Neur 603
Decision-making

- Decision variables, expected value, risk aversion
- Motivated procrastination and race models
- Optimal accumulation of evidence
- Neural correlates of decision variables

Decision-making

- **“Decision making** can be regarded as the cognitive process resulting in the selection of a course of action among several alternative scenarios.”

  – en.wikipedia.org (January 6, 2014)

- “Decide: *v.i.* To succumb to the preponderance of one set of influences over another set.”

  – Bierce A. (1906) *The Devil’s Dictionary*

How do humans make decisions?

- Value: The payoff of a given event
  - Ex: Play Loto-Québec and you can win $1,000,000 !!!
- Expected value: The payoff times its likelihood
  - The chance of winning is 1:10,000,000
  - Expected value is $\mathrm{EV} = p(win) \cdot Value$ = 10¢
- So which lottery would you choose?
  - A: 95% chance of winning $1,000,000
  - B: 50% chance of winning $3,000,000
  - Most people choose A (EV = $950,000) over B (EV = $1,500,000)
- Why?
  - The pain of regret!
  - Utility: The *subjective* payoff associated with a given event
- Expected utility: The subjective payoff times its likelihood
- Expected utility theory: Choose so to maximize expected utility

The "St. Petersburg Paradox"

- How much would you pay to play this game?
  1. R=1
  2. A coin is tossed
     - If tails, the game ends, and you receive R dollars
     - If heads, R = 2R and you go back to step 2

– What is the expected value?

$$EV = p(n=1)1 + p(n=2)2 + p(n=3)4 + p(n=4)8 + p(n=5)16 + \ldots$$

$$EV = \tfrac{1}{2}\cdot 1 + \tfrac{1}{4}\cdot 2 + \tfrac{1}{8}\cdot 4 + \tfrac{1}{16}\cdot 8 + \tfrac{1}{32}\cdot 16 + \ldots$$

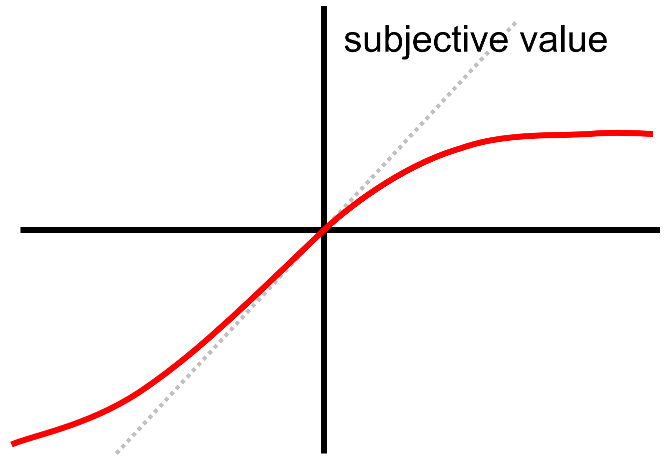
$$EV = \sum_{k=1}^{\infty} \frac{1}{2^k}\cdot 2^{(k-1)}$$

$$EV = \sum_{k=1}^{\infty} \tfrac{1}{2} = \infty \text{ !!!}$$

– What a great game! You should be willing to pay ANY amount

Risk Aversion

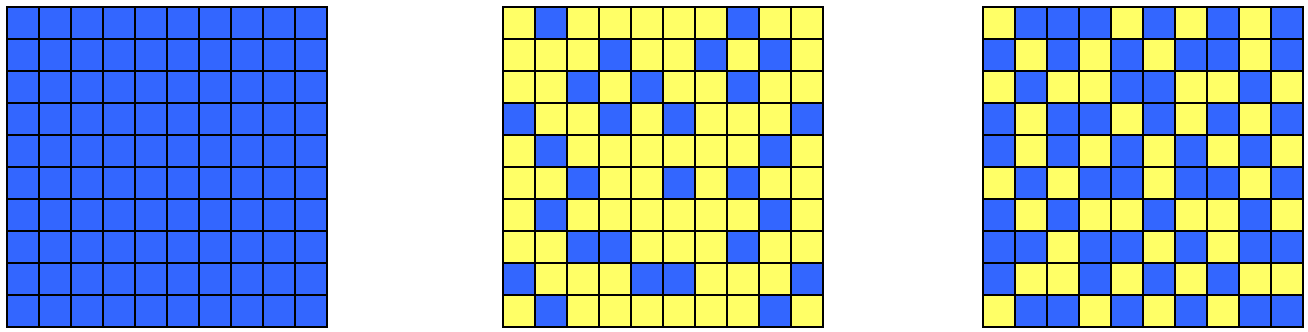
- People do not decide on basis of expected utility



- People exaggerate the negative value of a loss
- Why? Because “loss” in the jungle is often “loss of life”
- “Prospect Theory” (Kahneman & Tversky)

Motivated procrastination

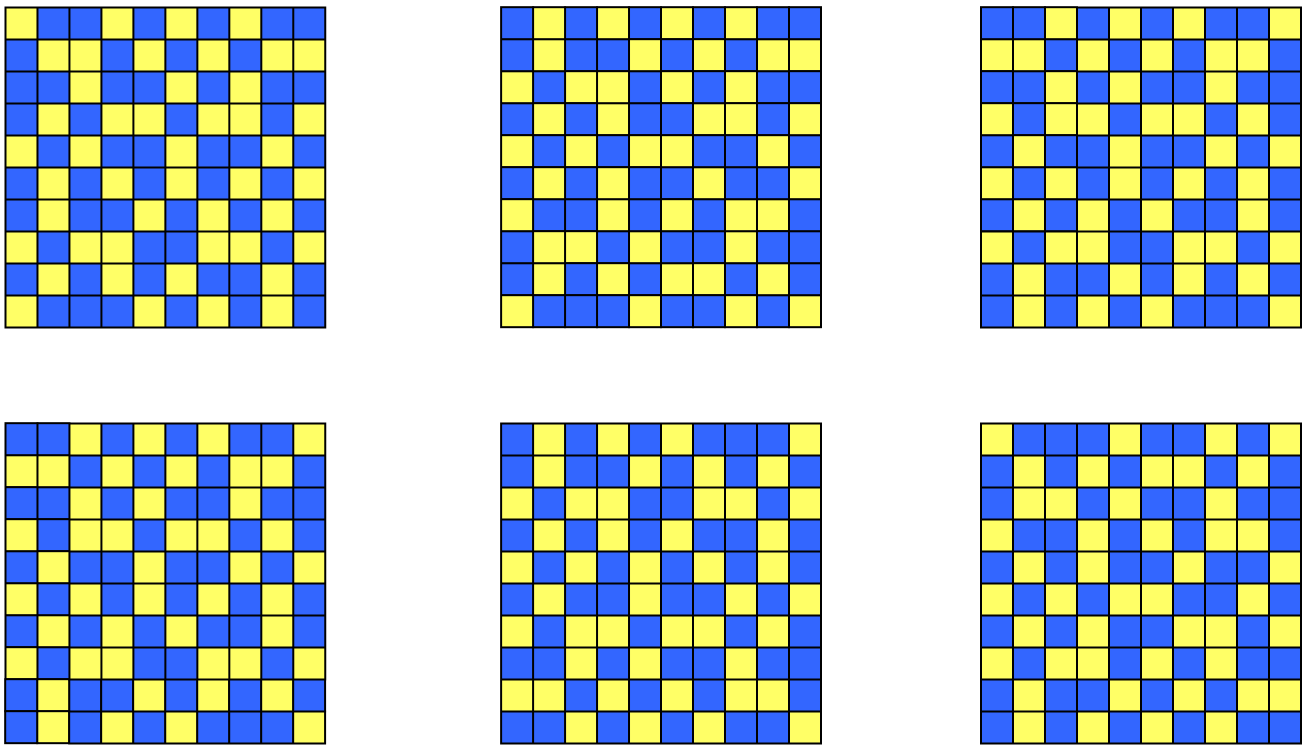
- Is the following pattern more blue or more yellow?



- Easy: Fast response
- Hard: Slow response
- Hard: More variability across the population

Motivated procrastination

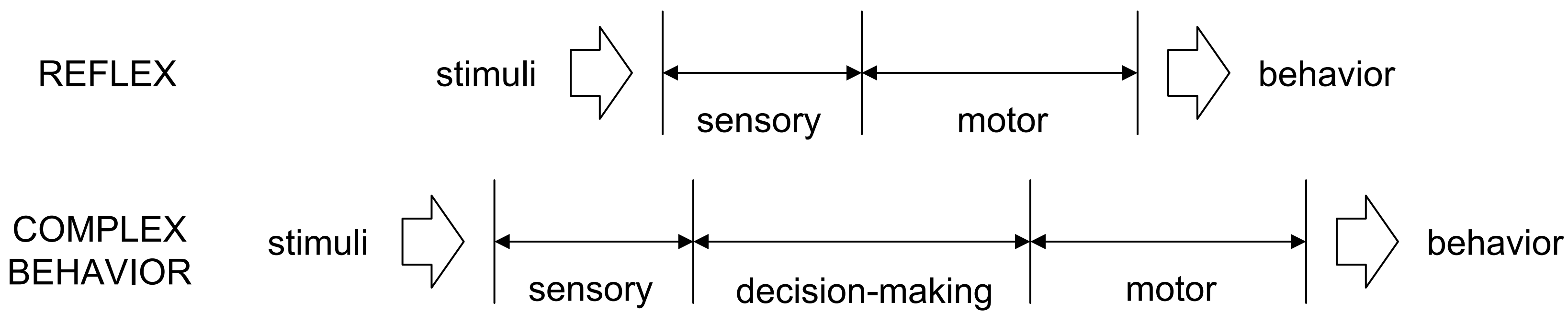
- Is the following pattern more blue or more yellow?



– More common → faster

Mental chronometry

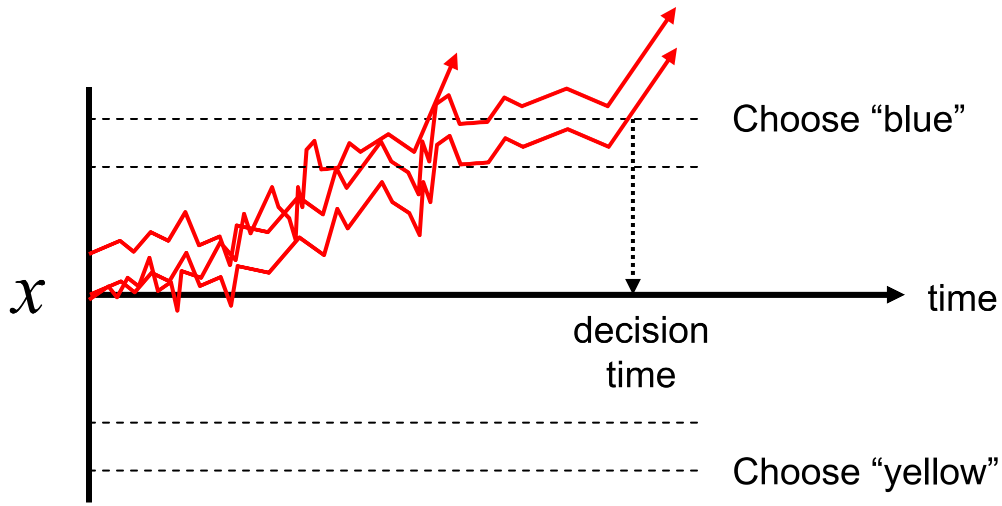
- Can we deduce something about the processes of decision-making by measuring the timing of decisions?



- Some phenomena:
  - Harder choices → slower, more variability
  - More common → faster
  - More choices → slower

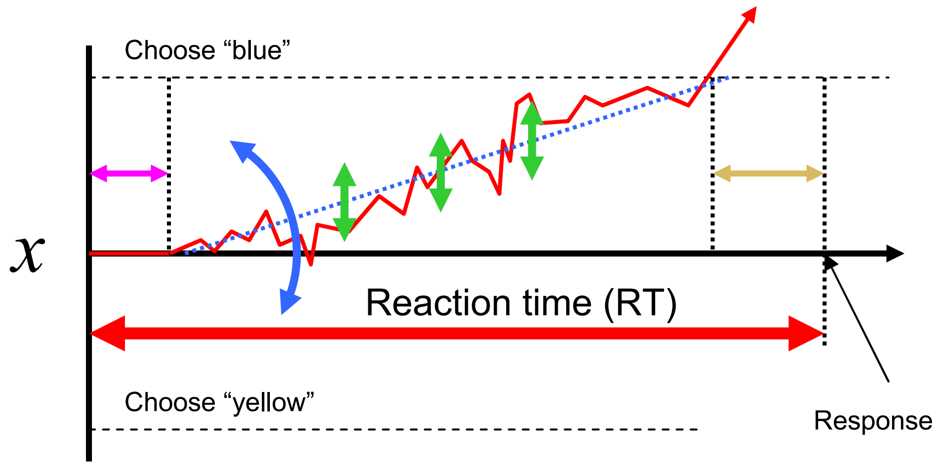
“Diffusion model”

- Hypothesis: Deliberation is similar to a random walk



- Some noisy mental variable (x) is changing in time, biased by sensory information, until it crosses one of two decision thresholds
  - The strength of the evidence determines the rate of drift
  - The desired accuracy determines the threshold
  - Any prior bias determines the starting point

Different kinds of noise



- Variation in non-decision processes
  - Delays in sensory processing
  - Delays in response initiation
- Variation between trials
  - E.g. Changes in arousal / attention
- Variation within a trial
  - E.g. Neural activity fluctuations

Diffusion model

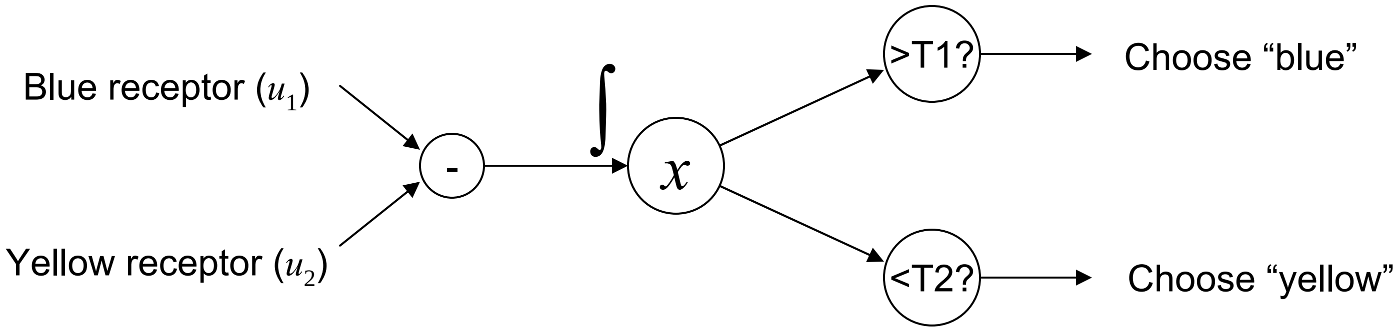
$$\boxed{x(N) = x(0) + \sum_{k=1}^{N} \alpha(u_1(k) - u_2(k)) + noise}$$

- Update your decision variable x as you sample sensory input
- $x(0)$: Initial state
- u_1 : Momentary sensory evidence for choice 1
- u_2 : Momentary sensory evidence for choice 2
- α : Rate of integration
- In a continuous form: $$x(t) = x(0) + \int_0^t \left(\alpha(u_1(\tau) - u_2(\tau)) + noise\right) d\tau$$

$$\boxed{\frac{dx}{dt} = \alpha(u_1 - u_2) + noise}$$

Integration of differential evidence

- "Diffusion model" (Ratcliff, 1978):



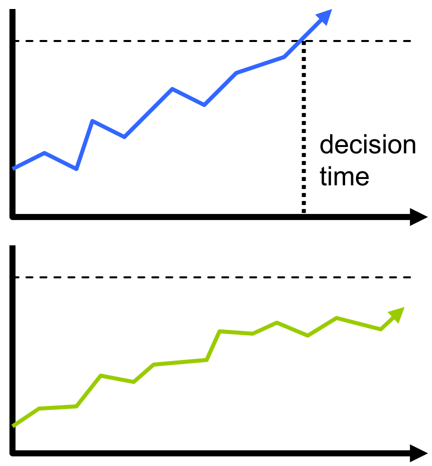
- Integration of the *difference* of evidence and comparison to thresholds

$$\frac{dx}{dt} = \alpha(u_1 - u_2) + noise$$

- Realistic?
  - Neural variables can't be negative
  - Inhibition and excitation are not naturally balanced
  - What about more than 2 options?

Independent integration

- "Race model" (Vickers, 1970)



$$\frac{dx_1}{dt} = \alpha u_1 + noise$$

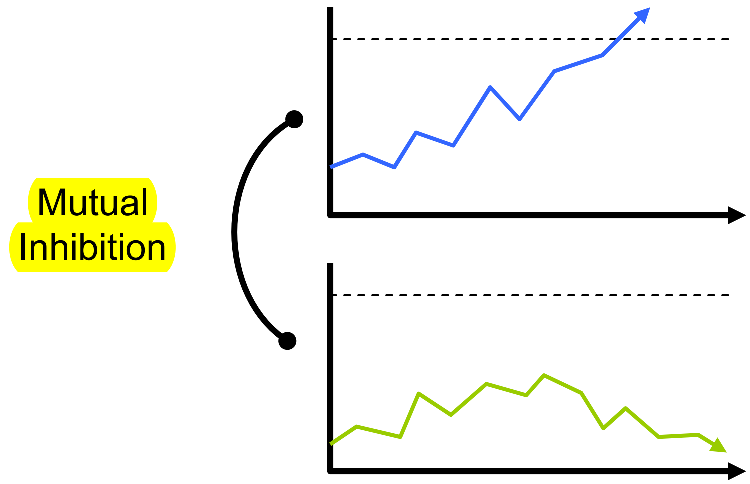
$$\frac{dx_2}{dt} = \alpha u_2 + noise$$

- Independent processes race to their individual thresholds, and whichever arrives first, wins the decision
- Perfect integration vs. leakage

$$\frac{dx_1}{dt} = \alpha u_1 - \beta x_1 + noise \qquad \frac{dx_2}{dt} = \alpha u_2 - \beta x_2 + noise$$

Competing integration

- "Leaky competing accumulator model" (Usher & McClelland, 2001)



$$\frac{dx_1}{dt} = \alpha u_1 - \beta x_1 - \gamma x_2 + noise$$

$$\frac{dx_2}{dt} = \alpha u_2 - \beta x_2 - \gamma x_1 + noise$$

- If $\beta = \gamma$ then it's equivalent to the diffusion model
- But it can handle multiple options, and is biologically plausible

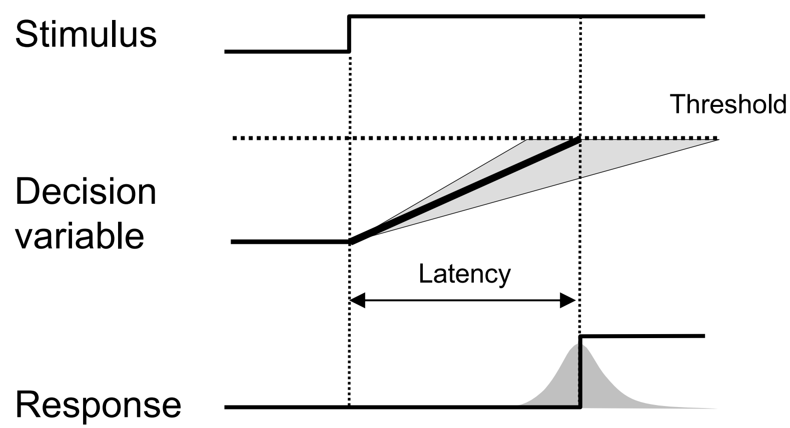
$$x_d = x_1 - x_2$$

$$\frac{dx_d}{dt} = \alpha u_1 - \beta x_1 - \gamma x_2 - \alpha u_2 + \beta x_2 + \gamma x_1 + noise$$

$$= \alpha(u_1 - u_2) + noise$$

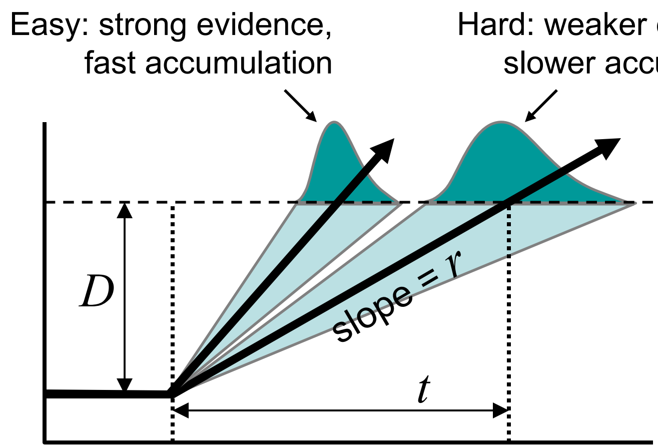
Behavioral predictions

- Accumulator models predict a specific relationship between strength of evidence and reaction times
- Ex: “LATER” model (Carpenter & Williams, 1995)



- Given Gaussian noise, what is the distribution of reaction times?

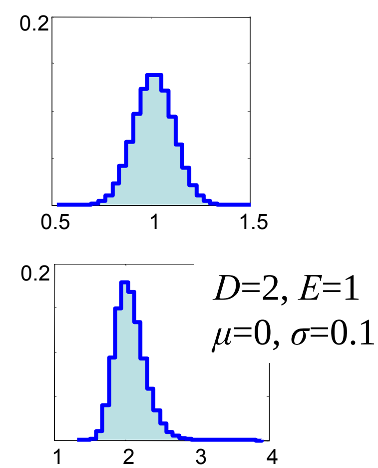
Timing of easy vs. hard choices



$$t = D / r$$

- As a decision gets harder, the reaction time distribution gets later and broader
- If rate is subject to Gaussian noise $r = E + G(\mu, \sigma)$
- Then the distribution of RTs is a skewed Gaussian

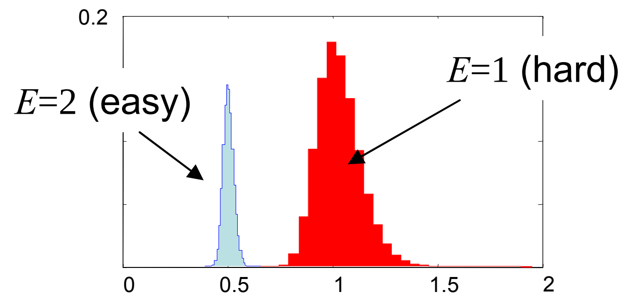
$$t = \frac{D}{E + G(\mu, \sigma)}$$



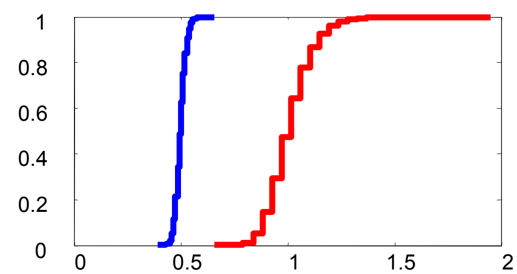
Reciprobit plot

- Suppose we have some distributions of reaction times

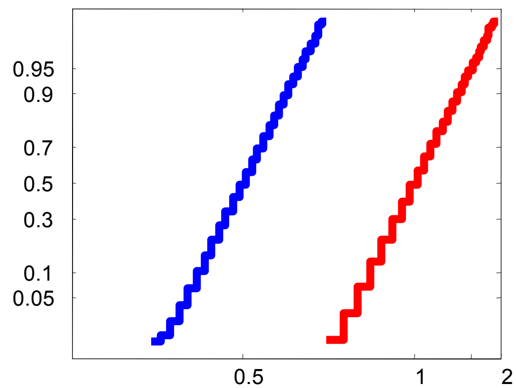
$$t = \frac{D}{E + G(\mu,\sigma)}$$



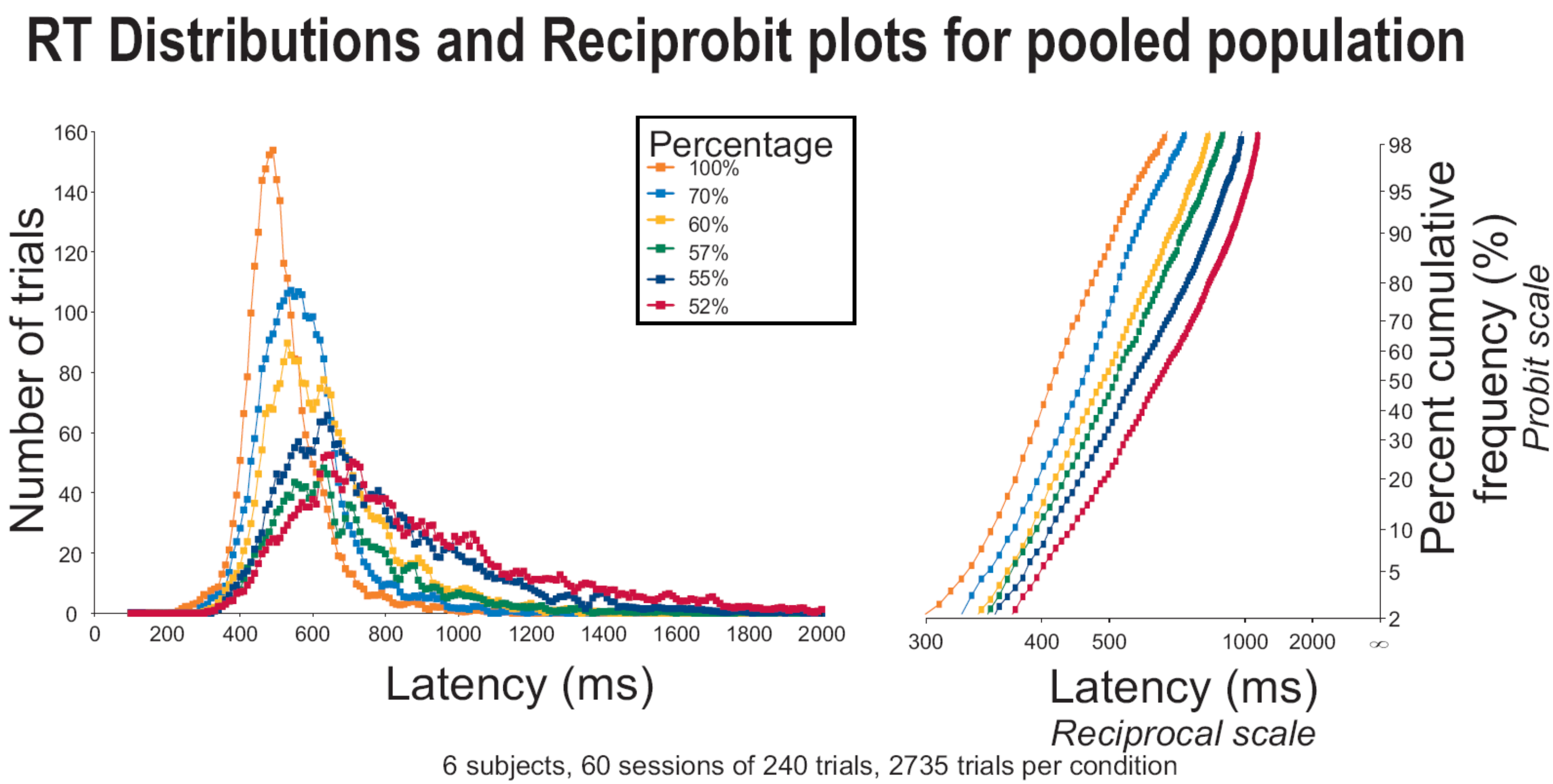
- We can plot these as cumulative RT distributions



- And replot on a "reciprobit" scale
  - Reciprocal x-axis
  - Inverse gaussian y-axis
- Different rates of accumulation produce different parallel lines



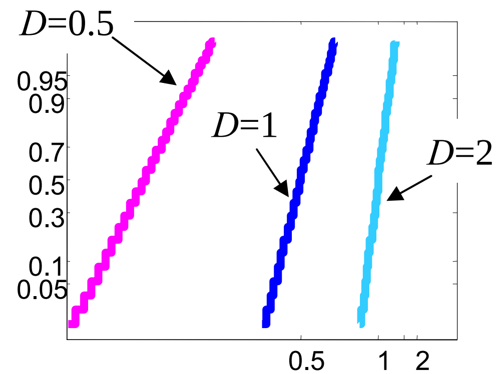
Change difficulty → Parallel lines



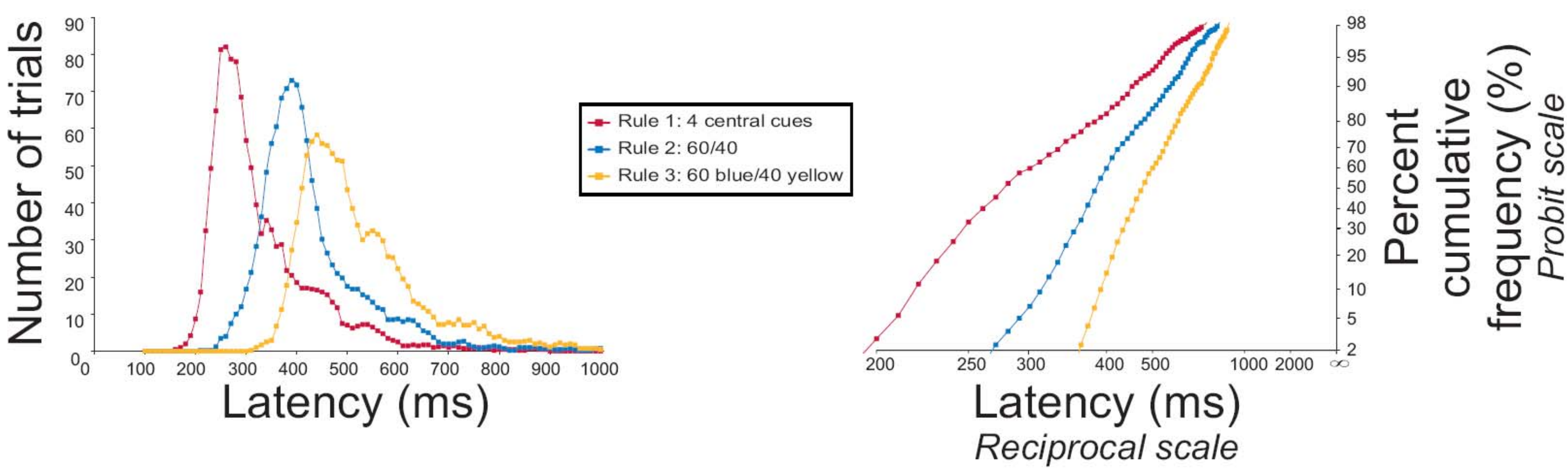
E. Coallier & J. Kalaska

Change threshold → Covergent lines

- Changing the threshold D changes the slopes of the lines so that they converge
- EX: Changing the number of possible conditions



RT Distributions and Reciprobit plots for pooled



4 subjects, 24 sessions of 200 trials, 1150 trials per condition

Why accumulation is a good idea

- Good way to control tradeoff between speed and accuracy
  - If you're getting reliable information, accumulate faster, reach threshold sooner
  - If you're getting weak information, accumulate slower, let more information come in with time…
  - In a situation of urgency, lower the threshold
  - In a situating requiring high accuracy, raise the threshold
- Accumulation maximizes the expected value of choices

Bayes' rule

- The probability of A: $p(A)$
- The probability of B: $p(B)$
- What is the probability of both A and B?

$$p(A \cap B) = p(A \mid B)\,p(B) = p(B \mid A)\,p(A)$$

posterior probability

likelihood

prior probability

$$\boxed{p(A \mid B) = \frac{p(B \mid A)\,p(A)}{p(B)}}$$

normalizing factor

How to maximize expected value?

- Suppose you have to guess between two mutually-exclusive hypotheses, h1 and h2, exactly one of which is correct
  - The consequences of being right or wrong
    - If you assume h1 and are correct, you win W1
    - If you assume h1 but are wrong, you win L1 (“loss”: it’s negative)
    - If you assume h2 and are correct, you win W2
    - If you assume h2 and are wrong, you win L2
- So the expected values are:
  - EV for assuming h1: $p(h_1)W_1 + p(h_2)L_1$
  - EV for assuming h2: $p(h_2)W_2 + p(h_1)L_2$
- Therefore, you should choose h1 when $EV_1 > EV_2$, or when

$$p(h_1)W_1 + p(h_2)L_1 > p(h_2)W_2 + p(h_1)L_2 \qquad \frac{p(h_1)(W_1 - L_2)}{p(h_2)(W_2 - L_1)} > 1$$

Taking evidence into account

- Suppose you have some sensory cue e which tells you something about how likely it is that h1 or h2 is true?

- So, given e, you can update your decision rule as follows

$$\frac{p(h_1)(W_1 - L_2)}{p(h_2)(W_2 - L_1)} > 1 \quad \Rightarrow \quad \frac{p(h_1 \mid e)(W_1 - L_2)}{p(h_2 \mid e)(W_2 - L_1)} > 1$$

- Bayes rule tells you that:

$$p(h_1 \mid e) = \frac{p(e \mid h_1)\, p(h_1)}{p(e)}$$

- So:

$$\frac{\dfrac{p(e \mid h_1)\, p(h_1)}{\cancel{p(e)}}(W_1 - L_2)}{\dfrac{p(e \mid h_2)\, p(h_2)}{\cancel{p(e)}}(W_2 - L_1)} > 1 \qquad \frac{p(e \mid h_1)\, p(h_1)(W_1 - L_2)}{p(e \mid h_2)\, p(h_2)(W_2 - L_1)} > 1$$

New evidence comes in, update again

- Previous evidence e_1
- New evidence e_2
- Replace $p(h_x)$ with $p(h_x|e_2)$

$$\frac{p(e_1 \mid h_1)\,p(h_1)(W_1 - L_2)}{p(e_1 \mid h_2)\,p(h_2)(W_2 - L_1)} > 1 \qquad \frac{p(e_1 \mid h_1)\,p(e_2 \mid h_1)\,p(h_1)(W_1 - L_2)}{p(e_1 \mid h_2)\,p(e_2 \mid h_2)\,p(h_2)(W_2 - L_1)} > 1$$

- In other words, each time new evidence comes in, multiply numerator by $p(e_k|h_1)$ and denominator by $p(e_k|h_2)$

$$\frac{\prod_{k=1}^{N}\big(p(e_k \mid h_1)\big)p(h_1)(W_1 - L_2)}{\prod_{k=1}^{N}\big(p(e_k \mid h_2)\big)p(h_2)(W_2 - L_1)} > 1$$

Take the log

- Turn $$\frac{\prod_{k=1}^{N}\left(p(e_k \mid h_1)\right)p(h_1)(W_1 - L_2)}{\prod_{k=1}^{N}\left(p(e_k \mid h_2)\right)p(h_2)(W_2 - L_1)} > 1$$

- Into

$$\sum_{k=1}^{N}\left(\log p(e_k \mid h_1)\right) + \log p(h_1) + \log(W_1 - L_2) > \sum_{k=1}^{N}\left(\log p(e_k \mid h_2)\right) + \log p(h_2) + \log(W_2 - L_1)$$

- i.e. choose h1 if the above relation is true

- Let's define "desirability" of a given choice i as

$$D_i = \sum_{k=1}^{N}\left(\log p(e_k \mid h_i)\right) + \log p(h_i) + \log(W_i - L_j)$$

Making choices in time

- How much evidence is enough evidence?
- Suppose you are receiving sensory evidence for one of two possible perceptual judgments, where one and only one is true, and they both offer the same payoff (e.g. W1=W2, L1=L2=0)
- Then simply choose h1 if

$$\log p(h_1) - \log p(h_2) + \sum_{k=1}^{N} \left(\log p(e_k \mid h_1)\right) - \sum_{k=1}^{N} \left(\log p(e_k \mid h_2)\right) > 0$$

- and h2 if

$$\log p(h_2) - \log p(h_1) + \sum_{k=1}^{N} \left(\underbrace{\log p(e_k \mid h_2)}_{\text{“log likelihood”}}\right) - \sum_{k=1}^{N} \left(\log p(e_k \mid h_1)\right) > 0$$

Do something or nothing

- Doing nothing can also be desirable!
  - Not just for the obvious reasons…
  - If you don't commit to a decision, more information might become available
- Quantify desirability of waiting using some constant D_0
  (*relative* desirability of doing nothing over making a choice)
- So choose h1 if

$$\log p(h_1) - \log p(h_2) + \sum_{k=1}^{N} \left(\log p(e_k \mid h_1)\right) - \sum_{k=1}^{N} \left(\log p(e_k \mid h_2)\right) > D_0$$

- or h2 if

$$\log p(h_2) - \log p(h_1) + \sum_{k=1}^{N} \left(\log p(e_k \mid h_2)\right) - \sum_{k=1}^{N} \left(\log p(e_k \mid h_1)\right) > D_0$$

- otherwise do nothing and wait for more information.

This policy reduces to the diffusion model

- Our policy:
  - Choose h1 if $\log p(h_1) - \log p(h_2) + \sum_{k=1}^{N}\left(\log p(e_k \mid h_1)\right) - \sum_{k=1}^{N}\left(\log p(e_k \mid h_2)\right) > D_0$
  - Else wait

- Note: If we define $u_i(t) = \frac{1}{\alpha}\log p(e(t) \mid h_i)$ "log likelihood"

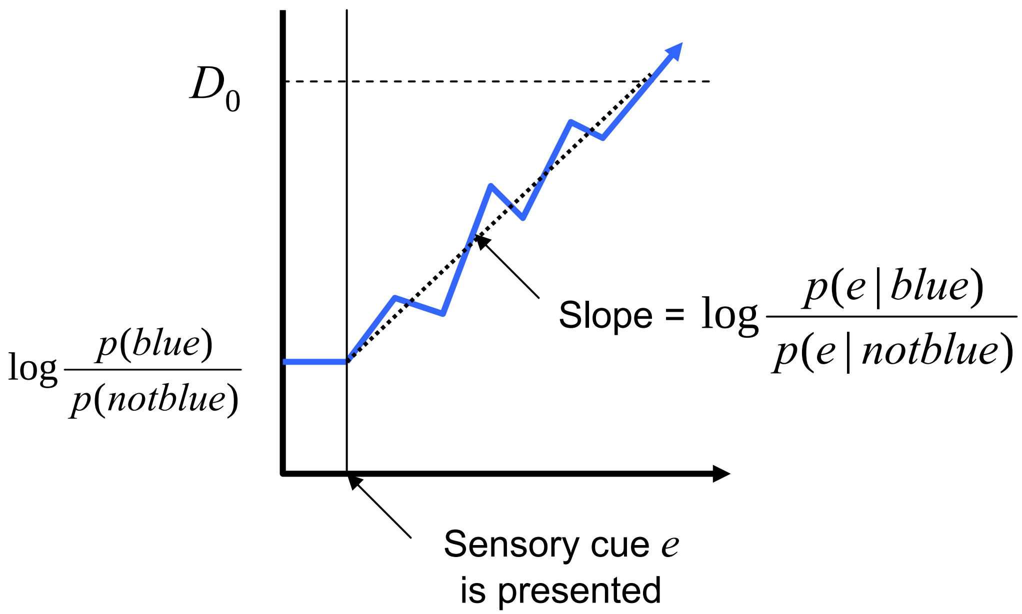
  and $x(0) = \log {}^{p(h_1)}\!/_{p(h_2)}$ "relative prior probability"

- Then this is equivalent to with threshold D_0

$$x(N) = x(0) + \sum_{k=1}^{N} \alpha \underbrace{(u_1(k) - u_2(k))}_{\text{"log likelihood ratio"}} + noise$$

- In other words… the diffusion / leaky competing accumulator model
- a.k.a. "Sequential probability ratio test" (Wald, 1945)

Race models re-examined

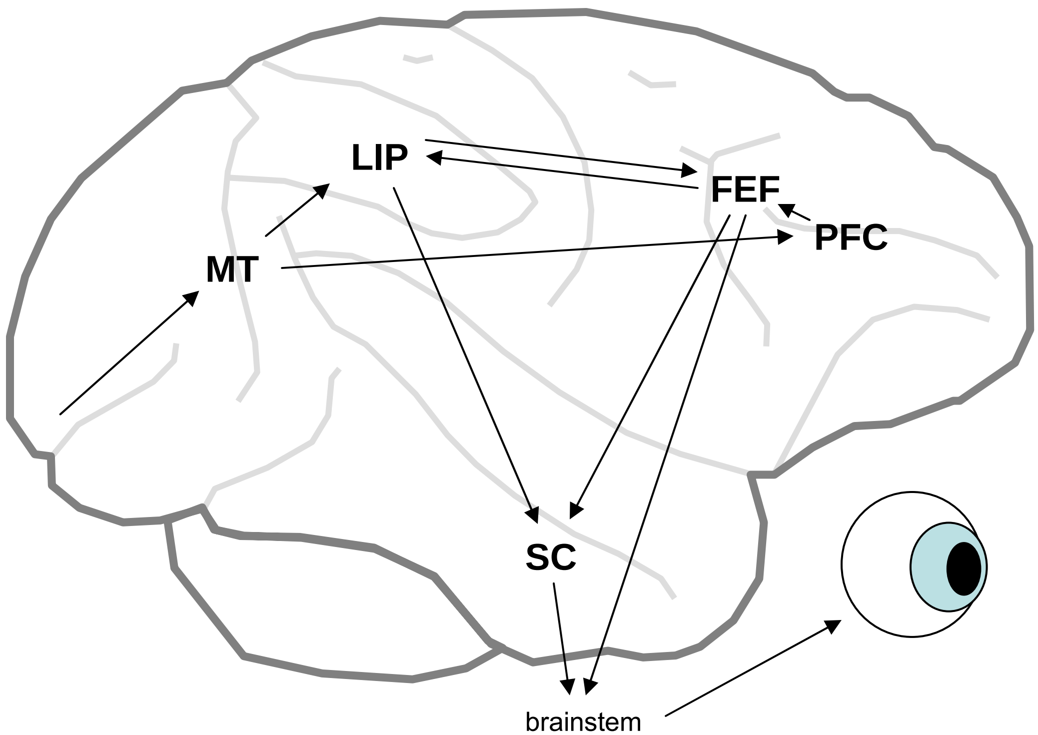


- Want to initialize the accumulation at a level of activity proportional to logarithm of relative prior probability of our hypothesis.
- Once a sensory cue appears, we want to accumulate activity at a rate proportional to the log of the likelihood ratio of that cue appearing if our hypothesis is true versus if it isn't.
- Threshold is proportional to the relative desirability of waiting versus taking a chance that our hypothesis is true (speed vs. accuracy tradeoff)

Caveats

- Again, people do not maximize expected value
  - Risk aversion
  - Exaggeration of small probabilities
- Nevertheless, the concept of accumulation to threshold is very successful at explaining a large variety of behavioral data
  - Reaction times
  - Error rates
- What is being accumulated?
  - Log likelihood ratios?
  - Subjective desirability?
- Where does this accumulation take place?
  - Example: Decisions about where to move the eye

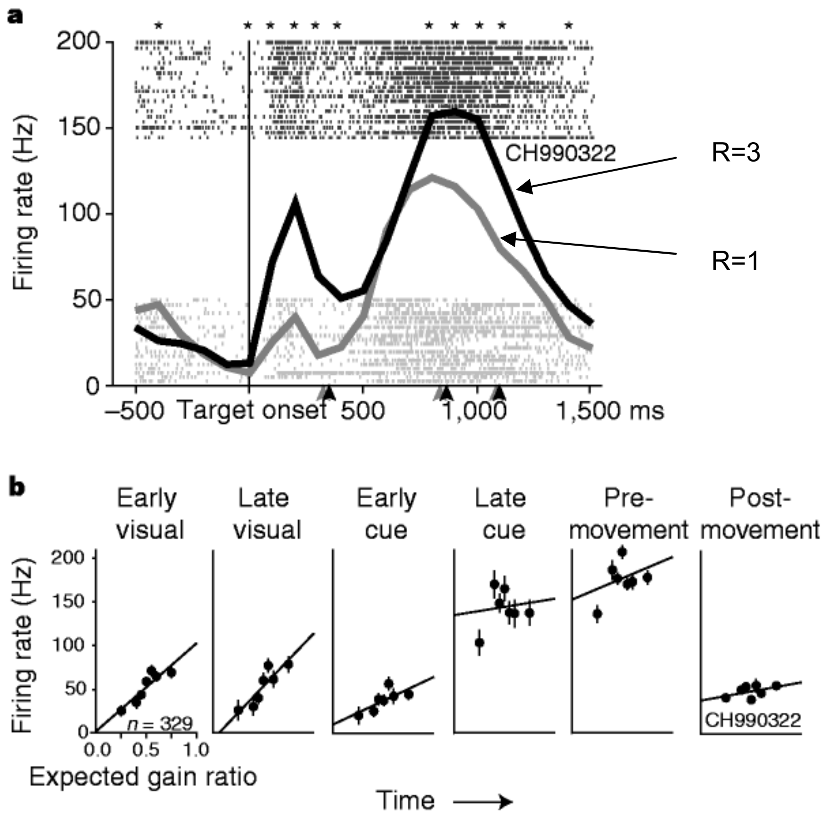
Cortical circuits for eye movement



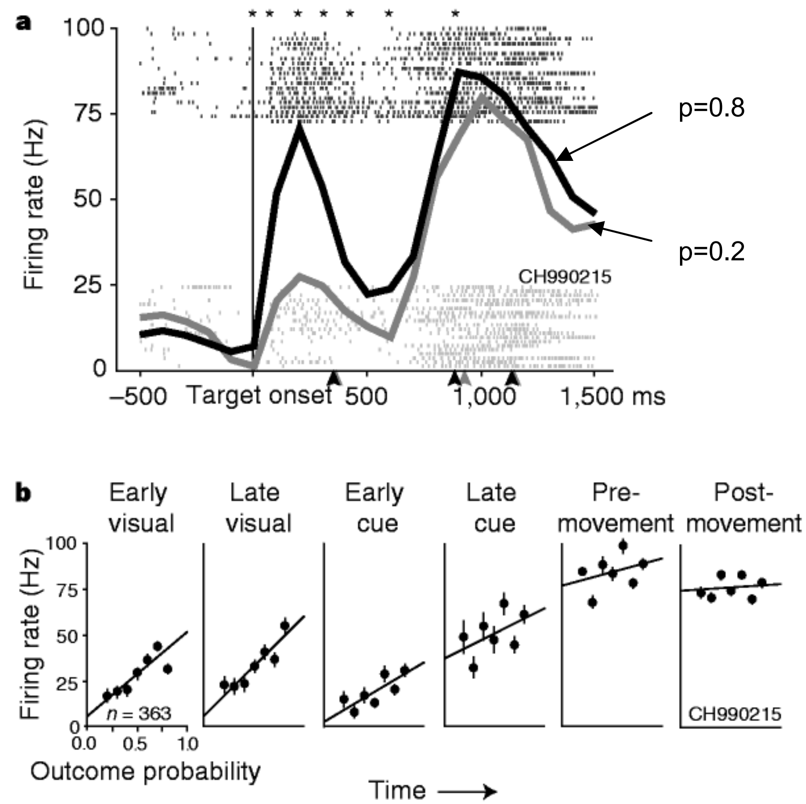
- Medial temporal area (MT)
- Lateral intraparietal area (LIP)
- Frontal eye fields (FEF)
- Prefrontal cortex (PFC)
- Superior colliculus (SC)
- Brainstem

Decision variables in LIP

- Different blocks of trials: Different rewards for moving into RF versus away from RF



- Different blocks: Different probability of movement target in RF versus away



Platt & Glimcher (1999) *Nature* “Neural correlates of decision variables in parietal cortex”

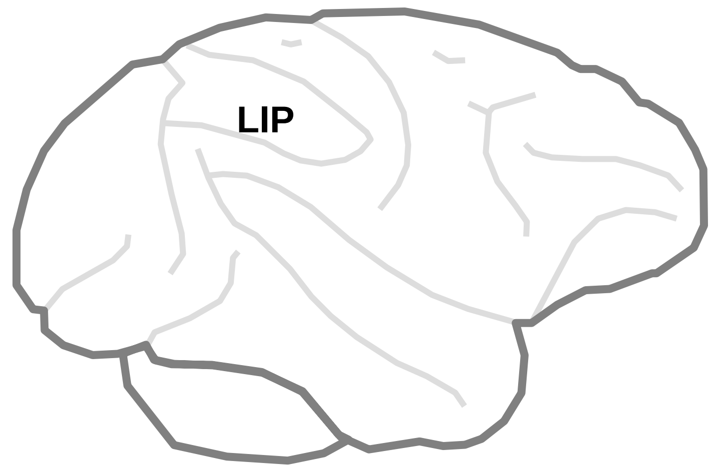
→ *“Neuroeconomics”*

So what does LIP do?

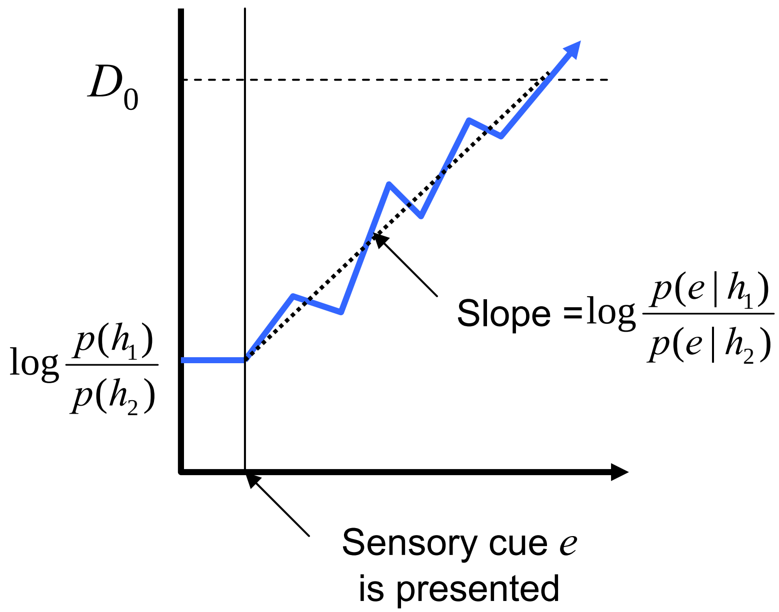
- In-between sensory and motor
- Representation of “expected value”?
- Recall:
  - Expected value

$$EV = p(V_1)V_1$$

  - The payoff (V_1) multiplied by its probability ($p(V_1)$)
  - LIP seems to respond to both kinds of information



Optimal policy



$$x(N) = x(0) + \sum_{k=1}^{N} \alpha(u_1(k) - u_2(k)) + noise$$

where $x(0) = \log \frac{p(h_1)}{p(h_2)}$

$$u_i(t) = \tfrac{1}{\alpha} \log p(e(t) \mid h_i)$$

and continue until $x(N) > D_0$

- Want to initialize the accumulation at a level of activity proportional to logarithm of relative prior probability of our hypothesis.
- Once a sensory cue appears, we want to accumulate activity at a rate proportional to the log of the likelihood ratio of that cue appearing if our hypothesis is true versus if it isn't.
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Including costs and risks

$$\frac{\prod_{k=1}^{N}\left(p(e_k \mid h_1)\right)p(h_1)(W_1 - L_2)}{\prod_{k=1}^{N}\left(p(e_k \mid h_2)\right)p(h_2)(W_2 - L_1)} > 1$$

$$x(N) = x(0) + R + \sum_{k=1}^{N} \alpha(u_1(k) - u_2(k)) + noise$$

where $x(0) = \log \frac{p(h_1)}{p(h_2)}$

$$R = \log \frac{(W_1 - L_2)}{(W_2 - L_1)}$$

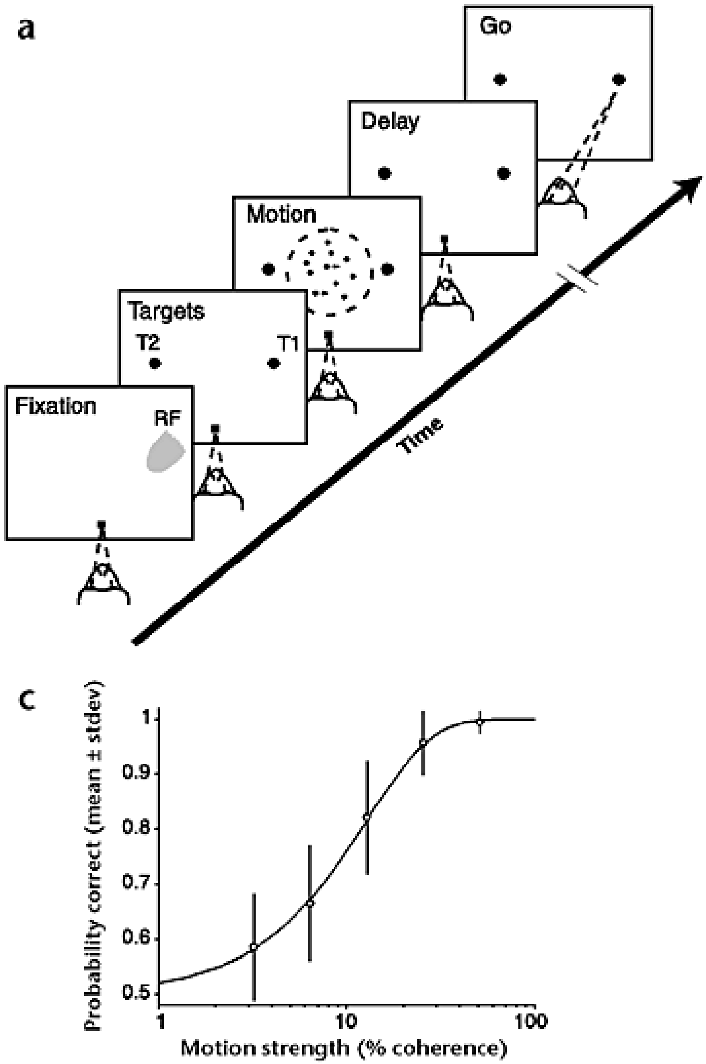
EV

$$u_i(t) = \tfrac{1}{\alpha} \log p(e(t) \mid h_i)$$

and continue until $x(N) > D_0$

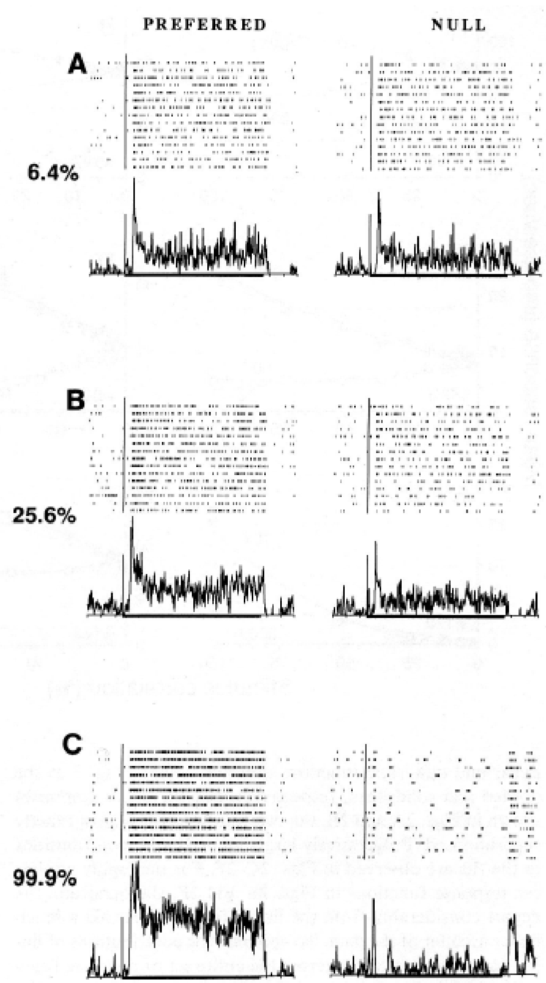
– We can include the costs as another term in the equation
– Both the probability and the payoff should be represented
– Can the model be used to interpret neural data?

Coherent motion discrimination task



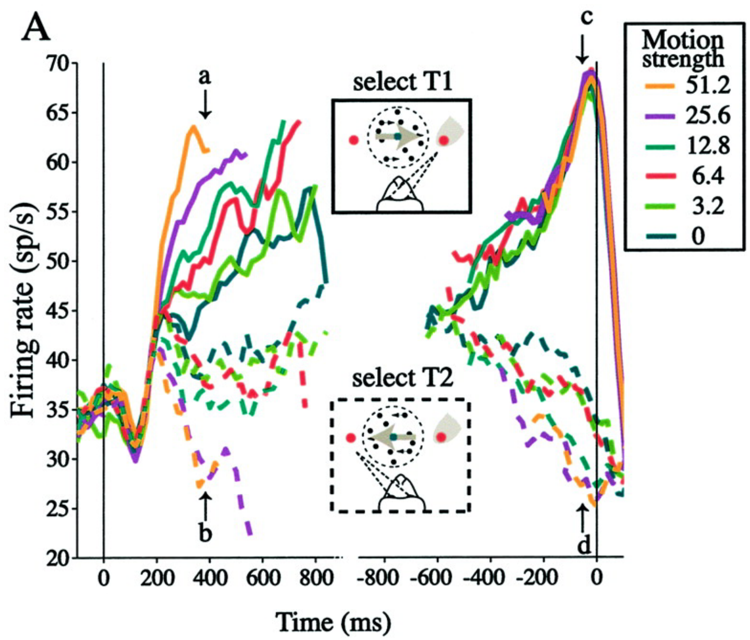
- Monkey trained to discriminate the direction of motion
- Perceptual report made by a saccade to a target in the direction of the motion
- Two versions
  - Fixed Duration
  - Reaction Time

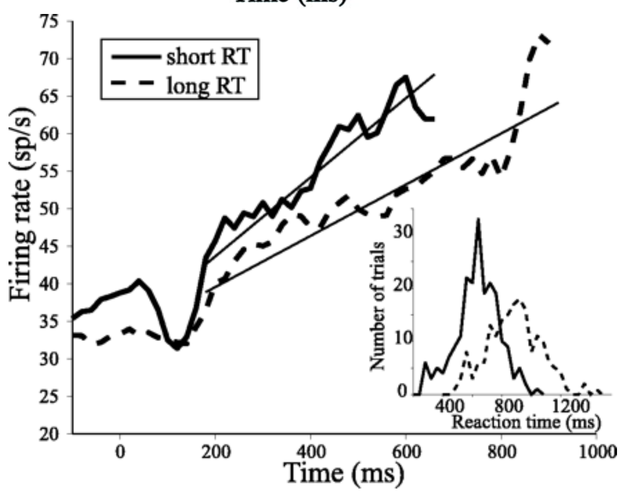
Activity in medial temporal area (MT)



- Britten, Shadlen, Newsome, & Movshon (1993) *Visual Neuroscience*
- Neurons in area MT are sensitive to the direction of visual motion signals
- During coherent motion viewing, the response of these neurons depends upon the coherence of the motion stimulus

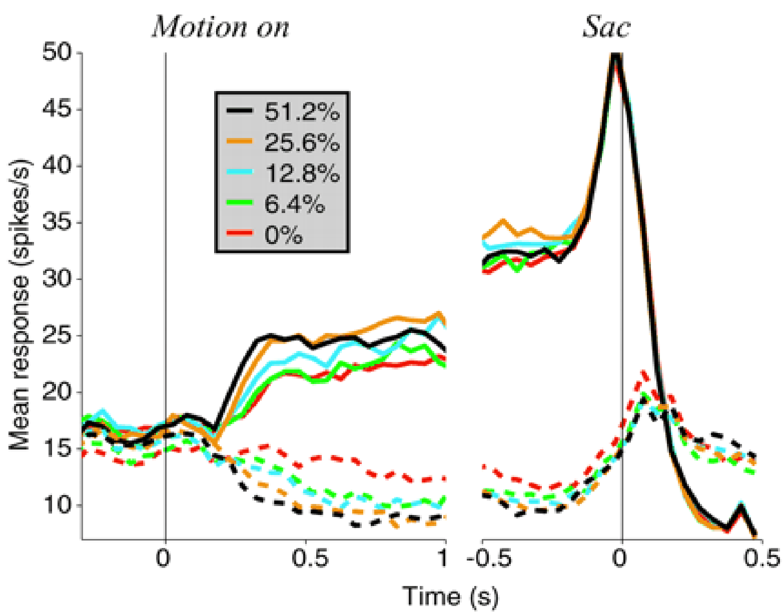
Activity in LIP (reaction time task)

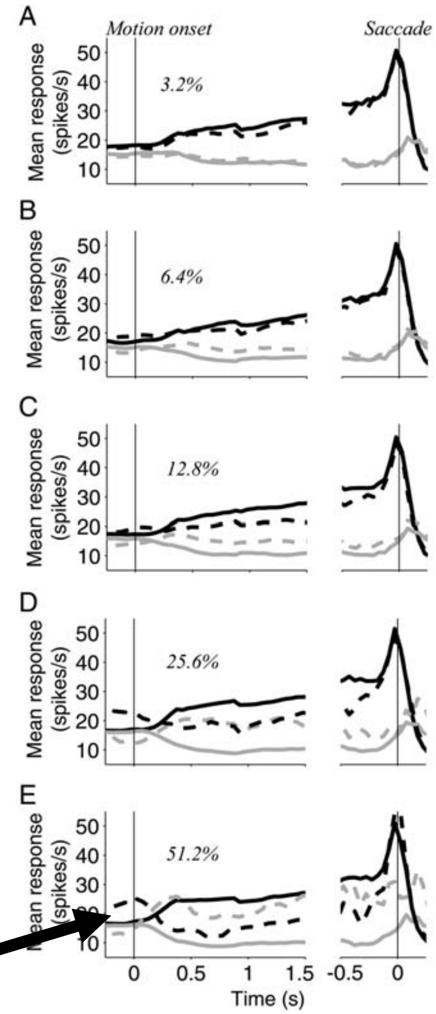




- Roitman & Shadlen (2002) *J. Neurosci*
- Neural activity in LIP grows at a rate related to the coherence of the motion stimulus
- Rate of activity growth predicts the reaction time

Activity in LIP (fixed duration task)

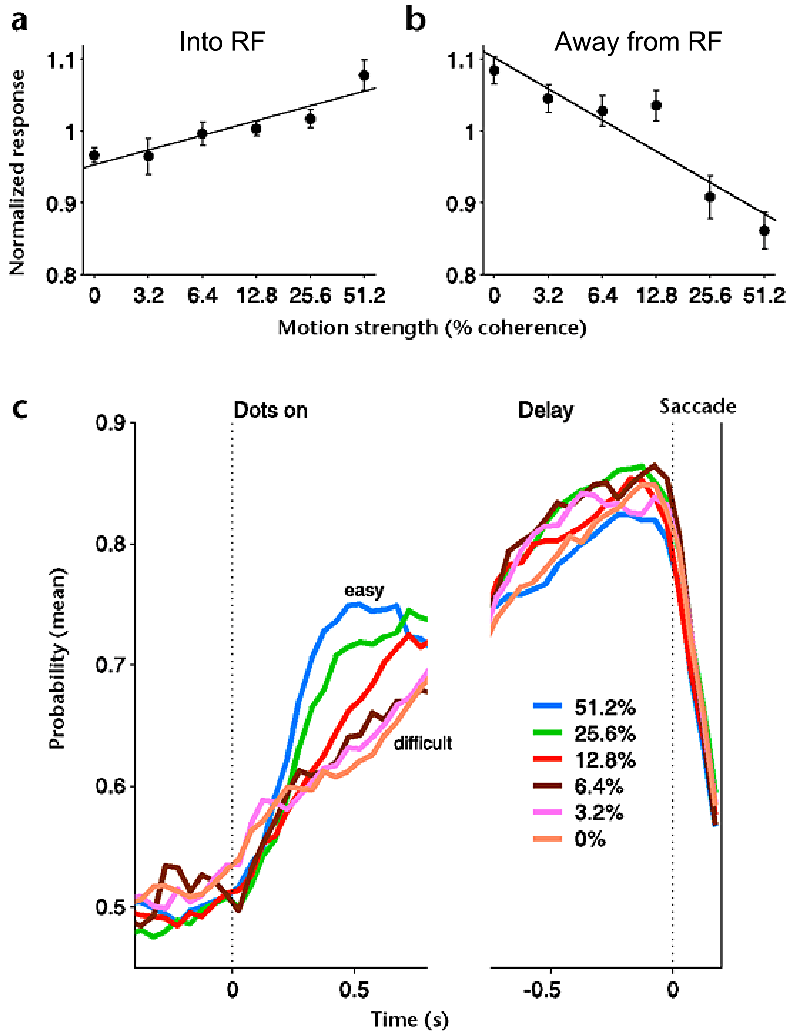




- Shadlen & Newsome (2001) *J. Neurophysiol*
- Neural activity in area LIP predicts the choice that the monkey will make, and reflects the strength of the motion signal

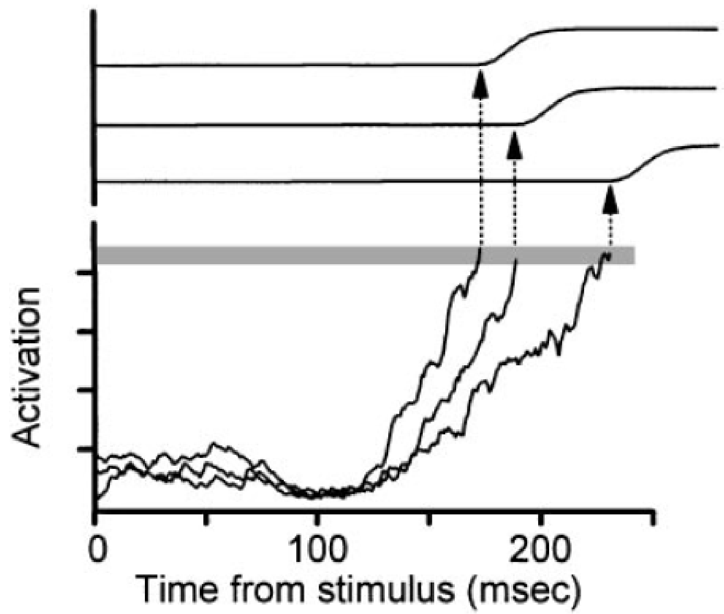
Note the bias!

Activity in prefrontal cortex (PFC)



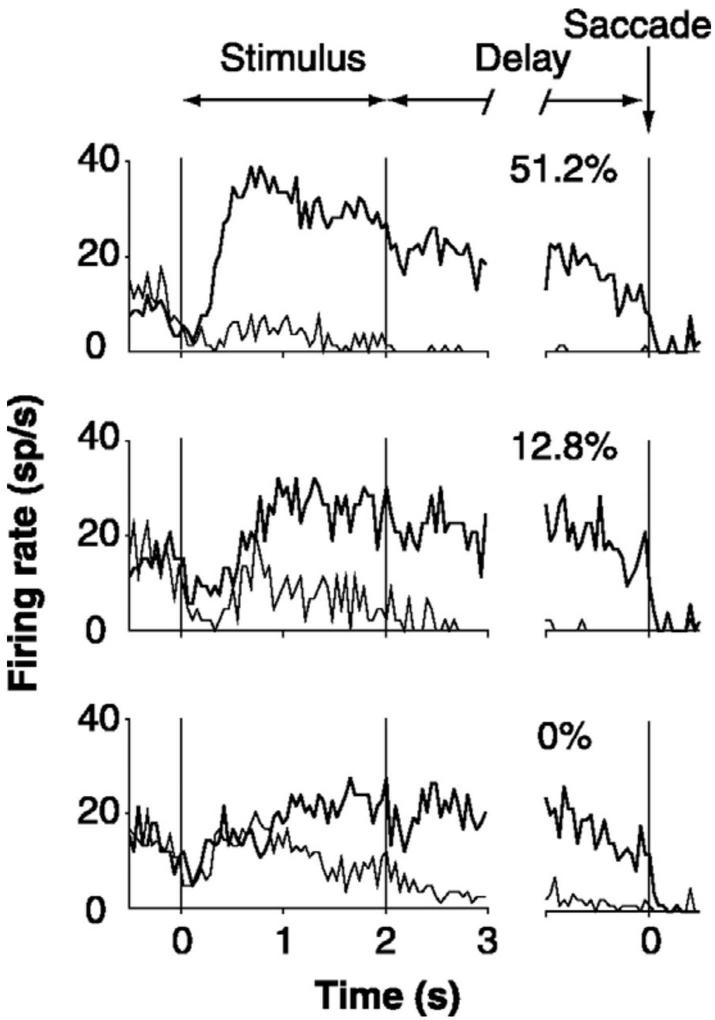
- Kim & Shadlen (1999) *Nature Neuroscience*
- Neural activity is proportional to the coherence
- Probability of predicting the monkey's choice on the basis of neural activity grows as a function of coherence.

Initiation threshold in FEF



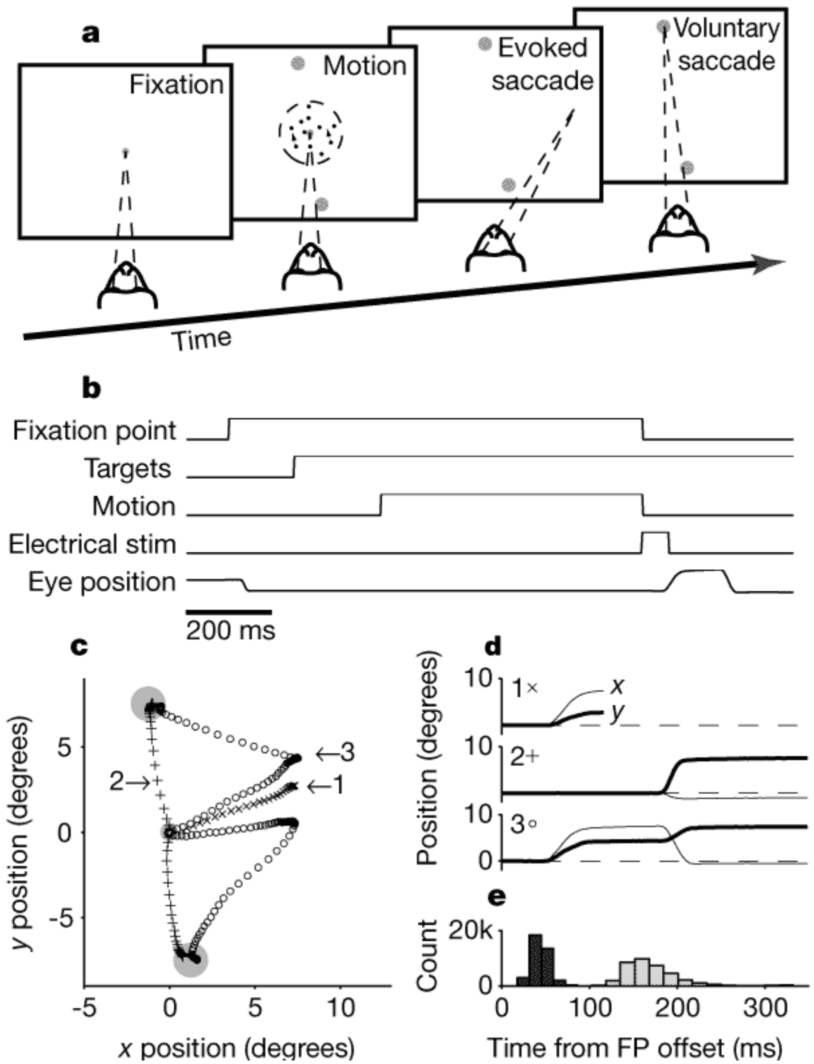
- Schall & Thompson (1999) *Annual Review of Neuroscience*
- FEF activity (of movement-related neuron) predicts the time of a saccade

Activity in the superior colliculus

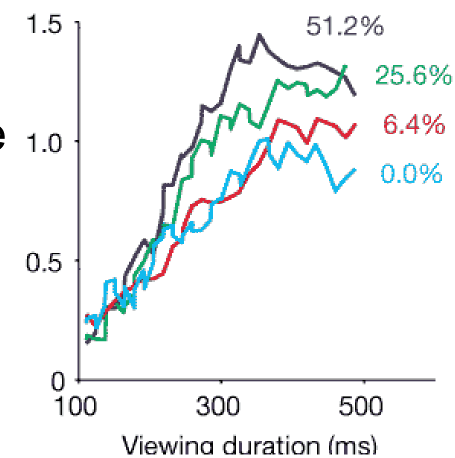


- Horwitz and Newsome (1999) *Science*
- Neural activity predicts the monkey's choice, even when the stimulus coherence is 0%
- Early activity dependent on coherence
- Late activity simply predicts the movement

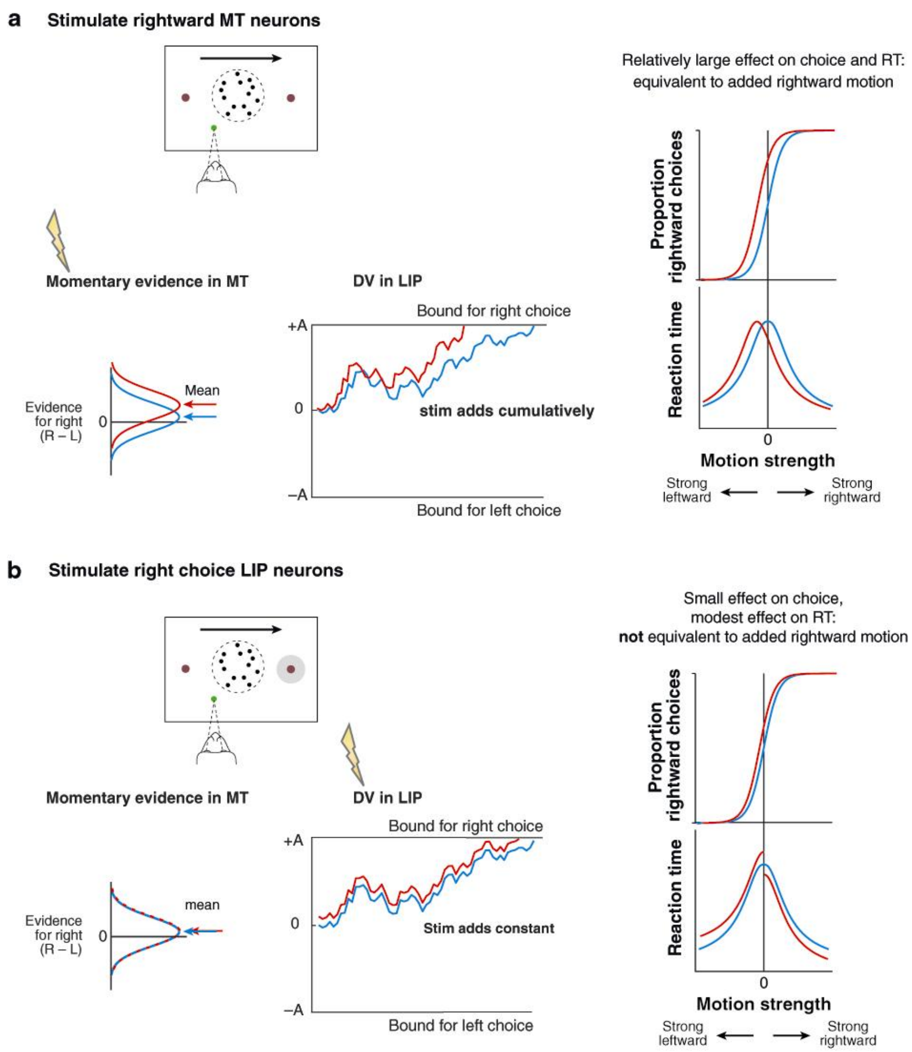
Decisions spilling into commands



- Gold & Shadlen (2000) *Nature*
- Microstimulation in frontal eye fields produces saccades
- If microstimulation occurs while the monkey is in the process of deciding between stimulus motion, the saccade is deviated in the direction of the developing motor command
- A function of coherence and viewing duration



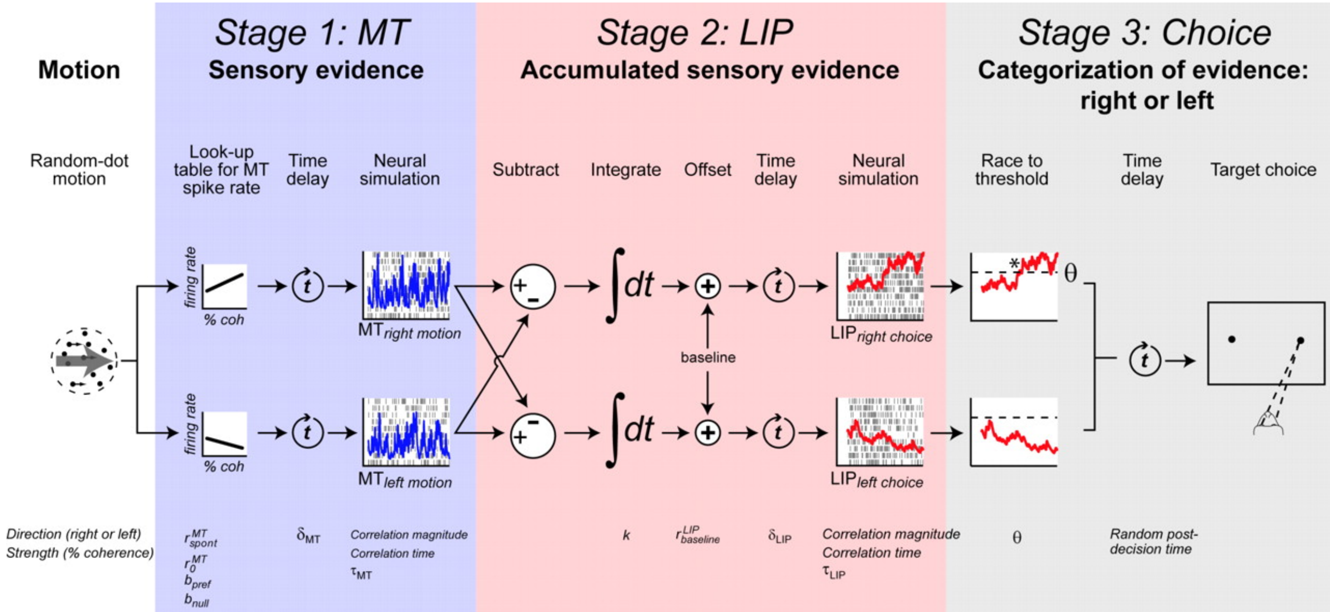
Changing the mind



Gold JI, Shadlen MN. 2007. Annu. Rev. Neurosci. 30:535–74

- Intracortical microstimulation
  - By stimulating rightward preferring cells we can bias the decision toward the right, and modify the RT
- Stimulation in MT
  - Shifts decision and timing
  - Acts like a change in the strength of rightward motion
- Stimulation in LIP
  - Small shift in decision, speed up rightward RT, slow down leftward RT
  - Acts like a change in the threshold

A neural model

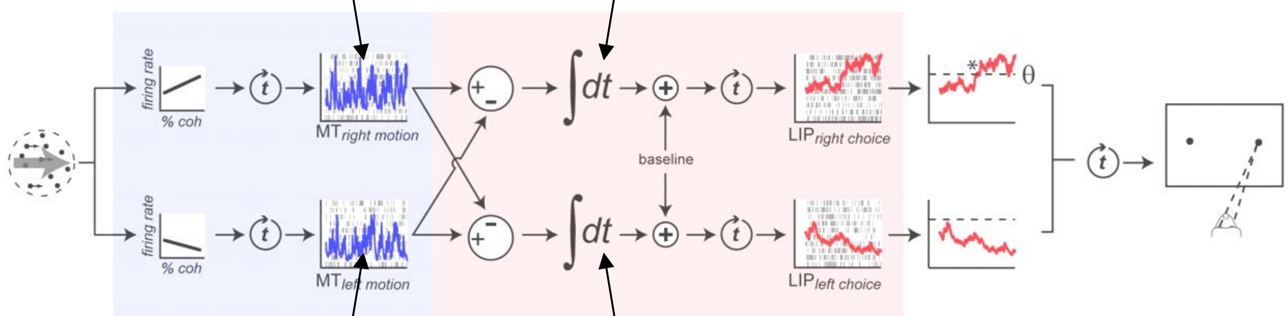


- Mazurek, Roitman, Ditterich, & Shadlen (2003) *Cerebral Cortex*
- Three stages:
  1. Detection of sensory evidence (area MT)
  2. Accumulation of sensory evidence for a given choice (LIP / PFC / FEF)
  3. Categorization of evidence (?)

A neural model

$$r_{right}^{MT}$$

$$r_{right}^{LIP}(T) = r_{baseline}^{LIP} + k \int_{\delta_{MT}}^{T-\delta_{LIP}} r_{right}^{MT}(t) - r_{left}^{MT}(t)\,dt$$



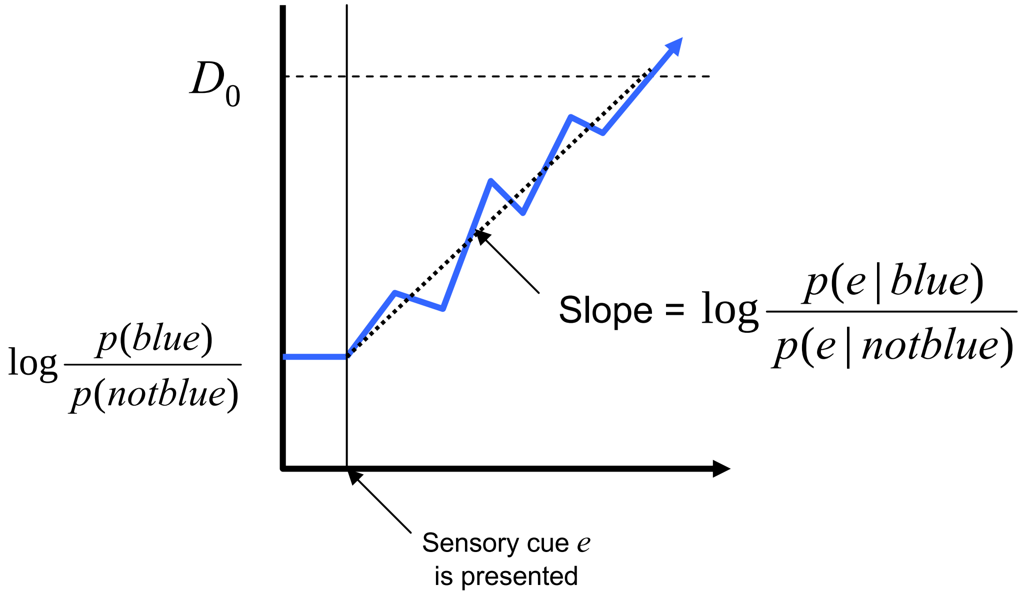
$$r_{left}^{MT}$$

$$r_{left}^{LIP}(T) = r_{baseline}^{LIP} + k \int_{\delta_{MT}}^{T-\delta_{LIP}} r_{left}^{MT}(t) - r_{right}^{MT}(t)\,dt$$

- Note: This is equivalent to the diffusion model

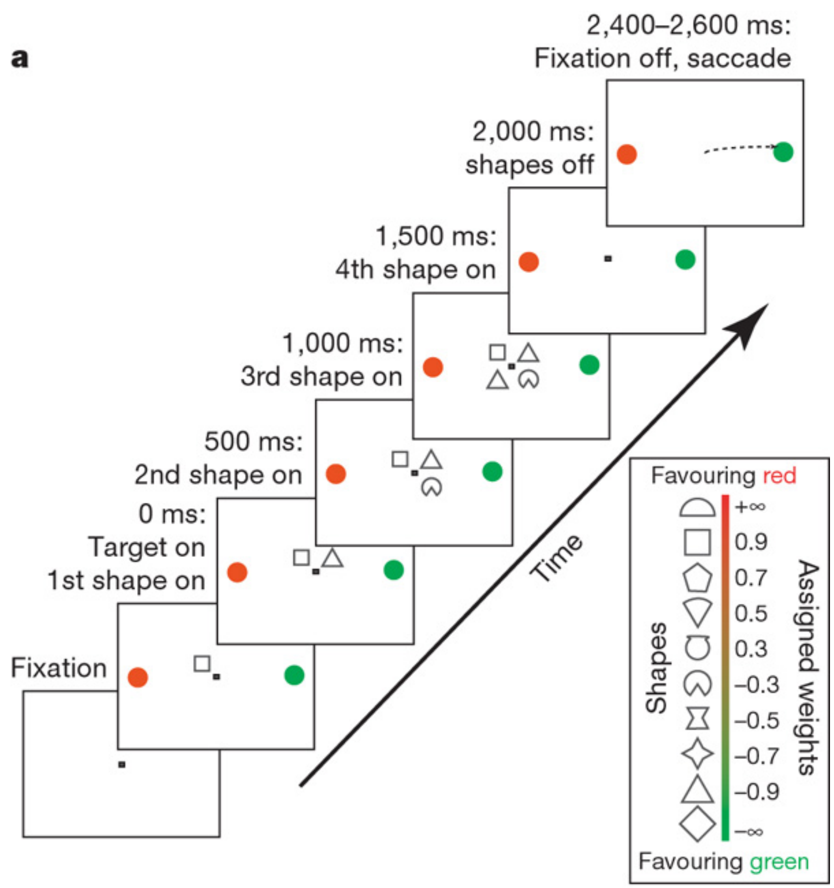
$$x(N) = x(0) + \sum_{k=1}^{N} \alpha(u_1(k) - u_2(k)) + noise$$

What's the best way to accumulate?



- Recall
  - Want to initialize at a level proportional to log of the prior probability ratios
  - Each time a new stimulus appears, want to increase activity by the **log likelihood ratio**
  - Is that what is being accumulated in area LIP?

Probabilistic inference by monkeys

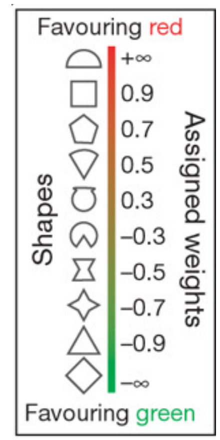


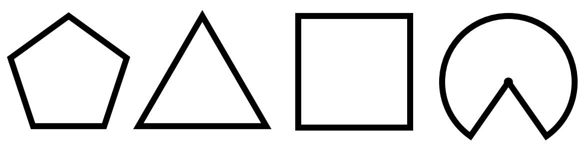
- Yang & Shadlen (2007) *Nature*
- Probabilistic categorization task
  1. Fixate
  2. Two targets (red and green) appear, and a symbol in the center
     - Each shape has a meaning: it favors either the red target or the green target
  3. After 500ms, another shape appears
  4. And another
  5. And another
  6. Now, move to the target which has more evidence
  7. If guessed correctly, receive reward

Weight of evidence

- At the end of the trial, the total evidence from all of the shapes is computed, and the rewarded target assigned by the rule

$$p(R \mid s_1, s_2, s_3, s_4) = \frac{10^S}{1+10^S} \qquad S = \sum_{i=1}^{4} w_i$$



- Example: 

0.7 **-0.9** **0.9** **-0.3**

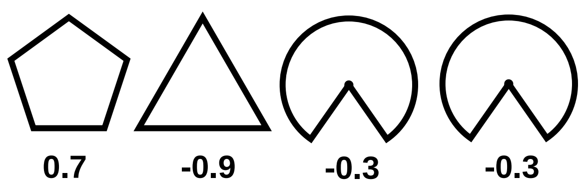
$$S = 0.4 \qquad p(R \mid s_1, s_2, s_3, s_4) = \frac{10^{0.4}}{1+10^{0.4}} = \frac{2.51}{3.51} = 0.71$$

- The monkey should guess “red”

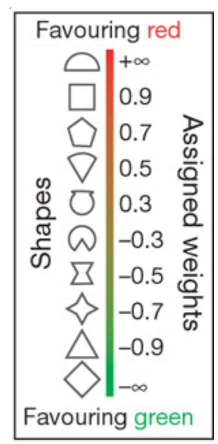
What's the right way to accumulate the evidence?

$$\log \frac{p(red)}{p(green)} + \sum_{i=1}^{N} \left(\log \frac{p(s_i \mid red)}{p(s_i \mid green)} \right) > 0$$

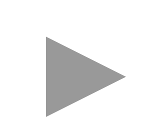
- Start at a level determined by priors (since they are equal, start unbiased)
- Add up the log likelihood ratios each time a symbol appears
- Example:



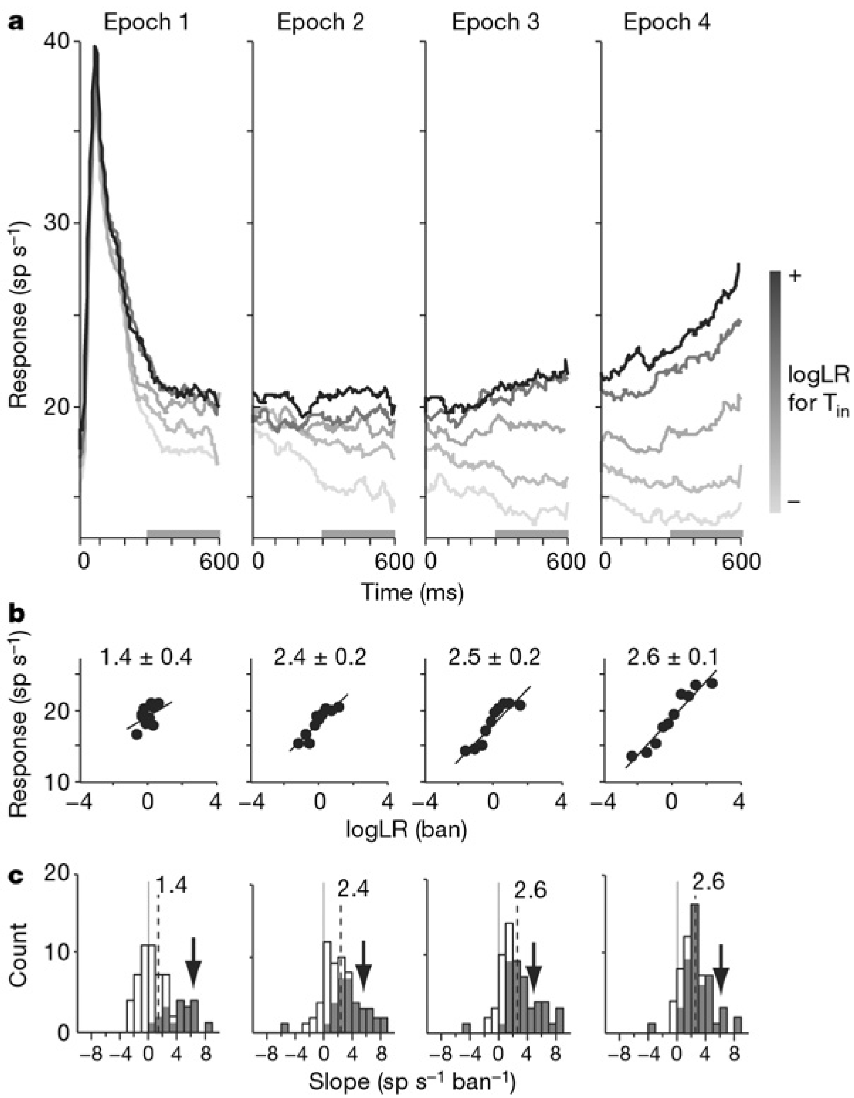
  – First favor red, then choose green



A cell which prefers lower right (red)



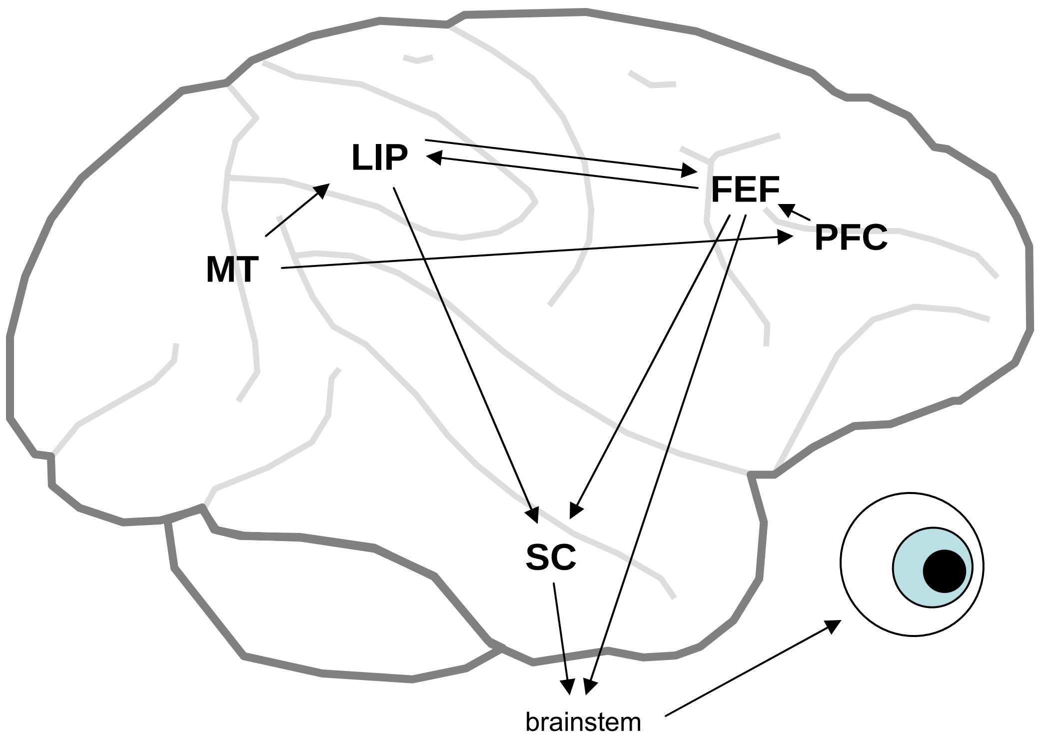
LogLR in LIP



- Each time a symbol appears, neural activity in LIP changes.
- The magnitude of the change is a function of the logLR conveyed by each symbol
  - Not quite a linear function, but pretty good
  - Slope ≈ 2.5 spikes/sec/ban
    - (1 ban = 10:1, 2 bans = 100:1)
- Neural activity reflects the subjective weight of evidence that the monkeys use to make their decisions

$$FR \approx b + \sum_{i=1}^{N} \left(\log \frac{p(s_i \mid red)}{p(s_i \mid green)} \right)$$

Brain regions again



- Medial temporal area (MT)
- Lateral intraparietal area (LIP)
- Frontal eye fields (FEF)
- Prefrontal cortex (PFC)
- Superior colliculus (SC)
- Brainstem

NOTE: “Decision variables” appear in nearly every structure studied, including those responsible for *movement control*

Summary

- Accumulator models
- Neural data on simple decisions supports the models
  - Activity in sensory area (MT) provides the input (u_i)
  - Activity in parietal area (LIP) reflects the evidence (x_i)
  - FEF initiates movement when threshold is reached
  - However…
    - What should we really maximize? EV? Reward rate?
    - What about redundant samples?
- Decision variables are everywhere, not just in “cognitive” regions
- Hypothesis: Simultaneous sensorimotor processing and biased competition
  - Applies to reach decisions
  - Applies to attention
- Suggests alternative ways of thinking about behavior

“Decide: *v.i.* To succumb to the preponderance of one set of influences over another set.”
– Bierce A. (1906) *The Devil’s Dictionary*