第七次作业

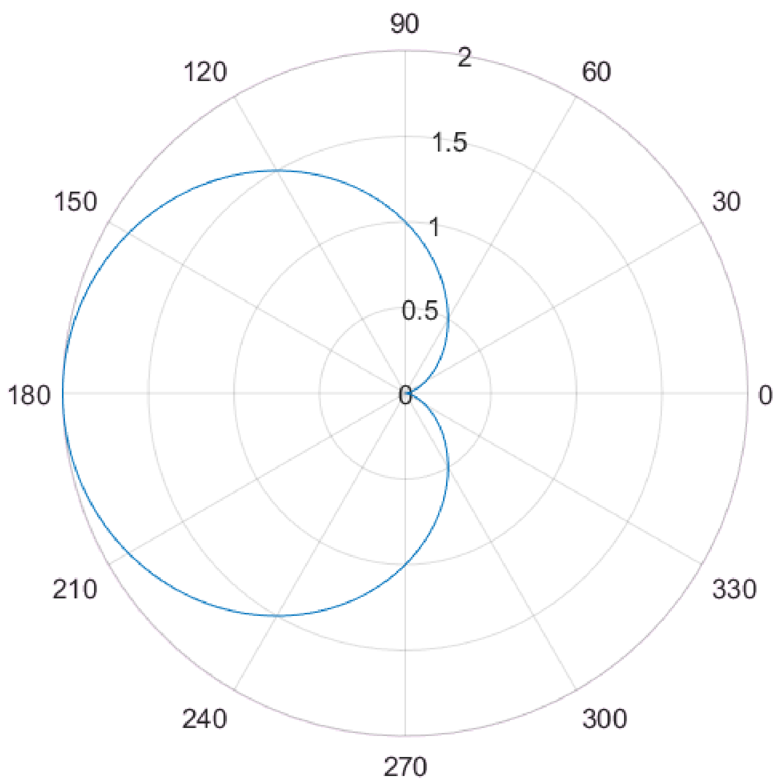
心形线（选做）

用polarplot函数作图。面积为

$$A = \int_0^{2\pi} \frac{1}{2} r \cdot r \mathrm{d}r.$$

注意面积微元是一个腰为r，底为$r\mathrm{d}r$ 的三角形。

```
dlm = 0.01*pi;
t = 0:dlm:2*pi;
r = 1 - cos(t);
figure; polarplot(t, r);
```




```
area = trapz(r.^2/2)*dlm
```

```
area =
   4.712388980384692
```

三个积分

第1个积分

```
integral3(@(x,y,z)x.*y.*z,1,2,@(x)x,@(x)2*x,@(x,y)x.*y,@(x,y)2*x.*y)
```

```
ans = 179.2969
```

```
integral(@(x)arrayfun(@(x)integral2(@(y,z)x.*y.*z,x,2*x,@(y)x.*y,@(y)2.*x.*y),x),1,2)
```

```
ans = 179.2969
```

```
integral(@(x)arrayfun(@(x)integral(@(y)arrayfun(@(y)integral(@(z)x.*y.*z,x.*y,2*x.*y),y),x,2*x
```

```
ans = 179.2969
```

第2个积分

```
integral(@(x)1./arrayfun(@(x)integral(@(y)y,x,x.^2),x),10,100)
```

```
ans = 6.7003e-04
```

第3个积分

```
integral(@(y)2*y.*exp(-y.^2).*arrayfun(@(y)integral(@(x)exp(-x.^2)./(x.^2+y.^2),-1,1),y).^2,0.
```

```
ans = 10.2135
```

1 符号和数值积分

符号法

```
syms x; int(x^4*(1-x)^4/(1+x^2), x, 0, 1)
```

ans =

$$\frac{22}{7} - \pi$$

数值法

```
X=0:0.1:1; trapz(X,X.^4.*(1-X).^4./(1+X.^2))
```

```
ans = 0.0013
```

```
integral(@(x)x.^4.*(1-x).^4./(1+x.^2), 0, 1)
```

```
ans = 0.0013
```

2 误差函数

```
format long; myerf = @(x)2/sqrt(pi)*integral(@(xi)exp(-xi.^2), 0, x); X=0.1:0.1:1.0; arrayfun(
```

```
ans =
   0.112462916018285   0.222702589210478   0.328626759459127   0.428392355046668   0.520499877813047   0
ans =
   0.112462916018285   0.222702589210478   0.328626759459127   0.428392355046668   0.520499877813047   0
```

3 室温下氢分子运动速率的 Maxwell 分布

以 v_p 为速度量纲做无量纲化运算。以 x 代表 $\frac{v}{v_p}$

```
f=@(x)4/sqrt(pi)*x.^2.*exp(-x.^2); integral(f,0,1),  integral(f,0,3.3),  integral(f,3e4/1578,3
```

```
ans =
   0.427593295529120
ans =
   0.999927519495740
ans =
    9.367342245784810e-157
```

4 均匀带电圆环的电场

如果绘制二维图,先计算电势，有:

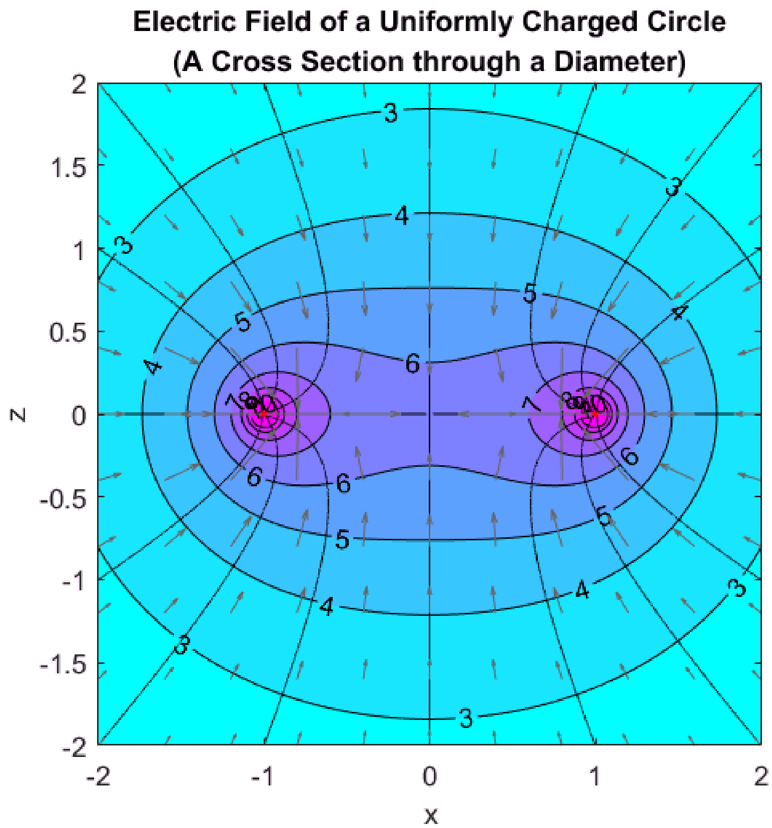
$$V(\vec{r}) = \oint_{C_a} \frac{\mathrm{d}q}{4\pi\varepsilon_0|\vec{r} - \vec{r}_{\mathrm{d}q}|}$$

令 $a = 1, q = 2\pi, y = 0$ ，则为

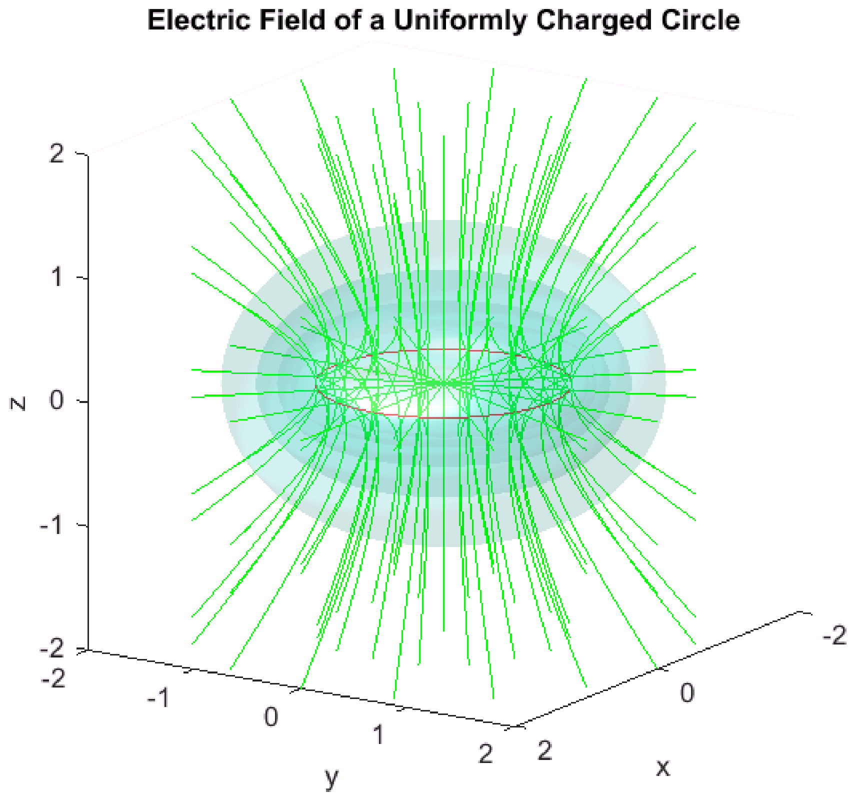
$$\int_0^{2\pi} \frac{\mathrm{d}\phi}{(x - \cos\phi)^2 + \sin^2\phi + z^2};$$

再用 gradient 即可计算出电场强度。

```
[x,z] = meshgrid(-2:.02:2);
phi = 0:0.1*pi:2*pi;
V = @(x,z)trapz(phi,1./sqrt((x-cos(phi)).^2+(sin(phi)).^2+z.^2));
Vdata = reshape(arrayfun(V, x, z), size(x));
[Ex, Ez] = gradient(Vdata);
figure; c = contourf(x, z, Vdata, 2:1:10, 'showtext', 'on'); colormap('cool');
hold on;
sl_left_bottom = streamline(x,z,Ex,Ez,[-2:1:2, -2*ones([1,5]), -.02], [-2*ones([1,5]), -2:1:2,
set(sl_left_bottom, 'Color', 'k');
sl_right_top = copyobj(sl_left_bottom, gca);
rotate(sl_left_bottom, [0,0,1], 180);
quiver(x(1:20:end,1:20:end), z(1:20:end,1:20:end), Ex(1:20:end,1:20:end), Ez(1:20:end,1:20:enc
plot([-1, 1], [0, 0], 'r.');
axis equal; xlabel('x'); ylabel('z');
title({'Electric Field of a Uniformly Charged Circle', '(A Cross Section through a Diameter)'}
```




```
figure; hold on;
s1 = copyobj(sl_left_bottom, gca); s2 = copyobj(sl_right_top, gca);
s0 = allchild(gca); s = copyobj(s0, gca); delete(s0);
rotate(s, [1,0,0], 90, [0,0,0]); set(s, 'Color', 'g');
for k = 1:4
    ax = copyobj(allchild(gca),gca);
    rotate(ax, [0,0,1], 11.25*2^k, [0,0,0]);
end
c = contourdata(c);
phi = linspace(0, pi);
for k = 3:length(c)
    rho = c(k).xdata; z = c(k).ydata;
    [~,Z] = meshgrid(phi,z); [Phi,R] = meshgrid(phi,rho);
    [x,y,z] = pol2cart(Phi,R,Z);
    patch(surf2patch(x,y,z,ones(size(x))), 'FaceAlpha', .1); shading interp; light;
end
plot(exp(1i*[phi, pi + phi]),'r-','LineWidth',.1);
view([3,2,1]); axis equal;
axis equal; xlabel('x'); ylabel('y'); zlabel('z');
title('Electric Field of a Uniformly Charged Circle');
```



以圆环所在平面为 xOy 坐标面，以环心为原点，则空间任意一点 $\mathrm{d}q$ 的电场强度为:

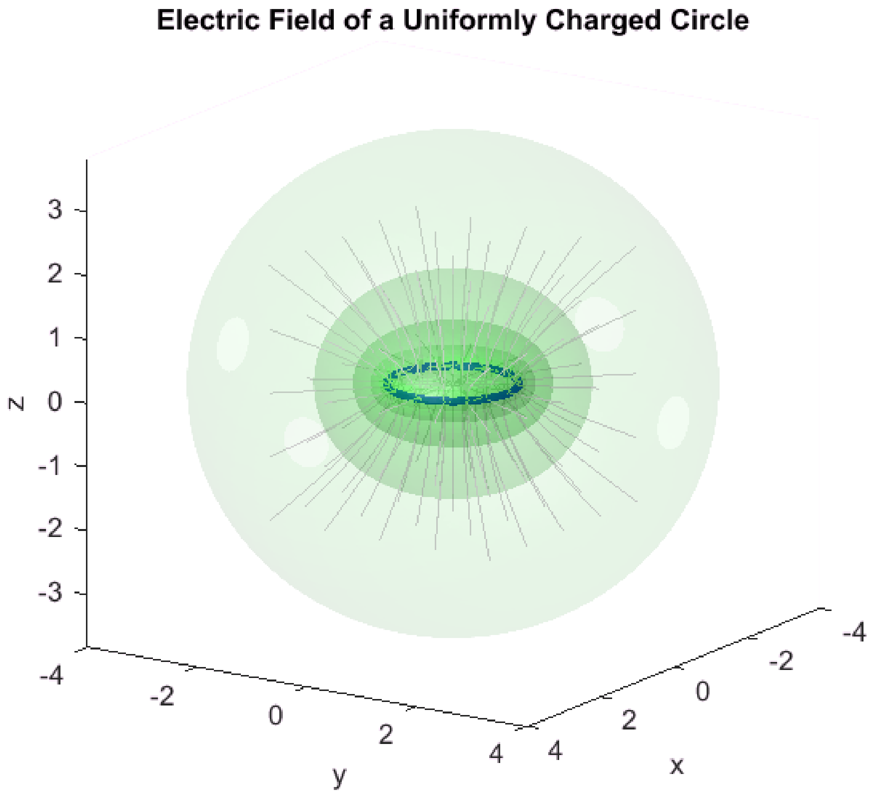
$$\vec{E}_p = \oint_{C_a} \frac{(\vec{r_P} - \overrightarrow{r_{\mathrm{d}Q}})\mathrm{d}(4\pi\varepsilon_0 q)}{4\pi\varepsilon_0 |\vec{r_P} - \overrightarrow{r_{\mathrm{d}Q}}|^3}$$

$$= \frac{q}{2\pi} \oint_{C_a} \frac{(\vec{r_P} - \overrightarrow{r_{\mathrm{d}Q}})}{|\vec{r_P} - \overrightarrow{r_{\mathrm{d}Q}}|^3} \mathrm{d}\phi$$

$$= \frac{q}{2\pi} \int_0^{2\pi} \frac{(x - a\cos\phi, y - a\sin\phi, z)}{\left[(x - a\cos\phi)^2 + (y - a\sin\phi)^2 + z^2\right]^{\frac{3}{2}}} \mathrm{d}\phi$$

```
[x,y,z]=meshgrid(-4:.1:4);
V = zeros(81, 81, 81, 100);
for k = 1:100
    V(:,:,:,k) = 1./sqrt((x-cos((2*pi+.05)/100*k)).^2+(y-sin((2*pi+.05)/100*k)).^2+z.^2);
end
V = trapz(V, 4);
[Ex,Ey,Ez] = gradient(V, x(1,2,1)-x(1,1,1));
figure; hold on;
    [startx,starty] = meshgrid(-2:1:2); startz = -2*ones(size(startx));
    sSide = streamline(x,y,z,Ex,Ey,Ez,startx,startz,starty);
    rotate(copyobj(allchild(gca), gca), [0,0,1], 180, [0,0,0]);
    rotate(copyobj(allchild(gca), gca), [0,0,1], 90, [0,0,0]);
    sBottom = streamline(x,y,z,Ex,Ey,Ez,startx,starty,startz);
    sTop = copyobj(sBottom, gca);  rotate(sTop, [0,1,0], 180, [0,0,0]);
    [startx, starty] = deal(.1*cos(linspace(0, 2*pi, 16)), .1*sin(linspace(0, 2*pi, 16))); sta
    streamline(x,y,z,Ex,Ey,Ez,startx,starty,startz);
```

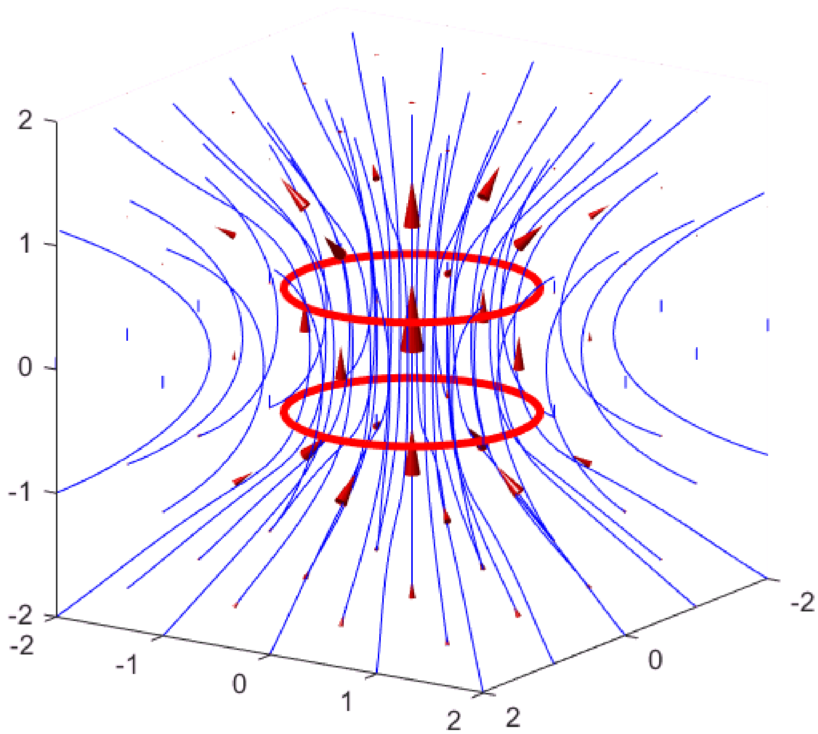
```
    set(allchild(gca), 'color', [.8,.8,.8]);
    t = 0:pi/20:2*pi; plot(exp(1i*t),'b-','LineWidth',3);
    for k = 1:5
        patch(isosurface(x,y,z,V,25*k),'FaceAlpha',0.05*k,'LineStyle','none','FaceColor','gree
    end
    light;
    view([3,2,1]); axis equal;
axis equal; xlabel('x'); ylabel('y'); zlabel('z');
title('Electric Field of a Uniformly Charged Circle');
```



5 Helmholtz

绘图

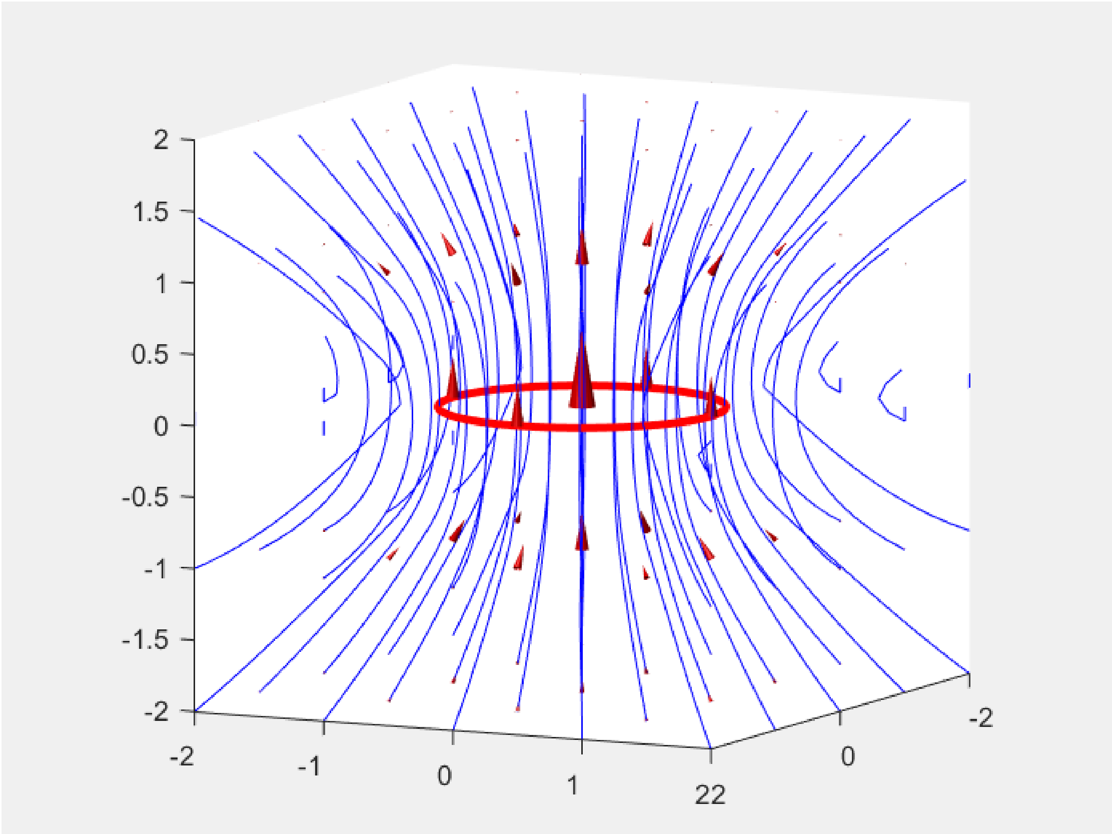
```
figure; helmholtzfun(1,1,1);
axis equal; axis([-2,2,-2,2,-2,2]); view([3,2,1]);
```



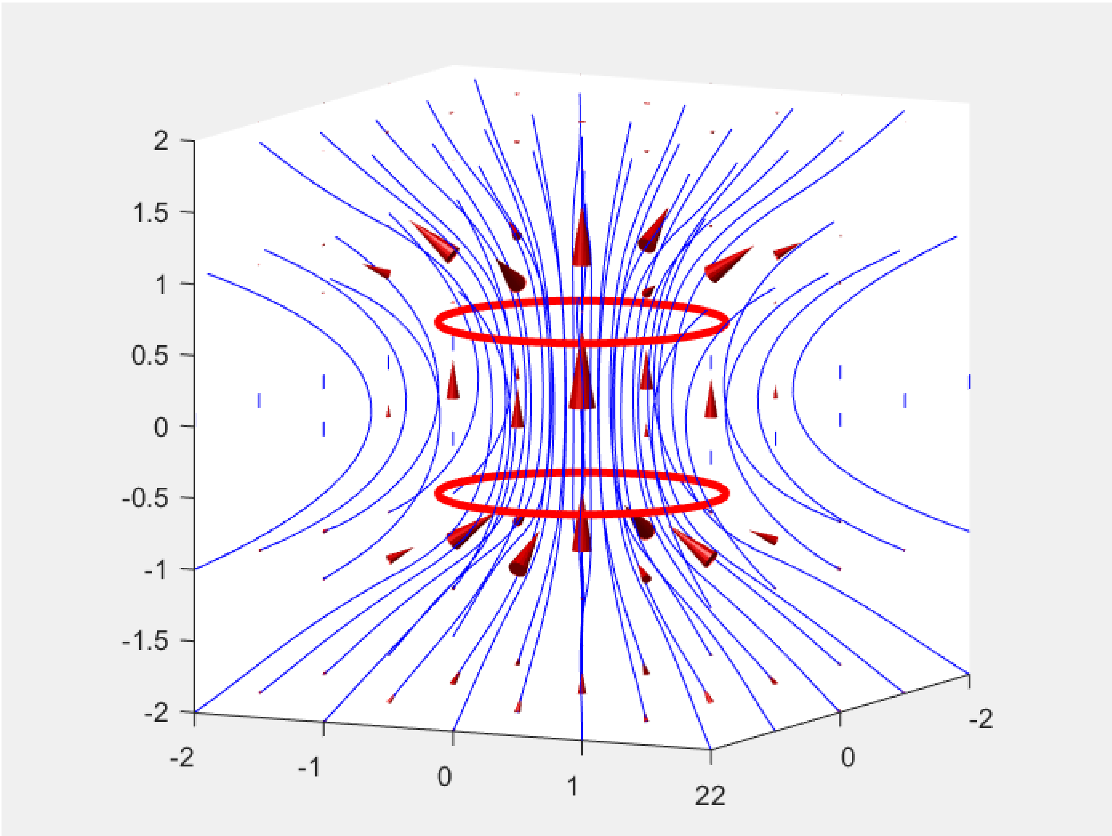

```

```

变化距离

```
hscale = 0:0.1:1.2;
figure('visible', 'on');
```

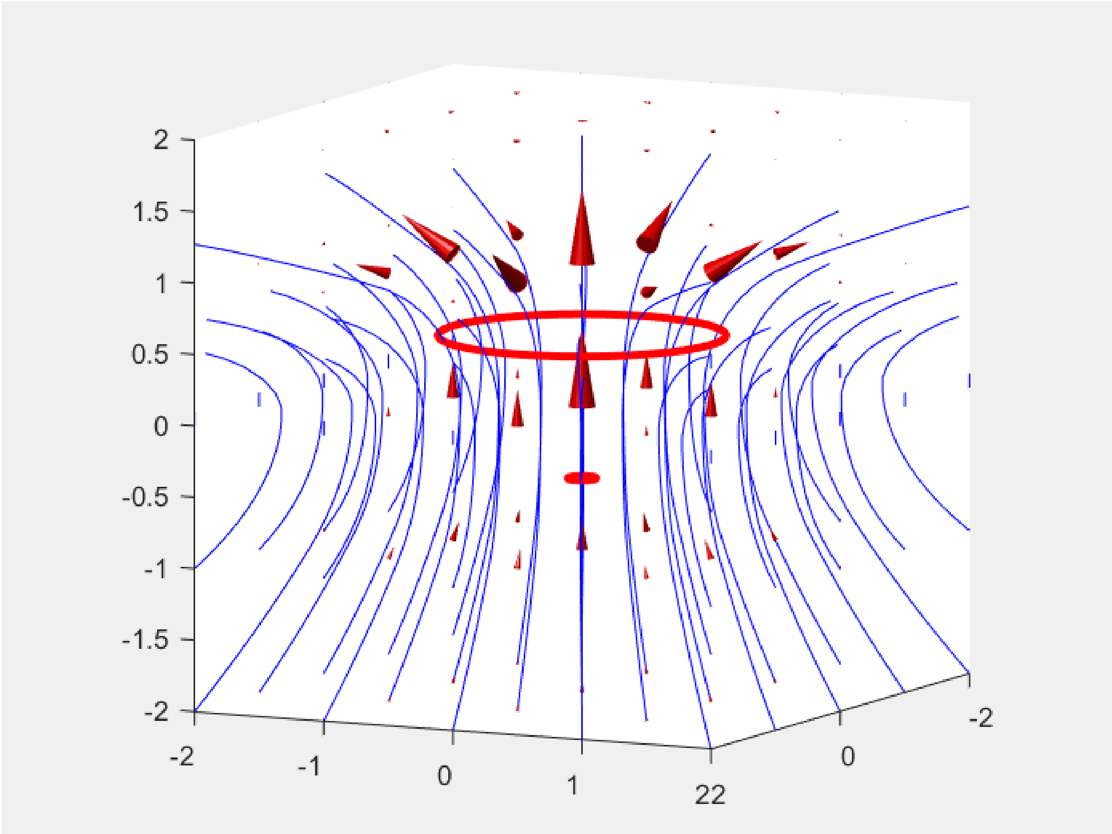



```
for k = 1:length(hscale)
    helmholtzfun(1,1,hscale(k));
    axis equal; axis([-2,2,-2,2,-2,2]);
    view([6,3,1]); getframe;
end
```

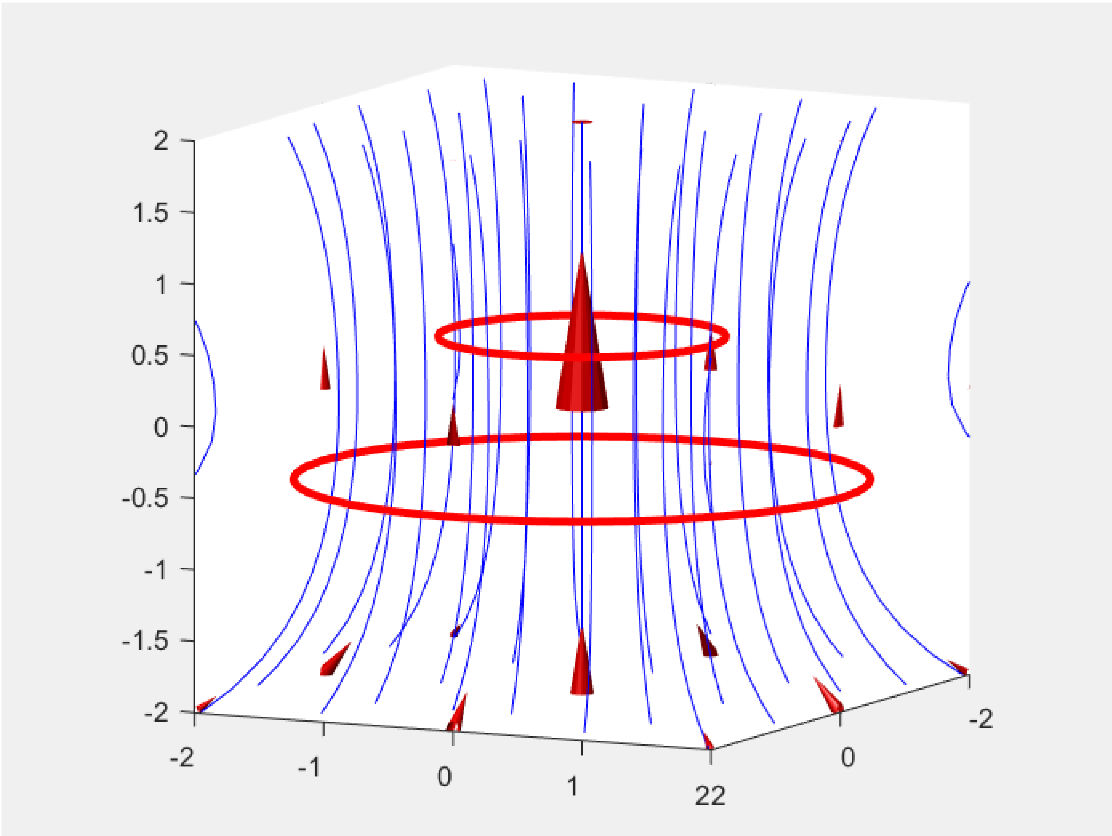


变化下环半径

```
Rdscale = .1:.1:2;
figure('visible', 'on');
```




```
for k = 1:length(Rdscale)
    helmholtzfun(1,Rdscale(k),1);
    axis equal; axis([-2,2,-2,2,-2,2]);
    view([6,3,1]); getframe;
end
```



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