



Reconstruction of Weak Gravitational Lensing with Galaxy Survey

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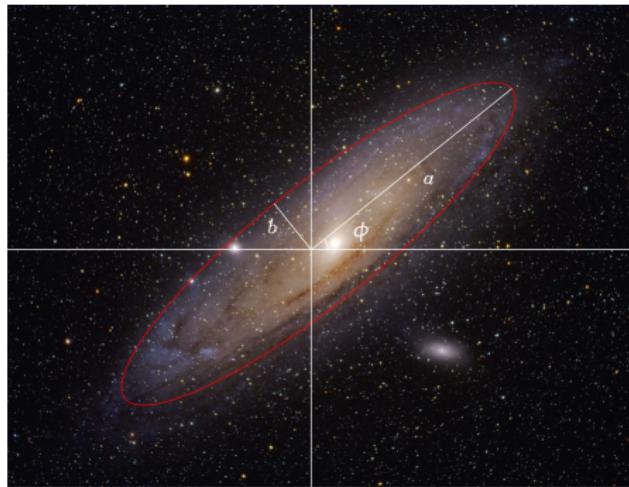
Galaxy ellipticity



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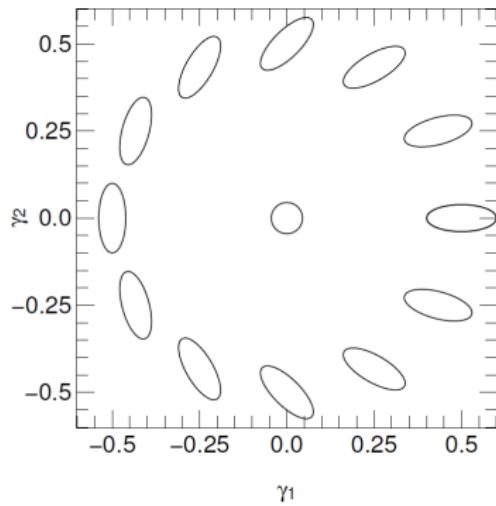
Galaxy ellipticity:

$$\varepsilon_s = \frac{a - b}{a + b} \exp(2i\phi) = \varepsilon_{s1} + i\varepsilon_{s2}$$

Weak lensing effect



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Observed ellipticity and Weak lensing approximation (Seitz & Schneider, 1997):

$$\varepsilon = \frac{\varepsilon_s + g}{1 + g^* \varepsilon_s} \approx \varepsilon_s + \gamma$$

Weak lensing experiment



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Distribution of intrinsic ellipticity (Bartelmann & Narayan, 1995):

$$p_e(\varepsilon_s) = \frac{\exp(-|\varepsilon_s|^2/\sigma_\varepsilon^2)}{\pi\sigma_\varepsilon^2[1 - \exp(-1/\sigma_\varepsilon^2)]}$$

$\sigma_\varepsilon \approx 0.2$ (e.g. Miralda-Escudé 1991b; Tyson & Seitzer 1988; Brainerd et al. 1996)

Galaxy number density distribution on redshift from CS82(Shan et al. 2014):

$$p_z(z) \propto \frac{z^a + z^{ab}}{z^b + c}$$

Where $a = 0.531$, $b = 7.810$, $c = 0.517$.

Angular Power Spectrum



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Get 3 angular power spectrum from CAMB

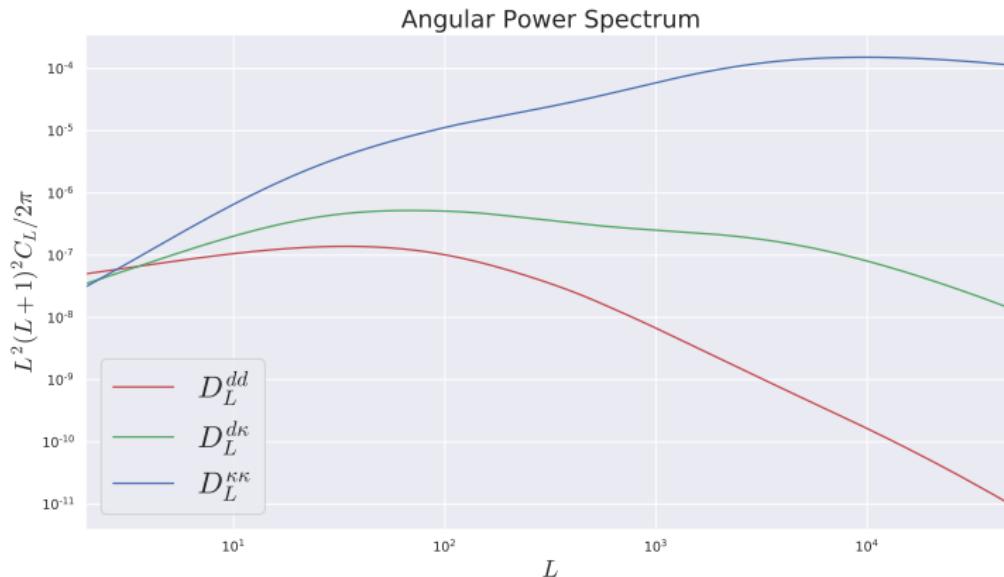
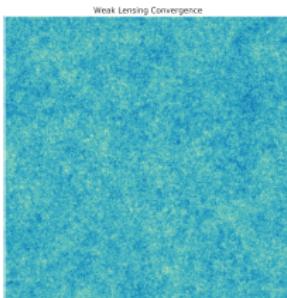
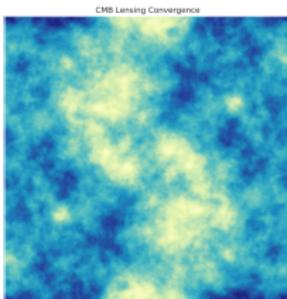


Figure: CMB Lensing deflection angle and Weak Lensing convergence and cross

Weak Lensing Convergence



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$$C_L^{\phi\phi} = \frac{L(L+1)}{4} C_L^{dd}$$

$$\kappa_\phi(\ell) = \zeta_1(C_L^{\phi\phi})^{1/2}$$

$$\kappa(\ell) = \zeta_1(\ell) \frac{C_L^{\kappa\phi}}{(C_L^{\phi\phi})^{1/2}} + \zeta_2(\ell) \left[C_L^{\kappa\kappa} - \frac{(C_L^{\kappa\phi})^2}{C_L^{\phi\phi}} \right]^{1/2}$$

κ_ϕ :convergence of CMB lensing

κ : convergence of weak lensing

$\zeta_{1,2}$ are two complex numbers drawn from a Gaussian distribution with unit variance (Kamionkowski et al.1997).

Weak Lensing shear

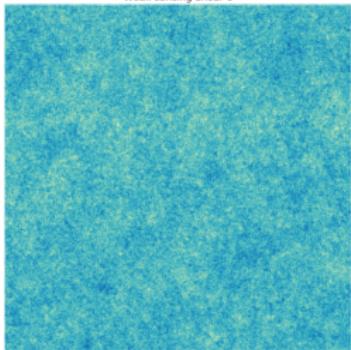


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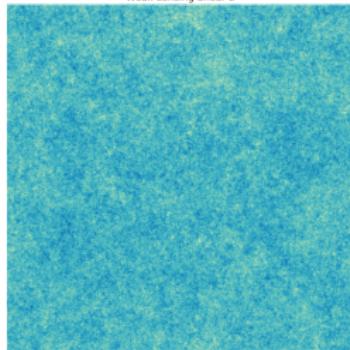
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Weak Lensing Shear 1



Weak Lensing Shear 2



The Fourier transform $\gamma(\ell)$ of the shear is related to $\kappa(\ell)$ through (Schneider et al. 2002):

$$\gamma(\ell) = \left(\frac{\ell_1^2 - \ell_2^2 + 2i\ell_1\ell_2}{|\ell|^2} \right) \kappa_g(\ell) = \kappa_g(\ell) e^{2i\beta}$$

Where β is the polar angle of ℓ



Define tangential shear and cross shear:

$$\begin{aligned}\gamma_t &= -\mathcal{R}e[\gamma \exp(-2i\phi_\theta)] \\ \gamma_x &= -\mathcal{I}m[\gamma \exp(-2i\phi_\theta)]\end{aligned}$$

Where ϕ_θ is the polar angle of the separation vector ϑ . Define correlation function ξ_+ and ξ_- (Schneider et al. 2002):

$$\begin{aligned}\xi_+(\vartheta) &= \langle \gamma_t \gamma_t \rangle + \langle \gamma_x \gamma_x \rangle = \langle \kappa \kappa \rangle(\vartheta) \\ \xi_-(\vartheta) &= \langle \gamma_t \gamma_t \rangle - \langle \gamma_x \gamma_x \rangle = \mathcal{R}e[\langle \gamma \gamma \rangle e^{-4i\phi_\theta}](\vartheta)\end{aligned}$$

Where $\vartheta = |\vartheta|$, ϕ_θ is the polar angle of the separation vector ϑ . ξ_\pm can be computed as following 1D Bessel integral:

$$\xi_\pm(\vartheta) = \int_0^\infty \frac{\ell d\ell}{2\pi} J_{0,4}(\ell\vartheta) C_\ell^{\kappa\kappa}$$

Correlation Function



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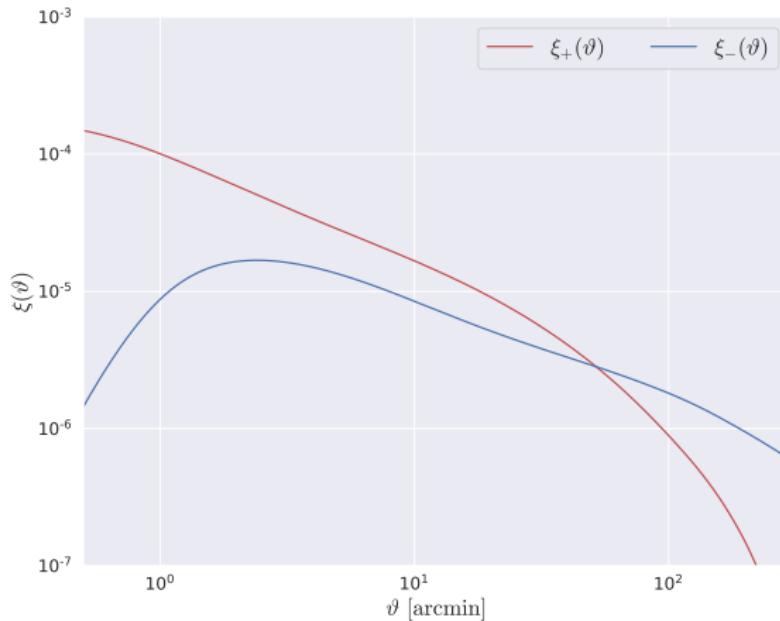


Figure: correlation function computed by convergence power spectrum



Lensing effect on galaxy ellipticity:

$$\varepsilon(\boldsymbol{\theta}) = \varepsilon_s(\boldsymbol{\theta}) + \gamma(\boldsymbol{\theta})$$

Ensemble average of lensed ellipticity:

$$\begin{aligned}\langle \varepsilon(\boldsymbol{\theta}) \varepsilon^*(\boldsymbol{\theta}') \rangle &= \langle [\varepsilon_s(\boldsymbol{\theta}) + \gamma(\boldsymbol{\theta})][\varepsilon_s(\boldsymbol{\theta}') + \gamma(\boldsymbol{\theta}')]^* \rangle \\ &= \langle \gamma(\boldsymbol{\theta}) \gamma^*(\boldsymbol{\theta}') \rangle + \text{Noise}\end{aligned}$$

An estimator of the ξ_{\pm} (Schneider et al. 2002a) is:

$$\hat{\xi}_{\pm}(\vartheta) = \frac{\sum_{ij} \omega_i \omega_j (\varepsilon_{t,i} \varepsilon_{t,j} \pm \varepsilon_{\times,i} \varepsilon_{\times,j})}{\sum_{ij} \omega_i \omega_j}$$

Shear Reconstruction and Noise



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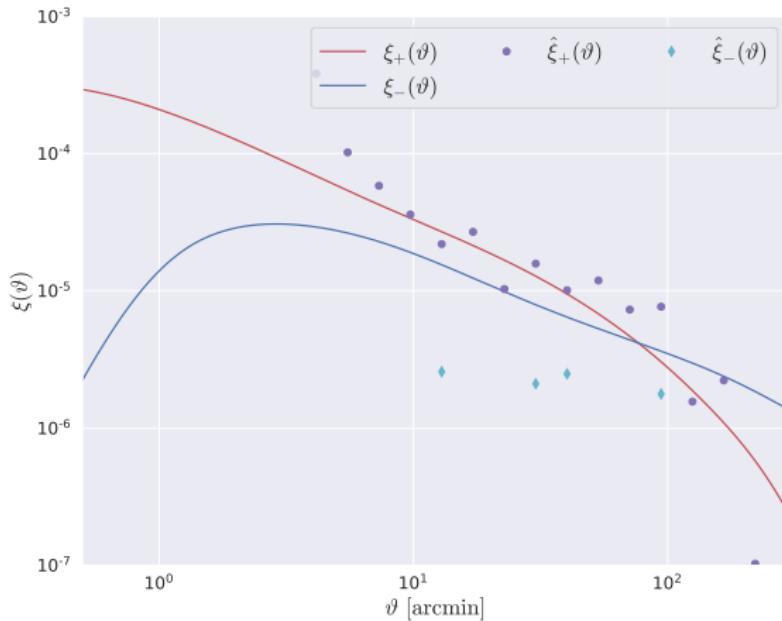


Figure: ξ_+ and ξ_- computed by TreeCorr



Two-point ensemble average of lensed ellipticity in Fourier space:

$$\langle \varepsilon(\ell) \varepsilon^*(\ell') \rangle = (2\pi)^2 \delta_D(\ell - \ell') \hat{C}_\ell^{\kappa\kappa}$$

Estimation of power spectrum

$$\hat{C}_\ell^{\kappa\kappa} = 2\pi \int_0^\infty \vartheta d\vartheta J_{0,4}(\ell\vartheta) \xi_\pm(\vartheta)$$

Ellipticity Noise in the Gaussian approximation:

$$\Delta C_\ell^{\kappa\kappa} = \frac{1}{(2\ell + 1)f_{\text{sky}}} \left(\frac{\sigma_\varepsilon^2}{2n} + C_\ell^{\kappa\kappa} \right)$$



THANKS!