Reconstruction of Weak Gravitational Lensing with Galaxy Survey

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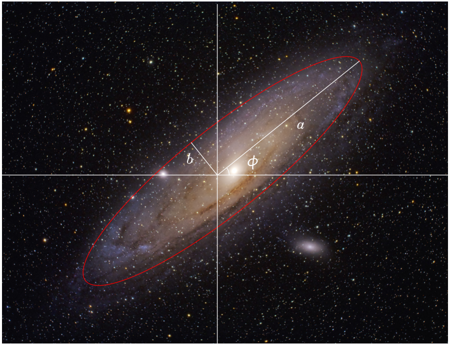
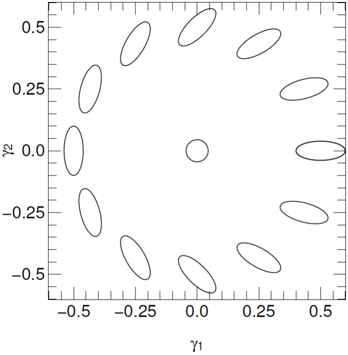


Galaxy ellpiticity:

$$\varepsilon_s = \frac{a-b}{a+b}\exp(2i\phi) = \varepsilon_{s1} + i\varepsilon_{s2}$$



Observed ellipticity and Weak lensing approximation (Seitz & Schneider, 1997):

$$\varepsilon = \frac{\varepsilon_s + \mathrm{g}}{1 + \mathrm{g}^* \varepsilon_s} \approx \varepsilon_s + \gamma$$

Distribution of instrinsic ellpiticity (Bartelmann & Narayan, 1995):

$$p_e(\varepsilon_s) = \frac{\exp(-|\varepsilon_s|^2/\sigma_\varepsilon^2)}{\pi\sigma_\varepsilon^2[1-\exp(-1/\sigma_\varepsilon^2)]}$$

$\sigma_\varepsilon \approx 0.2$(e.g. Miralda-Escudé 1991b; Tyson & Seitzer 1988; Brainerd et al. 1996)
Galaxy number density distribution on redshift from CS82(Shan et al. 2014):

$$p_z(z) \propto \frac{z^a + z^{ab}}{z^b + c}$$

Where $a = 0.531, b = 7.810, c = 0.517.$

Angular Power Spectrum

Get 3 angular power spectrum from CAMB

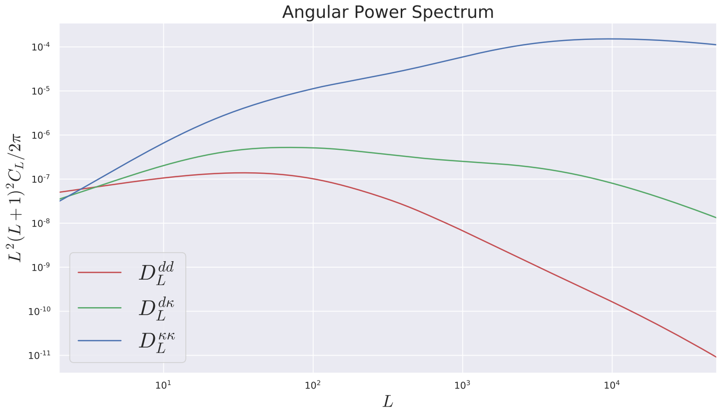
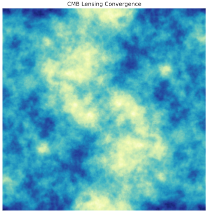
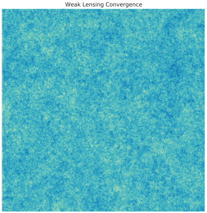


Figure: CMB Lensing deflection angle and Weak Lensing convergence and cross

Weak Lensing Convergence





$$C_L^{\phi\phi} = \frac{L(L+1)}{4} C_L^{dd}$$

$$\kappa_\phi(\boldsymbol{\ell}) = \zeta_1 (C_L^{\phi\phi})^{1/2}$$

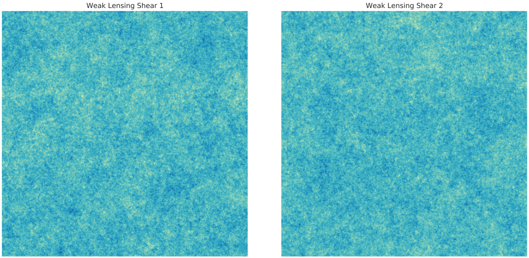
$$\kappa(\boldsymbol{\ell}) = \zeta_1(\boldsymbol{\ell}) \frac{C_L^{\kappa\phi}}{(C_L^{\phi\phi})^{1/2}} + \zeta_2(\boldsymbol{\ell}) \left[C_L^{\kappa\kappa} - \frac{(C_L^{\kappa\phi})^2}{C_L^{\phi\phi}} \right]^{1/2}$$

κ_ϕ :convergence of CMB lensing

κ : convergence of weak lensing

$\zeta_{1,2}$ are two complex numbers drawn from a Gaussian distribution with unit variance (Kamionkowski et al.1997).

Weak Lensing shear



The Fourier transform $\gamma(\boldsymbol{\ell})$ of the shear is related to $\kappa(\boldsymbol{\ell})$ through (Schneider et al.2002):

$$\gamma(\boldsymbol{\ell}) = \left(\frac{\ell_1^2 - \ell_2^2 + 2i\ell_1\ell_2}{|\boldsymbol{\ell}|^2} \right) \kappa_g(\boldsymbol{\ell}) = \kappa_g(\ell) e^{2i\beta}$$

Where β is the polar angle of $\boldsymbol{\ell}$

Define tangential shear and cross shear:

$$\gamma_t = -\mathcal{R}e[\gamma \exp(-2i\phi_\theta)]$$
$$\gamma_\times = -\mathcal{I}m[\gamma \exp(-2i\phi_\theta)]$$

Where ϕ_θ is the polar angle of the separation vector $\boldsymbol{\vartheta}$ Define correlation function ξ_+ and ξ_- (Schneider et al.2002):

$$\xi_+(\vartheta) = \langle\gamma_t\gamma_t\rangle + \langle\gamma_\times\gamma_\times\rangle = \langle\kappa\kappa\rangle(\vartheta)$$
$$\xi_-(\vartheta) = \langle\gamma_t\gamma_t\rangle - \langle\gamma_\times\gamma_\times\rangle = \mathcal{R}e[\langle\gamma\gamma\rangle e^{-4i\phi_\theta}](\vartheta)$$

Where $\vartheta = |\boldsymbol{\vartheta}|$, ϕ_θ is the polar angle of the separation vector $\boldsymbol{\vartheta}$. $\xi_\pm$ can be computed as following 1D Bessel integral:

$$\xi_\pm(\vartheta) = \int_0^\infty \frac{\ell \mathrm{d}\ell}{2\pi} J_{0,4}(\ell\vartheta) C_\ell^{\kappa\kappa}$$

Correlation Function

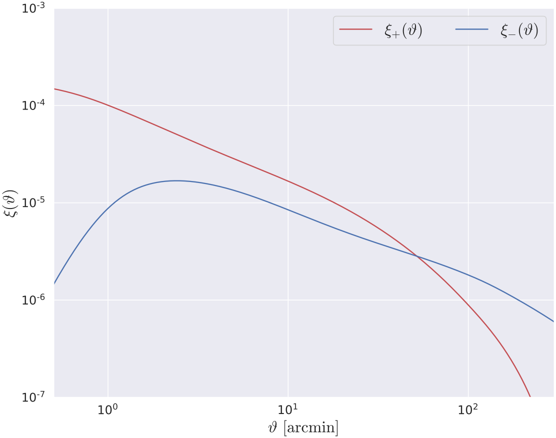


Figure: correlation function computed by convergence power spectrum

Shear Reconstruction and Noise

Lensing effect on galaxy ellipticity:

$$\varepsilon(\boldsymbol{\theta}) = \varepsilon_s(\boldsymbol{\theta}) + \gamma(\boldsymbol{\theta})$$

Ensemble average of lensed ellipticity:

$$\begin{aligned}\langle\varepsilon(\boldsymbol{\theta})\varepsilon^*(\boldsymbol{\theta}')\rangle &= \langle[\varepsilon_s(\boldsymbol{\theta}) + \gamma(\boldsymbol{\theta})][\varepsilon_s(\boldsymbol{\theta}') + \gamma(\boldsymbol{\theta}')]^*\rangle \\ &= \langle\gamma(\boldsymbol{\theta})\gamma^*(\boldsymbol{\theta}')\rangle + \text{Noise}\end{aligned}$$

An estimator of the $\xi_\pm$ (Schneider et al. 2002a) is:

$$\hat{\xi}_\pm(\vartheta) = \frac{\sum_{ij}\omega_i\omega_j(\varepsilon_{t,i}\varepsilon_{t,j} \pm \varepsilon_{\times,i}\varepsilon_{\times,i})}{\sum_{ij}\omega_i\omega_j}$$

Shear Reconstruction and Noise

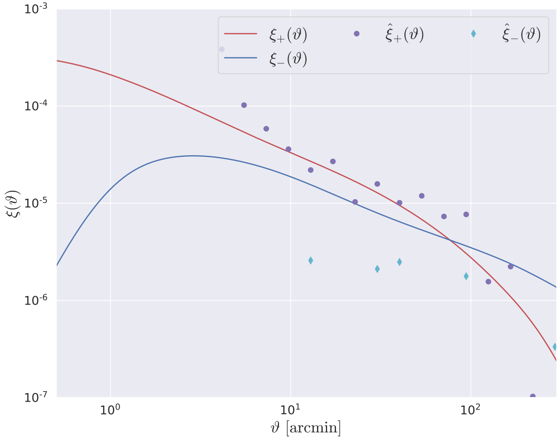


Figure: ξ_+ and ξ_- computed by TreeCorr

Two-point ensemble average of lensed ellipticity in Fourier space:

$$\langle \varepsilon(\boldsymbol{\ell})\varepsilon^*(\boldsymbol{\ell}')\rangle = (2\pi)^2 \delta_D(\boldsymbol{\ell} - \boldsymbol{\ell}')\,\hat{C}_\ell^{\kappa\kappa}$$

Estimation of power spectrum

$$\hat{C}_\ell^{\kappa\kappa} = 2\pi \int_0^\infty \vartheta \mathrm{d}\vartheta\, J_{0,4}(\ell\vartheta)\xi_\pm(\vartheta)$$

Ellipticity Noise in the Gaussian approximation:

$$\Delta C_\ell^{\kappa\kappa} = \frac{1}{(2\ell+1)f_{\rm sky}}\left(\frac{\sigma_\varepsilon^2}{2n} + C_\ell^{\kappa\kappa}\right)$$

THANKS!