Lista 08 Cálculo II

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Questão 01 a) $z = x^2 + y^2 + x\cdot y$; $x = \operatorname{sen} t$ e $y = e^t$

$$\frac{dz}{dt} = \frac{\partial z}{\partial x}\cdot\frac{dx}{dt} + \frac{\partial z}{\partial y}\cdot\frac{dy}{dt} = (2x+y)\cdot\cos t + (2y+x)\cdot e^t$$

$$\Rightarrow \frac{dz}{dt} = (2\operatorname{sen} t + e^t)\cdot\cos t + (2e^t + \operatorname{sen} t)\cdot e^t$$

b) $z = \cos(x+4y)$; $x = 5t^4$ e $y = 1/t$

$$\frac{dz}{dt} = -\operatorname{sen}(x+4y)\cdot 20t^3 - 4\cdot\operatorname{sen}(x+4y)\cdot(-1/t^2)$$

$$\Rightarrow \frac{dz}{dt} = \left(\frac{4}{t^2} - 20t^3\right)\cdot\operatorname{sen}\left(5t^4 + \frac{4}{t}\right)$$

c) $z = \sqrt{1+x^2+y^2}$; $x = \ln t$ e $y = \cos t$

$$\frac{dz}{dt} = \frac{x}{\sqrt{1+x^2+y^2}}\cdot\frac{1}{t} + \frac{y}{\sqrt{1+x^2+y^2}}\cdot(-\operatorname{sen} t)$$

$$\Rightarrow \frac{dz}{dt} = \frac{\ln t}{t\sqrt{1+\ln^2 t+\cos^2 t}} - \frac{\operatorname{sen} t\cos t}{\sqrt{1+\ln^2 t+\cos^2 t}}$$

d) $z = \operatorname{arctg}(y/x)$; $x = e^t$ e $y = 1 - e^{-t}$

$$\operatorname{tg} z = \frac{y}{x} = \frac{1-e^{-t}}{e^t} \Rightarrow \operatorname{tg} z = e^{-t} - e^{-2t}$$

$$\Rightarrow \frac{dz}{dt} = \frac{1}{1+(e^{-t}-e^{-2t})^2}\cdot(-e^{-t} + 2\cdot e^{-2t})$$

e) $w = x\cdot e^{y/z}$; $x = t^2$, $y = 1-t$ e $z = 1+2t$

$$\frac{dw}{dt} = e^{\frac{y}{z}}\cdot 2t + t^2\cdot e^{\frac{y}{z}}\cdot\frac{1}{z}\cdot(-1) + x\cdot e^{y/z}\cdot\left(-\frac{y}{z^2}\right)\cdot 2$$

$$\frac{dw}{dt} = e^{\frac{1-t}{1+2t}}\cdot\left(2t - \frac{t^2}{1+2t} - \frac{2t^2(1-t)}{(1+2t)^2}\right.$$

f) $w = \ln(x^2+y^2+z^2)$; $x = \operatorname{sen} t$, $y = \cos t$ e $z = \operatorname{tg} t$

$$w = \ln(c^2+s^2+t^2) = \ln(1+\operatorname{tg}^2 t) = \ln(\sec^2 t)$$

$$\Rightarrow \frac{dw}{dt} = \frac{1}{\sec^2 t}\cdot 2\sec t\cdot\sec t\cdot\operatorname{tg} t \Rightarrow \frac{dw}{dt} = 2\operatorname{tg} t$$

Questão 02|

a) $z = x^2 \cdot y^3$; $x = s \cdot \cos t$ e $y = s \cdot \text{sen}\, t$

$\cdot \frac{\partial z}{\partial s} = \frac{\partial z}{\partial x} \cdot \frac{\partial x}{\partial s} + \frac{\partial z}{\partial y} \cdot \frac{\partial y}{\partial s} = 2xy^3 \cdot \cos t + 3x^2y^2 \cdot \text{sen}\, t$

$= 2s \cdot \cos t \cdot s^3 \cdot \text{sen}^3 t \cos t + 3s^2 \cos^2 t \cdot s^2 \text{sen}^2 t \cdot \text{sen}\, t$

$= 5s^4 \cos^2 t \cdot \text{sen}^3 t$

$\cdot \frac{\partial z}{\partial t} = \frac{\partial z}{\partial x} \cdot \frac{\partial x}{\partial t} + \frac{\partial z}{\partial y} \cdot \frac{\partial y}{\partial t} = 2x \cdot y^3 \cdot (-s \cdot \text{sen}\, t) + 3x^2y^2 \cdot (s \cos t)$

$= 2 \cdot (s \cos t) \cdot (s^3 \text{sen}^3 t) \cdot (-s\, \text{sen}\, t) + 3 \cdot (s^2 \cos^2 t) \cdot (s^2 \text{sen}^2 t) \cdot (s \cos t)$

$= -2s^5 \cos t \cdot \text{sen}^4 t + 3s^5 \cdot \cos^3 t \cdot \text{sen}^2 t = s^5 \cdot (3 \text{sen}^2 t \cos^3 t - 2 \cos t \cdot \text{sen}^4 t)$

c) $z = \text{sen}\, \theta \cdot \cos \varphi$; $\theta = s \cdot t^2$ e $\varphi = s^2 t$

$\cdot \frac{\partial z}{\partial s} = \frac{\partial z}{\partial \theta} \cdot \frac{\partial \theta}{\partial s} + \frac{\partial z}{\partial \varphi} \cdot \frac{\partial \varphi}{\partial t} = \cos \theta \cdot \cos \varphi \cdot t^2 - \text{sen}\, \theta \cdot \text{sen}\, \varphi \cdot 2st$

$= \cos(st^2) \cdot \cos(s^2 t) \cdot t^2 - \text{sen}(st^2) \cdot \text{sen}(s^2 t) \cdot 2st$

$\cdot \frac{\partial z}{\partial t} = \cos(st^2) \cdot \cos(s^2 t) \cdot 2st - \text{sen}(st^2) \cdot \text{sen}(s^2 t) \cdot s^2$

e) $z = e^r \cdot \cos \theta$, $r = s \cdot t$ e $\theta = \sqrt{s^2 + t^2}$

$\cdot \frac{\partial z}{\partial s} = \frac{\partial z}{\partial r} \cdot \frac{\partial r}{\partial s} + \frac{\partial z}{\partial \theta} \cdot \frac{\partial \theta}{\partial s} = e^r \cos \theta \cdot t - e^r \text{sen}\, \theta \cdot \frac{s}{\sqrt{s^2+t^2}}$

$\cdot \frac{\partial z}{\partial t} = \frac{\partial z}{\partial r} \cdot \frac{\partial r}{\partial t} + \frac{\partial z}{\partial \theta} \cdot \frac{\partial \theta}{\partial t} = e^r \cos \theta \cdot s - e^r \text{sen}\, \theta \cdot \frac{t}{\sqrt{s^2+t^2}}$

f) $z = tg\left(\frac{u}{v}\right)$; $u = 2s + 3t$ e $v = 3s - 2t$

Seja $k = \frac{u}{v} \Rightarrow z = tg\, k \Rightarrow \frac{\partial z}{\partial k} = \sec^2 k$

$\cdot \frac{\partial z}{\partial s} = \frac{\partial z}{\partial k} \cdot \frac{\partial k}{\partial s} = \sec^2 k \cdot \left(\frac{\partial k}{\partial u} \cdot \frac{\partial u}{\partial s} + \frac{\partial k}{\partial v} \cdot \frac{\partial v}{\partial s} \right)$

$= \sec^2\left(\frac{u}{v}\right) \cdot \left[\frac{1}{v} \cdot 2 + \left(-\frac{u}{v^2}\right) \cdot 3 \right]$

Analogamente:

$\frac{\partial z}{\partial t} = \sec^2\left(\frac{u}{v}\right) \cdot \left[\frac{1}{v} \cdot 3 + \frac{2u}{v^2} \right]$

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Questão 03 | $z = f(x(t), y(t))$

$\frac{dz}{dt} = \frac{\partial z}{\partial x} \cdot \frac{dx}{dt} + \frac{\partial z}{\partial y} \cdot \frac{dy}{dt} = 6 \cdot 5 + (-8) \cdot (-4) \Rightarrow \left.\frac{dz}{dt}\right|_{t=3} = 62$

Questão 04 | $w = f(u(s,t), v(s,t))$

- $\frac{\partial w}{\partial s} = \frac{\partial w}{\partial u} \cdot \frac{\partial u}{\partial s} + \frac{\partial w}{\partial v} \cdot \frac{\partial v}{\partial s} = (-1) \cdot (-2) + (10) \cdot (5) = 52$
- $\frac{\partial w}{\partial t} = \frac{\partial w}{\partial u} \cdot \frac{\partial u}{\partial t} + \frac{\partial w}{\partial v} \cdot \frac{\partial v}{\partial t} = (-1) \cdot (6) + (10) \cdot 4 = 34$

Questão 06 | a) $y \cdot \cos x = x^2 + y^2$

Seja $W = y \cdot \cos x - x^2 - y^2 \equiv 0$

$\frac{\partial w}{\partial x} = \frac{\partial w}{\partial x} \cdot \cancel{\frac{dx}{dx}}^{1} + \frac{\partial w}{\partial y} \cdot \frac{dy}{dx} = 0 \Rightarrow \frac{dy}{dx} = -\frac{f_x}{f_y} = \frac{+y \operatorname{sen} x + 2x}{\cos x - 2y}$

b) $\cos(x \cdot y) = 1 + \operatorname{sen} y$

$\frac{dy}{dx} = -\frac{f_x}{f_y} = -\frac{y \cdot \operatorname{sen} x}{-x \cos y + \cos y}$

d) $e^y \cdot \operatorname{sen} x = x + x \cdot y$

$y' = -\frac{f_x}{f_y} = \frac{-e^y \cos x + 1 + y}{e^y \operatorname{sen} x - x}$

e) $x^2 + 2y^2 + 3z^2 = 1 \equiv W$

- $\frac{\partial w}{\partial x} = \frac{\partial w}{\partial x} \cdot \cancel{\frac{\partial x}{\partial x}}^{1} + \frac{\partial w}{\partial y} \cdot \cancel{\frac{\partial y}{\partial x}}^{0} + \frac{\partial w}{\partial z} \cdot \frac{\partial z}{\partial x} = 0 \Rightarrow \frac{\partial z}{\partial x} = -\frac{W_x}{W_z}$

$\Rightarrow \frac{\partial z}{\partial x} = -\frac{2x}{6z}$ Analogamente $\frac{\partial z}{\partial y} = -\frac{4y}{6z}$