = -\dfrac{1}{(1+x)^2} \to f''(x_o) = -1$

$$P(x) = f(x_o) + f'(x_o)(x - x_o) + f''(x_o)\frac{(x-x_o)^2}{2!}$$

$$\boxed{P_2(x) = x - \frac{x^2}{2}} \approx \ln(1+x)$$

3.2 02 a)

$$\ln 1,3 = \ln(1 + 0,3)$$

$$\ln 1,3 \approx 0,3 - \frac{0,3^2}{2}$$

$$\ln 1,3 \approx 0,255$$

Função erro:

$$E(x) = |f(x) - P(x)|$$

$$E(x) \leq \left| f^{(n+1)}(c) \cdot \frac{(x-x_o)^{n+1}}{(n+1)!} \right|$$

$$E(x) \leq \left| f^{(3)}(c) \cdot \frac{(x-x_o)^3}{(3)!} \right|$$

Temos que:

$$1 < 1,3 < e$$

$$0 < \ln 1,3 < 1$$

Calculando a derivada terceira:

$$f^{(3)}(x) = -\frac{0(1+x)^2 - 1 \cdot 2(1+x)}{(1+x)^4} = \frac{2}{(1+x)^3} \to f^{(3)}(0) = 2 \ e \ f^{(3)}(1) = \frac{1}{4}$$

Logo:

$$E(x) \leq \left| f^{(3)}(c) \cdot \frac{(x-x_o)^3}{(3)!} \right| \leq 2 \cdot \frac{x^3}{3!} = \frac{x^3}{3}$$

$$E(x) \leq \frac{0,3^3}{3}$$

$$\boxed{E(0,3) \leq 0,009}$$