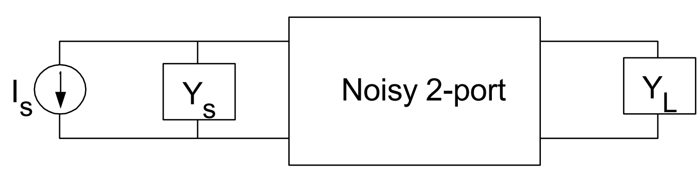
Active Microwave Circuits

Noise Analysis in Microwave Circuits

Part II

Dependence of Noise Figure on Source Admittance



Will show $F = F_{min} + (R_n / G_s)| Y_s - Y_{opt}|^2$
for any linear 2-port

F_{min}, R_n, G_{opt}, B_{opt} are the *noise parameters*

Thevenin-Norton Representations of Noisy 2-Ports

e_1 , e_2 are open circuit noise voltages

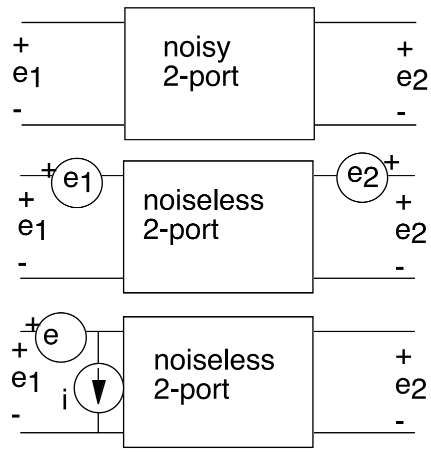
Thevenin equivalent

Thevenin-Norton equiv.

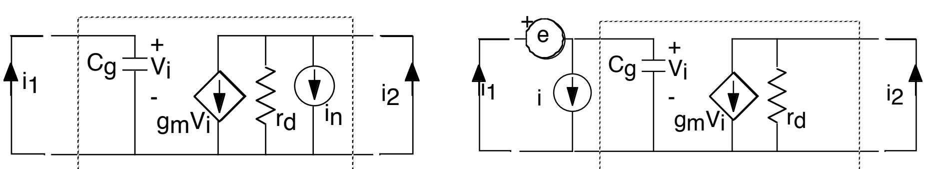
$e_2 = - i\, Z_{21}$

$e_1 = - i\, Z_{11} + e$

Solve for i, e in terms of e_1 , e_2



Equivalent Noise Sources - an example



$i_2 = i_n$ ⟷ $i_2 = g_m V_i = -g_m e$

equate and solve for e

$i_1 = 0$ ⟷ $i_1 = i - j\omega C_g e$

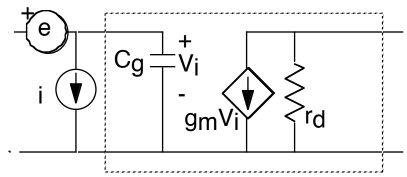
equate and solve for i

$e = \dfrac{-i_n}{g_m}$ $\quad i = j\omega C_g e = \dfrac{-j\omega C_g}{g_m} i_n$

Equivalent noise sources in terms of model noise sources

Example- continued

$$e = \frac{-i_n}{g_m} \qquad i = j\omega C_g e = \frac{-j\omega C_g}{g_m} i_n$$



Write in terms of noise notation

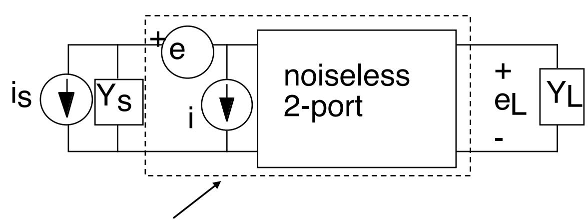
$$\overline{|e|^2} = \frac{\overline{|i_n|^2}}{{g_m}^2} \qquad \overline{|i|^2} = \left(\frac{\omega C_g}{g_m}\right)^2 \overline{|i_n|^2} \qquad \overline{ei^*} = \left(-j\omega C_g / g_m^2\right)\overline{|i_n|^2}$$

Derive for yourself the correlation coefficient, C.

Note e and i are uncorrelated since they are 90˚ out of phase.

This results in C being imaginary and Re(C) = 0 (see lecture 9)

Thevenin-Norton Equivalent



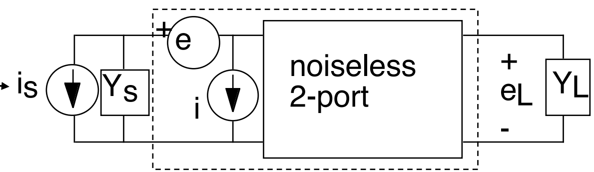
Thevenin-Norton equivalent of a noisy 2-port

•e and i are functions of impedances and noise sources internal to the 2-port

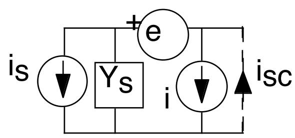
•e and i are specified by $\overline{|e|^2}, \overline{|i|^2}, \overline{ei^*}$

Calculation of Noise Temp. in terms of e and i

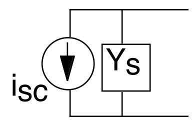
source
admittance
noise



First create Norton equivalent of everything on the input



all sources

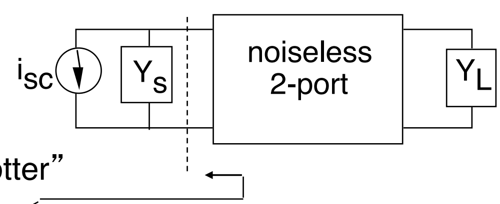


Norton equivalent

$$i_{sc} = i + i_s + eY_s$$

Noise temperature i/t/o sources i, e

$$i_{sc} = i + i_s + eY_s$$



Source appears “hotter”
because of e & i

$$P_{av} = \frac{\overline{|i_{sc}|^2}}{4\,\mathrm{Re}(Y_s)} = \frac{\overline{|i + i_s + eY_s|^2}}{4\,\mathrm{Re}(Y_s)} = \kappa(T_o + T_n)\Delta f$$

$$\text{noise temp.} = \frac{P_{av}\big|_{i_s=0}}{k\Delta f} = T_n = \frac{\overline{|i + eY_s|^2}}{4k\Delta f\,\mathrm{Re}(Y_s)}$$

Noise Factor i/t/o e and i

It's easy to go from noise temp to noise factor

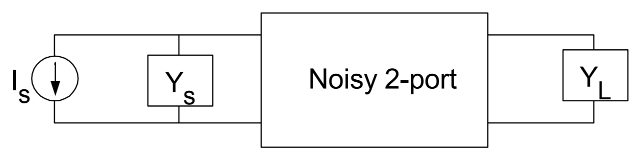
$$F = 1 + \frac{T_n}{T_o} = 1 + \frac{\overline{|i + eY_s|^2}}{4kT_o \Delta f \mathrm{Re}(Y_s)}$$

•Shows how F varies versus Y_s for <u>any</u> linear 2-port

•Need $\overline{|e|^2}, \overline{|i|^2}, \overline{ei^*}$ to specify the equation (4 numbers)

•Form is not convenient as it stands

A More Convenient Form



Will show $F = F_{min} + (R_n / G_s) | Y_s - Y_{opt} |^2$

F_{min}, R_n, G_{opt}, B_{opt} are the *noise parameters*

These four numbers are derived from the four numbers,

$$\overline{|e|^2}, \overline{|i|^2}, \overline{ei^*}$$

Derivation of Noise Equation

$$F = 1 + \frac{\overline{(i + eY_s)(i^* + e^* Y_s^*)}}{4kT_o \Delta f \, \mathrm{Re}(Y_s)}$$

$$= 1 + \frac{\overline{|i|^2} + \overline{ei^*}\, Y_s + \overline{e^* i}\, Y_s^* + \overline{|e|^2} |Y_s|^2}{4kT_o \Delta f \, \mathrm{Re}(Y_s)}$$

Remember correl coef.

$$C = C_r + jC_i \equiv \frac{\overline{ei^*}}{\sqrt{\overline{|e|^2}\;\overline{|i|^2}}}$$

Normalize admittance

$$\tilde{Y}_s = \tilde{G}_s + j\tilde{B}_s \equiv Y_s \sqrt{\frac{\overline{|e|^2}}{\overline{|i|^2}}}$$

$$F = 1 + \frac{\sqrt{\overline{|e|^2}\;\overline{|i|^2}}}{4kT_o \Delta f} \, \frac{\tilde{G}_s^2 + \tilde{B}_s^2 + 2C_r \tilde{G}_s - 2C_i \tilde{B}_s + 1}{\tilde{G}_s}$$

Choosing Admittance to Minimize F

$$F = 1 + \frac{\sqrt{\overline{|e|^2}\;\overline{|i|^2}}}{4kT_o \Delta f} \, \frac{\tilde{G}_s^2 + \tilde{B}_s^2 + 2C_r \tilde{G}_s - 2C_i \tilde{B}_s + 1}{\tilde{G}_s}$$

Find minimum versus B_s in usual manner (dF/dB_s)= 0

$$\tilde{B}_{s,min} = C_i \equiv \tilde{B}_{opt}$$

Find minimum versus G_s

$$\tilde{G}_{s,min} = \sqrt{1 - C_i^2} \equiv \tilde{G}_{opt}$$

Insert back in equation to get minimum F

$$F_{min} \equiv 1 + \frac{\sqrt{\overline{|e|^2}\;\overline{|i|^2}}}{2kT_o \Delta f} \left(\sqrt{1 - C_i^2} + C_r \right)$$

Rewrite Expression for F i/t/o minimum parameters

$$F - F_{min} = \frac{\sqrt{\overline{|e|^2}\,\overline{|i|^2}}}{4kT_o\Delta f} \frac{\tilde{G}_s^2 - \tilde{G}_s 2\sqrt{1-C_i^2} + \tilde{B}_s^2 - 2C_i\tilde{B}_s + 1}{\tilde{G}_s}$$

$$= \frac{\sqrt{\overline{|e|^2}\,\overline{|i|^2}}}{4k\Delta f} \frac{\left(\tilde{G}_s - \tilde{G}_{opt}\right)^2 + \left(\tilde{B}_s - \tilde{B}_{opt}\right)^2}{\tilde{G}_s}$$

un-normalize

$$\tilde{Y}_s \equiv Y_s\sqrt{\frac{\overline{|e|^2}}{\overline{|i|^2}}} \qquad \tilde{Y}_{opt} \equiv Y_{opt}\sqrt{\frac{\overline{|e|^2}}{\overline{|i|^2}}}$$

$$F - F_{min} = \frac{\overline{|e|^2}}{4kT_o\Delta f} \frac{\left(G_s - G_{opt}\right)^2 + \left(B_s - B_{opt}\right)^2}{B_s}$$

Final Form

$$F - F_{min} = R_n \frac{\left(G_s - G_{opt}\right)^2 + \left(B_s - B_{opt}\right)^2}{B_s}$$

Noise parameters

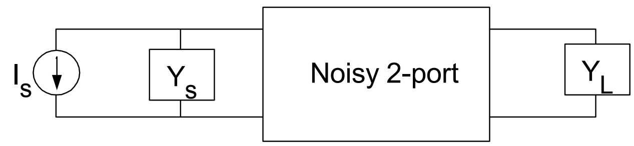
$$F_{min} \equiv 1 + \frac{\sqrt{\overline{|e|^2}\,\overline{|i|^2}}}{2kT_o\Delta f}\left(\sqrt{1-C_i^2} + C_r\right)$$

$$R_n \equiv \frac{\overline{|e|^2}}{4\kappa T_o\Delta f}$$

$$G_{opt} \equiv \sqrt{\frac{\overline{|i|^2}}{\overline{|e|^2}}}\sqrt{1-C_i^2} \qquad B_{opt} \equiv \sqrt{\frac{\overline{|i|^2}}{\overline{|e|^2}}}C_i$$

$$C_r + jC_i \equiv \frac{\overline{ei^*}}{\sqrt{\overline{|e|^2}\,\overline{|i|^2}}}$$

General Form Noise Figure of Two Port

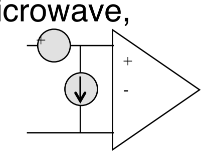


$$F = F_{min} + R_n \frac{\left|Y_s - Y_{opt}\right|^2}{G_s}$$

F_{min} , R_n, G_{opt}, B_{opt} are the *noise parameters*

Example: an OP-AMP

The noise spec. for op-amps often includes $\overline{|e|^2}, \overline{|i|^2}$.
These sources are uncorrelated at low, non microwave, frequencies.



For example at 10KHz a 741 opamp has

$$\overline{|e|^2}/\Delta f = 5x10^{-16}\ V^2/Hz \qquad \overline{|i|^2}/\Delta f = 3X10^{-25}\ A^2/Hz$$

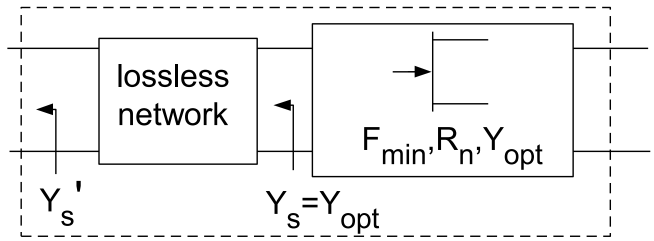
Using these numbers we get

$$F_{min} \equiv 1 + \frac{\sqrt{\overline{|e|^2}\ \overline{|i|^2}}}{2kT_o\Delta f}\left(\sqrt{1 - C_i^2} + C_r\right) = \underline{2.53}$$

Of course this ignores the feedback normally used with op-amps

Invariants

•Adding a lossless 2-port to a noisy 2-port will not change F_{min} . (A Y_s' can always be found to make $Y_s = Y_{opt}$)



•Adding a lossless 2-port to a noisy 2-port will not change the product (R_n X G_{opt}).

$$(R'_n \times G'_{opt}) = (R_n \times G_{opt}) = N$$

Invariants

The minimum noise measure, M_{min}, is invariant for any lossless embedding network (even including feedback)

(Haus & Adler, Proc. IEEE, Aug. 1958)

$T_{min} < 4NT_o$ (Pospieszalski, MTT Trans. Sept.1989)

Noise Figure Expression in terms of Γ

$$Y_s = Y_o \frac{1-\Gamma_s}{1+\Gamma_s} \qquad Y_{opt} = Y_o \frac{1-\Gamma_{opt}}{1+\Gamma_{opt}}$$

$$F = F_{min} + 4 r_n \frac{\left|\Gamma_s - \Gamma_{opt}\right|^2}{\left|1+\Gamma_{opt}\right|^2\left(1-\left|\Gamma_s\right|^2\right)} \quad , r_n \equiv \frac{R_n}{Z_o}$$

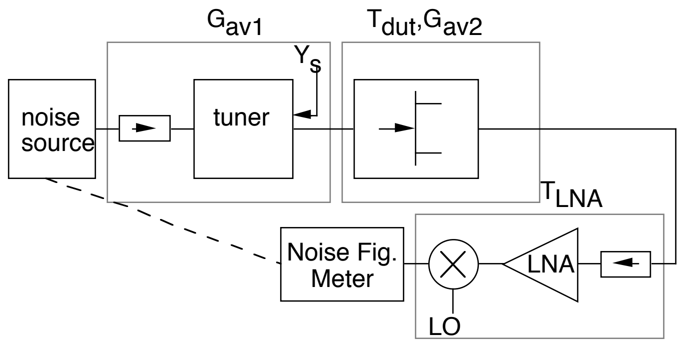
Find R_n, F_{min}, Γ_{opt} experimentally by measuring F for 4 or more known Γ_s

Fit equation to measurements.

Noise Parameter Measurement

Several Ys presented to device.

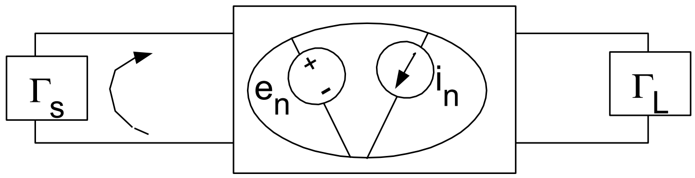
Noise fig. measured at each



$$F_{meas} = \frac{1}{G_{av1}} + \frac{F_{dut}-1}{G_{av1}} + \frac{F_{LNA}-1}{G_{av1}G_{av2}}$$

need to de-embed F_{dut} from measurement

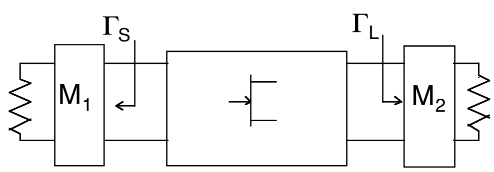
Interpretation of Effect of Source on Noise



To minimize noise delivered to load, Γ_s is adjusted so as to reflect noise through 2-port and partially cancel noise emitted from port 2

Low Noise Amplifier Design

Previously Γ_s, Γ_L chosen for gain and match



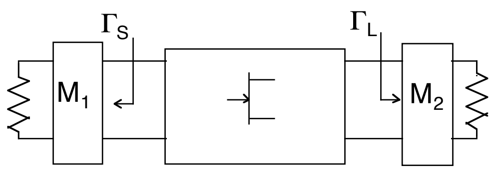
$$F = F_{min} + 4\, r_n \frac{\left|\Gamma_s - \Gamma_{opt}\right|^2}{\left|1 + \Gamma_{opt}\right|^2 \left(1 - \left|\Gamma_s\right|^2\right)} \quad , r_n \equiv \frac{R_n}{Z_o}$$

Amplifier vs. Device: Gain and Noise Figure

For lossless matching networks

$$F_{M1} = \frac{1}{G_{avM1}} = 1$$

$$F_{M2} = \frac{1}{G_{avM2}} = 1$$



$$G_{avLNA} = G_{avM1} G_{avdev} G_{avM1} = G_{avdev}$$

$$F_{LNA} = F_{M1} + \frac{F_{dev} - 1}{G_{avM1}} + \frac{F_{M2} - 1}{G_{avM1} G_{avdev}} = F_{dev}$$

If M_1, M_2 lossless, amplifier available gain and noise figure = device available gain and noise figure

Noise Circles

Noise equation gives noise figure resulting from Γ_S

$$F = F_{min} + 4\, r_n \frac{\left|\Gamma_S - \Gamma_{opt}\right|^2}{\left|1 + \Gamma_{opt}\right|^2 \left(1 - \left|\Gamma_S\right|^2\right)} \quad , r_n \equiv \frac{R_n}{Z_o}$$

The reverse problem is: Find a set of Γ_S that give a specified noise figure.

A locus of Γ_S points that all give the same noise figure form a <u>noise circle</u> in the Γ_S plane.

Noise Circles on the Smith Chart

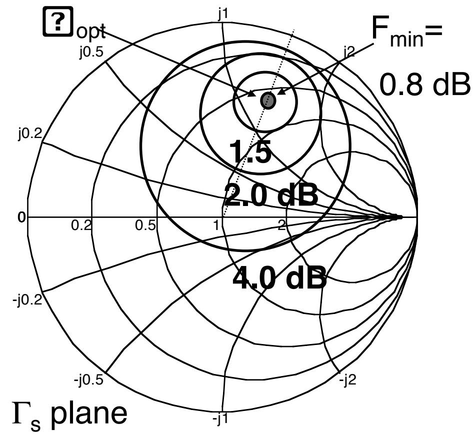
center of i' th circle

$$C_{fi} = \frac{\Gamma_{opt}}{1 + N_i}$$

radius of i' th circle

$$r_{fi} = \frac{1}{1 + N_i}\sqrt{N_i\left(1 - \left|\Gamma_{opt}\right|^2\right) + N_i^2}$$

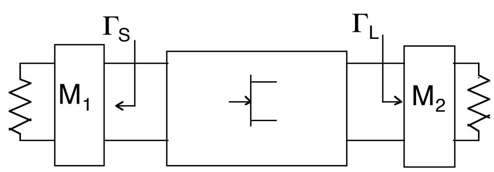
$$N_i \equiv \frac{(F_i - F_{min})\left|1 + \Gamma_{opt}\right|^2}{4 r_n}$$



Derivation is similar to previous derivations that use circles in the Smith Chart

Low Noise Amplifier Design

Previously Γ_S, Γ_L chosen for gain and match



Different choice for minimum noise

$\Gamma_S = \Gamma_{opt}$ (minimum noise)

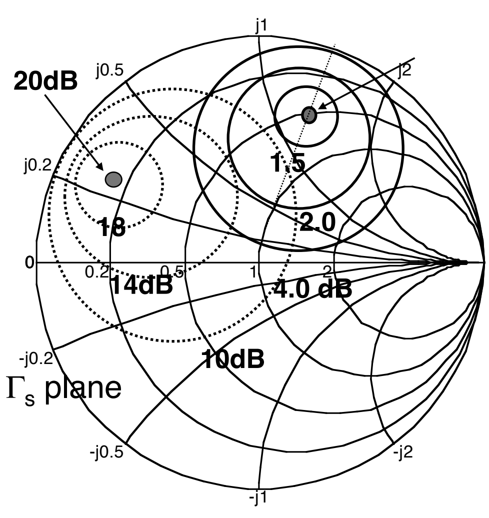
$$\Gamma_L = \left(S_{22} + \frac{S_{21}S_{12}\Gamma_S}{1 - S_{11}\Gamma_S}\right)^* \quad \text{(good output match)}$$

input match sacrificed

Gain Versus Noise Trade

- Choosing $\Gamma_s = \Gamma_{opt}$ trades (i) gain and (ii) input match for good noise performance
- The *associated gain* of a device is the gain achieved when $\Gamma_s = \Gamma_{opt}$ and $\Gamma_L = \Gamma_{out}{}^*$
- Often a designer will trade a little noise performance for improved gain
- To evaluate gain-noise trades plot noise and available gain circles on Smith charts

Noise and Available Gain Circles in Γ_s Plane



Note *associated gain*

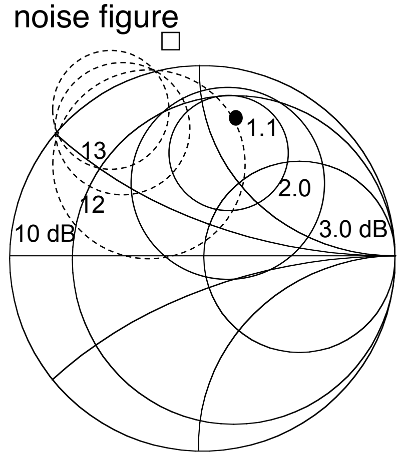
- Choosing $\Gamma_s = \Gamma_{opt}$ gives $F=F_{min}$=.8 dB and $G_T = G_{assoc}$< 10 dB
- Choosing $\Gamma_s = \Gamma_{sm}$ gives F > 4 dB and $G_T = G_{mag}$= 20 dB
- Choosing $\Gamma_s = 0.5 < 100^\circ$ gives F = 2dB and G_T = 14 dB

Equations for Available Gain circles

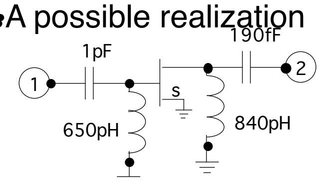
$$C_A = \frac{g_a\left(S_{11} - \Delta S_{22}^*\right)^*}{1 + g_a\left(|S_{11}|^2 - |\Delta|^2\right)}, \quad g_a \equiv \frac{G_{AV}}{|S_{21}|^2} \quad r_A = \frac{\left[1 - 2K|S_{21}S_{21}|g_a + |S_{21}S_{21}|^2 g_a{}^2\right]^{\frac{1}{2}}}{\left|1 + g_a\left(|S_{11}|^2 - |\Delta|^2\right)\right|}$$

LNA with Potentially Unstable Device

- Design to achieve at least 12 dB gain and < 2.5 dB noise figure



- Choosing $\Gamma_S = .45<110^\circ$ gives F= 2 dB, G_T = 12dB
- Choosing $\Gamma_L = 0.56<66^\circ$ matches output $\Gamma_L = \Gamma_{out}{}^*$
- A possible realization



- S parameters of over-all amplifier: IS21I =12.1 dB IS11I = -1.0 dB IS22I = -42 dB F =1.84 dB

Notes on LNA example

@ 10 GHz

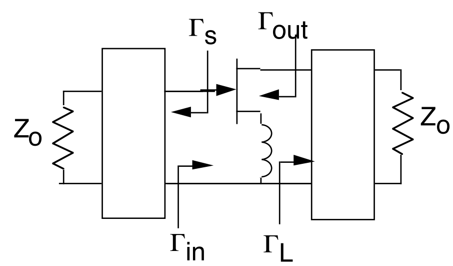
$S_{11} = 0.806 < -96$ $\quad$ $S_{21} = 2.24 < 99$ $\quad$ $F_{min} = 1.12$ dB

$S_{12} = 0.115 < 38$ $\quad$ $S_{22} = 0.553 < -43$ $\quad$ $\Gamma_{opt} = 0.71 < 74.0$

$r_n = 0.458$

Use of Source Inductance in LNA design

Find source inductance and Γ_L such that $\Gamma_{in} = \Gamma^*_{opt}$ with good gain



- Source inductance reduces F_{min}, but not much
- Source inductance reduces G_{mag} (or G_{msg})
- Goal is to get a good input match **and** a good noise figure

Procedure

- Pick a trial inductance
- Compute a new Γ_{opt}
- Is there a Γ_L that gives $\Gamma_{in} = \Gamma_{opt}{}^* = S_{11} + \dfrac{S_{12}S_{21}\Gamma_L}{1 - S_{22}\Gamma_L}$
- For this Γ_L compute gain - enough?
- Stability
- If not good, increase inductance and try again

Example

	S and noise parameters of FET at 10 GHz	After adding 0.2 nH series L
S11	0.76<-87°	0.53<-76°
S21	2.2<94°	2.08<94°
S12	0.11<52°	0.13<52°
S22	0.54<-50°	0.57<-28°
F_{min}	1.5 dB	1.47 dB ⟵
Γ_{opt}	0.50<81°	0.40<92°
R_n	0.37(50)	0.26(50) ⟵
Gmsg	13 dB	12 dB ⟵

Note: gain went down, noise circles spread, F_{min} stays same

Find Γ_L and check gain

$$\Gamma_{in} = S_{11} + \left.\frac{S_{21}S_{12}\Gamma_L}{1 - S_{22}\Gamma_L}\right| = \Gamma_{opt}^{\ *}$$

Solve for Γ_L

$$\Gamma_L = \left.\frac{\Gamma_{in} - S_{11}}{-\Delta + S_{22}\Gamma_{in}}\right|_{\Gamma_{in} = \Gamma_{opt}^{\ *}} \quad =.40<-92^{\circ}$$

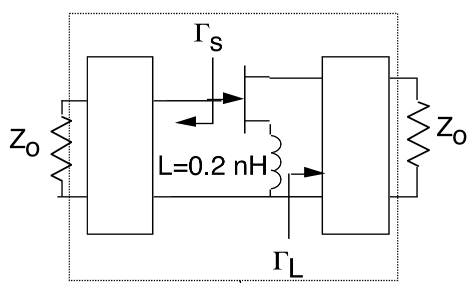
$$\Gamma_L = .58 < -22.7$$

Evaluate gain

$$G_p = \frac{1}{1 - |\Gamma_{in}|^2} |S_{21}|^2 \frac{1 - |\Gamma_L|^2}{|1 - S_{22}\Gamma_L|^2}$$

$$= \underline{\underline{7db}}$$

Final Results



$\Gamma_s = .40 < 92^{\circ}$

$\Gamma_L = .58 < -22.7$

- poor match at output
- good match at input
- good noise figure
- mediocre gain

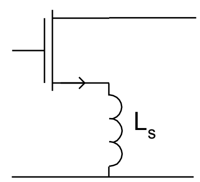
IS11I = .004 IS12I=.134

IS21I = 2.22 (6.9 dB) IS22I = .71

Fmin = 1.5 dB $\Gamma_s = .40 < 92^{\circ}$

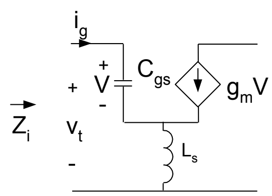
Inductive Source Degeneration in CMOS LNA

Adding inductance simplifies matching without reducing F



Input impedance

$$v_T = i_G \frac{1}{sC_{gs}} + sL_s\left(i_G + g_m V\right) \qquad V = \frac{i_G}{sC_{gs}}$$

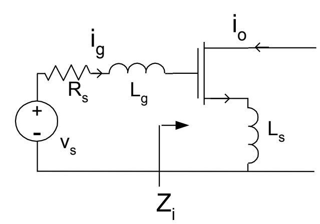


$$Z_{in} = \frac{v_T}{i_G} = \frac{1}{j\omega C_{gs}} + j\omega L_s + \frac{g_m L_s}{C_{gs}}$$

L_s set to make $\mathrm{Re}(Z_i)$ equal source resistance, R_s

L_g Added

L_g is added to resonate $\mathrm{Im}(Z_i)$



$$i_G = \frac{V_s}{R_S + j\omega L_g + Z_i} = \frac{V_s}{R_S + \frac{g_m L_s}{C_{gs}} + \cancel{j\left(\omega L_s + \omega L_g - \frac{1}{\omega C_{gs}}\right)}}$$

$$i_o = g_m V = g_m \frac{i_G}{j\omega C_{gs}} = \frac{g_m}{j\omega C_{gs}} \frac{V_s}{R_S + \frac{g_m L_s}{C_{gs}}}$$

$$i_o = \frac{g_m V_s}{j\omega C_{gs} 2R_S}$$

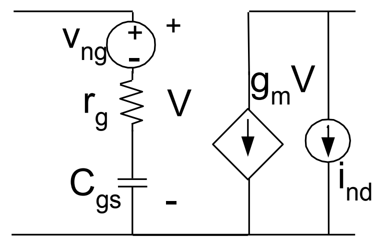
$$\left|i_o\right| = Q_{in} g_m V_s$$

The g_m of the circuit is enhanced by the input circuit Q

CMOS Noise Model

$$\overline{|i_{nd}|^2} = 4\kappa T\gamma g_{do}\Delta f$$

$$\overline{|v_{ng}|^2} = 4\kappa T\delta r_g \Delta f$$



$$C = \frac{\overline{v_{ng} i_{nd}^*}}{\sqrt{\overline{|v_{ng}|^2}\,\overline{|i_{nd}|^2}}}$$

For long channel CMOS

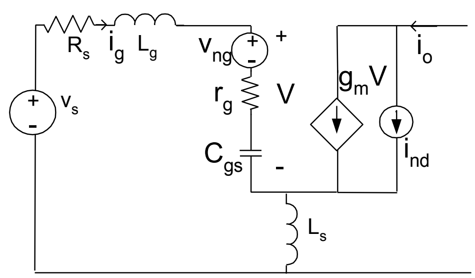
$$\delta \approx 4/3 \quad \gamma \approx 2/3 \quad C \approx -j(.39)$$

$$r_g \approx 1/5g_{do}$$

g_{do} is drain source conductance at $V_{ds} = V_{gs} - V_T$

$$g_{do} \approx g_m$$

Noise Figure Inductor Degenerated CMOS



To simplify algebra

$$V \approx \frac{i_g}{j\omega C_{gs}}$$

$$C \approx 0$$

$$\delta \approx 1$$

$$F = 1 + \frac{|i_o|^2_{v_s=0}}{|i_o|^2_{v_{ng}, i_{nd}=0}} = 1 + \frac{r_g}{R_s} + \frac{\omega^2}{\omega_T^2} R_s \gamma g_{do}$$

$$= 1 + \frac{1}{R_s 5 g_{do}} + \frac{\omega^2}{\omega_T^2} R_s \gamma g_{do}$$

$$\omega_T \equiv \frac{g_m}{C_{gs}}$$

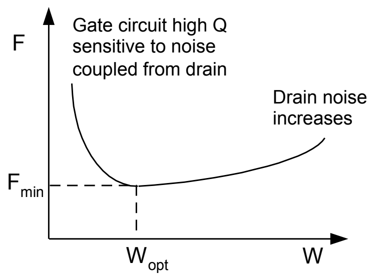
$$r_g \approx 1/5g_{do}$$

Derivation in HW solutions

Choosing width of device for F_{min}

$$F = 1 + \frac{1}{R_s 5 g_{do}} + \frac{\omega^2}{\omega_T^2} R_s \gamma g_{do} \qquad \omega_T \equiv \frac{g_m}{C_{gs}}$$

For a long channel device $\quad g_m = \frac{\partial i_d}{\partial v_{gs}} = \mu_n C_{ox} \frac{W}{L}(V_{GS} - V_T) \approx g_{do}$



$$(g_{do})_{min} = \frac{\omega_T}{\omega} \frac{1}{R_s \sqrt{5\gamma}}$$

$$F_{min} = 1 + \frac{\omega}{\omega_T} 2\sqrt{\frac{\gamma}{5}}$$

Example: $\omega/\omega_t = 0.25$, $R_s = 50$, $\gamma = 2$

$$(g_{do})_{min} = \frac{\omega_T}{\omega} \frac{1}{R_s \sqrt{5\gamma}} = .025\ \text{S} \qquad F_{min} = 1 + \frac{\omega}{\omega_T} 2\sqrt{\frac{\gamma}{5}} \rightarrow 1.1\text{dB}$$

For a long channel device

$$g_{do} = \mu_n C_{ox} \frac{W}{L}(\underline{V_{GS} - V_T})$$

μ_n: .04 m²/V-s $\quad C_{ox}$: .015 F/m² $\quad L$: .18 uM $\quad V_{OD}$ = .25 V

$\boxed{W_{opt} = 30\ \text{uM}}$

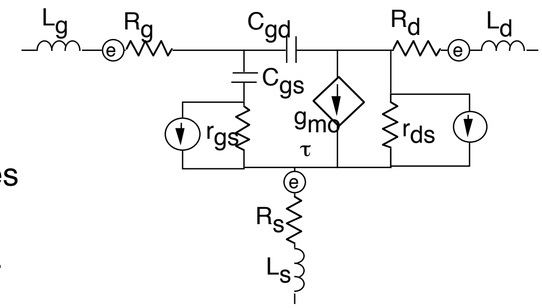
$$I_D = \frac{\mu_n C_{ox}}{2} \frac{W_{opt}}{L} V_{od}^2 = \underline{6\,\text{mA}}$$

Noise Models for Devices

Used to determine noise parameters for:

- different frequency
- different size devices

Used to diagnose device noise behavior



Noise sources not frequency dependent

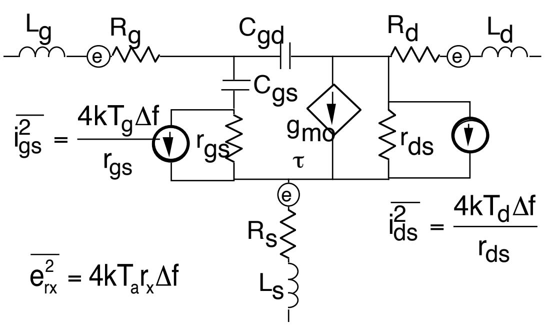
Noise sources depend on channel doping, device structure

Can be found by fitting to measurements or theoretically

FET Noise Model (Pospieszalski model)

r_{ds} and r_{gs} at temperatures, T_d and T_g

T_d and T_g set by noise measurements at one frequency

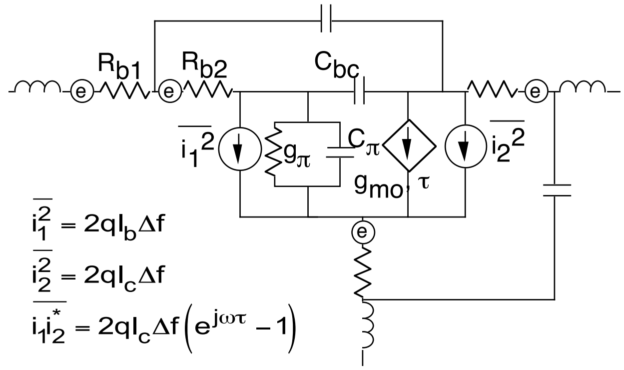


T_g usually near ambient, T_d is much higher (500-2000˚K)

Other resistors have a thermal noise source in series having a value $\overline{e_{rx}^2} = 4kT_ar_x\Delta f$ (T_a is ambient temp.)

HBT Noise Model

Model determined from physical characteristics - no fitting once circuit model is determined



$$\overline{i_1^2} = 2qI_b\Delta f$$

$$\overline{i_2^2} = 2qI_c\Delta f$$

$$\overline{i_1 i_2^*} = 2qI_c\Delta f\left(e^{j\omega\tau} - 1\right)$$

Each resistor has a thermal noise source in series with it having a value $\overline{|e_R|^2} = 4kTR\Delta f$

See Also Rudolph et. al. IEEE E.D. Letters Jan. 1999

Summary

- Transforming noise parameters
- Source inductance in LNA design
- Noise models for FET, HBT

References on CAD of Noisy Circuits

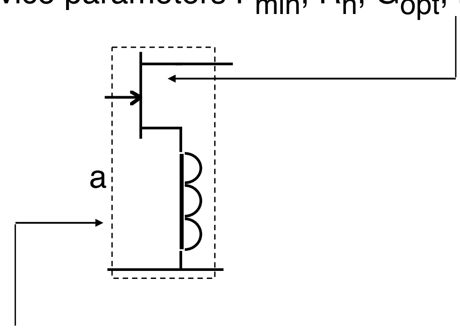
"An Efficient Method for Computer Aided Analysis of Linear Amplifier Networks," H. Hillbrand and P. H. Russer, IEEE Trans. on Circuits and Systems, April 1976

Computationally Efficient Electronic-Circuit Noise Calculations," R. Rohrer, L. Nagel, R. Meyer, and L, Weber, IEEE Journal of Solid State Circuits, August 1971

Supplemental Material

Transforming Noise Parameters

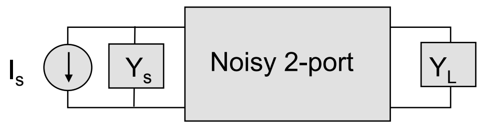
Given the device parameters F_{min}, R_n, G_{opt}, B_{opt}



Find the parameters F^a_{min}, R^a_n, G^a_{opt}, B^a_{opt}

To find the noise parameters for the “a” circuit from the device parameters use thevenin, norton sources

Reminder

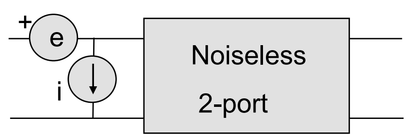


$F = F_{min} + (R_n / G_s)|Y_s - Y_{opt}|^2$

The *noise parameters ,* F_{min} , R_n, G_{opt}, B_{opt} can be derived from the four numbers,

$$\overline{|e|^2}, \overline{|i|^2}, \overline{ei^*}$$

Noise sources needed for combining noisy circuits



First go from noise parameters to equivalent sources

$$\begin{Bmatrix} F_{min} \\ R_n \\ Y_{opt} \end{Bmatrix} \Rightarrow \begin{Bmatrix} \overline{|e|^2} \\ \overline{|i|^2} \\ C \end{Bmatrix} \quad \text{where} \quad C = C_r + jC_i \equiv \frac{\overline{ei^*}}{\sqrt{\overline{|e|^2}}\sqrt{\overline{|i|^2}}}$$

Transformations

Sources ⇨ Noise Param. Noise Param. ⇨ Sources

$$R_n = \frac{\overline{|e|^2}}{4KT\Delta f}$$

$$Y_{opt} = \frac{\sqrt{\overline{|i|^2}}}{\sqrt{\overline{|e|^2}}}\left[\sqrt{1-C_i^2} + jC_i\right]$$

$$F_{min} = \frac{\sqrt{\overline{|i|^2}\,\overline{|e|^2}}}{2kT\Delta f}\left(\sqrt{1-C_i^2} + C_r\right) + 1$$

$$\overline{|e|^2} = 4KT\Delta f R_n$$

$$\overline{|i|^2} = 4KT\Delta f R_n |Y_{opt}|^2$$

$$C_i = \mathrm{Im}\left[Y_{opt}\right] / |Y_{opt}|$$

$$C_r = \frac{1}{|Y_{opt}|}\left[\frac{F_{min} - 1}{2R_n} - \mathrm{Re}\left(Y_{opt}\right)\right]$$

Where do transformations come from?

$$R_n = \frac{\overline{|e|^2}}{4KT\Delta f} \quad \Rightarrow \quad \underline{\overline{|e|^2} = 4KT\Delta f R_n}$$

$$Y_{opt} = \frac{\sqrt{\overline{|i|^2}}}{\sqrt{\overline{|e|^2}}}\left[\sqrt{1-C_i^2} + jC_i\right] \quad \Rightarrow \quad \overline{|i|^2} = |Y_{opt}|^2\,\overline{|e|^2}$$

$$\underline{= 4KT\Delta f R_n |Y_{opt}|^2}$$

$$\Rightarrow C_i = \sqrt{\frac{\overline{|e|^2}}{\overline{|i|^2}}}\,\mathrm{Im}\left[Y_{opt}\right]$$

$$\underline{= \mathrm{Im}\left[Y_{opt}\right] / |Y_{opt}|}$$

Algebra for Transformation (cont.)

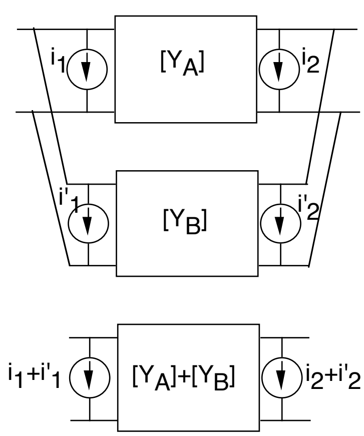
$$F_{min} = \frac{\sqrt{\overline{|i|^2}\,\overline{|e|^2}}}{2kT\Delta f}\left(\sqrt{1-C_i^2}+C_r\right)+1 \;\Rightarrow\; C_r = \left(F_{min}-1\right)\frac{2kT\Delta f}{\sqrt{\overline{|i|^2}\,\overline{|e|^2}}}-\sqrt{1-C_i^2}$$

$$= \frac{F_{min}-1}{2R_n\left|Y_{opt}\right|}-\sqrt{1-C_i^2}$$

$$\underline{C_r = \frac{1}{\left|Y_{opt}\right|}\left[\frac{F_{min}-1}{2R_n}-\mathrm{Re}\left(Y_{opt}\right)\right]}$$

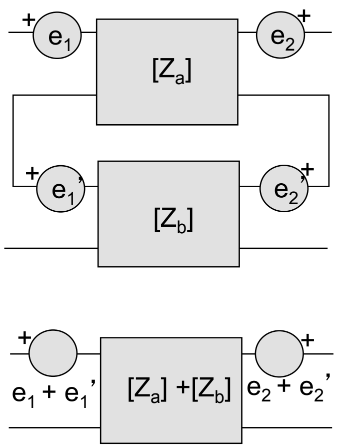
$$\sqrt{1-C_i^2} = \sqrt{1-\left(\frac{\mathrm{Im}\left(Y_{opt}\right)}{\left|Y_{opt}\right|}\right)^2} = \frac{1}{\left|Y_{opt}\right|}\sqrt{\left|Y_{opt}\right|^2-\mathrm{Im}\left(Y_{opt}\right)^2} = \frac{\mathrm{Re}\left(Y_{opt}\right)}{\left|Y_{opt}\right|}$$

Combined Noise Sources of Shunt Connected Circuit



Norton equivalent noise sources make shunt combinations easy

Combined Noise Sources of Series Connected Circuit

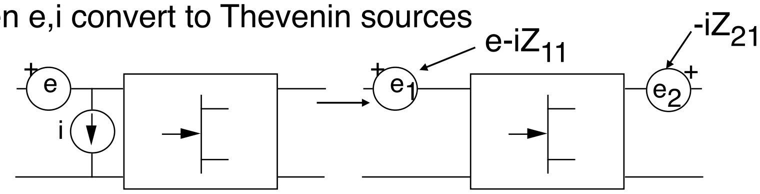


Thevenin equivalents make series combinations easy

(Thevenin-Norton sources are appropriate for cascaded connections of noisy 2-ports)

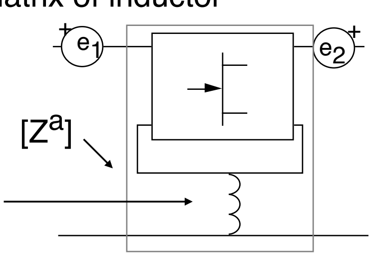
Example: Effect of Series Feedback Inductance

•Given e,i convert to Thevenin sources



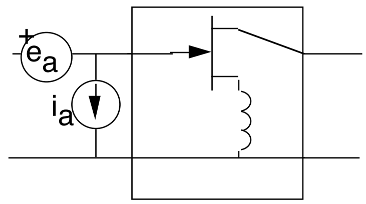
•sum Z matrix of FET to Z matrix of inductor

$$\left[Z^a\right] = \left[Z\ \right] + j\omega L \begin{bmatrix} 1 & 1 \\ 1 & 1 \end{bmatrix}$$



lossless inductor adds no noise

Convert Back to Thevenin-Norton



Solve for e_a, i_a i/t/o e_1, e_2 from last slide

Replace e_1, e_2 with device noise sources e, i

$$\left.\begin{aligned} e_a - i_a z^a_{11} &= e_1 \\ -i_a z^a_{11} &= e_2 \end{aligned}\right\} \rightarrow \left\{\begin{aligned} e_a &= e_1 + \frac{-e_2}{z^a_{21}} z^a_{11} \\ i_a &= \frac{-e_2}{z^a_{21}} \end{aligned}\right\} \rightarrow \left\{\begin{aligned} e_a &= e - i z_{11} + \frac{i z_{21}}{z^a_{21}} z_{11} \\ i_a &= \frac{i z_{21}}{z^a_{21}} \end{aligned}\right\}$$

Noise Transformation Matrix

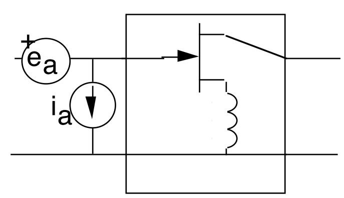
$$e_a = e + \left[-z_{11} + \frac{z^a_{11}}{z^a_{21}}\right] i$$

$$i_a = \frac{z_{21}}{z^a_{21}} i$$

$$\downarrow$$

$$\begin{bmatrix} e_a \\ i_a \end{bmatrix} = \underline{\begin{bmatrix} n_{11} & n_{12} \\ n_{21} & n_{22} \end{bmatrix}} \begin{bmatrix} e \\ i \end{bmatrix}$$

noise transformation matrix



$n_{11} = 1$

$$n_{12} = -z_{11} + \frac{z_{11} + j\omega L}{z_{21} + j\omega L}$$

$n_{21} = 0$

$$n_{22} = \frac{z_{21}}{z_{21} + j\omega L}$$

Noise Transformation Matrix

In this example, the circuit (inductor) added was lossless.

If the feedback circuit was lossy, the transformation matrix would look like,

$$\begin{bmatrix} e_a \\ i_a \end{bmatrix} = \begin{bmatrix} n_{11} & n_{12} \\ n_{21} & n_{22} \end{bmatrix} \begin{bmatrix} e \\ i \end{bmatrix} + \begin{bmatrix} n_{11}' & n_{12}' \\ n_{21}' & n_{22}' \end{bmatrix} \begin{bmatrix} e_n \\ i_n \end{bmatrix}$$

due to noise in feedback circuit

Calculate Noise Amplitudes &Correlation

$$\overline{|e^a|^2} = \overline{|n_{11}e + n_{12}i|^2}$$

$$= |n_{11}|^2\overline{|e|^2} + \overline{n_{11}e(n_{12}i)^*} + \overline{n_{12}i\,(n_{11}e)^*} + |n_{12}|^2\overline{|i|^2}$$

$$= |n_{11}|^2\overline{|e|^2} + 2\mathrm{Re}\left[n_{11}n_{12}{}^*\,C\sqrt{\overline{|e|^2}}\sqrt{\overline{|i|^2}}\right] + |n_{12}|^2\overline{|i|^2}$$

$$\overline{|i^a|^2} = \overline{|n_{21}e + n_{22}i|^2}$$

$$= |n_{21}|^2\overline{|e|^2} + 2\mathrm{Re}\left[n_{21}n_{22}{}^*\,C\sqrt{\overline{|e|^2}}\sqrt{\overline{|i|^2}}\right] + |n_{22}|^2\overline{|i|^2}$$

$$C^a = \frac{1}{\sqrt{\overline{|e^a|^2}}\sqrt{\overline{|i^a|^2}}}\,\overline{e^a i^{a\,*}} = \frac{1}{\sqrt{\overline{|e^a|^2}}\sqrt{\overline{|i^a|^2}}}\,\underbrace{\overline{(n_{11}e + n_{12}i)(n_{21}e + n_{22}i)^*}}$$

$$= n_{11}n_{21}{}^*\,\overline{|e|^2} + n_{12}n_{21}{}^*\,\overline{ie^*} + n_{11}n_{22}{}^*\,\overline{ei^*} + n_{12}n_{22}{}^*\,\overline{|i|^2}$$

Finally, Calculate New Noise Parameters

$$Y^a_{opt} = \frac{\sqrt{\overline{\left|i^a\right|^2}}}{\sqrt{\overline{\left|e^a\right|^2}}}\left[\sqrt{1-\left(C^a_i\right)^2} + jC^a_i\right]$$

$$F^a_{min} = \frac{\sqrt{\overline{\left|i^a\right|^2}\,\overline{\left|e^a\right|^2}}}{2kT\Delta f}\left[\sqrt{1-\left(C^a_i\right)^2} + C^a_r\right] + 1$$

$$R^a_n = \frac{\overline{\left|e^a\right|^2}}{4kT\Delta f}$$

Summary

- Start with F_{min}, R_n, G_{opt}, B_{opt} of FET
- Find $\overline{|e|^2}$, $\overline{|i|^2}, C$ from formulas
- Find noise transformation matrix for circuit under study

$$\begin{Bmatrix} e \\ i \end{Bmatrix} \rightarrow \begin{Bmatrix} e^a \\ i^a \end{Bmatrix}$$

- Form $\overline{\left|e^a\right|^2}$, $\overline{\left|i^a\right|^2}, C^a$
- Plug into formulas for F^a_{min}, R^a_n, G^a_{opt} B^a_{opt}

(usually done by CAD)