L1-6 Consider the following system:

$$10^{-3}x - y = 1$$

$$x + y = 0$$

(a) Use 3-digit arithmetic with no pivoting to solve this system.

$$\left(\begin{array}{cc|c} 10^{-3} & -1 & 1 \\ 1 & 1 & 0 \end{array}\right) \xrightarrow{R_2 - 10^3 R_1} \left(\begin{array}{cc|c} 10^{-3} & -1 & 1 \\ 0 & 10^3 & -10^3 \end{array}\right) \rightarrow \left(\begin{array}{cc|c} 1 & -10^3 & 10^3 \\ 0 & 1 & -1 \end{array}\right)$$

$x = 0$ $\quad y = -1$

(b) Now use partial pivoting and 3-digit arithmetic to solve the original system.

$$\left(\begin{array}{cc|c} 10^{-3} & -1 & 1 \\ 1 & 1 & 0 \end{array}\right) \rightarrow \left(\begin{array}{cc|c} 1 & 1 & 0 \\ 10^{-3} & -1 & 1 \end{array}\right) \xrightarrow{R_2 - 10^{-3} R_1} \left(\begin{array}{cc|c} 1 & 1 & 0 \\ 0 & -1 & 1 \end{array}\right)$$

$x = 1$ $\quad y = -1$

L2-4-b Determine the general solution for each of the following nonhomogeneous systems.

$$\begin{cases} 2x + y + z = 4 \\ 4x + 2y + z = 6 \\ 6x + 3y + z = 8 \\ 8x + 4y + z = 10 \end{cases} \quad A|b \left(\begin{array}{ccc|c} 2 & 1 & 1 & 4 \\ 4 & 2 & 1 & 6 \\ 6 & 3 & 1 & 8 \\ 8 & 4 & 1 & 10 \end{array}\right) \rightarrow \left(\begin{array}{ccc|c} 1 & \frac{1}{2} & \frac{1}{2} & 2 \\ 0 & 0 & 1 & 2 \\ 0 & 0 & 2 & 4 \\ 0 & 0 & 3 & 6 \end{array}\right)$$

$$\rightarrow \left(\begin{array}{ccc|c} 1 & \frac{1}{2} & \frac{1}{2} & 2 \\ 0 & 0 & 1 & 2 \\ 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 \end{array}\right) \rightarrow \left(\begin{array}{ccc|c} 1 & \frac{1}{2} & 0 & 1 \\ 0 & 0 & 1 & 2 \\ 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 \end{array}\right) = E|b$$

$$\begin{cases} x = 1 - \frac{1}{2}y \\ y = \text{free} \\ z = \cancel{0}\,2 \end{cases}$$

The general solution is: $\begin{pmatrix} x \\ y \\ z \end{pmatrix} = \begin{pmatrix} 1-\frac{1}{2}y \\ y \\ 2 \end{pmatrix} = \begin{pmatrix} 1 \\ 0 \\ 2 \end{pmatrix} + y \cdot \begin{pmatrix} -\frac{1}{2} \\ 1 \\ 0 \end{pmatrix}$

(a) $\begin{cases} x_1 + 2x_2 + x_3 + 2x_4 = 3 \\ 2x_1 + 4x_2 + x_3 + 3x_4 = 4 \\ 3x_1 + 6x_2 + x_3 + 4x_4 = 5 \end{cases}$ $\quad A|b = \left(\begin{array}{cccc|c} 1 & 2 & 1 & 2 & 3 \\ 2 & 4 & 1 & 3 & 4 \\ 3 & 6 & 1 & 4 & 5 \end{array}\right)$

$$\rightarrow \left(\begin{array}{cccc|c} 1 & 2 & 1 & 2 & 3 \\ 0 & 0 & -1 & -1 & -2 \\ 0 & 0 & -2 & -2 & -4 \end{array}\right) \rightarrow \left(\begin{array}{cccc|c} 1 & 2 & 1 & 2 & 3 \\ 0 & 0 & 1 & 1 & 2 \\ 0 & 0 & 1 & 1 & 2 \end{array}\right) \rightarrow \left(\begin{array}{cccc|c} 1 & 2 & 0 & 1 & 1 \\ 0 & 0 & 1 & 1 & 2 \\ 0 & 0 & 0 & 0 & 0 \end{array}\right)$$

$$\begin{cases} x_1 = 1 - 2x_2 - x_4 \\ x_2 = \text{free} \\ x_3 = 2 - x_4 \\ x_4 = \text{free} \end{cases}$$

The general solution is $\begin{pmatrix} x_1 \\ x_2 \\ x_3 \\ x_4 \end{pmatrix} = \begin{pmatrix} 1 \\ 0 \\ 2 \\ 0 \end{pmatrix} + x_2 \begin{pmatrix} -2 \\ 1 \\ 0 \\ 0 \end{pmatrix} + x_4 \begin{pmatrix} -1 \\ 0 \\ -1 \\ 1 \end{pmatrix}$