# Cross-correlation of CMB Lensing and Weak Gravitational Lensing

Wang Zheng-Yi

Department of Astronomy ,Beijing Normal University

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Supervisor: Prof. Hu Bin

# Purpose

- Reconstruction of CMB Lensing (AliCPT).
- Reconstruction of Weak Gravitational Lensing convergence field (CSST).
- Cross-correlation of the above two items.

# Sky fraction

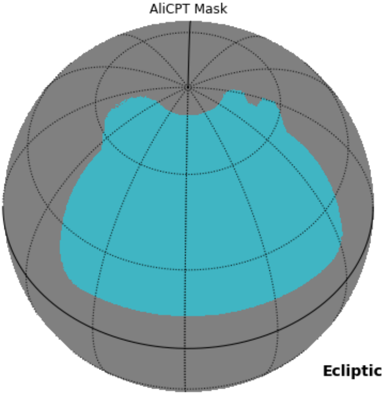



Figure: Sky fraction of AliCPT, around 12% of fullsky, cross-correlation area is part of it with area of $10^\circ \times 10^\circ$

# T-,Q- and U-mode



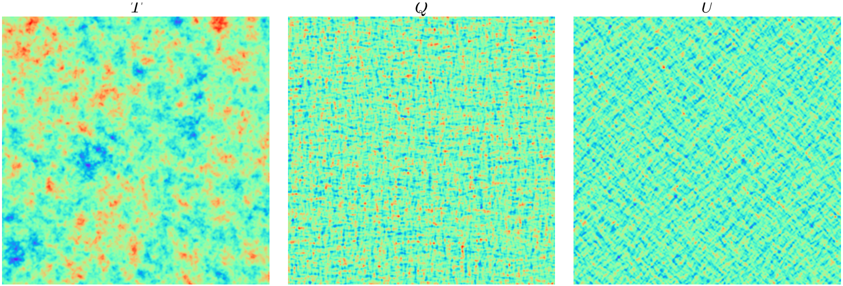



Figure: T-,Q- and U-mode from AliCPT

Lensing effect:

$$\tilde{\Theta}(\boldsymbol{\theta}) = \Theta(\boldsymbol{\theta} + \mathbf{d}(\boldsymbol{\theta})) = \Theta(\boldsymbol{\theta}) + \nabla^i \psi \nabla_i \Theta(\boldsymbol{\theta}) + \mathcal{O}(\psi^2)$$
$$[\tilde{Q} + i\tilde{U}](\boldsymbol{\theta}) = [Q + iU](\boldsymbol{\theta} + \mathbf{d}(\boldsymbol{\theta}))$$

Where $\mathbf{d}(\boldsymbol{\theta}) = \nabla \psi(\boldsymbol{\theta})$,and flat sky with plane wave mode:

$$\Theta(\boldsymbol{\theta}) = \int \frac{\mathrm{d}^2 \ell}{(2\pi)^2} \Theta(\boldsymbol{\ell}) \exp\left(i\boldsymbol{\ell} \cdot \boldsymbol{\theta}\right)$$
$$[Q \pm iU](\boldsymbol{\theta}) = \int \frac{\mathrm{d}^2 \ell}{(2\pi)^2} [E(\boldsymbol{\ell}) \pm iB(\boldsymbol{\ell})] \exp \pm 2i\varphi_\ell \exp\left(i\boldsymbol{\ell} \cdot \boldsymbol{\theta}\right)$$
$$\psi(\boldsymbol{\theta}) = \int \frac{\mathrm{d}^2 L}{(2\pi)^2} \psi(\boldsymbol{L}) \exp\left(i\boldsymbol{L} \cdot \boldsymbol{\theta}\right)$$

Define filter $f_\alpha(\boldsymbol{\ell}, \boldsymbol{\ell}')$:

$$\langle \tilde{x}(\boldsymbol{\ell})\tilde{x}'(\boldsymbol{\ell}')\rangle = f_\alpha(\boldsymbol{\ell}, \boldsymbol{\ell}')\psi(\boldsymbol{L})$$

Where:$x$ and $x'$ are two of $\Theta$, $E$, $B$.

$$\langle d_\alpha^*(\boldsymbol{L})d_\beta(\boldsymbol{L}')\rangle = (2\pi)^2\delta_D(\boldsymbol{L}-\boldsymbol{L}')[C_L^{dd} + N_{\alpha\beta}(L)]$$

Where $d$ is deflection angle.

$$d(\boldsymbol{L}) = \frac{A_\alpha}{L}\int \frac{\mathrm{d}^2\ell_1}{(2\pi)^2} x(\boldsymbol{\ell}_1)x'(\boldsymbol{\ell}_2)F_\alpha(\boldsymbol{\ell}_1, \boldsymbol{\ell}_2')$$

$$A_\alpha(L) = L^2\left[\frac{1}{(2\pi)^2}\int \mathrm{d}^2\ell_1 f_\alpha(\boldsymbol{\ell}_1, \boldsymbol{\ell}_2)F_\alpha(\boldsymbol{\ell}_1, \boldsymbol{\ell}_2)\right]^{-1}$$

After a little bit algebra:

$$F_\alpha(\boldsymbol{\ell}_1,\boldsymbol{\ell}_2)=\frac{C_{\ell_1}^{x'x'}C_{\ell_2}^{xx}f_\alpha(\boldsymbol{\ell}_1,\boldsymbol{\ell}_2)-C_{\ell_1}^{xx'}C_{\ell_2}^{xx'}f_\alpha(\boldsymbol{\ell}_2,\boldsymbol{\ell}_1)}{C_{\ell_1}^{xx}C_{\ell_2}^{x'x'}C_{\ell_1}^{x'x'}C_{\ell_2}^{xx}-(C_{\ell_1}^{xx'}C_{\ell_2}^{xx'})^2}$$

$$N_{\alpha\beta}(L)=L^{-2}A_\alpha(L)A_\beta(L)\int\frac{\mathrm{d}^2\ell_1}{(2\pi)^2}F_\alpha(\boldsymbol{\ell}_1,\boldsymbol{\ell}_2)$$

$$\times(F_\beta(\boldsymbol{\ell}_1,\boldsymbol{\ell}_2)C_{\ell_1}^{x_\alpha x_\beta}C_{\ell_2}^{x'_\alpha x'_\beta}+F_\beta(\boldsymbol{\ell}_2,\boldsymbol{\ell}_1)C_{\ell_1}^{x_\alpha x'_\beta}C_{\ell_2}^{x'_\alpha x_\beta})$$

Notice that for the minimum variance filter:

$$N_{\alpha\alpha}(L)=A_\alpha(L)$$

# Noise levels

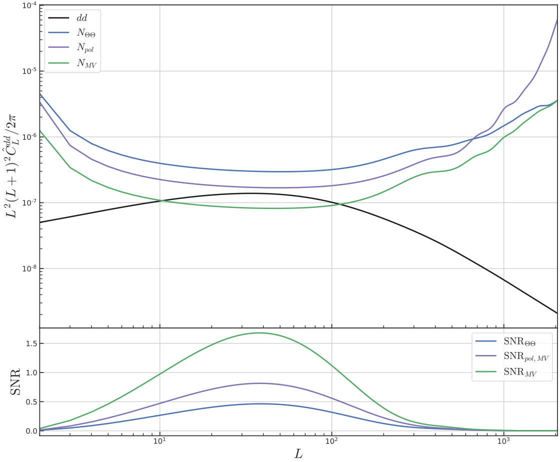
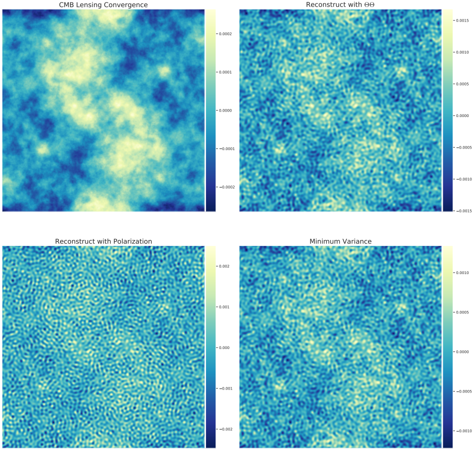



Figure: CMB Lensing Convergence Power Spectrum and Noise

# CMB Lensing Convergence Map





# Galaxy ellpiticity

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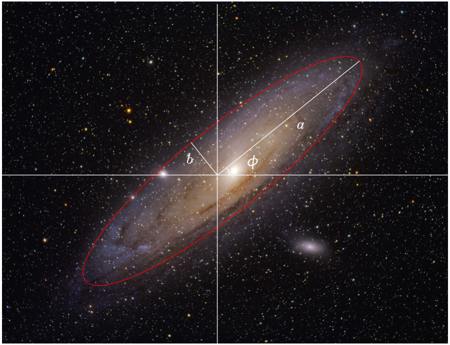
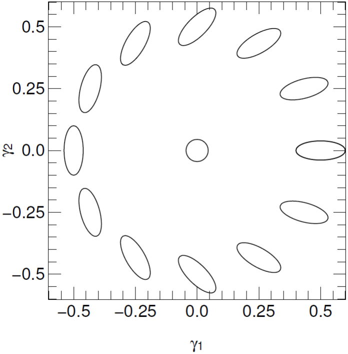



Galaxy ellpiticity:

$$\varepsilon_s = \frac{a-b}{a+b}\exp(2i\phi) = \varepsilon_{s1} + i\varepsilon_{s2}$$




Observed ellipticity and Weak lensing approximation (Seitz & Schneider, 1997):

$$\varepsilon = \frac{\varepsilon_s + g}{1 + g^* \varepsilon_s} \approx \varepsilon_s + \gamma$$

Distribution of instrinsic ellpiticity (Bartelmann & Narayan, 1995):

$$p_e(\varepsilon_s) = \frac{\exp(-|\varepsilon_s|^2/\sigma_\varepsilon^2)}{\pi\sigma_\varepsilon^2[1-\exp(-1/\sigma_\varepsilon^2)]}$$

$\sigma_\varepsilon \approx 0.2$(e.g. Miralda-Escudé 1991b; Tyson & Seitzer 1988; Brainerd et al. 1996)
Galaxy number density distribution on redshift from CS82(Shan et al. 2014):

$$p_z(z) \propto \frac{z^a + z^{ab}}{z^b + c}$$

Where $a = 0.531, b = 7.810, c = 0.517.$

Get 3 angular power spectrum from CAMB

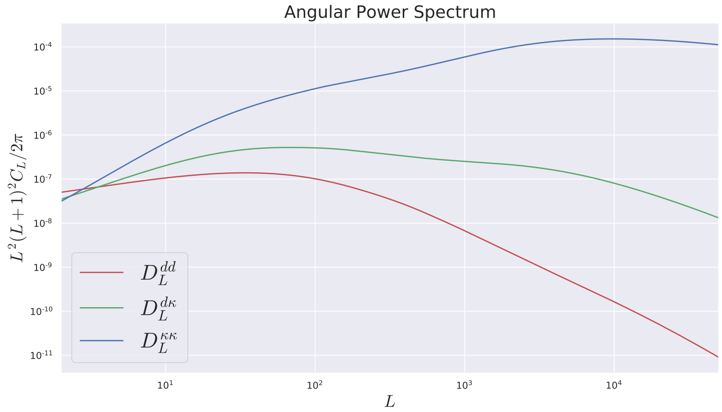
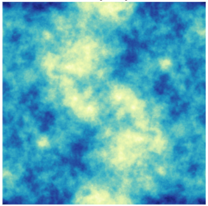



Figure: CMB Lensing deflection angle and Weak Lensing convergence and cross

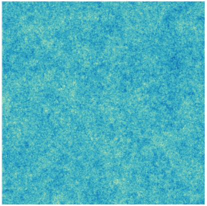
# Weak Lensing Convergence



CMB Lensing Convergence



Weak Lensing Convergence



$$C_L^{\phi\phi} = \frac{L(L+1)}{4} C_L^{dd}$$

$$\kappa_\phi(\boldsymbol{\ell}) = \zeta_1 (C_L^{\phi\phi})^{1/2}$$

$$\kappa(\boldsymbol{\ell}) = \zeta_1(\boldsymbol{\ell}) \frac{C_L^{\kappa\phi}}{(C_L^{\phi\phi})^{1/2}} + \zeta_2(\boldsymbol{\ell}) \left[ C_L^{\kappa\kappa} - \frac{(C_L^{\kappa\phi})^2}{C_L^{\phi\phi}} \right]^{1/2}$$

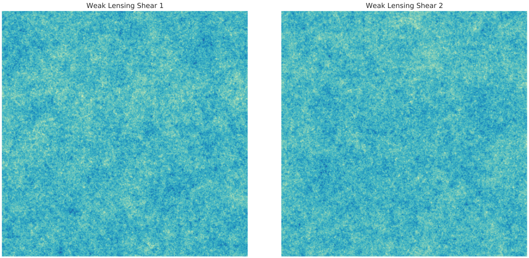
$\kappa_\phi$ :convergence of CMB lensing

$\kappa$ : convergence of weak lensing

$\zeta_{1,2}$ are two complex numbers drawn from a Gaussian distribution with unit variance (Kamionkowski et al.1997).

# Weak Lensing shear






The Fourier transform $\gamma(\boldsymbol{\ell})$ of the shear is related to $\kappa(\boldsymbol{\ell})$ through (Schneider et al.2002):

$$\gamma(\boldsymbol{\ell}) = \left( \frac{\ell_1^2 - \ell_2^2 + 2i\ell_1\ell_2}{|\boldsymbol{\ell}|^2} \right) \kappa(\boldsymbol{\ell}) = \kappa(\boldsymbol{\ell}) e^{2i\beta}$$

Where $\beta$ is the polar angle of $\boldsymbol{\ell}$

Define correlation function $\xi_+$ and $\xi_-$ (Schneider et al.2002):

$$\xi_+(\vartheta) = \langle\kappa\kappa\rangle(\vartheta)$$
$$\xi_-(\vartheta) = \mathcal{R}e[\langle\gamma\gamma\rangle e^{-4i\phi_\theta}](\vartheta)$$

Where $\vartheta = |\boldsymbol{\vartheta}|$, $\phi_\theta$ is the polar angle of the separation vector $\boldsymbol{\vartheta}$. $\xi_\pm$ can be computed as following 1D Bessel integral:

$$\xi_\pm(\vartheta) = \int_0^\infty \frac{\ell \mathrm{d}\ell}{2\pi} J_{0,4}(\ell\vartheta)\, C_\ell^{\kappa\kappa}$$

# Shear Reconstruction and Noise

Lensing effect on galaxy ellipticity:

$$\varepsilon(\boldsymbol{\theta}) = \varepsilon_s(\boldsymbol{\theta}) + \gamma(\boldsymbol{\theta})$$

Ensemble average of lensed ellipticity:

$$\begin{aligned}\langle \varepsilon(\boldsymbol{\theta})\varepsilon^*(\boldsymbol{\theta}')\rangle &= \langle [\varepsilon_s(\boldsymbol{\theta}) + \gamma(\boldsymbol{\theta})][\varepsilon_s(\boldsymbol{\theta}') + \gamma(\boldsymbol{\theta}')]^*\rangle \\ &= \langle \gamma(\boldsymbol{\theta})\gamma^*(\boldsymbol{\theta}')\rangle + \mathrm{Noise}\end{aligned}$$

# Shear Reconstruction and Noise

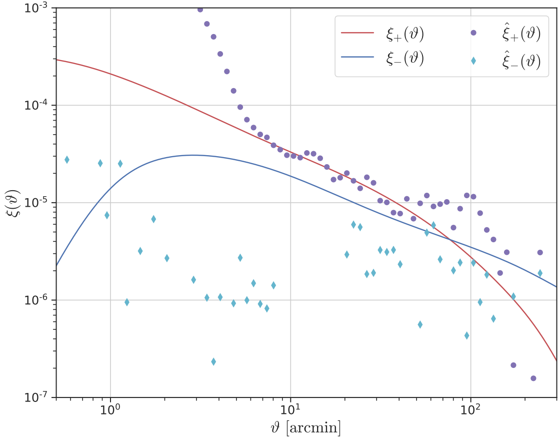



Figure: $\xi_+$ and $\xi_-$ computed by TreeCorr

# Shear Reconstruction and Noise

Estimation of power spectrum

$$\hat{C}_\ell^{\kappa\kappa} = 2\pi \int_0^\infty \vartheta \mathrm{d}\vartheta J_{0,4}(\ell\vartheta)\xi_\pm(\vartheta)$$

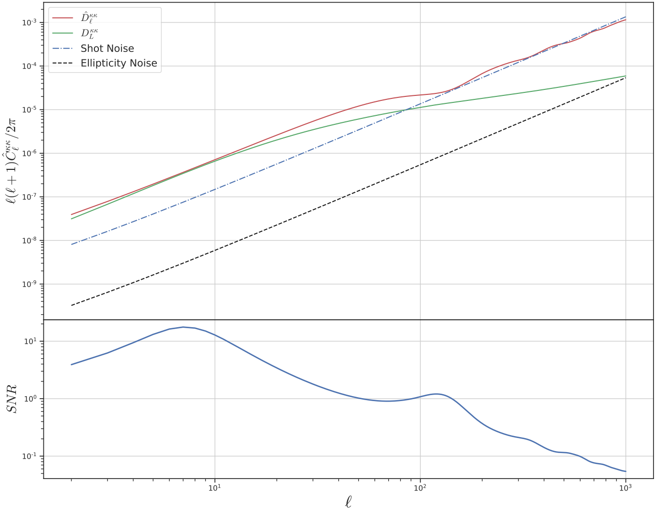
Noise in the Gaussian approximation:

$$\hat{C}_\ell^{\kappa\kappa} = \frac{4\pi f_{\text{sky}}}{N_g} + \frac{4\pi f_{\text{sky}}\sigma_\varepsilon^2}{N_g} + C_\ell^{\kappa\kappa}$$

Where $f_{\text{sky}} \simeq 0.24\%$, is the sky fraction, $N_g$ is the total number in the sky area.

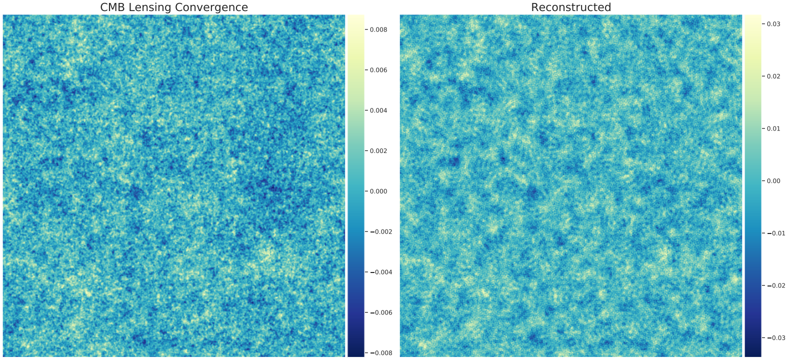
# Convergence Power Spectrum





# Weak lensing map





Correlation function

$$\xi(r) = \langle \kappa_\phi(\boldsymbol{\theta})\kappa(\boldsymbol{\theta} - \boldsymbol{\vartheta})\rangle$$

Cross spectrum(A. Lewis, 2006:31-32)

$$C_\ell^{\kappa\phi} = 2\pi \int \vartheta \mathrm{d}_0(\ell\vartheta)\xi(\vartheta)$$

# Discussion

- **N1 bias of CMB Lensing reconstruction:**
  comes form secondary contraction of lensing trispectrum, proportional to lensing power spectrum. This term is analytically estimated under flat sky approximation.
- **Alignment noise and $\varepsilon_s - \gamma$ noise:**
  Alignment noise and $\varepsilon_s - \gamma$ noise shall be under consideration at higher redshift with less galaxies
- **Mono-redshift to multi-redshift:**
  Galaxies have been fixed at $z = 1$ in the experiment.

# THANKS!