4. Let A and b be matrices

$$A=\begin{pmatrix}1&2&4&17\\3&6&-12&3\\2&3&-3&2\\0&2&-2&6\end{pmatrix}\qquad b=\begin{pmatrix}17\\3\\3\\4\end{pmatrix}$$

(a) Use Partial pivoting and find the permutation matrix P such that $PA=LU$

$$[A|P]=\left(\begin{array}{cccc|c}1&2&4&17&1\\3&6&-12&3&2\\2&3&-3&2&3\\0&2&-2&6&4\end{array}\right)\xrightarrow{R_2\leftrightarrow R_1}\left(\begin{array}{cccc|c}3&6&-12&3&2\\1&2&4&17&1\\2&3&-3&2&3\\0&2&-2&6&4\end{array}\right)\xrightarrow[R_3-\frac{2}{3}R_1]{R_2-\frac{1}{3}R_1}\left(\begin{array}{cccc|c}3&6&-12&3&2\\\frac{1}{3}&0&8&16&1\\\frac{2}{3}&-1&5&0&3\\0&2&-2&6&4\end{array}\right)$$

$$\xrightarrow{R_4\leftrightarrow R_2}\left(\begin{array}{cccc|c}3&6&-12&3&2\\0&2&-2&6&4\\\frac{2}{3}&-1&5&0&3\\\frac{1}{3}&0&8&16&1\end{array}\right)\xrightarrow{R_3-(-\frac{1}{2}R_2)}\left(\begin{array}{cccc|c}3&6&-12&3&2\\0&2&-2&6&4\\\frac{2}{3}&\frac{1}{2}&4&3&3\\\frac{1}{3}&0&8&16&1\end{array}\right)\xrightarrow{R_4\leftrightarrow R_3}\left(\begin{array}{cccc|c}3&6&-12&3&2\\0&2&-2&6&4\\\frac{1}{3}&0&8&16&1\\\frac{2}{3}&-\frac{1}{2}&4&3&3\end{array}\right)$$

$$\xrightarrow{R_4-\frac{1}{2}R_3}\left(\begin{array}{cccc|c}3&6&-12&3&2\\0&2&-2&6&4\\\frac{1}{3}&0&8&16&1\\\frac{2}{3}&-\frac{1}{2}&\frac{1}{2}&-5&3\end{array}\right)\quad\therefore P=\begin{pmatrix}0&1&0&0\\0&0&0&1\\1&0&0&0\\0&0&1&0\end{pmatrix}\quad L=\begin{pmatrix}1&0&0&0\\0&1&0&0\\\frac{1}{3}&0&1&0\\\frac{2}{3}&-\frac{1}{2}&\frac{1}{2}&1\end{pmatrix}$$

$$U=\begin{pmatrix}3&6&-12&3\\0&2&-2&6\\0&0&8&16\\0&0&0&-5\end{pmatrix}$$

(b) Use the information in P, L, U to solve $Ax=b$

$Ax=b$ $\quad PAx=Pb$ ~~$Ax=y$ $\quad Py=Pb$~~

$PA = LU$　　$LUx = Pb$　　令 $Ux = y$ 则 $Ly = Pb$

$$\begin{pmatrix} 1 & 0 & 0 & 0 \\ 0 & 1 & 0 & 0 \\ \frac{1}{3} & 0 & 1 & 0 \\ \frac{2}{3} & -\frac{1}{2} & \frac{1}{2} & 1 \end{pmatrix} \begin{pmatrix} y_1 \\ y_2 \\ y_3 \\ y_4 \end{pmatrix} = \begin{pmatrix} 0 & 1 & 0 & 0 \\ 0 & 0 & 0 & 1 \\ 1 & 0 & 0 & 0 \\ 0 & 0 & 1 & 0 \end{pmatrix} \begin{pmatrix} 17 \\ 3 \\ 3 \\ 4 \end{pmatrix} = \begin{pmatrix} 3 \\ 4 \\ 17 \\ 3 \end{pmatrix}$$

$\therefore y^T = (3, 4, 16, -5)$

$\because Ux = y \quad \therefore \begin{pmatrix} 3 & 6 & -12 & 3 \\ 0 & 2 & -2 & 6 \\ 0 & 0 & 8 & 16 \\ 0 & 0 & 0 & -5 \end{pmatrix} \begin{pmatrix} x_1 \\ x_2 \\ x_3 \\ x_4 \end{pmatrix} = \begin{pmatrix} 3 \\ 4 \\ 16 \\ -5 \end{pmatrix}$

$\therefore$ ~~$x^T = (0, 9, 4, 1)$~~　　$x^T = (2, -1, 0, 1)$

[redacted]