Cross-correlation of CMB Lensing and Weak Gravitational Lensing

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Contents

Purpose

- Reconstruction of CMB Lensing (AliCPT).
- Reconstruction of Weak Gravitational Lensing convergence field (CSST).
- Cross-correlation of the above two items.

Sky fraction

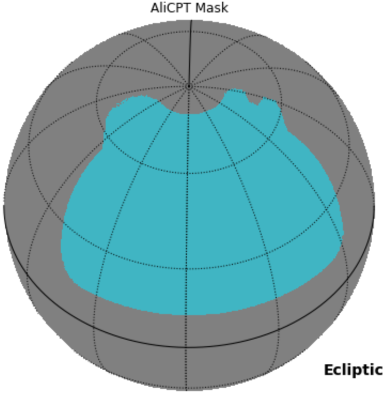


Figure: Sky fraction of AliCPT, around 12% of fullsky, cross-correlation area is part of it with area of $10^\circ \times 10^\circ$

Power Spectrum

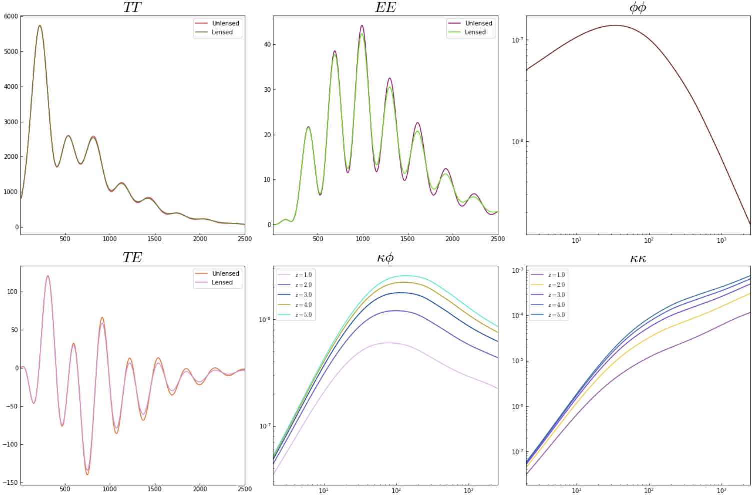


Figure: C_ℓs from CAMB×PLANCK 2018

Sky Maps

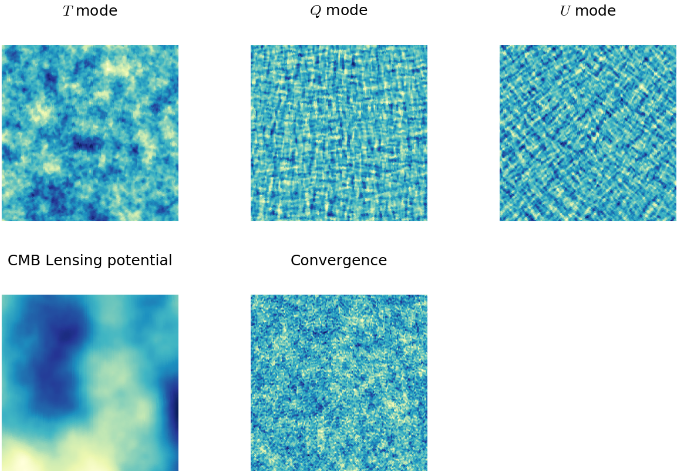


Figure: Skymaps from C_ℓs

For lensing power spectrum estimate

$$\hat{C}_\ell^{\phi\phi} = \tilde{C}_\ell^{\tilde{\phi}_1\tilde{\phi}_2} - \Delta\tilde{C}_\ell^{\tilde{\phi}_1\tilde{\phi}_2}|_{RDN0} - \Delta\tilde{C}_\ell^{\tilde{\phi}_1\tilde{\phi}_2}|_{N1} - \Delta\tilde{C}_\ell^{\tilde{\phi}_1\tilde{\phi}_2}|_{PS}$$

- **N0 bias:**
  comes from primary CMB. We evaluate this term by MC simulations.
- **N1 bias:**
  comes form secondary contraction of lensing trispe ctrum, proportional to lensing power spectrum. This term is analytically estimated under flat sky approximation.
- **mMC correction:**
  response is position dependent, corrected by MC simulations.

$$\frac{C_\ell^{\rm fid}}{\langle\tilde{C}_\ell\rangle_{\rm MC}}\tilde{C}_\ell, \quad \frac{C_\ell^{\rm fid}}{\langle\tilde{C}_\ell\rangle_{\rm MC}} \approx \left[\int\frac{\mathrm{d}\boldsymbol{n}}{4\pi}\left(\frac{R_\ell(\boldsymbol{n})}{R_\ell^{\rm fid}}\right)^2\right]^{-1}$$

Noise levels

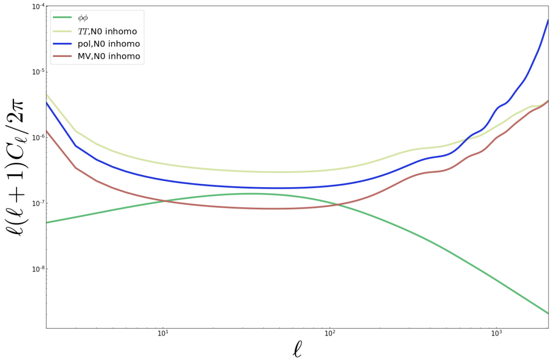
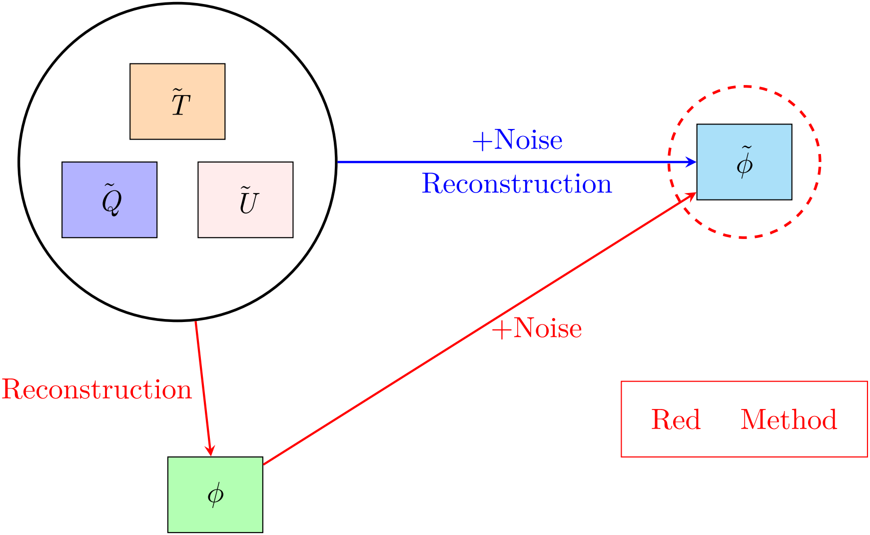


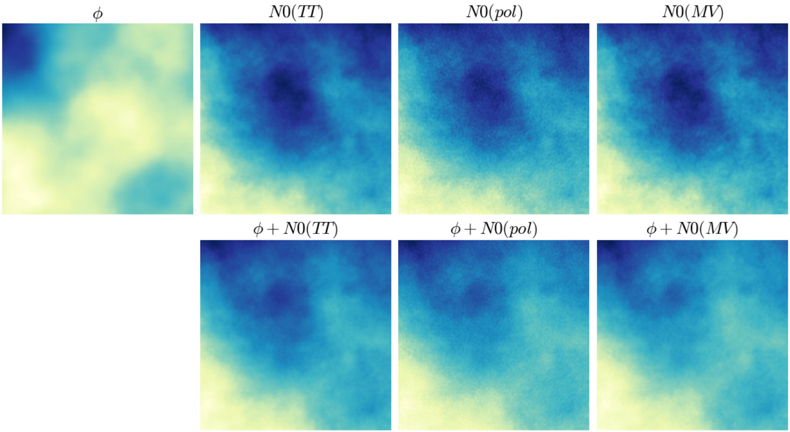
Figure: Potential and analytical N0 for inhomogeneous noise distribution.

$$N_\ell^{(0)} \approx \int \frac{\mathrm{d}\boldsymbol{n}}{4\pi}\left(\frac{R_\ell(\boldsymbol{n})}{R_\ell^{\mathrm{fid}}}\right)^2 N_\ell^{(0)}(\boldsymbol{n}) = \frac{1}{\sum N^{\mathrm{b}}(R_\ell^{\mathrm{fid}})^2}\sum_{\mathrm{b}}(R_\ell^{\mathrm{b}})^2 N_\ell^{\mathrm{b},(0)}$$

Reconstruction of ϕ



Noise simulation



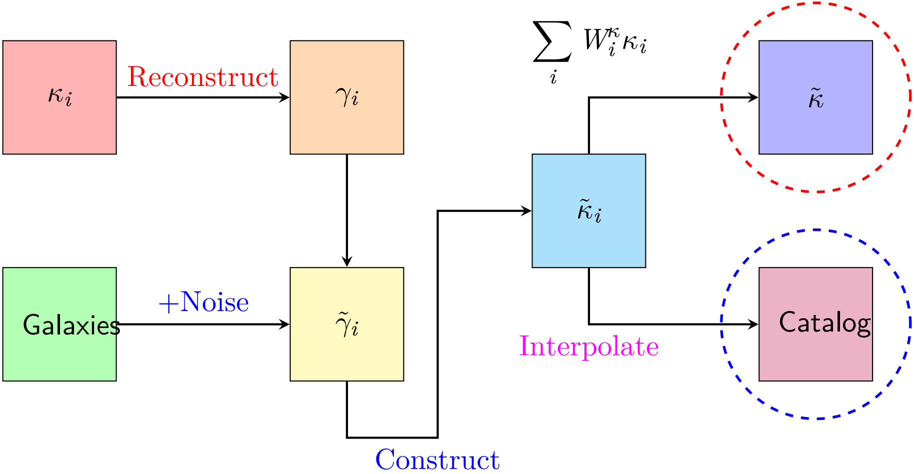
For lensing power spectrum estimate

- **Galaxy ellpiticity noise:**
  Galaxies distributing in a series of shear map makes galaxy ellpiticity noise of corresponding convergence maps.

Reconstruction of κ

κ:convergence

γ:shear



Reconstruction of κ (M.Bartelmann P. Schneider,1999)

$$\gamma(\vec{\theta}) = \frac{1}{\pi}\int_{\mathbb{R}^2} \mathrm{d}^2\theta' \mathcal{D}_\kappa(\vec{\theta} - \vec{\theta'})\kappa(\vec{\theta'})$$

$$\mathcal{D}_\kappa(\vec{\theta}) = -\frac{\theta_1^2 - \theta_2^2 + 2i\theta_1\theta_2}{|\vec{\theta}|^4} = -\frac{1}{(\theta_1 - i\theta_2)^2}$$

Fourier Space:

$$\hat{\gamma}(\vec{k}) = \pi^{-1}\hat{\mathcal{D}}_\kappa(\vec{k})\hat{\kappa}(\vec{k})$$

$$\hat{\kappa}(\vec{k}) = \pi^{-1}\hat{\mathcal{D}}_\gamma(\vec{k})\hat{\gamma}(\vec{k})$$

Algorithm check

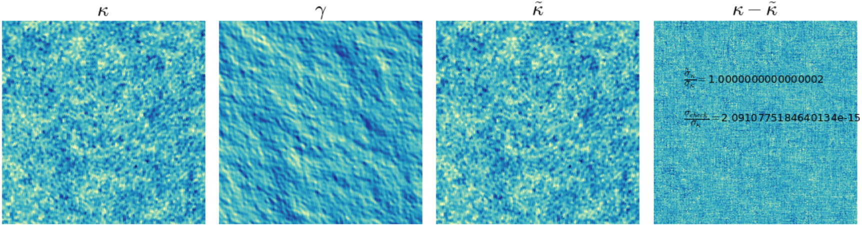
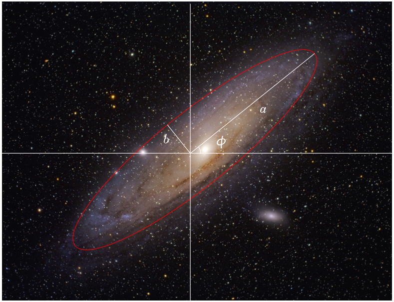


Figure: From left to right, κ map, γ map, reconstructed κ map, and their difference

Galaxy ellpiticity



Galaxy ellpiticity:

$$\varepsilon_s = |\varepsilon_s| \exp(2i\phi)$$
$$|\varepsilon_s| = \frac{a-b}{a+b}$$

Distribution of instrinsic ellpiticity(Bartelmann & Narayan, 1995):

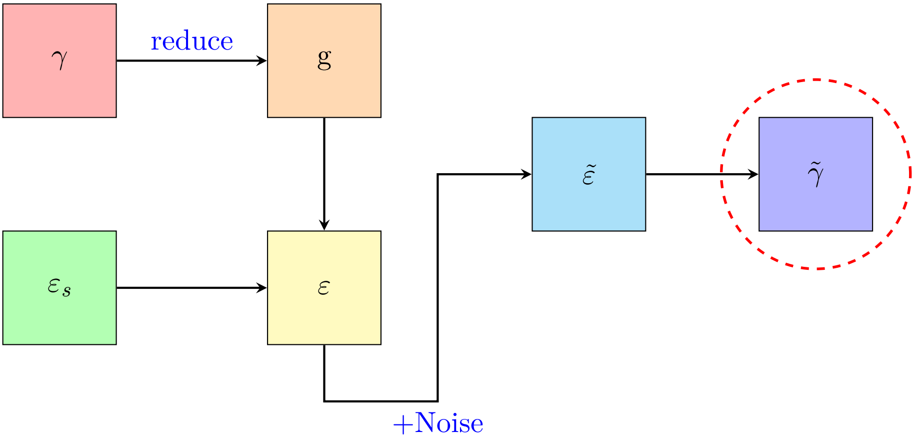
$$p_e(\varepsilon_s) = \frac{\exp(-|\varepsilon_s|^2/\sigma_\varepsilon^2)}{\pi\sigma_\varepsilon^2[1-\exp(-1/\sigma_\varepsilon^2)]}$$

$\sigma_\varepsilon \approx 0.2$(e.g. Miralda-Escudé 1991b; Tyson & Seitzer 1988; Brainerd et al. 1996)

Observed ellipticities(Seitz & Schneider, 1997):

$$\varepsilon = \frac{\varepsilon_s + \mathrm{g}}{1 + \mathrm{g}^* \varepsilon_s}$$

Galaxy ellpiticity noise



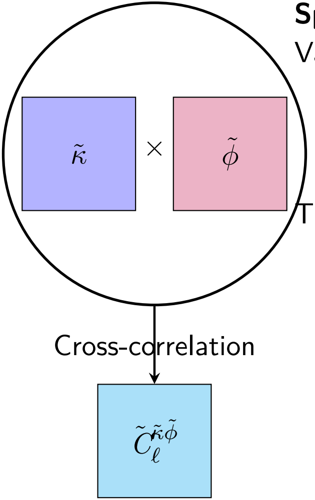
Correlation function

$$\xi(r) = \frac{1}{2\pi^2} \int_0^\infty k^2 P(k) \frac{\sin(kr)}{kr} \mathrm{d}k$$

Where $P(k)$ is power spectrum
Cross spectrum(A. Lewis, 2006:31-32)

$$C_\ell^{\kappa\phi} = \int d^2\vec{r} e^{i\vec{\ell}\cdot\vec{x}} \xi(r) = 2\pi \int r dr J_0(\ell r) \xi(r)$$

Cross-correlation



Spectrum and S/N

Varinace of $C_\ell^{\kappa\phi}$ (F. Bianchini et al.,2015):

$$(\Delta C_\ell^{\kappa\phi})^2 = \frac{(C_\ell^{\kappa\phi})^2 + (C_\ell^{\kappa\kappa} + N_\ell^{\kappa\kappa})(C_\ell^{\phi\phi} + N_\ell^{\phi\phi})}{2(\ell+1)f_{\rm sky}}$$

The signal to noise ratio at multipole ℓ is:

$$\left(\frac{S}{N}\right)_\ell = \frac{(C_\ell^{\kappa\phi})^2}{(\Delta C_\ell^{\kappa\phi})^2}$$

$$= \frac{2(\ell+1)f_{\rm sky}}{1 + (C_\ell^{\kappa\kappa} + N_\ell^{\kappa\kappa})(C_\ell^{\phi\phi} + N_\ell^{\phi\phi})/(C_\ell^{\kappa\phi})^2}$$

Appreciate your Criticism