STANDARDS FOR EFFICIENT CRYPTOGRAPHY

SEC 2: Recommended Elliptic Curve Domain Parameters

Certicom Research

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1 Introduction

1.1 Overview

This document lists example elliptic curve domain parameters at commonly required security levels for use by implementers of SEC 1 [SEC 1] and other ECC standards like ANSI X9.62 [X9.62], ANSI X9.63 [X9.63], and IEEE 1363 [1363] and IEEE 1363a [1363A].

It is strongly recommended that implementers select parameters from among the parameters listed in this document when they deploy ECC-based products in order to encourage the deployment of interoperable ECC-based solutions.

1.2 Compliance

Implementations may claim compliance with the recommended parameters specified in this document provided some subset of the recommended parameters is used by the cryptographic schemes based on elliptic curve cryptography included in the implementation.

It is envisioned that implementations choosing to comply with this document will typically choose also to comply with its companion document, SEC 1 [SEC 1].

It is intended to make a validation system available so that implementors can check compliance with this document—see the SECG website, www.secg.org, for further information.

1.3 Document Evolution

This document will be reviewed every five years to ensure it remains up to date with cryptographic advances. The next scheduled review will therefore take place in February 2015.

Additional intermittent reviews may also be performed from time-to-time as deemed necessary by the Standards for Efficient Cryptography Group.

1.4 Intellectual Property

The reader's attention is called to the possibility that compliance with this document may require use of an invention covered by patent rights. By publication of this document, no position is taken with respect to the validity of this claim or of any patent rights in connection therewith. The patent holder(s) may have filed with the SECG a statement of willingness to grant a license under these rights on fair, reasonable and nondiscriminatory terms and conditions to applicants desiring to obtain such a license. Additional details may be obtained from the patent holder and from the SECG website, www.secg.org.

1.5 Organization

This document is organized as follows.

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The main body of the document focuses on the specification of recommended elliptic curve domain parameters. Section 2 describes recommended elliptic curve domain parameters over \mathbb{F}_p , and Section 3 describes recommended elliptic curve domain parameters over \mathbb{F}_{2^m} .

The appendices to the document provide additional relevant material. Appendix A provides reference ASN.1 syntax for implementations to use to identify the parameters. Appendix B lists the references cited in the document.

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2 Recommended Elliptic Curve Domain Parameters over \mathbb{F}_p

This section specifies the elliptic curve domain parameters over \mathbb{F}_p recommended in this document.

The section is organized as follows. First Section 2.1 describes relevant properties of the recommended parameters over \mathbb{F}_p . Then Section 2.2 specifies recommended 192-bit elliptic curve domain parameters over \mathbb{F}_p , Section 2.3 specifies recommended 224-bit elliptic curve domain parameters over \mathbb{F}_p , Section 2.4 specifies recommended 256-bit elliptic curve domain parameters over \mathbb{F}_p , Section 2.5 specifies recommended 384-bit elliptic curve domain parameters over \mathbb{F}_p , Section 2.6 specifies recommended 521-bit elliptic curve domain parameters over \mathbb{F}_p ,

2.1 Properties of Elliptic Curve Domain Parameters over \mathbb{F}_p

Following SEC 1 [SEC 1], elliptic curve domain parameters over \mathbb{F}_p are a sextuple:

$$T = (p, a, b, G, n, h)$$

consisting of an integer p specifying the finite field \mathbb{F}_p , two elements $a, b \in \mathbb{F}_p$ specifying an elliptic curve $E(\mathbb{F}_p)$ defined by the equation:

$$E: y^2 \equiv x^3 + a.x + b \pmod{p},$$

a base point $G = (x_G, y_G)$ on $E(\mathbb{F}_p)$, a prime n which is the order of G, and an integer h which is the cofactor $h = \#E(\mathbb{F}_p)/n$.

When elliptic curve domain parameters are specified in this document, each component of this sextuple is represented as an octet string converted using the conventions specified in SEC 1 [SEC 1].

Again following SEC 1 [SEC 1], elliptic curve domain parameters over \mathbb{F}_p must have:

$$\lceil \log_2 p \rceil \in \{192, 224, 256, 384, 521\}.$$

This restriction is designed to encourage interoperability while allowing implementers to supply commonly required security levels—recall that elliptic curve domain parameters over \mathbb{F}_p with $\lceil \log_2 p \rceil = 2t$ supply approximately t bits of security—meaning that solving the logarithm problem on the associated elliptic curve is believed to take approximately 2^t operations.

Here recommended elliptic curve domain parameters are supplied at each of the sizes allowed in SEC 1.

All the recommended elliptic curve domain parameters over \mathbb{F}_p use special form primes for their field order p. These special form primes facilitate especially efficient implementations like those described in [Nat99]. Recommended elliptic curve domain parameters over \mathbb{F}_p which use random primes for their field order p may be added later if commercial demand for such parameters increases.

The elliptic curve domain parameters over \mathbb{F}_p supplied at each security level typically consist of examples of two different types of parameters—one type being parameters associated with a Koblitz curve and the other type being parameters chosen verifiably at random—although only verifiably random parameters are supplied at export strength and at extremely high strength.

Parameters associated with a Koblitz curve admit especially efficient implementation. The name Koblitz curve is best-known when used to describe binary anomalous curves over \mathbb{F}_{2^m} which have $a,b \in \{0,1\}$ [Kob92]. Here it is generalized to refer also to curves over \mathbb{F}_p which possess an efficiently computable endomorphism [GLV01]. The recommended parameters associated with a Koblitz curve were chosen by repeatedly selecting parameters admitting an efficiently computable endomorphism until a prime order curve was found.

Verifiably random parameters offer some additional conservative features. These parameters are chosen from a seed using SHA-1 as specified in ANSI X9.62 [X9.62]. This process ensures that the parameters cannot be predetermined. The parameters are therefore extremely unlikely to be susceptible to future special-purpose attacks, and no trapdoors can have been placed in the parameters during their generation. When elliptic curve domain parameters are chosen verifiably at random, the seed S used to generate the parameters may optionally be stored along with the parameters so that users can verify the parameters were chosen verifiably at random.

Here verifiably random parameters have been chosen either so that the associated elliptic curve has prime order, or so that scalar multiplication of points on the associated elliptic curve can be accelerated using Montgomery's method [Mon87]. The recommended verifiably random parameters were chosen by repeatedly selecting a random seed and counting the number of points on the corresponding curve until appropriate parameters were found. Typically the parameters were chosen so that a = p - 3 because such parameters admit efficient implementation. For a given p, approximately half the isomorphism classes of elliptic curves over \mathbb{F}_p contain a curve with a=p-3.

See SEC 1 [SEC 1] for further guidance on the selection of elliptic curve domain parameters over \mathbb{F}_p .

Parameters	Section	Strength	Size	RSA/DSA	Koblitz or ran-
					dom
secp192k1	2.2.1	96	192	1536	k
secp192r1	2.2.2	96	192	1536	r
secp224k1	2.3.1	112	224	2048	k
secp224r1	2.3.2	112	224	2048	r
secp256k1	2.4.1	128	256	3072	k
secp256r1	2.4.2	128	256	3072	r
secp384r1	2.5.1	192	384	7680	r
secp521r1	2.6.1	256	521	15360	r

Table 1: Properties of Recommended Elliptic Curve Domain Parameters over \mathbb{F}_p

The recommended elliptic curve domain parameters over \mathbb{F}_p have been given nicknames to enable them to be easily identified. The nicknames were chosen as follows. Each name begins with sec to denote 'Standards for Efficient Cryptography', followed by a p to denote parameters over

 \mathbb{F}_p , followed by a number denoting the length in bits of the field size p, followed by a k to denote parameters associated with a Koblitz curve or an r to denote verifiably random parameters, followed by a sequence number.

Table 1 summarizes salient properties of the recommended elliptic curve domain parameters over \mathbb{F}_p .

Information is represented in Table 1 as follows. The column labelled 'parameters' gives the nickname of the elliptic curve domain parameters. The column labelled 'section' refers to the section of this document where the parameters are specified. The column labelled 'strength' gives the approximate number of bits of security the parameters offer. The column labelled 'size' gives the length in bits of the field order. The column labelled 'RSA/DSA' gives the approximate size of an RSA or DSA modulus at comparable strength. (See SEC 1 [SEC 1] for precise technical guidance on the strength of elliptic curve domain parameters.) Finally the column labelled 'Koblitz or random' indicates whether the parameters are associated with a Koblitz curve — 'k' — or were chosen verifiably at random — 'r'.

Parameters	Section	ANSI X9.62	ANSI X9.63	echeck	IEEE P1363	IPSec	NIST	WAP
secp192k1	2.2.1	c	c	С	С	-	-	c
secp192r1	2.2.2	r	r	c	c	r	r	С
secp224k1	2.3.1	c	c	С	c	_	-	С
secp224r1	2.3.2	r	r	С	С	r	r	С
secp256k1	2.4.1	c	c	С	c	-	-	С
secp256r1	2.4.2	r	r	c	c	r	r	c
secp384r1	2.5.1	c	r	c	С	r	r	c
secp521r1	2.6.1	r	r	С	С	r	r	С

Table 2: Status of Recommended Elliptic Curve Domain Parameters over \mathbb{F}_p

Table 2 summarizes the status of the recommended elliptic curve domain parameters over \mathbb{F}_p with respect to their alignment with other standards.

Information is represented in Table 2 as follows. The column labelled 'parameters' gives the nickname of the elliptic curve domain parameters. The column labelled 'section' refers to the section of this document where the parameters are specified. The remaining columns give the status of the parameters with respect to various other standards which specify mechanisms based on elliptic curve cryptography: 'ANSI X9.62' refers to the ANSI X9.62 standard [X9.62], 'ANSI X9.63' refers to the ANSI X9.63 standard [X9.63], 'echeck' refers to the draft FSML standard [Fin99], 'IEEE P1363' refers to the IEEE 1363 standard [1363], 'IPSec' refers to the recent internet draft related to ECC [Int06] submitted to the IETF's IPSec working group, 'NIST' refers to the list of recommended parameters recently released by the U.S. government [Nat99], and 'WAP' refers to the Wireless Application Forum's WTLS standard [WTLS]. In these columns, a '-' denotes parameters non-

conformant with the standard, a 'c' denotes parameters conformant with the standard, and an 'r' denotes parameters explicitly recommended in the standard.

Note that ANSI X9.62 has been updated. The set of recommended parameters in the updated ANSI X9.62 [X9.62] is a subset of the set of recommended parameters in this document.

2.2 Recommended 192-bit Elliptic Curve Domain Parameters over \mathbb{F}_p

This section specifies the two recommended 192-bit elliptic curve domain parameters over \mathbb{F}_p in this document: parameters secp192k1 associated with a Koblitz curve, and verifiably random parameters secp192r1.

Section 2.2.1 specifies the elliptic curve domain parameters secp192k1, and Section 2.2.2 specifies the elliptic curve domain parameters secp192r1.

2.2.1 Recommended Parameters secp192k1

The elliptic curve domain parameters over \mathbb{F}_p associated with a Koblitz curve secp192k1 are specified by the sextuple T = (p, a, b, G, n, h) where the finite field \mathbb{F}_p is defined by:

The curve E: $y^2 = x^3 + ax + b$ over \mathbb{F}_p is defined by:

The base point G in compressed form is:

G=03 DB4FF10E C057E9AE 26B07D02 80B7F434 1DA5D1B1 EAE06C7D and in uncompressed form is:

G=04 DB4FF10E C057E9AE 26B07D02 80B7F434 1DA5D1B1 EAE06C7D 9B2F2F6D 9C5628A7 844163D0 15BE8634 4082AA88 D95E2F9D

Finally the order n of G and the cofactor are:

2.2.2 Recommended Parameters secp192r1

The verifiably random elliptic curve domain parameters over \mathbb{F}_p secp192r1 are specified by the sextuple T = (p, a, b, G, n, h) where the finite field \mathbb{F}_p is defined by:

The curve E: $y^2 = x^3 + ax + b$ over \mathbb{F}_p is defined by:

b = 64210519 E59C80E7 OFA7E9AB 72243049 FEB8DEEC C146B9B1

E was chosen verifiably at random as specified in ANSI X9.62 [X9.62] from the seed:

S = 3045 AE6F C8422F64 ED579528 D38120EA E12196D5

The base point G in compressed form is:

G=03188DA80E B03090F6 7CBF20EB 43A18800 F4FF0AFD 82FF1012 and in uncompressed form is:

 $G=04\,188{\rm DA80E\,B03090F6\,7CBF20EB\,43A18800\,F4FF0AFD\,82FF1012}$ 07192B95 FFC8DA78 631011ED 6B24CDD5 73F977A1 1E794811

Finally the order n of G and the cofactor are:

h = 01

2.3 Recommended 224-bit Elliptic Curve Domain Parameters over \mathbb{F}_p

This section specifies the two recommended 224-bit elliptic curve domain parameters over \mathbb{F}_p in this document: parameters secp224k1 associated with a Koblitz curve, and verifiably random parameters secp224r1.

Section 2.3.1 specifies the elliptic curve domain parameters secp224k1, and Section 2.3.2 specifies the elliptic curve domain parameters secp224r1.

2.3.1 Recommended Parameters secp224k1

The elliptic curve domain parameters over \mathbb{F}_p associated with a Koblitz curve secp224k1 are specified by the sextuple T = (p, a, b, G, n, h) where the finite field \mathbb{F}_p is defined by:

The curve $E: y^2 = x^3 + ax + b$ over \mathbb{F}_p is defined by:

The base point G in compressed form is:

and in uncompressed form is:

 $G=04~{\rm A}1455{\rm B}33~{\rm 4DF099DF}~30{\rm FC28A1}~69{\rm A}467{\rm E}9~{\rm E}47075{\rm A}9~0{\rm F7E}650{\rm E}$ ${\rm B}687{\rm A}45{\rm C}~7{\rm E}089{\rm FED}~7{\rm FBA}3442~82{\rm CAFBD}6~{\rm F7E}319{\rm F7}~{\rm C}080{\rm B}{\rm D}59~{\rm E}2{\rm CA}4{\rm B}{\rm D}{\rm B}$ ${\rm 5}56{\rm D}61{\rm A}5$

Finally the order n of G and the cofactor are:

 $n = 01 \ 00000000 \ 000000000 \ 00001DCE8 \ D2EC6184 \ CAF0A971 \ 769FB1F7$

h = 01

2.3.2 Recommended Parameters secp224r1

The verifiably random elliptic curve domain parameters over \mathbb{F}_p secp224r1 are specified by the sextuple T = (p, a, b, G, n, h) where the finite field \mathbb{F}_p is defined by:

The curve E: $y^2 = x^3 + ax + b$ over \mathbb{F}_p is defined by:

 $b = 84050A85 \ 0C04B3AB \ F5413256 \ 5044B0B7 \ D7BFD8BA \ 270B3943 \ 2355FFB4$

E was chosen verifiably at random as specified in ANSI X9.62 [X9.62] from the seed:

 $S = \mathtt{BD713447} \ \mathtt{99D5C7FC} \ \mathtt{DC45B59F} \ \mathtt{A3B9AB8F} \ \mathtt{6A948BC5}$

The base point G in compressed form is:

G=02 B70E0CBD 6BB4BF7F 321390B9 4A03C1D3 56C21122 343280D6 115C1D21

and in uncompressed form is:

G=04 B70E0CBD 6BB4BF7F 321390B9 4A03C1D3 56C21122 343280D6 115C1D21 BD376388 B5F723FB 4C22DFE6 CD4375A0 5A074764 44D58199 85007E34

Finally the order n of G and the cofactor are:

h = 01

2.4 Recommended 256-bit Elliptic Curve Domain Parameters over \mathbb{F}_p

This section specifies the two recommended 256-bit elliptic curve domain parameters over \mathbb{F}_p in this document: parameters secp256k1 associated with a Koblitz curve, and verifiably random parameters secp256r1.

Section 2.4.1 specifies the elliptic curve domain parameters secp256k1, and Section 2.4.2 specifies the elliptic curve domain parameters secp256r1.

2.4.1 Recommended Parameters secp256k1

The elliptic curve domain parameters over \mathbb{F}_p associated with a Koblitz curve secp256k1 are specified by the sextuple T = (p, a, b, G, n, h) where the finite field \mathbb{F}_p is defined by:

$$= 2^{256} - 2^{32} - 2^9 - 2^8 - 2^7 - 2^6 - 2^4 - 1$$

The curve $E: y^2 = x^3 + ax + b$ over \mathbb{F}_p is defined by:

The base point G in compressed form is:

G=02 79BE667E F9DCBBAC 55A06295 CE870B07 029BFCDB 2DCE28D9 59F2815B 16F81798

and in uncompressed form is:

G=04 79BE667E F9DCBBAC 55A06295 CE870B07 029BFCDB 2DCE28D9 59F2815B 16F81798 483ADA77 26A3C465 5DA4FBFC 0E1108A8 FD17B448 A6855419 9C47D08F FB10D4B8

Finally the order n of G and the cofactor are:

h = 01

2.4.2 Recommended Parameters secp256r1

The verifiably random elliptic curve domain parameters over \mathbb{F}_p secp256r1 are specified by the sextuple T = (p, a, b, G, n, h) where the finite field \mathbb{F}_p is defined by:

$$= 2^{224}(2^{32} - 1) + 2^{192} + 2^{96} - 1$$

The curve $E: y^2 = x^3 + ax + b$ over \mathbb{F}_p is defined by:

- b = 5 AC635 D8 AA3 A93 E7 B3EBBD55 769886 BC651 D06 B0 CC53 B0 F6 3BCE3 C3E 27 D2604 B

E was chosen verifiably at random as specified in ANSI X9.62 [X9.62] from the seed:

S = C49D3608 86E70493 6A6678E1 139D26B7 819F7E90

The base point G in compressed form is:

 $G=03\,6$ B17D1F2 E12C4247 F8BCE6E5 63A440F2 77037D81 2DEB33A0 F4A13945 D898C296

and in uncompressed form is:

 $G=04\,6$ B17D1F2 E12C4247 F8BCE6E5 63A440F2 77037D81 2DEB33A0 F4A13945 D898C296 4FE342E2 FE1A7F9B 8EE7EB4A 7C0F9E16 2BCE3357 6B315ECE CBB64068 37BF51F5

Finally the order n of G and the cofactor are:

 $n = {\sf FFFFFFF} \; {\sf O0000000} \; {\sf FFFFFFFF} \; {\sf FFFFFFF} \; {\sf BCE6FAAD} \; {\sf A7179E84} \; {\sf F3B9CAC2} \;$ FC632551

h = 01

2.5 Recommended 384-bit Elliptic Curve Domain Parameters over \mathbb{F}_p

This section specifies the recommended 384-bit elliptic curve domain parameters over \mathbb{F}_p in this document: verifiably random parameters secp384r1.

Section 2.5.1 specifies the elliptic curve domain parameters secp384r1.

2.5.1 Recommended Parameters secp384r1

The verifiably random elliptic curve domain parameters over \mathbb{F}_p secp384r1 are specified by the sextuple T = (p, a, b, G, n, h) where the finite field \mathbb{F}_p is defined by:

$$= 2^{384} - 2^{128} - 2^{96} + 2^{32} - 1$$

The curve $E: y^2 = x^3 + ax + b$ over \mathbb{F}_p is defined by:

- b = 83312FA7 E23EE7E4 988E056B E3F82D19 181D9C6E FE814112 0314088F 5013875A C656398D 8A2ED19D 2A85C8ED D3EC2AEF

E was chosen verifiably at random as specified in ANSI X9.62 [X9.62] from the seed:

 $S={\tt A335926A~A319A27A~1D00896A~6773A482~7ACDAC73}$ The base point G in compressed form is:

 $G = 03 \text{ AA87CA22 BE8B0537 8EB1C71E F320AD74 6E1D3B62 8BA79B98} \\ 59F741E0 82542A38 5502F25D BF55296C 3A545E38 72760AB7 \\ \text{and in uncompressed form is:}$

G=04 AA87CA22 BE8B0537 8EB1C71E F320AD74 6E1D3B62 8BA79B98 59F741E0 82542A38 5502F25D BF55296C 3A545E38 72760AB7 3617DE4A 96262C6F 5D9E98BF 9292DC29 F8F41DBD 289A147C E9DA3113 B5F0B8C0 0A60B1CE 1D7E819D 7A431D7C 90EA0E5F

Finally the order n of G and the cofactor are:

h = 01

2.6 Recommended 521-bit Elliptic Curve Domain Parameters over \mathbb{F}_p

This section specifies the recommended 521-bit elliptic curve domain parameters over \mathbb{F}_p in this document: verifiably random parameters secp521r1.

Section 2.6.1 specifies the elliptic curve domain parameters secp521r1.

2.6.1 Recommended Parameters secp521r1

The verifiably random elliptic curve domain parameters over \mathbb{F}_p secp521r1 are specified by the sextuple T = (p, a, b, G, n, h) where the finite field \mathbb{F}_p is defined by:

- - $= 2^{521} 1$

The curve $E: y^2 = x^3 + ax + b$ over \mathbb{F}_p is defined by:

- $b=0051\ 953{\rm EB961}\ 8{\rm E1C9A1F}\ 929{\rm A21A0}\ 868540{\rm EE}\ {\rm A2DA725B}\ 99{\rm B315F3}$ ${\rm B8B48991}\ 8{\rm EF109E1}\ 56193951\ EC7E937B\ 1652C0BD\ 3BB1BF07\ 3573DF88$ ${\rm 3D2C34F1}\ EF451FD4\ 6B503F00$

E was chosen verifiably at random as specified in ANSI X9.62 [X9.62] from the seed:

 $S = {\tt D09E8800~291CB853~96CC6717~393284AA~A0DA64BA}$

The base point G in compressed form is:

 $G=0200{\rm C}6$ 858E06B7 0404E9CD 9E3ECB66 2395B442 9C648139 053FB521 F828AF60 6B4D3DBA A14B5E77 EFE75928 FE1DC127 A2FFA8DE 3348B3C1 856A429B F97E7E31 C2E5BD66

and in uncompressed form is:

 $G=04\,00{\rm C6858E}\,06{\rm B70404}\,{\rm E9CD9E3E}\,{\rm CB662395}\,{\rm B4429C64}\,{\rm 8139053F}\,$ ${\rm B521F828}\,{\rm AF606B4D}\,{\rm 3DBAA14B}\,{\rm 5E77EFE7}\,{\rm 5928FE1D}\,{\rm C127A2FF}\,{\rm A8DE3348}\,$ ${\rm B3C1856A}\,{\rm 429BF97E}\,{\rm 7E31C2E5}\,{\rm BD660118}\,{\rm 39296A78}\,{\rm 9A3BC004}\,{\rm 5C8A5FB4}\,$ ${\rm 2C7D1BD9}\,{\rm 98F54449}\,{\rm 579B4468}\,{\rm 17AFBD17}\,{\rm 273E662C}\,{\rm 97EE7299}\,{\rm 5EF42640}\,$ ${\rm C550B901}\,{\rm 3FAD0761}\,{\rm 353C7086}\,{\rm A272C240}\,{\rm 88BE9476}\,{\rm 9FD16650}\,$

Finally the order n of G and the cofactor are:

h = 01

3 Recommended Elliptic Curve Domain Parameters over \mathbb{F}_{2^m}

This section specifies the elliptic curve domain parameters over \mathbb{F}_{2^m} recommended in this document.

The section is organized as follows. First Section 3.1 describes relevant properties of the recommended parameters over \mathbb{F}_{2^m} . Then Section 3.2 specifies recommended 163-bit elliptic curve domain parameters over \mathbb{F}_{2^m} , Section 3.3 specifies recommended 233-bit elliptic curve domain parameters over \mathbb{F}_{2^m} , Section 3.4 specifies recommended 239-bit elliptic curve domain parameters over \mathbb{F}_{2^m} , Section 3.5 specifies recommended 283-bit elliptic curve domain parameters over \mathbb{F}_{2^m} , Section 3.6 specifies recommended 409-bit elliptic curve domain parameters over \mathbb{F}_{2^m} , and Section 3.7 specifies recommended 571-bit elliptic curve domain parameters over \mathbb{F}_{2^m} .

3.1 Properties of Elliptic Curve Domain Parameters over \mathbb{F}_{2^m}

Following SEC 1 [SEC 1], elliptic curve domain parameters over \mathbb{F}_{2^m} are a septuple:

$$T = (m, f(x), a, b, G, n, h)$$

consisting of an integer m specifying the finite field \mathbb{F}_{2^m} , an irreducible binary polynomial f(x) of degree m specifying the polynomial basis representation of \mathbb{F}_{2^m} , two elements $a, b \in \mathbb{F}_{2^m}$ specifying an elliptic curve $E(\mathbb{F}_{2^m})$ defined by the equation:

$$E: y^2 + x.y = x^3 + a.x^2 + b \text{ in } \mathbb{F}_{2^m},$$

a base point $G = (x_G, y_G)$ on $E(\mathbb{F}_{2^m})$, a prime n which is the order of G, and an integer h which is the cofactor $h = \#E(\mathbb{F}_{2^m})/n$.

When elliptic curve domain parameters over \mathbb{F}_{2^m} are specified in this document, m is represented directly as an integer, f(x) is represented directly as a polynomial, and the remaining components are represented as octet strings converted using the conventions specified in SEC 1 [SEC 1].

Again following SEC 1 [SEC 1], elliptic curve domain parameters over \mathbb{F}_{2^m} must have:

$$m \in \{163, 233, 239, 283, 409, 571\}.$$

Furthermore elliptic curve domain parameters over \mathbb{F}_{2^m} must use the reduction polynomials listed in Table 3 below.

This restriction is designed to encourage interoperability while allowing implementers to supply efficient implementations at commonly required security levels.

Here recommended elliptic curve domain parameters are supplied at each of the sizes allowed by SEC 1.

The elliptic curve domain parameters over \mathbb{F}_{2^m} supplied at each security level typically consist of examples of two different types of parameters — one type being parameters associated with a Koblitz curve and the other type being parameters chosen verifiably at random — although only verifiably random parameters are supplied at export strength.

Field	Reduction Polynomial(s)
$\mathbb{F}_{2^{163}}$	$f(x) = x^{163} + x^7 + x^6 + x^3 + 1$
$\mathbb{F}_{2^{233}}$	$f(x) = x^{233} + x^{74} + 1$
$\mathbb{F}_{2^{239}}$	$f(x) = x^{239} + x^{36} + 1 \text{ or } x^{239} + x^{158} + 1$
$\mathbb{F}_{2^{283}}$	$f(x) = x^{283} + x^{12} + x^7 + x^5 + 1$
$\mathbb{F}_{2^{409}}$	$f(x) = x^{409} + x^{87} + 1$
$\mathbb{F}_{2^{571}}$	$f(x) = x^{571} + x^{10} + x^5 + x^2 + 1$

Table 3: Representations of \mathbb{F}_{2^m}

Parameters associated with a Koblitz curve admit especially efficient implementation. Koblitz curves over \mathbb{F}_{2^m} are binary anomalous curves which have $a, b \in \{0, 1\}$ [Kob92].

Verifiably random parameters offer some additional conservative features. These parameters are chosen from a seed using SHA-1 as specified in ANSI X9.62 [X9.62]. This process ensures that the parameters cannot be predetermined. The parameters are therefore extremely unlikely to be susceptible to future special-purpose attacks, and no trapdoors can have been placed in the parameters during their generation. When elliptic curve domain parameters are chosen verifiably at random, the seed S used to generate the parameters may optionally be stored along with the parameters so that users can verify the parameters were chosen verifiably at random.

The recommended verifiably random parameters were chosen by repeatedly selecting a random seed and counting the points on the corresponding curve using Schoof's algorithm until appropriate parameters were found. The parameters were chosen so that either a is random or a = 1. For a given m, approximately half the isomorphism classes of elliptic curves over \mathbb{F}_{2^m} contain a curve with a = 1.

See SEC 1 [SEC 1] for further guidance on the selection of elliptic curve domain parameters over \mathbb{F}_{2^m} .

The example elliptic curve domain parameters over \mathbb{F}_{2^m} have been given nicknames to enable them to be easily identified. The nicknames were chosen as follows. Each name begins with **sec** to denote 'Standards for Efficient Cryptography', followed by a t to denote parameters over \mathbb{F}_{2^m} , followed by a number denoting the field size m, followed by a k to denote parameters associated with a Koblitz curve or an \mathbf{r} to denote verifiably random parameters, followed by a sequence number.

Table 4 summarizes salient properties of the recommended elliptic curve domain parameters over \mathbb{F}_{2^m} .

Information is represented in Table 4 as follows. The column labelled 'parameters' gives the nickname of the elliptic curve domain parameters. The column labelled 'section' refers to the section of this document where the parameters are specified. The column labelled 'strength' gives the approximate number of bits of security the parameters offer. The column labelled 'size' gives the field size m. The column labelled 'RSA/DSA' gives the approximate size of an RSA or DSA modulus at comparable strength. (See SEC 1 [SEC 1] for precise technical guidance on the strength

	~ ·	Strength	Size	/	Koblitz
Parameters	Section			RSA/DSA	or ran-
					dom
sect163k1	3.2.1	80	163	1024	k
sect163r1	3.2.2	80	163	1024	r
sect163r2	3.2.3	80	163	1024	r
sect233k1	3.3.1	112	233	2240	k
sect233r1	3.3.2	112	233	2240	r
sect239k1	3.4.1	115	239	2304	k
sect283k1	3.5.1	128	283	3456	k
sect283r1	3.5.2	128	283	3456	r
sect409k1	3.6.1	192	409	7680	k
sect409r1	3.6.2	192	409	7680	r
sect571k1	3.7.1	256	571	15360	k
sect571r1	3.7.2	256	571	15360	r

Table 4: Properties of Recommended Elliptic Curve Domain Parameters over \mathbb{F}_{2^m}

of elliptic curve domain parameters.) Finally the column labelled 'Koblitz or random' indicates whether the parameters are associated with a Koblitz curve — 'k' — or were chosen verifiably at random — 'r'.

Parameters	Section	ANSI X9.62	ANSI X9.63	echeck	IEEE P1363	IPSec	NIST	WAP
sect163k1	3.2.1	r	r	r	c	r	r	r
sect163r1	3.2.2	c	c	r	c	-	-	c
sect163r2	3.2.3	r	r	r	c	r	r	c
sect233k1	3.3.1	r	r	c	c	r	r	c
sect233r1	3.3.2	r	r	c	c	r	r	c
sect239k1	3.4.1	С	c	c	С	-	-	c
sect283k1	3.5.1	r	r	r	c	r	r	c
sect283r1	3.5.2	r	r	r	С	r	r	c
sect409k1	3.6.1	r	r	c	С	r	r	c
sect409r1	3.6.2	r	r	С	c	r	r	c
sect571k1	3.7.1	r	r	c	С	r	r	c
sect571r1	3.7.2	r	r	с	c	r	r	c

Table 5: Status of Recommended Elliptic Curve Domain Parameters over \mathbb{F}_{2^m}

Table 5 summarizes the status of the recommended elliptic curve domain parameters over \mathbb{F}_{2^m} with respect to their alignment with other standards.

Information is represented in Table 5 as follows. The column labelled 'parameters' gives the nickname of the elliptic curve domain parameters. The column labelled 'section' refers to the section of this document where the parameters are specified. The remaining columns give the status of the parameters with respect to various other standards which specify mechanisms based on elliptic curve cryptography: 'ANSI X9.62' refers to the ANSI X9.62 standard [X9.62], 'ANSI X9.63' refers to the draft ANSI X9.63 standard [X9.63], 'echeck' refers to the draft FSML standard [Fin99], 'IEEE P1363' refers to the IEEE 1363 standard [1363], 'IPSec' refers to the recent internet draft related to ECC [Int06] submitted to the IETF's IPSec working group, 'NIST' refers to the list of recommended parameters recently released by the U.S. government [Nat99], and 'WAP' refers to the Wireless Application Forum's WTLS standard [WTLS]. In these columns, a '-' denotes parameters non-conformant with the standard, a 'c' denotes parameters conformant with the standard, and an 'r' denotes parameters explicitly recommended in the standard.

Note that ANSI X9.62 has been updated. The set of recommended parameters in the updated ANSI X9.62 [X9.62] is a subset of the set of recommended parameters in this document.

3.2 Recommended 163-bit Elliptic Curve Domain Parameters over \mathbb{F}_{2^m}

This section specifies the three recommended 163-bit elliptic curve domain parameters over \mathbb{F}_{2^m} in this document: parameters sect163k1 associated with a Koblitz curve, verifiably random parameters sect163r1, and verifiably random parameters sect163r2.

Section 3.2.1 specifies the elliptic curve domain parameters sect163k1, Section 3.2.2 specifies the elliptic curve domain parameters sect163r1, and Section 3.2.3 specifies the elliptic curve domain parameters sect163r2.

3.2.1 Recommended Parameters sect163k1

The elliptic curve domain parameters over \mathbb{F}_{2^m} associated with a Koblitz curve sect163k1 are specified by the septuple T=(m,f(x),a,b,G,n,h) where m=163 and the representation of $\mathbb{F}_{2^{163}}$ is defined by:

$$f(x) = x^{163} + x^7 + x^6 + x^3 + 1$$

The curve $E: y^2 + xy = x^3 + ax^2 + b$ over \mathbb{F}_{2^m} is defined by:

The base point G in compressed form is:

 $G = 0302 \text{ FE}13C053 \text{ 7BBC}11AC \text{ AAO7D793 DE}4E6D5E 5C94EEE8}$

and in uncompressed form is:

G = 0402FE 13C0537B BC11ACAA 07D793DE 4E6D5E5C 94EEE802 89070FB0 5D38FF58 321F2E80 0536D538 CCDAA3D9

Finally the order n of G and the cofactor are:

 $n = 04\ 00000000\ 00000000\ 00020108\ A2E0CCOD\ 99F8A5EF$

h = 02

3.2.2 Recommended Parameters sect163r1

The verifiably random elliptic curve domain parameters over \mathbb{F}_{2^m} sect163r1 are specified by the septuple T = (m, f(x), a, b, G, n, h) where m = 163 and the representation of $\mathbb{F}_{2^{163}}$ is defined by:

$$f(x) = x^{163} + x^7 + x^6 + x^3 + 1$$

The curve $E: y^2 + xy = x^3 + ax^2 + b$ over \mathbb{F}_{2^m} is defined by:

a = 07 B6882CAA EFA84F95 54FF8428 BD88E246 D2782AE2

b=07 13612DCD DCB40AAB 946BDA29 CA91F73A F958AFD9

E was chosen verifiably at random from the seed:

S = 24B7B137 C8A14D69 6E676875 6151756F D0DA2E5C

However for historical reasons the method used to generate E from S differs slightly from the method described in ANSI X9.62 [X9.62]. Specifically the coefficient b produced from S is the reverse of the coefficient that would have been produced by the method described in ANSI X9.62.

The base point G in compressed form is:

 $G=0303\ 69979697\ {\rm AB438977\ 89566789\ 567F787A\ 7876A654}$ and in uncompressed form is:

 $G = 040369 \ 979697 \text{AB} \ 43897789 \ 56678956 \ 7F787 \text{A}78 \ 76 \text{A}65400 \ 435 \text{EDB}42$ EFAFB298 9D51FEFC E3C80988 F41FF883

Finally the order n of G and the cofactor are:

n = 03 FFFFFFF FFFFFFF FFFF48AA B689C29C A710279B

h = 02

3.2.3 Recommended Parameters sect163r2

The verifiably random elliptic curve domain parameters over \mathbb{F}_{2^m} sect163r2 are specified by the septuple T = (m, f(x), a, b, G, n, h) where m = 163 and the representation of $\mathbb{F}_{2^{163}}$ is defined by:

$$f(x) = x^{163} + x^7 + x^6 + x^3 + 1$$

The curve $E: y^2 + xy = x^3 + ax^2 + b$ over \mathbb{F}_{2^m} is defined by:

b = 02 0A601907 B8C953CA 1481EB10 512F7874 4A3205FD

E was chosen verifiably at random from the seed:

S = 85E25BFE 5C86226C DB12016F 7553F9D0 E693A268

E was selected from S as specified in ANSI X9.62 [X9.62] in normal basis representation and converted into polynomial basis representation.

The base point G in compressed form is:

G=0303 F0EBA162 86A2D57E A0991168 D4994637 E8343E36 and in uncompressed form is:

G=0403F0 EBA16286 A2D57EA0 991168D4 994637E8 343E3600 D51FBC6C 71A0094F A2CDD545 B11C5C0C 797324F1

Finally the order n of G and the cofactor are:

 $n = 04 \ 00000000 \ 00000000 \ 000292$ FE 77E70C12 A4234C33

h = 02

3.3 Recommended 233-bit Elliptic Curve Domain Parameters over \mathbb{F}_{2^m}

This section specifies the two recommended 233-bit elliptic curve domain parameters over \mathbb{F}_{2^m} in this document: parameters sect233k1 associated with a Koblitz curve, and verifiably random parameters sect233r1.

Section 3.3.1 specifies the elliptic curve domain parameters sect233k1, and Section 3.3.2 specifies the elliptic curve domain parameters sect233r1.

3.3.1 Recommended Parameters sect233k1

The elliptic curve domain parameters over \mathbb{F}_{2^m} associated with a Koblitz curve **sect233k1** are specified by the septuple T=(m,f(x),a,b,G,n,h) where m=233 and the representation of $\mathbb{F}_{2^{233}}$ is defined by:

$$f(x) = x^{233} + x^{74} + 1$$

The curve $E: y^2 + xy = x^3 + ax^2 + b$ over \mathbb{F}_{2^m} is defined by:

The base point G in compressed form is:

G = 020172~32BA853A 7E731AF1 29F22FF4 149563A4 19C26BF5 0A4C9D6E EFAD6126

and in uncompressed form is:

 $G=04\,017232$ BA 853A7E73 1AF129F2 2FF41495 63A419C2 6BF50A4C 9D6EEFAD 612601DB 537DECE8 19B7F70F 555A67C4 27A8CD9B F18AEB9B 56E0C11056FAE6A3

Finally the order n of G and the cofactor are:

 $n = 80\ 00000000\ 000000000\ 000069D5B\ B915BCD4\ 6EFB1AD5$ F173ABDF

h = 04

3.3.2 Recommended Parameters sect233r1

The verifiably random elliptic curve domain parameters over \mathbb{F}_{2^m} sect233r1 are specified by the septuple T = (m, f(x), a, b, G, n, h) where m = 233 and the representation of $\mathbb{F}_{2^{233}}$ is defined by:

$$f(x) = x^{233} + x^{74} + 1$$

The curve $E: y^2 + xy = x^3 + ax^2 + b$ over \mathbb{F}_{2^m} is defined by:

 $b = 0066\ 647 {\rm EDE6C}\ 332 {\rm C7F8C}\ 0923 {\rm BB58}\ 213 {\rm B333B}\ 20 {\rm E9CE42}\ 81 {\rm FE115F}$ $7 {\rm D8F90AD}$

E was chosen verifiably at random from the seed:

$$S = 74D59FF0 7F6B413D 0EA14B34 4B20A2DB 049B50C3$$

E was selected from S as specified in ANSI X9.62 [X9.62] in normal basis representation and converted into polynomial basis representation.

The base point G in compressed form is:

$$G = 0300$$
 FA C9DFCBAC 8313BB21 39F1BB75 5FEF65BC 391F8B36 F8F8EB73 71FD558B

and in uncompressed form is:

$$G=04~00$$
FAC9DF CBAC8313 BB2139F1 BB755FEF 65BC391F 8B36F8F8 EB7371FD 558B0100 6A08A419 03350678 E58528BE BF8A0BEF F867A7CA 36716F7E 01F81052

Finally the order n of G and the cofactor are:

$$n = 0100 \ 00000000 \ 000000000 \ 0013E974 \ E72F8A69 \ 22031D26 \ 03CFE0D7$$

$$h = 02$$

3.4 Recommended 239-bit Elliptic Curve Domain Parameters over \mathbb{F}_{2^m}

This section specifies the recommended 239-bit elliptic curve domain parameters over \mathbb{F}_{2^m} in this document: parameters sect239k1 associated with a Koblitz curve.

Section 3.4.1 specifies the elliptic curve domain parameters sect239k1.

3.4.1 Recommended Parameters sect239k1

The elliptic curve domain parameters over \mathbb{F}_{2^m} associated with a Koblitz curve sect239k1 are specified by the septuple T=(m,f(x),a,b,G,n,h) where m=239 and the representation of $\mathbb{F}_{2^{239}}$ is defined by:

$$f(x) = x^{239} + x^{158} + 1$$

The curve $E: y^2 + xy = x^3 + ax^2 + b$ over \mathbb{F}_{2^m} is defined by:

The base point G in compressed form is:

and in uncompressed form is:

G=04 29A0B6A8 87A983E9 730988A6 8727A8B2 D126C44C C2CC7B2A 65551930 35DC7631 0804F12E 549BDB01 1C103089 E73510AC B275FC31 2A5DC6B7 6553F0CA

Finally the order n of G and the cofactor are:

 $n = 2000\ 00000000\ 00000000\ 0005$ A79FE C67CB6E9 1F1C1DA8 00E478A5

h = 04

3.5 Recommended 283-bit Elliptic Curve Domain Parameters over \mathbb{F}_{2^m}

This section specifies the two recommended 283-bit elliptic curve domain parameters over \mathbb{F}_{2^m} in this document: parameters sect283k1 associated with a Koblitz curve, and verifiably random parameters sect283r1.

Section 3.5.1 specifies the elliptic curve domain parameters sect283k1, and Section 3.5.2 specifies the elliptic curve domain parameters sect283r1.

3.5.1 Recommended Parameters sect283k1

The elliptic curve domain parameters over \mathbb{F}_{2^m} associated with a Koblitz curve sect283k1 are specified by the septuple T=(m,f(x),a,b,G,n,h) where m=283 and the representation of $\mathbb{F}_{2^{283}}$ is defined by:

$$f(x) = x^{283} + x^{12} + x^7 + x^5 + 1$$

The curve $E: y^2 + xy = x^3 + ax^2 + b$ over \mathbb{F}_{2^m} is defined by:

 $G=02\ 0503213$ F 78CA4488 3F1A3B81 62F188E5 53CD265F 23C1567A 16876913 B0C2AC24 58492836

and in uncompressed form is:

 $G=04\,0503213$ F 78CA4488 3F1A3B81 62F188E5 53CD265F 23C1567A 16876913 B0C2AC24 58492836 01CCDA38 0F1C9E31 8D90F95D 07E5426F E87E45C0 E8184698 E4596236 4E341161 77DD2259

Finally the order n of G and the cofactor are:

h = 04

3.5.2 Recommended Parameters sect283r1

The verifiably random elliptic curve domain parameters over \mathbb{F}_{2^m} sect283r1 are specified by the septuple T = (m, f(x), a, b, G, n, h) where m = 283 and the representation of $\mathbb{F}_{2^{283}}$ is defined by:

$$f(x) = x^{283} + x^{12} + x^7 + x^5 + 1$$

The curve $E: y^2 + xy = x^3 + ax^2 + b$ over \mathbb{F}_{2^m} is defined by:

b = 0278680 A C8B8596D A5A4AF8A 19A0303F CA97FD76 45309FA2 A581485A F6263E31 3B79A2F5

E was chosen verifiably at random from the seed:

S = 77E2B073 70EB0F83 2A6DD5B6 2DFC88CD 06BB84BE

E was selected from S as specified in ANSI X9.62 [X9.62] in normal basis representation and converted into polynomial basis representation.

The base point G in compressed form is:

G=03 05F93925 8DB7DD90 E1934F8C 70B0DFEC 2EED25B8 557EAC9C 80E2E198 F8CDBECD 86B12053

and in uncompressed form is:

 $G=04\,05$ F93925 8DB7DD90 E1934F8C 70B0DFEC 2EED25B8 557EAC9C 80E2E198 F8CDBECD 86B12053 03676854 FE24141C B98FE6D4 B20D02B4 516FF702 350EDDB0 826779C8 13F0DF45 BE8112F4

Finally the order n of G and the cofactor are:

h = 02

3.6 Recommended 409-bit Elliptic Curve Domain Parameters over \mathbb{F}_{2^m}

This section specifies the two recommended 409-bit elliptic curve domain parameters over \mathbb{F}_{2^m} in this document: parameters **sect409k1** associated with a Koblitz curve, and verifiably random parameters **sect409r1**.

Section 3.6.1 specifies the elliptic curve domain parameters sect409k1, and Section 3.6.2 specifies the elliptic curve domain parameters sect409r1.

3.6.1 Recommended Parameters sect409k1

The elliptic curve domain parameters over \mathbb{F}_{2^m} associated with a Koblitz curve **sect409k1** are specified by the septuple T = (m, f(x), a, b, G, n, h) where m = 409 and the representation of $\mathbb{F}_{2^{409}}$ is defined by:

$$f(x) = x^{409} + x^{87} + 1$$

The curve $E: y^2 + xy = x^3 + ax^2 + b$ over \mathbb{F}_{2^m} is defined by:

The base point G in compressed form is:

- G=030060F05F 658F49C1 AD3AB189 0F718421 0EFD0987 E307C84C 27ACCFB8 F9F67CC2 C460189E B5AAAA62 EE222EB1 B35540CF E9023746 and in uncompressed form is:
 - $G=04\,0060$ F05F 658F49C1 AD3AB189 0F718421 0EFD0987 E307C84C 27ACCFB8 F9F67CC2 C460189E B5AAAA62 EE222EB1 B35540CF E9023746 01E36905 0B7C4E42 ACBA1DAC BF04299C 3460782F 918EA427 E6325165 E9EA10E3 DA5F6C42 E9C55215 AA9CA27A 5863EC48 D8E0286B

Finally the order n of G and the cofactor are:

h = 04

3.6.2 Recommended Parameters sect409r1

The verifiably random elliptic curve domain parameters over \mathbb{F}_{2^m} sect409r1 are specified by the septuple T = (m, f(x), a, b, G, n, h) where m = 409 and the representation of $\mathbb{F}_{2^{409}}$ is defined by:

$$f(x) = x^{409} + x^{87} + 1$$

The curve $E: y^2 + xy = x^3 + ax^2 + b$ over \mathbb{F}_{2^m} is defined by:

b=0021A5C2 C8EE9FEB 5C4B9A75 3B7B476B 7FD6422E F1F3DD67 4761FA99 D6AC27C8 A9A197B2 72822F6C D57A55AA 4F50AE31 7B13545F

E was chosen verifiably at random from the seed:

S = 4099B5A4 57F9D69F 79213D09 4C4BCD4D 4262210B

E was selected from S as specified in ANSI X9.62 [X9.62] in normal basis representation and converted into polynomial basis representation.

The base point G in compressed form is:

 $G = \hbox{\tt 03~015D4860~D088DDB3~496B0C60~64756260~441CDE4A~F1771D4D}$ $\hbox{\tt B01FFE5B~34E59703~DC255A86~8A118051~5603AEAB~60794E54~BB7996A7}$ and in uncompressed form is:

 $G=04\ 015\mathrm{D}4860\ \mathrm{D}088\mathrm{D}B3\ 496\mathrm{B}0C60\ 64756260\ 441\mathrm{C}DE4A\ F1771\mathrm{D}4D$ B01FFE5B 34E59703 DC255A86 8A118051 5603AEAB 60794E54 BB7996A7 0061B1CF AB6BE5F3 2BBFA783 24ED106A 7636B9C5 A7BD198D 0158AA4F 5488D08F 38514F1F DF4B4F40 D2181B36 81C364BA 0273C706

Finally the order n of G and the cofactor are:

h = 02

3.7 Recommended 571-bit Elliptic Curve Domain Parameters over \mathbb{F}_{2^m}

This section specifies the two recommended 571-bit elliptic curve domain parameters over \mathbb{F}_{2^m} in this document: parameters sect571k1 associated with a Koblitz curve, and verifiably random parameters sect571r1.

Section 3.7.1 specifies the elliptic curve domain parameters sect571k1, and Section 3.7.2 specifies the elliptic curve domain parameters sect571r1.

3.7.1 Recommended Parameters sect571k1

The elliptic curve domain parameters over \mathbb{F}_{2^m} associated with a Koblitz curve sect571k1 are specified by the septuple T=(m,f(x),a,b,G,n,h) where m=571 and the representation of $\mathbb{F}_{2^{571}}$ is defined by:

$$f(x) = x^{571} + x^{10} + x^5 + x^2 + 1$$

The curve $E: y^2 + xy = x^3 + ax^2 + b$ over \mathbb{F}_{2^m} is defined by:

The base point G in compressed form is:

G=002 026EB7A8 59923FBC 82189631 F8103FE4 AC9CA297 0012D5D4 60248048 01841CA4 43709584 93B205E6 47DA304D B4CEB08C BBD1BA39 494776FB 988B4717 4DCA88C7 E2945283 A01C8972

and in uncompressed form is:

G=04026EB7A8 59923FBC 82189631 F8103FE4 AC9CA297 0012D5D4 60248048 01841CA4 43709584 93B205E6 47DA304D B4CEB08C BBD1BA39 494776FB 988B4717 4DCA88C7 E2945283 A01C8972 0349DC80 7F4FBF37 4F4AEADE 3BCA9531 4DD58CEC 9F307A54 FFC61EFC 006D8A2C 9D4979C0 AC44AEA7 4FBEBBB9 F772AEDC B620B01A 7BA7AF1B 320430C8 591984F6 01CD4C14 3EF1C7A3

Finally the order n of G and the cofactor are:

00000000 00000000 131850E1 F19A63E4 B391A8DB 917F4138 B630D84B E5D63938 1E91DEB4 5CFE778F 637C1001

h =04

3.7.2 Recommended Parameters sect571r1

The verifiably random elliptic curve domain parameters over \mathbb{F}_{2^m} sect571r1 are specified by the septuple T = (m, f(x), a, b, G, n, h) where m = 571 and the representation of $\mathbb{F}_{2^{571}}$ is defined by:

$$f(x) = x^{571} + x^{10} + x^5 + x^2 + 1$$

The curve $E: y^2 + xy = x^3 + ax^2 + b$ over \mathbb{F}_{2^m} is defined by:

00000000 00000000 00000000 00000001

b = 02F40E7E 2221F295 DE297117 B7F3D62F 5C6A97FF CB8CEFF1 CD6BA8CE4A9A18AD 84FFABBD 8EFA5933 2BE7AD67 56A66E29 4AFD185A 78FF12AA 520E4DE7 39BACAOC 7FFEFF7F 2955727A

E was chosen verifiably at random from the seed:

S = 2AA058F7 3A0E33AB 486B0F61 0410C53A 7F132310

E was selected from S as specified in ANSI X9.62 [X9.62] in normal basis representation and converted into polynomial basis representation.

The base point G in compressed form is:

03 0303001D 34B85629 6C16C0D4 0D3CD775 0A93D1D2 955FA80A A5F40FC8 DB7B2ABD BDE53950 F4C0D293 CDD711A3 5B67FB14 99AE6003 8614F139 4ABFA3B4 C850D927 E1E7769C 8EEC2D19

and in uncompressed form is:

G =04 0303001D 34B85629 6C16C0D4 0D3CD775 0A93D1D2 955FA80A A5F40FC8 DB7B2ABD BDE53950 F4C0D293 CDD711A3 5B67FB14 99AE6003 8614F139 4ABFA3B4 C850D927 E1E7769C 8EEC2D19 037BF273 42DA639B 6DCCFFFE B73D69D7 8C6C27A6 009CBBCA 1980F853 3921E8A6 84423E43 BAB08A57 6291AF8F 461BB2A8 B3531D2F 0485C19B 16E2F151 6E23DD3C 1A4827AF 1B8AC15B

Finally the order n of G and the cofactor are:

h = 02

ASN.1 Syntax \mathbf{A}

This section discusses the representation of elliptic curve domain parameters using ASN.1 syntax and specifies ASN.1 object identifiers for the elliptic curve domain parameters recommended in this document.

Syntax for Elliptic Curve Domain Parameters A.1

There are a number of ways of representing elliptic curve domain parameters using ASN.1 syntax. The following syntax is recommended in SEC 1 [SEC 1] for use in X.509 certificates and elsewhere (following [RFC 3279]).

```
Parameters{CURVES:IOSet} ::= CHOICE {
    ecParameters ECParameters,
    namedCurve CURVES.&id({IOSet}),
    implicitCA NULL
}
```

where

- ecParameters of type ECParameters indicates that the full elliptic curve domain parameters are given,
- namedCurve of type CURVES indicates that a named curve from the set delimited by CurveNames is to be used, and
- implicitCA of type NULL indicates that the curve is known implicitly, that is, the actual curve is known to both parties by other means.

The following syntax is then used to describe explicit representations of elliptic curve domain parameters, if need be.

```
ECParameters ::= SEQUENCE {
    version INTEGER { ecpVer1(1) } (ecpVer1),
    fieldID FieldID {{FieldTypes}},
    curve Curve,
    base ECPoint,
    order INTEGER,
    cofactor INTEGER OPTIONAL,
}
```

See SEC 1 [SEC 1] for more details on the explicit representation of elliptic curve domain parameters.

A.2 Object Identifiers for Recommended Parameters

This section specifies object identifiers for the elliptic curve domain parameters recommended in this document. These object identifiers may be used, for example, to represent parameters using the namedCurve syntax described in the previous section.

Parameters that have not previously been assigned object identifiers appear in the tree whose root is designated by the object identifier certicom-arc. It has the following value.

```
certicom-arc OBJECT IDENTIFIER ::= {
   iso(1) identified-organization(3) certicom(132)
}
```

Parameters that are given as examples in ANSI X9.62 [X9.62] appear in the tree whose root is designated by the object identifier ansi-X9-62. It has the following value.

```
ansi-X9-62 OBJECT IDENTIFIER ::= {
   iso(1) member-body(2) us(840) 10045
}
```

The values of the object identifiers of parameters given in ANSI X9.62 are duplicated here for convenience.

To reduce the encoded lengths, the parameters under certicom-arc appear just below the main node. The object identifier ellipticCurve represents the root of the tree containing all such parameters in this document and has the following value.

```
ellipticCurve OBJECT IDENTIFIER ::= { certicom-arc curve(0) }
```

The actual parameters appear immediately below this; their object identifiers may be found in the following sections. Section A.2.1 specifies object identifiers for the parameters over \mathbb{F}_p , and Section A.2.2 specifies object identifiers for the parameters over \mathbb{F}_{2^m} .

A.2.1 OIDs for Recommended Parameters over \mathbb{F}_p

The object identifiers for the recommended parameters over \mathbb{F}_p have the following values. The names of the identifiers agree with the nicknames given to the parameters in this document. In ANSI X9.62 [X9.62], the curve secp192r1 is designated prime192v1, and the curve secp256r1 is designated prime256v1.

```
-- Curves over prime-order fields:
-- secp192k1 OBJECT IDENTIFIER ::= { ellipticCurve 31 } secp192r1 OBJECT IDENTIFIER ::= { ansi-X9-62 curves(3) prime(1) 1 } secp224k1 OBJECT IDENTIFIER ::= { ellipticCurve 32 } secp224r1 OBJECT IDENTIFIER ::= { ellipticCurve 33 }
```

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```
secp256k1 OBJECT IDENTIFIER ::= { ellipticCurve 10 }
secp256r1 OBJECT IDENTIFIER ::= { ansi-X9-62 curves(3) prime(1) 7 }
secp384r1 OBJECT IDENTIFIER ::= { ellipticCurve 34 }
secp521r1 OBJECT IDENTIFIER ::= { ellipticCurve 35 }
```

A.2.2OIDs for Recommended Parameters over \mathbb{F}_{2^m}

The object identifiers for the recommended parameters over \mathbb{F}_{2^m} have the following values. The names of the identifiers agree with the nicknames given to the parameters in this document.

```
-- Curves over characteristic 2 fields.
sect163k1 OBJECT IDENTIFIER ::= { ellipticCurve 1 }
sect163r1 OBJECT IDENTIFIER ::= { ellipticCurve 2 }
sect163r2 OBJECT IDENTIFIER ::= { ellipticCurve 15 }
sect233k1 OBJECT IDENTIFIER ::= { ellipticCurve 26 }
sect233r1 OBJECT IDENTIFIER ::= { ellipticCurve 27 }
sect239k1 OBJECT IDENTIFIER ::= { ellipticCurve 3 }
sect283k1 OBJECT IDENTIFIER ::= { ellipticCurve 16 }
sect283r1 OBJECT IDENTIFIER ::= { ellipticCurve 17 }
sect409k1 OBJECT IDENTIFIER ::= { ellipticCurve 36 }
sect409r1 OBJECT IDENTIFIER ::= { ellipticCurve 37 }
sect571k1 OBJECT IDENTIFIER ::= { ellipticCurve 38 }
sect571r1 OBJECT IDENTIFIER ::= { ellipticCurve 39 }
```

A.2.3The Information Object Set SECGCurveNames

The following information object set SECGCurveNames of class CURVES may be used to delineate the use of a curve recommended in this document. When it is used to govern the component namedCurve of Parameters (defined in section A.1), the value of namedCurve must be one of the values of the set.

```
SECGCurveNames CURVES ::= {
    -- Curves over prime-order fields:
    { ID secp192k1 } |
```

```
{ ID secp192r1 } |
    { ID secp224k1 } |
    { ID secp224r1 } |
    { ID secp256k1 } |
    { ID secp256r1 } |
    { ID secp384r1 } |
    { ID secp521r1 } |
    -- Curves over characteristic 2 fields:
    { ID sect163k1 } |
    { ID sect163r1 } |
    { ID sect163r2 } |
    { ID sect233k1 } |
    { ID sect233r1 } |
    { ID sect239k1 } |
    { ID sect283k1 } |
    { ID sect283r1 } |
    { ID sect409k1 } |
    { ID sect409r1 } |
    { ID sect571k1 } |
    { ID sect571r1 } ,
     . . .
    }
The type CURVES used above is defined below.
CURVES ::= CLASS {
    &curve-id OBJECT IDENTIFIER UNIQUE
```

} WITH SYNTAX { ID &curve-id }

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